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Article — Published Version

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Journal of Public Economic Theory

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Suggested Citation: Windsteiger, Lisa (2021) : Monopolistic supply of sorting, inequality, and welfare, Journal of Public Economic Theory, ISSN 1467-9779, Wiley, Hoboken, NJ, Vol. 23, Iss. 5, pp. 801-821,
<https://doi.org/10.1111/jpet.12518>

This Version is available at:

<https://hdl.handle.net/10419/284779>

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Monopolistic supply of sorting, inequality, and welfare

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Abstract

Why is an increase in income inequality often accompanied by an increase in socioeconomic segregation? And what are the welfare implications of this comovement? This paper uses a theoretical model to analyze the relationship between income inequality and socioeconomic segregation. It shows that rising inequality can trigger sorting according to income, as a monopolist's profits from offering sorting increase with income inequality. It also examines the relationship between sorting and social welfare and shows that profit-maximizing sorting patterns are not necessarily optimal from a welfare perspective. In fact, for a broad field of income distributions (monopolist) profits increase with inequality, while at the same time total welfare from sorting decreases.

1 | INTRODUCTION

In recent years, we have observed a rise in socioeconomic segregation in many industrialized countries.¹ People tend to interact increasingly with others who are not too different from themselves in terms of income, education, and political beliefs.² Moreover, evidence suggests that segregation and income inequality move jointly. Several studies for the United States show that both income inequality and segregation have increased in most metropolitan areas over the past 40 years (see e.g., Reardon & Bischoff, 2011; Watson, 2009).

¹See for example, Forman and Koch (2012) and Bishop (2008) for evidence on the United States.

²The terms *segregation* and *sorting* are used interchangeably throughout this paper. Segregation hereby always means *socioeconomic* segregation and must not be confused with *racial* segregation or similar concepts.

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The reasons for this comovement have not been explored widely so far. While the presence of assortative matching and (positive) sorting has been extensively discussed in the economics and sociology literature, little research has been done so far on the supply side of segregation and the relationship between inequality and the supply of segregation. Who offers individuals the possibility to segregate and how does this depend on the shape of the income distribution?

Given the trend of mounting socioeconomic segregation, an important question is also the social desirability of sorting. If people benefit from interacting with wealthy and influential people, poor people who are deprived of these contacts due to social seclusion will suffer. But sorting might not be universally beneficial for the rich either: Especially if inequality is high, it might be the case that they have to pay huge sums to separate themselves off from the rest of society (e.g., via gated communities or private schools). While Becker (1974) shows that assortative matching always maximizes total surplus in society, Levy and Razin (2015) and Hoppe et al. (2009) demonstrate that segregation is not necessarily beneficial for welfare if we count “sorting fees” as deadweight loss (i.e., costly signals which enable the formation of groups) and subtract them from the surplus.

Finally, it is important to note that the interests of a supplier of the sorting technology might be different from society's interests, and that the way sorting is implemented need not be optimal for society. In addition, an increase in inequality is likely to have different effects on the supplier of the sorting technology and on welfare.

The present paper is a first attempt to analyze the relationship between income inequality and the supply of sorting and to examine how well the interests of the supplier of the sorting technology and of society as a whole are aligned, especially in the face of rising inequality. A simple theoretical model is used in which income is distributed unequally in society and people can pay a “fee” to join a group and interact only with members of that group henceforth. The model is used to examine how this fee will be set if a profit-maximizing monopolist offers the sorting technology, and analyze the monopolist's profits and society's total welfare resulting from this split into groups.

Sorting is not generally beneficial for welfare. In fact, members of the poor group never benefit from being separated from the rest of society. However, depending on the shape of the income distribution, sorting can increase welfare of the rich and their utility gains can compensate for the losses of the poor, such that total welfare in society is higher with sorting compared to random matching. An increase in inequality increases monopolist profits from offering people the possibility to segregate, and potentially also welfare from segregation. However, it turns out that there is often a conflict between welfare and monopolist profits, in the sense that different partitions of society would be optimal for profits and welfare—the way in which the monopolist splits up society is in general not efficient (i.e., welfare maximizing). This conflict tends to intensify as inequality increases: monopolist profits increase, while welfare from sorting decreases as income inequality climbs high. The paper concludes by arguing that there is a sense in which this finding holds also if the monopolist can offer segregation into more than just two groups.

Even when sorting is optimal for total welfare due to large benefits from sorting accruing to the rich, governments would most likely be hesitant to promote segregation because of the losses it entails on the lower end of the income distribution (at least without considering additional measures to redistribute some of the benefits from sorting from the rich to the poor). However, the calculations in this paper show that there are cases in which sorting is detrimental even for total welfare, because the benefits on the upper tail of the income distribution cannot compensate for the losses of the poor, or because even the rich would be better off without sorting. The analysis can thus help to identify situations in which any benevolent social planner (or government agency) should have an interest in counteracting sorting, in particular in the face of rising inequality, for instance via social housing or

rent control policies to prevent gentrification or via school (choice) reforms to avoid educational (as well as residential) segregation (see e.g., Xu, 2019).

The presented model can either be read “literally,” in the sense that sorting happens in the form of residential segregation (in the extreme even gated communities), private education, and exclusive clubs. The sorting fee might then be regarded as the club’s membership fee, the cost of private education, or the price mark-up for living in a certain area or community. On the other hand, the model can be interpreted along the lines of Rayo (2013), who regards the monopolist as a supplier of conspicuous goods to individuals who want to signal a hidden type, such as their wealth. Those conspicuous goods provide no intrinsic value—consumers want to purchase them simply to obtain social status (via signaling that they belong to a certain “club” of high-ranked individuals who can afford to purchase such goods). Examples might be publicized charity contributions whose cause the donor does not care about (see also Glazer & Konrad, 1996) or art purchases when the consumer does not value the piece of art per se.

Both the literal and the more abstract reading of the model justify to some extent the assumption that the sorting technology (or signal) is provided by a monopolist, as both situations are usually associated with a large degree of market power. In particular with conspicuous consumption it is often the case that big and well-established luxury brands have an advantage over small competitors, and consumers tend to coordinate around “fashionable” brands with a certain reputation. But also with the “literal” type of segregation, companies offering residential segregation such as gated communities, but also private schools (think for instance Eton) and exclusive clubs (or similar institutions that enable exclusive interactions, like Scientology) are usually able to act as “quasi”—monopolists given their limited number (at least for a given region). However, the supplier of sorting being a monopolist is clearly just a first, simplifying assumption for an analysis that can provide a starting point for a more thorough investigation into the benefits and welfare implications of sorting and how these depend on inequality.

The rest of the paper is organized as follows: Section 2 presents related literature, Section 3 introduces the model of sorting according to income and examines how changes in inequality affect monopolist profits and welfare. Section 4 demonstrates that there can be a conflict between monopolist profits and welfare as inequality increases. This section is accompanied by an extensive Supporting Information Appendix presenting detailed calculations for some of the results. Section 5 examines the effect of increasing inequality on monopolist profits and welfare if the monopolist can offer as many cutoffs as she wants and Section 6 concludes.

2 | RELATED LITERATURE

The standard model of sorting and assortative matching is outlined and analyzed in Becker (1974). Levy and Razin (2015) examine total welfare and preferences for redistribution in the presence of costly income sorting without explicitly modeling the supply side of the sorting technology. Rayo (2013) characterizes optimal sorting if a profit-maximizing monopolist without costs chooses the sorting schedule, while Damiano and Li (2007) analyze the case of two or more competing firms. The present paper carries elements of both Levy and Razin (2015) (in the sense that it analyzes the normative aspects of segregation, in particular its effects on welfare) and of Rayo (2013) (because the sorting technology is assumed to be offered by a profit-maximizing monopolist). The paper’s main contribution is a thorough examination of how optimal sorting varies with inequality and how this affects the (potential) conflict between welfare and monopolist profit.

