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## Environmental policy instruments for investments in backstop technologies under present bias - An application to the building sector

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# Environmental policy instruments for investments in backstop technologies under present bias - an application to the building sector

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## Abstract

Governments worldwide have set targets to reduce greenhouse gas emissions from the residential sector to zero. Policy instruments, such as carbon pricing or subsidies, are being discussed and implemented to achieve these targets. If individuals exhibit present bias, [Heutel \(2015\)](#) has shown that optimal policies targeting investments in the efficiency state of externality-producing durable goods and their usage consist of two components, one aimed at the externality and one aimed at the present bias. We generalize Heutel's theoretical model by defining a larger technology set. This allows us to represent the dependence of fuel prices and emission intensities on technologies used in the building sector and to include a zero emission backstop technology. We first examine the effect of this model generalization on Heutel's main propositions, assuming still that the backstop technology is not optimal. Second, we extend this examination to the case when the backstop technology is optimal. In a stylized case study for a representative building in Germany, we numerically estimate magnitudes of the present bias effect on investment and heating decisions, emissions, policies, and deadweight loss. We show that as long as social costs of carbon and the corresponding CO<sub>2</sub> price are not high enough to make the backstop technology optimal, Heutel's proposition holds that optimal policies must consist of two components. Contrary to Heutel's proposition, if the social costs of carbon and the CO<sub>2</sub> price are high enough, a single instrument can address present bias. While the level of this single instrument, i.e., a tax or subsidy, depends on the level of present bias, we find that there exists a tax-subsidy combination that is optimal regardless of the level of present bias.

*Keywords:* Present bias, policy, heating investments, durable goods, climate neutrality

JEL classification: D15, D62, D91, H23, Q48, Q58

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# 1. Introduction

## 1.1. Background and motivation

Governments of many countries have set themselves climate targets, i.e., emission reduction targets. These targets no longer aim at a mere partial reduction of greenhouse gas (GHG) emissions but rather at a reduction of GHG emissions to zero or close to zero. More than 70 countries pledged to reach net-zero emissions, including the countries of the European Union, China, and the USA ([United Nations, 2023](#)). Investments must be stimulated and carried out beyond efficiency improvements to achieve these climate neutrality goals. Therefore, in all sectors investments in zero emission technologies, i.e., backstop technologies, must be made.

An example is the residential building sector: Global GHG emissions from building operations, i.e., heating and hot water provision, have increased in recent years. In 2021 global direct CO<sub>2</sub> emissions from building operations accounted for around 8 % of global energy-related CO<sub>2</sub> emissions ([IEA, 2022](#)). In the residential building sector, decarbonization needs to be carried out by private households investing in new technologies (e.g., heating systems and refurbishment) and choosing their indoor temperature level. In this paper, the analysis is applied to the residential building sector, although the results are generalizable.

The prominent policy from classic economics to reach the first-best outcome in the presence of an environmental externality (i.e., the emitted emissions) is to introduce a price on said externality (i.e., a carbon or CO<sub>2</sub> price), internalizing the externality into the decision-making rationale of the households, like the Pigouvian tax ([Pigou, 1920](#)). Empirical literature suggests that individuals do not always behave according to classic rational choice theory. Behavioral issues, such as time-inconsistent discounting (e.g., present bias), could prevent individuals from investing optimally in time. [Heutel \(2015\)](#) has shown that if consumers experience present bias, a Pigouvian tax does not lead to welfare optimal investment decisions for externality-producing durable goods. Instead, the optimal policy mix consists of an instrument to correct the externality and another one aiming at the present bias, constituting an internality. Besides carbon taxation or pricing, these instruments can include subsidies, taxes based on efficiency, or mandates.

In his analysis, [Heutel \(2015\)](#) assumes that consumers can invest in technologies with different efficiencies. Sheer improvement of efficiencies in externality-producing durable goods, however, cannot reduce externalities to zero. Thus, by assumption, no backstop technology exists. In his analysis, the author finds that there exists a welfare-optimal amount of externalities (i.e., GHG emissions) resulting from the Pigouvian tax rate, representing the monetary damage of the externality and the utility drawn from the externality-producing good. In contrast, in many countries, climate neutrality is the declared political target. The implicitly assumed damage from GHG emissions thus is infinite, and correspondingly, the optimal amount of GHG emissions is zero. Put differently, the policy maker is interested in target-consistent CO<sub>2</sub> pricing and policy measures rather than taxing the externality at the rate of social costs of carbon ([Aldy et al., 2021](#)).

Building on the work of [Heutel \(2015\)](#), this raises the following questions. First: How can Heutel's model be generalized to account for the existence of zero emission backstop technologies with finite costs? Second: What does this generalization imply for the main propositions of the model? Third: What are optimal policies under present bias for externality-producing durable goods if the optimal investment decision is the investment in the backstop technology?

We generalize the analytical model of [Heutel \(2015\)](#) for investments in externality-producing durable goods under present bias by allowing for a greater technology space. In the generalized model, the investment may be accompanied by the substitution of the fuel used, for example, in the case of heating investments, switching from a gas heating system to an electric heat pump. The integration of fuel substitution into the investment decision allows us to depict the existence of a zero emissions backstop technology.

We first examine the effect of the model generalization on Heutel's main propositions, assuming still that there is a welfare-optimal inner solution, i.e., that the backstop technology is not optimal. We then discuss the implications of the situation when the investment in the backstop technology is optimal. This may be the case if the assumed damage of the externality is high enough so that the backstop technology is welfare-optimal, or due to politically set climate neutrality targets. In a stylized case study for a representative building of the German building

sector, we assume a politically set climate neutrality target. We numerically estimate real-world magnitudes of the present bias effect on investment and heating decisions, emissions, policies, and associated deadweight loss.

In our analysis, we show that as long as social damage of carbon and the corresponding CO<sub>2</sub> price is not high enough to make the backstop technology optimal, households in the optimum will still emit CO<sub>2</sub>. In this case, Heutel's propositions hold that to reach the social optimum, we need two policy instruments, one to address the internality and a second one to address the externality, that will still be present. Contrary to Heutel's propositions, if the social costs of carbon and the corresponding CO<sub>2</sub> price are high enough, a mark-up on the CO<sub>2</sub> price can also induce the social optimum. Therefore, present bias can be addressed by a tax or another single instrument when aiming at climate neutrality in the presence of a zero-emission backstop technology. In numerical simulations for a representative household in Germany and under the assumption of continuous investment choices, we quantify the target-consistent CO<sub>2</sub> at  $\tau_t^{neu} = 192\text{€}/\text{tCO}_2$ . Applying  $\tau_t^{neu}$  in the case of present bias leads to a welfare loss. In the case of a present-biased household, a higher CO<sub>2</sub> tax exists that reaches the target (in our exemplary building and an assumed present bias of  $\beta=0.7$ :  $235\text{€}/\text{tCO}_2$  including an internality-mark-up of  $43\text{€}/\text{tCO}_2$ ). While the optimal tax rate and subsidy depend on the level of present bias, we find that there exists an optimal tax-subsidy combination that is optimal regardless of the level of present bias.

## 1.2. Related literature and contribution

Ever since (Strotz, 1955) introduced the idea of time-inconsistent discounting with his theory of commitment, it has been recognized that consumers may deviate from the assumption of exponential, thus time-consistent, discounting.<sup>1</sup> In line with time-inconsistent discounting, Laibson (1997) coined the concept of present bias, i.e., agents' preference for immediate benefits over advantages in future periods beyond exponential discounting.<sup>2</sup> To represent this behavior,

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<sup>1</sup>Frederick et al. (2002) includes a critical review of the history and models of time discounting including time-consistent utility discounting models as well as time preferences and (quasi-)hyperbolic discounting models.

<sup>2</sup>See the reviews Frederick et al. (2002) and DellaVigna (2009) for empirical estimates for present bias in various circumstances and Imai et al. (2021) and Cheung et al. (2021) for recent meta studies of papers reporting present bias estimates.

the literature has introduced and applied models of quasi-hyperbolic discounting (Phelps and Pollak, 1968; Laibson, 1997; O'Donoghue and Rabin, 1999).

One metric for policy evaluation is welfare. Assuming time-inconsistent preferences implies that preferences change over time, complicating welfare analysis. Economists provide several welfare criteria to overcome this complication. The two most prominent criteria are the Pareto criterion, i.e., considering each period's perspective in overall utility, and the long-run criterion, i.e., evaluating the "true" utility from a long-run perspective (O'Donoghue and Rabin, 2015). O'Donoghue and Rabin (1999) argue that the Pareto criterion is a too strong assumption when applied to intertemporal choice. O'Donoghue and Rabin (2015) claim that both approaches, as well as other thinkable welfare criteria, frequently yield the same conclusions but argue for the usage of the long-run criterion.<sup>3</sup> As Heutel (2015) utilizes the long-run criterion in his model, we will also apply it.

Applying the long-run criterion deviates from standard social welfare analysis, relying on revealed preferences as information about the consumer's true utility. The paternalistic assumption that the consumer's choices do not optimize her welfare is as critical as it is controversial. Saint-Paul (2011) argues that taxes levied for inducing a particular behavior might only lead to consumers paying higher prices instead of changing behavior, reducing overall welfare. According to Whitman (2006), the justifications of policy interventions for addressing externalities are based on the idea of Pigouvian taxation, ignoring Coase's theorem (Coase, 1960). The theorem states that externalities can be resolved by negotiation between individual parties when transaction costs are low. Since externalities consist of choices within the individual, Whitman (2006) argues that Coase's theorem is better suited for dealing with externalities. The information required to find the least costly option addressing the damage from time-inconsistent discounting is only available to the individual. Moreover, Krusell et al. (2002) argues that to tackle consumers' time-inconsistent preferences, only an intervention by a time-consistent social planner is welfare enhancing.

We apply our analysis to the case of households' heating system investment decisions. The empirical literature regarding behavioral biases in energy efficiency decision-making is limited

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<sup>3</sup>Kang (2015) shows that improvements in the Pareto criterion are also welfare-improving from the long-run perspective.



(Gillingham et al., 2009). Schleich et al. (2019) investigated the role of present bias and other behavioral aspects in adopting energy-efficient technologies within different countries in the European Union. They provide evidence for the significance of present bias in reducing investments in energy-efficient appliances and building retrofitting. Werthschulte and Löschel (2021) find that present bias increases power consumption. Therefore, as households undervalue energy costs, price-based policies might fail to reduce household energy consumption. Furthermore, in the specific case of investment in household appliances in India, Fuerst and Singh (2018) find that present bias becomes more significant the larger the purchase object investigated. This finding is relevant to our work, as heating system replacement represents a particularly large investment decision for households. Overall, there is not yet a comprehensive empirical view on the effect of present bias on heating system investments. We account for this lack of estimates by considering a range of present bias factors in our numerical simulation.

