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Alternative Approaches to Labor Values and Prices of Production: Theory and Evidence

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Alternative Approaches to Labor Values and Prices of Production: Theory and Evidence*

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Summary

In this paper, we discuss three approaches to estimating classical prices of production (long run equilibrium prices) in both a circulating capital model and a model that includes capital stock: the Standard Interpretation of Marx's value theory, the New Interpretation of Marx's value theory, and the Sraffian approach to prices of production. We add two refinements to both models: (a) allowing for differential wages rates across industries; and (b) taking account of unproductive industries in labor value calculations. We implement (a) the circulating capital models using harmonized input-output data from the World Input Output Database for 37 countries for the period 2000–2014, and (b) the model with capital stock for the U.S. economy using input-output and other relevant data for 2020. For all models, we estimate labor values, prices of production and the uniform rate of profit. We test for deviation between relative labor values and relative prices of production using both regression and non-regression-based methods. For both the circulating capital model and the model with capital stock, we find that the vector of relative labor values and the vector of relative prices of production are far apart in terms of both regression and non-regression-based measures.

Keywords: labor value, price of production, uniform rate of profit, classical theory of prices.

1 Introduction

In capitalist economies, capital moves across industries in search of higher rates of profit. This mobility of capital in search of higher profit rates imparts a long run tendency of the profit rate to equalize across industries. Long run equilibrium prices that are consistent with a uniform rate of profit across industries are known in the classical tradition as 'prices of production'.

The classical tradition of Smith, Ricardo, and Marx also understands prices as being regulated by 'labor values' of commodities, the amount of direct and indirect human labor necessary to produce commodities at a given point in time with the existing average technology and intensity of work. Hence, the question arises: what is the relationship between labor values and prices of production?

To our mind, there are two different ways to conceptualize the 'regulation' of prices of production by labor values. The first conceptualization comes from the writings of David Ricardo, according to which relative prices of production are, on average, 'close' to relative labor values of commodities. Why prices of production would be 'close' to labor values was not very clearly explained by Ricardo. It was only in the early 1980s that Anwar Shaikh provided an answer, which we discuss below.

The second, and different, conceptualization of the regulation of prices by labor values emerges in the writings of Karl Marx. For Marx, the process through which prices of production emerged as long run equilibrium prices was primarily a process of distribution and redistribution of value and surplus value across industries. In Marx's understanding, the regulation of prices of production by labor values was an aggregate invariance principle, i.e., at the aggregate level the value added by labor to the newly produced bundle of commodities was expressed in monetary terms using prices. This aggregate relationship did not imply that prices of production and labor values would be close. If anything, the distribution and redistribution of surplus value (and value) across industries, and the existence of unproductive industries, seemed to imply that labor values and prices of production of

individual commodities would not be close–other than in exceptional cases.[1]

Shaikh (1984) offered a different reading of the classical tradition, arguing that even if actually observed organic composition of capital are different across industries, which it is, this need not imply large deviations between prices of production and labor values of commodities because the deviations depend on *integrated*, and not actual, capital-labor ratios. Integrated capital-labor ratios aggregate actual capital-labor ratios of all previous stages of production of any commodity. If different industries are connected to each other through complex supply linkages, captured by the input-output matrix of any economy, then ratios of *integrated* capital-labor ratios are likely to be similar for all industries. This would imply, argued Shaikh (1984), that deviation of *relative* labor values from *relative* prices of production would be relatively small even if actual capital-labor ratios were very different across industries.[2]

Starting from the pioneering contribution of Shaikh (1984), a large literature has reported results from empirical studies, across a wide range of countries and for different years, which seem to show that labor values and prices of production are indeed close to each other (Ochoa, 1984; Shaikh, 1984; Petrovic, 1987; Ochoa, 1989; Cockshott and Cottrell, 1997; Tsoulfidis and Maniatis, 2002; Tsoulfidis and Mariolis, 2007; Tsoulfidis, 2008; Fröhlich, 2012; Shaikh, 2016; Işikara and Mokre, 2022). The 'closeness' of prices of production and labor values has, thus, become an important part of a certain understanding of the labor theory of value.[3]

The current paper challenges this long and distinguished literature by revisiting the calculation of prices of production and showing that prices of production and labor values are not 'close' to each other. We estimate prices of production for both circulating capital models and for models with capital stock. For the circulating capital model, we use input-

[1] "Only for capitals ... in branches of production whose composition chanced to coincide with the social average, would the value and the price of production be the same." (Marx, 1993, p. 264).

[2] We explain this argument in detail in section 4.

[3] There are a few previous papers which have raised questions about the finding that deviations between values and prices are small (Steedman and Tomkins, 1998; Kliman, 2002). These papers have relied on using either one approach or one measure to draw their conclusions. Our paper is more comprehensive. We use a host of methods for computing prices of production and several measures to quantify the deviation between labor values and prices of production.

output data for 37 countries from the World Input Output Database (WIOD). For the model with capital stock, we use input-output and other relevant data for the US economy in 2020. For both models, our findings show that relative prices of production and relative labor values are much farther apart than has generally been reported in previous studies.

Why do we find large deviations between prices of production and labor values? There can be various reasons for this–though we are unable to completely pin it down. First, all scholars who have studied this issue understand and highlight the difference between complex and simple labor so far as labor values are concerned. But, when computing value and price of production vectors, the same scholars fail to correctly implement the difference. While this error is fatal for the Sraffian approach, as we explain in section 3.1.2, even scholars using the SI approach have not implemented the difference consistently.

Second, most scholars who have studied this issue have used high magnitudes of R-squared in a regression of log prices on log values as evidence that prices and values are close together. This is an incorrect argument, as we explain below–drawing on Basu (2017). While this criticism relates to regression-based measures, it does not concern the non-regression-based measures used by a large number of scholars. Since we find large deviation using the non-regression-based measures too, there must be other reasons–related to analytical methods and construction of variables–that are more important.

The most important reason why we cannot pin down the reasons behind the difference in our results from the vast majority of existing studies is because scholars have not transparently explained their methodologies–related both to the computation of the price of production vector and construction of their data sets. For many papers we read in this literature, we have been unable to *fully* understand how the system of equations relating to the vector of absolute price of production has been closed and how all the variables have been constructed. In the interest of taking this literature forward and to encourage replicability of research in classical economics, we are making all our data and code, with detailed instructions, publicly available.[4] We hope other researchers will use our data and also make

[4]The full replication package can be downloaded from here.

their data publicly available–so that we can clearly identify the sources of differences of our results with the vast majority of existing studies.

While challenging the existing literature on labor values and prices of production, we also make other contributions. First, we present a systematic treatment of the major alternative approaches to estimating prices of production and the uniform rate of profit within the classical tradition - the Standard Interpretation (SI), the Sraffian Approach (SR) and the New Interpretation (NI). While these three major approaches attempt to estimate the same entities – the prices of production and the uniform rate of profit – they rely on very different analytical methods. We highlight these key differences in our exposition.

Second, we implement a novel algorithm to estimate prices of production and the uniform rate of profit using the New Interpretation of Marx's labor theory of value. While this algorithm was presented in Basu (2021), to the best of our knowledge, this is the first implementation of this algorithm with actual input-output data. In general, we get the smallest deviations between relative labor values and relative prices of production using the NI approach. But even in these cases, our results show much higher deviation between relative labor values and relative prices of production than in most of the extant literature.