The paper is also related to the literature of educational segregation via private schools (see e.g., Epple & Romano, 1998; Fernandez & Rogerson, 2003; Levy & Razin, 2017), as well as the literature on costly signaling (see e.g., Hoppe et al., 2009) and conspicuous consumption (see e.g., Bagwell & Bernheim, 1996; Pesendorfer, 1995; Veblen, 1899). Hopkins and Kornienko (2004) analyze status good consumption and show that as society becomes richer, the proportion of income spent on conspicuous consumption increases and equilibrium utility falls at each level of income. Furthermore, an exogenous decrease in inequality can make the poor worse off. Slack and Ulph (2018) introduce reference consumption into an optimal tax model. They show that for low-productivity workers well-being decreases with net wage, which affects optimal redistributive taxation. In addition, reference consumption is distortionary, which gives a distortion-correcting role to taxation.

Furthermore, the paper is connected to the literature on endogenous jurisdiction formation in connection with sorting (see e.g., Ellickson, 1971; Tiebout, 1956). Gravel and Thoron (2007) examine a situation where unequally wealthy households with identical preferences sort into jurisdictions to produce a public good financed by proportional taxation and derive necessary and sufficient conditions for resulting wealth stratification.

Finally, the paper is related to the literature on club formation. While in the present model the utility of belonging to a group increases with own income and the income of the other members, Amir et al. (2014) examine a setup in which club membership, as well as carrying an idiosyncratic intrinsic benefit for each member, becomes more valuable as the size of the club increases. They analyze how positive network externalities affect the formation of thematic clubs and can lead to the disappearance of clubs whose intrinsic value only appeals to a minority of the population. They also compare non-cooperative equilibrium club formation to the optimal club structure a utilitarian social planner would set up and show that the welfare maximizing solution leads to the disappearance of the minority club more often than in the non-cooperative game.

3 | INEQUALITY, MONOPOLIST PROFIT, AND WELFARE

Let income y in an economy be distributed according to an income distribution $F(y)$ with support on the interval $Y = [0, \nu]$ (where $\nu < \infty$ unless explicitly mentioned otherwise). Assume furthermore that $F(y)$ is continuous and strictly monotonic on Y , with pdf $f(y)$ such that $F(z) = \int_0^z f(y)dy \forall z \in Y$. Suppose that an agent's utility is increasing not only in her own income but also in the average income of the people that she interacts with, which will henceforth be called her "reference group." Specifically, a person with income y_j gets utility $U_j = y_j E(y | y \in S_i)$, where S_i is individual j 's reference group. If society is not economically segregated, everybody's reference group is a representative sample of the whole population, such that $U_j = y_j E(y)$. However, suppose a person with income y_j can pay a fee $b > 0$ to join group S_b and interact henceforth (mainly) with members of that group. Joining group S_b yields utility

$$y_j E[y | y \in S_b] - b,$$

but the agent can also refrain from paying b and get utility

$$y_j E[y | y \in S_0]$$

instead, where S_b is the set of incomes y of people who have paid b and S_0 is the set of incomes y of people who haven't paid b . Then we can define the following:

Definition 1. A sorting equilibrium is a partition $[S_0, S_b]$ of Y and a sorting fee $b > 0$ such that

$$yE[y|y \in S_b] - b \leq yE[y|y \in S_0] \quad \forall y \in S_0, \quad (1)$$

$$yE[y|y \in S_b] - b \geq yE[y|y \in S_0] \quad \forall y \in S_b. \quad (2)$$

In a sorting equilibrium as defined above, people stay in the group that gives them the highest utility.

Appendix A.1 shows that in any sorting equilibrium, group S_b must have a higher average income than group S_0 , and that all sorting equilibria will be *monotone*, meaning that the groups S_0 and S_b are single intervals of Y (where group S_b must lie to the right of group S_0 on the Y scale). Therefore, I will from now on call people in S_b "the rich" and people in S_0 "the poor."

Furthermore, as all equilibria are monotone, the definition of a sorting equilibrium can be rewritten in terms of a cutoff \hat{y} , where everybody with income below the cutoff is in the poor group and everybody with income above the cutoff is in the rich group. For simplicity of notation, average income in the rich group, $E[y|y \in S_b]$, will be denoted by $\bar{E}(\hat{y})$ and average income in the poor group, $E[y|y \in S_0]$, by $\underline{E}(\hat{y})$.³ Appendix A.1 shows the following:

Proposition 1. Any sorting equilibrium is characterized by a cutoff $\hat{y} \in Y$ and a sorting fee $b > 0$ such that

$$\hat{y}\bar{E}(\hat{y}) - \hat{y}\underline{E}(\hat{y}) = b. \quad (3)$$

Proof. See Appendix A.1. □

A person with income \hat{y} just at the border of the two groups S_b and S_0 has to be exactly indifferent between joining either of the two groups in equilibrium. For the remainder of the paper I will choose the convention that people with income \hat{y} (who are indifferent between the two groups) stay in the poor group.

It can immediately be seen from (3) that the sorting fee is uniquely determined by the equilibrium cutoff \hat{y} , that is, for a given equilibrium partition $\{[0, \hat{y}], (\hat{y}, \nu]\}$, the sorting fee b is unique. The reverse statement is not true in general: For a given b , there might be multiple cutoffs \hat{y} that satisfy $\hat{y}(\bar{E}(\hat{y}) - \underline{E}(\hat{y})) = b$ (this could happen if the distribution is such that $\hat{y}(\bar{E}(\hat{y}) - \underline{E}(\hat{y}))$ is not strictly increasing or decreasing for all $\hat{y} \in Y$).⁴ For a given sorting fee, there could therefore be several monotone partitions of society that would be sorting equilibria given this fee. When modeling the supply side below, I thus require that whoever offers the sorting technology chooses the cutoff optimally and I implicitly assume that the supplier can then ensure that the agents coordinate on the equilibrium that yields the highest payoff for the supplier (which, in the case of a profit-maximizing firm, would always be the lowest cutoff \hat{y} such that $\hat{y}(\bar{E}(\hat{y}) - \underline{E}(\hat{y})) = b$, because it yields the largest mass of customers).

³Note that $\bar{E}(\hat{y}) = \frac{\int_{\hat{y}}^{\nu} yf(y)dy}{1-F(\hat{y})}$ and $\underline{E}(\hat{y}) = \frac{\int_0^{\hat{y}} yf(y)dy}{F(\hat{y})}$.

⁴It can be shown that a sufficient condition for $\hat{y}(\bar{E}(\hat{y}) - \underline{E}(\hat{y}))$ to be monotone is that the income distribution is *new worse than used in expectations* (NWUE). For a definition of the NWUE property see Section 3.2.

3.1 | Monopolist profit

The model outlined above shows how the sorting fee has to be set to generate a certain partition of society. But who determines how the groups in society look like? Who offers the sorting technology and chooses the cutoff? As mentioned in the introduction, both the “literal” reading of the model concerning residential segregation, private education, and clubs, and the “signaling” interpretation along the lines of Rayo (2013) point toward some degree of market power on the supply side. For the remainder of this paper I will thus assume that the sorting technology is offered by a profit-maximizing monopolist and examine the implications of an increase in inequality for the monopolist’s profits and for total welfare.

