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**Article — Published Version**

## Input Price Discrimination, Demand Forms, And Welfare

The Journal of Industrial Economics

**Provided in Cooperation with:**

John Wiley & Sons

*Suggested Citation:* Gaudin, Germain; Lestage, Romain (2023) : Input Price Discrimination, Demand Forms, And Welfare, The Journal of Industrial Economics, ISSN 1467-6451, Wiley, Hoboken, NJ, Vol. 70, Iss. 4, pp. 1033-1057, <https://doi.org/10.1111/joie.12306>

This Version is available at:

<https://hdl.handle.net/10419/287878>

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## INPUT PRICE DISCRIMINATION, DEMAND FORMS, AND WELFARE\*

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We analyse the effects of input price discrimination in the canonical model where an upstream monopolist sells to downstream firms with various degrees of efficiency. We first recast a series of existing results within our setting, extending previous findings related to discrimination in final-goods markets to the case of discrimination in input markets. Then, we examine the impact of input price discrimination on welfare. A key determinant of the effects of input price discrimination corresponds to the sum of demand curvature and pass-through elasticity. We provide examples relying on derived demands with constant curvature, including demands with constant pass-through rates.

### I. INTRODUCTION

UNDERSTANDING THE IMPACT OF THIRD-DEGREE PRICE DISCRIMINATION on prices, output, and welfare has been one of the most important topics in the field of industrial organization since at least the work of Pigou [1920] and Robinson [1933]. When firms operating at different levels of the supply chain set up vertical agreements, input (or wholesale) price discrimination may significantly affect market outcomes. This is the case, for instance, when considering discrimination by geographical markets. Such discrimination has recently been opposed by the European Commission for violating the rules of the European Single Market: AB InBev, the world's largest beer brewer, was fined for implementing so-called 'territorial supply constraints,' which

\*We thank the Editor and two anonymous referees for their helpful comments and suggestions. We also thank Taka Adachi, Marcus Asplund, Anette Boom, Bjørn Olav Johansen, Youping Li, Robert Ritz, John Thanassoulis, Jurre Thiel, as well as audiences at the 2018 CRESSE Conference, the 2020 Vfs Annual Conference, Nagoya University, GAEL, and the University of Bergen for valuable remarks and discussions. Open Access funding enabled and organized by Projekt DEAL

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facilitated price discrimination across countries by preventing cross-border sales at the wholesale level.<sup>1,2</sup>

Concerns over price discrimination strategies can generally be classified into four broad categories: exploitation of customers, distortions in related markets, exclusion of competitors, and concerns based on fairness or other policy goals (OECD [2016]). Price discrimination in input markets may distort related downstream markets in several respects. For instance, if the most-efficient firm downstream is charged a higher input price than its less-efficient counterparts, productive efficiency could be damaged.<sup>3</sup> From a theoretical perspective, however, a clear characterisation of the effects of input price discrimination on prices, output, and welfare, as well as the key drivers behind such effects, is often lacking.

In this paper, we uncover some relationships between the effects of input price discrimination by an upstream monopolist and key primitives of demand. We show that the effects of input price discrimination on prices, output, and welfare depend on the sum of the curvature of inverse demand and the quantity-elasticity of the pass-through rate.<sup>4</sup> Using such variables allows us to characterise the determinants of the effects of input price discrimination in a comprehensive manner. They also help us to understand better the economic intuitions behind these effects. Indeed, these variables indicate in which market the derived demand is more elastic and, hence, in which market the upstream firm should set higher input prices. They also relate to the pass-through rate of downstream firms' marginal costs to retail prices, thus revealing how changes in input prices affect output and welfare.

Our theoretical model builds on the classic framework introduced by Katz [1987] and DeGraba [1990], where an upstream monopolist sells an

<sup>1</sup> See the Commission Decision C(2019) 3465, case AT.40134 – AB InBev beer trade restrictions.

<sup>2</sup> Beside rules regulating discrimination by geographical markets, in various sectors, such as network industries, access pricing to essential infrastructures is often required to be uniform across buyers. Other instances of rules addressing input price discrimination originate from industry self-regulation, such as fair, reasonable, and non-discriminatory (FRAND) licensing.

<sup>3</sup> Another distortionary aspect of input price discrimination relates to its effects on competition downstream. This was emphasised recently both by the Supreme Court in the U.S. and the Court of Justice in the European Union. See, e.g., *Volvo Trucks North America, Inc. v. Reeder-Simco GMC, Inc.*, 546 U.S. 164 (2006), and *Serviços de Comunicações e Multimédia SA (MEO) v Autoridade da Concorrência*, Case C-525/16 (2018), respectively. In the U.S., the Robinson-Patman Act of 1936 condemns price discrimination 'to the extent that it threatens to injure competition' (see *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*, 509 U.S. 209 (1993)), while, in the EU, Article 102 of the TFEU prohibits dominant firms from applying 'dissimilar conditions to equivalent transactions with other trading parties, thereby placing them at a competitive disadvantage.'

<sup>4</sup> Our approach thus relates to recent work by Weyl and Fabinger [2013] or Mrázová and Neary [2017], which aim at categorising demand forms according to primitives such as curvature or pass-through rate, in order to appreciate various properties related to the shape of demand.

input to downstream firms with different levels of efficiency.<sup>5</sup> In contrast to most of the literature, we do not specify any particular functional form of demand at the downstream level, thus allowing for greater flexibility. For tractability, however, we assume that downstream firms operate in separate markets. We rely on this framework to investigate the effects of third-degree price discrimination by an upstream monopolist on input and retail prices, output, and welfare.

First, we show that the sum of demand curvature and the elasticity of the pass-through rate affects the upstream firm's decision of whether to discriminate against or in favour of the most-efficient firm downstream. We demonstrate that this sum corresponds to the curvature of the (inverse) *derived* demand for the input and, hence, governs how the elasticity of this derived demand varies across retailers. Relying on this feature, we apply existing techniques, in particular related to the analysis of price discrimination in final-goods markets, to determine the effects of input price discrimination on prices and output within our setting.

Second, we show that the effects of discrimination on total welfare depend on whether input price discrimination (i) increases total output or not, (ii) improves allocative efficiency by bringing retail prices closer to one another or not, as in Miklós-Thal and Shaffer [2021], and (iii) improves productive efficiency by shifting output from the high-cost market to the low-cost one or not, as in Chen and Schwartz [2015]. The sum of the demand curvature and the quantity-elasticity of the pass-through rate constitutes the main determinant behind those effects, and we provide sufficient conditions for input price discrimination to either raise or reduce total welfare. The effects of discrimination on consumer surplus are also investigated.

In order to illustrate our findings, we show how our results apply to demand forms with a constant curvature, which are commonly used in the literature and for which the elasticity of the pass-through rate equals zero. We also rely on specific demand forms that do not share the same curvature property in order to illustrate, for instance, that, in contrast to the standard case of third-degree price discrimination by a monopolist in final-goods markets, input price discrimination can raise welfare even when it does not increase total output.

*Related Literature.* After the seminal contribution by Katz [1987], who studied the effects of input price discrimination when downstream firms differ in their ability to integrate backwards and their marginal costs of production,

<sup>5</sup> Following most of the literature, including Katz [1987] and DeGraba [1990], we assume that the upstream monopolist makes take-it-or-leave-it offers to downstream firms, based on linear input prices. Linear, vertical contracts are widely used in practice in many industries, as, for instance, TV distribution (Crawford and Yurukoglu [2012]), medical supplies (Grennan [2013]), or between book publishers and resellers (Gilbert [2015]).

the literature has often followed DeGraba [1990], who analysed such effects on downstream firms' production technology choice in the long run, relying on a linear demand system to model the downstream market.

Amongst the authors discussing the impact of flexible functional forms of demand, Valletti [2003] found that the curvature of demand was a key driver of some welfare effects of input price discrimination under Cournot competition downstream. More recently, Li [2014] derived a condition on demand primitives determining whether a high-cost downstream firm would face a lower or greater input price than a low-cost firm. In related analyses, flexible demand forms were used by Li [2017] to determine key conditions on demand primitives driving the effects of differential pricing by an upstream monopolist,<sup>6</sup> and by Miklós-Thal and Shaffer [2021] to study discrimination across markets by an upstream supplier, when buyers are active in more than one market.

The effects of input price discrimination have been analysed under a broad range of scenarios, departing from the seminal contributions by Katz [1987] and DeGraba [1990]. These include, for instance, settings where firms rely on non-linear wholesale contracts (O'Brien and Shaffer [1994]; Inderst and Shaffer [2009]; Herweg and Müller [2014, 2016]) or bargain over input prices (O'Brien [2014]), as well as environments with a threat of demand-side substitution (Inderst and Valletti [2009]) or consumer search costs (Janssen and Reshidi [2018]). Others have studied the effects of heterogeneous levels of efficiency in converting inputs by the downstream firms (Yoshida [2000]), the effects of buyer power on other buyers' input prices (Inderst and Valletti [2011]), along with the effects of input price discrimination on the market structure downstream (Herweg and Müller [2012]). Moreover, some authors have analysed the effects of input price discrimination when buyers are active in several markets and discrimination can take place either across buyers (Arya and Mittendorf [2010]), or across markets when the upstream monopolist sets non-linear tariffs (Miklós-Thal and Shaffer [2021]).

In this paper, we mostly rely on techniques developed for the analysis of 'standard' third-degree price discrimination by a monopolist selling in final-goods markets. Our output and welfare analyses build on the approach first established by Schmalensee [1981] for final-goods markets.<sup>7</sup> Also, our main welfare result is derived by adapting the approach of Aguirre, Cowan and Vickers [2010] to vertically-related markets. In this literature focusing on final-goods markets, it is known, since the work of Robinson [1933], that demand curvature is a key driver of the output and welfare effects of price discrimination. In this respect, several authors have considered non-linear

<sup>6</sup> 'Differential pricing' consists in charging different prices for the same product or service to distinct customer groups that differ in their marginal costs of service.