This paper focuses on the consequence of present bias in agents' decision-making on policies for decarbonization. The model from Heutel (2015) constitutes the basis of our analysis. A detailed description of the model for analyzing optimal policy instruments for externality-producing durable goods under present bias can be found in Section 2.1. Heutel (2015) considers a technology space with efficiency and investment costs as dimensions. We expand this space by allowing technologies to differ in emission intensity and fuel price. As we will see, this generalization enables us to discuss the subject of climate neutral backstop technologies. Since Heutel (2015), recent work has deepened the understanding of present bias in economic policy design and welfare analysis (Drugeon and Wigniolle, 2021; Kotsogiannis and Schwager, 2022; Kang, 2022; Bar-Gill and Hayashi, 2021; Lades et al., 2021). Bar-Gill and Hayashi (2021) discuss the investment decisions for durable goods by present-biased agents. In contrast to our work, they focus on the effect of purchase financing. They find countervailing effects of present bias on the valuing of the benefits of an investment and the costs of financing said investment and derive recommendations for credit regulation. Since they discuss general durable goods, they do not consider the emission externalities from using energy technologies. Lades et al. (2021) examine investments from present-biased households in energy efficiency technologies. They illustrate particularly how administrative burden can reduce these

investments. Similar to our work, they apply a theoretical model and a simulation with exemplary building data. As we will see, our key point of departure from Heutel (2015) and Lades et al. (2021) is that we consider policies reaching climate neutrality combined with the availability of a backstop technology.

While there is literature on policies in the context of present bias, to the best of our knowledge, there is no literature addressing the subject of policies for externality-producing durable goods aiming at climate neutrality. In the present work, we aim to close this gap by (i) generalizing the model from Heutel to more complex technologies also differing in emission intensity and fuel price to be able to account for zero emission backstop technologies, (ii) analyzing the consequences of the existence of an optimal backstop technology, and (iii) illustrating the consequence of such policies in the residential building sector numerically.

## 2. Analytical model

In this Section, we first describe the representative agent model for investments in externality-producing durable goods under present-bias from Heutel (2015). Then we generalize the model and apply it to the building sector. By defining a larger technology set, we are able to represent technologies running on different fuels and thus zero emission backstop technologies. Based on the generalized model, we discuss two different cases: First, the case that the backstop technology is not optimal. Second, the case that the backstop technology is the optimal technology choice.

### *2.1. A representative agent model for investments in externality-producing durable goods under present-bias*

Heutel (2015) describes the investment and operation problem for externality-producing durable goods under present bias in a representative agent model. We present the model based on nomenclature for residential heating. The investment decision is made in the initial period ( $t = 0$ ) and the good lasts  $T$  periods. In each period after the investment ( $t = 1$  through  $t = T$ ), the household decides on the operating intensity of the good: the generated heat or indoor temperature.

The model is defined by the household's problem and the social planner's problem. In the household's problem, future utility and costs are discounted using quasi-hyperbolic discounting.

Quasi-hyperbolic discounting is a method for modeling the behavior of households who experience present bias, i.e., prefer immediate payoffs and undervalue future costs and payoffs.<sup>4</sup> To this end, two discount factors are introduced.  $\delta$  is called the "long-run" discount factor, and  $\beta$  represents the "present bias". If a household experiences present bias, then  $\beta < 1$ .

The present-biased household perspective is contrasted with the social planner's problem. Present bias is a behavioral anomaly that a social planner does not experience due to fully rational behavior. One way to solve the social planner's optimization problem is to directly apply the long-run criterion while disregarding the household's present bias, i.e., setting  $\beta = 1$ . The approach assumes that the household's utility maximization deviates from optimal welfare even from the household's perspective. Thus, the household "makes a mistake" and does not optimize its "true utility".<sup>5</sup>

In the initial period, the household chooses the heating systems ratio of fuel input and generated heat, the so-called effort coefficient  $fph$  (fuel per heat), representing the investment decision for the durable good. In the subsequent periods, the heat generated in each period  $h_t(t)$  is chosen, which translates into indoor temperature.  $U(h_t)$ , where  $U' > 0$  and  $U'' < 0$ , describes the utility from generated heat in monetary terms. The costs per  $kWh$  of fuel are calculated as the sum of the time-dependent fuel cost ( $p_t$ ) and a tax per  $kWh$  of fuel ( $\tau_t$ ). This fuel tax, hereinafter referred to as carbon tax, is intended to put a price on the GHG emissions. When choosing a level of  $fph$ , the household faces investment costs of  $c(fph)$ . It is assumed that  $c' < 0$ , meaning that less efficient goods (heating systems) are less expensive, and  $c'' > 0$ . The household's problem is thus described in Equation (1):

$$\max_{fph, \{h_t\}_{t=1}^T} -c(fph) + \beta \cdot \left[ \sum_{t=1}^T \delta^t \cdot \left[ U(h_t) - [p_t + \tau_t] \cdot fph \cdot h_t \right] \right] \quad (1)$$

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<sup>4</sup>Technically speaking, present-biased households discount utility and costs in the near future at a higher implicit discount rate than in the distant future (Laibson, 1997).

<sup>5</sup>Alternative welfare criteria in the case of time-inconsistent discounting include the Paretian approach (e.g. Bhattacharya and Lakdawalla (2004)), or the "dictatorship of the present" approach discussed in Gruber and Köszegi (2004) or Laibson (1997), which prioritizes the preferences of the current self over the preferences of all future selves. Analogous to the approach in Heutel's basic model and following the arguments of O'Donoghue and Rabin (1999), we apply the long-run criterion.

The social planner's problem is characterized by including the externality of fuel consumption. The external damage from fuel consumption, i.e., damage from GHG emissions, depends on  $h_t$ , the  $kWh$  of fuel used in each period  $t$ , times  $fph$ , the fuel used for producing the heat. The damage is denoted as  $d(h_t \cdot fph)$ , where  $d(0) = 0$ ,  $d' > 0$  and  $d'' = 0$ . The corresponding social planner's problem, using the long-run criterion for discounting and including external damages, is described in Equation (2):

$$\max_{fph, \{h_t\}_{t=1}^T} -c(fph) + \sum_{t=1}^T \delta^t \cdot [U(h_t) - p_t \cdot fph \cdot h_t - d(h_t \cdot fph)] \quad (2)$$

## 2.2. Model generalization

In the model described in the previous section, consumers invest in one technology and can decide on its efficiency. By assumption, no backstop technology exists because efficiency improvements cannot reduce externalities to zero, and fuel cost differences between technologies used are neglected. We extend the technology set by allowing technologies running on different fuels. Therefore the investment decision affects fuel costs and GHG emissions per unit of generated heat. This enables us to analyze how optimal investment decisions depend on fuel cost ratios and to include a zero emission backstop technology. An example for a zero emission backstop technology in the building sector would be the switch to renewably generated heat from solar thermal energy or to electric heating powered by renewably generated electricity.

By allowing technologies to vary in fuel price and emission intensity (down to zero), we extend the technology set and generalize the model. In this generalized model, the fuel price  $p_t(fph)$  and the CO<sub>2</sub> factor of the heating system  $epf(fph)$  are represented as functions of the effort coefficient  $fph$ .<sup>6</sup> The functional form of  $p_t(fph)$  is ambiguous: it is conceivable that the change to a more efficient heating system, e.g., from a gas boiler to an electric heat pump, is accompanied by decreasing fuel prices, in € per  $kWh_{fuel}$ , but also that the fuel price increases, if, for example, electricity is more expensive than gas. We incorporate a backstop technology with finite costs  $fph^{BS}$  by assuming that the emission function  $epf(fph)$  equals zero for all  $fph \leq fph^{BS}$ , and  $epf' > 0$

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<sup>6</sup>We model the choice of  $fph$ , the investment costs, the change in fuel price, and CO<sub>2</sub> factor as continuous. This serves the theoretical tractability of the model.

for  $fph \geq fph^{BS}$ . This means that when investing in the reduction of  $fph$ ,  $epf(fph)$  decreases linearly until the backstop technology  $fph^{BS}$  is reached, where emission intensity is zero. Further investments in reducing  $fph$  cannot further reduce the emission intensity.

The household's problem, including quasi-hyperbolic discounting as defined in Section 2.1, is thus described as follows:

$$\max_{fph, \{h_t\}_{t=1}^T} -c(fph) + \beta \cdot \left[ \sum_{t=1}^T \delta^t \cdot \left[ U(h_t) - [p_t(fph) + epf(fph) \cdot \tau_t] \cdot fph \cdot h_t \right] \right] \quad (3)$$

The household's problem differs from Heutel (2015), since the investment decision  $fph$  depends on  $p_t$  and the newly introduced CO<sub>2</sub> factor  $epf$ . This yields first-order conditions for  $fph$  and each  $h_t$ . Assume that there exists a unique interior solution.<sup>7</sup> The solutions to the household's problem are called  $fph^*$  and  $h_t^*$ .

$$\begin{aligned} & -c'(fph^*) \\ & - \beta \cdot \sum_{t=1}^T \delta^t \cdot h_t^* \cdot [p_t(fph^*) + epf(fph^*) \cdot \tau_t] \\ & - \beta \cdot \sum_{t=1}^T \delta^t \cdot h_t^* \cdot [[p'_t(fph^*) + epf'(fph^*) \cdot \tau_t] \cdot fph^*] = 0 \end{aligned} \quad (4)$$

$$U'(h_t^*) - [p_t(fph^*) + epf(fph^*)\tau_t] \cdot fph^* = 0, \forall t \quad (5)$$

In Equation 4, considering the negative sign, the first term  $-c'(fph^*)$  is positive. The term represents the benefit of a marginal increase in  $fph$ . Since  $c' > 0$ , it is cheaper to choose a system with higher  $fph$  and hence, lower efficiency. Similar to Heutel (2015), the first sum represents the discounted cost of a marginal increase in  $fph$  due to the decrease in efficiency: the utility in each future period decreases as heating costs increase. The second sum adds the changes in fuel prices  $p'_t(fph)$  and changes in emission costs  $epf'(fph) \cdot \tau_t$ . While  $epf'_t$  is positive,  $p'_t$  can be positive or negative, depending on the constellation of fuel prices.<sup>8</sup> If both  $p'_t(fph) = 0$  and  $epf'(fph) = 0$ , the third summand equals zero. In this case, neither the fuel price nor the emission intensity of

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<sup>7</sup>It is assumed that  $\lim_{h_t \rightarrow 0} U'(h_t) = \textit{infinity}$  to ensure a unique interior solution.

<sup>8</sup>We will discuss the implications of this relationship under Proposition 2.

the heating system depend on the investment decision, obtaining Heutels application with limited technology set. Equation 5 sets equal the marginal increase in utility of an additional  $kWh$  of heat with the marginal increase in costs for each period  $t$ .

