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## RESEARCH ARTICLE

# Outsourcing via reverse auction with a built-in menu of change orders

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## Abstract

Many firms invent and design products while outsourcing their production to independent contractors. We consider a dominant strategy mechanism that selects a contractor using a reverse auction, combined with a menu of permitted change orders from which the contractor can choose after updated cost information has become available. That mechanism maximizes the gain from trade, allows the firm to extract the second highest surplus, and induces the contractor to make efficient adjustments to output after updated cost information has emerged.

## KEYWORDS

auctions, change orders, mechanism design, outsourcing, procurement

## JEL CLASSIFICATION

D02, D44, D47, D82, L10, L24

## 1 | INTRODUCTION

In accordance with Peter Drucker's dictum: "Do what you do the best and outsource the rest," many firms invent and design products while outsourcing their production to independent contractors. A clear-cut example is the U.S. technology giant Apple Inc. that designs, develops, and sells consumer electronics, like the Mac, the iPhone, and the iPad, but completely outsources their production to foreign contractors like Foxconn in Shenzhen, China.

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Outsourcing production allows Apple to focus on innovation and design and to be uncompromising about quality. However, outsourcing puts a heavy burden on procurement methods and may hazard infringement of intellectual property.

A perennial concern in procurement is cost-overruns that trigger change orders. Change orders often pose a hold-up problem that tends to erode the procurer's profit (see, e.g., Bajari et al., 2014). This suggests that clear-cut rules to control change orders should be an integral part of every procurement mechanism.

In the present paper, we consider a dominant strategy mechanism that selects a contractor using a reverse auction, combined with a menu of permitted change orders from which the contractor can choose after updated cost information has become available.

The proposed mechanism selects the contractor that maximizes the gain from trade, allows the firm to extract the second highest expected surplus, and induces the contractor to make efficient adjustments of output after updated cost information has become available, without affecting the firm's profit.<sup>1</sup>

The plan of the paper is as follows: The model and the proposed procurement mechanism are stated in Section 2. Section 3 solves the change order subgame that is played after updated cost information is available and then proceeds to solve the bidding game. Section 4 relates the proposed mechanism to the literature on scoring auctions and points out where they differ.

## 2 | MODEL

A firm sells a patented good to a market described by an inverse demand function,  $P(q)$ , that is decreasing in the number of units sold,  $q$ , and exhibits strict concavity of the revenue function  $P(q)q$ . The firm outsources the production of the good to one of  $n \geq 2$  potential contractors.<sup>2</sup> Potential contractors' cost functions are  $C(q) = cq + F$ , where  $c$  denotes marginal cost and  $F$  the fixed cost.

Cost profiles are independently and identically distributed random variables drawn from a continuous probability distribution. Their realizations are potential contractors' private information; they are subject to an error term,  $\eta$ , with finite variance and  $E[\eta] = 0$ .

Potential contractors cannot observe the firm's demand, revenue, and profit which are the firm's private information.

The firm and all potential contractors are risk neutral.

After the contractor has been selected, but before production is finalized, the contractor draws new information that reveals (or reduces) the error of the first draw and thus leads to an updated and more accurate cost estimate. At that time, the contractor has the option to submit a change order that allows him to adjust output.

The firm employs a two-stage procurement mechanism  $M := (S(w, f), q(w), T_1, T_2)$  that is announced at the outset of the game.

In the first stage, potential contractors submit two-dimensional bids,  $(w, f)$ . In the spirit of direct mechanisms,  $w$  represents the reported unit cost estimate and  $f$  the reported fixed cost. Bids are evaluated by a scoring rule  $S(w, f)$ , and the highest scoring bidder is selected as the contractor. Depending upon his reported unit cost, the contractor is obliged to deliver  $q(w)$  output units and is entitled to be paid according to the transfer rule  $T_1$ .

<sup>1</sup> For a survey of alternative procurement methods and relevant issues, see Dimitri et al. (2006).

<sup>2</sup> Multiple contractors would duplicate fixed costs.

In the second stage, after new information has become available and before production has been finalized, the contractor has the option to ask for a change order to adjust his output by revising his reported unit cost, and the contractor is paid according to the transfer rule  $T_2$  in lieu of  $T_1$ .

The mechanism  $M$  is specified as follows:

*Scoring rule:* The two-dimensional bids,  $(w, f)$ , are mapped into a one-dimensional score:

$$S(w, f) = (P(q(w)) - w)q(w) - f. \quad (1)$$

*Allocation rule:* The bidder with the highest scoring bid is selected as the contractor.

*Output rule:* Depending upon his bid, the contractor is obliged to deliver  $q(w)$  output units. That output rule is implicitly defined as follows:

$$q(w) = \arg \max_q (P(q) - w)q. \quad (2)$$

The concavity of the revenue function assures the uniqueness of  $q(w)$  and  $q'(w) < 0$ .

