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# Minimum wages in an automating economy

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## Abstract

We explore the suitability of the minimum wage as a policy instrument for reducing emerging income inequality created by new technologies. For this, we implement a binding minimum wage in a task-based framework, in which tasks are conducted by machines, low-skill, and high-skill workers. In this framework, an increasing minimum wage reduces the inequality between the low-skill wage and the other factor prices, whereas the share of income of low-skill workers in the national income is nonincreasing. Then, we analyze the impact of an automating economy along the extensive and intensive margins. In a setting with a minimum wage, it can be shown that automation at the extensive margin and the creation of new, labor-intensive tasks do not increase the aggregate output in general, as the displacement of low-skill workers counteracts the positive effects of cost-savings. Finally, we highlight a potential trade-off between less inequality of the factor prices and greater inequality of the income distribution when a minimum wage is introduced into an automating economy.

## KEYWORDS

automation, displacement effects, employment, inequality, labor demand, minimum wage, tasks, wages

## JEL CLASSIFICATION

E25, J20, J31, J38

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## 1 | INTRODUCTION

In the last decades, new technologies, such as artificial intelligence and robots and other forms of automation, have been rapidly developing. These new technologies will likely have a significant impact on the economy. In particular, the labor market will change fundamentally in the future (e.g., Brynjolfsson & McAfee, 2014; Ford, 2015). Frey and Osborne (2017) explore the susceptibility of jobs in relation to automation and estimate that approximately 47% of current jobs in the United States (US) could be automated within one or two decades. Empirical studies show that automation has a substantial impact on routine tasks, leads to a polarized labor force, and increases inequality in the economy (e.g., Acemoglu & Restrepo, 2020a; Autor, 2015; Autor & Dorn, 2013; Autor et al., 2003, 2015; Goos & Manning, 2007; Graetz & Michaels, 2018). Moreover, Goos et al. (2019) emphasize that the adjustment costs from automation on unemployed job seekers are unequally distributed between low-skill and high-skill workers. To reduce the emerging inequality various policy instruments, such as taxing robots, a basic universal income, or a minimum wage, are discussed (e.g., Acemoglu et al., 2020; Costinot & Werning, 2018; Freeman, 2015; Furman, 2019; Guerreiro et al., 2017; McAfee & Brynjolfsson, 2016; Thuemmel, 2018).

Little is known, however, about the effects of the minimum wage in conjunction with automation. In one of the few existing studies, Lordan and Neumark (2018) empirically show that higher minimum wages reduce employment in automatable jobs. Moreover, they emphasize that there are groups of workers, such as older and less-skilled workers, that are frequently ignored in the empirical literature on the effects of minimum wages. However, it appears that scarcely any theoretical work exists on the effects of a minimum wage in a task-based framework, in which tasks are increasingly conducted by machines replacing low-skill workers. One exception is the work by Aaronson and Phelan (2019), who develop a theoretical framework based on tasks to test for the labor market consequences of minimum wages.

The aim of this paper is to explore the effects of a binding minimum wage on aggregate output, employment, factor prices, and various measures of the income distribution in an automating economy.

To analyze the labor market effects of a minimum wage in conjunction with automation, we build on the work of Acemoglu and Restrepo (2018a, 2018b, 2018d) and that of Acemoglu and Autor (2011), both of which are interconnected, and are based on Zeira (1998) and Acemoglu and Zilibotti (2001). A task-based framework takes up the notion of labor markets that can be empirically characterized by the task content of jobs (e.g., Goos et al., 2019). Theoretically, a task-based framework allows us to model automation along intensive and extensive margins (Acemoglu & Restrepo, 2018c), also with respect to the effects that may arise in the interplay with the introduction of a minimum wage. In our task-based framework, tasks in a unit interval are conducted by machines, low-skill, and high-skill workers. The range of tasks that machines and low-skill workers can produce is bounded by exogenous thresholds. The assumption of comparative advantage for each production factor on a subset of the tasks leads to a simple allocation of the factors. Hence, our task interval is divided into three intervals with increasing complexity, where machines produce the tasks in the first interval, low-skill workers produce the tasks in the middle interval, and high-skill workers produce the tasks in the last interval. By assuming a fixed and inelastic supply of machines, low-skill, and high-skill workers, we implement a minimum wage that is higher than the equilibrium low-skill wage and determine the new equilibrium.

Under an automating economy, we understand the impact of technological progress on changing exogenously the thresholds of tasks, the productivity of production factors, and the task interval. More precisely, we consider the consequences of increasing automation, whereby we distinguish between automation at the extensive and intensive margins. Automation at the extensive margin increases the measure of tasks that machines are able to produce whereas automation at the intensive margin (which we call the “deepening of automation”) means that the productivity of machines on the tasks increases (e.g., by replacing older machines with newer ones). Analogously to the increasing automation, technological progress can enable low-skill and high-skill workers to raise their productivity in their tasks and their range of tasks. Furthermore, technological progress can lead to the creation of new, more complex tasks that are labor-intensive, as assumed above.

First, we analyze the effects of an increase in the minimum wage and show that the low-skill employment, and thus the aggregate output, decrease. Moreover, a higher minimum wage reduces the inequality between the low-skill and the high-skill wage, as well as the rental rate of machines. In other words, machines and high-skill workers become relatively cheaper than low-skill workers: this can lead to a displacement of low-skill workers wherein the share of their income in the national income decreases. Furthermore, the expected low-skill wage decreases. Here, we consider the expected low-skill wage as a group-specific welfare measure of low-skill workers, as the income of the low-skill workers who retain their job increases, whereas the income of those who become unemployed falls to zero.

Subsequently, we explore the effects of an automating economy in the presence of a minimum wage. We start by increasing automation at the extensive margin. Machines displace low-skill workers if they are relatively cheaper in performing the new tasks. On the one hand, this increases the aggregate output that has a positive impact on the labor demand. On the other hand, the displacement of low-skill workers from the tasks, which are automated, reduces the demand for low-skill workers and, consequently, the low-skill employment that has a negative impact on the aggregate output. We observe two counteracting effects of the displacement of low-skill workers on the aggregate output and on low-skill employment. The magnitude of these effects decides whether the impacts on the aggregate output and low-skill employment are positive or negative. In particular, it is possible that the impact on the aggregate output is positive and the impact on low-skill employment is negative. Therefore, in contrast to Acemoglu and Restrepo (2018a, 2018b, 2018d), the impact on aggregate output and low-skill employment is ambiguous whenever there is a displacement of low-skill workers in the economy with a binding minimum wage.

We show that automation at the intensive margin raises the aggregate output, the rental rate, and the high-skill wage. Consequently, as the low-skill wage is fixed by the minimum wage, the inequality between the low-skill wage and the other factor prices increases. Furthermore, low-skill workers can displace high-skill workers in some cases and this can be greater than the displacement of machines. This is contrary to the model without a minimum wage (e.g., Acemoglu & Restrepo, 2018a). Moreover, the sign of the effect of the deepening of automation on the employment is ambiguous. Analogously, we can consider an increase in the productivity of low-skill or high-skill workers. If the productivity of high-skill workers increases, the mechanisms are similar and lead to the same results. If the productivity of low-skill workers increases, their employment increases and the share of income of low-skill workers in the national income is nondecreasing, as low-skill workers can displace machines and high-skill workers in some cases.

Moreover, we consider the effects of an expansion of the range of tasks that low-skill workers are able to perform. We show that this expansion raises the aggregate output, the employment, and the share of income of low-skill workers in the national income if low-skill workers displace high-skill workers by expanding their skills. The sign of the effect on the ratio between the low-skill and the high-skill wage is ambiguous.

By analyzing the impact of the creation of new tasks, we show that the impact on the aggregate output and low-skill employment is ambiguous, similar to the behavior in the case of automation at the extensive margin. For the same reason as before, this result also differs from the model without a minimum wage (Acemoglu & Restrepo, 2018a).

Our contribution relates to a large literature that theoretically explores the labor market effects of minimum wages in the standard neoclassical model (e.g., Borjas, 2012), emphasizing the possible employment-enhancing effects of minimum wages in monopsony (e.g., Manning, 1995; Robinson, 1969), or focusing on the interplay of minimum wages with search frictions (e.g., Burdett & Mortensen, 1998; Pissarides, 2000). For the most part, minimum wage effects are analyzed with respect to employment, but there are also contributions that look into more encompassing welfare measures (e.g., Gerritsen & Jacobs, 2020; Lavecchia, 2020). We contribute to the existing literature by focusing on the interplay of minimum wages and automation in a task-based framework. In addition to showing the effects of a minimum wage on employment, we also analyze the effects of a minimum wage on various measures of wage and income distribution.

There is only a limited number of contributions to the literature on the effects of minimum wages using a task-based framework. Building on the task-based framework of Autor et al. (2003), Aaronson and Phelan (2019) explore the effect of an increase in the minimum wage on the employment of routine and nonroutine workers. They show that an increase in the minimum wage decreases the employment of routine workers, who are either displaced by nonroutine workers or capital. Chu et al. (2020) endogenize R&D and automation to analyze the impact of a minimum wage on unemployment, high-skill workers, economic growth, R&D, and automation in a Schumpeterian growth model.

Our contribution differs from the aforementioned approaches to analyze the effects of a minimum wage in a task-based framework. Our tasks do not distinguish between routine and nonroutine work, but may be conducted by machines, low-skill, or high-skill workers. Moreover, R&D is not endogenous in our model, but automation may occur in different forms, which yields distinct labor market effects when a minimum wage is introduced. In particular, this paper explores, in contrast to the above-mentioned papers, the effects of five different types of automation on aggregate output, employment, factor prices, and various measures of the income distribution in the presence of a minimum wage.

The paper is organized as follows. In Section 2 we introduce our model with a binding minimum wage. The comparative statics are explored in Section 3, while Section 4 discusses generalizations. In Section 5 we conclude. Details of the proofs can be found in Appendix A and the Online Appendix.

## 2 | THE MODEL

This section introduces the theoretical framework for exploring the economic consequences of introducing a minimum wage in an automating economy. We use a task-based framework similar to Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018a, 2018b).

The unique final good  $Y$  is aggregated by combining the outputs of tasks in a unit measure task interval  $[N - 1, N]$ ,  $N \geq 1$ , according to a Cobb-Douglas function that is given by

$$Y = \exp\left(\int_{N-1}^N \ln y(x) dx\right), \quad (1)$$

where  $y(x)$  denotes the output of a task  $x$ . We postulate higher-indexed tasks as more complex. This enables us to model the creation of new, more complex tasks by shifting our task interval through an increase in  $N$  (Acemoglu & Restrepo, 2018a, 2018d).

The final good is produced by machines (capital), low-skill, and high-skill workers in a competitive market. We assume that the supply of machines, denoted by  $K$ , as well as the supply of low-skill and high-skill workers, denoted by  $L$  and  $H$ , respectively, are positive, fixed, and inelastic. Moreover, there are thresholds  $I$  and  $S$  (with  $N - 1 < I < S < N$ ), where  $I$  denotes the technological frontier of automation and  $S$  denotes the different abilities between low-skill and high-skill workers. High-skill workers can produce each task.

The following output function summarizes our previous assumptions:

$$y(x) = \begin{cases} \gamma_H(x)h(x) + \gamma_L(x)l(x) + \gamma_K(x)k(x) & \text{if } x \in [N - 1, I], \\ \gamma_H(x)h(x) + \gamma_L(x)l(x) & \text{if } x \in (I, S], \\ \gamma_H(x)h(x) & \text{if } x \in (S, N], \end{cases}$$

where  $\gamma_i(x)$ ,  $i \in \{K, L, H\}$ , denotes the productivity function of the corresponding factor and  $k(x)$ ,  $l(x)$ ,  $h(x)$  denote the demand for machines, low-skill, and high-skill workers, respectively, for some task  $x \in [N - 1, N]$ .

The next assumption introduces comparative advantages into our model.

**Assumption 2.1** (Comparative advantage). Assume that the productivity functions  $\gamma_i$ ,  $i \in \{K, L, H\}$ , and the ratios  $\gamma_1 := \frac{\gamma_L}{\gamma_K}$  and  $\gamma_2 := \frac{\gamma_H}{\gamma_L}$  are continuously differentiable and strictly increasing. Moreover, assume the following domains and ranges:

$$\begin{aligned} \gamma_K, \gamma_L, \gamma_H, \gamma_1: [0, \infty) &\rightarrow (0, \infty), \\ \gamma_2: [0, \infty) &\rightarrow (1, \infty).^1 \end{aligned}$$

The increasing productivity functions correspond to the postulation of the increased complexity of higher-indexed tasks. The properties of the ratios imply that high-skill workers have a comparative advantage relative to machines and low-skill workers in higher-indexed tasks, as well as that low-skill workers have a comparative advantage relative to machines in higher-indexed tasks. This Assumption 2.1 also simplifies the allocation of tasks with respect to all production factors within the task interval. The feature that  $\gamma_2$  is greater than 1 will guarantee that the high-skill wage is always greater than the low-skill wage.