The next sections will focus on the model of sorting with two groups as described above. The monopolist can therefore only decide between offering one cutoff or staying inactive, but she cannot offer more than one cutoff. This could be modeled explicitly by assuming that the costs of offering more than one cutoff are prohibitively high. The last section of this paper discusses what happens if the monopolist’s costs are negligible and she can therefore offer as many cutoffs as she wants.

If the monopolist faces fixed costs $c > 0$ of operating, her profits from offering sorting are

$$\Pi(\hat{y}^*) = R(\hat{y}^*) - c,$$

where $R(\hat{y}^*)$ is the revenue from offering sorting at cutoff \hat{y}^* and \hat{y}^* is chosen optimally, such that

$$\hat{y}^* = \arg \max_{\hat{y}} R(\hat{y}).$$

Revenue at cutoff \hat{y} is given by

$$R(\hat{y}) = \hat{y}(\bar{E}(\hat{y}) - \underline{E}(\hat{y}))(1 - F(\hat{y})) = \hat{y}(\bar{y} - \underline{E}(\hat{y})),$$

where \bar{y} is average income. It is straightforward to see that any solution to the revenue maximization problem must be interior, because $R(0) = R(\nu) = 0$ whereas $R(\hat{y})$ is strictly positive for any interior \hat{y} . The monopolist’s optimal cutoff is thus implicitly defined via the first order condition,⁵

$$R'(\hat{y}^*) = \bar{y} - \underline{E}(\hat{y}^*) - \hat{y}^* \frac{\partial \underline{E}(\hat{y}^*)}{\partial \hat{y}} = 0. \quad (4)$$

Suppose that the income distribution and the fixed costs c are such that $\Pi(\hat{y}^*) > 0$ and hence it is profitable for the monopolist to offer the sorting technology. What happens to her profits as inequality increases? The following proposition states that the monopolist’s profits always rise if inequality increases in the form of a particular type of mean-preserving spread of the income distribution. I shall say that a mean-preserving spread is *monotone* if $\bar{E}(\hat{y})$ increases and $\underline{E}(\hat{y})$ decreases for any interior cutoff \hat{y} (while of course, as implied by the definition of a mean-preserving spread, average income \bar{y} does not change). Note that any mean-preserving spread of the income distribution necessarily implies an increase in the Gini coefficient (see Cowell, 2000; Dalton, 1920), and thus in inequality.

⁵ As $\underline{E}(\hat{y}) = \frac{\int_0^{\hat{y}} y f(y) dy}{F(\hat{y})}$, we get that $\frac{\partial \underline{E}(\hat{y})}{\partial \hat{y}} = \frac{f(\hat{y})(\hat{y} - \underline{E}(\hat{y}))}{F(\hat{y})}$ and thus (4) can be written as

$$\bar{y} - \underline{E}(\hat{y}^*) = \hat{y}^* \frac{f(\hat{y}^*)(\hat{y}^* - \underline{E}(\hat{y}^*))}{F(\hat{y}^*)}.$$

Proposition 2. *An increase in inequality in the form of a monotone mean-preserving spread of the income distribution **increases** the monopolist's profits from offering sorting.*

Proof. If inequality increases in the form of a monotone mean-preserving spread of the income distribution, the difference $\bar{y} - \underline{E}(\hat{y})$ will increase. This implies a rise in $\hat{y}^*(\bar{y} - \underline{E}(\hat{y}^*))$, keeping \hat{y}^* constant at the optimal choice for the initial income distribution. It is very likely that the optimal cutoff will also change for the monopolist, but even with keeping the old cutoff, her revenues increase, and they will do even more so if the monopolist also chooses the cutoff optimally. \square

It is easy to see from the above proof that, in order for the monopolist's profits to increase, the mean-preserving spread does not have to be such that $\bar{E}(\hat{y})$ increases and $\underline{E}(\hat{y})$ decreases for *any* cutoff—it suffices if this holds for the initially optimal cutoff. Proposition 2 therefore states sufficient conditions for an increase in the monopolist's profits.

If the income distribution and the fixed cost are initially such that $\Pi(\hat{y}^*) < 0$, an increase in inequality can have an effect on the monopolist's decision of whether or not to offer sorting at some \hat{y} , where she compares the profits from offering the sorting technology to 0 (the profits she would make if she stays inactive). An increase in inequality of the form described above, if it is large enough, will make the monopolist's profits positive, which in turn leads the monopolist to become active. As a result, society can become economically segregated due to an increase in inequality in the form of a mean-preserving spread of the income distribution.

Corollary 1. *If society is not segregated initially, a **sufficiently high increase in inequality** in the form of a monotone mean-preserving spread will make it profitable for a monopolist to offer sorting and will thus **trigger segregation**.*

A mean-preserving spread is not the only type of increase in inequality that increases the monopolist's profits from offering sorting. In fact, from examining the expression for the monopolist's profits, $\hat{y}^*(\bar{y} - \underline{E}(\hat{y}^*)) - c$, it is straightforward to see that any increase in inequality that increases $\bar{y} - \underline{E}(\hat{y}^*)$ for the initially optimal cutoff \hat{y}^* will raise the monopolist's profits.

3.2 | Welfare

The above section shows that an increase in inequality in the form of a mean-preserving spread increases a monopolist's profit from offering sorting. But what happens to welfare in society? Total welfare under no sorting (and thus random matching) is simply

$$W^P = \int_0^y y \bar{y} f(y) dy = \bar{y}^2.$$

Total welfare from sorting at cutoff \hat{y} can be written as⁶

⁶As in Levy and Razin (2015), welfare from sorting at a particular cutoff takes into account the sorting fee paid as a deadweight loss to society, or benefiting only a negligible proportion of society (which is the case if the supplier is a monopolist). If the sorting fee would not be considered, perfect sorting would always yield maximal total welfare, because the utility from a match is supermodular (see Becker, 1974).

$$\begin{aligned}
 W(\hat{y}) &= \underline{E}(\hat{y}) \int_0^{\hat{y}} y f(y) dy + \bar{E}(\hat{y}) \int_{\hat{y}}^{\bar{y}} y f(y) dy - (1 - F(\hat{y})) \hat{y} (\bar{E}(\hat{y}) - \underline{E}(\hat{y})) \\
 &= F(\hat{y}) (\underline{E}(\hat{y}))^2 + (1 - F(\hat{y})) (\bar{E}(\hat{y}))^2 - \hat{y} (\bar{y} - \underline{E}(\hat{y})).
 \end{aligned} \tag{5}$$

It is immediate to see that sorting is never beneficial for the poor group—they would always obtain higher utility on average if they could interact randomly with everybody in society. However, welfare can be higher for people in the rich group (and in particular for the richest among those) compared with random matching, despite the sorting fee they have to pay to separate themselves off from the rest of society. Whether total welfare, which is just the sum of utilities in the poor and the rich group, is higher under sorting compared to random matching depends on the shape of the income distribution (and, in general, on the cutoff).