<sup>7</sup> See also the work of Varian [1985, 1989], and Schwartz [1990], for instance.

demand forms in order to understand the link between demand curvature and the effects of price discrimination. See, for instance, Cheung and Wang [1994], Cowan [2007, 2012, 2016], and Aguirre, Cowan and Vickers [2010] for analyses of the effects of discrimination by a monopolist, or Adachi and Fabinger [2019] under oligopolistic competition.

Several of our results closely relate to the literature on differential pricing in final-goods markets, and, in particular, to the work of Chen and Schwartz [2015], who studied the welfare effects of enforcing uniform *retail* prices across markets when a monopolist faces different marginal costs of services in distinct markets.<sup>8</sup> Finally, our results also relate to the literature on trade policy, tariffs, and the most-favoured nation clause (Choi [1995]; Saggi [2004]), and the literature on relative price regulation (Vickers [2020]).

The remainder of the paper is as follows. In Section II, we introduce our framework of analysis. We recast, in Section III, a series of existing results related to the effects of discrimination on prices and output within our setting, notably extending previous findings related to discrimination in final-goods markets to the case of discrimination in input markets. Then, in Section IV, we analyse the effects of input price discrimination on welfare. In Section V, we explain which of our results also apply to asymmetric demands downstream. Finally, Section VI concludes.

## II. THE MODEL

We follow one of the workhorse models to analyse third-degree price discrimination in input markets.<sup>9</sup> An input monopolist serves two downstream firms (two retailers, for instance), labelled 1 and 2, at linear input prices  $w_1$  and  $w_2$ , respectively. Linear input prices are common in several industries, such as cable TV (Crawford and Yurukoglu [2012]), medical supplies (Grennan [2013]), or book retailing (Gilbert [2015]), for instance.<sup>10</sup> We assume that retailers 1 and 2 face downstream, constant marginal costs  $c_1 \geq 0$  and  $c_2 > c_1$ , respectively, such that retailer 2 is intrinsically less efficient than retailer 1.

<sup>8</sup> See also Liu, Niu and White [2021] and Chen, Li and Schwartz [2021]. Our approach differs from that of Chen and Schwartz [2015], because we consider the case in which the upstream monopolist sells not directly to final consumers but indirectly via gatekeeping retailers, and because we study the welfare effects of enforcing uniform *input* prices across markets, leading to different retail prices in each market when downstream firms' marginal costs are asymmetric.

<sup>9</sup> A similar model was used by Katz [1987] and DeGraba [1990], for instance.

<sup>10</sup> See, instead, Inderst and Shaffer [2009] for an analysis of input price discrimination when the upstream firm can set two-part tariffs. They show that, with independent markets downstream, an unconstrained upstream monopolist would set  $w_1 = w_2 = 0$  and extract the entire industry profit through fixed fees, whereas it would set a strictly positive uniform input price if discrimination were prohibited.

We consider that markets downstream are independent, and we refer to these as markets 1 and 2.<sup>11</sup> We denote by  $\pi_i(p_i) \equiv (p_i - w_i - c_i) q_i(p_i)$  the profit of retailer  $i$ ,  $\forall i \in \{1, 2\}$ , given its own price  $p_i$  and demand  $q_i(\cdot)$ . We assume that firm  $i$ 's (perceived) marginal cost is constant and given by  $w_i + c_i$ . This means that retailer  $i$  demands one unit of input in order to produce one unit of output. We define  $q'_i(p_i) \equiv \partial q_i / \partial p_i$ ,  $q''_i(p_i) \equiv \partial^2 q_i / \partial p_i^2$ ,  $\pi'_i(p_i) \equiv \partial \pi_i / \partial p_i$ , and  $\pi''_i(p_i) \equiv \partial^2 \pi_i / \partial p_i^2$ ,  $\forall i$ . We assume that demands faced by retailers are 'symmetric,' in that they have equal elasticities at any common price.<sup>12</sup> This requires, following Chen and Schwartz [2015], that demands be proportional; i.e.,  $q_1(p) = \lambda q(p)$  and  $q_2(p) = (1 - \lambda)q(p)$ ,  $\forall p$ , with  $\lambda \in (0, 1)$ . This 'symmetry' assumption, which can be relaxed for some of our results, also ensures that the demand curvature, pass-through rate, and quantity-elasticity of the pass-through rate (as defined below) are equal across markets at any common price.

We denote by  $\pi_u(w_1, w_2) \equiv (w_1 - u_1) q_1(p_1^*(w_1; c_1)) + (w_2 - u_2) q_2(p_2^*(w_2; c_2))$  the upstream firm's profit resulting from sales made to downstream retailers 1 and 2, where  $p_i^*(w_i; c_i) \equiv \operatorname{argmax}_p (p - w_i - c_i) q_i(p)$ . We assume that the upstream firm's marginal costs,  $u_i$ ,  $\forall i$ , are symmetric,<sup>13</sup> and set them to zero, without loss of generality. We also define  $\partial_i \pi_u \equiv \partial \pi_u / \partial w_i$  and  $\partial_{ii} \pi_u \equiv \partial^2 \pi_u / \partial w_i^2$ ,  $\forall i \in \{1, 2\}$ .

The timing of the game is as follows. In the first stage, the upstream firm sets a linear wholesale price  $w_i$  for each retailer. In the second stage of the game, each retailer takes as given the input prices set by the upstream monopolist. Given the input price  $w_i$  it faces, as well as its own per-unit marginal costs,  $c_i$ , retailer  $i$  selects its price,  $p_i$ , in order to maximise its profit,  $\pi_i$ . We analyse the subgame perfect equilibrium of this game.

Given the above-mentioned timing, the equilibrium price is determined by the corresponding first-order condition in the second stage of the game,  $\forall i$ :

$$(1) \quad \frac{\partial \pi_i}{\partial p_i} \Big|_{p_i=p_i^*(w_i; c_i)} = 0 \Leftrightarrow q_i(p_i^*(w_i; c_i)) + [p_i^*(w_i; c_i) - w_i - c_i] q'_i(p_i^*(w_i; c_i)) = 0.$$

<sup>11</sup> Independent markets downstream provide a natural starting point for the analysis of input price discrimination under flexible demand forms, as done, e.g., by Li [2014] or in the main analysis of Miklós-Thal and Shaffer [2021].

<sup>12</sup> In Section V, we show that our main results regarding the effects of input price discrimination on output and welfare also extend to the cases where demands are neither symmetric, nor proportional.

<sup>13</sup> Symmetry in the upstream monopolist's marginal costs of service ensures that we rule away concerns related to differential pricing. See Chen and Schwartz [2015], Li [2017], Chen, Li and Schwartz [2021] or Liu, Niu and White [2021] for recent studies on differential pricing.



Differentiating this equality with respect to  $w_i$ , we find that the pass-through rate of (perceived) marginal cost to the equilibrium price in market  $i$ , defined by  $\rho_i(p_i^*) \equiv \partial p_i^* / \partial c_i = \partial p_i^* / \partial w_i$ , is equal to  $1 / [2 - \sigma_i(p_i^*)]$ , where  $\sigma_i(p) \equiv q_i(p) q_i''(p) / [q_i'(p)]^2$  corresponds to the curvature (i.e., the elasticity of the slope) of inverse demand in market  $i$ , expressed as a function of price.<sup>14</sup>

In the first stage of the game, the upstream firm's choice depends on whether it has the possibility to discriminate between the various retailers or not. In this paper, we analyse and compare both cases under the assumption that both retailers are served in equilibrium, even when the upstream monopolist cannot discriminate. The upstream firm's problem is equivalent to that of a firm setting  $w_i$  in a market characterized by the *derived* demand function  $q_i(p_i^*(w_i; c_i))$ ,  $\forall i$ . The curvature of the corresponding inverse derived demand, defined by  $q_i(p_i^*(w_i; c_i)) [\partial^2 q_i(p_i^*(w_i; c_i)) / \partial w_i^2] / [\partial q_i(p_i^*(w_i; c_i)) / \partial w_i]^2$ , is equal to  $\sigma_i(p_i^*(w_i; c_i)) + \mu_i(p_i^*(w_i; c_i))$ , where  $\mu_i(p) \equiv q_i(p) \rho_i'(p) / [q_i'(p) \rho_i(p)]$  represents the quantity-elasticity of the pass-through rate, with  $\rho_i'(p) \equiv \partial \rho_i / \partial p$ .<sup>15</sup>

Finally, we make the following technical assumptions in order to ensure the existence and uniqueness of the equilibrium.

*Assumption 1.*  $2 > \sigma_i, \forall i$ .

*Assumption 2.*  $2 > \sigma_i + \mu_i, \forall i$ . Moreover,  $\partial_{ii} \pi_u < 0, \forall i$ .

Assumption 1 ensures that retail prices which maximise retailers' profits are interior solutions, and that Equation 1 thus holds, in equilibrium. It is equivalent to  $\pi_i'' < 0$  evaluated at the equilibrium, and to  $\rho_i > 0$ . We assume that it holds over a range covering equilibrium retail prices both when discrimination is allowed and when it is prohibited, at given marginal costs. Assumption 2 implies that the maximisation problem of the upstream firm is well defined, both when it sets its input prices freely or when it faces some constraints. We assume that the first part (respectively, second part) of this assumption holds over an interval which includes equilibrium retail (resp., input) prices whether discrimination is allowed or not, in market  $i$ , at given marginal costs. It can be easily shown that, when the upstream firm is free to set input prices,  $\partial_{ii} \pi_u < 0 \Leftrightarrow 2 > \sigma_i + \mu_i$  in equilibrium. The second part of Assumption 2, indicating that the upstream firm's profit is locally concave in input prices, is thus relevant only when the upstream firm is constrained in setting such prices.