We denote the equilibrium rental rate (or the cost of machines) by  $R$ , the equilibrium low-skill and high-skill wage by  $W_L$  and  $W_H$ , respectively. Moreover, we introduce a binding minimum wage  $W_m \in (W_L, W_0)$ .<sup>2</sup>

<sup>1</sup>For our considerations, it is not necessary that the productivity functions  $\gamma_K, \gamma_L, \gamma_H$  are increasing. Moreover, this assumption is not necessary, but it makes the comparative statics easier.

<sup>2</sup>The low-skill wage is bounded above by  $W_0$  (see Appendix A).

Appendix A shows (under the assumption that we choose the final good as numeraire) that for any  $K, L, H > 0, N \geq 1, I, S$  with  $N - 1 < I < S < N$  there is a unique equilibrium that is characterized by thresholds  $(I^*, S^*)$  with  $N - 1 < I^* < S^* < N$ , the demand for low-skill workers

$$L_m = B \cdot \left( \frac{1}{W_m} \right)^{\frac{1}{1-S^*+I^*}} \cdot (S^* - I^*), \quad (2)$$

and the aggregate output

$$Y = B \cdot \left( \frac{1}{W_m} \right)^{\frac{S^*-I^*}{1-S^*+I^*}}, \quad (3)$$

where

$$B = \left( \exp \left( \int_{N-1}^{I^*} \ln \gamma_K(x) dx + \int_{I^*}^{S^*} \ln \gamma_L(x) dx + \int_{S^*}^N \ln \gamma_H(x) dx \right) \cdot \left( \frac{K}{I^* - N + 1} \right)^{I^*-N+1} \cdot \left( \frac{H}{N - S^*} \right)^{N-S^*} \right)^{\frac{1}{1-S^*+I^*}}.$$

Moreover, the rental rate, the minimum wage, and high-skill wage satisfy the equations:

$$R = Y \cdot \frac{I^* - N + 1}{K}, \quad W_m = Y \cdot \frac{S^* - I^*}{L_m}, \quad W_H = Y \cdot \frac{N - S^*}{H}. \quad (4)$$

The market-clearing conditions are given by

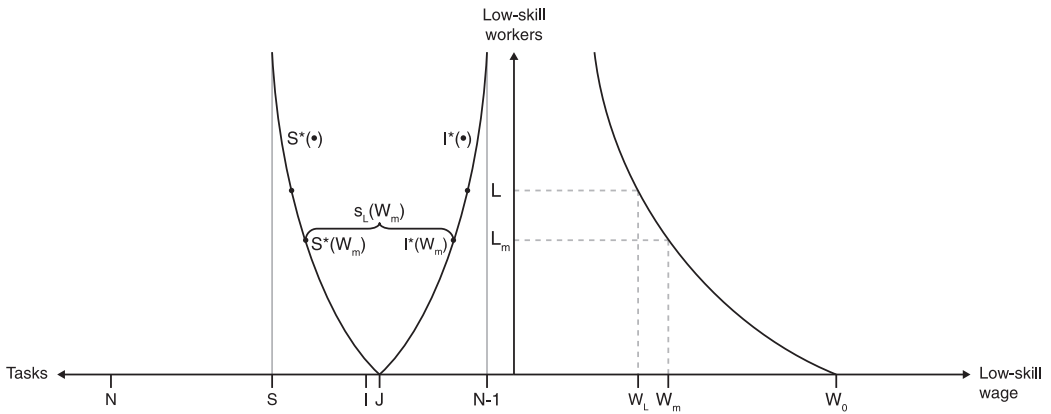
$$K = \int_{N-1}^{I^*} k(x) dx, \quad L > L_m = \int_{I^*}^{S^*} l(x) dx, \quad H = \int_{S^*}^N h(x) dx. \quad (5)$$

Finally, Appendix A indicates that the machines produce the tasks within  $[N - 1, I^*]$ , the low-skill workers within  $(I^*, S^*]$ , and the high-skill workers within  $(S^*, N]$ . The measure of these sets are the respective shares in the national income  $s_i, i \in \{K, L, H\}$ , namely:

$$\begin{aligned} s_K &= \frac{K \cdot R}{Y} = I^* - N + 1, \\ s_L &= \frac{L_m \cdot W_m}{Y} = S^* - I^*, \\ s_H &= \frac{H \cdot W_H}{Y} = N - S^*, \end{aligned} \quad (6)$$

where the terms on the right-hand side are obtained from (4).

Figure 1 illustrates the equilibrium outcomes of our model. Computing the equilibrium in our model without minimum wage, we can show that the equilibrium low-skill wage is continuous and strictly decreasing with respect to the supply of low-skill workers (i.e., equal to the demand). This is visualized on the right-hand side of Figure 1. Moreover, Figure 1 indicates that the low-skill wage is bounded above by  $W_0$ , which is defined as the low-skill wage where the relative prices of low-skill and high-skill workers are equal at some particular task  $J$ . Here,  $J$  divides the sets of tasks in an economy where only machines and high-skill workers produce the final good. The allocation of tasks to the production factors characterized by the functions  $I^*$  and  $S^*$  with respect to low-skill employment are plotted on the left-hand side in Figure 1. We can see there that the measure of low-skill tasks (which is equal to  $s_L$ ) is nondecreasing if the low-skill employment increases and zero if there is no low-skill employment. Introducing a binding minimum wage  $W_m \in (W_L, W_0)$  into our model, we obtain



**FIGURE 1** The low-skill wage and the corresponding allocation of factors (represented by the thresholds  $I^*$  and  $S^*$ ) with respect to the supply of (or demand for) low-skill workers

a unique demand for low-skill workers  $L_m$  and corresponding thresholds ( $I^*$ ,  $S^*$ ) in the new equilibrium (see Figure 1). Now, the demand for low-skill workers  $L_m$  is smaller than the supply of low-skill workers  $L$  and, consequently, the low-skill labor market is not clearing (see Equation 5). Contrary to this, the market for high-skill workers is clearing as the minimum wage is not binding for high-skill workers ( $W_H > W_m$ ). This is a consequence of Assumption 2.1 that high-skill workers have a higher productivity than low-skill workers for each task ( $\gamma_H > \gamma_L$ ).

### 3 | THE EFFECTS OF A MINIMUM WAGE

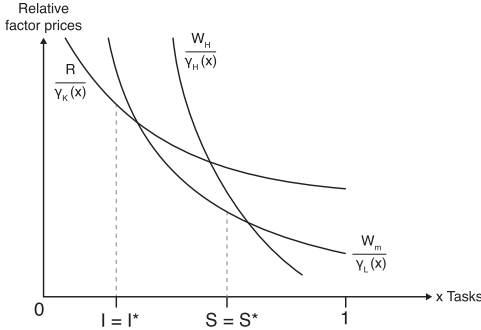
This section analyzes the model introduced in the last section. We are interested in the effects of a binding minimum wage on the aggregate output, employment, factor prices, and various measures of the income distribution.

The analysis requires us to distinguish the following four cases concerning the relationship between the factor prices and the ratio of the productivity functions as the corresponding thresholds ( $I^*$ ,  $S^*$ ) are not differentiable in general at the transitions between the cases. These cases are illustrated in Figure 2.

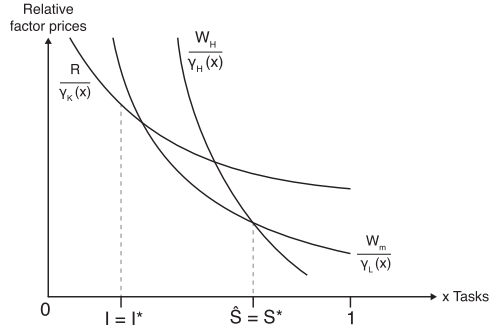
- In the first case, machines and low-skill workers are limited by their exogenous thresholds. This means that machines are relatively cheaper than (or equally expensive as) low-skill and high-skill workers for each task in the set  $[0, I]$  and low-skill workers are relatively cheaper than (or equally expensive as) high-skill workers for each task in the set  $(I, S]$ . Then, the inequalities  $\frac{W_m}{R} \geq \gamma_1(I)$  and  $\frac{W_H}{W_m} \geq \gamma_2(S)$  hold and  $(I^*, S^*) = (I, S)$  follows.
- In the second case, only machines are limited by their exogenous threshold  $I$  and there is a threshold  $\hat{S} < S$  such that the relative prices of low-skill and high-skill workers are equal at the task  $\hat{S}$ . Then, it holds that  $\frac{W_m}{R} \geq \gamma_1(I)$  and  $\frac{W_H}{W_m} = \gamma_2(\hat{S})$ . The allocation of tasks is characterized by  $(I^*, S^*) = (I, \hat{S})$ .
- The third case is similar to the second. Here, the low-skill workers are limited by their exogenous threshold  $S$  and there is a threshold  $\hat{I} < I$  such that the relative prices of



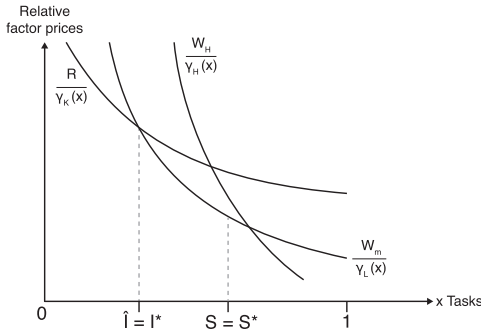
**First case**



**Second case**



**Third case**



**Fourth case**

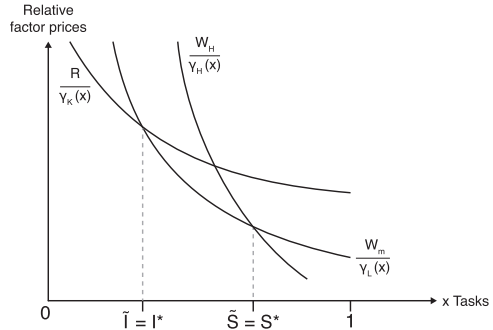


FIGURE 2 The tasks are allocated to the factor that has the lowest relative price

machines and low-skill workers are equal at the task  $\hat{I}$ . Therefore, it holds that  $\frac{W_m}{R} = \gamma_1(\hat{I})$  and  $\frac{W_H}{W_m} \geq \gamma_2(S)$  and  $(I^*, S^*) = (\hat{I}, S)$ .

- In the fourth case, there are thresholds  $\tilde{I} < I$  and  $\tilde{S} < S$  such that the relative prices of machines and low-skill workers are equal at the task  $\tilde{I}$  and the relative prices of low-skill and high-skill workers are equal at the task  $\tilde{S}$ . The equations  $\frac{W_m}{R} = \gamma_1(\tilde{I})$  and  $\frac{W_H}{W_m} = \gamma_2(\tilde{S})$  are satisfied. This allocation of tasks is characterized by  $(I^*, S^*) = (\tilde{I}, \tilde{S})$ .

The values  $\hat{I}$ ,  $\hat{S}$ ,  $\tilde{I}$ , and  $\tilde{S}$  are unique (for fixed  $I$  and  $S$ ) and the unique endogenous thresholds are given by

$$(I^*, S^*) = \min\{(I, S), (I, \hat{S}), (\hat{I}, S), (\tilde{I}, \tilde{S})\}^3$$

Note that from now on we consider strict inequalities in the first, second, and third case because, as already mentioned, the transitions between the cases are not differentiable in general.

In the sequel, we analyze the effects of a change in the minimum wage and an automating economy. As described in the introduction, we think of an automating economy along several dimensions; namely, changing exogenous thresholds, productivity functions, and the task interval (creation of new, labor-intensive tasks). Each analysis starts with the consideration of the

<sup>3</sup>The minimum min is defined component-by-component and corresponds to one of the four couples.

effects on the endogenous thresholds  $(I^*, S^*)$ , the so-called ripple effects. These ripple effects are necessary to compute the impact on the other objects if  $(I^*, S^*) \neq (I, S)$  as they depend on  $(I^*, S^*)$  and their changes (see Equations 3, 4, and 6). Then, we focus on the development of the output and the employment effect. Finally, we examine the distributional effects by exploring the impacts on ratios of factor prices (rental rate, minimum wage, and high-skill wage) and the income distribution; namely, the shares in the national income.

We will, without loss of generality, consider  $N = 1$  except for the creation of new tasks.<sup>4</sup>

### 3.1 | An increase in the minimum wage

Our first analysis shows the effects of an increase in the minimum wage  $W_m$  on employment, aggregate output, factor prices, and the income distribution. We start by considering the consequences of increasing  $W_m$  for the allocation of tasks in the following lemma.