Levy and Razin (2015) characterize distributions for which sorting is always more efficient (in a utilitarian sense, i.e., yielding higher total welfare) than no sorting, irrespective of the cutoff. They show that the difference between welfare of sorting at cutoff \hat{y} and welfare of no sorting can be written as

$$W(\hat{y}) - W^P = (\bar{y} - \underline{E}(\hat{y})) (\bar{E}(\hat{y}) - \bar{y} - \hat{y}) \tag{6}$$

and thus sorting yields higher total welfare than no sorting for any \hat{y} iff the income distribution is such that

$$\bar{E}(\hat{y}) - \bar{y} > \hat{y} \quad \forall \hat{y}. \tag{7}$$

This condition is what in reliability theory has been termed the *new worse than used in expectations* (NWUE) property. A distribution F is NWUE if condition (7) is satisfied, and *new better than used in expectations* (NBUE) if the opposite holds, that is

$$\bar{E}(\hat{y}) - \bar{y} < \hat{y} \quad \forall \hat{y}.$$

It is immediate to conclude the following:

1. If F is NWUE, sorting at any cutoff yields higher total welfare than no sorting.
2. If F is NBUE, no sorting yields higher total welfare than sorting at any \hat{y} .
3. If F is not NBUE, then there will always exist some cutoff \hat{y} at which sorting yields higher total welfare than no sorting.

To see the intuition behind 1. and 2., note that the positive assortativity benefits from sorting outweigh the welfare costs (in form of the sorting fee that needs to be paid) if the distribution is sufficiently unequal (such that random matching would lead to much lower utilities for rich individuals). NWUE distributions exhibit a greater degree of variability than NBUE distributions.⁷ Hence, the utility benefits of sorting outweigh the costs for NWUE distributions, while the opposite holds in the NBUE case.

In light of these considerations, it is not surprising that a monotone mean-preserving spread increases total welfare at certain cutoffs \hat{y} .

Proposition 3. *An increase in inequality in the form of a monotone mean-preserving spread of the income distribution increases total welfare from sorting at those cutoffs where $\bar{E}(\hat{y}) - \bar{y} > \hat{y}$.*

⁷ All NWUE distributions have a coefficient of variation $CV = \frac{\sqrt{\text{Var}(y)}}{\bar{y}}$ larger than (or equal) 1, whereas the NBUE property implies that $CV \leq 1$ (see Hall & Wellner, 1981).

Proof. If $\bar{E}(\hat{y}) - \bar{y} > \hat{y}$ then (6) tells us that the difference between welfare of sorting at \hat{y} and welfare of no sorting increases due to this mean-preserving spread (both $\bar{y} - \underline{E}(\hat{y})$ and $\bar{E}(\hat{y}) - \bar{y} - \hat{y}$ increase). As welfare of no sorting is \bar{y}^2 and thus does not change due to a mean-preserving spread, this implies that welfare of sorting at \hat{y} must increase. \square

From Proposition 3 we can immediately deduce the following

Corollary 2. *If F is NWUE, an increase in inequality in the form of a monotone mean-preserving spread of the income distribution increases total welfare from sorting at any given cutoff.*

Note that from Proposition 3 no general predictions can be made for welfare at those cutoffs where $\bar{E}(\hat{y}) - \bar{y} < \hat{y}$: On the one hand, $\bar{y} - \underline{E}(\hat{y})$ increases, but on the other hand $\bar{E}(\hat{y}) - \bar{y} - \hat{y}$ is negative (even though the mean-preserving spread will decrease this term in absolute value). The total effect of the mean-preserving spread on (6) is thus ambiguous and will depend on the shape of the analyzed income distribution.

If F is NBUE and hence there is no cutoff such that $\bar{E}(\hat{y}) - \bar{y} > \hat{y}$, a mean-preserving spread can make sorting efficient (i.e., better for welfare than no sorting) for some cutoffs.

Proposition 4. *If F is initially NBUE, a sufficiently large monotone mean-preserving spread of the income distribution will make sorting efficient at some cutoff \hat{y} .*

Proof. The mean-preserving spread will increase $\bar{E}(\hat{y}) - \bar{y}$ for all \hat{y} , which will eventually make $\bar{E}(\hat{y}) - \bar{y} - \hat{y}$ positive for some \hat{y} . \square

An increase in inequality will therefore increase total welfare from sorting at those cutoffs for which $\bar{E}(\hat{y}) - \bar{y} - \hat{y} > 0$ and can make sorting at *some* cutoff efficient (from a utilitarian point of view) if F is initially NBUE. Importantly, though, it is not necessarily the case that sorting at the cutoff that the monopolist chooses after the increase in inequality yields higher welfare than before. As described above, a mean-preserving spread of the income distribution increases welfare of sorting at those cutoffs for which $\bar{E}(\hat{y}) - \bar{y} - \hat{y} > 0$, but what happens to total welfare from sorting at the other cutoffs depends on the shape of the income distribution. Furthermore, even if the monopolist's optimal cutoff is initially such that $\bar{E}(\hat{y}) - \bar{y} - \hat{y} > 0$, the change in the shape of the income distribution can imply that the monopolist chooses a different cutoff after the mean-preserving spread, at which total welfare is lower than before.

Finally, it is important to emphasize that, as mentioned above, even if the income distribution is such that sorting yields higher total welfare than random matching, the poor group never benefits from sorting. Sorting can be efficient (in a utilitarian sense) for some income distributions (in particular for those which exhibit the NWUE property) due to large utility gains for people on the upper end of the income distribution, who profit from not having to interact with the poor part of society. However, poor people suffer from sorting because they lose the possibility to interact with high-income individuals. Hence, even though from a utilitarian perspective a social planner should be in favor of sorting whenever it maximizes social welfare, this is probably not what any real-life government agency would promote (at least not without considering additional measures aimed at redistributing some of the accruing benefits from sorting from the rich to the poor). But the calculations in this section and the following can

identify cases in which sorting (at any cutoff, or at the monopolist's optimally chosen one) is detrimental even for total welfare, because the utility gains accruing to the rich cannot compensate for the losses of the poor (or because even the rich would be better off without sorting). The analysis can thus help pinpoint situations in which sorting is a particularly bad idea from a welfare perspective and any benevolent social planner (or government agency) should have an interest in counteracting it, in particular as inequality increases (which, as Proposition 2 demonstrates, ups the attractiveness of offering sorting from a monopolist's point of view).

The relationship between the monopolist's profit and welfare and how this relationship changes as inequality increases will be the focus of the next section.

4 | INCREASING INEQUALITY AND THE CONFLICT BETWEEN MONOPOLIST PROFIT AND WELFARE

The analysis so far shows that an increase in inequality in the form of a monotone mean-preserving spread always increases the monopolist's profit and can also raise total welfare from sorting for some cutoffs \hat{y} . However, the cutoffs at which monopolist profits increase are not necessarily the same as the ones where welfare increases. Indeed, a monopolist's and a utilitarian social planner's interests are in general not aligned. As demonstrated below, total welfare from sorting at the monopolist's optimal cutoff often declines with inequality.

Section 1 in the Supporting Information Appendix analyzes a stylized income distribution to illustrate the potential conflict between monopolist profits and welfare due to increasing inequality. I decided to call this distribution the *symmetric atoms distribution*.⁸ In the Supporting Information Appendix I show that (unless inequality is very small) total welfare at the monopolist's optimal cutoff is decreasing in inequality, and that both welfare of the richest in society and average welfare in the poor group decline with rising inequality. The following section examines, which of these findings apply to a more general class of distributions.

First, five stylized types of income distributions are analyzed. These distributions have the same average income, but differ in their implied degree of inequality (measured as $\bar{E}(\hat{y}) - \underline{E}(\hat{y})$ for any cutoff). I analyze what these different degrees of inequality imply for monopolist profits and resulting total welfare. The income distributions range from full equality (where everybody in society has the same income) to a distribution that could be classified as "high inequality" (where half of the population have nothing, and half have the maximum possible income).