Third, we estimate prices of production for both circulating capital models and for models with capital stock. Most papers in the literature treat either one or the other. Our analysis is, therefore, more comprehensive than most existing studies.

Finally, we extend the classical model of relative prices in two directions. We allow wage differentials across industries and we take account of unproductive industries when computing labor values. These extensions are conceptually important but our main findings about large deviations between relative labor values and relative prices of production hold for all specifications.

The rest of the paper is organized as follows: in section 2, we introduce the concepts of labor values and prices of production; in section 3, we discuss the alternative approaches to estimating prices of production; in section 4, we discuss both the theory and the empirical

measures to quantify the deviation between vectors of relative prices of production and relative labor values; in section 5, we discuss our data sources and construction of variables; in section 6, we discuss our results; in section 7, we conclude the discussion and highlight some implications of our findings. In Appendix A, we provide details of how we construct the data for the capital stock model; in Appendix B, we illustrate our analytical approaches with a 3-industry example.

2 Labor Values and Prices of Production

2.1 Using Physical Input-Output Data

2.1.1 Circulating Capital Model

Consider a circulating capital model of a capitalist economy, where technology of production is captured by the $n \times n$ input-output matrix, A, and the $1 \times n$ direct labor input vector, l. The i,j-th element of the input-output matrix, a_{ij}, and the i-th element of the direct labor vector, l_i, denote, respectively, the *physical* amount of commodity i and *physical* amount of direct labor input (e.g., measured in hours) needed to produce one *physical* unit of commodity j.

Mobility of capital across the n industries will give rise to a long run tendency for the rate of profit to equalize across industries. The set of long run equilibrium prices that ensure the same, uniform rate of profit across industries are known as prices of production. The $1 \times n$ vector of prices of production, q, is given by the following system of equations,

$$q = qA + wl + r\left(qA + wl\right) = \left(1 + r\right)\left(qA + wl\right), \tag{1}$$

where w and r denote, respectively, the uniform nominal wage rate and the uniform profit rate.

The direct labor input vector, l, is the vector of what the Marxian tradition calls *complex labor*. This labor input registers the differentials in average levels of skills and

training associated with the labor inputs across industries. Thus, the direct labor input vector, measured in hours, will represent hours of labor of different degrees of complexity. We can contrast complex labor with *simple labor*, which is the labor performed by workers with basic skills available at any point in time to all members of a society. Complex labor, then, requires additional skill acquisition and training, in comparison to simple labor. If we follow Adam Smith in assuming that the differentials of complexity of labor inputs is captured by differentials in the wage rates, then we can convert the vector of complex labor, l, into the vector of simple labor, l_s, by multiplying each element of the former with the ratio of that industry's average wage rate and the minimum of the average nominal wage rate across industries, i.e.

$$l_{s,i} = \left(\frac{w_i}{w_{min}}\right) l_i, \tag{2}$$

where $i = 1, 2, \ldots, n$, denotes the i-th industry and the minimum is taken over all i.

The distinction between complex and simple labor is important for conceptualizing labor values because each hour of complex labor will add a multiple of the value that is added by an hour of simple labor – which is captured quantitatively by (2). Hence, for the circulating capital model of the capitalist economy with technology given by A and l, the $1 \times n$ vector of labor values of the n commodities, γ, is defined by

$$\gamma = \gamma A + l_s, \tag{3}$$

where l_s is the $1 \times n$ vector of direct labor inputs accounting for differential complexity of labor across industries, i.e. l_s is the direct labor input l converted into units of *simple labor*.

Two important points are worth highlighting at this point. First, the labor input that enters the value determination equation, l_s (units of simple labor), is different from the labor input vector involved in the price of production system, l (units of complex labor). The capitalist firm pays for units of complex labor power at the market wage rates, which is why l is used in (1). When this complex labor power is used in the production process, it

adds different amounts of value for every hour of work. This differential value addition for every hour of work is captured by expressing the direct labor input in hours of simple labor or using a "reduction coefficient". Many scholars have explicitly noted the importance of the reduction coefficient (Ochoa, 1984, 1989; Tsoulfidis and Rieu, 2006; Tsoulfidis and Mariolis, 2007; Tsoulfidis, 2008; Işikara and Mokre, 2022). It is precisely because of this reason that l_s (units of simple labor) enters in (3). It is important to note that many of the same scholars who have emphasized the conceptual importance of the difference between simple and complex labor have also, paradoxically, ignored this difference by using simple labor in *both* value and price calculations (Ochoa, 1984, 1989; Tsoulfidis and Rieu, 2006; Tsoulfidis and Mariolis, 2007; Tsoulfidis, 2008; Işikara and Mokre, 2022). This is a conceptual mistake. We will consistently use l in price of production calculations and l_s in value calculations.

The second point relates to the use of a uniform nominal wage rate in the price of production system, (1). This is an abstraction from the fact of differential wage rates across industries. Since the latter is used to convert complex to simple labor, it seems incorrect to then ignore it when writing the price of production system. One of the contributions of this paper will be to do away with the assumption of a uniform nominal wage rate in the price of production system and allow for industry-level differentials of wage rates. But we will also report results with a uniform nominal wage rate to facilitate comparison with the existing literature.

2.1.2 Model with Capital Stock

Consider a model with capital stock, as outlined in Ochoa (1984, pp. 54-55). The presence of stocks of capital has implications for the definitions of labor values and prices of production.

In the circulating capital model, the labor value of a commodity was the sum of value transferred by the material inputs used up and the value added by labor. Once we include capital stock in the model, the depreciation of the fixed capital is an additional part of the value transferred by material inputs. Hence, in the model with capital stock, the vector of

labor values is given by

$$\gamma = \gamma\,(A + D) + l_s, \tag{4}$$

where D is the matrix of depreciation of fixed capital.

In the model with capital stocks, capitalist firms compute the rate of profit on the stock of capital advanced. Hence, the price of production for any commodity is the sum of two components, the flow cost of production and the flow of profit income on the stock of capital advanced (Ochoa, 1984, p. 54). Thus, the vector of prices of production is given by

$$q = qA + wl + qD + r\left[qK + (qA + wl) < t >\right], \tag{5}$$

where r is the uniform rate of profit, K is the matrix of capital stock coefficients and $< t >$ is the diagonal matrix of turnover times. In (5), rqK denotes the profit income earned on the stock of fixed capital and $r\,(qA + wl) < t >$ captures the profit income earned on the stock of circulating capital advanced (which includes material inputs and the wage fund).

2.2 Using Nominal Input-Output Data

2.2.1 Nominal-Real Conversions

Input-output tables constructed by statistical agencies across the world do not provide data on physical units of commodities or labor needed to produce each physical unit of any commodity. Instead, they provide data on monetary value (e.g. measured in US dollars) of different commodities or labor needed to produce each monetary unit (e.g. measured in US dollars) of any commodity.