**Lemma 3.1** (Ripple effects). *Let Assumption 2.1 hold. Further, let  $N = 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ .*

- If  $(I^*, S^*) = (I, \hat{S})$ , then it holds that

$$\frac{d\hat{S}}{dW_m} = -\frac{\frac{1-\hat{S}}{1-\hat{S}+I} \cdot \frac{1}{W_m}}{\gamma_2'(\hat{S}) \cdot \frac{1-\hat{S}}{\gamma_2(\hat{S})} + \frac{I}{1-\hat{S}+I}} < 0.$$

- If  $(I^*, S^*) = (\hat{I}, S)$ , then it holds that

$$\frac{d\hat{I}}{dW_m} = \frac{\gamma_1(\hat{I}) \cdot \frac{\hat{I}}{1-S+\hat{I}} \cdot \frac{1}{W_m}}{\gamma_1(\hat{I}) \cdot \frac{1-S}{1-S+\hat{I}} + \gamma_1'(\hat{I}) \cdot \hat{I}} > 0.$$

- If  $(I^*, S^*) = (\tilde{I}, \tilde{S})$ , then it holds that

$$\begin{aligned} \frac{d\tilde{I}}{dW_m} &= c(W_m, \tilde{I}, \tilde{S}) \cdot \left( \gamma_2'(\tilde{S}) \cdot H \cdot \frac{W_m}{Y} + 1 \right) \cdot \gamma_1(\tilde{I}) \cdot \tilde{I} > 0, \\ \frac{d\tilde{S}}{dW_m} &= -c(W_m, \tilde{I}, \tilde{S}) \cdot (1 - \tilde{S}) \cdot \left( \gamma_1(\tilde{I}) + \gamma_1'(\tilde{I}) \cdot \tilde{I} \right) < 0, \end{aligned}$$

where  $c(W_m, \tilde{I}, \tilde{S})$  is a positive function defined in the Online Appendix (see Proof of Lemma 3.1 in Part IV).

*Proof.* The proof follows by totally differentiating the equation  $\frac{W_H}{W_m} = \gamma_2(\hat{S})$  with respect to  $W_m$  and  $\hat{S}$ , the equation  $\frac{W_m}{R} = \gamma_1(\hat{I})$  with respect to  $W_m$  and  $\hat{I}$ , and the equations  $\frac{W_m}{R} = \gamma_1(\tilde{I})$  and  $\frac{W_H}{W_m} = \gamma_2(\tilde{S})$  with respect to  $W_m$ ,  $\tilde{I}$ , and  $\tilde{S}$ . The details can be found in Part IV of the Online Appendix.  $\square$

If the minimum wage increases, the low-skill workers become too expensive to produce the tasks at the endogenous thresholds. We can see in Lemma 3.1 that, consequently, machines or

<sup>4</sup>Otherwise, we can shift the productivity functions.

high-skill workers displace the low-skill workers if the thresholds are not at the extensive margins. This mechanism corresponds to an upshift in the curve of the relative low-skill wage  $\frac{W_m}{\gamma_L(x)}$  in Figure 2.

Moreover, the change in the relative factor prices and the ripple effects have an effect on the production of the aggregate output and the labor demand.

**Proposition 3.2.** *Let Assumption 2.1 hold. Further, let  $N = 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, it holds that*

$$\frac{dL_m}{dW_m} = -\frac{1}{1 - S^* + I^*} \cdot \frac{Y}{W_m} \cdot \left( \frac{S^* - I^*}{W_m} + \frac{dI^*}{dW_m} - \frac{dS^*}{dW_m} \right) < 0,$$

$$\frac{dY}{dW_m} = -\frac{Y}{1 - S^* + I^*} \cdot \left( \frac{S^* - I^*}{W_m} + \frac{dI^*}{dW_m} - \frac{dS^*}{dW_m} \right) < 0.$$

*Proof.* The proof follows by differentiating (3), the fact that  $L_m = Y \cdot \frac{S^* - I^*}{W_m}$ , and from Lemma 3.1. The details can be found in Part IV of the Online Appendix.  $\square$

This proposition implies that an increasing minimum wage reduces the demand for low-skill workers and thus their employment. Further, the aggregate output decreases, because one input factor of the production function (low-skill workers) decreases. The ripple effects strengthen these negative effects on the demand for low-skill workers and, consequently, also on the output. We have already seen this negative effect on the demand for low-skill workers in Figure 1.

These results reflect the findings of introducing a minimum wage in the standard model (e.g., Borjas, 2012). The results of the following two corollaries are more interesting. Here, we characterize the distributional effects of an increase in the minimum wage. From Lemma 3.1 and Proposition 3.2, we obtain by differentiation the following statement.

**Corollary 3.3.** *Let Assumption 2.1 hold. Further, let  $N = 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, it holds that*

$$\frac{d\left(\frac{W_H}{W_m}\right)}{dW_m} = -\frac{1}{1 - S^* + I^*} \cdot \frac{W_H}{W_m} \cdot \left( \frac{1}{W_m} + \frac{dI^*}{dW_m} + \frac{I^*}{1 - S^*} \cdot \frac{dS^*}{dW_m} \right) < 0,$$

$$\frac{d\left(\frac{R}{W_m}\right)}{dW_m} = -\frac{1}{1 - S^* + I^*} \cdot \frac{R}{W_m} \cdot \left( \frac{1}{W_m} - \frac{dS^*}{dW_m} - \frac{1 - S^*}{I^*} \cdot \frac{dI^*}{dW_m} \right) < 0.$$

Moreover, the signs of

$$\frac{dW_H}{dW_m}, \frac{dR}{dW_m}, \text{ and } \frac{d\left(\frac{W_H}{R}\right)}{dW_m}$$

are ambiguous.

The main takeaway from this corollary is that an increasing minimum wage reduces the inequality between the low-skill wage and the high-skill wage and the inequality between the low-skill wage and the rental rate. The signs of the effects on the high-skill wage, the rental rate and their ratio are ambiguous and depend on the thresholds. As the high-skill wage

depends on the aggregate output (see Equation 4) that decreases (see Proposition 3.2), only a displacement of low-skill workers can lead to an increase in the rental rate or the high-skill wage. To enable machines or high-skill workers to replace low-skill workers, these prices cannot increase more than the minimum wage. Grossman (1983) also shows a wage compression. In his work, a substitution from low-skill workers to high-skill workers implies an increase in the high-skill wage. This is not necessarily the case in our framework as it is possible that the high-skill wage decreases even if high-skill workers displace low-skill workers if the aggregate output decreases sharply (see Equation 4).

In the following corollary that follows from Lemma 3.1 and Proposition A.3, we consider the redistribution between the factor groups and within the low-skill workers.

**Corollary 3.4.** *Let Assumption 2.1 hold. Further, let  $N \geq 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, the share of income of low-skill workers in the national income  $s_L$  is nonincreasing in  $W_m$  and it holds that*

$$\lim_{W_m \nearrow W_0} s_L = 0.$$

*Furthermore, the share of income of machines and high-skill workers in the national income are nondecreasing in  $W_m$ .*

Here, and in Figure 1, we can see that the lower aggregate output is now redistributed more unequally between the low-skill workers and the other production factors as the share of income of low-skill workers in the national income decreases. This corresponds to the redistribution between the low-skill workers, as some low-skill workers become unemployed, whereas the low-skill workers who are still employed gain from an increased minimum wage. Moreover,  $s_L$  approaches zero as the minimum wage approaches  $W_0$  as the low-skill employment approaches zero.

Due to the above-described polarization between employed and unemployed low-skill workers, we can consider the expected low-skill wage

$$(1 - u) \cdot W_m = \frac{L_m}{L} \cdot W_m = \frac{Y \cdot s_L}{L} \quad (7)$$

as a group-specific measure of the welfare of low-skill workers where  $u$  is the unemployment rate among low-skill workers. From Proposition 3.2 and Corollary 3.4 it follows that the expected low-skill wage decreases when the minimum wage increases. Consequently, the increase in the minimum wage cannot compensate the negative employment effect.

Further, we look from a different perspective on the income distribution in the following corollary by exploring the effects on the high-skill wage and the rental rate relative to the expected low-skill wage. They are proportional to the effects on the ratios of the shares in the national income.

**Corollary 3.5.** *Let Assumption 2.1 hold. Further, let  $N \geq 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, it holds that*

$$\begin{aligned} \frac{d\left(\frac{W_H}{(1-u) \cdot W_m}\right)}{dW_m} &= \frac{L}{H} \cdot \frac{1}{s_L^2} \cdot \left( s_L \cdot \frac{ds_H}{dW_m} - s_H \cdot \frac{ds_L}{dW_m} \right) \geq 0, \\ \frac{d\left(\frac{R}{(1-u) \cdot W_m}\right)}{dW_m} &= \frac{L}{K} \cdot \frac{1}{s_L^2} \cdot \left( s_L \cdot \frac{ds_K}{dW_m} - s_K \cdot \frac{ds_L}{dW_m} \right) \geq 0. \end{aligned}$$

This corollary illustrates the redistribution effects between the factor groups, as mentioned before.

Finally, this leads to the interim conclusion that there is a trade-off between the inequality of the factor prices and the inequality of the shares in the national income, meaning that an increasing binding minimum wage reduces the inequality between the factor prices and raises the inequality in the national income if  $(I^*, S^*) \neq (I, S)$ .

### 3.2 | A minimum wage in an automating economy

In this subsection, we explore the effects of an automating economy in our framework with a binding minimum wage  $W_m \in (W_L, W_0)$ . We compare it to the effects on various outcome measures in the model without a (binding) minimum wage (see also Table A1 in Appendix A). Here, the low-skill wage is fixed and inelastic. Consequently, a change in factor price has the same sign as its ratio with the low-skill wage (minimum wage). Therefore, we will only consider the changes in factor prices.

#### 3.2.1 | Automation at the extensive margin

We start by considering the expansion of automatable tasks—represented by an increase in the threshold  $I$ . Therefore, we focus only on the case  $I^* = I$ , where the machines are relatively cheaper than the low-skill workers at the task  $I$ , namely,  $\frac{R}{\gamma_K(I)} < \frac{W_m}{\gamma_L(I)}$ .<sup>5</sup>

We start our analysis with the following ripple effect.

**Lemma 3.6** (Ripple effect). *Let Assumption 2.1 hold. Further, let  $N = 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . If  $(I^*, S^*) = (I, \hat{S})$ , then it holds that*

$$\frac{d\hat{S}}{dI} = \frac{\frac{1-\hat{S}}{1-\hat{S}+I} \cdot \left( \ln \left( \frac{W_m}{Y \cdot \frac{I}{K}} \cdot \frac{1}{\gamma_L(I)} \right) - 1 \right)}{\gamma_2'(\hat{S}) \cdot H \cdot \frac{W_m}{Y} + \frac{I}{1-\hat{S}+I}}.$$

*The sign of this ripple effect is ambiguous and its absolute value is not bounded.*

*Proof.* The proof follows by totally differentiating the equation  $\frac{W_H}{W_m} = \gamma_2(\hat{S})$  with respect to  $I$  and  $\hat{S}$ . The details can be found in Part IV of the Online Appendix.  $\square$

The statement of Lemma 3.6 is different to the model without a minimum wage, where we have a positive ripple effect that is smaller than one. To explain this effect, we need the results of the following proposition and corollary.

**Proposition 3.7.** *Let Assumption 2.1 hold. Further, let  $N = 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ .*

<sup>5</sup>The case  $I^* \neq I$  is self-explanatory. Then, all effects of an increase in  $I$  are zero.

- If  $(I^*, S^*) = (I, S)$ , then it holds that

$$\frac{dL_m}{dI} = \frac{L_m}{1 - S + I} \cdot \left( \ln \left( \frac{W_m}{Y \cdot \frac{I}{K}} \cdot \frac{1}{\gamma_1(I)} \right) - \frac{1}{S - I} \right),$$

$$\frac{dY}{dI} = \frac{Y}{1 - S + I} \cdot \left( \ln \left( \frac{W_m}{Y \cdot \frac{I}{K}} \cdot \frac{1}{\gamma_1(I)} \right) - 1 \right).$$

- If  $(I^*, S^*) = (I, \hat{S})$ , then it holds that

$$\frac{dL_m}{dI} = \frac{L_m}{1 - \hat{S} + I} \cdot \left( \ln \left( \frac{W_m}{Y \cdot \frac{I}{K}} \cdot \frac{1}{\gamma_1(I)} \right) - 1 \right)$$

$$\cdot \left( 1 + \frac{\frac{1}{\hat{S} - I} \cdot \frac{1 - \hat{S}}{1 - \hat{S} + I}}{\gamma_2'(\hat{S}) \cdot H \cdot \frac{W_m}{Y} + \frac{I}{1 - \hat{S} + I}} \right) - \frac{L_m}{\hat{S} - I},$$

$$\frac{dY}{dI} = \frac{Y}{1 - \hat{S} + I} \cdot \left( \ln \left( \frac{W_m}{Y \cdot \frac{I}{K}} \cdot \frac{1}{\gamma_1(I)} \right) - 1 \right)$$

$$\cdot \left( 1 + \frac{\frac{1 - \hat{S}}{1 - \hat{S} + I}}{\gamma_2'(\hat{S}) \cdot H \cdot \frac{W_m}{Y} + \frac{I}{1 - \hat{S} + I}} \right).$$

The signs of the output effect and the employment effect are ambiguous. In particular, it is possible that the output effect is positive and the employment effect is negative.

