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The tax advantage of public firms over private firms in the German household solid waste disposal industry

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Abstract

We measure the tax advantage of public firms over private firms, which operate at municipality level in the German household solid waste disposal industry. Public firms with sovereign duties pay no taxes, but equivalent private firms have to. In a simple risk-free setting, we develop a measure of the percentage difference of the charges of both types of firms demanded under their respective tax treatments. We model a cost-covering public firm and a net present value maximizing private firm. For sensible model parameters from the German waste disposal industry the private firm has to demand an about 16% to 18% higher charge. The by far biggest impact on the measure has the value added tax, with revenues as a much larger tax base than profits. Tax savings, which directly affect pre-tax profits, only alleviate the disadvantage bit. There is some evidence that at least one type of private firms—that is, private law firms that are also majority privately owned, are productive enough to overcome the tax advantage of public firms and be able to charge a lower price than public firms.

KEYWORDS

corporate tax, private firm, public firm, trade tax, value added tax

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1 | INTRODUCTION

In Germany there are public companies with sovereign duties, which do not pay value-added, corporate and trade taxes, and which carry out services that private companies can also provide. This is true especially for providers of basic public services such as household solid waste disposal. However, private companies providing the same service have to pay the mentioned taxes. Despite this obvious tax-related disadvantage, we find both types, untaxed public and taxed private companies, providing waste disposal services in Germany.

In this light, we aim to analyze the main drivers of the tax disadvantage and possible ways for the private company to reduce it to be more competitive with a public company. More specifically, we want to know under which conditions a private firm, which maximizes firm value and pays taxes, can be competitive with a public firm, which pays no tax, has zero profit expectations and minimizes the charge for the offered service. By competitive we mean that the tax-paying private firm is able to provide the same output as the public firm at a price per unit of output close to the price of the public firm and still earn an adequate return on capital.

We use an infinite horizon model for public firm and private firm decisions based on a classic Cobb–Douglas production function. In this way we obtain a direct relationship between tax rates and decisions on capital and labor, which feed back into company output. We assume the public and the private firm to be mostly equal except in tax treatment. All variables that matter are certain.

We parameterize our model with current tax rates and with realistic parameter values for the production functions of public and private companies in the German solid waste disposal industry. For parameterization of the production function, we use Frieese et al. (2020). For tax rates, we use the current tax rates. We define the measure of the tax advantage of the public firm over the private firm as the percentage differential of the charge demanded by the private firm over the charge demanded by the public firm for the same output (a unit of waste disposal). The public firm sets the charge so that costs are covered and minimized. The private firm maximizes its net present value, and uses as baseline charge the one that sets the firm's net present value to zero. This model-based measure of tax advantage has the benefit that changing model parameters such as a tax rate is propagated through the model so that all model variables adjust to the new inputs using the decision rules of the private and the public firm.

Our model defines the factors that influence the tax advantage of the public firm to be the tax rates, productivity, accounting depreciation rate, leverage and the risk-free rate.

We find that the value added tax rate (currently 19%) causes the biggest tax advantage for the public firm versus the private firm. For sensible model parameters we derive a tax advantage for the public firm of about 16% just from the value added tax alone and setting other tax rates to zero. That is, looking only at the effect of the value added tax, the public firm could demand a 16% lower charge from its clients. This is less than the 19% tax rate, because the advantage of the public firm is limited a bit since it cannot deduct the value added tax from acquired capital but the private firm can.

The impact of the about 30% tax rate on profits on the measure of the tax advantage depends on the profit margin—that is, the share of profits per unit of revenue. Without value added taxes, we arrive at profit margins of 4% to 10%. They depend on the sensitivity of the product with respect to labor versus capital, the risk-free rate and the accounting depreciation. Profits are usually low compared to revenues, which makes the tax on profits relatively low compared to value added taxes.

A lower risk-free rate, higher leverage of the private firm and a higher accounting depreciation rate of the private firm decrease the measure of the tax advantage through the debt interest and depreciation tax shields. Those effects are related to taxes on profits and are therefore relatively small.

The huge productivity advantage of the private firms that fall under private law and are also privately owned versus public firms of about 16% as the most conservative measure from Frieese et al. (2020) pushes down the measure of the tax advantage to about -3% to -1%. The exact value depends on parameters such as leverage and the risk-free rate. That leaves some room for the private firm to increase charges and to turn the zero net present value positive. Without looking at other factors, productivity is the most important driver that makes the private firm competitive versus the public one based on charges.

Finally, we explore the tax redistribution to some extent. Including the redistribution of paid taxes back to the municipality into the computations reduces the tax advantage by a couple of percentage points. However, we suggest a more refined investigation of the rather complex tax redistribution system in Germany as further research.

There are some proposals for how to measure the tax advantage of a public firm with sovereign duties versus private firms, see Wassenaar and Gradus (2004) and Wassenaar et al. (2010). Wassenaar and Gradus (2004) state that if the private and the public firm incur the same costs, then, for the final price of the private firm to be lower than the one of the public firm, its value added including taxes must be less than the value added of the public firm. We believe that additional insights can be gained with a model with more structure in which more primitive parameters can be changed so that their effects propagate through the model. We also argue that the lower price of the private firm has to come from efficiency gains (more product with the same inputs—i.e. lower cost per unit of output), but not from lower value added. In addition, we use actual data from the German market.

The value added tax, which has the biggest impact on the charge of the service, is often an issue when it comes to public firms not paying taxes. This type of tax is taken up, for example, in Ambrosios (1996) within a historical context. Initiatives to bring private and public companies onto a level playing ground have long been discussed in the literature. Having public waste disposal taxed with value added taxes was said by public firms to bring a charge increase of 12%.¹ According to Lorenz (2015), private businesses see competition as distorted, with an advantage towards the tax-free public companies over taxable private firms that could provide the same service. Public firms with sovereign duties remain tax-free to the present day.

Our modeling tools are the discounted cash flow (DCF) method together with a standard production function. For DCF methods we stay close to Kruschwitz and Löffler (2006). However, we include a production function with capital and labor as main inputs and optimizing behavior. For more on this we refer to Heer and Maussner (2009) as a reference from the real business cycle literature.

¹ The value added tax rate was 19% at that time. böl/dpa (2009) wrote about the idea of a value added tax on public firms in waste disposal and sewage industry in the weekly journal *Der Spiegel*.

2 | HISTORICAL CONTEXT AND LEGAL SITUATION

After a trend towards privatization of state-run industries in Germany lasting several decades, since the mid-2000s there has been a reversal, with a trend away from privatization and towards an increase in newly founded public companies at communal level. Between 2007 and 2012, the Association of Communal Enterprises recorded more than 70 new foundations of communal firms and 170 reclaims of concessions through communal firms. The Federal Cartel Office (Bundeskartellamt) identifies the following drivers of this reversed trend at the communal level: the desire to regain and retain control of certain industry sectors, the creation of employment, financing of underfunded communal sectors, and environmental considerations. Even though there are exceptions, efficiency of public companies is often lower than for equivalent private ones due to different incentive structures (Bundeskartellamt, 2014, pp. 17–19).

Household solid waste disposal and several other services are categorized as sovereign duties. This was confirmed in the ruling by the Federal Financial Court² and by the German parliament (Bundestag)³. Public companies fulfilling sovereign duties are among the ones listed in sect. 4 para. 5 of the Corporation Tax Act (Körperschaftsteuergesetz) and therefore do not have to pay taxes on profits even with the intent to make profits (Meyer, 2010, p. 77).⁴ The charges that municipalities demand for their services usually preclude profits and require them to be cost covering.⁵ The costs to be covered are computed according to principles of cost accounting. Apart from the costs for inputs and labor they also include actual depreciation and interest for debt and for equity holders.

Thus, municipalities can choose to set up a company with sovereign duties (“Hoheitsbetrieb”), which does not pay taxes on profits as mentioned before. According to sect. 2 b of the Value Added Tax Law (Umsatzsteuergesetz) public companies with sovereign duties are exempt from value added taxes as long as no significant distortions of competition are prevalent.⁶ A company not paying value added taxes is not allowed to deduct value added taxes included in input prices. The price including value added taxes of capital goods is used to compute the book value of capital.⁷ Finally, public companies with sovereign duties are also exempt from the trade tax as per sect. 2 para. 2 of the Regulation on the Trade Tax Act (Gewerbesteuerdurchführungsverordnung (GewStDV)).

² BFH ruling, 23 October 1996 - I R 1-2/94

³ Deutscher Bundestag Fachbereich WD 4, Haushalt und Finanzen (2016) provides further information.

⁴ There is opposition to this profit intent criterion. Franz (2005, p. 458) opines that the absence of an entrepreneurial activity (unternehmerische Tätigkeit) also implies the absence of a profit motive.

⁵ Some examples of where to find cost covering requirements of communal charges within the municipal charges acts of the federal states, “Kommunalabgabengesetz” (KAG), are here: sect. 8 para. 2 cl. 1 BayKAG, sect. 10 para. 1 cl. 1 HessKAG, sect. 6 para. 1 cl. 3 KAG NRW, sect. 5 para. 1 cl. 2 NKAG, sect. 7 para. 1 cl. 1 KAG Rheinland Pfalz and sect. 6 para. 1 KAG Saarland. Examples of where to find the cost covering requirements in local statutes on the communal level are here: sect. 1 Satzung zur Erhebung von Benutzungsgebühren Landkreis Kaiserslautern, sect. 1 Satzung über die Kostendeckung der Abfallentsorgung in der Stadt Minden vom 20.04.1995, sect. 1 para. 1 cl. 2 Satzung über die Erhebung von Gebühren für die Abfallentsorgung (Abfallgebührensatzung) der Stadt Cottbus.