Instead of the physical input-output matrix, A, what statistical agencies report and researchers use is the nominal input-output matrix, $\hat{A}$, where

$$\hat{a}_{ij} = \frac{m_i x_i}{m_j x_j} = \frac{x_i}{x_j}\frac{m_i}{m_j} = a_{ij}\frac{m_i}{m_j},$$

where, for $i = 1, 2, \ldots, n$, m_i and x_i denote, respectively, the market price and real gross

output of commodity i. Written in terms of matrices, we therefore have,

$$\hat{A} =< m > A < m^{-1} >, \tag{6}$$

where $< m >$ is the $n \times n$ diagonal matrix of market prices. We have similar relationships between the nominal and physical units-denominated capital stock coefficient matrix

$$\hat{K} =< m > K < m^{-1} >, \tag{7}$$

and the depreciation matrix,

$$\hat{D} =< m > D < m^{-1} > . \tag{8}$$

Turning to the labor input vector, note that instead of the physical direct labor input vector l per physical unit of any commodity, what the statistical agencies report is $\hat{l}$, which is the direct labor input (measured in hours) per US dollar gross output of the commodity, i.e.,

$$\hat{l}_i = \frac{L_i}{m_i x_i} = \frac{L_i}{x_i}\frac{1}{m_i} = l_i \frac{1}{m_i},$$

so that in terms of matrices, we have,

$$\hat{l} = l < m^{-1} >= L < X^{-1} >, \tag{9}$$

where L is the total direct labor input and $< X^{-1} >=< X >^{-1}$ is the inverse of the $n \times n$ diagonal matrix of nominal gross outputs, i.e.

$$X_i = m_i x_i, \text{ for } i = 1, 2, \ldots, n, \tag{10}$$

where X_i is nominal gross output, x_i is real gross output and m_i is the market price, all referring to the output of industry i. A similar expression holds for the labor input vector

measured in units of simple labor,

$$\hat{l}_s = l_s < m^{-1} >= L_s < X^{-1} > . \tag{11}$$

2.2.2 Circulating Capital Model

Using (6) and (9), we can now re-write the equations for the labor value and price of production vectors in such a way that they can be estimated with input-output data provided by statistical agencies across the world. Post-multiplying (3) by the matrix $< m^{-1} >$ we get

$$\lambda = \lambda \hat{A} + \hat{l}_s, \tag{12}$$

where $\lambda = \gamma < m^{-1} >$ is the value vector per US dollar (or any other national currency) of gross output. Since $\hat{A}$ and $\hat{l}_s$ are observed, we can estimate λ.

Turning to the equation for the vector of prices of production, let us, once again, post-multiply through by the matrix $< m^{-1} >$ to get

$$q < m^{-1} >= (1 + r) \left(q < m^{-1} >< m > A < m^{-1} > + wl < m^{-1} > \right).$$

This gives us

$$p = (1 + r) \left(p\hat{A} + w\hat{l} \right), \tag{13}$$

where $p = q < m^{-1} >$ is the price of production vector per US dollar (or any other national currency) of gross output. Since $\hat{A}$ and $\hat{l}$ are observed, we can estimate p.

2.2.3 Model with Capital Stock

Using (6), (8) and (9), we can rewrite the equation for the labor value vector. Post-multiplying (4) by the matrix $< m^{-1} >$ we get

$$\lambda = \lambda \left(\hat{A} + \hat{D} \right) + \hat{l}_s, \tag{14}$$

where $\lambda = \gamma < m^{-1} >$ is the value vector per US dollar (or any other national currency) of gross output. Since $\hat{A}$, $\hat{D}$ and $\hat{l}$ are observed, we can estimate λ in the model with capital stock.

To derive the equation for the price of production vector, we post-multiply (5) by $< m^{-1} >$ and rearrange to get

$$p = pA + w\hat{l} + p\hat{D} + r\left[p\hat{K} + \left(p\hat{A} + w\hat{l}\right) < t >\right], \tag{15}$$

where $p = q < m^{-1} >$ is the price of production vector per US dollar (or any other national currency) of gross output. Since $\hat{A}, \hat{D}, \hat{K}$ and $\hat{l}$ are observed, we can estimate p in the model with capital stock.

3 Alternative Approaches to Estimating Prices of Production

3.1 Circulating Capital Model

There are at least three different methods for computing the vector of prices of production, and the uniform rate of profit, in the circulating capital model given by (13): the Standard Interpretation (SI) of Marx's labor theory of value (Ochoa, 1984; Shaikh, 1984, 2016); the Sraffian approach to prices of production (Steedman and Tomkins, 1998; Işikara and Mokre, 2022); and the New Interpretation (NI) of Marx's labor theory of value (Foley, 1982; Basu, 2021).

3.1.1 Standard Interpretation

Scholars following the SI take the $n \times 1$ vector of *nominal* consumption expenditure on the n commodities by the average worker per hour of labor input, $\hat{b}$, *as given.* Using the classical savings assumption, they posit that workers spend their wage income completely

on consumption, so that

$$w = p\hat{b}, \text{ and } \hat{b} =< m > \frac{B}{L}, \tag{16}$$

where B is the $n \times 1$ vector of total (personal) consumption and $L = \sum_i L_i$ is the (scalar) sum of total labor inputs (measured in hours) across all industries.[5] Substituting (16) in (13), we get

$$p = (1 + r)\, p\hat{M}, , \tag{17}$$

where

$$\hat{M} = \hat{A} + \hat{b}\hat{l} \tag{18}$$

is the augmented input matrix per US dollar of gross output.

Assumption 1. *$\hat{M}$ is nonnegative and irreducible.*

If assumption 1 holds, then we can apply the Perron-Frobenius theorem (Pasinetti, 1977, Mathematical appendix) to get

$$r = \frac{1}{\rho(\hat{M})} - 1, \tag{19}$$

where $\rho(\hat{M})$ refers to the maximal eigenvalue of the matrix $\hat{M}$. The eigenvector of $\hat{M}$ corresponding to $\rho(\hat{M})$ gives the *relative* price of production vector, p_{rel}.

The vector of *absolute* prices of production is then arrived at, following (Ochoa, 1989, equation 10, p. 417), by multiplying the relative price vector with the following constant:

$$\kappa = \frac{X}{p_{rel}X}. \tag{20}$$

Thus, κ is the ratio of the nominal gross output vector, X, aggregated with the vector market prices (the numerator) and the same vector, X, aggregated with the relative price

[5] SI scholars actually follow a slightly different process to construct the b vector. They divide the elements of the total consumption vector with the sum of the elements of that vector and then multiply the resulting vector by the minimum nominal wage rate across industries. "To obtain the real unit wage vetor b, we use the sectoral proportions in the Personal Consumption Expenditure component of final demand ... By multiplying it with the wage rate in current dollars of the lowest-wage sector, we obtain the dollar amount of each consumer good required per unit of reduced labor." (Ochoa, 1989, p. 428).

vector (the denominator). In the numerator, the vector of market prices is just the unit vector–because X is already expressed in market prices (Ochoa, 1989, section 3.2, p. 416). Thus,

$$p^{SI} = \kappa p_{rel}, \tag{21}$$

where p^{SI} is the $1 \times n$ vector of absolute prices of production estimated by following the SI. This procedure is an implementation of the 'normalization' that the gross output aggregated with prices of production is equal to the gross output aggregated with the market prices because

$$p^{SI}X = \kappa p_{rel}X = \frac{X}{p_{rel}X} p_{rel}X = X = mX,$$

where, by construction, m is the unit vector representing market prices.[6]

It is important to note that the 'normalization' used by the SI approach is conceptually different from 'invariance principles'. An invariance principle posits a relationship between labor values and prices of production at the aggregate level to capture some key intuition of the labor theory of value; a normalization condition, on the other hand, is used to close the system of equations. It does not attempt to capture any intuition of the *labor* theory of value so far as derivation of prices of production are concerned.[7] In this sense, it akin to choosing a numeraire to close the system of equations governing the prices of production.[8]

3.1.2 Sraffian Approach

In the Sraffian approach to estimating prices of production, the price of production equation is written slightly differently from (13) by assuming that *profit income is earned only on material costs* and not on wage costs. Hence, the Sraffian price of production system is

[6]There is a certain awkwardness built into this normalization, which is also reflected in the definition of the constant κ in (21), because what is being aggregated is the nominal gross output and not the real gross output. This cannot be helped because input-output tables do not provide information on the real gross output vector; it only provides data on the nominal gross output vector.