<sup>14</sup> Denoting the price-elasticity of demand in market  $i$  by  $\epsilon_i(p) \equiv -p q_i'(p) / q_i(p)$ , we have:  $\sigma_i(p) = 1 + 1 / \epsilon_i(p) + q_i(p) e_i'(p) / [q_i'(p) \epsilon_i(p)]$ .

<sup>15</sup> Given that  $\rho_i(p) = 1 / [2 - \sigma_i(p)]$ , we have  $\mu_i(p) = q_i(p) \rho_i(p) \sigma_i'(p) / q_i'(p)$ , with  $\sigma_i'(p) \equiv \partial \sigma_i / \partial p$ .



III. PRELIMINARY RESULTS: PRICE AND OUTPUT EFFECTS

In this section, we build on the existing literature in order to provide an overview of the effects of input price discrimination on prices and output.

III(i). *Price Effects*

First, we investigate the effect of an infinitesimal change in marginal costs when the upstream firm is free to choose the input prices, in order to derive sufficient conditions for comparing the equilibrium input prices across markets under a discriminatory regime,  $w_1^*$  and  $w_2^*$ , given  $c_1 < c_2$ . The first-order condition of the upstream firm leads to the following, in each market  $i$ , with  $\rho_i^* \equiv \rho_i(p_i^*(w_i^*))$ :<sup>16</sup>

$$(2) \quad \frac{\partial \pi_u}{\partial w_i} \Big|_{w_i=w_i^*} = 0 \Leftrightarrow q_i + w_i^* q_i' \rho_i^* \Big|_{p_i^*(w_i^*)} \Leftrightarrow w_i^* = \frac{-q_i}{q_i' \rho_i^*} \Big|_{p_i^*(w_i^*)}.$$

Moreover, a marginal change in  $c_i$  will have two effects on the equilibrium retail price: one direct effect, captured by the pass-through rate  $\partial p_i^* / \partial c_i$ , and one indirect effect, triggered by the strategic reaction of the upstream firm to a change in  $c_i$ :  $(\partial p_i^* / \partial w_i) (\partial w_i^* / \partial c_i)$ . Given that downstream firms require one unit of input in order to produce one unit of output, we have:  $\partial p_i^* / \partial w_i = \partial p_i^* / \partial c_i$ . Hence, we obtain:

$$(3) \quad \frac{\partial p_i^*(w_i^*(c_i); c_i)}{\partial c_i} = \frac{\partial p_i^*}{\partial c_i} + \frac{\partial w_i^*}{\partial c_i} \frac{\partial p_i^*}{\partial w_i} \Leftrightarrow \frac{\partial w_i^*}{\partial c_i} = \frac{\partial p_i^* / \partial c_i}{\rho_i^*} - 1.$$

Finally, we can total-differentiate Equation 1, evaluated at  $w_i^*$  given by Equation 2, with respect to  $c_i$ . This gives, with  $\sigma_i^* \equiv \sigma_i(p_i^*(w_i^*))$  and  $\mu_i^* \equiv \mu_i(p_i^*(w_i^*))$ :

$$(4) \quad \frac{dp_i^*}{dc_i} = \frac{\rho_i^*}{2 - (\sigma_i^* + \mu_i^*)}.$$

We can already observe that, in equilibrium, the upstream monopolist always sets its input prices  $w_1^*$  and  $w_2^*$  such that  $w_1^* + c_1 < w_2^* + c_2$  whenever  $c_1 < c_2$ . (All proofs are relegated to Appendix A.)

*Lemma 1.* In equilibrium,  $w_1^* + c_1 < w_2^* + c_2$  if and only if  $c_1 < c_2$ .

Lemma 1 simply states that the upstream monopolist never has an incentive to set input prices such that they alter the identity of the most-efficient

<sup>16</sup> Note that here and below, we simply denote  $p_i^*(w_i^*(c_i); c_i)$  by  $p_i^*(w_i^*)$ .

retailer overall, even when it discriminates in favour of the least-efficient firm.<sup>17</sup> Lemma 1 hence implies that, when the upstream firm is allowed to price-discriminate, the least-efficient retailer always sets the highest retail price in equilibrium.

We can also express the impact of a cost shock on  $w_i^*$  by relying on our key variables. In particular, from Equations 3 and 4 above, we obtain:

$$(5) \quad \frac{\partial w_i^*}{\partial c_i} = \frac{-[1 - (\sigma_i^* + \mu_i^*)]}{2 - (\sigma_i^* + \mu_i^*)},$$

where the denominator is strictly positive, following Assumption 2. We can now state the following result, when the sign of  $1 - (\sigma_i + \mu_i)$  remains unchanged over the relevant interval, defined here by  $p \in [p_1^*(w_1^*), p_2^*(w_2^*)]$ .

*Result 1.* (Li [2014]). When the upstream monopolist is allowed to price-discriminate, it charges a higher (respectively, lower) input price to the most-efficient downstream firm if  $\sigma_i + \mu_i < 1$  (resp.,  $\sigma_i + \mu_i > 1$ ) over the relevant interval,  $\forall i$ .

Result 1 was first stated by Li [2014], albeit relying on different variables.<sup>18</sup> The economic intuition is as follows: the derived demand for the input becomes less elastic with an infinitesimal reduction in downstream marginal costs if and only if  $1 > \sigma_i + \mu_i$ . In this case, the upstream monopolist has an incentive to set a higher input price to the most-efficient firm downstream. Indeed, the upstream firm’s problem is equivalent to that of a monopolist setting  $w_i$  in a market characterized by the derived demand function  $q_i(p_i^*(w_i; c_i))$ . The semi-elasticity of this derived demand is given by  $-(\partial q_i / \partial w_i) / q_i = -q_i' \rho_i / q_i$ , and the (inverse) derived demand curvature by  $q_i (\partial^2 q_i / \partial w_i^2) / (\partial q_i / \partial w_i)^2 = \sigma_i + \mu_i$ . Therefore, whether the derived demand is log-concave (i.e.,  $\sigma_i + \mu_i < 1$ ) or log-convex (i.e.,  $\sigma_i + \mu_i > 1$ ) determines whether the semi-elasticity of the derived demand increases or decreases in the (input) price.

It is important to note that the sign of  $1 - (\sigma + \mu)$ , remains constant in price for commonly used demand forms. Bulow and Pflleiderer [1983] identified the various demand forms for which the pass-through rate,  $\rho$ , remains constant under monopoly. These can be divided into three families: (i) constant-elasticity demand forms, with inverse demands given by

<sup>17</sup> A similar result was already stated by Katz [1987], under Cournot competition downstream.

<sup>18</sup> More precisely, Li [2014] showed that the upstream monopolist charges a higher input price to the most-efficient downstream firm if the inverse demand satisfies  $2P'(q) + 4qP''(q) + q^2P'''(q) < 0$ . Our key variables,  $\sigma$  and  $\mu$ , allow us to express this result in a way that may be more intuitive to some. They will also prove to be fundamental determinants of output and welfare effects, as demonstrated in our analysis below.

$P(q) = \beta q^{-1/\epsilon}$ , where  $\beta > 0$  and  $\epsilon > 1$ ; (ii) generalised ‘linear’ demand forms, with  $P(q) = \alpha - \beta q^\delta$ , for  $\alpha, \beta, \delta > 0$ ; and (iii) exponential demand forms, with  $P(q) = \alpha - \beta \ln(q)$ , for  $\alpha, \beta > 0$  and  $0 < q < e^{\alpha/\beta}$ . In our framework, these demand forms imply that the curvature,  $\sigma$ , is constant, and that  $\mu = 0$  everywhere. Therefore, for these demand forms we find that  $\partial w^*/\partial c < 0$  is simply equivalent to  $1 > \sigma$ . Computing the curvature for each of the three families listed above, we find that (i) constant-elasticity demand forms always lead to  $\partial w^*/\partial c > 0$ , as  $\sigma = 1 + 1/\epsilon$ ; that (ii) generalised ‘linear’ demand forms imply instead that the wholesale price is decreasing in the downstream marginal cost, i.e.,  $\partial w^*/\partial c < 0$ , because  $\sigma = 1 - \delta$ ; and (iii) that  $\sigma = 1$  and the upstream firm would never adjust its equilibrium input price after a shock on  $c$ , i.e.,  $\partial w^*/\partial c = 0$ , when demand is of the exponential form.

We can now compare equilibrium prices under the discriminatory and non-discriminatory regimes. A ban on price discrimination would force the upstream firm to set the same input price to each retailer, which we denote by  $w^0$  in equilibrium. We follow the analysis of Schmalensee [1981], established in order to analyse output and welfare effects of third-degree price discrimination in final-goods markets. We define the variable  $r$  as the scope for input price discrimination by the upstream monopolist. We consider that the upstream monopolist maximises its profit subject to  $|w_1 - w_2| \leq r$ , where  $r \geq 0$ . This limit on the scope to price-discriminate is binding when  $r$  is small enough, that is,  $r \leq |w_1^* - w_2^*| \equiv r^*$ .<sup>19</sup>

We denote by  $w_i^{**}(r)$  the equilibrium input price in market  $i$ , for any given  $r$  in  $[0, r^*]$ . We thus have  $w_1^{**}(0) = w_2^{**}(0) = w^0$  as well as  $w_1^{**}(r^*) = w_1^*$  and  $w_2^{**}(r^*) = w_2^*$ . Defining the retail price set by retailer  $i$  when  $r = 0$  (i.e., when  $w_i = w^0, \forall i$ ) as  $p_i^0 \equiv p_i^*(w^0)$ , we obtain the following.