⁶ Deutscher Bundestag Fachbereich WD 4, Haushalt und Finanzen (2016) explains the exemption for public waste disposal companies.

⁷ Sect. 15 para. 1 defines when a deduction of value added taxes is possible. Especially an entrepreneurial activity is necessary, which is not given for public companies with sovereign duties. Sect. 41 no. 4 GemHVO (Gemeindehaushaltsverordnung) Hessen is an example in which the inclusion of value added taxes in the book value of assets is codified on federal level.

According to sect. 22 of the Waste and Recycling Act (Kreislaufwirtschaftsgesetz (KrWG)), another option for the municipality is to employ a private company to provide the service. This private company would have a profit motive and would be fully taxable. We see both types in Germany, and there are many questions around having one or the other type. Frieese et al. (2020) find in their Table (A4) that some German federal states use primarily public law and others private law waste disposal companies, whereas the latter would be subject to taxes. However, we will focus here only on the tax-related advantage that a public company that is set up as a tax-free company has over a taxable private company.

3 | BASIC NOTATION AND ASSUMPTIONS

We start to define the basic content for our measure of the tax advantage. The measure we derive is proprietary and has not been used in the literature to our best knowledge. However, the ingredients of the measure (free cash flow modeling, production functions, etc.) are widely used in economics and finance models.

3.1 | General assumptions

We make several general assumptions to form our model framework. The assumptions abstract to some degree from reality, as in every model. However, we still obtain enough structure to give reasonable indication of the severity of the tax advantage and to serve as a baseline for more granular models.

We regard an infinite time horizon. We have constant prices and constant demand, reflecting the stationary nature of long-term considerations as in the real business cycle (RBC) literature.

We take prices for labor and capital, as given. The price for one unit of labor is the constant wage rate w and the price for capital is p_k^n , which we set equal to one (superscript “n” for net of value added taxes).

There is no uncertainty in our model. We find a model under certainty already rich enough in interesting results. We expect uncertainty to introduce risk premiums. Additionally, we would have to answer new questions such as what exactly should be risky.⁸ The risk-free rate r_f is constant over time and greater than zero. The requirement of a risk-free rate greater than zero is necessary in this infinite horizon model for discounted cash flows to converge to a finite value. We believe that the risk-free rate is a good proxy for the discount rate of cash flows from household waste disposal. It is an essential service with few innovations and little sensitivity to economic shocks. Rates for long-term risk-free assets are usually seen as appropriate discount rates.

The private firm will use the net present value (NPV), which we denote \mathcal{N} , as its decision criterion. The NPV is defined here as the difference between the value of the discounted cash flows of an investment and the price of the initial capital investment. The private firm will invest if the NPV is not negative.⁹

⁸ Risk and uncertainty is used here interchangeably.

⁹ A positive NPV under full certainty means that the firm would earn more than the risk-free rate on its capital. Under tradeability of the firm this would imply that an immediate risk-less profit is possible—that is, an arbitrage opportunity.

3.2 | Taxes

We regard three types of taxes: value added taxes, augmented corporate taxes and trade taxes. The respective tax rates are τ_v , τ_c and τ_g . The augmented corporate tax rate τ_c (without an asterisk) includes the solidarity surcharge τ_u and is computed as $\tau_c = \tau_c^*(1 + \tau_u)$, in which τ_c^* is the corporate tax rate without the solidarity surcharge.

Tax rates are certain and constant. Therefore the tax rates do not feature time subscripts. This assumption is a simplification. In reality tax rates are adjusted from time to time. We will take a quick look at tax rate changes when we introduce parameter values in Section 6.

Taxes are paid immediately. Tax bases are always non-negative.¹⁰ The tax bases are the underlying quantities on which the tax rates are applied.

We define the tax bases as follows. Revenues at time t , Y_t , are subject to value added taxes. Revenue Y_t is gross of value added taxes. The tax rate has to be applied on the revenue net of value added taxes, which is $\frac{1}{1+\tau_v}Y_t$. Value added taxes without deductibles are $\frac{\tau_v}{1+\tau_v}Y_t$. However, the value added tax paid on (capital) products can be deducted from that amount. We specify for our analysis that costs for investments in capital I_t are subject to value added taxes. Investments I_t are also gross of taxes. The value added tax that is included in investments is $\frac{\tau_v}{1+\tau_v}I_t$. They need to be deducted from the value added taxes paid on revenues. Thus, the value added tax result is $\mathcal{T}_{v,t} = \frac{\tau_v}{1+\tau_v}(Y_t - I_t)$. Corporate profits Π_t are taxed at the corporate tax rate τ_c . Corporate taxes are $\mathcal{T}_{c,t} = \tau_c\Pi_t$.

In Germany, there is an additional trade tax. We denote the respective tax rate τ_g . The application is very similar to corporate taxes. For simplification, we leave out the details of additions and subtractions of items for trade taxation purposes.¹¹ Thus, the trade tax has the same tax base as corporate taxes. The trade tax is $\mathcal{T}_{g,t} = \tau_g\Pi_t$.

For ease of notation we combine corporate and trade tax rates to the tax rate on profits τ_p so that $\tau_p = \tau_c + \tau_g$.

3.3 | EBIT, profit and leveraged free cash flows

We derive a free cash flow equation in which all taxes are visible. We start with earnings before interest and taxes (EBIT), denoted E_t , with

$$E_t = \frac{Y_t}{1 + \tau_v} - L_t - \frac{\mathbb{D}_{a,t}}{1 + \tau_v}, \quad (3.1)$$

This opportunity would be limited up to the amount of capital that the investment can absorb. We want to notice this here, however, we do not want to analyze the workings of a perfect capital market, which we most probably do not have in this industry. Therefore, we allow for positive NPVs under certainty.

¹⁰ In reality, negative tax bases—that is, tax losses—may be carried forward to offset future tax bases. However, in our risk-free and infinite horizon setting negative tax bases do not show up, so that we do not formally model such cases.

¹¹ For example, a part of the interest payments on long-term debt that finances certain types of investments is added back to the tax base to compute the trade tax. This would partly decrease the interest tax shield. The effect would be relatively small and would make the model less tractable, so that we leave this out.

in which L_t is the amount spent on labor, and $\mathbb{D}_{a,t}$ is the depreciation used for accounting and tax reasons. We divide the depreciation term by the value added tax rate term to express that capital at prices net of value added taxes are depreciated. EBIT is defined net of value added taxes.

To capture the effects of the corporate and the trade tax, we need to state the equation for profits. Pre-tax profits Π_t are EBIT E_t subtracted by interest payments I_t , so that we obtain

$$\Pi_t = E_t - I_t.$$

Notice that there are no accounting gains or losses in this model since the price of capital is constant over time. As outlined in the previous section, after-tax profits $\Pi_{\tau,t}$ are

$$\Pi_{\tau,t} = \Pi_t(1 - \tau_p). \quad (3.2)$$

At time $t - 1$, the firm may take out risk-free debt at face value B_{t-1} and with a statutory interest rate r_f . Since default is not possible, interest payments at t are $I_t = B_{t-1}r_f$.

To obtain leveraged free cash flows, we use after-tax profits, add back interest payments, subtract investments net of value added taxes $\frac{1}{1+\tau_v}I_t$, and add non-cash expenses $\mathbb{D}_{a,t}\frac{1}{1+\tau_v}$. This leads to

$$F_{l,t} = Y_t \frac{1 - \tau_p}{1 + \tau_v} - L_t(1 - \tau_p) + \tau_p I_t + \tau_p \mathbb{D}_{a,t} \frac{1}{1 + \tau_v} - I_t \frac{1}{1 + \tau_v}. \quad (3.3)$$

Taxes are saved on actually paid interest I_t , and tax savings are $\tau_p I_t$. By the same token, the depreciation tax savings are $\tau_p \mathbb{D}_{a,t}$.

3.4 | Functional specifications

We go on to specify functions for the different variables that we have seen before. Most of the specifications are regularly used in the RBC literature. They provide structure to our model which we will use to quantify the tax advantage in a later step.

Capital stock evolution. At time t the firm has a capital stock k_t . The capital stock is reduced every period by actual depreciation d_t . At the end of time t new capital i_t is bought and leads to the capital stock k_{t+1} , which is available at the beginning of $t + 1$. Thus, we assume capital to evolve as

$$k_{t+1} = k_t - d_t + i_t, \quad (3.4)$$

in which d_t is the actual depreciation of capital. We regard capital as one homogeneous mass so that there is no difference between old and new capital and that it is infinitely divisible. The private firm can deduct value added taxes of investments from value added taxes comprised in the final service. Therefore, the net price is applicable for depreciation. The public firm does not charge value added taxes for its final service and therefore must not deduct value added taxes for investments. It has to include value added taxes for depreciation purposes. To avoid confusion we denote $K_t = p_k k_t = p_k^n(1 + \tau_v)k_t = k_t(1 + \tau_v)$ when we use gross prices and $K_t^n = p_k^n k_t = k_t$ when we use net prices for capital.

Actual depreciation as well as the underlying valuation of capital are usually different from the figures used for accounting and tax reasons. For accounting depreciation we assume that capital evolves as follows:

$$k_{a,t+1} = k_{a,t} - d_{a,t} + i_t. \quad (3.5)$$

Actual depreciation d_t and depreciation for tax reasons $d_{a,t}$ may be different. We can exclude any realized accounting gains or losses from the equation since capital will have a constant price over time and no capital will be sold.

Actual depreciation at initial cost. Usually, linear depreciation is suggested for accounting and management accounting purposes. Instead of linear depreciation, we use geometric degressive depreciation. This simplifies the math quite a bit versus linear depreciation. The type of depreciation is not that important as long as we can model different rates for tax reasons and actual depreciation. Therefore, we decide for the mathematically simpler degressive depreciation.