[7]SI scholars use another normalization when they derive "direct prices", i.e. prices which are proportional to labor values: gross output aggregated with the vector of direct prices is equal to the gross output aggregated with the vector of market prices. This seems to provide a form of invariance principle that captures one of Marx's intuitions.

[8]For a detailed discussion on the importance of invariance principles in the labor theory of value, see Basu (2021, section 7.2.5 and 7.2.6).

given by

$$p = p\hat{A} + w\hat{l} + rp\hat{A}, \tag{22}$$

so that $p(I - A) = w\hat{l} + rp\hat{A}$, and thus, post-multiplying by $(I - A)^{-1}$, we get

$$p = w\lambda + rp\hat{H}, \tag{23}$$

where

$$\hat{H} = \hat{A}\left(I - \hat{A}\right)^{-1} \tag{24}$$

is the sum of direct and indirect capital input.

Assumption 2. *$\hat{H}$ is nonnegative and irreducible.*

Let R denote the maximal rate of profit, which occurs when $w = 0$ in (23), we have $p = Rp\hat{H}$. If assumption 2 holds, then we can apply the Perron-Frobenius theorem (Pasinetti, 1977, Mathematical appendix), we get

$$R = \frac{1}{\rho(\hat{H})}. \tag{25}$$

Note that the aggregate maximal profit income is equal to the aggregate value added because the former arises when the wage rate, and hence total wage income, is zero. Hence, the aggregate profit share is equal to the ratio r/R because

$$\frac{r}{R} = \frac{\text{total profit}/K}{\text{maximal profit}/K} = \frac{\text{total profit}/K}{\text{value added}/K} = \frac{\text{total profit}}{\text{value added}} = \pi, \tag{26}$$

where K denotes aggregate capital stock. In the Sraffian approach, π (share of profits in aggregate value added) is *taken as given from aggregate data* and using (26), (23) is rewritten to give us the absolute price of production vector,

$$p^{SR} = w\lambda\left(I - \pi R\hat{H}\right)^{-1}. \tag{27}$$

The Sraffian approach presented above suffers from a key conceptual problem. In moving from (22) to (23), we have used $\lambda = l(I-A)^{-1}$. This is incorrect. The correct expression for the vector of values is $\lambda = l_s(I-A)^{-1}$, i.e. value calculation should use the labor input measured in units of simple labor hours, l_s, and not labor input measured in units of complex labor hours, l. The Sraffian approach ignores the important conceptual distinction between simple and complex labor, and is thus flawed. While we report results from the Sraffian approach for completeness, we note that those results are incorrect and flawed. Thus, the results reported in Table 4 should be ignored.

3.1.3 New Interpretation

In the New Interpretation (NI) of Marx's labor theory of value (Foley, 1982), the value of labor power, v, and the uniform nominal wage rate, w, are *taken as given*, instead of the consumption bundle $\hat{b}$. The value of labor power is the product of the nominal wage rate and the value of money:

$$v = w \times \mu^{NI}. \tag{28}$$

In contrast to the SI, the NI defines the value of money with the net output vector measured in monetary units, Y:

$$\mu^{NI} = \frac{\lambda Y}{p^{NI}Y} = \frac{\hat{l}_s\left(I-\hat{A}\right)^{-1}Y}{p^{NI}Y} = \frac{\hat{l}_s X}{p^{NI}Y}, \tag{29}$$

where Y and X are the $n \times 1$ vectors of net and gross outputs in monetary units, p^{NI} and μ^{NI} denote, respectively, the absolute price of production vector and the value of money, according to the NI. This definition embeds an invariance principle with respect to the net output: the vector of net output aggregated with the prices of production and multiplied with the value of money (to covert it into units of labor hours) gives the same magnitude as when the net output vector is aggregated with labor values. The price of production

system can, in turn, be written as

$$p^{NI} = (1+r)\left(p^{NI}\hat{A} + w\hat{l}\right), \tag{30}$$

and so the NI can be captured by the following two equation systems: (30) and (28). We can now solve this equation system for the uniform rate of profit, r, and the price of production vector, p^{NI}, using the algorithm given in Basu (2021, Appendix 7.A.4).

Assumption 3. *$\hat{A}$ is nonnegative and irreducible, and the value of labor power, v, satisfies:*

$$0 < v < \frac{\hat{l}_s X}{\hat{l} X},$$

where X is the vector of nominal gross outputs.

The first step of the algorithm involves computing the maximal rate of profit, R. If the first part of assumption 3 holds, then we can use the Perron-Frobenius theorem on (30) to get $R = (1/\rho(\hat{A})) - 1$, where $\rho(\hat{A})$ is the maximal eigenvalue of the matrix A.

In the second step of the algorithm, we define the following univariate function of r,

$$f(r) = (1+r)\, w\hat{l}\left[I - (1+r)\,\hat{A}\right]^{-1}\left(I - \hat{A}\right)X - \frac{w\hat{l}_s X}{v}, \tag{31}$$

and find the unique zero of this function over the range, $0 \leq r < R$, i.e. the value of $r = r^*$ for which $f(r^*) = 0$; this gives us the uniform rate of profit, r^*. To understand why, note that (30) gives us

$$p^{NI} = (1+r)\, w\hat{l}\left[I - (1+r)\,\hat{A}\right]^{-1}$$

and since, using (28), we have $\mu^{NI} w = v$, using the expression for μ^{NI} and the expression for the price of production above, we get

$$(1+r)\, w\hat{l}\left[I - (1+r)\,\hat{A}\right]^{-1}\left(I - \hat{A}\right)X = \frac{w\hat{l}_s X}{v}.$$

This implies that the uniform rate of profit is the zero of the function defined in (31). To

see why this is unique, note that $f(0) = w(\hat{l}X - (\hat{l}_s X/v)) < 0$ (using the second part of assumption 3), $\lim_{r \to R} f(r) = +\infty$ and $f' > 0$. That is why there is a unique value of $r = r^* > 0$ at which $f(r^*) = 0$.

In the third, and final, step of the algorithm, we solve for the absolute price of production vector as

$$p^{NI} = (1 + r^*)\, w\hat{l} \left[I - (1 + r^*)\, \hat{A} \right]^{-1}$$

using the uniform rate of profit that we got in the previous step.[9]

3.2 Two Extensions of the Circulating Capital Model

We have so far dealt with the basic circulating capital model with a uniform wage rate across industries. Now we discuss two extensions of this basic model. In the first extension, we allow nominal wage rates to be different across sectors, and in the second extension, we take account of unproductive (in the sense of labor value production) industries. We can, of course, bring the two extensions together to consider a model with differential wage rates across industries that also takes account of unproductive industries in the value system.

3.2.1 Allowing for Differential Wage Rates

The first extension is motivated by the recognition that different conditions of production and bargaining power prevail in different industries. Differential technologies of production, intensities of work, unionization rates and bargaining power might give rise to differential nominal wage rates. It therefore makes sense to allow for differential nominal wage rates across industries in the price of production system.

$$p = (1 + r) \left(p\hat{A} + \hat{l} < w > \right),$$

where $< w >$ is the $n \times n$ diagonal matrix of nominal wage rates in the n industries.