The social planner's problem uses the long-run criterion and omits the term  $\beta$ . The external damage  $d$  depends on the GHG emissions emitted, which are calculated as the product of the emission intensity, the system efficiency, and the provided heat:  $d(epf(fph) \cdot fph \cdot h_t)$ , where  $d(0) = 0$ ,  $d' > 0$  and  $d'' = 0$ .

The social planner's problem is:

$$\max_{fph, \{h_t\}_{t=1}^T} -c(fph) + \left[ \sum_{t=1}^T \delta^t \cdot \left[ U(h_t) - [p_t(fph) \cdot fph \cdot h_t] - d(epf(fph) \cdot fph \cdot h_t) \right] \right] \quad (6)$$

The solutions to the social planner's problem are  $fph^{opt}$  and  $h_t^{opt}$ . The first-order conditions of the social planner's problem are:

$$-c'(fph^{opt}) - \sum_{t=1}^T \delta^t \cdot h_t^{opt} \cdot \left[ p_t(fph^{opt}) + epf(fph^{opt}) \cdot d'(epf(fph^{opt}) \cdot fph^{opt} \cdot h_t^{opt}) \right] \quad (7)$$

$$- \sum_{t=1}^T \delta^t \cdot h_t^{opt} \cdot \left[ p'_t(fph^{opt}) + epf'(fph^{opt}) \cdot d'(epf(fph^{opt}) \cdot fph^{opt} \cdot h_t^{opt}) \right] \cdot fph^{opt} = 0$$

$$U'(h_t^{opt}) - [p_t(fph^{opt}) + epf(fph^{opt}) \cdot d'(epf(fph^{opt}) \cdot fph^{opt} \cdot h_t^{opt})] \cdot fph^{opt} = 0, \forall t \quad (8)$$

### 2.3. Analysis

#### 2.3.1. Optimal inner solution

First, we analyze the case that there is a welfare-optimal inner solution, i.e., that the backstop technology is not optimal and  $fph^{opt} > fph^{BS}$ . This is the case when the assumed external damage from GHG emissions is smaller than necessary for the backstop technology to be welfare-optimal. Thus, there exists a welfare-optimal quantity of GHG emissions that is greater than zero. The optimal solution is therefore found in the non-zero linear part of the emission intensity function  $epf(fph)$ .

From the first-order conditions of the household and the social planner, it follows that if  $\beta = 1$ , the household chooses the first-best outcome if  $\tau_t = d'(epf(fph^{opt}) \cdot fph^{opt} \cdot h_t^{opt}) \forall t$ . This is the Pigouvian tax rate, called  $\tau_t^{pig}$ , which internalizes fossil fuel usage's external damage.

As in [Heutel \(2015\)](#), the following holds for the adapted model:

**Proposition 1.**

Let  $\beta < 1$ : No set of emissions tax  $\tau_t$  for all  $t \in [1, \dots, T]$  exists that leads to the first-best outcome  $fph^{opt}$  and  $h_t^{opt}$ .<sup>9</sup>

If the Pigouvian tax rate is applied, [Heutel \(2015\)](#) obtains that too little is invested and the good is underutilized compared to the optimum of the social planner. In our case, when looking at heating investments, the question of whether too much or too little is invested due to present bias depends on how fuel and emission costs are affected by the investment decision, leading to Proposition 2:

**Proposition 2.**

Let  $\beta < 1$ : If  $\tau_t = \tau_t^{pig}$  for all  $t \in [1, \dots, T]$  and  $\sum_{t=1}^T \delta^t \cdot h_t^* \cdot [p_t(fph^*) + epf(fph^*) \cdot \tau_t] + [p'_t(fph^*) + epf'(fph^*) \cdot \tau_t] \cdot fph^* \geq 0$  then  $fph^* > fph^{opt}$  and  $h_t^* < h_t^{opt}$  for all  $t \in [1, \dots, T]$ .<sup>10</sup>

Proposition 2 states that present-biased households under-invest in the investment period and heat less than optimal in the subsequent periods, as long as the change in discounted future costs for a marginal increase in  $fph$  (marginally less efficient heating system) is greater or equal to zero. The scenario occurs if  $p' \geq 0$ , meaning that the price for heating increases for less efficient heating systems. Then, the optimal solution is found as the trade-off between  $fph$  decreasing investment costs on the one side and the marginal increase in future heating costs consisting of the effects of the decreased efficiency, increasing emission costs, and increasing fuel costs on the other side. In

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<sup>9</sup>The proof is presented in [Appendix A.1](#).

<sup>10</sup>The proof is presented in [Appendix A.2](#).

the other scenario, in which  $p' < 0$ , the described relationship only continues to hold as long as the emission tax and lower efficiency offset the decrease in fuel costs for a marginal increase in  $fph$ . If this does not hold, i.e.,  $p'_t \ll 0$ , a marginal increase in  $fph$  leads to lower investment costs and a decrease in future costs. In such a case, it would be optimal to invest as little as possible, thus  $fph \rightarrow \infty$ .

In the case of present bias, no set of CO<sub>2</sub> tax rates produces the first-best outcome regarding investments, fuel usage, and GHG emissions. A second-best policy solely based on CO<sub>2</sub> tax rates must consist of higher tax rates than the Pigouvian tax rate. The tax must address the usage of the heating system while also unfolding high incentives in the investment period, compensating for the present bias. A tax that incentivizes efficient investment despite present bias cannot be optimal because the tax is too high in subsequent periods to incentivize optimal heating use, given the optimal efficiency level, the utility of heating, and GHG emission damage. A second policy instrument has to be introduced to achieve the first-best result. In [Heutel \(2015\)](#), the author discusses a tax on the goods effort coefficient in the starting period and a fuel effort coefficient standard. In the case of the building sector, there is another policy measure often applied by the regulator: A subsidy on capital expenditures. We define this subsidy as a monetary benefit  $\sigma$  that is scaled with  $\frac{1}{fph_{min} - fph_{max}} \cdot fph + \frac{1}{1 - \frac{fph_{min}}{fph_{max}}}$ . The subsidy thus decreases linearly in  $fph$ . The household gets the full value of the subsidy if the household chooses  $fph_{min}$  and no subsidy if the household chooses  $fph_{max}$ :

**Proposition 3.**

Let  $\beta < 1$ : The first best is achieved by setting  $\tau_t = \tau_{pig}$  in each period  $t > 0$  and setting a technology subsidy in the form of  $(\frac{1}{fph_{min} - fph_{max}} \cdot fph + \frac{1}{1 - \frac{fph_{min}}{fph_{max}}}) \cdot \sigma$  with  $\sigma = (fph_{min} - fph_{max}) \cdot (\beta - 1) \cdot \sum_{t=1}^T \delta^t \cdot h_t^{opt} \cdot [p_t(fph^{opt}) + epf(fph^{opt}) \cdot \tau_t^{pig}] + [p'_t(fph^{opt}) + epf'(fph^{opt}) \cdot \tau_t^{pig}] \cdot fph^{opt}$ .<sup>11</sup>

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<sup>11</sup>The proof is presented in [Appendix A.3](#).



Since the household is present-biased, in the investment period, the household only considers a share of  $\beta$  of the future discounted benefits from decreasing  $fph$ . The subsidy is therefore composed of the benefit from investing in lower  $fph$ , expressed in the sum, times  $(\beta - 1)$ , offsetting the present bias in the investment period. In contrast to the fuel economy tax in Heutel (2015), our model's subsidy depends not only on  $h_t^{opt}$  but also on  $fph^{opt}$ . The intuition behind this is that due to the fuel switch when investing,  $fph^{opt}$  is relevant for determining the marginal benefit from an increase in  $fph$  as it defines fuel and emission costs in the optimal case. This means that for the optimal design of the subsidy, it is crucial to know how the optimal investment changes the fuel and emission costs. Besides the level of fuel and emission costs at  $fph^{opt}$ , also their slopes at this point determine the optimal subsidy. In contrast to Heutel (2015), marginal changes in investment affect the fuel price and the emission intensity. Therefore, the subsidy accounts for the marginal variable cost changes, composed of fuel price and emission intensity, induced by investment decisions under present bias. The term can both increase or decrease the optimal subsidy, as the fuel price can have a positive or negative slope in  $fph^{opt}$ .

### 2.3.2. Optimal backstop technology

In the model so far, we assumed that there exists a welfare optimal amount of GHG emissions that relates to the Pigouvian tax rate, internalizing the external emission damage. It was not optimal to invest in the backstop technology. In contrast, in the current political debate around the decarbonization of the building sector, the declared target is climate neutrality. Thus the implicit damage from GHG emissions is infinite, and correspondingly, the optimal amount of GHG emissions is zero:  $fph$  is optimal if  $epf$  is zero. The same applies to a situation where the zero emission backstop technology is optimal due to sufficiently high assumed external emission damage. In the following, we discuss the case that  $fph^{opt} = fph^{BS}$ .

For the case without present bias, i.e.,  $\beta = 1$ , there exists a set of tax rates  $\tau_t^{neu}$  that is defined as the set of lowest tax rates, which achieves climate neutrality, which is thus the target-consistent set of CO<sub>2</sub> prices. Following the discussion in Section 2.3.1, this set of tax rates is insufficient to reach climate neutrality in the case of present bias, as it only addresses the externality.

In contrast to Proposition 1 in Section 2.3.1, if the backstop technology is the optimal investment choice, a set of emission taxes  $\tau_t > \tau_t^{neu}$  can be used to address both the externality and the internality. This is the case because of the added property of the emission function: We can choose taxes high enough for optimal investment, which induces climate neutrality. But since  $epf(fph^{opt}) = epf(fph^{BS}) = 0$ , the taxes do not influence the heating decision of households anymore. That means that any set of taxes high enough to induce investments in climate neutral heating technologies would be optimal since it does not affect heating decisions in the subsequent periods, as the heating decision becomes independent of the tax. This finding is, in principle, unaffected by the presence of present bias. Present bias only increases the minimum level of the taxes, inducing investments in climate neutral heating technologies.

Following that logic, the optimal investment decision can be derived from a set of taxes or a subsidy alone, a combination of both, or a command-and-control policy, i.e., a ban on new investments in conventional technologies. In our stylized model framework, except for the distributional effects of subsidies, there is no difference between those policies regarding the household's welfare, investment, or heating.<sup>12</sup>

### 3. Numerical simulation

In the numerical case study, we estimate real-world magnitudes of the present bias effect on under-investment, under-consumption of thermal energy, over-emissions, and associated deadweight loss. We obtain optimal single instrument magnitudes of CO<sub>2</sub> prices and subsidies representing the internality-mark-ups. Lastly, we show how present bias differentially affects CO<sub>2</sub> prices and subsidies and therefore numerically obtain optimal policy mixes.