For degressive depreciation, actual capital stock evolves as

$$k_{t+1} = k_t(1 - \delta) + i_t, \quad (3.6)$$

in which δ is the (constant) rate of depreciation, with $\delta \in [0, 1)$. At time $t = 0$ the initial capital stock is purchased. That means that $k_0 = 0$ and $d_0 = 0$. With $i_0 = k_1$, we can solve the equation forward to obtain

$$k_t = \sum_{s=0}^{t-1} i_s(1 - \delta)^{t-1-s}. \quad (3.7)$$

Actual depreciation uses the true depreciation rate δ . Depreciation at time t of an investment made n periods ago i_{t-n} is $i_{t-n}\delta(1 - \delta)^{n-1}$. The first depreciation of the investment i_{t-n} takes place at $t - n + 1$. Cumulative depreciation—that is, the sum of all depreciations of this investment over all periods from $t - n + 1$ to t of the same investment—is $i_{t-n}(1 - (1 - \delta)^n)$. The equation is derived from the geometric sum $i_{t-n}\delta \sum_{s=0}^{n-1} (1 - \delta)^s = i_{t-n}(1 - (1 - \delta)^n)$.

Monetary depreciation at the price of capital gross of value added taxes at time t for all investments made prior to time t is

$$\mathbb{D}_t = p_k^n(1 + \tau_v) \sum_{s=0}^{t-1} i_s\delta(1 - \delta)^{t-1-s}. \quad (3.8)$$

The price of capital gross of value added is relevant for the public firm (i.e., $p_k = p_k^n(1 + \tau_v) = 1 + \tau_v$).

Adding up all of those depreciated units from time zero to time t , we obtain cumulative depreciation:

$$\mathbb{D}_{cum,t} = p_k^n(1 + \tau_v) \sum_{s=0}^{t-1} i_s(1 - (1 - \delta)^{t-s}). \quad (3.9)$$

Depreciation for tax and accounting reasons. In this case the depreciation rate changes. Newly purchased capital is still valued at initial cost. The capital stock for accounting tax reasons

is

$$k_{a,t+1} = k_{a,t}(1 - \delta_a) + i_t, \quad (3.10)$$

in which δ_a is the depreciation rate used for accounting and tax reasons, with $\delta_a \in [0, 1)$. Initially both capital stocks are equal: $i_0 = k_{a,1} = k_1$.

Equations (3.8) and (3.9) with accounting depreciation also hold for depreciation for tax reasons. With gross prices, they are only applicable for the public firm. For the private firm, we use the net prices for capital:

$$\mathbb{D}_{a,t}^n = \mathbb{D}_{a,t} \frac{1}{1 + \tau_v} = p_k^n \sum_{s=0}^{t-1} i_s \delta_a (1 - \delta_a)^{t-1-s} \quad (3.11)$$

$$\mathbb{D}_{cum,a,t}^n = \mathbb{D}_{cum,a,t} \frac{1}{1 + \tau_v} = p_k^n \sum_{s=0}^{t-1} i_s (1 - (1 - \delta_a)^{t-s}). \quad (3.12)$$

The depreciation rate is not prescribed by law. However, the Federal Ministry of Finance issues tables that strongly suggest expected useful lives and depreciation rates for different types of assets and industries.¹²

Capital asset valuation. For the public firm, book value of capital is investments minus depreciation, all valued at initial prices gross of value added taxes:

$$V_{bk,t} = p_k \sum_{s=0}^t i_s - \mathbb{D}_{cum,a,t}.$$

Substituting in Equation (3.9) and simplifying we obtain

$$V_{bk,t} = \sum_{s=0}^t I_s (1 - \delta_a)^{t-s}. \quad (3.13)$$

Besides the book value, which is based on depreciated initial values, the replacement value of capital $V_{rk,t}$ is the current capital stock valued at current prices for capital

$$V_{rk,t} = p_k k_{t+1}. \quad (3.14)$$

This is the total cost at which all capital assets can be bought or sold at time t . Apart from capital valuations, we also have the firm value V_t , which is the sum of all discounted free cash flows of the firm. We will turn to that later. The replacement value of capital and the firm value should be the same in case that the firm is really just the sum of its capital assets.¹³

Again, capital asset prices net of value added taxes are relevant for the private firm.

Production function. With a given capital stock k_t , the firm uses labor l_t in hours worked, as well as a production technology $f(\cdot)$ to produce constant output y : $f(l_t, k_t) = y$. We use a Cobb–Douglas production function:

$$f(l_t, k_t) = A l_t^\alpha k_t^\beta, \quad (3.15)$$

¹² The table on useful lives and depreciation rates can be found here: https://www.bundesfinanzministerium.de/Web/DE/Themen/Steuern/Steuerverwaltung-Steuerrecht/Betriebspruefung/Afa_Tabellen/afa_tabellen.html.

¹³ That would imply a Tobin's Q of one.

in which α and β are constants between zero and one. They are the output elasticities of labor and capital, respectively. Furthermore, $\alpha + \beta >, =, < 1$ means increasing, constant, or decreasing returns to scale.¹⁴ A is total factor productivity (TFP).

4 | PUBLIC FIRM

The public firm provides a service under sovereign duty and is therefore tax exempt as explained before. For value added taxes that means that the public company must not charge value added taxes. Consequently, the firm cannot deduct value added taxes from purchased investments. It has to carry the value of those taxes for its cost accounting. Thus, the price of capital for depreciation purposes is $p_k = 1 + \tau_v$, namely the net price of one plus value added taxes.

We denote the charge for one unit of the firm's service by the public firm as $p_t^{n,pu}$, which we later compare to the charge of the private firm, which is gross of value added taxes. We assume here that the public firm is fully debt financed. This is not an uncommon case.¹⁵ The municipality is liable for any debt of the public firm. However, according to sect. 12 para. 1 InsO (Insolvency Code) public companies and any public entity are not subject to insolvency rulings as are private companies. Even with risk, a default of the public firm would be unlikely.

Cost-covering charge. The decision criteria of the untaxed firm are as follows: It has to produce sufficient output to satisfy a given demand of household solid waste disposal $y_{d,t}$. The charge for one unit of output at t is $p_t^{n,pu}$. In the interest of the people living in the municipality and having to pay for the service unconditionally, the public firm minimizes this charge subject to the condition that it has relevant costs C_t covered. The sum of the relevant costs C_t are the cost of labor L_t , actual depreciation \mathbb{D}_t , and imputed interest \mathbb{I}_t . The respective equation reads

$$C_t = L_t + \mathbb{D}_t + \mathbb{I}_t. \quad (4.1)$$

Covering costs means that revenue is equal to the relevant costs: $Y_t = C_t$, with $Y_t = y_d p_t^{n,pu}$. Since output is given through the demand of households in the municipality y_d , the price is computed through

$$p_t^{n,pu} = \frac{C_t}{y_d}. \quad (4.2)$$

Imputed interest. There are two ingredients: the interest rate and the basis on which it is paid. Interest is paid on debt and equity. Only book values are used. In our case we only have debt. Therefore, the imputed interest is $B_{t-1}r_f$ payable at t . In case of interest on equity being applicable, this interest should be the average interest rate for long-term bonds of the last ten years. The averaging over ten years of this rate is probably in place to avoid bigger jumps in interest and the derived charges. However, there does not seem to be a substantial difference between interest on debt and interest on equity, since the latter is actually derived from markets for debt

¹⁴ To see that, scale production inputs by a : $f(al_t, ak_t) = (al_t)^\alpha (ak_t)^\beta = a^{\alpha+\beta} l_t^\alpha k_t^\beta$. The scalar is greater, equal to or less than one if $\alpha + \beta$ is greater, equal to or less than one.

¹⁵ As per GPA Baden Württemberg (2005), this is not an unrealistic assumption and often the case for local sewage companies. Wörner (2011, p. 79) names an example for a sewage company for the town Bad Friedrichshall in the federal state Baden-Württemberg. If there is only debt and no equity, the actual contractual debt interest payments are used as imputed interest.

instruments. Thus, our assumption of a fully debt financed firm is not really different from other forms of financing.

Public firm decisions. At time $t = 0$ (optimal) capital is determined and fully financed through debt: $I_0 = K_1 = B_0$. There is no production at $t = 0$ so that there is no depreciation and the complete capital stock is available at $t = 1$: $i_0 = k_1$.

At $t = 1$ the firm employs (optimal) labor and capital to produce output that matches demand y_d , and which is sold at charges $p_1^{n,pu}$ that cover (minimized) costs. Revenues are $Y_1 = y p_1^{n,pu}$. Revenues pay labor costs L_1 and interest payments $B_0 r_f$. As per Equation (4.1) after paying for those items, only \mathbb{D}_1 remains from revenues. This is the depreciated capital valued at gross prices at the time of purchase. The public firm determines optimal capital necessary for the next period k_2 . New capital depends on the demand in the next period y_d . Investments need to be made to reach that level of capital stock. Investments and principal payments on debt have to be paid by the amount remaining from revenues—that is, the value of depreciation—and by new debt that is taken out: $I_1 = \mathbb{D}_1 + B_1 - B_0$. Generally, the equation for investment is

$$I_t = \mathbb{D}_t + B_t - B_{t-1}. \quad (4.3)$$

Rearranging to $I_t - \mathbb{D}_t = B_t - B_{t-1}$ we obtain investments net of depreciation at the lhs and net debt taken out on the rhs. When investments are greater than depreciation, new debt has to be taken out. If new investments are less than depreciation, debt is repaid. When both figures are equal, debt remains constant. From Equation (4.3), the level of debt B_1 is: $B_1 = I_0 - \mathbb{D}_1 + I_1$, in which $B_0 = I_0$.