[9] We use the same logic to estimate prices of production for all variants of the model with the NI approach. The only thing that changes in the definition of the wage vector and the vector of labor inputs used to produce value-added.

Implementing the differential wage rate model given above with the SI would mean finding industry-specific average real wage bundles. Such detailed information on consumption patters is not available in input-output data or labor statistics. Hence, we will not pursue the SI approach for the wage differential model. On the other hand, the wage differential model can be implemented with the NI approach because data on industry-specific nominal wage rates are readily available.

Following the NI approach, the price of production system with differential wages can be written as

$$p^{NI} = (1+r)\left(p^{NI}\hat{A} + \hat{l} < w >\right). \tag{32}$$

To complete the system, we revisit the definition of the value of labor power. Instead of using a uniform nominal wage rate, we define the value of labor power as the product of the *average* nominal wage rate, $\bar{w}$, and the value of money, μ^{NI}. Thus, the system with differential wage rates is given by (32) and

$$v = \bar{w} \times \mu^{NI}, \tag{33}$$

where μ^{NI} is defined in (29) and

$$\bar{w} = \frac{\hat{l} < w > X}{\hat{l}X} = \frac{\sum_{i=1}^{n} \hat{l}_i w_i X_i}{\sum_{i=1}^{n} \hat{l}_i X_i}. \tag{34}$$

If assumption 3 holds, then the system of equations given by (32) and (33) can be solved for the uniform rate of profit, r, and the price of production vector, p^{NI}, using the same three-step algorithm that we used above (Basu, 2021, Appendix 7.A.4).

If the first part of assumption 3 holds, then we can use the Perron-Frobenius theorem on (32) to get the maximal rate of profit, $R = (1/\rho(\hat{A})) - 1$, where $\rho(\hat{A})$ is the maximal eigenvalue of the matrix $\hat{A}$. In the next step, we define the univariate function of r,

$$f(r) = (1+r)\,\hat{l} < w > \left[I - (1+r)\,\hat{A}\right]^{-1}\left(I - \hat{A}\right)X - \frac{\hat{l}_s X \hat{l} < w > X}{v\hat{l}X}, \tag{35}$$

and find the unique zero of this function over the range, $0 \leq r < R$, i.e. the value of $r = r^*$ for which $f(r^*) = 0$; this gives us the uniform rate of profit, r^*. If the second part of assumption 3 holds, then a unique r^* exists because $f(0) = \hat{l} < w > X(1-(\hat{l}_s X)/(v\hat{l}X)) < 0$, $f' > 0$ and $\lim_{r \to R} = +\infty$. In the final step, we find the vector of prices of production using

$$p^{NI} = (1+r^*)\,\hat{l} < w > \left[I - (1+r^*)\,\hat{A}\right]^{-1}.$$

3.2.2 Accounting for Unproductive Industries

The second extension is motivated by the Marxian understanding that labor value (in the sense of Marx) is only generated in the production of commodities. In a modern capitalist economy, there are many industries that are not directly involved in producing commodities, e.g. trade, finance, real estate, etc. These industries do not produce value (or surplus value) and their income flows are a reflection of the redistribution of value and surplus value in the economy. Therefore, it makes sense to take account of unproductive industries (in the sense of labor value production) in the analysis.

The distinction between productive and unproductive sectors can be incorporated in the analysis at the point where we define invariance principles. Different invariance principles are ways of implementing the key intuition of the labor theory of value that labor value is created in the sphere of production and conserved in the sphere of exchange and distribution. Once we distinguish between productive and unproductive industries, the relevant invariance principle can be implemented by restricting value production to the former, while allowing for price of production to arise in both the former and the latter.

The key to implementing this extension is to revisit computation of labor values and the definition of the value of money. Suppose there are $m \leq n$ productive industries. Then, the vector of labor values is defined as

$$\lambda_p = \lambda_p \hat{A}_p + \hat{l}_{sp}, \tag{36}$$

where λ_p, $\hat{A}_p$ and $\hat{l}_{sp}$ are m-dimensional vectors and matrices for the m productive industries, and, in particular, $\hat{l}_{sp}$ refers to direct labor inputs measured in units of *simple* labor hours.

New Interpretation. Using (36), the value of money is redefined as

$$\mu_p^{NI} = \frac{\lambda_p Y_p}{p^{NI} Y}. \tag{37}$$

We can take account of unproductive industries either in the basic circulating capital model, which has uniform wage rates across industries, or in the extended circulating capital model that allows for differential wage rates across industries. In the former case, the system is given by

$$p^{NI} = (1+r)\left(p^{NI}\hat{A} + w\hat{l}\right), \tag{38}$$

and

$$v = w \times \mu_p^{NI}; \tag{39}$$

in the latter case, the system is given by

$$p^{NI} = (1+r)\left(p^{NI}\hat{A} + < w > \hat{l}\right), \tag{40}$$

and

$$v = \bar{w} \times \mu_p^{NI}. \tag{41}$$

In both cases, we can solve for the uniform rate of profit, r, and the price of production vector, p^{NI}, using the same algorithm that we have used above (Basu, 2021, Appendix 7.A.4).

Assumption 4. *Suppose $p \leq n$ denotes the number of productive industries.*

(a) $\hat{A}$ is nonnegative and irreducible.

(b) The value of labor power, v, satisfies the following condition:

$$0 < v < \frac{\hat{l}_{sp} X_p}{\hat{l} X}, \tag{42}$$

where $\hat{l}_{sp}$ and X_p are $1 \times p$ and $p \times 1$ vectors of direct labor input (measured in units of simple labor hours) and gross output in the subset of productive industries, respectively.

In the former case, i.e. with uniform wage rates across industries, the univariate function of r, whose zero gives us the uniform rate of profit, is given by,

$$f(r) = (1+r)\, w\hat{l} \left[I - (1+r)\, \hat{A}\right]^{-1} \left(I - \hat{A}\right) X - \frac{w\hat{l}_{sp} X_p}{v}, \tag{43}$$

where $\hat{l}_{sp}$ and X_p refer to the direct labor input in units of simple labor hours and the gross output (in monetary terms) of the subset of productive industries, respectively. If assumption 4 (a) and (b) are satisfied, then there exists a unique zero of the univariate function in (43). This is because $f(0) = w(\hat{l}X - (\hat{l}_{sp} X_p)/v) < 0$. Since $f' > 0$ and $\lim_{r \to R} f(r) = +\infty$, there is a unique zero of the function.

A similar argument shows that, in the latter case, i.e. when we allow for differential wage rates across industries, the corresponding univariate function of r is given by

$$f(r) = (1+r)\, \hat{l} < w > \left[I - (1+r)\, \hat{A}\right]^{-1} \left(I - \hat{A}\right) X - \frac{\hat{l}_{sp} X_p \hat{l} < w > X}{v \hat{l} X} \tag{44}$$

If assumption 4 (a) and (b) are satisfied, then there exists a unique zero of the univariate function in (44) and the algorithm for computing prices of production can be implemented. Note, once again, that if the condition in assumption 4 (b) is satisfied, then $f(0) = \hat{l} < w > X(1 - (\hat{l}_{sp} X_p)/(v\hat{l}X)) < 0$. Since $f' > 0$ and $\lim_{r \to R} f(r) = +\infty$, there is a unique zero of the function.