#### 3.1. Case study set-up

##### 3.1.1. Metric for evaluation of sub-optimal policies

We numerically investigate the effects of present bias under policy measures aimed at reaching the politically set emission target with the help of a two-step procedure. First, excluding present bias internalities, we determine the minimal target-consistent carbon tax rate inducing investments

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<sup>12</sup>See Section 4 for a discussion of the distributional effects of different policies.

suitable for climate neutrality goals, i.e., in the climate neutral backstop technology.<sup>13</sup> In the reference case, this carbon tax rate  $\tau_t^{neu}$  is 192€/tCO<sub>2</sub>.<sup>14</sup> Second, we use this carbon tax rate as the implied damage to evaluate social welfare and deadweight loss.

### 3.1.2. Building and system characteristics

The functional form and parametrization of the utility of heating determine the household’s choices. [Mertesacker \(2021\)](#) develops a utility function for domestic heating accounting for the properties of the technical heating system and building envelope. He estimates the utility’s parameters within a German case study. We utilize this function and its estimated parameters. Equation (9) shows the utility function of our household  $U(T_t)$  depending on the ideal indoor temperature  $\bar{T}_t$  of 21 °C and chosen indoor temperature  $T_t$ . The utility function thus reflects the willingness to pay for the heating temperature.  $\gamma$  expresses the marginal utility of indoor temperature. We also refer to  $\gamma$  as *valuation factor* since it expresses the valuation of a specific household for indoor temperature.

$$U(T_t) = -\gamma \cdot (\bar{T}_t - T_t)^2 \quad (9)$$

We specify the utility function for the case study by defining an example household by its estimated marginal utility of indoor temperature  $\gamma$ , and an ideal indoor temperature. The characteristics and estimates of the corresponding marginal utility from indoor temperature stem from the median household in [Mertesacker \(2021\)](#). For this household, we obtain a  $\gamma$  of 25€/ΔT<sup>2</sup>.<sup>15</sup> Figure 1 shows the resulting utility function and variations for the exemplary household.

The heat demand in the case study, associated with the indoor temperature choice, is based on a representative building from [Diefenbach et al. \(2015\)](#) and [IWU \(2016\)](#).<sup>16</sup>

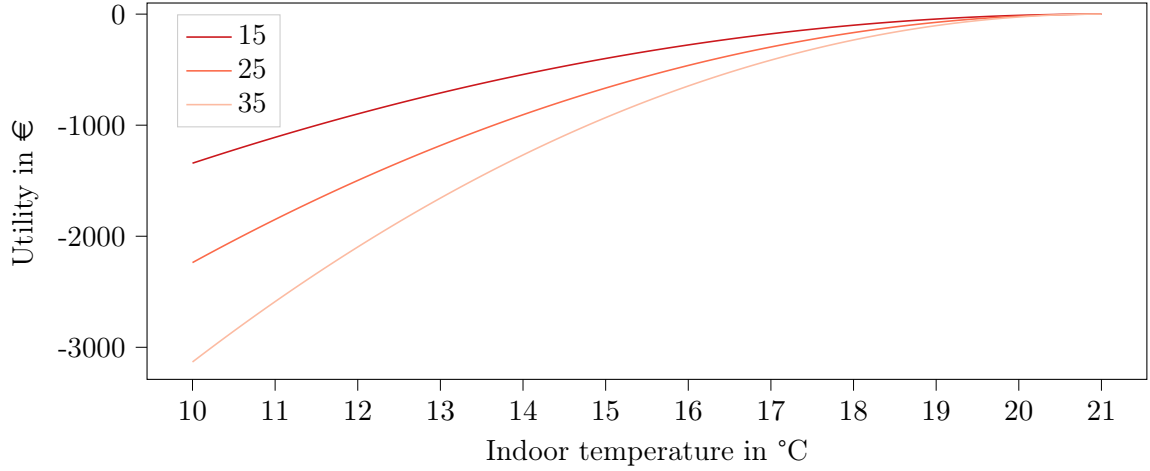
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<sup>13</sup>In our numerical simulation, we define the backstop technology with finite cost as an air-to-water heat pump. This applies under the assumption of continuously climate neutral electricity generation or based on political determinations that anticipate long-term decarbonization of power generation.

<sup>14</sup>In the numerical simulation, for simplicity, we assume constant fuel prices over time and utilize the same CO<sub>2</sub> tax rate in each year of the heating system lifetime.

<sup>15</sup>The underlying assumption of the household characteristics, the marginal utility estimates, and the computation of the valuation factor is described in [Appendix B.1](#).

<sup>16</sup>In [Appendix B.2](#), the computation of the heat demand and the underlying assumptions is described in detail.



**Figure 1:** Utility Functions of indoor temperature for varying valuation factors

To evaluate a continuous investment choice of households, we estimate functions for investment costs, CO<sub>2</sub> emissions, and fuel prices based on real data (Danish Energy Agency, 2021; Pickert et al., 2022; BAFA, 2021). The heating technologies include oil and gas condensing boilers with and without solar thermal support and an air-source heat pump. As in the theoretical model, the functions are formulated concerning the heating technology’s energy intensity level  $fph$ . We assume a system lifetime and an assessment period of 20 years. Table 1 shows the resulting technology functions.<sup>17</sup>

**Table 1:** Estimated continuous functions of investment costs, CO<sub>2</sub> emissions, and variable costs.

	Unit	Function
Investment costs	€	$c(fph) = 14,100e^{-1.019 \cdot fph}$
CO <sub>2</sub> emissions	kg/kWh	$epf(fph) = -0.110 + 0.358 \cdot fph$
Fuel price	€/kWh	$p(fph) = 0.394 - 0.331 \cdot fph$

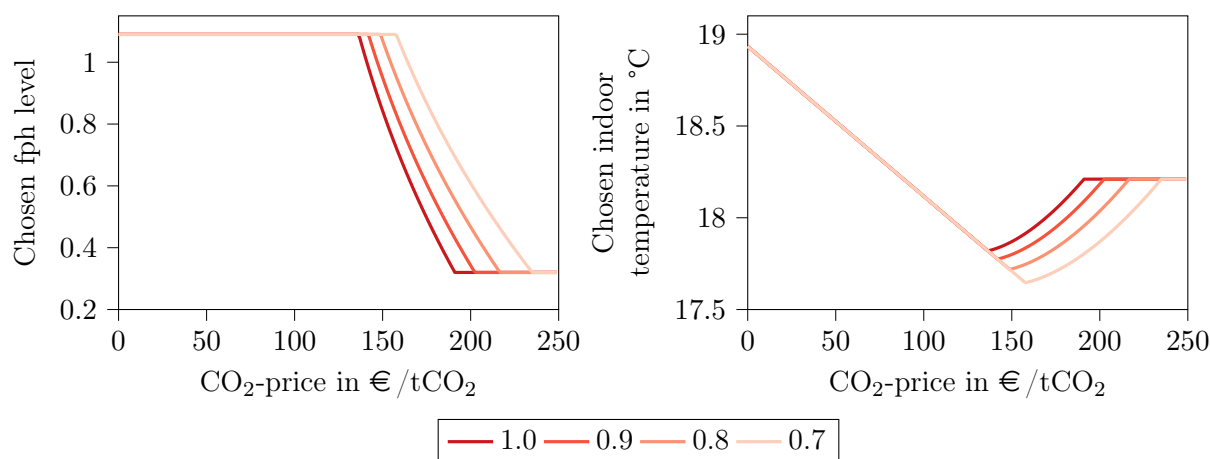
### 3.2. Results

#### 3.2.1. Continuous model

From the negative gradient of the fuel price in Table 1 follows that  $p'(fph) < 0$ . Considering Proposition 2 from Section 2,  $p'(fph) < 0$  means that a CO<sub>2</sub> price (or a subsidy) has to at least

<sup>17</sup>Appendix B.3 presents the underlying data and the computation of the technology functions.

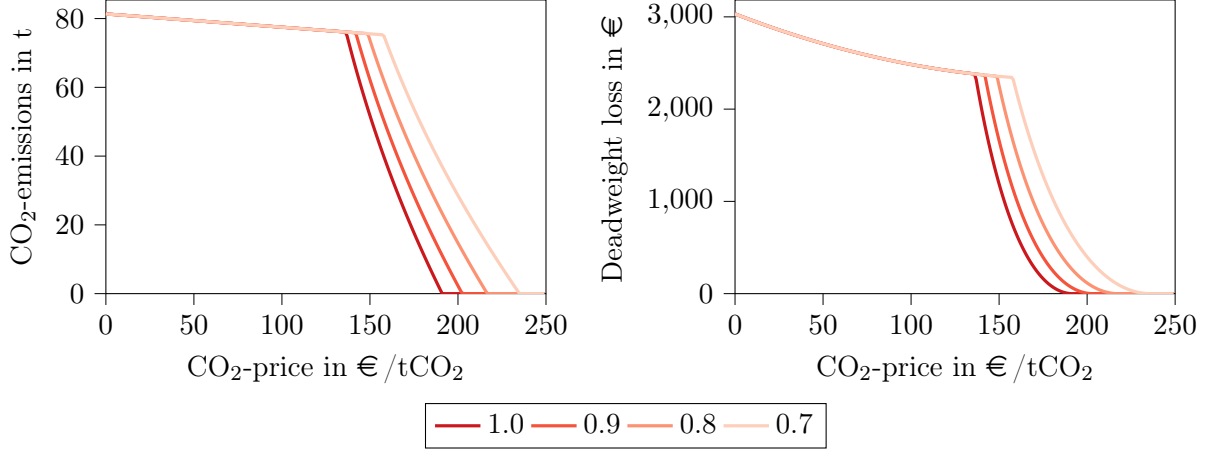
offset the decrease of the heating system’s variable fuel price induced by increasing  $fph$ . For such a CO<sub>2</sub> price, present bias leads to under-investment and, consequently, under-consumption of thermal energy. The numerical results replicate this finding on the relationship between present bias, investment, and consumption choices as illustrated in Figure 2. In case of no present bias,  $\beta = 1.0$ , there is no investment up to a CO<sub>2</sub> price of 137€/tCO<sub>2</sub>. The chosen indoor temperature at this price is 17.8°C. With an increasing CO<sub>2</sub> price, the investments in lower  $fph$  increase. As heating costs decrease, the indoor temperature increases, which is commonly referred to as the rebound effect. At a CO<sub>2</sub> price of 192€/tCO<sub>2</sub>, the household invests in climate neutral heating technology and reaches the corresponding indoor temperature of 18.2°C. As described in Section 3.1.1, we interpret this carbon tax rate  $\tau_t^{neu}$  as the implied emission damage to evaluate social welfare and consequently deadweight loss. In the presence of present bias, the household invests less and chooses a lower indoor temperature.