**Result 2.** The price effects of input price discrimination are:

- (i)  $w_1 > w^0 > w_2$  and  $p_1^0 < p_1 < p_2 < p_2^0$  if  $\sigma_i + \mu_i < 1, \forall i, \forall r \in (0, r^*]$ ;
- (ii)  $w_1 < w^0 < w_2$  and  $p_1 < p_1^0 < p_2^0 < p_2$  if  $\sigma_i + \mu_i > 1, \forall i, \forall r \in (0, r^*]$ ;
- (iii)  $w_1^* = w^0 = w_2^*$  and  $p_1^* = p_1^0 < p_2^0 = p_2^*$  if  $\sigma_i + \mu_i = 1, \forall i$ .

This result implies that the non-discriminatory input price always lies between  $w_1^*$  and  $w_2^*$ . The ranking of input prices can be obtained by following Nahata, Ostaszewski and Sahoo [1990], who show that, in final-goods markets, concavity of the profit functions implies that the non-discriminatory price lies between the two discriminatory prices. In our setting, we can consider that the input supplier’s problem is equivalent to that of a monopolist

<sup>19</sup> Note that an important difference in the case of third-degree price discrimination in final-goods markets (i.e., absent any vertical relation) is that, even when  $r = 0$ , the retail prices in both markets differ in our setting, as long as  $c_1 \neq c_2$ .

setting prices  $w_i, \forall i$ , in markets characterized by the derived demand functions  $q_i(p_i^*(w_i; c_i))$ . Moreover, Result 2 also allows us to understand how input and retail prices rank when the upstream firm faces some regulation over the relative difference between the prices it sets – a topic analysed by Vickers [2020] in final-goods markets.

Result 2 has straightforward implications for the demand forms identified by Bulow and Pfleiderer [1983], for which  $\mu = 0$ . A ban on discrimination or a binding, relative price regulation at the input level would always raise input prices for the most-efficient retailer and reduce them for the least-efficient one, for demand forms with a constant elasticity. By contrast, demands of the generalised ‘linear’ form would see relative price regulation or a ban on discrimination benefiting retailer 1 and hurting retailer 2, because  $c_2 > c_1$ . Instead, when the (inverse) demand is of the form  $P(q) = \alpha - \beta \ln(q)$  we obtain  $\sigma = 1, \forall q$ , and whether input price discrimination is allowed or not does not affect the equilibrium allocation.<sup>20</sup>

III(ii). *Output Effects*

We now study how input price discrimination affects total output. We use the subscripts  $\ell$  (for ‘low’) and  $h$  (for ‘high’) to denote the ‘weak’ and ‘strong’ input markets, respectively. The weak (respectively, strong) input market is that where  $w_i \leq w^0$  (resp.,  $w_i \geq w^0$ ), that is, where the wholesale price is lower (resp., higher) when input price discrimination is allowed.

The impact of an increase in  $r$  on total output,  $Q \equiv q_h + q_\ell$ , is given by:  $\partial Q/\partial r = (\partial w_\ell/\partial r) q'_\ell \rho_\ell + (\partial w_h/\partial r) q'_h \rho_h$ . Moreover, from the first-stage, first-order condition,  $\partial_\ell \pi_u + \partial_h \pi_u|_{w_\ell=w_h-r} = \partial_h \pi_u + \partial_\ell \pi_u|_{w_h=w_\ell+r} = 0$  and the binding constraint  $w_h - w_\ell \leq r$ , we obtain the following equilibrium input prices,  $\forall r \in [0, r^*]$ :

$$(6) \quad \begin{cases} w_h^{**}(r) = \frac{r q'_\ell \rho_\ell - (q_h + q_\ell)}{q'_h \rho_h + q'_\ell \rho_\ell} \\ w_\ell^{**}(r) = \frac{-r q'_h \rho_h - (q_h + q_\ell)}{q'_h \rho_h + q'_\ell \rho_\ell}, \end{cases}$$

together with the marginal effects of a change in  $r$  on input prices,  $\forall r \in [0, r^*]$ :

$$(7) \quad \begin{cases} \frac{\partial w_h^{**}}{\partial r} = \frac{\partial_\ell \ell \pi_u}{\partial_{hh} \pi_u + \partial_\ell \ell \pi_u} > 0 \\ \frac{\partial w_\ell^{**}}{\partial r} = \frac{-\partial_{hh} \pi_u}{\partial_{hh} \pi_u + \partial_\ell \ell \pi_u} < 0. \end{cases}$$

<sup>20</sup> Table B1, in Appendix B, provides an overview of how our results apply to the examples we mention in the main text.

Naturally, we obtain  $w_h^{**}(0) = w_\ell^{**}(0) = w^0$  as well as  $w_h^{**}(r^*) = w_h^*$  and  $w_\ell^{**}(r^*) = w_\ell^*$ . This allows us to characterise the output effect of price discrimination through our key variables. We obtain,  $\forall r \in [0, r^*]$ :

$$(8) \quad \frac{\partial Q}{\partial r} = \zeta \left\{ \underbrace{(\bar{r} - r)}_{\geq 0} \underbrace{\left( \frac{\sigma_\ell + \mu_\ell}{q_\ell} + \frac{\sigma_h + \mu_h}{q_h} \right)}_A + \underbrace{[\sigma_\ell + \mu_\ell - (\sigma_h + \mu_h)]}_B \underbrace{\left( \frac{1}{-q'_\ell \rho_\ell} + \frac{1}{-q'_h \rho_h} \right)}_{> 0} \right\},$$

with  $\zeta \equiv \left( q'_h \rho_h q'_\ell \rho_\ell \right)^2 / \left[ (\partial_{hh} \pi_u + \partial_{\ell\ell} \pi_u) \left( q'_h \rho_h + q'_\ell \rho_\ell \right) \right] > 0$ , and  $\bar{r}(r) \equiv -q_h / \left( q'_h \rho_h \right) + q_\ell / \left( q'_\ell \rho_\ell \right) \geq r$ .<sup>21</sup>

Conditions on  $A$  and  $B$ , as defined in Equation 8, can help us to determine the sign of  $\partial Q / \partial r$ , given that both  $\zeta$  and the difference  $\bar{r} - r$  are positive. Hence, when both  $A$  and  $B$  are positive (respectively, negative), a marginal increase in  $r$  raises (resp., decreases) total output.

*Result 3.* Input price discrimination raises total output if  $\sigma_\ell + \mu_\ell \geq \sigma_h + \mu_h > 0, \forall r \in [0, r^*]$ . By contrast, input price discrimination reduces total output if  $\sigma_\ell + \mu_\ell \leq \sigma_h + \mu_h < 0, \forall r \in [0, r^*]$ . Also, when  $\sigma_h + \mu_h = 0$ , input price discrimination raises (respectively, reduces) total output if  $\sigma_\ell + \mu_\ell > 0$  (resp.,  $\sigma_\ell + \mu_\ell < 0$ ),  $\forall r \in [0, r^*]$ .

Result 3 relates to the analysis of Cheung and Wang [1994], who derive conditions on demand curvature for total output to increase or decrease with discrimination, in final-goods markets.<sup>22</sup> They show that, absent any vertical relation, output increases if  $\sigma_\ell \geq \sigma_h > 0$  and falls when  $\sigma_\ell \leq \sigma_h < 0$ . In our setting, we can consider that the input supplier’s problem is equivalent to that of a monopolist setting prices  $w_i, \forall i$ , in markets characterized by the derived demand functions  $q_i \left( p_i^* \left( w_i; c_i \right) \right)$  with curvatures  $\sigma_i + \mu_i$ .

We can now analyse how total output changes with input price discrimination in greater details. First, we focus on the demand forms listed by Bulow and Pfleiderer [1983] for which the curvature remains constant in each market,  $\forall r$ , and also across markets with symmetric demands (i.e.,  $\sigma_i = \sigma, \forall i$ ) and, hence,  $\mu_i = 0, \forall i$ . This implies that  $B = 0$ , and we obtain  $\partial Q / \partial r = \zeta (\bar{r} - r) \sigma \left( 1 / q_\ell + 1 / q_h \right)$ . Therefore,  $\partial Q / \partial r$  has the same sign as  $\sigma$  for any  $r \in [0, r^*]$ , when  $\sigma$  is constant. For instance, input price discrimination

<sup>21</sup> We observe that  $\bar{r}(r) \geq r$ , because  $\bar{r}(r) - r$  is continuous and decreasing in  $r$  towards zero, given Assumption 2,  $\forall r \in [0, r^*]$ . Indeed, we obtain:  $\partial [\bar{r}(r) - r] / \partial r = - \left( \partial w_h^{**} / \partial r \right) [2 - (\sigma_h + \mu_h)] + \left( \partial w_\ell^{**} / \partial r \right) [2 - (\sigma_\ell + \mu_\ell)] \leq 0, \forall r \in [0, r^*]$ , with  $\bar{r}(r^*) - r^* = 0$ .

<sup>22</sup> See also Aguirre, Cowan and Vickers [2010] and Cowan [2016].

under linear demands (for which  $\sigma = 0$ ) leaves total output unchanged.<sup>23</sup> When demand is strictly concave (i.e.,  $\sigma < 0$ ), input price discrimination reduces total output. By contrast, if demand is strictly convex (that is, if,  $\sigma > 0$ ) as for instance with constant-elasticity demand forms, total output increases with input price discrimination.<sup>24</sup>

A similar reasoning can be applied for demand forms for which the sum  $\sigma + \mu$  remains constant,  $\forall p$ , even though  $\mu \neq 0$ . In this case,  $\partial Q/\partial r$  and  $\sigma + \mu$  have the same sign,  $\forall r \in [0, r^*]$ , because  $B = 0$ . For instance, the inverse demand form given by  $P(q) = 1 - q + 1/q$ , which gives  $\sigma + \mu = 0$ , would leave total output unaffected by input price discrimination. By contrast, the inverse demand form given by  $P(q) = 1 - 2\sqrt{q} + 1/q$ , for which  $\sigma + \mu = 1/2$ , would see input price discrimination increasing total output. We summarize how Results 1–3 apply to all the examples mentioned above in Table B1, in Appendix B.