*Proof.* The proof follows by differentiating (3), the fact that  $L_m = Y \cdot \frac{S^* - I^*}{W_m}$ , and from Lemma 3.6. The details can be found in Part IV of the Online Appendix.  $\square$

Also, the ambiguity of the sign of the output effect is contrary to the model without a minimum wage, where the automation at the extensive margin creates a positive output effect, which is equal to the cost-saving effect (e.g., Acemoglu & Restrepo, 2018a, 2018b). Moreover, Acemoglu and Restrepo (2018a) show that automation can reduce labor demand. Due to the assumption of elastic wages as well as fixed and inelastic labor supply, this reduces the wage and not employment. This is the main reason why there is only a positive effect on the output, in contrast to the case with a (binding) minimum wage.

Further, the low-skill wage in the model without a minimum wage is related to the expected low-skill wage as can be seen in (7) (e.g., Acemoglu & Restrepo, 2018a). The expected low-skill wage and the low-skill wage in a model without a minimum wage are driven by the demand for low-skill workers. In particular, the expected low-skill wage is proportional to the employment of low-skill workers as  $\frac{W_m}{L}$  is fixed by assumption.

The mechanism of our result summarized in Proposition 3.7 is the following: the machines displace low-skill workers because the machines are relatively cheaper than the low-skill workers. On the one hand, this creates a cost-saving effect that raises aggregate output and thus labor demand. On the other hand, the displacement effect reduces labor demand and low-skill employment, because

the low-skill wage is inelastic. Consequently, the displacement effect also reduces the aggregate output.

We identify two counteracting effects, the cost-saving effect and the displacement effect. Therefore, the sign of the derivatives in the above proposition depends on the difference in these effects. In particular, it is possible that the aggregate output increases and the low-skill employment decreases at the same time. Further, the aggregate output decreases if the cost-saving effect is too small to balance the displacement effect, meaning that machines are "... only marginally better than labor [here, low-skill workers, Author's note] at the newly-automated tasks" (Acemoglu & Restrepo, 2018a, p. 19). Acemoglu and Restrepo (2018a) call such new technologies "so-so."<sup>6</sup> In other words, a small technological advantage can lead to a smaller aggregate output in an economy with a minimum wage as before, because the low-skill wage is not flexible and cannot compensate for the reduction of the demand for low-skill workers. Figure 3 illustrates the above discussion.

Now, we consider the ripple effect stated in Lemma 3.6, which is positive (negative) if the output effect is positive (negative). The reason for this is that low-skill workers displace high-skill workers because low-skill workers are relatively cheaper than the high-skill workers at the threshold  $\hat{S}$  (and vice versa if the ripple effect is negative). As a consequence, the ripple effect strengthens the output and employment effect because the ripple effect counteracts (strengthens) the displacement effect.

In the following two corollaries, we consider the distributional effects of an increasing automation and see the reason for the second cost-saving effect above.

**Corollary 3.8.** *Let Assumption 2.1 hold. Further, let  $N = 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . If  $(I^*, S^*) \in \{(I, S), (I, \hat{S})\}$ , then it holds that*

$$\begin{aligned} \frac{dW_H}{dI} &= \frac{1 - S^*}{H} \cdot \frac{dY}{dI} - \frac{Y}{H} \cdot \frac{dS^*}{dI}, \\ \frac{dR}{dI} &= \frac{I}{K} \cdot \frac{dY}{dI} + \frac{Y}{K} > 0, \\ \frac{d\left(\frac{W_H}{R}\right)}{dI} &< 0. \end{aligned}$$

*In particular, the signs of  $\frac{dW_H}{dI}$  and  $\frac{dY}{dI}$  are the same.*

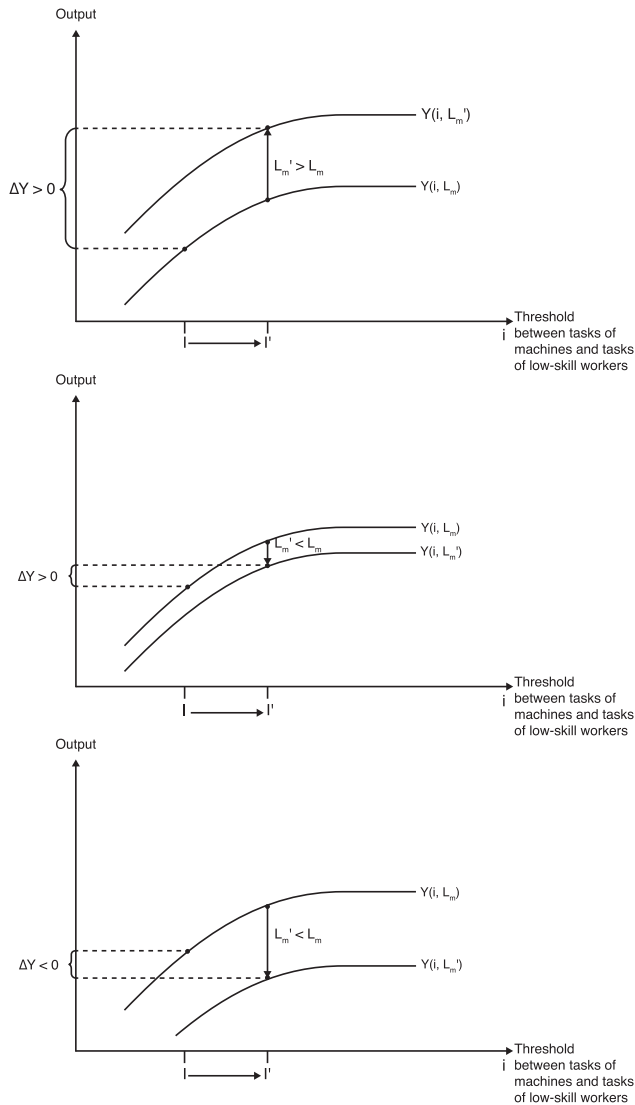
After considering the effects on the factor prices, the next corollary shows the effects on the income distribution.

**Corollary 3.9.** *Let Assumption 2.1 hold. Further, let  $N = 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . If  $(I^*, S^*) \in \{(I, S), (I, \hat{S})\}$ , then it holds that*

$$\frac{ds_L}{dI} = \frac{dS^*}{dI} - 1, \quad \frac{ds_H}{dI} = -\frac{dS^*}{dI}, \quad \text{and} \quad \frac{ds_K}{dI} = 1.$$

The main result of both corollaries is that the inequality in the remuneration between machines and workers increases if the automation increases at the extensive margin. First, we

<sup>6</sup>For this reason, Acemoglu and Restrepo (2019) analyze the "right" kind of automation.



**FIGURE 3** The movement on the curve describes the cost-saving effect. If the cost-saving effect increases the demand for low-skill workers more strongly than it is reduced by the displacement effect, the curve of the aggregate output is shifted up and pushes the output effect up. If this is not the case, the curve of the aggregate output is shifted down and will be either still above or below its initial value

note that the rental rate increases more strongly than the high-skill wage (if it increases at all). Second, the share of income of machines in the national income increases, too.

Moreover, we get distributional effects between the low-skill and high-skill workers. If the output effect is negative, the high-skill wage decreases. This reduces the inequality between the wages. Then, the employment effect is negative and, consequently, the expected low-skill wage decreases. If  $S^* = \hat{S}$ , high-skill workers displace the low-skill workers, which further reduces the low-skill share in the national income and raises the high-skill share in the national income. When the output effect is positive, the wage inequality increases and in the additional case where  $S^* = \hat{S}$ , the low-skill workers displace the high-skill workers. Consequently, this



reduces the inequality in the income distribution. Because the ripple effect is not bounded, it is possible that it is larger than one and, therefore, the share of income of low-skill workers in the national income increases.

However, as seen in the previous subsection, there is a trade-off between the inequality of the factor prices and the inequality of the income distribution. Further, from the above discussion it follows that the signs of the effects on the ratio between the high-skill wage or the rental rate and the expected low-skill wage are ambiguous, which is summarized in the following corollary.

**Corollary 3.10.** *Let Assumption 2.1 hold. Further, let  $N = 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . If  $(I^*, S^*) \in \{(I, S), (I, \hat{S})\}$ , then it holds that*

$$\frac{d\left(\frac{W_H}{(1-u) \cdot W_m}\right)}{dI} = \frac{L}{H} \cdot \frac{1}{s_L^2} \cdot \left( s_L \cdot \frac{ds_H}{dI} - s_H \cdot \frac{ds_L}{dI} \right),$$

$$\frac{d\left(\frac{R}{(1-u) \cdot W_m}\right)}{dI} = \frac{L}{K} \cdot \frac{1}{s_L^2} \cdot \left( s_L \cdot \frac{ds_K}{dI} - s_K \cdot \frac{ds_L}{dI} \right).$$

### 3.2.2 | Deepening of automation

The second type of automation is automation at the intensive margin, meaning that the productivity (function) of machines performing tasks increases. As in Acemoglu and Restrepo (2018a), we call it the “deepening of automation”. It occurs when machines are developed further or replaced with newer, more productive machines.

Here, we do not limit ourselves to  $I^* = I$ , in contrast to the previous case. We assume that the productivity function is proportional to the factor-augmenting technologies, that is,

$$\gamma_i(x) = A_i \cdot \psi_i(x) \quad \text{for } i \in \{K, L, H\}, \quad (8)$$

where  $A_i > 0, i \in \{K, L, H\}$ , denotes the factor-augmenting technology, respectively, and  $\psi_i, i \in \{K, L, H\}$ , has the same properties as  $\gamma_i$  in Assumption 2.1. Moreover, we denote  $\psi_1 = \frac{\psi_L}{\psi_K}$  and  $\psi_2 = \frac{\psi_H}{\psi_L}$ . Consequently, when analyzing the deepening of automation, we have an increase in  $A_K$ .

We now consider the ripple effects in the following lemma.

**Lemma 3.11.** *Let Assumption 2.1 hold. Further, let  $N = 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ .*

• *If  $(I^*, S^*) = (I, \hat{S})$ , then it holds that*

$$\frac{d\hat{S}}{dA_K} = \frac{(1 - \hat{S}) \cdot \frac{I}{1 - \hat{S} + I} \cdot \frac{Y}{A_K}}{\frac{I}{1 - \hat{S} + I} \cdot Y + \gamma_2'(\hat{S}) \cdot H \cdot W_m} > 0.$$

- If  $(I^*, S^*) = (\hat{I}, \hat{S})$ , then it holds that

$$\frac{d\hat{I}}{dA_K} = \frac{\frac{1-S}{1-S+\hat{I}} \cdot \hat{I} \cdot \gamma_1(\hat{I})}{A_L \cdot \left( \psi'(\hat{I}) \cdot \hat{I} + \psi_1(\hat{I}) \cdot \frac{1-S}{1-S+\hat{I}} \right)} > 0.$$

- If  $(I^*, S^*) = (\tilde{I}, \tilde{S})$ , then it holds that

$$\frac{d\tilde{I}}{dA_K} > 0 \text{ and } \frac{d\tilde{S}}{dA_K} > 0.$$

*Proof.* The proof follows by totally differentiating the equation  $\frac{W_H}{W_m} = \gamma_2(\hat{S})$  with respect to  $A_K$  and  $\hat{S}$ , the equation  $\frac{W_m}{R} = \gamma_1(\hat{I})$  with respect to  $A_K$  and  $\hat{I}$ , and the equations  $\frac{W_m}{R} = \gamma_1(\tilde{I})$  and  $\frac{W_H}{W_m} = \gamma_2(\tilde{S})$  with respect to  $A_K$ ,  $\tilde{I}$ , and  $\tilde{S}$ . The details can be found in Part IV of the Online Appendix.  $\square$

Lemma 3.11 shows that the thresholds of the tasks are increasing if they are not at the extensive margins. This is intuitive as the machines become better at their tasks and we expect them to displace low-skill workers as in the model without a minimum wage. To understand the mechanism behind this lemma, we need the results of the following proposition and corollary.