Note that all of these distributions are NBUE (see Appendix A.2.2) and thus no sorting yields higher total welfare than sorting at any cutoff (except for the case of total equality, where sorting and no sorting yield the same welfare). Furthermore, as average income is the same for all the distributions in this analysis, total welfare without sorting doesn't vary with inequality and is $\frac{v^2}{4}$ in all cases.⁹

- **Egalitarian distribution:** If the income distribution is completely egalitarian, that is, everybody has income $\frac{v}{2}$, then the monopolist's profits from sorting will be 0 (because offering sorting will not be profitable with fixed costs or yield a profit of 0 without fixed costs). Total welfare in this case is $\frac{v^2}{4}$.

⁸This distribution, and also some of the distributions analyzed later in this paper, do not satisfy all the conditions required in the initial setup of the model, that is, $F(\cdot)$ is in general not continuous and strictly monotonic. However, this is not a problem for most of the below calculations.

⁹All the background calculations for the results below can be found in Section 2 of the Supporting Information Appendix.

- Triangle distribution: If income is distributed in a triangular (isosceles) shape on $[0, \nu]$ such that the density is

$$f(y) = \frac{4}{\nu^2}y \quad \text{if } y \in \left[0, \frac{\nu}{2}\right]$$

$$f(y) = \frac{4}{\nu} - \frac{4}{\nu^2}y \quad \text{if } y \in \left[\frac{\nu}{2}, \nu\right],$$

the profit-maximizing cutoff for the monopolist is $\hat{y}^* = \frac{3\nu}{8}$ and the resulting profits are $\frac{3\nu^2}{32}$.

Total welfare from sorting at this cutoff amounts to $\frac{3059}{529} \frac{\nu^2}{32} < \frac{\nu^2}{4}$.

- Uniform distribution: If income is uniformly distributed on $[0, \nu]$, the monopolist's profit maximizing cutoff is $\hat{y}^* = \frac{\nu}{2}$ and the resulting profit is $\frac{\nu^2}{8}$. Total welfare from sorting at this cutoff is $\frac{3\nu^2}{16} < \frac{\nu^2}{4}$.

- Reverse triangle distribution: If income is distributed in a reverse-triangular (isosceles) shape on $[0, \nu]$ such that the density is

$$f(y) = \frac{2}{\nu} - \frac{4}{\nu^2}y \quad \text{if } y \in \left[0, \frac{\nu}{2}\right]$$

$$f(y) = -\frac{2}{\nu} + \frac{4}{\nu^2}y \quad \text{if } y \in \left[\frac{\nu}{2}, \nu\right]$$

the monopolist's optimal cutoff is $\hat{y}^* = 0.64\nu$, which yields a profit of $0.1935\nu^2$. Total welfare from sorting at this cutoff is $0.163\nu^2 < \frac{\nu^2}{4}$.

- Binary inegalitarian distribution: If half of the population has 0 income and half of them earn ν , the optimal cutoff for the monopolist is $\hat{y} = \nu$ with corresponding sorting fee $\frac{\nu}{2}$. Note that due to the jump in F at ν (F is not continuous here!) the sorting fee is not uniquely determined, any $b \in (0, \frac{\nu}{2}]$ would work, and the monopolist will choose the highest in this interval to maximize her profits (and therefore the profits will be $\frac{\nu^2}{2}$). Resulting total welfare in this case would be 0. Welfare would be maximized with the same partition, that is, a poor group with zero income and a rich group with income ν , but with the lowest of feasible sorting fees, that is, b being just ϵ over 0. Resulting welfare would be $\frac{\nu}{2} - \frac{\epsilon}{2}$. If the mass at both endpoints is not equal, this last result holds as well, because it is always efficient to separate rich and poor if the sorting fee is negligible, due to the supermodularity of utility from sorting (see Becker, 1974). The monopolist's profit in that latter case is increasing in the mass of rich people relative to poor people.

From this simple analysis the following can be concluded for these five distributions:

1. As **inequality increases** (in terms of discrete jumps from one distribution to another), **the monopolist's profits increase**.
2. As inequality increases, the monopolist's optimal cutoff increases.
3. Total welfare is independent of inequality in the absence of sorting, it depends only on average income. For all the above analyzed distributions, no sorting is more efficient than sorting at any cutoff \hat{y} .

4. If the monopolist chooses the cutoff to maximize her profits, then total welfare is highest in the case of the egalitarian distribution (because the sorting fee is 0 in that case and the situation is equal to no sorting, which maximizes welfare for all the distributions discussed above). The next highest welfare would be achieved in the uniform case, followed by the triangular and then the reverse triangular case, and the binary inegalitarian distribution would be worst for welfare (given the sorting fee that the monopolist would charge). Hence—if we exclude the case of the egalitarian distribution and start from a triangular distribution—**total welfare from sorting** at the monopolist's optimal cutoff **initially increases with inequality**, but **as inequality climbs too high** the monopolist can claim a huge part of the gross benefits from sorting for herself and **total welfare decreases**.

For the symmetric atoms distribution, Section 1 in the Supporting Information Appendix shows that welfare from sorting at the monopolist's optimal cutoff is decreasing in inequality. Here, we see that if we do not only look at mean-preserving spreads of the uniform distribution, but start from a situation where inequality is smaller than for a uniform distribution, the picture is different: Welfare increases with inequality for small rates of inequality, and decreases thereafter. Section 2.4 in the Supporting Information Appendix shows that this is true not only for the above discrete jumps in inequality but also for continuous changes in inequality for these types of distributions. In particular, the Supporting Information Appendix analyzes a distribution that is, for low levels of inequality, shaped like a house, and then as inequality increases becomes uniform and in the end looks like a reverse house (or trough). The two extreme cases are thus the triangle distribution (low inequality) and the reverse triangle distribution (high inequality) from above. The analysis finds the same results for this continuous version of the stylized distributions above: welfare of sorting at the monopolist's optimal cutoff y^* increases in inequality for low rates of inequality, and decreases for high rates. In a sense, there is thus less of a conflict between profit maximization and welfare for low rates of inequality than for high rates. However, note that all these distributions, ranging from the triangle to the reverse triangle one and all degrees of inequality in between, are NBUE and hence no sorting yields higher welfare than sorting at any cutoff (see Appendix A.2.2). Even though, for low rates of inequality, an increase in inequality increases total welfare from sorting at the monopolist's optimal cutoff, random matching would be superior from a utilitarian perspective in all those cases.

5 | MULTIPLE GROUPS

The previous sections examine how increasing inequality affects welfare and profits if the monopolist can choose one cutoff and thus offer segregation into two groups. The above analysis shows that the interests of a profit-maximizing monopolist and a utilitarian social planner are generally not aligned, and that the conflict between those interests increases with inequality. In the following section these results are compared to a situation where the monopolist does not face any costs of offering segregation and can therefore offer infinitely many groups (i.e., perfect sorting) if she wants. It turns out that the findings from the previous sections hold in some sense also for this more general setting: There is a way in which an increase in inequality increases the conflict between monopolist's profits and welfare (and lets the monopolist extract more surplus, if she can set the menu of sorting fees).