IV. WELFARE EFFECTS

We now analyse the effects of input price discrimination on total welfare. In each market  $i$ , for any given  $r$ , the sum of consumer surplus and firms’ profits equals:

$$(9) \quad [p_i^*(w_i^{**}(r)) - c_i] q_i(p_i^*(w_i^{**}(r))) + \int_{p_i^*(w_i^{**}(r))}^{+\infty} q_i(x) dx.$$

Three effects

The change in total welfare (across markets),  $W(r)$ , due to a marginal increase in  $r$  is given by:

$$(10) \quad \frac{\partial W}{\partial r} = [p_1^*(w_1^{**}(r)) - c_1] \frac{\partial w_1^{**}}{\partial r} q_1' \rho_1 + [p_2^*(w_2^{**}(r)) - c_2] \frac{\partial w_2^{**}}{\partial r} q_2' \rho_2.$$

We can disentangle the following effects, for any  $r \in [0, r^*]$ :

$$(11) \quad \frac{\partial W}{\partial r} = \underbrace{- [p_2^*(w_2^{**}(r)) - p_1^*(w_1^{**}(r))] q_1' \rho_1 \frac{\partial w_1^{**}}{\partial r}}_{\text{consumption allocation effect}} + \underbrace{(c_2 - c_1) q_1' \rho_1 \frac{\partial w_1^{**}}{\partial r}}_{\text{cost allocation effect}} + \underbrace{(p_2 - c_2) \frac{\partial Q}{\partial r}}_{\text{(value of) output effect}}.$$

<sup>23</sup> This is a well-known outcome. See, for instance, related results by Katz [1987], DeGraba [1990], Yoshida [2000], and Valletti [2003]. Pigou [1920] was the first to highlight a similar result in the case of third-degree price discrimination in final-goods markets.

<sup>24</sup> With exponential demand forms, for which  $\sigma = 1$  everywhere, we obtain  $w^0 = w_1^* = w_2^*$  and, thus,  $r^* = 0$ . Whether discrimination is allowed or not thus has no effect on prices or quantities.

Welfare changes in the upstream monopolist's scope to discriminate thus rely on three different effects. Recall that  $c_1 < c_2$ , which implies  $p_1 < p_2$ ,  $\forall r \in [0, r^*]$ , under symmetric demands (see Result 2). First, the 'consumption allocation effect' represents the welfare gains (respectively, losses) stemming from improved (resp., worsened) allocative efficiency across markets, when discrimination makes *retail prices* less (resp., more) discriminatory. This effect, which is also discussed by Miklós-Thal and Shaffer [2021], is positive when  $\sigma_i + \mu_i < 1$  (equivalent to  $\partial w_1^{**}/\partial r > 0$ ,  $\forall r \in [0, r^*]$ ) and negative when  $\sigma_i + \mu_i > 1$ ,  $\forall i$ . Second, the 'cost allocation effect' represents the welfare gains (respectively, losses) arising when discrimination improves (resp., deteriorates) productive efficiency by shifting output from the high-cost market (i.e., market 2) to the low-cost one. This effect, similar to the 'cost saving' effect highlighted by Chen and Schwartz [2015] in their welfare analysis of differential pricing, is positive when  $\sigma_i + \mu_i > 1$  and negative when  $\sigma_i + \mu_i < 1$ . Finally, the '(value of) output effect' relies on whether total demand is expanding or contracting due to discrimination, as detailed in Equation 8, and is well-known from the literature on discrimination in final-goods markets.<sup>25</sup>

The sum of the consumption allocation and cost allocation effects, equal to  $\{p_1^*(w_1^{**}) - c_1 - [p_2^*(w_2^{**}) - c_2]\} q_1' \rho_1 (\partial w_1^{**}/\partial r)$ , can be either negative or positive. In particular, it is negative when both the final-goods and derived demands are log-concave (i.e.,  $\sigma < 1$  and  $\sigma + \mu < 1$ ) or when they both are log-convex (i.e.,  $\sigma > 1$  and  $\sigma + \mu > 1$ ). In the former case, log-concavity of the final and derived demands leads to  $p_1^*(w_1) - w_1 - c_1 > p_2^*(w_2) - w_2 - c_2$ , and  $w_1 > w_2$  and  $\partial w_1/\partial r > 0$ , respectively,  $\forall r \in [0, r^*]$ . In the latter case, instead, log-convexity of the final-goods demand implies  $p_1^*(w_1) - w_1 - c_1 < p_2^*(w_2) - w_2 - c_2$ , and that of the derived demand  $w_1 < w_2$  and  $\partial w_1/\partial r < 0$ ,  $\forall r \in [0, r^*]$ . We thus obtain the following result.

*Proposition 1.* A strictly positive output effect is a necessary condition for input price discrimination to improve welfare if either (i)  $\sigma_i < 1$  and  $\sigma_i + \mu_i < 1$ , or (ii)  $\sigma_i > 1$  and  $\sigma_i + \mu_i > 1$ ,  $\forall i$ ,  $\forall r \in [0, r^*]$ .

This result has direct implications for the demand forms with a constant curvature, listed by Bulow and Pfleiderer [1983]. Indeed, when  $\mu = 0$ , we see from Proposition 1 that a positive output effect becomes a *necessary condition* for input price discrimination to increase total welfare, because the sum of the consumption allocation and cost allocation effects is always negative.<sup>26</sup> Following Result 3, this implies that input price discrimination always

<sup>25</sup> See, e.g., Schmalensee [1981], Varian [1985], and Schwartz [1990].

<sup>26</sup> In the literature on third-degree price discrimination by a monopolist serving final consumers, Schmalensee [1981], Varian [1985], and Schwartz [1990] have demonstrated that a positive



(weakly) reduces total welfare for concave demand forms with constant curvature. Below, however, we depart from demand forms with constant curvature and we also discuss cases where input price discrimination can increase welfare, without any positive output effect.

*Welfare Analysis*

We now follow Aguirre, Cowan and Vickers [2010] to analyse changes in total welfare due to input price discrimination. Following the previous section, we rely on our notation of ‘weak’ and ‘strong’ input markets. From Equations 7 and 10, we obtain:

$$(12) \quad \frac{\partial W}{\partial r} = \underbrace{\frac{-\partial_{hh}\pi_u}{\partial_{hh}\pi_u + \partial_{\ell\ell}\pi_u} \frac{\partial_{\ell\ell}\pi_u}{\partial_{\ell\ell}\pi_u}}_{>0} [z_\ell(w_\ell(r)) - z_h(w_h(r))],$$

where  $z_i(w_i) \equiv [p_i^*(w_i; c_i) - c_i] q'_i(p_i^*(w_i; c_i)) \rho_i(p_i^*(w_i; c_i)) / \partial_{ii}\pi_u > 0, \forall i$ .

We can now rely on  $z_i(w_i)$ , the ratio of the marginal effect of an *input* price increase on social welfare in market  $i$  to the second derivative of the *upstream* firm’s profit function in this market, in order to state the following condition.<sup>27</sup>

*Condition 1.* (Upstream increasing ratio condition (UIRC)).  $z_i(w_i)$  is increasing in  $w_i$  in each market  $i$ .

Our definition of the ‘upstream increasing ratio condition’ (UIRC) mirrors that of the ‘increasing ratio condition,’ provided by Aguirre, Cowan and Vickers [2010] in their study of price discrimination in final-goods markets. A notable difference with their approach, however, is that our variable  $z_i(w_i)$  is defined differently and takes the input price  $w_i$  as an argument.<sup>28</sup> As our focus is on input price discrimination, and because the timing of the game is such that the upstream firm sets wholesale prices before downstream firms set their prices,  $z_i(w_i)$  depends on the first, second and third derivatives of demand.

output effect is necessary, in order to obtain a positive total welfare impact of price discrimination. Alternative settings considering, for instance, oligopolistic competition (Galera and Zaratiegui [2006]) or income effects (Galera, Garcia-del Barrio and Mendi [2019]) may overturn this result. In the context of input price discrimination, when downstream firms possess different levels of efficiency in the number of units of input they require per unit of output, a positive output effect may be sufficient for welfare to fall with discrimination (Yoshida [2000]).

<sup>27</sup> In their study of discrimination in final-goods markets, Aguirre, Cowan and Vickers [2010] relied on the ratio of the marginal effect of a (downstream) price increase on social welfare to the second derivative of the monopolist’s profit function in one (downstream) market:  $(p_i - c_i)q'_i/\pi''_i$ .

<sup>28</sup> The ‘increasing ratio condition’ stated by Aguirre, Cowan and Vickers [2010] corresponds to the ratio  $(p_i - c_i)q'_i/\pi''_i$  being increasing in  $p_i, \forall i$ . Our UIRC, instead, is defined as the ratio  $[p^*(w_i; c_i) - c_i] q'_i \rho_i / \partial_{ii}\pi_u$  being increasing in  $w_i, \forall i$ , thus considering the downstream price reactions to any change in input prices.

The UIRC is always satisfied for linear and generalised ‘linear’ demand forms with  $\delta < 1$ , for instance, as well as for some demand forms with  $\mu \neq 0$ , such as the one given by  $P(q) = 1 - q + 1/q$ . For constant-elasticity demand forms, a sufficient condition for the UIRC to hold over the relevant price range is that the cost disadvantage of retailer 2 is not too large:  $2c_1 > c_2$ .

The UIRC implies that  $W(\cdot)$  is strictly quasi-concave in  $r$ .<sup>29</sup> Therefore, when the UIRC is satisfied, welfare can either (i) always decrease with  $r$ , (ii) always increase in  $r$ , or (iii) first increase and then decrease. We thus obtain the following result on the welfare effects of input price discrimination.

*Proposition 2.* Given the UIRC, input price discrimination increases total welfare if  $[p_\ell^*(w_\ell^*) - c_\ell] / [2 - (\sigma_\ell^* + \mu_\ell^*)] > [p_h^*(w_h^*) - c_h] / [2 - (\sigma_h^* + \mu_h^*)]$ .