**Proposition 3.12.** *Let Assumption 2.1 hold. Further, let  $N = 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, it holds that*

$$\begin{aligned} \frac{dL_m}{dA_K} &= \frac{1}{1 - S^* + I^*} \cdot \frac{Y}{W_m} \cdot \left( \frac{I^* \cdot (S^* - I^*)}{A_K} - \frac{dI^*}{dA_K} + \frac{dS^*}{dA_K} \right), \\ \frac{dY}{dA_K} &= \frac{Y}{1 - S^* + I^*} \cdot \left( \frac{I^*}{A_K} - \frac{dI^*}{dA_K} + \frac{dS^*}{dA_K} \right) > 0. \end{aligned}$$

*In particular, if  $(I^*, S^*) \in \{(I, S), (I, \hat{S})\}$ , then it follows that  $\frac{dL_m}{dA_K} > 0$ .*

*Proof.* The proof follows by differentiating (3), the fact that  $L_m = Y \cdot \frac{S^* - I^*}{W_m}$ , and from Lemma 3.11. The details can be found in Part IV of the Online Appendix.  $\square$

In contrast to our previous findings, introducing a minimum wage does not alter the economic consequences of an increase in the aggregate output. Deepening of automation creates a cost-saving effect. Consequently, the aggregate output increases. The impact on the employment of low-skill workers is ambiguous due to the possible displacement of low-skill workers by machines that decreases the demand for low-skill workers and counteracts the output effect.

In the following corollary, we consider the effects on the factor prices.

**Corollary 3.13.** *Let Assumption 2.1 hold. Further, let  $N = 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, it holds that*

$$\begin{aligned}\frac{dW_H}{dA_K} &= \frac{1}{1 - S^* + I^*} \cdot \frac{Y}{H} \cdot \left( (1 - S^*) \cdot \left( \frac{I^*}{A_K} - \frac{dI^*}{dA_K} \right) - I^* \cdot \frac{dS^*}{dA_K} \right) > 0, \\ \frac{dR}{dA_K} &= \frac{1}{1 - S^* + I^*} \cdot \frac{Y}{K} \cdot \left( I^* \cdot \left( \frac{I^*}{A_K} + \frac{dS^*}{dA_K} \right) + (1 - S^*) \cdot \frac{dI^*}{dA_K} \right) > 0, \\ \frac{d\left(\frac{W_H}{R}\right)}{dA_K} &= -\frac{K}{H} \cdot \frac{1}{(I^*)^2} \cdot \left( I^* \cdot \frac{dS^*}{dA_K} + (1 - S^*) \cdot \frac{dI^*}{dA_K} \right) \leq 0.\end{aligned}$$

First, we note that the high-skill wage increases due to the positive output effect. This explains the displacement of the high-skill workers at the endogenous threshold  $\hat{S}$  or  $\hat{S}$ . Besides, the wage inequality increases. Second, the rental rate does not increase as strongly as the factor-augmenting technology.<sup>7</sup> This explains the other ripple effect. The sign of the effect on the ratio between the rental rate and high-skill wage is ambiguous.

Figure 4 presents the allocation of tasks (and the ripple effects). There, the curves of the relative factor prices are plotted. An increase in  $A_K$  leads first to a downshift in the curve of the relative price of the machines. The positive output effect raises the rental rate and shifts this curve upwards but it still remains below the initial curve. The same holds for the curve of the relative price of high-skill workers. Consequently, we can conclude that machines become relatively cheaper than the other production factors as their relative price curve is below the initial curve. Moreover, low-skill workers become relatively cheaper than high-skill workers as their relative price curve does not change.

A further implication of Lemma 3.11 is that the share of income of machines (high-skill workers) in the national income is nondecreasing (nonincreasing) as long as the minimum wage is binding. However, the sign of the effect on the share of income of low-skill workers in the national income is ambiguous. Thus, it can also be positive, which cannot happen in the model without a minimum wage and leads to an increasing low-skill employment.<sup>8</sup> The following corollary summarizes these results.

**Corollary 3.14.** *Let Assumption 2.1 hold. Further, let  $N = 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, it holds that*

$$\frac{ds_L}{dA_K} = \frac{dS^*}{dA_K} - \frac{dI^*}{dA_K}, \quad \frac{ds_H}{dA_K} = -\frac{dS^*}{dA_K} \leq 0, \quad \text{and} \quad \frac{ds_K}{dA_K} = \frac{dI^*}{dA_K} \geq 0.$$

As the sign of the effect on the share of income of low-skill workers in the national income is ambiguous, the signs of the effect on the ratios between it and the other shares of income are ambiguous, too, see Corollary 3.15.

**Corollary 3.15.** *Let Assumption 2.1 hold. Further, let  $N = 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, it holds that*

<sup>7</sup>It holds that  $d\left(\frac{R}{A_K}\right)/dA_K < 0$ .

<sup>8</sup>In the model without a minimum wage, it holds that  $\frac{dI^*}{dA_K} \geq \frac{dS^*}{dA_K} \geq 0$ .

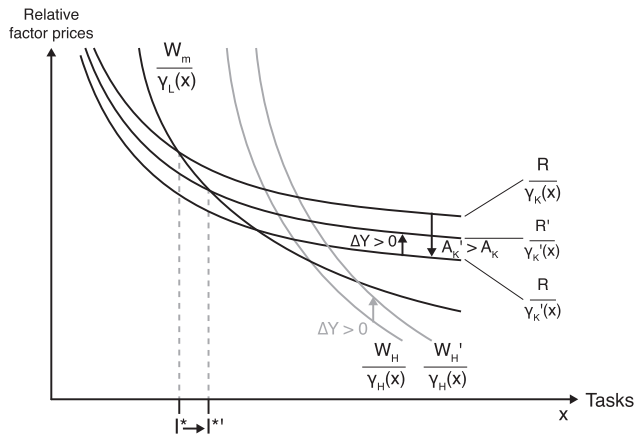


FIGURE 4 The development of the relative factor prices for increasing  $A_K$  and the consequences on the allocation of the factors to tasks

$$\frac{d\left(\frac{W_H}{(1-u) \cdot W_m}\right)}{dA_K} = \frac{L}{H} \cdot \frac{1}{s_L^2} \cdot \left( s_L \cdot \frac{ds_H}{dA_K} - s_H \cdot \frac{ds_L}{dA_K} \right),$$

$$\frac{d\left(\frac{R}{(1-u) \cdot W_m}\right)}{dA_K} = \frac{L}{K} \cdot \frac{1}{s_L^2} \cdot \left( s_L \cdot \frac{ds_K}{dA_K} - s_K \cdot \frac{ds_L}{dA_K} \right).$$

### 3.2.3 | Deepening of skills

Analogously to the deepening of automation, we can also consider an upshift in the productivity function of low-skill or high-skill workers. By assuming the same representation of the productivity functions as in (8), we model these shifts with an increase in the labor-augmenting technologies  $A_L$  and  $A_H$ .

The mechanisms are analogous to the case of deepening of automation. An increase in a labor-augmenting technology ( $A_L$  or  $A_H$ ) decreases the relative price of the corresponding factor. This creates a cost-saving effect that raises the aggregate output. On the one hand, increasing  $A_H$  reverses the effects of the deepening of automation on the thresholds of the tasks and thus on the shares in the national income. The other effects remain the same. On the other hand, if  $A_L$  increases, low-skill workers displace machines if  $I^* \neq I$ , and they displace high-skill workers if  $S^* \neq S$ . This leads to a nondecreasing share of income of low-skill workers in the national income. Moreover, the sign of the effects on the high-skill wage and the rental rate are ambiguous, whereas the employment effect is positive. The expected low-skill wage defined in Equation (7) increases at least as much as the rental rate or the high-skill wage.

### 3.2.4 | Expanding skills

In our model, we assume that low-skill workers cannot perform tasks above  $S$ . Now we explore what happens when an increase in  $S$  occurs, which corresponds to an expansion of the skill

range of low-skill workers. Therefore, we limit our considerations to the essential case  $S^* = S$ , where the low-skill workers are relatively cheaper than the high-skill workers at the task  $S$ , namely,  $\frac{W_m}{\gamma_L(S)} < \frac{W_H}{\gamma_H(S)}$ .<sup>9</sup>

We start to determine the ripple effect in the case  $I^* = \hat{I}$ .

**Lemma 3.16** (Ripple effect). *Let Assumption 2.1 hold. Further, let  $N = 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ .*

*If  $(I^*, S^*) = (\hat{I}, S)$ , then it holds that*

$$\frac{d\hat{I}}{dS} = -\frac{\frac{\gamma_1(\hat{I}) \cdot \hat{I}}{1-S+\hat{I}} \cdot \left( \ln \left( \frac{Y \cdot \frac{1-S}{H}}{W_m} \cdot \frac{1}{\gamma_2(S)} \right) + 1 \right)}{\gamma_1'(\hat{I}) \cdot \hat{I} + \gamma_1(\hat{I}) \cdot \frac{1-S}{1-S+\hat{I}}} < 0.$$

*Proof.* The proof follows by totally differentiating the equation  $\frac{W_m}{R} = \gamma_1(\hat{I})$  with respect to  $S$  and  $\hat{I}$ . The details can be found in Part IV of the Online Appendix.  $\square$

To explain this negative ripple effect, which differs from the framework without a (binding) minimum wage, we need the following proposition.<sup>10</sup>

**Proposition 3.17.** *Let Assumption 2.1 hold. Further, let  $N = 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ .*

• *If  $(I^*, S^*) = (I, S)$ , then it holds that*

$$\begin{aligned} \frac{dL_m}{dS} &= \frac{S-I}{W_m} \cdot \frac{dY}{dS} + \frac{Y}{W_m} > 0, \\ \frac{dY}{dS} &= \frac{Y}{1-S+I} \cdot \left( \ln \left( \frac{Y \cdot \frac{1-S}{H}}{W_m} \cdot \frac{1}{\gamma_2(S)} \right) + 1 \right) > 0. \end{aligned}$$

• *If  $(I^*, S^*) = (\hat{I}, S)$ , then it holds that*

$$\begin{aligned} \frac{dL_m}{dS} &= \frac{S-\hat{I}}{W_m} \cdot \frac{dY}{dS} + \frac{Y}{W_m} \cdot \left( 1 - \frac{d\hat{I}}{dS} \right) > 0, \\ \frac{dY}{dS} &= \frac{Y}{1-S+I} \cdot \left( \ln \left( \frac{Y \cdot \frac{1-S}{H}}{W_m} \cdot \frac{1}{\gamma_2(S)} \right) + 1 \right) \cdot \left( 1 + \frac{\frac{\gamma_1(\hat{I}) \cdot \hat{I}}{1-S+\hat{I}}}{\gamma_1'(\hat{I}) \cdot \hat{I} + \gamma_1(\hat{I}) \cdot \frac{1-S}{1-S+\hat{I}}} \right) > 0. \end{aligned}$$

*Proof.* The proof follows by differentiating (3), the fact that  $L_m = Y \cdot \frac{S^*-I^*}{W_m}$ , and from Lemma 3.16. The details can be found in Part IV of the Online Appendix.  $\square$

This proposition implies that we get a positive output and employment effect by increasing  $S$ . The reason for this is twofold: a cost-saving effect, and a reinstatement effect of the low-skill workers. Both raise the output, the demand for low-skill workers, and thus the employment.

<sup>9</sup>The case  $S^* \neq S$  is self-explanatory. Then, all effects of an increase in  $S$  are zero.

<sup>10</sup>In the model without a minimum wage, it holds that  $\frac{d\hat{I}}{dS} \in (0, 1)$ .

The mechanism is as follows. Due to the cost-saving effect, the production of the final good becomes cheaper, leading to an increase in the aggregate output. This raises the demand for low-skill workers (i.e., the employment), which again raises the aggregate output. At the same time, the reinstatement effect, which is the opposite of the displacement effect, increases the demand for low-skill workers, too. Due to the increase in the low-skill employment, there is a further positive output effect. Figure 5 shows these output effects.

The ripple effect strengthens both effects of Proposition 3.17 as the displacement of machines by low-skill workers creates an employment effect and thus an output effect. It follows from an increase in the rental rate, which we see together with the other effects of the factor prices in the following corollary.