**Figure 2:** The chosen  $fph$  and indoor temperature levels depending on the CO<sub>2</sub> price for present biases of 1.0, 0.9, 0.8, and 0.7.

Figure 3 shows that the total CO<sub>2</sub> emissions over the 20 years of heating system lifetime follow the household’s investment and consumption choices. As discussed in Section 2.3.1, the investment in lower  $fph$  impacts emissions more than decreasing indoor temperature. In case of  $\beta = 0.7$ , emissions decrease from 81 tCO<sub>2</sub> to 75 tCO<sub>2</sub> for a CO<sub>2</sub> price increase from 0€/tCO<sub>2</sub> to 157€/tCO<sub>2</sub>, due to the decrease in temperature. The emission decline turns more significant once the investments in

lower  $fph$  start at  $158\text{€}/\text{tCO}_2$ . At an emission price of  $235\text{€}/\text{tCO}_2$ , the household invests in the climate neutral technology so that total  $\text{CO}_2$  emissions are 0.

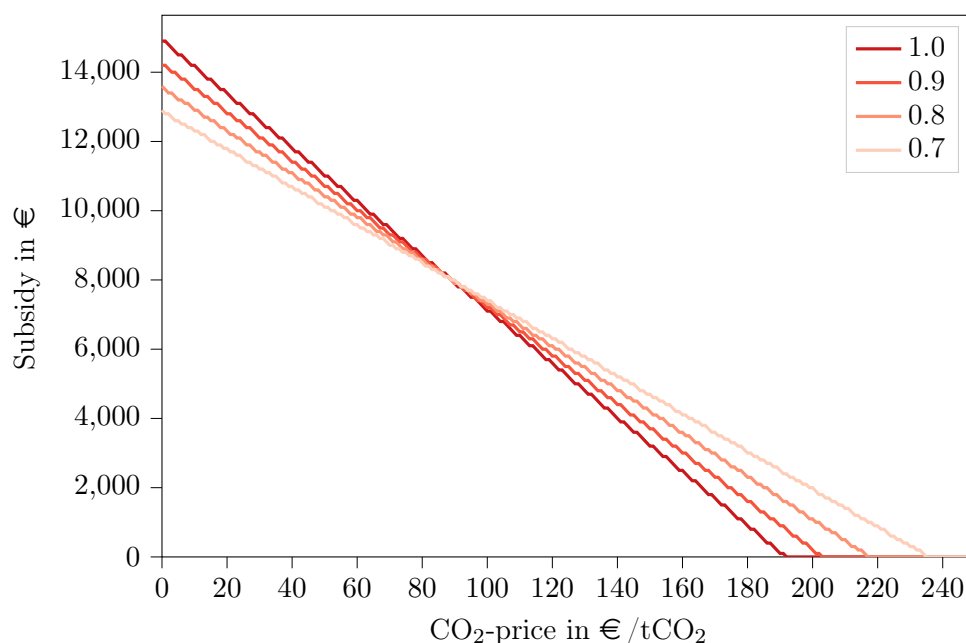


**Figure 3:** Total emissions and deadweight loss over the heating system’s lifetime of 20 years depending on the  $\text{CO}_2$  price for present biases of 1.0, 0.9, 0.8, and 0.7.

The deadweight loss over the heating system’s lifetime of 20 years is illustrated in Figure 3. It is based on the two-step procedure described in Section 3.1.1 and follows the chosen  $fph$  level. Without a  $\text{CO}_2$  price, the household invests in the option with the highest  $fph$ , leading to a deadweight loss above  $3,000\text{€}$  due to the emissions. With an increasing  $\text{CO}_2$  price, the indoor temperature first decreases slightly and with it consequently the emissions. Once the household invests in lower  $fph$ , the deadweight loss decreases convexly. Without present bias, the carbon tax rate  $\tau_t^{neu}$  of  $192\text{€}/\text{tCO}_2$  is sufficient to incentivize investment in the climate neutral technology. As Proposition 2 in Section 2.3.1 suggests, present bias leads to a deadweight loss caused by under-investment and, consequently, under-consumption. For  $\beta = 0.9$ ,  $\beta = 0.8$  and  $\beta = 0.7$  the deadweight loss at  $\tau_t^{neu}$  is  $58\text{€}$ ,  $252\text{€}$ , and  $613\text{€}$ , respectively. The loss results from the present bias externality as the  $\tau_t^{neu}$  addresses the emission externality. In Section 2.3.2, we argue that under a climate neutrality regime, a mark-up on top of the  $\text{CO}_2$  price which addresses the externality can address the internality and reach the climate neutral technology. The required mark-ups in the case study for  $\beta = 0.9$ ,  $\beta = 0.8$  and  $\beta = 0.7$  are  $11\text{€}/\text{tCO}_2$ ,  $25\text{€}/\text{tCO}_2$ , and  $43\text{€}/\text{tCO}_2$ , respectively.

According to Proposition 3 in Section 2.3.1, a subsidy is an alternative to a mark-up on the carbon tax. If the subsidy is high enough to induce investments in heat pumps, no  $\text{CO}_2$  price is needed

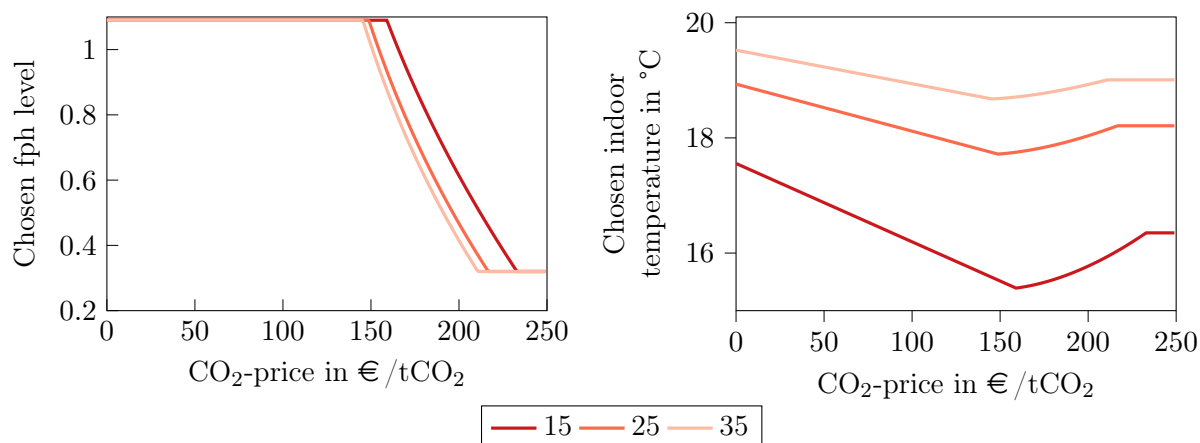
since subsequent heating does not emit CO<sub>2</sub>. If the mark-up is high enough to induce investments in climate neutral heat pumps, no subsidy is needed. Consequently, a negative relationship exists between the two policies. All policy combinations that lead to the social optimum are illustrated in Figure 4. The function's slope describing the relationship between policies depends on the level of present bias and is lower for a high present bias. The slope differences originate from the differing times at which subsidies and CO<sub>2</sub> prices affect the household. Present bias hinders households from fully considering the CO<sub>2</sub> price in their optimal choice problem. Subsidies take effect directly at the time of the investment. The higher the level of present bias, the more the CO<sub>2</sub> price must increase to reduce the required subsidy.



**Figure 4:** Combinations of CO<sub>2</sub> price and subsidy that lead to the social optimum for present biases of 1.0, 0.9, 0.8, and 0.7.

At a CO<sub>2</sub> price of 89€/t and a subsidy of 8,000€, there is an intersection of the functions for the different levels of present biases. Thus, at this combination of CO<sub>2</sub> price and subsidy, the required policy for the social optimum is independent of the level of present bias. The policy combination's CO<sub>2</sub> price creates parity between the variable costs of all technology options. In other words, the variable costs become independent of the chosen *fph*. We identify this intersection in Proposition 2 in Section 2.3.1 by stating that present bias leads to under-investment and, consequently, under-

consumption as long as total future discounted heating costs, including fuel and emission costs, for a marginal increase in  $fph$  are greater or equal to zero. If this is not the case, i.e., less efficient heating systems have lower future heating costs, present bias will lead to over-investment. At the intersection between both cases, when future discounted heating costs are equal for all  $fph$ , the investment costs determine the investment choice. As present bias affects the household's weighting between marginal changes in investment costs and marginal changes in total future discounted costs, it does not affect the household's decision for equal future discounted costs. 89€/t is the CO<sub>2</sub> price, which offsets the differences in the fuel costs. In this case, the subsidy must compensate households for the difference in investment costs between CO<sub>2</sub>-emitting and climate neutral technologies. This subsidy is 8,000€. As a result, the policy mix at the intersection of the functions is optimal, independent of the level of present bias.<sup>18</sup>



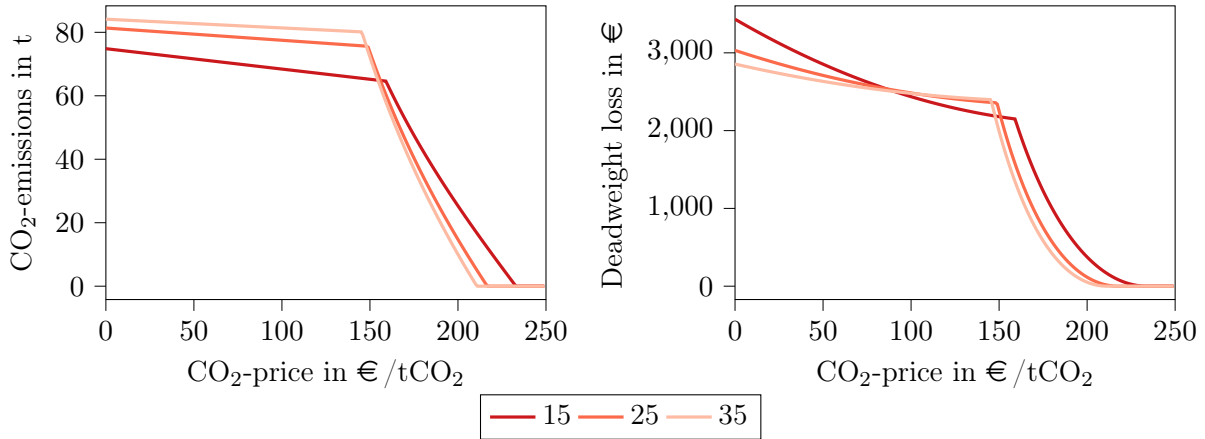
**Figure 5:** The chosen  $fph$  and indoor temperature levels depending on the CO<sub>2</sub> price for valuation factors of 15€/ΔT<sup>2</sup>, 25€/ΔT<sup>2</sup>, and 35€/ΔT<sup>2</sup>.