Proposition 2 allows us to identify demand forms for which welfare increases with input price discrimination.<sup>30</sup> Let us consider, for example, the (inverse) demand form given by  $P(q) = 1 - q + 1/q$ , for which  $\sigma + \mu = 0$  everywhere in both markets, with  $0 < c_1 < c_2 < 1$ . This implies that input price discrimination leaves total output unchanged, from Result 3, and that the consumer allocation effect is positive, whereas the cost allocation effect is negative. Given that  $\sigma_\ell + \mu_\ell = \sigma_h + \mu_h = 0$ , we only need to compare the numerators of the ratios used in the condition stated in Proposition 2. We know that market 2 corresponds to the weak input market (as  $\sigma + \mu = 0 < 1$ ), and, for any  $c \in (0, 1)$ , we find that  $d[p^*(w^*) - c]/dc > 0$  and, hence, that  $p_2^*(w_2^*) - c_2 > p_1^*(w_1^*) - c_1$ . Therefore, Proposition 2 applies in this case, and we find that input price discrimination raises total welfare. This example thus illustrates that input price discrimination can improve welfare without increasing the total output. This result is driven by the positive consumption allocation effect, presented in Equation 11, which dominates the (negative) cost allocation effect.

Proposition 2 above mirrors Proposition 2 of Aguirre, Cowan and Vickers [2010], albeit with three important differences, which reflect the facts that our result applies to *input* price discrimination, and not to discrimination in final-goods markets. First, the UIRC is defined by taking into consideration how input prices affect retail prices. Second, the curvature of the (inverse) *derived* demand appears in the denominators of the ratios used in the condition stated in Proposition 2. Finally, our approach allows for cost asymmetries:  $c_h \neq c_\ell$ .

<sup>29</sup> See Lemma 2, in Appendix A.

<sup>30</sup> Note that the sufficient condition for welfare to increase with discrimination stated in Proposition 2 never applies to symmetric demand forms with a constant pass-through rate. Indeed, for demand forms with  $\mu = 0$ ,  $\forall p$ , we have  $d[p^*(w^*) - c]/dc = \rho^2 - 1$  and, hence,  $p_2^*(w_2^*) - c_2 > p_1^*(w_1^*) - c_1 \Leftrightarrow \rho > 1 \Leftrightarrow \sigma > 1$ , implying that  $p_\ell^*(w_\ell^*) - c_\ell < p_h^*(w_h^*) - c_h$ .

In addition, we can build on the analysis of Chen and Schwartz [2015] to determine some conditions under which welfare must be reduced when input price discrimination is allowed. We obtain the following result.

*Proposition 3.* Given the UIRC, if  $\sigma_i + \mu_i > 1$  and  $\sigma'_i + \mu'_i > -(2 - \sigma_i - \mu_i) \left[ \frac{(3 - \sigma_i)^2 - (2 - \sigma_i - \mu_i) - \mu_i(2 - \sigma_i)}{(3 - \sigma_i)(q_i/q'_i)} \right]$ ,  $\forall i, \forall p \in [p_1^*(w_1^*), p_2^*(w_2^*)]$ , then  $W(r^*) < W(0)$ , that is, discrimination reduces welfare compared to homogeneous input prices.

In contrast to Proposition 2, which provided sufficient conditions for total welfare to increase with input price discrimination, Proposition 3 presents sufficient conditions for total welfare to decrease with discrimination, compared to the case with uniform input prices. When  $\sigma + \mu > 1$ , the consumption allocation effect is negative whereas the cost allocation effect is positive. The value of output effect can be either positive or negative. The second condition stated in Proposition 3 ensures that the negative effects prevail.<sup>31</sup>

Chen and Schwartz [2015] introduced a sufficient condition, related to that stated in Proposition 3, for total welfare to increase with differential pricing. Indeed, with variables defined as functions of prices, as in our setting, their condition is equivalent to:  $\sigma' \leq -(3 - \sigma)(2 - \sigma)/(q/q')$ . The main difference between their condition and ours, stated in Proposition 3, is that theirs is sufficient for welfare to be convex in the upstream firm’s own marginal cost, whereas ours ensures that welfare is concave in the downstream firms’ own marginal costs.

### Consumer Surplus Analysis

The results related to the effects of discrimination on output mentioned above can also guide our understanding of the effects of input price discrimination on consumers surplus and industry profits. In each market  $i$ , for any given  $r$ , consumer surplus is equal to  $\int_{p_i^*(w_i^*(r))}^{+\infty} q_i(x)dx$ . This implies that the change in consumer surplus (considering both markets together),  $CS(r)$ , due to a marginal increase in  $r$  is given by:

$$(13) \quad \frac{\partial CS}{\partial r} = \left( \frac{q_h}{\partial q_h / \partial p_h} - \frac{q_\ell}{\partial q_\ell / \partial p_\ell} \right) \frac{\partial q_\ell}{\partial r} - \frac{q_h}{\partial q_h / \partial p_h} \frac{\partial Q}{\partial r}.$$

We obtain the following result.

<sup>31</sup> Note that Proposition 3 does not apply to demand forms for which  $\sigma_i + \mu_i$  is constant in prices. Indeed, Assumptions 1 and 2, together with the condition  $\sigma_i + \mu_i > 1$ , ensure that the right-hand side of the last condition stated in this proposition is strictly positive. Hence, that last condition is never satisfied when  $\sigma'_i + \mu'_i = 0$ .

*Proposition 4.* Input price discrimination increases consumer surplus if (i)  $\sigma_i > 1$  and  $1 > \sigma_\ell + \mu_\ell \geq \sigma_h + \mu_h > 0$ , or if (ii)  $\sigma_i < 1$  and  $\sigma_\ell + \mu_\ell \geq \sigma_h + \mu_h > 1$ ,  $\forall i, \forall r \in [0, r^*]$ . By contrast, input price discrimination reduces consumer surplus if  $\sigma_i < 1$  and  $\sigma_\ell + \mu_\ell \leq \sigma_h + \mu_h < 0$ ,  $\forall i, \forall r \in [0, r^*]$ .

Proposition 4 thus provides sufficient conditions for consumer surplus to either increase or decrease with input price discrimination. Note that it relies on the conditions stated in Result 3 about the effects of discrimination on output, in order to determine the sign of the last term on the right-hand side of Equation 13. However, consumer surplus could also increase with discrimination even when total output falls with discrimination, for example if the necessary condition  $q_h / (\partial q_h / \partial p_h) - q_\ell / (\partial q_\ell / \partial p_\ell) > 0$  is satisfied. In Table B1, in Appendix B, we show that Proposition 4 applies to some of the examples mentioned above.

Finally, we can briefly discuss the effects of input price discrimination of firms' profits. By revealed preference, the upstream firm's profit must increase with input price discrimination. Also, we can consider industry profits, which are equal to  $\Pi(r) \equiv [p_h^*(w_h^{**}(r)) - c_h] q_h + [p_\ell^*(w_\ell^{**}(r)) - c_\ell] q_\ell$ . We find that  $\partial \Pi / \partial r = (w_h^{**} - w_\ell^{**}) q'_h \rho_h (\partial w_h^{**} / \partial r) + w_\ell^{**} (\partial Q / \partial r)$  and, hence, that  $\partial Q / \partial r > 0$  is a necessary condition for industry profits to increase with input price discrimination.

#### V. ASYMMETRIC DEMANDS: OUTPUT AND WELFARE EFFECTS

In this extension, we allow for demands in each market to be asymmetric (i.e., non-proportional), assuming that all markets are served under both input price discrimination and when the upstream firm is constrained to set a uniform price.

First, let us explain why some of our results *do not* hold for demands which are not proportional. Because our Result 1 is derived by studying the sign of  $1 - [\sigma(p) + \mu(p)]$  for a given demand, it can be used across markets only when demands are proportional, as proportionality keeps  $\sigma(p)$  and  $\mu(p)$  unchanged for a given  $p$ .<sup>32</sup> Hence, every result that builds on Result 1 and relies on the sign of  $1 - [\sigma(p) + \mu(p)]$  to identify the strong and weak input markets applies to proportional demands only. However, as explained below, Result 3 and Proposition 2 also apply to asymmetric demands across markets, as long as the 'weak' and 'strong' input markets have been respectively determined.

#### *Output and Welfare Effects*

In relation to output effects, Equation 8 also applies to asymmetric demands, and Result 3 also carries on to such case. When demands are asymmetric,

<sup>32</sup> The same holds for Lemma 1.

even the forms identified by Bulow and Pfleiderer [1983] do not necessarily lead to  $B = 0$ , in sharp contrast to the cases discussed in Subsection III(ii) above. Indeed, even if curvature remains constant within a market, it needs not remain constant across markets when demands are not proportional.

Similarly, parts of the analysis related to the welfare impact of input price discrimination derived in Section IV also apply directly to asymmetric demands, once the ‘weak’ and ‘strong’ input markets have been respectively determined. The various welfare effects we presented in Equation 11 also exist when demands are asymmetric.<sup>33</sup> In particular, Proposition 2 also applies to asymmetric demands.

*Example.* In order to illustrate our findings and the robustness of our results to asymmetric demands, we rely on the following example. Demands in markets 1 and 2 are given by  $q_1(p_1) = 1 - p_1$  for  $p_1 \in [0, 1]$  and  $q_2(p_2) = 4 - 4p_2 + p_2^2$  for  $p_2 \in [0, 2]$ , respectively, and equal to zero elsewhere. We assume that  $c_1 = 0$  and  $c_2 \in (0.5, 1)$ . Given  $c_1 = 0$ , the condition  $c_2 < 1$  ensures that all profit functions are single-peaked, whereas  $c_2 > 0.5$  implies that market 2 is the weak input market.<sup>34</sup>

Both demands belong to the family of generalised ‘linear’ forms (Bulow and Pfleiderer [1983]), and demand curvatures are constant in prices over the relevant ranges, with  $\sigma_1 = 0$  and  $\sigma_2 = 1/2$  (and, thus,  $\mu_1 = \mu_2 = 0$ ). Input price discrimination decreases the quantity sold in the strong input market (i.e., market 1) and increases that sold in the weak input market (i.e., market 2). Result 3 applies, because  $\sigma_\ell = 1/2 > \sigma_h = 0$ . It indicates that, on balance, discrimination raises total output.