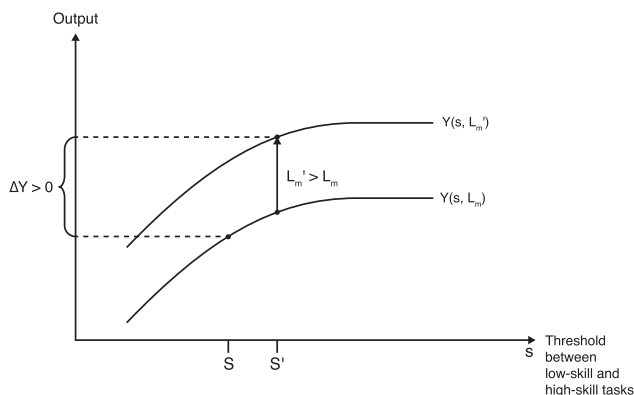
**Corollary 3.18.** *Let Assumption 2.1 hold. Further, let  $N = 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . If  $(I^*, S^*) \in \{(I, S), (\hat{I}, S)\}$ , then it holds that*

$$\begin{aligned} \frac{dW_H}{dS} &= \frac{1-S}{H} \cdot \frac{dY}{dS} - \frac{Y}{H}, \\ \frac{dR}{dS} &= \frac{I^*}{K} \cdot \frac{dY}{dS} + \frac{Y}{K} \cdot \frac{dI^*}{dS} > 0, \\ \frac{d\left(\frac{W_H}{R}\right)}{dS} &= -\frac{K}{H} \cdot \frac{1}{(I^*)^2} \cdot \left(1 + (1-S) \cdot \frac{dI^*}{dS}\right). \end{aligned}$$

*In particular, it holds that  $\frac{d\left(\frac{W_H}{R}\right)}{dS} < 0$  if  $I^* = I \neq \hat{I}$ .*

As emphasized before, the effect on the rental rate is positive, which explains the ripple effect in Lemma 3.16. Consequently, the inequality between the low-skill wage and the rental rate increases. The signs of the effects on the high-skill wage and the ratio between the rental rate and the high-skill wage are ambiguous. These depend on the size of the output effect.

Due to the displacement of the high-skill workers and perhaps also the machines, the share of income of the low-skill workers in the national income increases. The other shares in the national income are nonincreasing, whereas the share of income of machines in the national income is nondecreasing in the model without a minimum wage. We summarize this in the following corollary.



**FIGURE 5** First, the movement on the curve describes the cost-saving effect. The upshift in the curve is a consequence of the employment effect as a result of the first output effect and the reinstatement effect

**Corollary 3.19.** *Let Assumption 2.1 hold. Further, let  $N = 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . If  $(I^*, S^*) \in \{(I, S), (\hat{I}, S)\}$ , then it holds that*

$$\frac{ds_L}{dS} = 1 - \frac{dI^*}{dS} \geq 1, \quad \frac{ds_H}{dS} = -1, \quad \text{and} \quad \frac{ds_K}{dS} = \frac{dI^*}{dS} \leq 0.$$

As a consequence of Corollary 3.19, the expected low-skill wage increases more strongly than the rental rate and the high-skill wage. This result is summarized in the following corollary, which also shows that an increase in  $S$  reduces the inequality in the income distribution.

**Corollary 3.20.** *Let Assumption 2.1 hold. Further, let  $N = 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . If  $(I^*, S^*) \in \{(I, S), (\hat{I}, S)\}$ , then it holds that*

$$\begin{aligned} \frac{d\left(\frac{W_H}{(1-u) \cdot W_m}\right)}{dS} &= \frac{L}{H} \cdot \frac{1}{s_L^2} \cdot \left( s_L \cdot \frac{ds_H}{dS} - s_H \cdot \frac{ds_L}{dS} \right) < 0, \\ \frac{d\left(\frac{R}{(1-u) \cdot W_m}\right)}{dS} &= \frac{L}{K} \cdot \frac{1}{s_L^2} \cdot \left( s_L \cdot \frac{ds_K}{dS} - s_K \cdot \frac{ds_L}{dS} \right) < 0. \end{aligned}$$

### 3.2.5 | Creation of new tasks

Acemoglu and Restrepo (2018a, 2018d) implement the creation of new, labor-intensive tasks in their task-based framework by an increase in  $N$  and show a powerful counteracting force of these new tasks regarding the rapid automation.

We implement the creation of new tasks in our framework with a binding minimum wage. For the analysis, we need a further assumption.

**Assumption 3.21.** Let Assumption 2.1 hold. Further, let  $N \geq 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$  hold such that

$$\frac{W_H}{\gamma_H(N)} < \frac{R}{\gamma_K(N-1)}$$

holds.

This assumption ensures that an increase in  $N$  creates a cost-saving effect and thus the possibility of a positive output effect. After an infinitesimal right-shift of the task interval, the new tasks can be produced relatively cheaper than the old destroyed tasks. The power of this cost-saving effect is very important for the following ripple effects.

**Lemma 3.22** (Ripple effects). *Let Assumption 2.1 and 3.21 hold. Further, let  $N \geq 1, K, L, H > 0, 0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ .*

• If  $(I^*, S^*) = (I, \hat{S})$ , then it holds that

$$\frac{d\hat{S}}{dN} = \frac{\frac{Y}{W_m} \cdot \left( 1 + \frac{N-\hat{S}}{1-\hat{S}+I} \cdot \ln \left( \frac{H}{N-\hat{S}} \cdot \frac{I-N+1}{K} \cdot \frac{\gamma_H(N)}{\gamma_K(N-1)} \right) \right)}{\gamma_2'(\hat{S}) \cdot H + \frac{Y}{W_m} \cdot \frac{1+I-N}{1-\hat{S}+I}} > 0.$$

- If  $(I^*, S^*) = (\hat{I}, S)$ , then it holds that

$$\frac{d\hat{I}}{dN} = -\gamma_1(\hat{I}) \cdot \frac{-1 + \frac{\hat{I} - N + 1}{1 - S + \hat{I}} \cdot \ln\left(\frac{H}{N - S} \cdot \frac{\hat{I} - N + 1}{K} \cdot \frac{\gamma_H(N)}{\gamma_K(N - 1)}\right)}{\gamma_1(\hat{I}) \cdot \frac{N - S}{1 - S + \hat{I}} + \gamma_1'(\hat{I}) \cdot (\hat{I} - N + 1)}.$$

- If  $(I^*, S^*) = (\tilde{I}, \tilde{S})$ , then it holds that the sign of  $\frac{d\tilde{I}}{dN}$  is ambiguous and the sign of  $\frac{d\tilde{S}}{dN}$  is positive.

*Proof.* The proof follows by totally differentiating the equation  $\frac{W_H}{W_m} = \gamma_2(\hat{S})$  with respect to  $N$  and  $\hat{S}$ , the equation  $\frac{W_m}{R} = \gamma_1(\hat{I})$  with respect to  $N$  and  $\hat{I}$ , and the equations  $\frac{W_m}{R} = \gamma_1(\tilde{I})$  and  $\frac{W_H}{W_m} = \gamma_2(\tilde{S})$  with respect to  $N$ ,  $\tilde{I}$ , and  $\tilde{S}$ . The details can be found in Part IV of the Online Appendix.  $\square$

If  $S^* \neq S$ , low-skill workers displace high-skill workers. Furthermore, the sign of the effect on  $I^*$  is ambiguous due to the relation between the cost-saving effect and the displacement effect of machines from the old destroyed tasks. This relation corresponds to the change in the rental rate and can be seen with the following proposition and corollary.

**Proposition 3.23.** *Let Assumption 2.1 and 3.21 hold. Further, let  $N \geq 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, it holds that*

$$\frac{dL_m}{dN} = \frac{1}{1 - S^* + I^*} \cdot \frac{Y}{W_m} \cdot \left( (S^* - I^*) \cdot \ln\left(\frac{H}{N - S^*} \cdot \frac{I^* - N + 1}{K} \cdot \frac{\gamma_H(N)}{\gamma_K(N - 1)}\right) - \frac{dI^*}{dN} + \frac{dS^*}{dN} \right),$$

$$\frac{dY}{dN} = \frac{Y}{1 - S^* + I^*} \cdot \left( \ln\left(\frac{H}{N - S^*} \cdot \frac{I^* - N + 1}{K} \cdot \frac{\gamma_H(N)}{\gamma_K(N - 1)}\right) - \frac{dI^*}{dN} + \frac{dS^*}{dN} \right).$$

*In particular, if  $(I^*, S^*) \in \{(I, S), (I, \hat{S})\}$ , then it follows that*

$$\frac{dY}{dN} > 0 \text{ and } \frac{dL_m}{dN} > 0.$$

*Proof.* The proof follows by differentiating (3), the fact that  $L_m = Y \cdot \frac{S^* - I^*}{W_m}$ , and from Lemma 3.22. The details can be found in Part IV of the Online Appendix.  $\square$

The consequence of this proposition is that the output effect, as well as the employment effect, are not positive in general. If  $I^* = I$ , then they are positive, whereas their signs are ambiguous in the other cases. These results are contrary to the framework without a (binding) minimum wage, where the output effect is equal to the cost-saving effect and, consequently, positive by assumption. Moreover, in the model without minimum wage, both ripple effects are positive and the flexible low-skill wage absorbs the change in the demand for low-skill workers.

Due to the right-shift of the task interval, the set of tasks conducted by machines decreases. This reduces the rental rate if the cost-saving effect cannot balance the right-shift. If this is the case (the rental rate decreases) and  $I^* \neq I$ , machines displace low-skill workers. Without the cost-saving effect, this displacement effect reduces the employment and, thus, the aggregate output. Therefore, the signs of the output and the employment effect depend on the power of the cost-saving and the displacement effect and on the question of which of the two dominates



(as they are counteracting). In particular, it is possible that the aggregate output increases and the low-skill employment decreases at the same time. This is illustrated in Figure 6. Analogously to the “so-so” automation, we talk about the “so-so” new tasks if the cost-saving effect is too small to create an output effect.

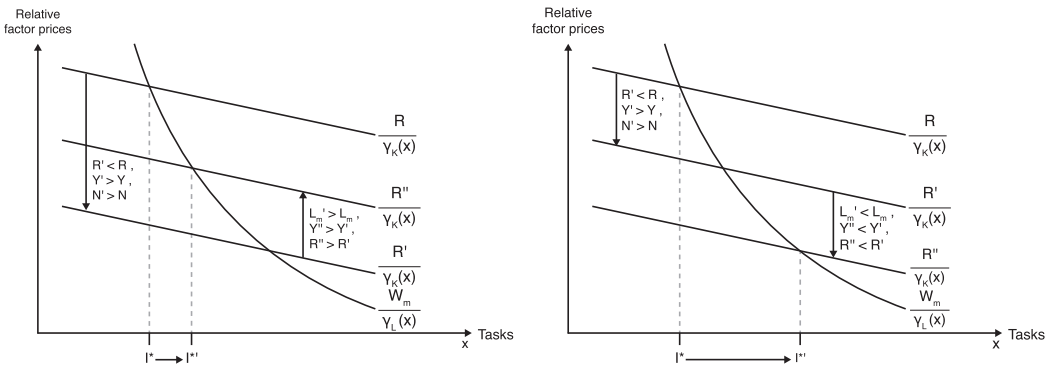
In the other cases, the output and the employment effect are positive as there is no displacement of low-skill workers. Moreover, high-skill workers benefit from the creation of new tasks as in the model without a minimum wage because their wage increases. Consequently, there is a reinstatement effect of the low-skill workers if  $S^* = \hat{S}$  or  $S^* = \tilde{S}$ . Therefore, low-skill employment increases and thus also the aggregate output.

The next corollary summarizes this discussion.