The utility function of households is a critical assumption. As shown in [Appendix B.1](#), we identify different valuation levels for indoor temperature. Figure 5 shows the  $fph$  level and chosen indoor temperature over CO<sub>2</sub> price for three different valuation factors given a present-bias of  $\beta = 0.8$ . A higher valuation factor implies a lower necessary CO<sub>2</sub> price to incentivize investments in  $fph$ . In the case of a low valuation factor, the household reacts first with decreasing indoor temperature, as

<sup>18</sup>The values of the optimal policy mix depend on the assumptions fed into the model, like fuel prices, heating efficiencies, and the utility function.



this yields lower utility loss compared to the additional costs of investing, as is shown in the right part of Figure 5. As soon as investments in more efficient technologies are profitable, efficiency increases, and the household increases the indoor temperature until the installation of the climate neutral backstop technology.



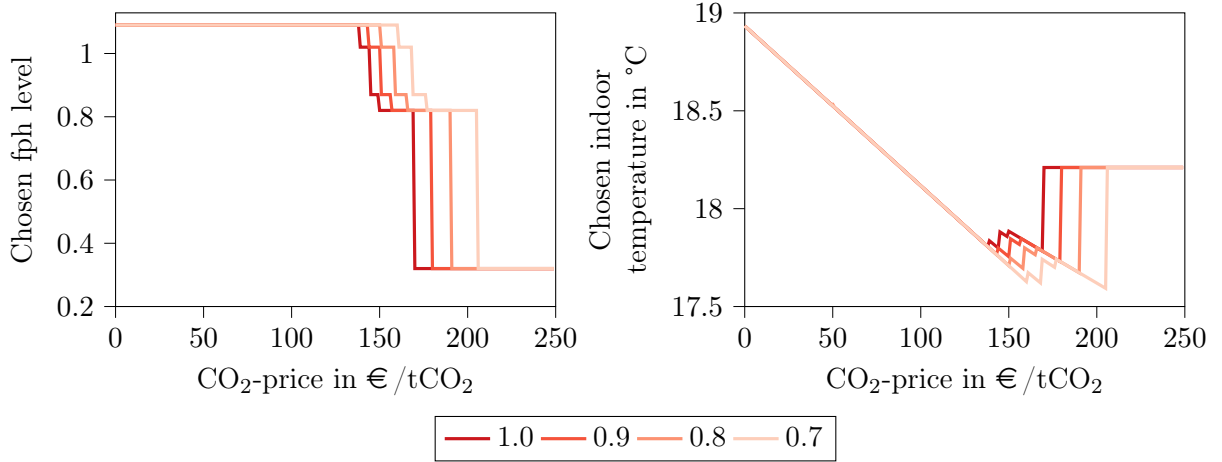
**Figure 6:** Total emissions and the deadweight loss over the heating system’s lifetime of 20 years depending on the CO<sub>2</sub> price for valuation factors of  $15\text{€}/\Delta T^2$ ,  $25\text{€}/\Delta T^2$ , and  $35\text{€}/\Delta T^2$  and a present bias of 0.8.

The CO<sub>2</sub> emissions and deadweight losses over the heating system’s lifetime of 20 years illustrated in Figure 6 show a slight decline until the start of investments in lower  $fph$  followed by a convex decline until the investment into the climate neutral technology. Before investments in more efficient technologies start, the deadweight loss is the highest for the high valuation factor since the under-consumption of indoor temperature weighs the most. The same logic also applies to why households with a high valuation factor start investing in more efficient heating systems at lower CO<sub>2</sub> prices than households with lower valuation factors. As a higher level of investments decreases the deadweight loss not only through increased efficiency but also through reduced fuel costs, they exhibit a quadratic effect on the deadweight loss. Thus, the decline in welfare is less significant for lower valuation households, which still react by decreasing indoor temperature.

### 3.2.2. Discrete model

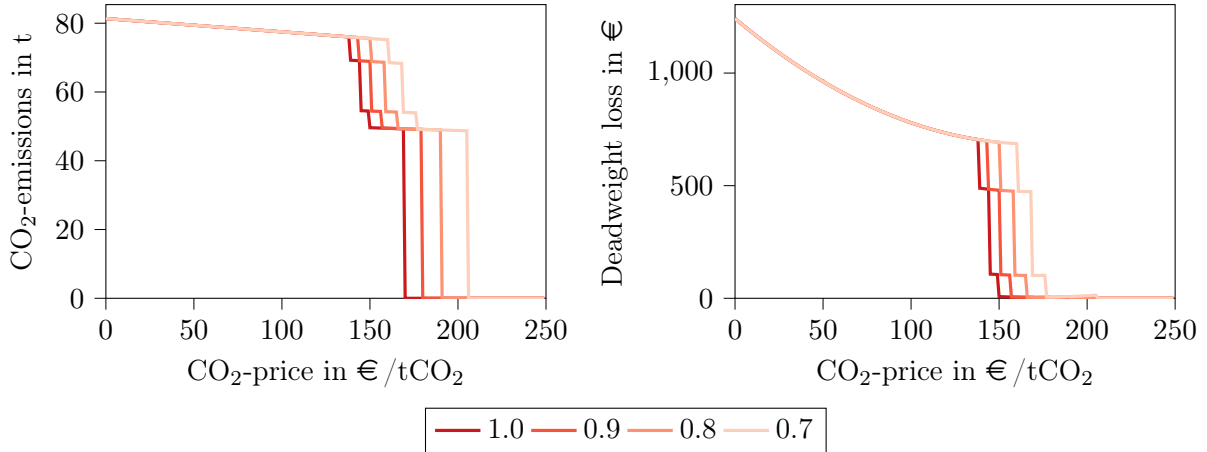
So far, the presented theoretical and numerical results assume continuous technology options so that all  $fph$  levels are feasible between the climate neutral and the least efficient option. In reality, there is only a limited set of heating technologies. Figure 7 illustrates the household’s investment

and consumption choices, given a discrete technology set, including an oil condensing boiler, a gas condensing boiler, both boiler combined with solar thermal, and an air-to-water heat pump. We define the set of technologies as the available  $fph$  levels from Section 3.1 and choose the cost and emission levels according to the functions from the continuous model (see Appendix B.3).



**Figure 7:** The chosen  $fph$  and indoor temperature levels in case of discrete technology options depending on the CO<sub>2</sub> price for present biases of 1.0, 0.9, 0.8, and 0.7.

For each present bias level, there are four break-even CO<sub>2</sub> prices that lead to a technology switch. In case of no present bias, i.e.,  $\beta = 1.0$ , the household invests in the highest  $fph$  of 1.09, i.e., the oil condensing boiler, until a CO<sub>2</sub> price of 139€/tCO<sub>2</sub>. For higher prices, the household chooses a  $fph$  of 1.02, i.e., the gas condensing boiler. The break-even points for investing in the oil condensing boiler combined with solar thermal and the gas condensing boiler combined with solar thermal are at 145€/tCO<sub>2</sub> and 150€/tCO<sub>2</sub> respectively. At a CO<sub>2</sub> price of 170€/tCO<sub>2</sub>, the household invests in the heat pump. Thus, in the discrete case 170€/tCO<sub>2</sub> is the carbon tax rate  $\tau_t^{neu}$  that induces investment in the climate neutral technology. The  $\tau_t^{neu}$  is lower in the discrete case than in the continuous case because the CO<sub>2</sub> price only has to create a break-even between the gas condensing boiler with solar thermal and the heat pump, and not between an infinitesimal less efficient heating technology and the climate neutral backstop technology. Analogously to the continuous case, present bias leads to under-investment and under-consumption.



**Figure 8:** The total emissions and deadweight loss over the heating system’s lifetime of 20 years in case of discrete technology options depending on the CO<sub>2</sub> price for present biases of 1.0, 0.9, 0.8, and 0.7.

The total CO<sub>2</sub> emissions over the heating system’s lifetime of 20 years in Figure 8 mirror the step function of  $f_{ph}$ . There is a nearly linear decrease in CO<sub>2</sub> emissions following the household’s temperature decreases and a more significant step whenever the CO<sub>2</sub> price causes a switch between two heating technologies. At the carbon tax rate  $\tau_t^{neu}$ , there are zero CO<sub>2</sub> emissions in case of no present bias. The under-investment, due to present bias, leads to CO<sub>2</sub> emissions increases. These increases are for present biases of  $\beta = 0.9$ ,  $\beta = 0.8$ , and  $\beta = 0.7$ , 49tCO<sub>2</sub>, 49tCO<sub>2</sub>, and 54tCO<sub>2</sub>. This step is significantly higher than in the continuous case as the next available technology is a gas condensing boiler with solar thermal compared to a technology with infinitesimal higher emission intensity. A mark-up on the CO<sub>2</sub> price can address the internality and incentivize investment in the heat pump as stated in Section 2.3.2. For  $\beta = 0.9$ ,  $\beta = 0.8$ , and  $\beta = 0.7$ , the mark-up is 10€/tCO<sub>2</sub>, 21€/tCO<sub>2</sub>, and 36€/tCO<sub>2</sub>, respectively. Following the two-step procedure described in Section 3.1.1, the implied damage from CO<sub>2</sub> emissions  $\tau_t^{neu}$  is lower than in the continuous case, naturally resulting in a lower total level of deadweight loss. The deadweight loss due to present bias from under-investment and under-consumption is 101€ for a present bias of  $\beta = 0.7$ . The household chooses the oil condensing boiler with solar thermal instead of a heat pump. For present biases of  $\beta = 0.9$  and  $\beta = 0.8$  the household chooses a gas condensing boiler with solar thermal, which leads to neglectable deadweight loss since  $\tau_t^{neu}$  is defined as the necessarily implied damage to break-even between the two heating technologies.

## 4. Discussion

In our stylized model, we find that single-instrument policies can be welfare optimal and target-consistent even if the household is present biased. We implicitly assume that all households, their valuation factors, and their level of present bias are homogeneous. Accounting for household heterogeneity, however, implications of policy instruments can differ, especially in distributional effects.

According to our analysis, the lower the valuation for heat, the higher the CO<sub>2</sub> price must be to induce investment in the zero-emission backstop technology. Assuming the policymaker introduces a CO<sub>2</sub> price sufficient for incentivizing investment into the zero-emission backstop technology for a household with an average valuation factor, low-valuation households would not invest in the climate neutral technology. Instead, they would pay the CO<sub>2</sub> price and heat less, while high-valuation households invest in the zero-emission backstop technology. Similarly, if instead of a CO<sub>2</sub> price, the policymaker sets a target-consistent subsidy for average households, low-valuation households will not invest sufficiently. For households with higher valuations, however, the subsidy is not only sufficient but too high: they receive more money from the state than would have been necessary to stimulate the investment. The empirical literature suggests that high-income households have a higher valuation for thermal energy than low-income households (Cayla et al., 2011; Mertesacker, 2021). This would imply that a single, uniform subsidy, which aims to reach households with low valuation factors as well, favors high-income households.