Standard Interpretation. Allowing for unproductive industries is possible in the SI approach with a uniform wage rate. The only change to the steps outlined in section 3.1.1

is that the value of money is redefined as

$$\mu_p^{SI} = \frac{\lambda_p X_p}{p_{rel} X}, \tag{45}$$

where the numerator is restricted to the productive industries (and hence the subscript p). All the other steps remain unchanged and allow us to compute the price of production vector and the uniform rate of profit.

3.3 Model with Capital Stock

There are at least two different ways to solve the model with capital stock. We can either use the SI or the NI. In both cases, the steps for computing the vector of labor values, the uniform rate of profit and the vector of prices of production are very similar in the model with capital stock to the corresponding method outlined above for the circulating capital model.

3.3.1 Standard Interpretation

To compute the uniform rate of profit and the vector of prices of production, we start from (15), and use (16) to get

$$p = p\hat{A} + p\hat{b}\hat{l} + p\hat{D} + r\left[p\hat{K} + p\hat{B} + p\hat{b}\hat{l} < t >\right],$$

which can be rearranged to give

$$\frac{1}{r}p = p\left[\hat{K} + \hat{B} + \hat{b}\hat{l} < t >\right]\left[I - \hat{A} - \hat{b}\hat{l} - \hat{D}\right]^{-1}. \tag{46}$$

Assumption 5. *$\hat{N}$ is nonnegative and irreducible, where*

$$\hat{N} = \left[\hat{K} + \hat{B} + \hat{b}\hat{l} < t >\right]\left[I - \hat{A} - \hat{b}\hat{l} - \hat{D}\right]^{-1}. \tag{47}$$

If assumption 5 holds, then we can use the Perron-Frobenius theorem to see that the uniform rate of profit, r, is the reciprocal of the maximal eigenvalue of $\hat{N}$. The corresponding eigenvector of $\hat{N}$ is the vector of relative prices of production, p_{rel}. the vector of *absolute* prices of production is arrived at by multiplying p_{rel} with the following constant:

$$\kappa = \frac{X}{p_{rel} < m^{-1} > X}, \tag{48}$$

where $< m^{-1} >$ is the diagonal matrix of market prices. Thus,

$$p^{SI} = \kappa p_{rel}, \tag{49}$$

where p^{SI} is the $1 \times n$ vector of prices of production estimated by following the SI.

If the model does not allow for unproductive industries, then we compute the vector of labor values using (12); if the model allows for unproductive industries, then the vector of labor values is computed with (36).

3.3.2 New Interpretation

Just like in the SI approach, if the model does not allow for unproductive industries, then we compute the vector of labor values using (12); if the model allows for unproductive industries, then the vector of labor values is computed with (36). To compute the uniform rate of profit and the vector of prices of production, we follow the same steps as in the circulating capital model – with some changes for the relevant matrices.

The price of production system is given by

$$p = pA + w\hat{l} + p\hat{D} + r\left[p\hat{K} + \left(p\hat{A} + w\hat{l}\right) < t >\right]$$

Assumption 6. *$\hat{M}$ is nonnegative and irreducible, where*

$$\hat{M} = \left[\hat{K} + \hat{A} < t >\right]\left[I - \hat{A} - \hat{D}\right]^{-1}, \tag{50}$$

and the value of labor power, v, satisfies the condition:

$$0 < v < \frac{l_s X}{lX}.$$

In the first step, we find the maximal rate of profit, R (which arises when $w = 0$ in the above equation system). If assumption 6 holds, then we can apply the Perron-Frobenius theorem to get

$$R = \frac{1}{\rho(\hat{M})}. \tag{51}$$

In the second step, we define the univariate function of r,

$$f(r) = \left[w\hat{l} + rw\hat{l} < t >\right] \left[I - \hat{A} - \hat{D} - r\hat{K} - r\hat{A} < t >\right]^{-1} \left(I - \hat{A} - \hat{D}\right) X - \frac{wl_s X}{v}, \tag{52}$$

and find its unique zero over the range $0 \leq r < R$, i.e. the unique value of $r = r^*$ such that $f(r^*) = 0$. If the second part of assumption 6 is satisfied, then $f(0) = wl\hat{X} - w\hat{l}_s X/v < 0$. Since $f' > 0$, this gives us a unique magnitude of uniform rate of profit, r^*. In the final step, we find the vector of price of production using

$$p = \left[w\hat{l} + r^* w\hat{l} < t >\right] \left[I - \hat{A} - \hat{D} - r^*\hat{K} - r^*\hat{A} < t >\right]^{-1}. \tag{53}$$

We can easily extend the analysis to (a) allow for wage differentials across industries (as in section 3.2.1) and (b) to allow for unproductive industries (as in section 3.2.2), and (c) to allow for both.

Assumption 7. *The following conditions hold.*

(a) $\hat{M}$ is nonnegative and irreducible, where $\hat{M} = \left[\hat{K} + \hat{A} < t >\right] \left[I - \hat{A} - \hat{D}\right]^{-1}$,

(b) The value of labor power, v, satisfies the following condition:

$$0 < v < \frac{\hat{l}_{sp} X_p}{\hat{l}X}. \tag{54}$$

For the model with capital stock, wage differentials across industries but not taking account of unproductive industries, we need assumption 6 to be satisfied. If this assumption is satisfied then solving the unique root of

$$f(r) = \left[\hat{l} < w > +r\hat{l} < w >< t >\right] \left[I - \hat{A} - \hat{D} - r\hat{K} - r\hat{A} < t >\right]^{-1} \left(I - \hat{A} - \hat{D}\right) X - \frac{l_s X \hat{l} < w > X}{v\hat{l}X}, \tag{55}$$

will give us the unique magnitude of the uniform rate of profit, $r = r^*$, such that $f(r^*) = 0$. This is because, if the second part of assumption 6 is satisfied, then $f(0) = \hat{l} < w > X(1 - (\hat{l}_s X)/(v\hat{l}X)) < 0$. Since $f' > 0$, this ensures a unique zero of the function. We can then compute the vector of prices of production as

$$p = \left[\hat{l} < w > +r^*\hat{l} < w >< t >\right] \left[I - \hat{A} - \hat{D} - r^*\hat{K} - r^*\hat{A} < t >\right]^{-1}. \tag{56}$$

For the model with capital stock that takes account of unproductive industries in value calculations, we have two sub-cases. In the first sub-case, we assume a uniform wage rate across industries. In this sub-case, if assumption 7 (a) and (b) are satisfied, then finding the unique root of

$$f(r) = \left[\hat{l}w + r\hat{l}w < t >\right] \left[I - \hat{A} - \hat{D} - r\hat{K} - r\hat{A} < t >\right]^{-1} \left(I - \hat{A} - \hat{D}\right) X - \frac{wl_{sp}X_p}{v}, \tag{57}$$

over $0 < r < R$ (where R is the maximal profit rate, as before), will give us the uniform rate of profit, $r = r^*$. This is because, if assumption 7(b) is satisfied, then $f(0) = w\hat{l}X - w\hat{l}_s X/v < 0$. Since $f' > 0$, we have a unique root, r^*. We can then compute the vector of prices of production as

$$p = \left[\hat{l}w + r^*\hat{l}w < t >\right] \left[I - \hat{A} - \hat{D} - r^*\hat{K} - r^*\hat{A} < t >\right]^{-1}.$$