**Corollary 3.24.** *Let Assumptions 2.1 and 3.21 hold. Further, let  $N \geq 1$ ,  $K, L, H > 0$ ,  $0 < I < S < 1$ , and  $W_m \in (W_L, W_0)$ . Then, it holds that*

$$\begin{aligned} \frac{dW_H}{dN} &= \frac{N - S^*}{H} \cdot \frac{dY}{dN} + \frac{Y}{H} \cdot \left(1 - \frac{dS^*}{dN}\right) > 0, \\ \frac{dR}{dN} &= \frac{I^* - N + 1}{K} \cdot \frac{dY}{dN} + \frac{Y}{K} \cdot \left(\frac{dI^*}{dN} - 1\right), \\ \frac{d\left(\frac{W_H}{R}\right)}{dN} &= \frac{K}{H} \cdot \left( \frac{1}{I^* - N + 1} \cdot \left(1 - \frac{dS^*}{dN}\right) - \frac{N - S^*}{(I^* - N + 1)^2} \cdot \left(\frac{dI^*}{dN} - 1\right) \right). \end{aligned}$$

In particular, if  $(I^*, S^*) \in \{(I, S), (\hat{I}, S), (\tilde{I}, \tilde{S})\}$ , it holds that



**FIGURE 6** Both panels plot the allocation of tasks. If  $I^* \neq I$  and the rental rate decreases ( $N$  increases to  $N'$  and  $Y$  increases to  $Y'$ ), the curve of the relative price of machines is shifted down and  $I^*$  increases to  $I^*$ . This creates a displacement effect of low-skill workers. In the left panel, the cost-saving effect dominates the displacement effect by focusing on the demand for low-skill workers. Then, the low-skill employment increases  $L_m$  to  $L'_m$ . Therefore, the aggregate output increases further to  $Y''$ . Consequently, the rental rate also increases and shifts the curve of the relative price of machines up. Then, the low-skill workers displace machines and generate a reinstatement effect that raises the low-skill employment and the aggregate output, and so on. Finally, there is a positive output and employment effect. In the right panel, the displacement effect dominates the cost-saving effect by focusing on the demand for low-skill workers. Therefore, the low-skill employment  $L_m$  decreases to  $L'_m$  and the aggregate output also decreases to  $Y''$ . Consequently, the rental rate decreases further and shifts the curve of relative price of machines downwards. Then, the mechanism starts again, and so on. At the end, there is a negative-employment effect and the output effect can be either positive or negative

$$\frac{d\left(\frac{W_H}{R}\right)}{dN} > 0.$$

As discussed, the high-skill wage increases. The signs of the other effects on the factor prices are ambiguous. In addition, the signs of the effects on the shares in the national income are also ambiguous in general (see Lemma 3.22). Thus, the signs of the effects on the ratios between the expected low-skill wage and the other factor prices are also ambiguous.

### 3.2.6 | Summarizing the results

Our task-based model allows for different types of automation, which may result in distinct labor market outcomes. The analysis in Section 3.2 has derived several results which we summarize here, focusing on the effects of automation on output, employment of low-skill workers, and their income share.

In principle, all types of automation come with cost-saving effects in production. Whether the saved costs lead to higher production, however, is determined by a counteracting displacement effect of automation on the employment of the low-skill workers. For the case where the cost-saving effect is not large enough compared to the strength of the displacement effect, we find that output decreases as a consequence of automation. For the case where the cost-saving effect is large enough compared to the strength of the displacement effect, we find that output increases as a consequence of automation.

Let us first summarize what follows for the case of a decreasing output. Here, we can unambiguously determine that employment for the low-skill workers will also decline. However, as output and employment of the low-skill workers decline, the effect of automation on the share of income of the low-skill workers in the national income is ambiguous.

What follows after an increase in output is ambiguous, in contrast to what follows from a decrease in output. Then, employment increases as long as automation does not cost too many of the low-skill workers' jobs, as these tasks are now completed by machines or high-skill workers. If too many low-skill workers are substituted by machines, higher output comes with a decrease in low-skill employment. When employment decreases while output increases, the effect on the share of income of the low-skill workers in the national income is unambiguous: it will decrease. If higher output is accompanied by higher low-skill employment, then the share of income of the low-skill workers in the national income is nondecreasing.

The main reason for these mechanisms is that the minimum wage is fixed. Therefore, the low-skill wage cannot absorb the impact of automation on the demand for low-skill workers and clear the market, which occurs in the model without a minimum wage. Consequently, the effects on aggregate output, ratios of factor prices, and shares in the national income are different. This is summarized in Table A1 in Appendix A.

So far, we have grouped together five different types of automation. At least two of these automation processes (i.e., expanding skills and the deepening of skill) may also be interpreted as policy parameters rather than a type of automation. Specifically designed training programs may enhance the productivity of low-skill workers on tasks to which they are assigned (the deepening of skills) or may make them capable of doing tasks that could previously only be assigned to high-skill workers (expanding skills). Thus, our analysis may also be a guide for policymakers to learn about the labor market effects of training policies in a task-based framework with minimum wages.

## 4 | GENERALIZATIONS

This section discusses the possible generalizations of our model.

The elasticity of substitution  $\sigma$  of our production function (1) is  $\sigma = 1$ . However, we can choose another  $\sigma \in (0, \infty)$  instead, as in Acemoglu and Restrepo (2018b, 2018d). This should not change our main results and mechanisms as the displacement of low-skill workers has a negative impact on the aggregate output and the positive cost-saving effect can be small.

Moreover, we also do not need a task interval with measure one (e.g., Acemoglu & Restrepo, 2020b). In particular, the measure of the newly created tasks could be smaller than the measure of the destroyed tasks in general. For example, we could consider a task interval  $[f(N), g(N)]$  for some functions  $f, g$ , where  $g - f > 0$ , where  $N$  increases through the creation of new, more complex tasks in the economy. All results, apart from the consideration concerning the creation of new tasks, also remain valid for this kind of interval as the analysis does not depend on the length of the task interval. Considering the creation of new tasks, the results will then depend on the derivatives of  $f$  and  $g$  as they have an impact on the displacement of low-skill workers and the cost-savings.

Another way to generalize our framework is to include high-skill automation as considered in Acemoglu and Restrepo (2018b), where the task interval is divided into two intervals and machines are able to produce the first part of both intervals. This models the progress of artificial intelligence technologies that can perform relatively complex human tasks. However, this does not change the considered environment and, consequently, does not affect our results, but gives us only one more variable to analyze. We can model a high-skill automation by withdrawing Assumption 2.1 and changing the set of possible tasks of the machines (e.g., Acemoglu & Restrepo, 2020b). In particular, without the assumption  $\gamma_2(S^*) > 1$ , only  $W_H \geq W_m$  is guaranteed (instead of  $W_H > W_m$ ) because the minimum wage is binding. Consequently, high-skill workers can also become unemployed. If the demand for high-skill workers is smaller than the supply, all employed workers earn the same wage, namely, the minimum wage, and the threshold between low-skill and high-skill tasks is  $S^*$ , satisfying  $\gamma_2(S^*) = 1$  provided that there are employed low-skill workers.

Finally, another option to model the capital market or the long-run equilibrium is by fixing the rental rate and considering a variable capital stock (e.g., Acemoglu & Restrepo, 2018b, 2020b).

## 5 | CONCLUSION

It is an open question whether the use of minimum wages is the right policy to reduce the emerging inequality in an automating economy. This paper explores the effects of a binding minimum wage on aggregate output, employment, factor prices, and various measures of the income distribution in a task-based framework, in which tasks are conducted by machines, low-skill, and high-skill workers. We analyze an increase in the minimum wage and an automating economy in the presence of a minimum wage and compare the results with a model without a minimum wage.

We first show that an increase in the minimum wage reduces the aggregate output and low-skill employment. While the effect of an increasing minimum wage on machine prices and wages of high-skill workers is ambiguous, inequality between the low-skill wage and these two other factor prices decreases. The minimum wage may result in ripple effects where machines or high-skill workers displace low-skill workers. As a consequence, the share of income of the low-skill workers in the

national income is nonincreasing. Moreover, a minimum wage reduces the expected low-skill wage, which is defined as the minimum wage weighted with the share of low-skill employment.

We explore the effects of five different margins of automation on labor market outcomes in the presence of a minimum wage regulation. A variety of interesting effects stand out. All the findings are the result of an interplay of a cost-saving effect of automation and the effects that automation has on the employment of the low-skill workers. Whether cost-saving leads to higher or lower aggregate output depends on how strongly workers are displaced by automation. For a relatively weak cost-saving effect, output will decrease, and we can show that low-skill employment also decreases. Consequently, the effect on the share of income of the low-skill workers in the national income is ambiguous. When automation increases output, employment may increase or fall. A relatively high number of low-skill workers may be displaced, as their jobs are taken over by high-skill workers or by machines. Then employment and the share of income of the low-skill workers in the national income decrease. If only a few low-skill workers are substituted, however, employment increases with output. In this case, the share of income of low-skill workers in the national income is nondecreasing.

We can interpret these effects as follows. Automation may increase the demand for low-skill workers or decrease the demand for low-skill workers. In the first case, the equilibrium wage of the low-skill workers increases, while in the second case the equilibrium wage of the low-skill workers decreases. Thus, a minimum wage in an automating economy may have more or less bite. It will have more bite in the case when automation decreases labor demand for low-skill workers, and it will have less bite in the case where automation increases labor demand for low-skill workers.

We can also show that factor prices and income shares are affected differently by an automating economy, depending on whether a minimum wage exists or does not exist. There are also differences in the effects of automation on aggregate output. In particular, we show that automation at the extensive margin or the creation of new tasks affects aggregate output ambiguously in the presence of a minimum wage, while the effect on aggregate output is positive when there is no minimum wage. Table A1 in Appendix A summarizes the effects of all types of automation on labor market outcomes, differentiating between the models with and without minimum wages.

Finally, we conclude with further research topics related to this study that could be addressed in the future. There is a rapidly increasing empirical literature on job polarization that has been showing the distinct effects of automation on workers in different occupations (e.g., Autor & Dorn, 2013; Autor et al., 2003; Goos & Manning, 2007). In particular, it appears that there is a “hollowing out” of employment opportunities for workers engaged to perform routine tasks. It might be an interesting question to consider to what extent a minimum wage affects the employment opportunities of workers in a model of automation that explicitly models routine tasks or an additional group of workers with medium skills. We believe that our framework can be extended by including medium-skill workers to explore job polarization in conjunction with minimum wages.

In addition, while our analysis focuses on minimum wages, other redistributive instruments may be discussed and compared. In particular, one might consider a tax and transfer scheme that taxes machines and redistributes the revenue to workers negatively affected by automation. It would also be interesting to learn how the taxation of machines affects the speed of automation, and whether the revenues generated by a tax on machines would help to counterbalance the negative wage and employment effects on low-skill workers. To explore the consequences of taxing machines would require extending the current model substantially along different margins. We would not only have to define a tax schedule on machines, but we would also have to determine

how tax revenues are redistributed among workers. Moreover, the beneficiaries of the returns on the machines would have to be defined (i.e., who owns the capital) and how these returns are affected by introducing a tax on machines.

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## SUPPORTING INFORMATION

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## APPENDIX A

### A.1 | Proofs of Section 2

Here, we prove the unique equilibrium that is characterized by thresholds ( $I^*$ ,  $S^*$ ) and Equations (2)–(4). Before we start with the proof, we note that Assumption 2.1 implies that the

minimum wage does not affect high-skill employment if there are employed low-skill workers due to  $W_H > W_m$ .

We start with the following proposition that computes the equilibrium in an economy without a minimum wage.

**Proposition A.1.** *Let  $N \geq 1$  and Assumption 2.1 hold. Then, there exists for any  $K, L, H > 0$ ,  $I \in (N - 1, N)$ ,  $S \in (I, N)$  a unique equilibrium that is characterized by thresholds  $(I^*, S^*) \in (N - 1, N)^2$  with  $I^* < S^*$  and the aggregate output that is given by*

$$Y = \exp\left(\int_{N-1}^{I^*} \ln \gamma_K(x) dx + \int_{I^*}^{S^*} \ln \gamma_L(x) dx + \int_{S^*}^N \ln \gamma_H(x) dx\right) \cdot \left(\frac{K}{I^* - N + 1}\right)^{I^* - N + 1} \cdot \left(\frac{L}{S^* - I^*}\right)^{S^* - I^*} \cdot \left(\frac{H}{N - S^*}\right)^{N - S^*}. \quad (\text{A1})$$

In particular, it holds that  $(I^*, S^*) = \min\{(I, S), (I, \hat{S}), (\hat{I}, S), (\tilde{I}, \tilde{S})\}^{11}$ , where  $\hat{I}, \hat{S}, \tilde{I}, \tilde{S}$  are defined such that they satisfy the following equations:

$$\begin{aligned} \frac{S - \hat{I}}{\hat{I} - N + 1} \cdot \frac{K}{L} &= \gamma_1(\hat{I}), & \frac{N - \hat{S}}{\hat{S} - I} \cdot \frac{L}{H} &= \gamma_2(\hat{S}), \\ \frac{\tilde{S} - \tilde{I}}{\tilde{I} - N + 1} \cdot \frac{K}{L} &= \gamma_1(\tilde{I}), & \text{and } \frac{N - \tilde{S}}{\tilde{S} - \tilde{I}} \cdot \frac{L}{H} &= \gamma_2(\tilde{S}). \end{aligned}$$

Moreover, it holds that

$$\begin{aligned} l(x) = h(x) = 0 & \quad \text{if } x \in [N - 1, I^*], \\ k(x) = h(x) = 0 & \quad \text{if } x \in (I^*, S^*], \\ k(x) = l(x) = 0 & \quad \text{if } x \in (S^*, N],^{12} \end{aligned}$$

and

$$R = Y \cdot \frac{I^* - N + 1}{K}, \quad W_L = Y \cdot \frac{S^* - I^*}{L}, \quad W_H = Y \cdot \frac{N - S^*}{H}. \quad (\text{A2})$$

*Proof.* The proof is analogous to Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018a, 2018b). The details can be found in Part I of the Online Appendix.  $\square$

To obtain a unique equilibrium after introducing a minimum wage, we have to show that the relationship between the demand and the wage of low-skill workers is unambiguous. Because the demand is equal to the supply, we consider the following lemma.