Households also show heterogeneity with respect to their level of present bias. Assume the policymaker sets a CO<sub>2</sub> price that is target-consistent for households with average present bias. As shown in Section 3.2.1, households with stronger present bias ( $\beta < \bar{\beta}$ ) would underinvest and pay the CO<sub>2</sub> price in future periods, heating less than optimal. Literature estimations of the correlation between income and the present bias level range between no correlation and a negative correlation, suggesting that low-income households experience higher levels of present bias (Meier and Sprenger, 2010; Can and Erdem, 2013; Filippini et al., 2021). We show in Section 3.2.1 that an optimal policy combination of a CO<sub>2</sub> price and subsidy exists that can account for different (unknown) levels of present bias.

Further real-world issues that our stylized model does not consider are investment distortions, hindering households from investing. Possible distortions and, thus, obstacles to investment include budget constraints, lack of access to capital, technological or regional circumstances, or split incentives between landlords and tenants. In cases where households cannot invest, otherwise target-consistent CO<sub>2</sub> prices may lead to high costs. Subsidies can help to overcome budget constraints and lack of access to capital. Suppose a subsidy is introduced as a single instrument. In that case, there is no price signal to at least partially internalize the externalities of households that cannot invest and whose heating is still associated with GHG externalities.

The policy instruments also differ regardless of households' heterogeneity. In the case of a singular implementation of a CO<sub>2</sub> price or bans on GHG emitting technologies, the households bear the full costs. With (supplementary) subsidies, by contrast, the state pays (part of) the costs. The latter may make sense from a social justice point of view or to increase acceptance among the population. Given the heterogeneity of households and their potential investment constraints, as well as the possible desirability of distributing costs between households and the state, there are arguments in favor of combining taxes (or bans) with subsidies. By distinguishing the effects of policies on investment and utilization decisions, our analysis can support a nuanced discussion of appropriate policy mixes.

## 5. Conclusion

The present paper examines the impact of present bias on optimal environmental policies aimed at achieving climate neutrality. The study generalizes Heutel's model for policy design for externality-producing durable goods when externalities are present. Besides increasing efficiency, investments in a new heating system may substitute the fuel used. Accounting for this substitution adds the dimensions of fuel price and emission intensity to our technology space. The generalization allows us to include a backstop technology with finite cost and analyze policy choices that reach climate neutrality.

This work contributes to the scientific literature in three ways. First, we generalize Heutel's model by allowing technologies to differ in fuel price and emission intensity. Second, we introduce a model framework for developing target-consistent environmental policies given a backstop technology.

It can serve as one element within a toolbox for welfare analysis given political targets beyond externality pricing. Third, we apply the model framework to the case of decarbonization in the German heating sector of private households under present bias and derive numerical magnitudes of the present bias effects.

We find that contrary to Heutel's propositions, one instrument can be sufficient to address both externality and internality. Still, a combination of subsidies and taxes can be advantageous as we show that there exists a tax-subsidy combination that is optimal regardless of the present bias level. This finding can be applied to comparable investment decisions in externality (GHG emission) producing durable goods, such as private mobility investments. The existence of the optimal policy mix is particularly relevant because the level of present bias is private information unknown to the policymaker and heterogeneous among households. Policymakers could avoid distributional effects by utilizing the present bias agnostic optimal policy mix. There are further arguments supporting policy mixes that fall short in our stylized model, including heterogeneity in the valuation of heating, investment distortions, and the costs' distribution between households and the state.

Based on our analysis, there remains room for further research. In contrast to our greenfield analysis with constant prices, in reality, households already own heating systems, and the heating system stock's age structure is heterogeneous. Therefore, households are faced not only with the question of which technology to invest in but also whether it is worth investing in a new heating system early on before the existing one breaks down. This raises questions about the timing of policy instruments, e.g., concerning the interdependencies of price paths of CO<sub>2</sub> taxes or fuel prices over time. Here, as well, the question arises as to what constitutes target-consistent policy instruments. The issue could prove complicated, as it is difficult to determine under which circumstances early heating systems replacement is required to achieve climate targets. Further, we discussed the role of household heterogeneity in our findings qualitatively. Households differ in their level of present bias, their current heating systems, and their financial capabilities. A more detailed examination of these properties could, in addition to theoretical analyses, e.g., concerning optimal policy mixes across households, also quantify effects at the level of the entire German building stock.

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## Appendix A. Proofs

*Appendix A.1. Proof 1:*

Proposition:

There does not exist any set of emission tax rates  $\tau_t$  for all  $t \in [1, \dots, T]$  that leads to the first-best outcome  $fph^{opt}$  and  $h_t^{opt}$ .

Proof analogous to [Heutel \(2015\)](#):

Suppose the contradiction: there exists a set of tax rates  $\tau_t^{opt\beta}$  that lead to  $m_t^* = m_t^{opt}$  for all  $t > 0$  and  $gpm^* = gpm^{opt}$ . The first-order condition for choice of  $m_t$  in period  $t$  is  $U'(m_t^*) = (p_t(fph^*) + epf(fph^*) \cdot \tau_t^{opt\beta}) \cdot fph^*$  or  $U'(m_t^{opt}) = (p_t(fph^{opt}) + epf(fph^{opt}) \cdot \tau_t^{opt\beta}) \cdot fph^{opt}$ . Since  $U''$  is strictly negative the only optimal solution, when  $\beta = 1$ , is  $\tau_t^{opt\beta} = \tau_t^{pig}$ . When  $\beta < 1$ , the optimal solution does not change, since the planner does not consider the quasi-hyperbolic discount factor, but first order conditions of the consumer change. Thus, it does not equal the planner's solution.

*Appendix A.2. Proof 2:*

Proposition:

If  $\tau_t = \tau_t^{pig}$  for all  $t \in [1, \dots, T]$  and  $\sum_{t=1}^T \delta^t \cdot h_t^* \cdot [p_t(fph^*) + epf(fph^*) \cdot \tau_t] + [p'_t(fph^*) + epf'(fph^*) \cdot \tau_t] \cdot fph^*$  then  $fph^* > fph^{opt}$  and  $h_t^* < h_t^{opt}$  for all  $t \in [1, \dots, T]$ .

Proof analogous to [Heutel \(2015\)](#):

Note that the consumer's choice of  $fph^*$  is given by her first-order condition; call this equation F:

$$F = -c'(fph^*) - \beta \cdot \sum_{t=1}^T \delta^t \cdot h_t^* \cdot \left[ p_t(fph^*) + epf(fph^*) \cdot \tau_t + [p'_t(fph^*) + epf'(fph^*) \cdot \tau_t] \cdot fph^* \right] \quad (\text{A.1})$$

The implicit function theorem can be used to show how  $fph^*$  varies with  $\beta$ :

$$\frac{dfph^*}{d\beta} = \frac{-dF/d\beta}{dF/dfph} = \frac{\sum_{t=1}^T \delta^t \cdot h_t^* \cdot \left[ p_t(fph^*) + epf(fph^*) \cdot \tau_t + [p'_t(fph^*) + epf'(fph^*) \cdot \tau_t] \cdot fph^* \right]}{dF/dfph} \quad (\text{A.2})$$

The denominator is negative from the second-order condition of the consumer's optimization problem. The numerator is positive (or zero) if

$$\sum_{t=1}^T \delta^t \cdot h_t^* \cdot \left[ [p_t(fph^*) + epf(fph^*) \cdot \tau_t] + [p'_t(fph^*) + epf'(fph^*) \cdot \tau_t] \cdot fph^* \right] \geq 0 \quad (\text{A.3})$$

If the numerator is positive, then  $dfph^*/d\beta < 0$ . Since when  $\beta = 1$   $fph^* = fph^{opt}$ , it follows that in the case of  $\beta < 1$   $fph^* > fph^{opt}$ .

The consumer's choice of  $h_t^*$  in each period is a function of the total price of one kWh of heating,  $[p_t(fph) + epf(fph) \cdot \tau_t] \cdot fph$ , from the first-order condition  $U'(h_t^*) = [p_t(fph) + epf(fph) \cdot \tau_t] \cdot fph$ . Since  $U'' < 0$ ,  $dh_t^*/dfph < 0$ . When  $fph = fph^{opt}$ ,  $h_t^* = h_t^{opt}$ . But when  $\beta < 1$ ,  $fph^* > fph^{opt}$ , so  $h_t^* < h_t^{opt}$  for each period  $t > 0$  (and vice versa).

*Appendix A.3. Proof 3:*

Proposition:

Let  $\beta < 1$ : The first best is achieved by setting  $\tau_t = \tau_{pig}$  in each period  $t > 0$  and setting a technology subsidy in the form of  $(\frac{1}{fph_{min} - fph_{max}} \cdot fph + \frac{1}{1 - \frac{fph_{min}}{fph_{max}}}) \cdot \sigma$  with  $\sigma = (fph_{min} - fph_{max}) \cdot (\beta - 1) \cdot \sum_{t=1}^T \delta^t \cdot h_t^{opt} \cdot [ [p_t(fph^{opt}) + epf(fph^{opt}) \cdot \tau_t^{pig}] + [p'_t(fph^{opt}) + epf'(fph^{opt}) \cdot \tau_t^{pig}] \cdot fph^{opt} ]$ .

Proof:

The subsidy is defined as a monetary benefit  $\sigma$  that is scaled with  $\frac{1}{fph_{min} - fph_{max}} \cdot fph + \frac{1}{1 - \frac{fph_{min}}{fph_{max}}}$ .