Before looking at the monopolist's optimization problem, let me first examine what is best for total welfare if multiple groups are possible.¹⁰ In a first step we can compare welfare from no sorting to welfare arising from perfect sorting. Hoppe et al. (2009) show that if the income distribution is such that the coefficient of variation, which is given by

$$CV(y) = \frac{\sqrt{\text{Var}(y)}}{\bar{y}},$$

is larger than 1 then perfect sorting yields higher welfare than no sorting, and if $CV(y) \leq 1$, the opposite holds:

Proposition 5 (Hoppe et al., 2009). *Perfect sorting is more (less) efficient than no sorting iff $CV(y) \geq (\leq) 1$.*

Proof. The proof from Hoppe et al. (2009) is reproduced in Section 3.1 of the Supporting Information Appendix. \square

The coefficient of variation can be regarded as a measure of inequality - it is high if the difference between the standard deviation and the average is high, and it increases due to a mean-preserving spread of the income distribution. Hence, another way to interpret Proposition 5 is in terms of inequality: For low rates of inequality, no sorting is more efficient than perfect sorting, whereas if inequality is high, perfect sorting yields higher welfare than random matching. The intuition behind this result is that the decision between pooling the entire population and perfect sorting involves a trade-off: Rich people achieve lower utility if they have to interact with the whole population than if they can separate themselves off, while poor people attain higher utility if they can interact with the rich. For high rates of inequality, the former effect dominates the latter due to the supermodularity of the utility function.

It can be shown that the NBUE property, introduced in the previous section, is intricately connected to another property for distributions called IFR. A distribution exhibits an *increasing failure rate (IFR)* iff $\frac{f(y)}{1-F(y)}$ is increasing for all y . Two things can be shown for distributions with IFR:

Proposition 6.

- If $F(y)$ is IFR $\Rightarrow F(y)$ is NBUE (Marshall & Proschan, 1972)
- If $F(y)$ is NBUE $\Rightarrow CV(y) \leq 1$ (Hall & Werner, 1981)

Corollary 3. *All IFR distributions must have a coefficient of variation smaller than 1.*

Proof. The proof of the first part of Proposition 6 is a well-known fact from reliability theory, proved for instance in Marshall and Proschan (1972). Hall and Wellner (1981) show that if $F(y)$ is NBUE then the coefficient of variation is smaller than unity. Together, these two facts imply that all IFR distributions must have a coefficient of variation smaller than 1. \square

¹⁰Note again that examining welfare in society as just the sum of individual utilities (i.e., total welfare) does not shed light onto who in society benefits or loses from sorting. With two groups (like in the previous sections), it is clear that the poor group is always worse off with sorting compared to random matching, whereas it depends on the shape of the income distribution whether the rich benefit from sorting. With multiple groups, matters are not so simple. Depending on how the groups are chosen (and, again, on the shape of the income distribution), even people at the lower end of the income distribution could be better off with sorting compared to random matching, if the sorting schedule is such that it prevents them from interacting with poorer people.

Appendix A.2.2 shows that the triangle distribution, the uniform distribution, and the reverse triangle distribution discussed in the previous section and the distribution discussed in the Supporting Information Appendix (which encompasses all the others) are NBUE. As the coefficient of variation is smaller than 1 for all distributions which are NBUE, no sorting yields higher welfare than perfect sorting for these distributions. The symmetric atoms distribution is not NBUE—indeed Appendix A.2.1 shows that for small \hat{y} sorting yields higher welfare than no sorting. However, the symmetric atoms distribution has $CV(y) \leq 1$ and therefore perfect sorting always yields lower welfare than no sorting (see Appendix A.2.1).

For NBUE distributions, Levy and Razin (2015) also prove an additional feature:

Proposition 7 (Levy & Razin, 2015). *No sorting yields higher welfare than any finite sorting (i.e., any finite incentive compatible partition of society) iff $F(y)$ is NBUE.*

Proof. The main intuition behind the proof in Levy and Razin (2015) is reproduced in Section 3.2 of the Supporting Information Appendix. \square

For the class of NBUE distributions, we can hence conclude the following:

Corollary 4. *If the income distribution is NBUE, no sorting yields higher welfare than perfect sorting or any type of finite sorting.*

Proof. This follows immediately from Propositions 5, 6, and 7. \square

Note that, as NBUE is a weaker property than IFR, Corollary 4 also holds for the class of IFR distributions.

After characterizing the class of distributions for which no sorting yields higher welfare than perfect sorting and any finite sorting, we can now analyze the monopolist's optimization problem: What is the monopolist's optimal sorting schedule if she does not face any costs of offering the technology? Rayo (2013) characterizes the optimal placement of regions of pooling and perfect sorting, depending on the shape of the income distribution.¹¹ In the following I want to examine the implications of changing inequality on the monopolist's optimal sorting schedule and total welfare.

Rayo shows that if (and only if) the function $h(y) = y - \frac{1-F(y)}{f(y)}$ is nondecreasing everywhere, perfect sorting is the profit-maximizing sorting schedule. If there are regions of y for which $h(y)$ is decreasing, perfect sorting is not optimal for the monopolist and she will want to introduce intervals of y for which she pools everybody into one joint group.¹²

It is immediate to see that $h(y)$ is always decreasing if the distribution has an increasing failure rate (IFR). Hence, if a distribution exhibits IFR, perfect sorting is optimal for the monopolist. We can therefore conclude the following:

Corollary 5. *If the income distribution exhibits IFR, a monopolist and a utilitarian social planner have conflicting interests: No sorting is more efficient than perfect sorting or any*

¹¹Section 3.3 of the Supporting Information Appendix reproduces the main intuition behind his findings.

¹²Note however that there are never two pooling intervals next to each other (i.e., pooling intervals are always maximal) and that pooling is never optimal at the top end of the distribution.

type of finite sorting, but the monopolist wants to implement perfect sorting to maximize her profits.

What happens within the class of distributions for which perfect sorting is optimal for the monopolist (note that this class contains the family of IFR distributions, which are characterized by low inequality in terms of the coefficient of variation, because $IFR \Rightarrow CV \leq 1$) if inequality increases in the sense of a mean-preserving spread of the income distribution? We know that total welfare and monopolist profit are both

$$\int \frac{y^2}{2} f(y) dy = \frac{E(y^2)}{2}$$

under perfect sorting, that is, both the monopolist and the citizens get half of the total surplus from perfect sorting (see Appendix A.2.1). Hence, whenever a change to the distribution happens such that perfect sorting is still optimal for the monopolist afterwards, welfare and profits are affected in the same way, that is, a utilitarian social planner's and a monopolist's interests are aligned. For instance, look at the effects of a mean-preserving spread: The variance increases but average income doesn't change. Because of

$$\text{Var}(y) = E(y^2) - \bar{y}^2$$

this implies that $E(y^2)$ must increase due to a mean-preserving spread, which means that a mean-preserving spread increases both total welfare and the monopolist's profits in this case.

Proposition 8. *If the income distribution is such that perfect sorting is optimal for the monopolist, total welfare and monopolist profits benefit equally from an increase in inequality in the form of a mean-preserving spread.*

Proof. See above. □

The conflict between monopolist profits and welfare is thus not further intensified as inequality increases within the class of distributions for which perfect sorting is optimal for the monopolist: From a welfare perspective, no sorting would always be preferred to perfect sorting, but as inequality increases, both welfare and profits increase equally.

Importantly, the above result applies to small (infinitesimal) increases in inequality, such that perfect sorting still remains optimal for the monopolist. If the shape of the distribution changes too much, perfect sorting might no longer be the optimal sorting schedule for the monopolist. For instance, it can be shown in simulations that in case of the lognormal distribution, the function $h(\cdot)$ is everywhere increasing in y for small σ (below 1), and hence perfect sorting is optimal for the monopolist. However, as σ increases further, there is an increasing interval of Y for which h is decreasing, which implies that pooling some regions of Y is optimal for the monopolist. In that case, welfare and profits don't necessarily increase in tandem as inequality increases.