**Lemma A.2.** *Let Assumption 2.1 hold. Further, let  $N \geq 1$ ,  $K, L, H > 0$ ,  $N - 1 < I < S < N$ . Then, there is a unique function*

$$\begin{aligned} \omega: (0, \infty) &\rightarrow (0, \infty), \\ \omega: L &\mapsto W_L. \end{aligned}$$

In particular,  $\omega$  is continuous and strictly decreasing.

<sup>11</sup>Here, the minimum is defined component-by-component.

<sup>12</sup>This holds almost everywhere on the interval  $[N - 1, N]$ .

*Proof.* We set, without loss of generality,  $N = 1$ . As the existence and uniqueness of the function  $\omega$  is clear from Proposition A.1, we have to show that  $\omega$  is continuous and strictly decreasing. For this, we consider the pairs  $(I, S), (I, \hat{S}), (\hat{I}, S), (\hat{I}, \hat{S})$  from Proposition A.1. If these pairs are continuous, then  $(I^*, S^*)$  and  $\omega$  are continuous because  $\omega(L) = W_L$  is a composition of continuous functions. The continuity of each pair follows from the implicit function theorem. In addition, the second property follows by calculating the slope of  $W_L$ .<sup>13</sup>  $\square$

Now, to determine the inverse function of  $\omega$ , we show that the low-skill wage is bounded and that the equilibrium of the model with low-skill workers converges to the equilibrium of the model without low-skill workers.

**Proposition A.3.** *Let  $N \geq 1$ ,  $L = 0$ , and Assumption 2.1 hold. Then, there exists for any  $K, H > 0$ ,  $I \in (N - 1, N)$  a unique equilibrium that is characterized by a threshold  $J \in (N - 1, I]$  and the aggregate output*

$$Y_0 = \exp\left(\int_{N-1}^J \ln \gamma_K(x) dx + \int_J^N \ln \gamma_H(x) dx\right) \cdot \left(\frac{K}{J-N+1}\right)^{J-N+1} \cdot \left(\frac{H}{N-J}\right)^{N-J}.$$

Moreover, it holds that

$$R_0 = Y_0 \cdot \frac{J - N + 1}{K} \quad \text{and} \quad W_{H,0} = Y_0 \cdot \frac{N - J}{H}.$$

In particular, it holds that

$$\begin{aligned} \lim_{L \searrow 0} I^* &= J = \lim_{L \searrow 0} S^*, \\ \lim_{L \searrow 0} Y &= Y_0 \end{aligned}$$

and

$$\lim_{L \searrow 0} W_L = \frac{W_{H,0}}{\gamma_2(J)} = : W_0, \tag{A3}$$

where  $Y$ ,  $I^*$ , and  $S^*$  are taken from the model in Proposition A.1 with  $L > 0$ .

*Proof.* The first part is analogous to the proof of Proposition A.1.

To show the convergence, we have to study the behavior of the threshold  $(I^*, S^*)$  if  $L$  increases. Similar to Lemma 3.1 (consider Lemma A.2) it follows that  $I^*$  is nonincreasing and  $S^*$  is nondecreasing. Thus,  $S^* - I^*$  is nondecreasing, too. It holds that

$$\frac{I^*}{K} \cdot \gamma_1(I^*) \leq \frac{S^* - I^*}{L} \leq \frac{1 - S^*}{H} \cdot \frac{1}{\gamma_2(S^*)}.$$

As  $\frac{S^* - I^*}{L}$  is bounded by  $\frac{1}{H \cdot \gamma_2(0)}$ , the convergences

$$\lim_{L \searrow 0} (S^* - I^*) = 0 \quad \text{and} \quad \lim_{L \searrow 0} \frac{S^* - I^*}{L} \in (0, \infty)$$

<sup>13</sup>The function  $W_L$  is not differentiable in general at the transitions between the pairs  $(I, S), (\hat{I}, S), (I, \hat{S}), (\hat{I}, \hat{S})$ .



follow. From the uniqueness of the equilibrium, we conclude

$$\lim_{L \searrow 0} I^* = J = \lim_{L \searrow 0} S^*$$

and thus

$$\lim_{L \searrow 0} Y = Y_0.$$

For small  $L$ , it even holds that

$$\frac{S^* - I^*}{L} = \frac{1 - S^*}{H} \cdot \frac{1}{\gamma_2(S^*)}.$$

Consequently, it holds that

$$\lim_{L \searrow 0} \frac{S^* - I^*}{L} = \lim_{L \searrow 0} \frac{1 - S^*}{H \cdot \gamma_2(S^*)} = \frac{1 - J}{H \cdot \gamma_2(J)}$$

and thus (A3) follows.  $\square$

Lemma A.2 and Proposition A.3 imply the following corollary that gives the (unique) necessary supply of low-skill workers in a competitive market for a fixed arbitrary low-skill wage between 0 and  $W_0$ .

**Corollary A.4.** *Let Assumption 2.1 hold. Further, let  $N \geq 1$ ,  $K, H > 0$ ,  $N - 1 < I < S < N$ . Then, there is a unique function*

$$\begin{aligned} \omega^{-1}: (0, W_0) &\rightarrow (0, \infty), \\ \omega^{-1}: W_L &\mapsto L. \end{aligned}$$

*In particular,  $\omega^{-1}$  is continuous and strictly decreasing.*

### A.1.1 | Proof of Equations (2)–(4)

Let  $N \geq 1$ ,  $K, L, H > 0$ ,  $I \in (N - 1, N)$ ,  $S \in (I, N)$  and  $W_m \in (W_L, W_0)$ . As the supply and demand for low-skill workers are identical in an equilibrium of a competitive market, Corollary A.4 implies that the demand for low-skill workers  $L_m = \omega^{-1}(W_m) \in (0, L)$  if the minimum wage  $W_m$  is binding. By construction, we get a new equilibrium  $(I^*, S^*)$  and  $Y$  for  $K, L_m, H, I$  and  $S$ , where  $(I^*, S^*) = \min\{(I, S), (I, \hat{S}), (\hat{I}, S), (\tilde{I}, \tilde{S})\}$  and  $\hat{I}, \hat{S}, \tilde{I}, \tilde{S}$  satisfy

$$\begin{aligned} \frac{S - \hat{I}}{\hat{I} - N + 1} \cdot \frac{K}{L_m} &= \gamma_1(\hat{I}), & \frac{N - \hat{S}}{\hat{S} - I} \cdot \frac{L_m}{H} &= \gamma_2(\hat{S}), \\ \frac{\tilde{S} - \tilde{I}}{\tilde{I} - N + 1} \cdot \frac{K}{L_m} &= \gamma_1(\tilde{I}), & \text{and } \frac{N - \tilde{S}}{\tilde{S} - \tilde{I}} \cdot \frac{L_m}{H} &= \gamma_2(\tilde{S}), \end{aligned}$$

$$Y = \exp\left(\int_{N-1}^{I^*} \ln \gamma_K(x) dx + \int_{I^*}^{S^*} \ln \gamma_L(x) dx + \int_{S^*}^N \ln \gamma_H(x) dx\right) \cdot \left(\frac{K}{I^* - N + 1}\right)^{I^* - N + 1} \cdot \left(\frac{L_m}{S^* - I^*}\right)^{S^* - I^*} \cdot \left(\frac{H}{N - S^*}\right)^{N - S^*}, \quad (\text{A4})$$

and

$$R = Y \cdot \frac{I^* - N + 1}{K}, \quad W_m = Y \cdot \frac{S^* - I^*}{L_m}, \quad W_H = Y \cdot \frac{N - S^*}{H}$$

analogously to before. Consequently, (4) holds. Then, it holds that

$$W_m = \exp\left(\int_{N-1}^{I^*} \ln \gamma_K(x) dx + \int_{I^*}^{S^*} \ln \gamma_L(x) dx + \int_{S^*}^N \ln \gamma_H(x) dx\right) \cdot \left(\frac{K}{I^* - N + 1}\right)^{I^* - N + 1} \\ \cdot \left(\frac{L_m}{S^* - I^*}\right)^{S^* - I^*} \cdot \left(\frac{H}{N - S^*}\right)^{N - S^*} \cdot \frac{S^* - I^*}{L_m}.$$

Thus, we get (2) if we solve it for  $L_m$ . Moreover, we get (3) from (A4) and (2).

## A.2 | Comparison of the model with and without minimum wage

Table A1 summarizes the differences between the model with and without a minimum wage.

**TABLE A1** Differences of partial effects of automation between the model with and without a (binding) minimum wage

With a minimum wage	Without a minimum wage
<i>Automation at the extensive margin</i>	
$\text{sign}\left(\frac{d\hat{S}}{dI}\right)$ is ambiguous	$\frac{d\hat{S}}{dI} \in (0, 1)$
$\text{sign}\left(\frac{dY}{dI}\right)$ is ambiguous	$\frac{dY}{dI} > 0$
$\text{sign}\left(\frac{d\left(\frac{W_H}{W_m}\right)}{dI}\right)$ is ambiguous	$\frac{d\left(\frac{W_H}{W_L}\right)}{dI} > 0$
$\text{sign}\left(\frac{ds_L}{dI}\right)$ is ambiguous	$\frac{ds_L}{dI} < 0$
<i>Deepening of automation</i>	
$\frac{d\hat{S}}{dA_K} > 0$	$\frac{d\hat{S}}{dA_K} = 0$
$\frac{d\left(\frac{W_H}{W_m}\right)}{dA_K}, \frac{d\left(\frac{R}{W_m}\right)}{dA_K} > 0$	$\frac{d\left(\frac{W_H}{W_L}\right)}{dA_K}, \frac{d\left(\frac{R}{W_L}\right)}{dA_K} \geq 0$
$\text{sign}\left(\frac{ds_L}{dA_K}\right)$ is ambiguous	$\frac{ds_L}{dA_K} \leq 0$
<i>Deepening of skills</i>	
$\text{sign}\left(\frac{d\left(\frac{W_H}{W_m}\right)}{dA_L}\right)$ is ambiguous	$\frac{d\left(\frac{W_H}{W_L}\right)}{dA_L} \leq 0$
$\text{sign}\left(\frac{d\left(\frac{R}{W_m}\right)}{dA_L}\right)$ is ambiguous	$\frac{d\left(\frac{R}{W_L}\right)}{dA_L} \leq 0$
$\frac{d\hat{I}}{dA_H} < 0$	$\frac{d\hat{I}}{dA_H} = 0$
$\frac{d\left(\frac{W_H}{W_m}\right)}{dA_H}, \frac{d\left(\frac{R}{W_m}\right)}{dA_H} > 0$	$\frac{d\left(\frac{W_H}{W_L}\right)}{dA_H}, \frac{d\left(\frac{R}{W_L}\right)}{dA_H} \geq 0$
$\text{sign}\left(\frac{ds_L}{dA_H}\right)$ is ambiguous	$\frac{ds_L}{dA_H} \leq 0$

**Expanding skills**

$$\frac{dI^*}{dS} \leq 0$$

$$\text{sign} \left( \frac{d \left( \frac{W_H}{W_m} \right)}{dS} \right) \text{ is ambiguous}$$

$$\frac{ds_K}{dS} \leq 0$$

$$\frac{dI^*}{dS} \in [0, 1)$$

$$\frac{d \left( \frac{W_H}{W_L} \right)}{dS} < 0$$

$$\frac{ds_K}{dS} \geq 0$$

**Creation of new tasks**

$$\text{sign} \left( \frac{dI^*}{dN} \right) \text{ is ambiguous}$$

$$\text{sign} \left( \frac{dY}{dN} \right) \text{ is ambiguous}$$

$$\text{sign} \left( \frac{d \left( \frac{R}{W_m} \right)}{dN} \right) \text{ is ambiguous}$$

$$\text{sign} \left( \frac{ds_K}{dN} \right) \text{ is ambiguous}$$

$$\text{sign} \left( \frac{ds_H}{dN} \right) \text{ is ambiguous}$$

$$\frac{dI^*}{dS} \in [0, 1)$$

$$\frac{dY}{dS} > 0$$

$$\frac{d \left( \frac{R}{W_L} \right)}{dN} < 0$$

$$\frac{ds_K}{dN} < 0$$

$$\frac{ds_H}{dN} > 0$$

*Notes:* This table shows differences of partial effects of automation on selected outcome variables for the cases where partial effects differ between a model with and without a minimum wage. Whenever partial effects do not differ, results are not compared.