*Transfer rule  $T_1$ :* The contractor who won the auction with the bid  $(w, f)$  is entitled to receive the transfer  $T_1$ :

$$T_1 = P(q(w))q(w) - S_{(-)}, \quad (3)$$

where  $S_{(-)}$  denotes the highest score of the  $n - 1$  losing bidders (whose score is by definition lower than  $S(w, f)$ ).

*Change orders:* If the contractor revises his reported unit cost, say to  $w_2$ , he is obliged to deliver  $q(w_2)$  output units (in lieu of  $q(w)$ ) and is paid the transfer  $T_2$  (in lieu of  $T_1$ ):

$$T_2 = P(q(w_2))q(w_2) - S_{(-)}. \quad (4)$$

Losers of the auction receive no transfer, and the firm's expected profit is evidently equal to the expected value of the second highest score:  $E[S_{(2:n)}]$ .<sup>3</sup>

To avoid a possible misunderstanding, we emphasize that potential contractors do *not* observe the firm's revenue. They only know the stipulated mechanism which is not contingent on unobservable variables such as the firm's revenue.

For example, the mechanism may stipulate the following:

$$q(w) = \frac{1-w}{2}, \quad S(w, f) = \frac{(1-w)^2}{4} - f \quad (5)$$

$$T_1 = \frac{1-w^2}{4} - S_{(-)}, \quad T_2 = \frac{1-w_2^2}{4} - S_{(-)} \quad (6)$$

<sup>3</sup>This notation is borrowed from order statistics; there, the 2 stands for "second highest" and  $n$  for the size of the sample of scores.

(which the firm would choose if  $P(q) = 1 - q$ ). The mechanism is not contingent on the firm's revenue. Making the mechanism contingent on revenue would not make sense because potential contractors cannot verify revenue.

The revenue function enters into the firm's design of the mechanism, but the mechanism itself depends only on  $w$  and  $f$ . Potential contractors do not know how the firm has computed these functions. The interpretation of the rules is only relevant for the firm and the analyst's assessment of the equilibrium outcome.

### 3 | RESULTS

We first consider the change order subgame which is played after the contractor has drawn an updated and more accurate cost estimate denoted by  $(c', F')$ .

**Proposition 1.** *In the change order subgame, the contractor exercises the option to change his reported unit cost to  $w_2$  equal to his updated estimate of his marginal cost,  $w_2 = c'$ , thus improving efficiency, increasing his expected profit, and leaving the firm's profit unchanged.*

*Proof.* For convenience define,  $\pi_0(w) := (P(q(w)) - w)q(w)$  and note that, by the envelope theorem,<sup>4</sup>  $\pi'_0(w) = -q(w)$ .

When the contractor asks for a change order and sets  $w_2$  his expected payoff is

$$\begin{aligned}\Pi(w_2) &= T_2 - c'q(w_2) - F' \\ &= (P(q(w_2)) - c')q(w_2) - F' - S_{(-)} \\ &= (w_2 - c')q(w_2) + \pi_0(w_2) - F' - S_{(-)}.\end{aligned}\tag{7}$$

The optimal  $w_2$  must solve the first-order condition:

$$0 = \Pi'(w_2) = q(w_2) + (w_2 - c')q'(w_2) + \pi'_0(w_2) = (w_2 - c')q'(w_2).\tag{8}$$

Because  $q'(w) < 0$ , it follows that the contractor's updated  $w$  is  $w_2 = c'$ . Moreover, because confirming the original  $w$  is a special case of a change order, it follows that the contractor always submits a change order and increases his expected profit (unless there is no payoff relevant updating of information). Efficiency is assured, and the firm's profit is not affected:

$$\Pi_0 = P(q(w_2))q(w_2) - T_2 = P(q(w_2))q(w_2) - (P(q(w_2))q(w_2) - S_{(-)}) = S_{(-)}.\tag{9}$$

□

Note that the contractor always exercises the option to revise the requested unit price and the bid revision always increases his profit. Of course, it is possible that the initial cost estimate was more optimistic and the initially predicted profit was higher than the terminal profit. Nevertheless, the optimal bid revision always increases the contractor's profit conditional on the most

<sup>4</sup> Recall, by (2) the output rule  $q(w)$  is the maximizer of  $\pi_0(w)$ .

up-to-date information available to him at the time when he submits his change order. (Evaluating the terminal profit conditional on obsolete information would not be sensible.)

We now turn to the first-round bidding game which selects the contractor. For this purpose, it is useful to first look at bidders' strategy as a score,  $S(c, F)$ .

Note that the contractor's first-round bid does not affect the contractor's second-round payoff function and vice versa. The only link between the first-round play and the contractor's second-round payoff is that the latter depends upon the competing bidders' highest score, which is of course not controlled by the contractor's own bid. This implies that we can solve the first-round bidding game without reference to the solution of the change order subgame.