What if the income distribution is such that perfect sorting is not optimal for the monopolist, and she therefore implements a different sorting schedule? The following proposition shows that perfect sorting is always better for total welfare than any other sorting schedule that the monopolist would design.

Proposition 9. *If the income distribution is such that the monopolist doesn't want to implement perfect sorting, a utilitarian social planner would always prefer perfect sorting to the monopolist's sorting schedule. With her optimal sorting schedule, the monopolist can rake more than half of the total surplus from sorting.*

Proof. Total surplus (i.e., the sum of all utilities) is always maximized with perfect sorting, due to supermodularity of the utility function (see Becker, 1974): Pooling everybody yields a total surplus of \bar{y}^2 while perfect sorting yields $E(y^2)$, which is always larger because $E(y^2) = \text{Var}(y) + \bar{y}^2$. The same holds for pooling intervals of y . As total surplus is maximized with perfect sorting, anything else must yield either the same surplus or less. With perfect sorting, citizens and the monopolist share the surplus equally. If the monopolist decides that she would rather not offer perfect sorting, it means she must expect a higher surplus with another sorting schedule, which must mean that the citizens get less than half of total surplus (and that total surplus might even be lower than that of perfect sorting). Hence, perfect sorting is always better for total welfare than any other sorting schedule that the monopolist would design. \square

To conclude, the conflict between monopolist profits and total welfare is multifaceted in the case of multiple groups: If the distribution exhibits IFR (which implies that $CV \leq 1$ and thus inequality is low), total welfare is maximized with random matching, while the monopolist wants perfect sorting, but the conflict does not intensify with inequality: As inequality increases (in the form of a mean-preserving spread) but we stay within the class of distributions such that the monopolist wants perfect sorting (IFR is a sufficient condition for that), welfare and monopolists profits increase equally. Starting from a situation where perfect sorting is optimal for the monopolist and inequality increases such that the monopolist wants to implement a different sorting schedule (and pool some intervals of Y), monopolist profits will increase by more and welfare will increase by less than if sorting would still be perfect.

The question of what happens to profits and total welfare if we already start from a situation where perfect sorting is not optimal for the monopolist and then see an increase in inequality is left open for future research.

6 | DISCUSSION AND CONCLUSION

This paper discusses how changes in inequality affect socioeconomic segregation and resulting welfare in society. A simple two-group model is employed to show that a rise in inequality always increases profits of a monopolist who offers the sorting technology and thus also makes the supply of segregation profitable in the presence of fixed costs. Through this channel, an increase in inequality can lead to a (more) segregated society. Corresponding welfare in society, however, doesn't typically increase in line with profits. In particular when inequality is high, a conflict between welfare and profits arises and intensifies as inequality increases, and welfare decreases with inequality if the monopolist implements sorting to maximize her profits.

The paper also analyzes how sorting affects different parts of society. In particular, while sorting always lowers welfare in the poor group, welfare among the rich can be higher under sorting compared to random matching. Depending on the shape of the income distribution, the benefits of the rich can compensate for the losses of the poor, such that sorting yields higher

total welfare than no sorting. However, I show that this is not necessarily the case and that for some income distributions even the rich would be better off under random matching. The intuition is that they may have to pay very high sorting fees to separate themselves off from the poor, eating up most of their utility gains from interacting only with other rich people. In those cases, sorting is thus a particularly bad idea from a welfare perspective, as nobody in society benefits from it.

The last section of the paper discusses how these findings generalize if the monopolist is not restricted to offer only one group. If the income distribution is such that perfect sorting is optimal for the monopolist initially, the prediction is clear: there is a conflict between total welfare and profits, because no sorting would be welfare maximizing. The conflict does not intensify for small increases in inequality, as long as perfect sorting remains optimal, but the monopolist is able to capture more than half of the total surplus if pooling for some income intervals becomes optimal. The case where perfect sorting is not optimal for the monopolist to begin with remains to be explored in future research.

This paper provides a first tentative analysis of how the welfare benefits of segregation vary with inequality if a monopolist offers the opportunity to segregate. I find that the conflict between profits and social welfare from sorting intensifies as inequality increases. While caution should be applied in translating these theoretical results into policy recommendations, it generally seems to be the case that an investigation into the benefits and losses of segregation is more relevant the higher the prevailing rate of inequality. This is due to two reasons. First, the likelihood that segregation is provided increases with inequality, and second, the analysis shows that the optimal type of segregation from a profit-maximizing and from a welfare-maximizing perspective tend to diverge as inequality rises. On a related note, the analysis in this paper can also be used to address the question as to whether, from a social welfare perspective, the government (i.e., the social planner) wants to support the provision of segregation by a monopolist or try to actively counteract it (for instance via offering social housing or employing rent support or rent control policies to prevent gentrification).

ACKNOWLEDGMENTS

I thank Rabah Amir, Matt Levy, Ronny Razin, and the participants of the LSE Microeconomic Theory Work in Progress Seminar for their helpful suggestions and comments. Open Access funding enabled and organized by Projekt DEAL.

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How to cite this article: Windsteiger, L. (2021). Monopolistic supply of sorting, inequality, and welfare. *J Public Econ Theory*. 23, 801–821.
<https://doi.org/10.1111/jpet.12518>

APPENDIX A

A.1 | Sorting equilibria

Lemma 1. *In any sorting equilibrium, group S_b will have higher average income than group S_0 .*

Proof. This immediately follows from 2 and from the fact that $b > 0$. \square

Lemma 2. *All sorting equilibria will be monotone.*^{13,14}

Proof. Suppose w.l.o.g. that a sorting equilibrium exists where $y_2 \in S_0$ and $y_1 \in S_b$, and $y_2 > y_1$. Then we must have

$$y_2 E[y|y \in S_b] - y_2 E[y|y \in S_0] \leq b$$

and

$$y_1 E[y|y \in S_b] - y_1 E[y|y \in S_0] \geq b$$

and hence

$$y_1 E[y|y \in S_b] - y_1 E[y|y \in S_0] \geq y_2 E[y|y \in S_b] - y_2 E[y|y \in S_0].$$

But given that $E[y|y \in S_b] > E[y|y \in S_0]$ (see Proposition 1), this is a contradiction to $y_2 > y_1$. \square

Proposition 1. *Any sorting equilibrium is characterized by a cutoff $\hat{y} \in Y$ and a sorting fee $b > 0$ such that*

$$\hat{y} \bar{E}(\hat{y}) - \hat{y} \underline{E}(\hat{y}) = b. \quad (\text{A1})$$

Proof. (Note that, as in the main part, for simplicity of notation average income in the rich group, $E[y|y \in S_b]$, is denoted by $\bar{E}(\hat{y})$ and average income in the poor group, $E[y|y \in S_0]$, by $\underline{E}(\hat{y})$.)

Given the equilibrium conditions, it follows that both

$$yE[y|y \in S_b] - yE[y|y \in S_0] \leq b \quad \forall y \in [0, \hat{y}]$$

and

$$yE[y|y \in S_b] - yE[y|y \in S_0] \geq b \quad \forall y \in [\hat{y}, \nu]$$

need to hold in any sorting equilibrium. This implies that a person with income \hat{y} just at the border of the two groups has to be exactly indifferent between joining either of the two groups in equilibrium. Hence, we get

$$\hat{y} E[y|y \in S_b] - \hat{y} E[y|y \in S_0] = b. \quad \square$$

A.2 | Calculations for Section 5

A.2.1 | Proof that for the atoms distribution no sorting is more efficient than perfect sorting, that is, that it has $CV \leq 1$

With perfect sorting, everybody is matched only with people who earn the same income than them. The sorting fee at every income level y has to be chosen to ensure incentive

¹³By *monotone* I mean that the groups S_0 and S_b are single intervals of Y .