For $t > 1$ the same logic applies as for the first period. We can solve the debt equation forward and write

$$B_t = \sum_{s=0}^t I_s - \mathbb{D}_{cum,t} \quad (4.4)$$

$$= \sum_{s=0}^t I_s (1 - \delta)^{t-s}, \quad (4.5)$$

in which the second equality uses Equation (3.9).

We use those relations to formulate the minimization problem.

Minimization problem. The decision problem of the public firm is

$$\min_{l_t, k_t, B_{t-1}} L_t + \mathbb{D}_t + B_{t-1} r_f \quad (4.6)$$

for $t = 1, 2, \dots$

We minimize the price with respect to l_t , k_t and B_{t-1} subject to

$$f(l_t, k_t) = y, \quad (4.7)$$

$$y = y_d, \quad (4.8)$$

$$B_t = (1 + \tau_v) \sum_{s=0}^t i_s (1 - \delta)^{t-s} \quad (4.9)$$

$$\mathbb{D}_t = (1 + \tau_v) \sum_{s=0}^{t-1} i_s \delta (1 - \delta)^{t-1-s} \quad (4.10)$$

$$i_t = k_{t+1} - k_t(1 - \delta) \quad (4.11)$$

$$k_0 = 0. \quad (4.12)$$

We can use the first two constraints and the production function to express labor in terms of demand and capital:

$$l_t = \left(\frac{y_d^d}{A} \right)^{\frac{1}{\alpha}} k_t^{-\frac{\beta}{\alpha}}. \quad (4.13)$$

We also substitute the last condition into the third and forth to express everything in terms of capital. Additionally, we replace investment by Equation (3.4) rearranged for investments. We obtain:

$$\min_{k_t} w \left(\frac{y_d^d}{A} \right)^{\frac{1}{\alpha}} k_t^{-\frac{\beta}{\alpha}} + (1 + \tau_v)(\delta + r_f) \sum_{s=0}^{t-1} (k_{s+1} - k_s(1 - \delta))(1 - \delta)^{t-1-s} \quad (4.14)$$

for $t = 1, 2, \dots$

We restate the sums and use that $k_0 = 0$ so that the problem becomes

$$\min_{k_t} w \left(\frac{y_d}{A k_t^\beta} \right)^{\frac{1}{\alpha}} + (1 + \tau_v)(\delta + r_f) \left(\sum_{s=0}^{t-1} k_{s+1}(1 - \delta)^{t-1-s} - \sum_{s=1}^{t-1} k_s(1 - \delta)^{t-s} \right), \quad (4.15)$$

for $t = 1, 2, \dots$

Solution of the problem. Setting the first derivative with respect to k_t to zero and rearranging for capital we obtain

$$k = \left(\frac{w}{(1 + \tau_v)(\delta + r_f)} \frac{\beta}{\alpha} \right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{y_d}{A} \right)^{\frac{1}{\alpha+\beta}}. \quad (4.16)$$

Since capital does not depend on time, we leave out the time subscript. From constant capital and Equation (3.6) follow that investments are constant and equal to actual depreciation: $I = i(1 + \tau_v) = \delta k(1 + \tau_v) = \mathbb{D}$. Debt is taken out only initially and then held constant. The public firm's revenue pays the actual depreciation and interest payments after paying for labor as per Equation (4.1). Using the demand constraint and Equation (4.13) we obtain

$$l = \left(\frac{(1 + \tau_v)(\delta + r_f) \alpha}{w} \frac{\beta}{\alpha} \right)^{\frac{\beta}{\alpha+\beta}} \left(\frac{y_d}{A} \right)^{\frac{1}{\alpha+\beta}}. \quad (4.17)$$

The relations are intuitive. More output y_d requires more capital and more labor. Higher wages w increase capital and decrease labor employed. Any cost increase related to capital, namely an increase in the rate of depreciation or interest rate on debt, will decrease capital employed and increase labor. Increasing the value added tax rate shifts resources away from capital towards labor. Increasing τ_v would increase the price for capital by the respective tax so that labor becomes relatively cheaper.

Charge. The private firm has to take the charge of the public firm as its maximum charge. Based in this, it can make a decision whether it can compete in that market. Therefore, we derive

an expression for the charge from Equation (4.2) and from optimal capital. We start with

$$\begin{aligned} p^{n,pu} &= \frac{1}{y_d} \left[w \left(\frac{y^d}{A} \right)^{\frac{1}{\alpha}} k^{-\frac{\beta}{\alpha}} + (1 + \tau_v)(\delta + r_f)k \right] \\ &= \frac{k}{y_d} (1 + \tau_v)(\delta + r_f) \frac{\alpha + \beta}{\beta}. \end{aligned} \quad (4.18)$$

For the second equality we use that

$$k^{-\frac{\beta}{\alpha}} = k \left(\frac{y}{A} \right)^{-\frac{1}{\alpha}} w^{-1} (r_f + \delta) (1 + \tau_v) \frac{\alpha}{\beta}. \quad (4.19)$$

Substituting in the equation for capital and simplifying, the equation turns to

$$p^{n,pu} = \frac{1}{y_d} \left(\frac{y_d}{A} w^{\alpha} (1 + \tau_v)^{\beta} (\delta + r_f)^{\beta} \right)^{\frac{1}{\alpha+\beta}} \left[\left(\frac{\alpha}{\beta} \right)^{\frac{\beta}{\alpha+\beta}} + \left(\frac{\beta}{\alpha} \right)^{\frac{\alpha}{\alpha+\beta}} \right]. \quad (4.20)$$

Since there is no time dependence in this case, we drop time subscripts here. From this equation we can observe the following. The charge increases with wages. It also increases with the value added tax rate, which makes the price of capital relatively higher. However, it depends on the output elasticities of capital β and labor α , how much wages and the tax rate¹⁶ contribute to charges. For example a high elasticity of output with respect to capital will make the charge more susceptible to the price of capital than to wages. The charge is also positively related to depreciation and the rate of interest. Both are costs to be paid on capital. The charge needs to cover depreciated capital and interest on debt, which financed capital. For a constant-returns-to-scale production function—that is, with $\alpha + \beta = 1$ —the charge does not depend on demand. With increasing returns to scale ($\alpha + \beta > 1$), the charge decreases with increasing demand and output. Doubling inputs for capital and labor would more than double output. Economies of scale are realized, which reduce costs. For decreasing returns to scale ($\alpha + \beta < 1$), the opposite is true.

We continue with the analysis of the private firm.

5 | PRIVATE FIRM

The private firm faces the same price of capital as the public firm $p_k = 1 + \tau_v$. It also has to provide output y^d . The private company has the same production function as the public one. Private firms are often stated to offer lower wage rates than public firms (Kraemer et al., 2017, p. 24). Wage rate differences are important regarding general working conditions. However, herein we want to distill the tax advantage and therefore we assume equal wage rates for the public and the private firm.¹⁷

Decision criterion. We assume that the private company maximizes the present value of leveraged free cash flows net of the initial investment $\mathcal{N}_{l,0}$. The initial investment is represented by the

¹⁶ The term $1 + \tau_v$ really is the gross price of capital.

¹⁷ An extension of the model with respect to wage could be to state the wage rate of the private firm as a multiple of the rate of the public firm: $w_{pr} = f w_{pu}$, with f as a factor.

initial cash flow $F_{l,t} = -I_0 \frac{1}{1+\tau_v}$ (further explained below). The basic valuation equation under certainty is

$$\mathcal{N}_0 = \sum_{t=0}^{\infty} \frac{F_{l,t}}{(1+r_f)^t}, \quad (5.1)$$

For the investment to make sense the net present value must not be negative—that is, it should at least earn the risk-free rate of return. Thus, when the firm optimizes inputs but still has a negative net present value, it will not continue the effort.

At $t = 0$ the initial investment I_0 is made. Nothing is produced at time zero so that at time one $k_1 = I_0 \frac{1}{1+\tau_v}$ is available ($k_0 = 0$). The leveraged free cash flow from Equation (3.3) is

$$\begin{aligned} F_{l,0} &= -I_0 \frac{1}{1+\tau_v} \\ &= -k_1 \frac{1}{1+\tau_v}. \end{aligned}$$

The equation implies that value added taxes paid on investments are refunded immediately.

Leverage. We impose on the private firm that it has to follow an exogenously given and constant leverage ratio. We define the leverage ratio γ in terms of book values as follows:

$$\gamma = \frac{B_t}{V_{bk,t}^n}. \quad (5.2)$$

Thus, debt can be represented by

$$\begin{aligned} B_t &= \gamma V_{bk,t}^n \\ &= \frac{\gamma}{1+\tau_v} \sum_{s=0}^t I_s (1-\delta_a)^{t-s} \\ &= \gamma \sum_{s=0}^t (k_{s+1} - (1-\delta)k_s)(1-\delta_a)^{t-s}. \end{aligned} \quad (5.3)$$

We impose that $\gamma \in [0, 1)$. Even though the private firm has to pay for capital the price gross of value added taxes, since the firm gets an immediate tax refund, there is no need to finance this amount.