**Proposition 2.** *The bidding game has an equilibrium in (weakly) dominant score strategies. Depending upon their cost profile, each potential contractor bids a score,  $S$ , equal to*

$$S = S(c, F) = (P(q(c)) - c)q(c) - F, \quad (10)$$

and the contractor's first-round equilibrium payoff is  $\Pi = S(c, F) - S_{(-)} > 0$ .

*Proof.* Consider a bidder with cost estimate  $(c, F)$  and denote the competing bidders' highest score by  $S_{(-)}$ . Conditional on winning, the expected payoff of that bidder (who bids  $(w, f)$  and achieves the score  $S \geq S_{(-)}$ ) is, after substituting  $T_1$  and  $S$ , equal to

$$\begin{aligned} \Pi &= (P(q(w)) - c)q(w) - F - S_{(-)} \\ &= (w - c)q(w) - (F - f) + (S - S_{(-)}). \end{aligned} \quad (11)$$

In equilibrium, bidders must be indifferent between winning and losing in the event when  $S = S_{(-)}$ . Therefore,

$$(w - c)q(w) - (F - f) = 0, \quad (12)$$

and, hence, conditional on winning, bidders' first-round expected equilibrium payoff is

$$\Pi = S - S_{(-)}. \quad (13)$$

To show that bidding  $S = (P(q(c)) - c)q(c) - F$  is a weakly dominant strategy, suppose a bidder deviates and bids  $S' > S$ . This does not affect his payoff as long as  $S' \leq S_{(-)}$ . However, it affects his payoff if  $S' > S_{(-)}$  and  $S < S_{(-)}$ , because then that bidder wins and earns a nonzero payoff but would have lost if he had bid  $S$ . The increased score can only be achieved by reducing the requested transfer by  $\delta = S' - S$  or more. But then, altogether, that bidder suffers a loss, because

$$\Pi' = (S' - \delta) - S_{(-)} \leq S - S_{(-)} < 0. \quad (14)$$

Therefore, bidding a score greater than  $S$  is never profitable, regardless of other bidders' play.

Similar reasoning shows that bidders cannot benefit from bidding less than  $S$  either, and we conclude that, reminiscent of a standard single-unit Vickrey auction with one-dimensional bids, bidding "truthfully," that is, bidding a score equal to  $S(c, F)$  is a weakly dominant strategy.  $\square$

While this proves that bidding a score equal to  $S(c, F)$  is a weakly dominant strategy, it does not imply that submitting the two-dimensional bid  $(w, f) = (c, F)$  is the only equilibrium consistent with submitting that score. Indeed, there are other combinations of  $w$  and  $f$  that yield the same score. However, even if the contractor submits any of these other bids after the change order subgame has been played the unique equilibrium outcome is  $w = c$ . This indicates once more the importance of the change order subgame.

Altogether, the terminal expected profit of the firm is equal to  $E[S_{(2:n)}]$  and the equilibrium expected payoff of the contractor with score  $S$  is equal to

$$S - E[S_{(1:n-1)} | S_{(1:n-1)} < S] \quad (15)$$

plus the increment in surplus realized by adjusting output in the change order subgame.

#### 4 | RELATIONSHIP TO THE LITERATURE ON SCORING AUCTIONS

The proposed mechanism employs a scoring auction rule to select the contractor and to reveal information about potential contractors' costs. This is similar to Che (1993), who considers the procurement of one unit of a good with variable quality in a second-score auction.

However, whereas Che (1993) assumes a transfer rule where the winner is allowed to match the second highest score,<sup>5</sup> in the present model, the transfer is, by the design of the mechanism, indirectly linked to the firm's revenue *and* the second highest score. This peculiar feature is reminiscent of a "share auction," in which the transfer to the winner of the auction is linked to the firm's performance measure (see the seminal contribution by Hansen, 1985; and the survey of more recent contributions by Skrzypacz, 2013).

Another peculiar feature is that the contractor has the option to submit a change order after updated cost information is available. Together, these features assure that the overall equilibrium implements efficient output levels.

While the considered procurement mechanism is efficient, the procurer may be able to increase his expected profit by employing a strategic minimum score (in the spirit of Myerson, 1981, and Che, 1993) or by using a mechanism that is based on a strategically distorted inverse demand function, as in Arozamena et al. (2018). Adding a minimum score does not fundamentally change the analysis, except that procurement may fail with positive probability, whereas distorting inverse demand functions distorts efficiency even if procurement takes place. This suggests that using a minimum score is more profitable for the procurer, yet it remains an open question.

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<sup>5</sup> Che (1993) considers two ways in which the winner is permitted to match the highest score: by adjusting the payment and by adopting the two-dimensional bid that achieved the second highest score. He shows that the former has more desirable properties.

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