In the second sub-case, we allow wage differentials across industries. In this sub-case, if assumption 7 (a) and (b) are satisfied, then finding the unique root of

$$f(r) = \left[\hat{l} < w > +r\hat{l} < w >< t >\right] \left[I - \hat{A} - \hat{D} - r\hat{K} - r\hat{A} < t >\right]^{-1} \left(I - \hat{A} - \hat{D}\right) X - \frac{l_{sp} X_p \hat{l} < w > X}{v\hat{l}X}, \tag{58}$$

over $0 < r < R$ (where R is the maximal profit rate, as before), will give us the uniform rate of profit, $r = r^*$. This is because, if assumption 7(b) is satisfied, then $f(0) = \hat{l} < w > X(1 - (\hat{l}_{sp} X_p)/(\hat{l}X)) < 0$. Since $f' > 0$, we have unique root, r^*. We can then compute the vector of prices of production as

$$p = \left[\hat{l} < w > +r^*\hat{l} < w >< t >\right] \left[I - \hat{A} - \hat{D} - r^*\hat{K} - r^*\hat{A} < t >\right]^{-1}.$$

4 Measuring Deviation between Labor Values and Prices of Production

4.1 The Theory

The theory for understanding the deviation between prices of production and labor values was developed in Shaikh (1984, pp. 65–71) and explained in Basu (2017, section 4 and online appendix). Here we follow these presentations.

Consider the commodity produced the i-th industry. The price per unit of output of this commodity can be decomposed into unit labor cost, profit per unit of output and intermediate costs per unit of output,

$$p_i = wL_i + \pi_i + M_i, \tag{59}$$

where p_i is the price of the commodity, w is the uniform nominal wage rate, L_i is the amount of labor needed to produce one unit of the commodity, and M_i is the intermediate

input cost per unit of the commodity. The material cost is the price received by some other capitalist firm. Hence, it can, in turn, be decomposed into the sum of three analogous terms as

$$M_i = wL_i^{(1)} + \pi_i^{(1)} + M_i^{(1)}, \tag{60}$$

where the subscript $^{(1)}$ refers to a previous stage of production. Substituting (60) in to (59), we get

$$p_i = wL_i + \pi_i + wL_i^{(1)} + \pi_i^{(1)} + M_i^{(1)}. \tag{61}$$

If we carry out this process of substitution for an infinite number of steps, we get

$$p_i = W_i^T + \pi_i^T, \tag{62}$$

where W^T is the total, i.e. sum of direct and indirect (taking account of all previous stages of production), labor cost per unit of the commodity,

$$W_i^T = wL_i^T = w\left(L_i + L_i^{(1)} + L_i^{(2)} + \cdots\right), \tag{63}$$

and π_i^T is the total, i.e. sum of direct and indirect, profit income per unit of the commodity,

$$\pi_i^T = \left(\pi_i + \pi_i^{(1)} + \pi_i^{(2)} + \cdots\right). \tag{64}$$

Since the sum of direct and indirect labor required to produce the commodity is its labor value,

$$\lambda_i = L_i + L_i^{(1)} + L_i^{(2)} + \cdots \tag{65}$$

substituting (65) into (63) and (62) gives us the link between labor values and prices,

$$p_i = W_i^T + \pi_i^T = wL_i^T + \pi_i^T = wL_i^T\left(1 + \frac{\pi_i^T}{wL_i^T}\right) = w\lambda_i\left(1 + y_i\right), \tag{66}$$

where y is the integrated profit-wage ratio

$$y_i = \frac{\pi_i^T}{wL_i^T} = \frac{\pi_i + \pi_i^{(1)} + \pi_i^{(2)} + \cdots}{w\left(L_i + L_i^{(1)} + L_i^{(2)} + \cdots\right)} \tag{67}$$

The same analysis for industry j will give us

$$p_j = w\lambda_j\left(1 + y_j\right), \tag{68}$$

and hence we have,

$$\frac{p_i}{p_j} = \frac{\lambda_i}{\lambda_j}\left(\frac{1+y_i}{1+y_j}\right). \tag{69}$$

So far, the analysis has been carried out for any set of prices. When we deal with prices of production, i.e. long run equilibrium prices, we have

$$\pi_i = rK_i \text{ and } \pi_i^T = rK_i^T = r\left(K_i + K_i^{(1)} + K_i^{(2)} + \cdots\right), \tag{70}$$

where r is the uniform rate of profit. Thus, the integrated profit-wage ratio becomes

$$y_i = \frac{\pi_i^T}{wL_i^T} = \frac{\pi_i + \pi_i^{(1)} + \pi_i^{(2)} + \cdots}{w\left(L_i + L_i^{(1)} + L_i^{(2)} + \cdots\right)} = \frac{r}{w}\left(\frac{K_i + K_i^{(1)} + K_i^{(2)} + \cdots}{L_i + L_i^{(1)} + L_i^{(2)} + \cdots}\right) = \frac{r}{w}\frac{K_i^T}{L_i^T},$$

where K_i^T is the integrated, i.e. sum of direct and indirect, capital

$$K_i^T = K_i + K_i^{(1)} + K_i^{(2)} + \cdots$$

Using this, we get the key relationship between *relative* labor values and *relative* prices of production

$$p_{i,j} = \lambda_{i,j} z_{i,j}, \quad i = 1, 2, \ldots, n; j = 1, 2, \ldots, n, \tag{71}$$

where

$$p_{i,j} = \frac{p_i}{p_j}, \lambda_{i,j} = \frac{\lambda_i}{\lambda_j}, z_{i,j} = \frac{z_i}{z_j}, \tag{72}$$

and

$$z_i = 1 + \frac{r}{w}\frac{K_i^T}{L_i^T}, \quad i = 1, 2, \ldots, n. \tag{73}$$

According to Shaikh (1984), equations (71), (72), and (73) show that deviation oof relative prices of production from relative labor values will be small. This is because, for any i and j, $z_i \approx z_j$, which implies that $z_{i,j} \approx 1$. Hence, from (71), we see that $p_{i,j} \approx \lambda_{i,j}$. But, why is $z_i \approx z_j$? This is because "in an actual economy with its extensive network of industrial interconnections ... even large variations in direct profit-wage ratios ... can be reduced to relatively moderate variations in integrated profit-wage ratios." (Shaikh, 1984, p. 68). The latter follows because, "as long as the economy is composed of basic goods in the sense of Sraffa", the integrated profit-wage ratios of each commodity is a weighted averages of all direct profit-wage ratios in the economy.

4.2 Regression-Based Measure

4.2.1 Log-Log Regression

The basic claim about small deviations between relative prices of production and relative values can be tested using a simple log-log regression (Shaikh, 1984, pp. 70). Taking the logarithms of both sides of (71) and treating $\ln z_{i,j}$ as a stochastic disturbance term, we get

$$\ln p_{i,j} = a_0 + a_1 \ln \lambda_{i,j} + u_{i,j}, \tag{74}$$

where $u_{i,j} = \ln z_{i,j}$. As argued in Basu (2017, pp. 1369), the claim that relative prices of production will be close to relative labor values can perhaps be captured by the following joint null hypothesis:

$$H_0 : a_0 = 0, a_1 = 1. \tag{75}$$

We can test this null hypothesis with a F-test. Almost all existing work on this issue has, instead, used the R-squared to test the claim that relative prices of production will be close to relative labor values. As pointed out in Basu (2017, pp. 1369), this is a mistake because

the claim about closeness of the vector of relative prices of production and the vector of relative labor values does not have any implication on the R-squared of the log-log regression in (74).