Maximization problem. We express all terms from the free cash flow equation in terms of capital and write it into the valuation equation. The firm chooses k_t for $t = 1, 2, \dots$ to maximize the firm's net present value. Expressing the elements of free cash flows in terms of capital, the private firm maximizes the following expression:

$$\begin{aligned} \max_{k_t} \quad & -k_1 + \sum_{t=1}^{\infty} \frac{1}{R_f^t} \left\{ p_t y_d \frac{1-\tau_p}{1+\tau_v} - \left(\frac{y_d}{A} \right)^{\frac{1}{\alpha}} k_t^{\frac{\beta}{\alpha}} w (1-\tau_p) \right. \\ & \left. + r_f \tau_p \gamma \sum_{s=0}^{t-1} (k_{s+1} - (1-\delta)k_s)(1-\delta_a)^{t-s} \right\} \end{aligned}$$

$$+ \tau_p \sum_{s=0}^{t-1} (k_{s+1} - k_s(1 - \delta)) \delta_a (1 - \delta_a)^{t-1-s} - (k_{t+1} - k_t(1 - \delta)) \Big\}. \quad (5.4)$$

Optimal capital with tax effects. We leave the derivation of optimal capital to the Appendix. It turns out to be

$$k = \frac{\left[\frac{\beta}{\alpha} \frac{w}{(r_f + \delta)} \right]^{\frac{\alpha}{\alpha + \beta}} \left(\frac{y_d}{A} \right)^{\frac{1}{\alpha + \beta}}}{\left[\frac{1}{1 - \tau_p} \left(1 - \tau_p \frac{r_f \gamma (1 - \delta_a) + \delta_a}{r_f + \delta_a} \right) \right]^{\frac{\alpha}{\alpha + \beta}}}. \quad (5.5)$$

The numerator is the same as Equation (4.16), the equation for optimal capital for the public firm. Since output and its price are given, the optimization is also a cost minimizing problem. In this case, maximization of firm value without taxes leads to the same result as minimization of costs. All the tax terms are in the denominator. If the tax rates are all zero, the equation for capital is equal to the one for the public firm.

The denominator shows that optimal capital does not depend on the value added tax rate. Any value added taxes paid are returned from the tax office.¹⁸

Taking the derivative of the denominator with respect to τ_p and requiring it to be greater than zero leads to

$$1 > \frac{r_f \gamma (1 - \delta_a) + \delta_a}{r_f + \delta_a},$$

which must be true for admissible parameter values for γ , δ_a and r_f . Thus, the denominator increases (decreases) with increasing (decreasing) τ_p so that optimal capital decreases (increases). A higher tax rate τ_p favors more labor instead of capital. The intuition here is that after-tax labor—that is, labor after the tax rate on profits applied to it—is deducted in the leveraged free cash flows equation. A higher tax rate decreases this after-tax value and the firm can employ more labor to arrive at the same free cash flow. The debt interest tax and the accounting depreciation tax shield represented by the term $\frac{r_f \gamma (1 - \delta_a) + \delta_a}{r_f + \delta_a}$ dampen that effect a bit.

Charge for zero net present value. The charge for the waste disposal service should provide for a net present value of at least zero. We set the valuation Equation (5.4) to zero and use the fact that capital is constant¹⁹ to arrive at the following equation for the charge:

$$p^{pr} = \frac{1 + \tau_v}{y_d} \left[\left(\frac{y^d}{A} \right)^{\frac{1}{\alpha}} k^{-\frac{\beta}{\alpha}} w + \frac{k}{1 - \tau_p} \left((\delta + r_f) - \tau_p \frac{\delta + r_f}{r_f + \delta_a} (r_f \gamma (1 - \delta_a) + \delta_a) \right) \right]. \quad (5.6)$$

We make that equation more comparable to Equation (5.5) in that we set its numerator equal to $k^{(N)}$ and its denominator equal to $k^{(D)}$. With equal parameters, $k^{(N)}$ of the private firm is equal to optimal capital of the public firm times $(1 + \tau_v)^{\frac{\alpha}{\alpha + \beta}}$. Using this and Equation (4.19) we arrive at

¹⁸ The term in the round brackets is positive since the fraction is less than one. The denominator of the fraction contains $r_f + \delta_a$. The numerator contains the same figures multiplied by expressions in $[0, 1]$, so that the fraction is a positive number less than one.

¹⁹ The exception is k_0 , which is zero, and has to be taken care of.

$$p^{pr} = (1 + \tau_v) \frac{k^{(N)}}{y_d} (\delta + r_f) \frac{\alpha + \beta}{\beta} (k^{(D)})^{\frac{\beta}{\alpha}} \quad (5.7)$$

$$= (1 + \tau_v) \frac{k^{(N)}}{y_d} (\delta + r_f) \frac{\alpha + \beta}{\beta} \left[\frac{1}{1 - \tau_p} \left(1 - \tau_p \frac{r_f \gamma (1 - \delta_a) + \delta_a}{r_f + \delta_a} \right) \right]^{\frac{\beta}{\alpha + \beta}}. \quad (5.8)$$

Keeping parameters for the public and the private firm the same, the effect of taxes is only expressed through the term: $(1 + \tau_v) \left[\frac{1}{1 - \tau_p} \left(1 - \tau_p \frac{r_f \gamma (1 - \delta_a) + \delta_a}{r_f + \delta_a} \right) \right]^{\frac{\beta}{\alpha + \beta}}$.

The charge increases (decreases) when τ_v increases (decreases). Additionally the charge also increases (decreases) with increasing (decreasing) τ_p . The equation also shows that increasing (decreasing) leverage and accounting depreciation will decrease (increase) the charge.

The effect of tax rates and other parameters on the charge depend a lot on the interaction with other parameters. We will continue to use reasonable basis parameters and compute measures of the tax advantage for varying parameter combinations.

6 | ANALYSIS OF THE TAX ADVANTAGE

6.1 | The measure of the tax advantage

We will measure the tax advantage using the prices that the private and public firms command, when they employ inputs optimally. Specifically, we use the percentage difference of the charges:

$$r_{pr,pu} = \frac{p^{pr}}{p^{pu}} - 1. \quad (6.1)$$

A positive $r_{pr,pu}$ means that the private firm has to command a higher price than the public firm, so that the public firm has a tax advantage. A negative $r_{pr,pu}$ means the opposite. Keeping all parameters for the public and the private firm the same the percentage difference of charges is expressed by the following equation:

$$\begin{aligned} r_{pr,pu} &= (1 + \tau_v)^{\frac{\alpha}{\alpha + \beta}} (k^{(D)})^{\frac{\beta}{\alpha}} - 1 \\ &= (1 + \tau_v)^{\frac{\alpha}{\alpha + \beta}} \left[\frac{1}{1 - \tau_p} \left(1 - \tau_p \frac{r_f \gamma (1 - \delta_a) + \delta_a}{r_f + \delta_a} \right) \right]^{\frac{\beta}{\alpha + \beta}} - 1 \end{aligned} \quad (6.2)$$

The measure $r_{pr,pu}$ depends on tax rates, leverage, the risk-free rate and accounting depreciation. There are some relations that are immediately obvious.

The risk-free rate only acts on the tax advantage measure $r_{pr,pu}$ through accounting depreciation δ_a . Setting δ_a to zero changes the equation to

$$r_{pr,pu} = (1 + \tau_v)^{\frac{\alpha}{\alpha + \beta}} \left(\frac{1 - \tau_p \gamma}{1 - \tau_p} \right)^{\frac{\beta}{\alpha + \beta}} - 1. \quad (6.3)$$

Setting δ_a to zero reduces the term $\frac{r_f \gamma (1 - \delta_a) + \delta_a}{r_f + \delta_a}$ to γ . This is because in this infinite horizon model and with $\delta_a = 0$, the risk-free rates cancel in the equation, since the discount rate and the interest

rate on debt are equal. Leverage γ remains and expresses the effect of the interest tax shield. Higher leverage decreases the tax advantage measure. Accounting depreciation affects taxes through the depreciation tax shield. Increasing δ_a increases the term $\frac{r_f \gamma (1 - \delta_a) + \delta_a}{r_f + \delta_a}$, which in turn decreases $r_{pr,pu}$.

An increasing risk-free rate decreases $\frac{r_f \gamma (1 - \delta_a) + \delta_a}{r_f + \delta_a}$, which increases $r_{pr,pu}$. The reason is that the risk-free rate acts as future rate of interest payments, which provide tax savings. But it also acts as the discount rate not only on future interest payments, but also on future tax savings through accounting depreciation. A higher risk-free rate means a higher discount rate and a lower value of the depreciation tax shield. This increases the tax advantage of the public firm.

After those general results, we will continue to set the stage for a quantification of the tax advantage measured as $r_{pr,pu}$. To do this, we will set basic parameters in line with the current tax environment in Germany.

6.2 | Basic parameter choice

Tax rates. Not regarding the reduced rate, the tax rate for value added taxes increased from 10% in 1968 over several steps to currently 19%.²⁰ The trend is increasing and adjustments are very infrequent. Historically, there have been several years between adjustments. As the baseline tax rate we use 19% for τ_v .

The corporate tax rate τ_c is flat at 15% since 2008. It was 25% from 2001 to 2007 (one exception with 26.5% in 2003) and 40% and higher throughout the 1990s and before. Adjustments are also infrequent. The trend of the rate is downwards and adjustments were often large, for example the last one with a 10% decrease in the rate.²¹ For the corporate tax rate we use 15% as the baseline rate.

The solidarity surcharge τ_u is 5.5% since 1998. From 2021 onward, it is being largely abolished for natural persons. However, companies as legal persons will still have to pay the surcharge.²² We will use the rate of 5.5%.

The tax rates above are determined at a federal level. However, the trade tax is different. The trade tax rate has two components. We use 3.5% for the base rate for municipal trade tax τ_{bg} . This rate is applicable for all municipalities in Germany since 2008. The assessment rate τ_{ag} , is determined by each municipality. It has a minimum of 200%. We use an assessment rate of 400%, which is the average rate over municipalities for 2018. The trade tax rate τ_g is then computed as the product of the two rates: $\tau_g = \tau_{bg} \tau_{ag} = 14\%$. Using the mentioned parameters the combined rate of trade and corporate tax rate is $\tau_p = 29.825\%$.