¹⁴If $b = 0$, then there exist trivial non-monotone sorting equilibria where the average income in both groups is the same, so that people are indifferent about which of these groups to join. Those cases are excluded by requiring that $b > 0$.

compatibility, that is, to make sure nobody would prefer to be matched with people of a different income (and pay their sorting fee). Hence, we need

$$\begin{aligned} y^2 - b(y) &\geq yy' - b(y') \\ y'^2 - b(y') &\geq y'y - b(y) \quad \forall y, y' \in Y. \end{aligned}$$

Combining the two and dividing by $y - y'$ (assuming w.l.o.g. that $y > y'$) yields

$$\frac{y'y - y'^2}{y - y'} \leq \frac{b(y) - b(y')}{y - y'} \leq \frac{y^2 - yy'}{y - y'}$$

Letting $y' \rightarrow y$ yields $b'(y) = y$. Using the fact that $b(0)$ must be 0 we get that $b(y) = \int_0^y zd(z) = \frac{y^2}{2}$. Hence, welfare from perfect sorting is given by

$$\int_0^y \left(y^2 - \frac{y^2}{2} \right) f(y) dy = \int_0^y \frac{y^2}{2} f(y) dy = \frac{E(y^2)}{2}$$

Note that, as the monopolist's optimal sorting fee under perfect sorting amounts to $b(y) = \frac{y^2}{2}$, her revenue from offering perfect sorting amounts to $\int_0^y \frac{y^2}{2} f(y) dy = \frac{E(y^2)}{2}$ and the total surplus from perfect sorting $\int_0^y y^2 f(y) dy = E(y^2)$ is split equally between the monopolist and welfare.

For the atoms distribution on $[0, 1]$ we can calculate that

$$E(y^2) = \int_0^1 y^2(1 - 2z) dy + z = \frac{1}{3} + \frac{z}{3}$$

Therefore we see that, as described in Section 5, $E(y^2)$ (the total surplus of perfect sorting) and total welfare from perfect sorting (which is just half of it) are increasing in inequality z . However, welfare from perfect sorting is smaller than welfare from random matching for all z :

$$\frac{E(y^2)}{2} = \frac{1}{6} + \frac{z}{6} \leq \frac{1}{4} \Leftrightarrow z \leq 0.5$$

Another way to see this is to calculate the coefficient of variation:

$$CV = \frac{\sqrt{\text{Var}(y)}}{\bar{y}} = \frac{\sqrt{\frac{1}{3} + \frac{z}{3} - \frac{1}{4}}}{\frac{1}{2}} = 2\sqrt{\frac{1}{12} + \frac{z}{3}}$$

It is straightforward to see that $CV \leq 1 \forall z \in [0, 0.5]$ and that it reaches its maximum of 1 where $z = 0.5$. If $z = 0.5$, perfect sorting would yield the same welfare than no sorting if the sorting fee is set at $\frac{1}{2}$, such that the total surplus is split in half. However, the sorting fee is not uniquely determined in this case and a profit-maximizing monopolist would set it as high as possible, which would be 1 in this case, such that total welfare is 0 and the monopolist gets all the surplus from sorting (which is 0.5) for herself.

A.2.2 | Proof that the distributions analyzed in Section 4 are NBUE

In order to prove that the distributions analyzed in Section 4 are NBUE, we need to show that

$$\bar{E} - \bar{y} - \hat{y} < 0 \quad \forall \hat{y}, \quad \forall z \in [-2, 2].$$

The general specification of the pdf of these distributions that encompasses all of them is

$$f(y) = x - \frac{2z}{\nu}y \quad \text{if } y \in \left[0, \frac{\nu}{2}\right]$$

$$f(y) = x - 2z + \frac{2z}{\nu}y \quad \text{if } y \in \left[\frac{\nu}{2}, \nu\right]$$

where

$$x = \frac{1}{\nu} + \frac{z}{2} \tag{A2}$$

in order for $F(\nu) = 1$ and

$$z \in \left[-\frac{2}{\nu}, \frac{2}{\nu}\right]$$

which implies $x \in \left[0, \frac{2}{\nu}\right]$.

If $\hat{y} \leq 0.5$ we have that

$$\begin{aligned} \bar{E} - \bar{y} - \hat{y} &= \frac{6 - 6\hat{y}^2 - 3z\hat{y}^2 + 8z\hat{y}^3}{12 - 12\hat{y} - 6z\hat{y} + 12z\hat{y}^2} - \frac{1}{2} - \hat{y} \\ &= \frac{-6\hat{y} + 6\hat{y}^2 + z(-4\hat{y}^3 + 3\hat{y} - 3\hat{y}^2)}{12 - 12\hat{y} - 6z\hat{y} + 12z\hat{y}^2}. \end{aligned}$$

The denominator is always positive, so we just need to analyze the numerator: $-6\hat{y} + 6\hat{y}^2$ is always negative, and $-4\hat{y}^3 + 3\hat{y} - 3\hat{y}^2$ is positive for $\hat{y} \leq 0.5$, hence if z is negative, the whole expression is negative for sure. If z is positive, then the numerator reaches its maximum at $z = 2$, where it becomes $-6\hat{y} + 6\hat{y}^2 - 8\hat{y}^3 + 6\hat{y} - 6\hat{y}^2 = -8\hat{y}^3$ which is always negative. Hence, $\bar{E} - \bar{y} - \hat{y} < 0$ if $\hat{y} \leq 0.5$.

If $\hat{y} \geq 0.5$ we have that

$$\begin{aligned} \bar{E} - \bar{y} - \hat{y} &= \frac{6 - z - 6\hat{y}^2 + 9z\hat{y}^2 - 8z\hat{y}^3}{12 - 6z - 12\hat{y} + 18z\hat{y} - 12z\hat{y}^2} - \frac{1}{2} - \hat{y} \\ &= \frac{-6\hat{y} + 6\hat{y}^2 + z(2 + 4\hat{y}^3 - 3\hat{y} - 3\hat{y}^2)}{12 - 6z - 12\hat{y} + 18z\hat{y} - 12z\hat{y}^2}. \end{aligned}$$

The denominator is again positive, and the first term of the numerator, $-6\hat{y} + 6\hat{y}^2$ is always negative. $2 + 4\hat{y}^3 - 3\hat{y} - 3\hat{y}^2$ reaches its minimum at $\frac{1}{4} + \sqrt{\frac{5}{16}}$ where it is negative, and hence $z(2 + 4\hat{y}^3 - 3\hat{y} - 3\hat{y}^2)$ is positive if $z < 0$, and maximal at $z = -2$. Combined with $-6\hat{y} + 6\hat{y}^2$ evaluated at $\frac{1}{4} + \sqrt{\frac{5}{16}}$ the total expression is negative. $2 + 4\hat{y}^3 - 3\hat{y} - 3\hat{y}^2$ reaches its maximum at 0.5 where it is positive and hence $z(2 + 4\hat{y}^3 - 3\hat{y} - 3\hat{y}^2)$ is maximal at $z = 2$. Again combined with $-6\hat{y} + 6\hat{y}^2$ evaluated at 0.5 the whole expression is negative. Hence $\bar{E} - \bar{y} - \hat{y} < 0$ if $\hat{y} > 0.5$, and thus the examined distribution is NBUE for all z .