Total welfare effects depend on the value of output effect, which is thus positive, a positive consumption allocation effect, and a negative cost allocation effect. It can be shown that the UIRC is satisfied for both demands and, also, that  $p_1^*(w_1^*) = 3/4$  and  $p_2^*(w_2^*) = (2/9)(5 + 2c_2)$ . Hence, we have  $p_\ell^*(w_\ell^*) - c_\ell = (5/9)(2 - c_2)$  and  $p_h^*(w_h^*) - c_h = 3/4$ . We can rely on Proposition 2 if  $(p_\ell^*(w_\ell^*) - c_\ell) / (2 - \sigma_\ell) > (p_h^*(w_h^*) - c_h) / (2 - \sigma_h)$ , which is equivalent to  $c_2 < 79/80$ . Therefore,  $c_2 < 79/80$  is a sufficient condition for input price discrimination to increase total welfare.

<sup>33</sup> Note, however, that the interpretation of the various economic effects, in the paragraphs following Equation 11, reflects the facts that  $c_1 < c_2$  and  $p_1 < p_2$ ,  $\forall r \in [0, r^*]$  in our baseline setting. With asymmetric demands, instead, the least-efficient retailer does not necessarily set the highest retail price and, hence, it could be that  $c_1 < c_2$  and  $p_1 > p_2$ .

<sup>34</sup> We obtain  $w_1^* = 1/2 > w_2^* = (2 - c_2) / 3$ ,  $\forall c_2 \in (0.5, 1)$ . Moreover, with such demands and costs, the upstream monopolist always serves both markets, even when facing a ban on input price discrimination.

## VI. CONCLUSION

In this paper, we studied the effects of input price discrimination by an upstream monopolist selling to firms facing asymmetric marginal costs, building on the seminal work of Katz [1987] and DeGraba [1990]. We demonstrated that the upstream monopolist has an incentive to reduce the difference in (perceived) marginal cost between retailers when the sum of curvature of inverse demand,  $\sigma$ , and the quantity-elasticity of the pass-through rate,  $\mu$ , lies below unity. Instead, it would prefer to discriminate in favour of the most-efficient retailer, when  $\sigma + \mu > 1$ .

We also showed that this sum,  $\sigma + \mu$ , plays, in the analysis of output and welfare effects of input price discrimination, a role similar to that played by the curvature alone in the analysis of such effects when a monopolist seller discriminates in final-goods markets, as studied by Aguirre, Cowan and Vickers [2010]. We decomposed the welfare effects of input price discrimination and showed that these depend on whether input price discrimination (i) increases total output or not, (ii) improves allocative efficiency by bringing retail prices closer to one another or not, as in Miklós-Thal and Shaffer [2021], and (iii) improves productive efficiency by shifting output from the high-cost market to the low-cost one or not, as in Chen and Schwartz [2015]. We showed how our findings apply to various demand forms with a constant curvature (and, thus, with  $\mu = 0$ ) commonly used in the literature. We also provided particular examples with demand forms with variable demand curvature that possess special features due to their values of  $\sigma + \mu$ , in order to illustrate some of our results. Importantly, our key result about the welfare effects of input price discrimination carries over the case of asymmetric demands.

Our analysis could be further extended along various dimensions. An interesting avenue for future work would be to depart from the case of independent monopolists downstream and to incorporate competition between retailers. For instance, considering within-market competition, in the spirit of Miklós-Thal and Shaffer [2021], could help alleviate the potential tractability issues related to modelling competition across markets under cost asymmetries, in our general setting.

## APPENDIX A

## PROOFS

*Proof of Lemma 1.* We find that  $dp_i^*(w_i^*)/dc_i = \rho_i^* / [2 - (\sigma_i^* + \mu_i^*)]$  is strictly positive, due to Assumptions 1 and 2. This implies that  $\partial(w_i^* + c_i)/\partial c_i > 0$  in equilibrium, as  $\partial w_i^*/\partial c_i = [dp_i^*(w_i^*)/dc_i] / \rho_i^* - 1$ . Hence,  $w_i^* + c_i < w_k^* + c_k$  if and only if  $c_i < c_k$ ,  $\forall i, k \neq i$ , when demands are proportional. ■

*Proof of Result 1.* Immediate from Equation 5, given that  $2 > \sigma_i + \mu_i$  from Assumption 2, when demands are proportional. ■

*Proof of Result 2.* Consider the case where  $w_1 \geq w_2$ . The upstream firm's profit can be written as:  $\pi_u(w_1, w_2) = \pi_u(w_1, w_2)|_{w_2=w_1-r} = \pi_u(w_1, w_2)|_{w_1=w_2+r}$ . The first-order condition of the upstream firm's maximisation problem is:  $\partial_1\pi_u + \partial_2\pi_u|_{w_2=w_1-r} = \partial_1\pi_u + \partial_2\pi_u|_{w_1=w_2+r} = 0$ , at the equilibrium prices  $w_1^{**}(r)$  and  $w_2^{**}(r)$ , given  $r$ . Differentiating this first-order condition by  $r$  and reorganising, it follows that:  $\partial w_1^{**}(r)/\partial r = \partial_{22}\pi_u / (\partial_{11}\pi_u + \partial_{22}\pi_u) > 0$  and  $\partial w_2^{**}(r)/\partial r = -\partial_{11}\pi_u / (\partial_{11}\pi_u + \partial_{22}\pi_u) < 0$ , given Assumption 2. This means that, when  $w_1 \geq w_2$ , as  $r$  increases the upstream firm raises  $w_1$  and decreases  $w_2$ ,  $\forall r \in [0, r^*]$ . Moreover, at  $r = r^*$ , we have  $\sigma_i + \mu_i < 1$ ,  $\forall i$ , implying  $w_1^* > w_2^*$  for proportional demands. Therefore,  $w_1^* > w^0 > w_2^*$ . Retail prices rankings follow from Lemma 1. The opposite case where  $w_1 \leq w_2$  follows the same logic. ■

*Proof of Result 3.* From Equation 8, we see that (i)  $\sigma_\ell + \mu_\ell \geq \sigma_h + \mu_h \Leftrightarrow B \geq 0$  and (ii)  $\sigma_i + \mu_i > 0$ ,  $\forall i$ , implies  $A > 0$ . Hence,  $\partial Q/\partial r > 0$  when  $\sigma_\ell + \mu_\ell \geq \sigma_h + \mu_h > 0$ . Moreover, we also have (i)  $\sigma_\ell + \mu_\ell \leq \sigma_h + \mu_h \Leftrightarrow B \leq 0$  and (ii)  $\sigma_i + \mu_i < 0$ ,  $\forall i$ , implies  $A < 0$ . Therefore,  $\partial Q/\partial r < 0$  when  $\sigma_\ell + \mu_\ell \leq \sigma_h + \mu_h < 0$ . Finally, if  $\sigma_h + \mu_h = 0$ , both  $A$  and  $B$  have the sign of  $\sigma_\ell + \mu_\ell$ , which thus determines the sign of  $\partial Q/\partial r$ . ■

*Proof of Proposition 1.* When  $\sigma_i < 1$ , over the relevant interval,  $\forall i$ , we have  $\rho_i < 1$  and, hence,  $p_1^*(w_1; c_1) - w_1 - c_1 > p_2^*(w_2; c_2) - w_2 - c_2$  for proportional demands, as  $c_1 < c_2$ . Also, when  $\sigma_i + \mu_i < 1$ ,  $\forall i$ , we have  $\partial w_1^{**}/\partial r > 0$ ,  $\partial w_2^{**}/\partial r < 0$ , and  $w_1^{**} > w_2^{**}$ ,  $\forall r \in (0, r^*]$ . Hence,  $\{p_1^*(w_1^{**}) - c_1 - [p_2^*(w_2^{**}) - c_2]\} q_1^* \rho_1 (\partial w_1^{**}/\partial r) < (w_1^{**} - w_2^{**}) q_1^* \rho_1 (\partial w_1^{**}/\partial r) < 0$ ,  $\forall r \in (0, r^*]$ . Then, from Equation 11, given that the sum of both allocation effects is negative, a strictly positive output effect is a necessary condition to obtain  $\partial W/\partial r > 0$ . The case where  $\sigma_i > 1$  and  $\sigma_i + \mu_i > 1$ ,  $\forall i$ , follows the same logic. ■

*Proof of Proposition 2.* First, following a similar result in Aguirre, Cowan and Vickers [2010], we demonstrate in Lemma 2 below that the UIRC implies that  $W(\cdot)$  is strictly quasi-concave in  $r$ . ■

*Lemma 2.* Given the UIRC, if there exists an  $\tilde{r}$  such that  $W'(\tilde{r}) = 0$ , then  $W''(\tilde{r}) < 0$ .