Production function. For a quantitative analysis of the tax advantage of public firms in the waste disposal industry, we need some sensible inputs for our parameters. We find those in the work of Frieese et al. (2020). They estimate the production function for firms in the German solid waste disposal industry. They use a sample of 865 firms. Firms are categorized into five different types: municipal firms, public institutions, public limited liability firms, semipublic limited liability firms, and private limited liability firms. They assume that production is represented by the

²⁰ For more information see Statista (2020). The rate decrease in 2020 due to the Corona pandemic was already reversed back to 19% in 2021.

²¹ Deutscher Bundestag Fachbereich WD 4, Haushalt und Finanzen (2018, p. 6) shows the development of the corporate tax rate over time.

²² Law for the reversal of the solidarity surcharge from 1995, "Gesetz zur Rückführung des Solidaritätszuschlags 1995"

same relationship across the different types of firms. We will also follow this approach and assume that the parameters α and β of the production function are the same for the public and the private firm. Friese et al. (2020) estimate a valued added production function. Inputs are labor in terms of the number of employees working in the firm and capital as the value of tangible assets. Output is measured as revenue minus cost of material. This output is termed value added output. That actually suits our analysis, since we do not explicitly account for input materials. Friese et al. (2020) choose not to estimate with quantities alone since output is not that homogeneous (different types of waste and services) and capital may also have different forms (buildings, machines, vehicles).

Their estimation from their Table 2 leads to $\alpha = 0.8245$ and $\beta = 0.1507$ for the approach used in Akerberg et al. (2015). Both values are highly significant. We use as baseline values $\alpha = 0.82$ and $\beta = 0.15$.

Friese et al. (2020) find TFP values for the five different types of firms. For example, their regression output from their Table 3 (specification one), contains the regression constant and coefficients of the natural log of TFP on dummy variables for four of the five different types of firms (the fifth type covered by the constant). For municipal firms expected log of TFP is 3.5736. For private limited liability firms the expected log of TFP is 3.8211.²³ We ignore the effect from $\log(\cdot)$ being a concave function²⁴, which is of a magnitude that does not influence results much, and use as TFP for the private firm $A^{pr} = \exp(3.8211)$ for the public firm and $A^{pu} = \exp(3.5736)$. The resulting TFPs imply an about 28% higher productivity of the private firm. In other regressions they find more conservative numbers with the lowest being about 19% for regressions (5) and (6). This finding in Friese et al. (2020) only holds for private law firms that are also majority privately owned. They find that private law firms that are publicly owned have even worse productivity than public law firms. Additionally, as mentioned in Syverson (2011), the use of value added instead of gross output to estimate the production function is problematic especially in a setting of monopolistic competition, and waste disposal can be regarded as a regional monopoly. Value added output uses revenues, which, in turn, use prices together with gross output. Increased productivity measured from value added production functions may also stem from a price markup, if such markups cannot be excluded due to competition. It is sensible to assume that a private company would not be employed by a municipality if it would introduce substantial price markups on its services. Eventually, the 28% (19%) in productivity increase need to be looked at with caution.

Remaining parameters. Depreciation varies widely for diverse reasons. Thus what we provide here is a crude measure. For accounting depreciation we use a rate of 10% per year as the basic parameter. To perform a sanity check of whether this is a realistic value, we looked at a sample of 11 companies and derived the depreciation rate by dividing current year's additions to depreciation by the prior year's ending book value of assets. For the companies and periods used the average rate was 10.90% and a median of 9.83%.²⁵ One can come up with many issues of those number, however, we are not interested in exact values but in a realistic and representative one.

²³ We use the constant 3.8211, which is the TFP for private firms, and the coefficient for public firms of -0.2475 . Both added up gives the expected log TFP for the private limited liability firm.

²⁴ The expected value of the log of a variable is greater than or equal to the log of the expected value of the same variable: $E(\log(X)) \geq \log(E(X))$. Equality holds when X is a constant.

²⁵ We used depreciation of non-financial assets as share of non-financial assets of the prior year for: Abfallverwertungs- und Entsorgungsbetrieb Kreis Paderborn (A.V.E. Eigenbetrieb) 2016, Abfallwirtschaftsbetrieb des Landkreises Rastatt 2016, Abfallwirtschaftsbetrieb Landkreis Calv 2014, AWISTA Gesellschaft für Abfallwirtschaft und Stadtreinigung mbH 2016, Eigenbetrieb Abfallwirtschaft Landkreis Göppingen 2017, Entsorgungsbetriebe Essen GmbH 2018, Entsorgungsbetriebe Lübeck 2017, Entsorgungsbetrieb der Gemeinde Eitorf 2016, Greifswald Entsorgung GmbH (GEG) 2014, Remondis Vorpommern GmbH 2016, Stadtwerke Overath - Entsorgungsbetrieb - 2014.

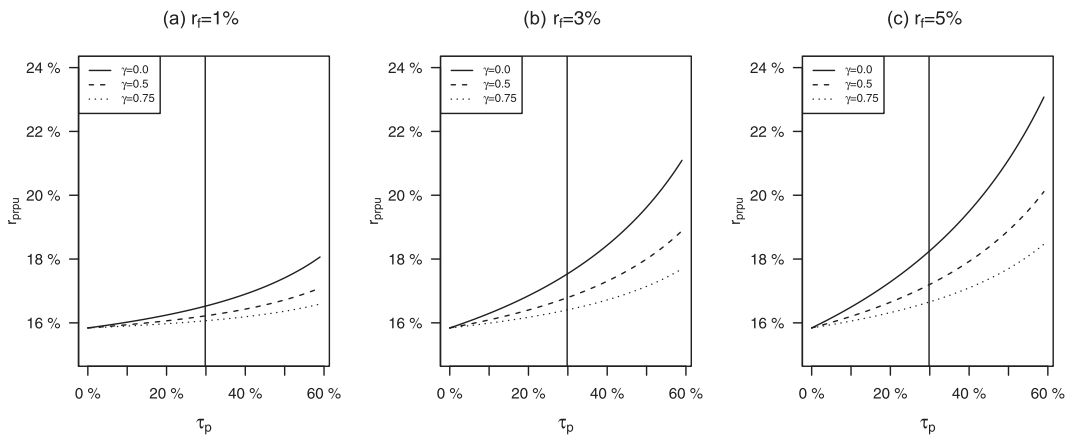


FIGURE 1 The graphs show the tax advantage for varying τ_p for different leverages γ and for different risk-free rates r_f . All other parameters have their baseline values. The vertical line is drawn at the baseline value for the tax rate on profits—that is, at $\tau_p = 29.825\%$

We assume the same actual depreciation rate for the public and the private firm. Therefore, it does not appear in our measure for the tax advantage and we do not worry about its value here.

6.3 | Measuring the tax advantage

We continue to quantify the tax advantage using Equation (6.2) under varying parameters values.

6.3.1 | Varying the tax rate on profits τ_p

We start analyzing the tax advantage relative to τ_p for leverage $\gamma \in \{0.0, 0.5, 0.75\}$ and for the risk-free rate $r_f \in \{0.01, 0.03, 0.05\}$. Figure 1 shows the results.

For a tax rate τ_p equal to zero, the equation for the tax advantage becomes

$$r_{pr,pu} = (1 + \tau_v)^{\frac{\alpha}{\alpha+\beta}} - 1, \quad (6.4)$$

which is about 16% for our parameters. Figure 1 shows that the tax advantage measure rises with τ_p . Intuitively, a higher tax rate on profits increases the advantage of the public firm, which pays no taxes, versus the private firm, which pays the taxes. The term $(1 + \tau_v)^{\frac{\alpha}{\alpha+\beta}}$ that includes the representation of the value added tax rate τ_v in the equation for the tax advantage—that is, in Equation (6.2)—is just a scalar in the function $r_{pr,pu}(\tau_p)$.

The increase of $r_{pr,pu}$ for increasing τ_p is very modest for a low risk-free rate of 1% in part a) of the figure and becomes more pronounced for higher rates (3% in part b) and 5% in part c)). In Equation (6.2), for r_f going to zero, the term $[\frac{1}{1-\tau_p}(1 - \tau_p \frac{r_f \gamma (1-\delta_a) + \delta_a}{r_f + \delta_a})]^{\frac{\beta}{\alpha+\beta}}$ approaches one, which cancels out the effect of τ_p . Increasing the risk-free rate generally increases the tax advantage. As

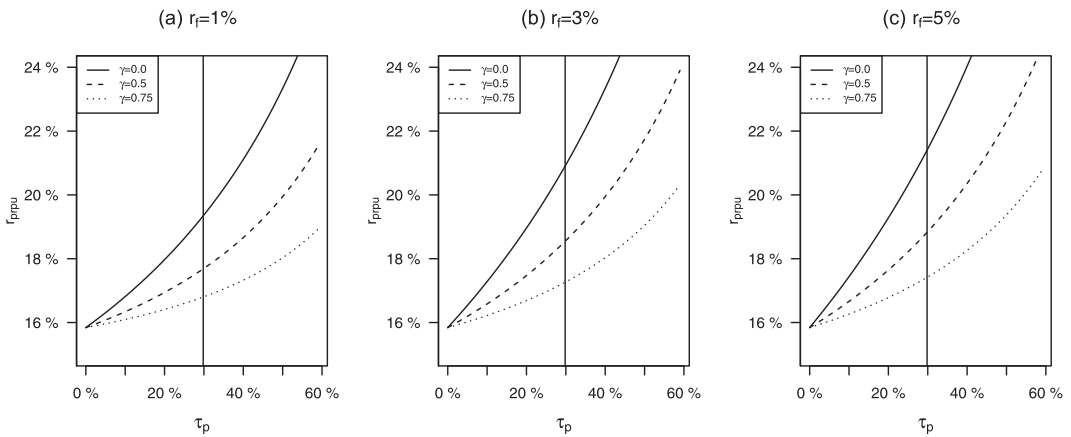


FIGURE 2 The graphs show the tax advantage for varying τ_p for different leverages γ and for different risk-free rates r_f . The accounting depreciation rate is set from 10% down to 1%. All other parameters have their baseline values. The vertical line is drawn at the baseline value for the combined corporate and trade tax rate—that is, at $\tau_p = 29.825\%$

we mentioned before, this is due to the discount rate effect of the risk-free rate. With a higher rate, depreciation and interest related tax savings are discounted at a higher rate and have less value. Due to non-linearities the tax advantage increases faster the higher the tax rate τ_p is. We stop the graphs at $\tau_p = 60\%$, at which, with zero leverage, the tax advantage measures are around 18%, 21% and 23% for risk-free rates of 1%, 3% and 5%, respectively. At the actual tax rate of $\tau_p = 29.825\%$, the tax advantage measures are slightly above 16%, about 17.5% and about 18%, for the three risk-free rates of 1%, 3% and 5%, respectively.