The consumer's problem is:

$$\begin{aligned} & \max_{fph, \{h_t\}_{t=1}^T} -c(fph) + \left( \frac{1}{fph_{min} - fph_{max}} \cdot fph + \frac{1}{1 - \frac{fph_{min}}{fph_{max}}} \right) \cdot \sigma \\ & + \beta \cdot \left[ \sum_{t=1}^T \delta^t \cdot \left[ U(h_t) - [p_t(fph) + epf(fph) \cdot \tau_t^{pig}] \cdot fph \cdot h_t \right] \right] \end{aligned} \quad (\text{A.4})$$

Subject to

$$U'(h_t) - [p_t(fph) + epf(fph) \cdot \tau_t^{pig}] \cdot fph = 0, \forall t \quad (\text{A.5})$$

Consider this problem's Lagrangian, where the constraint from the period  $t$  choice of  $h_t^*$  has a multiplier  $\lambda_t$ . The first-order condition with respect to  $h_t$  is:

$$\beta \cdot \delta^t \left[ U'(h_t^*) - [p_t(fph^*) + epf(fph^*) \cdot \tau_t^{pig}] \cdot fph^* \right] + \lambda_t \cdot U''(h_t^*) = 0 \quad (\text{A.6})$$

The term in brackets is zero from the first-order condition from the static choice of  $h_t$ , Since  $U$  is strictly negative,  $\lambda_t = 0$  for all  $t > 0$ . Then, the first-order condition for  $fph$  is:

$$\begin{aligned}
& -c'(fph^*) + \frac{1}{fph_{min} - fph_{max}} \cdot \sigma \\
& - \beta \cdot \left[ \sum_{t=1}^T \delta^t \cdot h_t^* \cdot [p_t(fph^*) + epf(fph^*) \cdot \tau_t^{pig}] + [p'_t(fph^*) + epf'(fph^*) \cdot \tau_t^{pig}] \cdot fph^* \right] \quad (A.7) \\
& = 0
\end{aligned}$$

With the value of  $\sigma$  as given, this first-order condition can be written as:

$$\begin{aligned}
& -c'(fph^*) - \sum_{t=1}^T \delta^t \cdot \left[ \beta \cdot h_t^* \cdot [p_t(fph^*) + epf(fph^*) \cdot \tau_t^{pig}] + [p'_t(fph^*) + epf'(fph^*) \cdot \tau_t^{pig}] \cdot fph^* \right] \\
& + (1 - \beta) \cdot h_t^{opt} \cdot [p_t(fph^{opt}) + epf(fph^{opt}) \cdot \tau_t^{pig}] + [p'_t(fph^{opt}) + epf'(fph^{opt}) \cdot \tau_t^{pig}] \cdot fph^{opt} \right] \\
& = 0 \quad (A.8)
\end{aligned}$$

When  $fph^* = fph^{opt}$ , then  $h_t^* = h_t^{opt}$  for all  $t > 0$  since  $\tau_t = \tau_t^{pig}$ . Plugging  $fph^* = fph^{opt}$  and  $h_t^* = h_t^{opt}$  in Equation A.8 makes it equal to the social planner's first-order condition. So  $fph^{opt}$  and  $h_t^{opt}$  solve the consumer's problem, and by the second-order condition this is a unique solution.

## Appendix B. Numerical simulation

### Appendix B.1. Household's heating valuation

Mertesacker (2021) provides a utility function and estimates of its parameters for a case study on energy consumers in Germany. The utility function from Equation (9) includes a household's indoor temperature valuation factor  $\gamma$ .  $\gamma$  is obtained by multiplying household characteristics  $x$  as binary vector and the estimated coefficient for each characteristic  $\delta$ . We specify the utility function for the case study by defining an example household by its characteristics  $x$  and corresponding estimated marginal utility of indoor temperature  $\delta$ . The characteristics and estimates of the corresponding marginal utility from indoor temperature stem from Mertesacker (2021) as listed in Table B.2. For the case study in this paper, we construct three sample households covering the potential spread of valuations as shown in Table B.2. The base household has a valuation of  $25\text{€}/\Delta T^2$ , while

the minimum obtained value is  $13\text{€}/\Delta T^2$  and the maximum  $38\text{€}/\Delta T^2$ . For simplicity, we utilize valuations of  $15\text{€}/\Delta T^2$ ,  $25\text{€}/\Delta T^2$ , and  $35\text{€}/\Delta T^2$  in the valuation sensitivity.

**Table B.2:** Estimates of a household's characteristic's effect on marginal utility from indoor temperature from [Mertesacker \(2021\)](#).

Parameter		$\delta$	x		
		Coefficient	Base	Min	Max
Constant		12,256	1	1	1
Age	30-39	1.262	0	0	0
	40-49	0.094	1	1	0
	50-59	3.918	0	0	0
	$\geq 60$	4.811	0	0	1
# adults	2	3.014	1	1	0
	3	6.654	0	0	1
	$\geq 4$	3.850	0	0	0
# children	1	-0.368	0	1	0
	$\geq 2$	2.964	1	0	1
Is employed		-2.234	1	1	0
Has Abitur		3.271	1	0	1
Is owner		-0.033	1	1	0
Income	$< 1,500\text{€}$	2.329	0	0	1
	$\geq 3,500\text{€}$	0.320	0	1	0
Dwelling size	Small: $< 1\text{st}$ tercile	-1.103	0	1	0
	Large: $> 2\text{nd}$ tercile	5.322	1	0	1
Valuation $\beta x$ in $\text{€}/\Delta T^2$			25	13	38

### Appendix B.2. Building's heat demand

The heat demand for the representative building used in the numerical case study is based on the building "SFH 1" from the building stock model of [Diefenbach et al. \(2015\)](#) and [IWU \(2016\)](#)<sup>19</sup>. The building definition in [Diefenbach et al. \(2015\)](#) is part of a model for the German building stock of the year 2009 via six representative average buildings. In this model "SFH 1" represents the most common average building in Germany. It is a single-family home with  $147.1\text{ m}^2$  of floor area. Based on the calculations implemented in [IWU \(2016\)](#), the specific heat demand of the household (including domestic water supply, storage and distribution losses) for the ideal room temperature of  $21^\circ\text{C}$  is set to  $224\text{ kWh}/\text{m}^2$ . This value includes an assumed reduction factor of

<sup>19</sup>Specified with the code DE.National.2009.002.01

0.86 which corrects for the heated area and the reduction of temperatures during the night. Using the calculation methods and definitions from IWU (2016), the heat demand is determined for different temperature levels. Theoretical calculation methods for determining heat demand tend to overpredict the real heat demand (Mertesacker, 2021; Loga et al., 2012). To account for this we assume an adaptation factor of 0.8 for our representative building, based on Loga et al. (2012) and IWU (2016). The choice of this factor is consistent with the results of Mertesacker (2021), who estimates lower adaptation factors, but does not take into account domestic hot water generation. The heat demand in our model  $h$  [kWh] is then approximated as a linear function, depending on the chosen temperature  $T$  [°C]:

$$h = 0.8 \cdot 147.1 \text{ m}^2 \cdot \left( 11.61 \frac{\text{kWh}}{\text{m}^2 \cdot \text{°C}} \cdot T - 19.85 \frac{\text{kWh}}{\text{m}^2} \right) \quad (\text{B.1})$$

### Appendix B.3. Heating technologies

Within the case study, we utilize stylized continuous functions of investment costs, CO<sub>2</sub> emissions, and fuel prices depending on the chosen  $fph$ -level. These functions are based on technical and economic heating system parameters from Danish Energy Agency (2021), emission intensities from BAFA (2021), and fuel price trajectories from Pickert et al. (2022). Danish Energy Agency (2021) provides data on real heating systems including reference capacities, efficiencies, and cost components as shown in Table B.3. We combine the solar thermal system with both an oil boiler and a gas boiler to obtain in total five investment options for the household in the case study.  $fph$  is the reciprocal of the efficiency. The efficiency of the combined heating system of a boiler and solar thermal results from the assumption that the solar thermal accounts for 20% of heat demand (BDEW, 2020). The investment costs of for each technology are obtained by scaling the equipment costs from Table B.3 to the uniform heating system size of 12.5 kW for the conventional technologies and 6.25 kW for the heat pump and adding the corresponding installation costs. Figure B.9 illustrates the resulting investment costs depending on  $fph$ , both for the discrete heating systems and a fitted exponential function.

The fuel data in Table B.4 contains the CO<sub>2</sub>-intensities from BAFA (2021) and the average fuel prices over the next 20 years from Pickert et al. (2022). Note that the fuel prices are final consumption prices accounting for grid fees and other levies. Accounting for the efficiencies of the heating

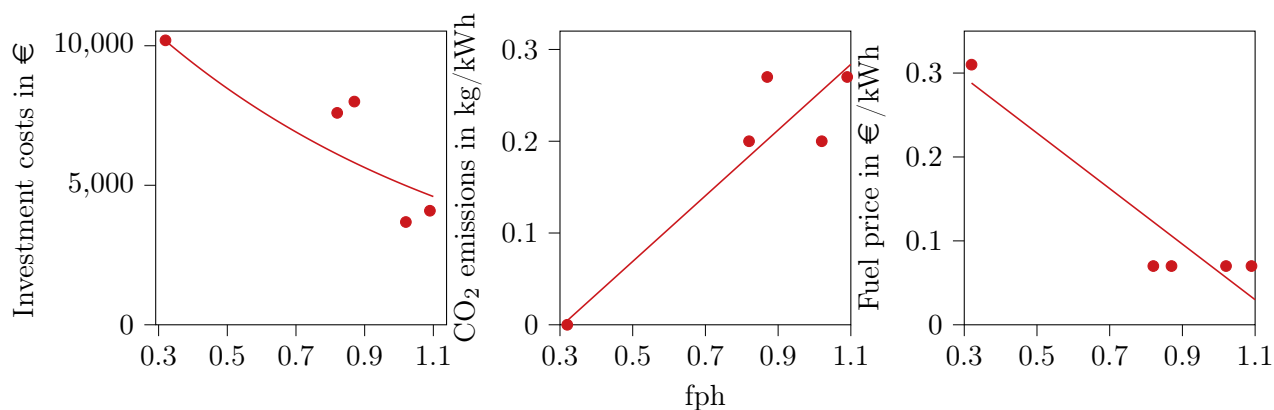
systems, Figure B.9 shows the resulting discrete emission intensities and fuel prices in dependence of  $fph$  as well as fitted linear functions.

**Table B.3:** Technical and economic properties of oil boiler, gas boiler, air-to-water heatpump, and solar thermal systems (Danish Energy Agency, 2021).

Heating technology	Capacity $\text{kW}_{\text{th}}$	Efficiency	Equipment costs €	Installation costs €
Oil boiler	20	0.92	4260	1300
Gas boiler	14	0.98	2702	1158
Air-to-Water heatpump	7	3.15	6780	3830
Solar thermal	4.2		2872	1228

**Table B.4:** Energy prices based on Pickert et al. (2022) and  $\text{CO}_2$ -intensities based on BAFA (2021).

Fuel	Fuel price €/kWh	$\text{CO}_2$ -intensity $\text{kgCO}_2/\text{kWh}$
Oil	0.0738	0.266
Gas	0.0743	0.201
Electricity	0.3142	0



**Figure B.9:** Investment costs,  $\text{CO}_2$  emissions, and fuel prices for different heating technologies, including gas and oil condensing boiler, gas and oil boiler combined with a solar thermal system respectively, and an air-source heat pump. Both the discrete technologies as well as fitted functions are displayed.