*Proof of Lemma 2.* From Equation 12, we have:

$$(14) \quad \frac{\partial^2 W}{\partial r^2} = \overbrace{\frac{-\partial_{hh}\pi_u \partial_{\ell\ell}\pi_u}{\partial_{hh}\pi_u + \partial_{\ell\ell}\pi_u}}^{>0} \left[ z'_\ell(w_\ell^{**}(r)) \frac{\partial w_\ell^{**}}{\partial r} - z'_h(w_h^{**}(r)) \frac{\partial w_h^{**}}{\partial r} \right] + \underbrace{\left[ z_\ell(w_\ell^{**}(r)) - z_h(w_h^{**}(r)) \right]}_{=0 \text{ at } \tilde{r}} \frac{\partial}{\partial r} \left( \frac{-\partial_{hh}\pi_u \partial_{\ell\ell}\pi_u}{\partial_{hh}\pi_u + \partial_{\ell\ell}\pi_u} \right),$$

where  $\partial w_\ell^{**}/\partial r < 0$ ,  $\partial w_h^{**}/\partial r > 0$ , implying  $w_h^{**}(r) > w_\ell^{**}(r)$ ,  $\forall r > 0$ . Hence, if  $z'_i(w_i^{**}) > 0$  and  $W'(\tilde{r}) = 0$  (i.e.,  $z_\ell(w_\ell^{**}(\tilde{r})) = z_h(w_h^{**}(\tilde{r}))$ ), then  $W''(\tilde{r}) < 0$ . Therefore, when the UIRC is satisfied, welfare can either (i) always decrease with  $r$ , (ii) always increase in  $r$ , or (iii) first increase when  $r \leq \tilde{r}$  and then decrease. ■



Then, in order to complete the proof of Proposition 2, consider that, at  $r^*$ ,  $W'(r)$  has the sign of:

$$\begin{aligned} z_\ell(w_\ell^{**}(r^*)) - z_h(w_h^{**}(r^*)) &= z_\ell(w_\ell^*) - z_h(w_h^*) \\ &= \frac{p_\ell^*(w_\ell^*) - c_\ell}{2 - (\sigma_\ell^* + \mu_\ell^*)} - \frac{p_h^*(w_h^*) - c_h}{2 - (\sigma_h^* + \mu_h^*)}, \end{aligned}$$

Given the UIRC,  $W(\cdot)$  is strictly quasi-concave in  $r$ , from Lemma 2. Therefore, if  $W'(r^*) > 0$ , then  $W'(r) > 0$  for all  $r \in [0, r^*]$ . It follows that  $W(r^*) > W(0)$  for the condition laid down in the proposition, as it ensures that  $z_\ell(w_\ell^{**}(r^*)) - z_h(w_h^{**}(r^*)) > 0$ . ■

*Proof of Proposition 3.* Define  $\omega_i(c_i)$  as the total welfare in market  $i$  at the equilibrium, absent any constraint on input prices. For proportional demands, we have  $W(r^*) = \omega_1(c_1) + \omega_2(c_2) = \lambda\omega(c_1) + (1 - \lambda)\omega(c_2)$ , with  $\omega(c) \equiv \omega_1(c)/\lambda = \omega_2(c)/(1 - \lambda), \forall c$ . Define  $r_0$  as the hypothetical difference between input prices which would enforce  $w_1 + c_1 = w_2 + c_2$  and, hence,  $p_1 = p_2$  at the retail level. As  $\sigma_i + \mu_i > 1$ , we have  $r_0 \equiv c_1 - c_2 < 0$ . Redefining  $r$  as  $w_2 - w_1$  and allowing it to take negative values, we can follow Lemma 2 and show that the UIRC implies the quasi-concavity of  $W(\cdot)$  in  $r$  and, hence, that  $W(r_0) > W(r^*) \Rightarrow W(0) > W(r^*)$ . Moreover, when the function  $\omega_i(\cdot)$  is concave,  $\forall i$ ,  $\omega(\cdot)$  is also concave, and we obtain:  $\omega_1(c_1) + \omega_2(c_2) = \lambda\omega(c_1) + (1 - \lambda)\omega(c_2) < \omega(\bar{c})$ , with  $\bar{c} \equiv \lambda c_1 + (1 - \lambda)c_2$ . Also,  $W(r_0) = \omega(\bar{c})$ , because  $w_i^*(\bar{c}) + \bar{c} = w_1^*(r_0) + c_1 = w_2^*(r_0) + c_2$  and, hence,  $p_i^*(w_i^*(r_0); c_i) = p_i^*(w_i^*(\bar{c}); \bar{c}), \forall i$ . Therefore, if  $\omega_i(\cdot)$  is concave,  $\forall i$ , we have  $W(r_0) > W(r^*)$ , and the UIRC implies  $W(0) > W(r^*)$ . Finally:

$$\begin{aligned} \frac{\partial^2 \omega_i}{\partial c_i^2} &= -2q_i' \frac{dp_i^*(w_i^*)}{dc_i} + \{q_i' + [p_i^*(w_i^*) - c_i] q_i''\} \left[ \frac{dp_i^*(w_i^*)}{dc_i} \right]^2 \\ &+ [p_i^*(w_i^*) - c_i] q_i' \frac{d^2 p_i^*(w_i^*)}{dc_i^2}. \end{aligned}$$

It follows that  $\omega_i(\cdot)$  is concave in  $c_i$  (i.e.,  $\partial^2 \omega_i / \partial c_i^2 < 0$ ) if and only if  $\sigma_i' + \mu_i' > - (2 - \sigma_i - \mu_i) \left[ (3 - \sigma_i)^2 - (2 - \sigma_i - \mu_i) - \mu_i(2 - \sigma_i) \right] / [(3 - \sigma_i)(q_i/q_i')]$ . ■

*Proof of Proposition 4.* First, recall that  $\partial q_\ell / \partial r > 0, \forall r \in [0, r^*]$ . Then, observe that when (i) demands are proportional, (ii)  $\sigma_i + \mu_i < 1$ , and (iii)  $\sigma_i > 1, \forall i$ , we have:  $q_h / (\partial q_h / \partial p_h) - q_\ell / (\partial q_\ell / \partial p_\ell) = q_1 / (\partial q_1 / \partial p_1) - q_2 / (\partial q_2 / \partial p_2) > 0$ , from Result 1. Similarly, when  $\sigma_i + \mu_i > 1$  and  $\sigma_i < 1, \forall i$ , we find:  $q_h / (\partial q_h / \partial p_h) - q_\ell / (\partial q_\ell / \partial p_\ell) = q_2 / (\partial q_2 / \partial p_2) - q_1 / (\partial q_1 / \partial p_1) > 0$ . Besides, when  $\sigma_\ell + \mu_\ell \geq \sigma_h + \mu_h > 0$ , we have  $\partial Q / \partial r > 0$ ; see Result 3. Therefore,  $\partial CS / \partial r > 0$  if  $\sigma_i > 1$  and  $1 > \sigma_\ell + \mu_\ell \geq \sigma_h + \mu_h > 0$ , or if  $\sigma_i < 1$  and  $\sigma_\ell + \mu_\ell \geq \sigma_h + \mu_h > 1, \forall i, \forall r \in [0, r^*]$ . Conversely, when  $\sigma_i + \mu_i < 1$  and  $\sigma_i < 1, \forall i$ , we have:  $q_h / (\partial q_h / \partial p_h) - q_\ell / (\partial q_\ell / \partial p_\ell) = q_1 / (\partial q_1 / \partial p_1) - q_2 / (\partial q_2 / \partial p_2) < 0$ . Also,  $dQ/dr < 0$  if  $\sigma_\ell + \mu_\ell \leq \sigma_h + \mu_h < 0$ . Hence,  $\partial CS / \partial r < 0$  when  $\sigma_i < 1$  and  $\sigma_\ell + \mu_\ell \leq \sigma_h + \mu_h < 0, \forall i, \forall r \in [0, r^*]$ . ■

APPENDIX B  
EXAMPLES

TABLE B1  
EXAMPLES OF INVERSE DEMANDS WITH CONSTANT  $\sigma(p) + \mu(p)$ , AND CORRESPONDING EFFECTS OF INPUT PRICE DISCRIMINATION FOR PROPORTIONAL DEMANDS, WITH  $c_1 < c_2$ .

Inverse demand	$\sigma(p)$	$\mu(p)$	$\sigma(p) + \mu(p)$	Price effect (Results 1 & 2)	Output effect (Result 3)	Total welfare effect	Cons. surplus effect (Proposition 4)
$P(q) = \alpha - \beta q^\epsilon$ with $\alpha, \beta, \delta > 0$ and $q \in [0, (\alpha/\beta)^{1/\epsilon}]$	$1 - \delta$	0	$1 - \delta$	$w_1^* > w^0 > w_2^*$ $p_1^0 < p_1^* < p_2^* < p_2^0$	Positive if $\delta < 1$ Null if $\delta = 1$ Negative if $\delta > 1$	Negative if $\delta \geq 1$ (Proposition 1)	Negative if $\delta > 1$
$P(q) = \beta q^{-1/\epsilon}$ with $\beta > 0, \epsilon > 1$ and $q \in (0, +\infty)$	$1 + 1/\epsilon$	0	$1 + 1/\epsilon$	$w_1^* < w^0 < w_2^*$ $p_1^* < p_1^0 < p_2^0 < p_2^*$	Positive	<i>n.a.</i>	<i>n.a.</i>
$P(q) = \alpha - \beta \ln(q)$ with $\alpha, \beta > 0$ and $q \in (0, e^{\alpha/\beta}]$	1	0	1	No effect	-	-	-
$P(q) = 1 - q + 1/q$ with $q \in (0, (1 + \sqrt{5})/2)$	$\frac{2}{1+(q\phi)^2}$	$\frac{-2}{1+(q\phi)^2}$	0	$w_1^* > w^0 > w_2^*$ $p_1^0 < p_1^* < p_2^* < p_2^0$	Null	Positive (Proposition 2)	<i>n.a.</i>
$P(q) = 1 - 2\sqrt{q} + 1/q$ with $q \in (0, 1]$	$\frac{4+4(q\phi)^{3/2}}{2+2(q\phi)^{3/2}}$	$\frac{-3}{2+2(q\phi)^{3/2}}$	1/2	$w_1^* > w^0 > w_2^*$ $p_1^0 < p_1^* < p_2^* < p_2^0$	Positive	<i>n.a.</i>	Positive

Notes: *n.a.* indicates that our results are not applicable.

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