A higher leverage helps to decrease the tax advantage. This decrease is more pronounced for higher τ_p and for higher r_f . The three graphs in Figure 1 show that increasing leverage by the same amount decreases the tax advantage of public firms much more for higher τ_p and for a higher risk-free rate. The same leverage increase decreases the tax advantage relatively more the higher the level of the tax advantage is upfront.

Figure 2 shows what happens when the depreciation tax shield is nearly taken out. In this graph we lower δ_a from 10% down to 1%. This potentially increases profits—that is, the tax base. The tax advantage of public firms dramatically increases. The pattern, however, is the same as for Figure 1. Leverage will help much more to decrease the tax advantage of the public firm. For example, an increase in leverage from zero to 0.75 with a risk-free rate of 5% would decrease the tax advantage measure by about 5%.

In summary, the analysis shows that the private firm has an incentive to increase leverage and to increase the accounting depreciation rate. The incentive is stronger for a higher risk-free rate and for lower depreciation rates, because the relative advantage of both measures increases with a higher risk-free rate and a lower depreciation rate. That is, a private company that initially faces a low accounting depreciation rate would seek to increase or fully use measures of tax avoidance to raise accounting depreciation. The private company would also seek to increase leverage, at least within the relatively risk-free range, to increase interest related tax savings.

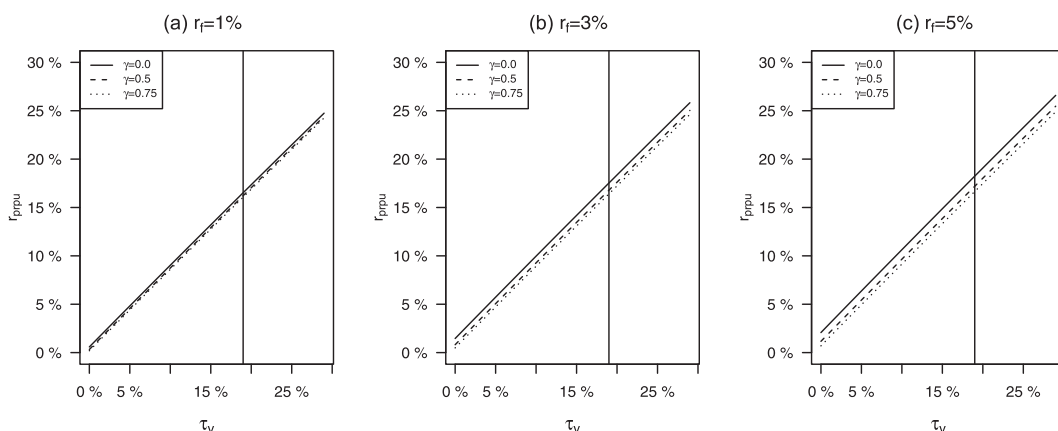


FIGURE 3 The graphs show the tax advantage for varying τ_v for different leverages γ and for different risk-free rates r_f . All other parameters have their baseline values. The vertical line is drawn at the baseline value for value added tax rate with $\tau_v = 19\%$

6.3.2 | Varying the tax rate on value added τ_v

Figures 1 and 2 show that, with zero taxes on profits, the impact of the value added tax rate of 19% on the tax advantage is about 16%. Equation (4.20) shows that value added taxes also increase the charge of the public firm, since the public firm cannot deduct value added taxes paid for capital, and it needs to pass those taxes through via the charge. On the other hand, the private firm can deduct value added taxes paid on capital. This decreases the tax advantage of the public firm a bit, so that it is less than 19%. Figure 3 shows that changes in τ_v are passed through to the charge nearly one on one. Mathematically, this is expressed in the term $\left[\frac{1}{1-\tau_p} (1 - \tau_p \frac{r_f \gamma (1-\delta_a) + \delta_a}{r_f + \delta_a}) \right]^{\frac{\beta}{\alpha+\beta}}$, which will be close to one for basic parameters and also for slight variations of them and the term $(1 + \tau_v)^{\frac{\alpha}{\alpha+\beta}}$ in Equation (6.2), which has an exponent of 0.845, which is close to one. Figure 3 also shows that the leverage and the risk-free rate do not make much of a difference in the measure of the tax advantage. Those parameters only change the first term a bit.

With a value added tax rate of zero and the tax rate on profits at its basic level, the tax advantage is minute. For example with a risk-free rate of one percent, a leverage of zero and the standard parameters, the tax advantage measure is only 0.4%. The tax rate on profits τ_p does not make much of an impact to increase the public firm's tax advantage. The reason for the small impact of taxes on profits is that profits are a relatively small part of revenues, and with that a small part in the charge, whereas the value added tax rate is applied to the net price as a whole. To see this, we keep τ_v at zero and use pre-tax profits Π and divide them by revenue Y , to calculate the profit margin. For simplicity, we set leverage to zero. The resulting profit margin of the private firm is:

$$\frac{\Pi^{pr}}{Y^{pr}} = \frac{Y^{pr} - L^{pr} - \mathbb{D}_{a,t}^{pr}}{Y^{pr}} = \frac{\beta \left(1 - \frac{\delta_a}{r_f + \delta_a} \left[\frac{1}{1-\tau_p} \left(1 - \tau_p \frac{\delta_a}{r_f + \delta_a} \right) \right] \right)^{-\frac{\alpha}{\beta}}}{\alpha + \beta}. \quad (6.5)$$

The profit margin is around 4% for a risk-free rate of 1% and accounting depreciation δ_a of 10%. Decreasing δ_a down to 1% and increasing the risk-free rate to 5% raises the profit margin to around

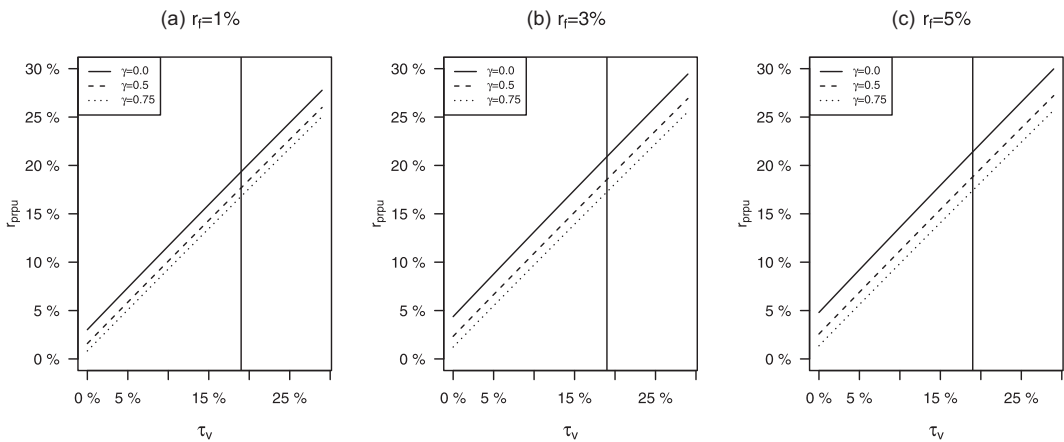


FIGURE 4 The graphs show the tax advantage for varying τ_v for different leverages γ and for different risk-free rates r_f . The accounting depreciation rate was set to 1%. All other parameters have their baseline values. The vertical line is drawn at the baseline value for value added with $\tau_v = 19\%$

10%. The depreciation is a non-cash expense and is only deducted from revenues to compute taxes on profits. The depreciation expense is added back to compute free cash flows, which eventually are distributed to providers of capital. Therefore, capital providers require less profit if accounting depreciation is high and they earn more on the depreciation tax shield. With low accounting depreciation profits need to be increased so that providers of capital still earn the risk-free rate because the depreciation tax shield is low. A tax rate on profits of 30% on a 10% profit margin is still small compared to a tax rate of 19% applied to 100% of the net price.

Figure 4, which sets δ_a to 1% and therefore diminishes the depreciation tax shield, shows that the tax advantage measure starts at higher levels. The term $\left[\frac{1}{1-\tau_p} (1 - \tau_p \frac{r_f \gamma (1-\delta_a) + \delta_a}{r_f + \delta_a}) \right]^{\frac{\beta}{\alpha+\beta}}$ in the tax advantage measure is scaled up a bit and the tax advantage is more elevated. However, the effect of this huge decline in accounting depreciation is still small versus the effect of changes in τ_v .

6.4 | Overcoming the tax advantage through higher productivity

As mentioned before, one type of private firm—namely private law firms with majority private ownership—show on average a higher productivity than public firms. We use the values from Table (3) Friese et al. (2020). For example their regression (1) delivers with $A^{pr} = \exp(3.8211)$ for the public firm and $A^{pu} = \exp(3.5736)$ for the private firm, which implies a productivity gain of about 28% of the private firm versus the public one. Friese et al. (2020) present further regressions controlling for additional effects. The most conservative productivity gains are about 19% for regressions (5) and (6). As mentioned before, the productivity difference between the public and the private firm assumes that output prices are about the same. We adjust Equation (6.2) to allow for different TFPs:

$$r_{pr,pu} = \left(\frac{A^{pu}}{A^{pr}} \right)^{\frac{1}{\alpha+\beta}} (1 + \tau_v)^{\frac{\alpha}{\alpha+\beta}} \left[\frac{1}{1-\tau_p} \left(1 - \tau_p \frac{r_f \gamma (1-\delta_a) + \delta_a}{r_f + \delta_a} \right) \right]^{\frac{\beta}{\alpha+\beta}} - 1 \quad (6.6)$$

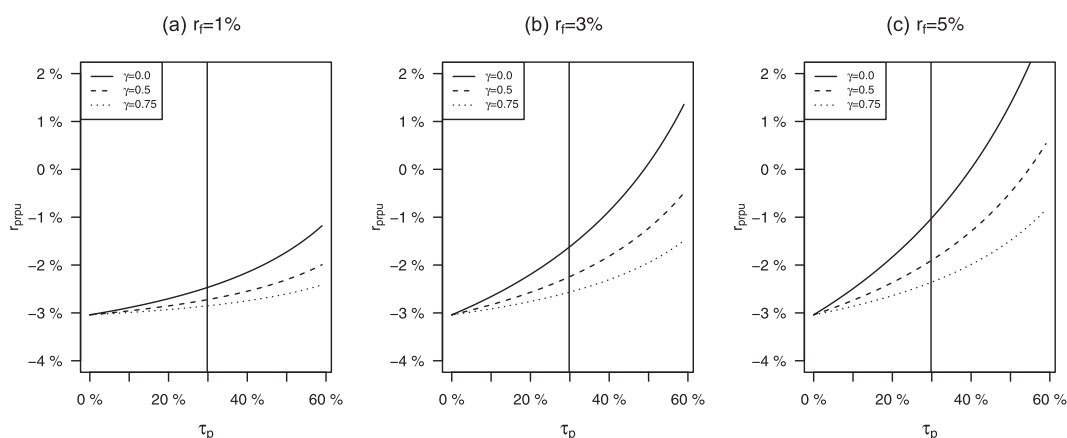


FIGURE 5 The graph shows the tax advantage for varying risk-free rates for different leverages γ . It takes into account the productivity gain of the private firm versus the public firm. All other parameters have their baseline values

The most conservative productivity gains of about 19% lead to an about 16% lower charge if we ignore taxes.

Next, we include the tax effects. Figure 5 shows for the most conservative productivity differential, that the tax advantage of the public firm is now negative, with a charge of the private firm about 1% to 3% less than the one for the public firm.

We add here that private law firms that are publicly owned do not show that productivity gain. In fact, Friese et al. (2020) show that they score worse than public firms.

6.5 | Limitations of the analysis and scope for further research

Our method describes the tax advantage of public companies versus private ones along a few variables that we selected. We have a production function as our basic element and we took a simplifying approach here in using several assumptions to reduce the mathematical overhead and to concentrate on some fundamental factors such as tax rates, the discount rate, depreciation rates and leverage.

An alternative would be to compute the tax advantage purely based on accounting equations and to have a more detailed approach to modeling tax payments. However, in our approach we introduce a direct relationship between taxes and the factor inputs capital and labor. Thus, variations of any parameters in the model propagate through the whole model in a consistent way.

Important extension to our model could be the inclusion of risk since we only look at the risk-free case. On the empirical side, the productivity advantage of the private law firm with private ownership is stunning. It would be helpful to have alternative estimates of the productivity advantage and also more knowledge on exact sources of higher productivity.

Eventually, one could also consider the tax redistribution of private firms back to the municipality. This is an implicit gain of the municipality and potentially reduces the tax advantage of a public firm. We did some first rough, computations regarding this issue. There are five steps on

how taxes are distributed to the state, the federal states and the municipalities. The most basic model would only consider the first step that distributes zero corporate, close to zero value added tax, and 100% trade tax back to the municipality. With a more refined view, on average, more than 10% value and corporate, but less than 100% of trade taxes are redistributed.²⁶ For a risk-free rate of 1%, zero leverage, and no consideration of productivity gains, this reduces the measure of the tax advantage about 3% for the basic parameterization. This still leaves the private firm with a substantial disadvantage. This number can be made more certain with more precise models of the tax redistribution mechanism.

7 | CONCLUSION

Our goal is to measure the advantage related to taxes of public firms with sovereign duties versus private firms for the household solid waste disposal industry in Germany. Therefore, we develop a measure of the tax advantage of those public firms versus private firms that would provide the same service. We compare a public firm that covers its costs and a private firm that maximizes its net present value. We baseline the private firm's charge at a value to provide for a net present value of zero. We assume the two types of firms to be largely the same except for taxation, and, in a second step, for productivity. Our parameters are current tax rates on value added and profits, as well as parameters for the production function from the household solid waste disposal industry in Germany.

Assuming the same productivity of the private and the public firm, the 19% value added tax rate leads to the biggest advantage of the public firms. The private firm needs to charge about 16% more just for the value added tax alone. The tax rate on profits generally leads to a smaller impact since it taxes only profits, which is just a fraction of the total charge for the service. For different parameters we obtain values from less than one percent up to a few percentage points.

Increasing tax shields through an increase in leverage or the depreciation rate helps to reduce the advantage of the public firm. However, those measures work directly only on profits and therefore mostly reduce the advantage caused by taxes on pre-tax profits. They cannot do much about the reduction of value added taxes except that they reduce pre-tax profits, which in turn, reduces revenues and value added taxes a bit.

None of the prior measures makes the private firm competitive in terms of the charge it has to demand. Private firms may overcome the tax advantage mostly through higher productivity. Data suggests that this happens at least for private law firms with private ownership. However, productivity seems to be worse for private law firms with public ownership than for public law firms.

From a pure productivity perspective, the municipality may be better off with the more productive private firm even with the tax disadvantage. However, since we see both types of setups in the German market, productivity seems not to be the decisive factor for many decision makers.

²⁶ Here we use data for 2018. To compute the actual percentages for the redistribution of taxes, we use a proprietary model that considers the redistribution on federal level, the horizontal redistribution among federal states, horizontal redistribution among financially sound and weak federal states, transfers from the state to financially weak federal states, and financial transfers from the federal state to financially weak municipalities.

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APPENDIX A: DERIVATION OF OPTIMAL CAPITAL FOR THE PRIVATE FIRM

We join the sums so that the expression turns to

$$\max_{k_t} -k_1 + \sum_{t=1}^{\infty} \frac{1}{R_f^t} \left\{ y_d p_t \frac{1-\tau_p}{1+\tau_v} - \left(\frac{y_d}{A k_t^\beta} \right)^{\frac{1}{\alpha}} w (1-\tau_p) \right\} \quad (\text{A.1})$$

$$+ \tau_p(r_f\gamma(1 - \delta_a) + \delta_a) \sum_{s=0}^{t-1} (k_{s+1} - k_s(1 - \delta))(1 - \delta_a)^{t-1-s} - (k_{t+1} - k_t(1 - \delta)) \Big\}.$$

For $t = 1, 2, \dots$, for k_t the derivative set to zero is

$$\begin{aligned} & -\frac{1}{R_f^{t-1}} + \frac{1}{R_f^t} \left\{ w(1 - \tau_p) \left(\frac{y_d}{A} \right)^{\frac{1}{\alpha}} \frac{\beta}{\alpha} k_t^{-\frac{\alpha+\beta}{\alpha}} + (1 - \delta) \right\} \\ & + \tau_p(r_f\gamma(1 - \delta_a) + \delta_a) \left\{ \frac{1}{R_f^t} + \frac{1 - \delta_a}{R_f^{t+1}} \left(1 - \frac{1 - \delta}{1 - \delta_a} \right) + \frac{(1 - \delta_a)^2}{R_f^{t+2}} \left(1 - \frac{1 - \delta}{1 - \delta_a} \right) + \dots \right\} = 0. \end{aligned} \quad (\text{A.2})$$

Multiplying by R_f^t and collecting terms in sums we obtain

$$\begin{aligned} & -(r_f + \delta) + w(1 - \tau_p) \left(\frac{y_d}{A} \right)^{\frac{1}{\alpha}} \frac{\beta}{\alpha} k_t^{-\frac{\alpha+\beta}{\alpha}} \\ & + \tau_p(r_f\gamma(1 - \delta_a) + \delta_a) \left\{ \left(1 - \frac{1 - \delta}{1 - \delta_a} \right) \sum_{s=0}^{\infty} \frac{(1 - \delta_a)^s}{R_f^s} + \frac{1 - \delta}{1 - \delta_a} \right\} = 0. \end{aligned} \quad (\text{A.3})$$

We rearrange for capital, which yields

$$k = \left[\frac{\beta}{\alpha} (1 - \tau_p) \right]^{\frac{\alpha}{\alpha+\beta}} (w^\alpha y_d A^{-1})^{\frac{1}{\alpha+\beta}} \left[(r_f + \delta) - \frac{\tau_p}{r_f + \delta_a} ((r_f\gamma(1 - \delta_a) + \delta_a)(r_f + \delta)) \right]^{-\frac{\alpha}{\alpha+\beta}}. \quad (\text{A.4})$$