Shaikh (2016, pp. 389) has argued that the log-log regression in (74) is invalid because logarithms are only meaningful for dimensionless variables and some of the terms in (71) are not dimensionless. Let us evaluate the dimensions of each of the term in (71).

The first term in (71) is the relative price ratio, $p_{i,j} = p_i/p_j$. This is the ratio of the prices of production of commodity i and j. As we have seen earlier, p_i is measured in units of dollar per dollar of gross output. Hence p_i is dimensionless. So is p_j. Hence, $p_{i,j}$ is dimensionless. The second term in (71) is the relative labor value ratio, $\lambda_{i,j} = \lambda_i/\lambda_j$, which is the ratio of the labor value of commodity i and j. As we have seen above, λ_i is measured in units of hours per dollar of gross output. In the same way, λ_j is measured in units of hours per dollar of gross output. Hence, $\lambda_{i,j}$ is dimensionless. The third term in (71) is $z_{i,j} = z_i/z_j$. It is easy to see from (73) that z_i is dimensionless. Hence, $z_{i,j}$ is dimensionless. Thus, all the three terms in (71) are dimensionless and it is perfectly legitimate to use the log-log regression in (74).

4.2.2 Level-Level Regression

A better way to test the key claim about closeness of the vector of relative labor values and relative prices of production is to use a level-level regression.

$$p_{i,j} = a_0 + a_1\lambda_{i,j} + u_{i,j}, \tag{76}$$

and test by the following joint null hypothesis:

$$H_0 : a_0 = 0, a_1 = 1. \tag{77}$$

There are at least three advantages to using the level-level regression instead of the log-log regression.

First, under the joint null hypothesis that $a_0 = 0, a_1 = 1$, the model in (74) implies that the expected value of $\ln p_{i,j}$ is equal to the expected value of $\ln \lambda_{i,j}$. Since the expected value of the log is not equal to the log of the expected value, the log-log regression and the F-test does not exactly test what we might be interested in: is the expected value of $p_{i,j}$ is equal to the expected value of $\lambda_{i,j}$? This hypothesis is directly tested by the level-level regression. This is because under the joint null hypothesis that $a_0 = 0, a_1 = 1$, the model in (76) implies that the expected value of $p_{i,j}$ is equal to the expected value of $\lambda_{i,j}$.

Second, the discussion in Shaikh (1984, pp. 68) leads to the conclusion that the integrated profit-wage ratios of each commodity is a weighted averages of all direct profit-wage ratios in the economy. i.e. $z_{i,j} \approx 1$. One way to implement this idea is to note that the factor $z_{i,j}$ multiplying $\lambda_{i,j}$ in (71) is not stochastic but rather is a constant which is close to unity. This idea can then be tested with the hypothesis that $a_0 = 0, a_1 = 1$ in (76).

Third, since logarithms are not used, the issue of dimensions of the variables do not arise. Of course, this is a minor issue so far as empirical analyses are concerned. This is because, as argued above, when researchers use actual input-output data, both labor values and prices of production are measured per monetary unit (the unit of measurement of the gross output). Once a researcher decides to measure gross output of all commodities in the same monetary units, e.g. millions of US dollars, then the problem of units are immediately resolved.

4.3 Non-Regression-Based Measures

While many scholars have used the log-log regression to quantify the degree of deviation between relative prices of production and relative labor values, others have used non-regression-based measures. Here, we discuss several such measures that have been used in the literature. The basic idea behind most of these measures is to compute some aggregate of the element by element deviation between the vector of relative price of production and the vector of relative labor values.

Letting $p_{i,j} = p_i/p_j$ denote the relative price of production and $\lambda_{i,j} = \lambda_i/\lambda_j$ denote the

relative labor values, the first measure used in the literature is the root mean squared error and is defined as

$$RMSE = \sqrt{\frac{1}{n\,(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{p_{i,j}}{\lambda_{i,j}} - 1 \right)^2} \tag{78}$$

Since $(p_{i,j}/\lambda_{i,j}) - 1 = (p_{i,j} - \lambda_{i,j})/\lambda_{i,j}$ is the deviation between an element of the relative price of production vector and the corresponding element of the relative value vector as a fraction of the same element of the relative value vector, $RMSE$ is the square root of the average of the squared deviation. The sum will have $C_2^n = n(n-1)$ nonzero terms because when $i = j$, the corresponding term is zero: $(p_{i,i}/\lambda_{i,i}) - 1 = (1/1) - 1 = 0$. This measure was used by Petrovic (1987).

The second measure is the mean absolute distance and is defined as

$$MAD = \frac{1}{n\,(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| \frac{p_{i,j}}{\lambda_{i,j}} - 1 \right| \tag{79}$$

Whereas $RMSE$ uses the mean of the square of the difference in the deviations between elements of the vector of relative prices of production and the vector of relative values, MAD uses mean of the absolute value of the difference. This measure was used by Shaikh (1984); Ochoa (1984, 1989); Tsoulfidis and Maniatis (2002); Tsoulfidis and Mariolis (2007).

The third measure is a variation on the second and is known as the mean absolute weighted distance and is defined as

$$MAWD = \frac{1}{n\,(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| \frac{p_{i,j}}{\lambda_{i,j}} - 1 \right| \omega_i \omega_j, \tag{80}$$

where, for $j = 1, 2, \ldots, n$, $\omega_j = p_j Q_j / \sum_j p_j Q_j$ is the weight of industry j measured in terms of gross output. The only difference between MAD and $MAWD$ is that the latter uses a weighted average, while the former uses an unweighted average. This measure has been used by Ochoa (1984, 1989); Tsoulfidis and Maniatis (2002); Tsoulfidis (2008); Shaikh (2016).

The fourth measure is known as the classical distance measure and is defined as

$$CDM = \frac{1}{n\left(n-1\right)} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| \frac{p_{i,j}}{w_{i,j}\lambda_{i,j}} - 1 \right| \omega_i \omega_j, \tag{81}$$

where $w_{i,j} = w_i / w_j$ is the ratio of industry i and industry j's wage rate. This measure has been developed by Shaikh (2016, pp. 392–393). The motivation for developing the CDM was to propose a method that was unit and scale-independent, much like the next two measures: α and D_α.

The fifth measure is the angle, α, between the relative labor value and relative price of production vectors and is defined as

$$\alpha = \arctan\left(\frac{\sqrt{\text{Var}\left(s\right)}}{\text{E}\left(s\right)} \right), \tag{82}$$

where s is the $1 \times n(n-1)$ vector, with typical element given by $p_{i,j}/\lambda_{i,j}$, $\text{Var}(s)$ refers to the variance of s and $\text{E}(s)$ refers to the mean of s. The final measure is the "distance" between the relative labor value and relative price of production vectors defined using α, i.e.

$$D_\alpha = \sqrt{2\left(1 - \cos\alpha\right)} \tag{83}$$

Both α and D_α were proposed by Steedman and Tomkins (1998) and has been used in Steedman and Tomkins (1998); Tsoulfidis and Maniatis (2002); Tsoulfidis and Mariolis (2007); Tsoulfidis (2008); Fröhlich (2012).[10]

[10] In Appendix B, we have illustrated the methodology presented in this paper for estimating prices of production and measuring deviation between relative prices of production and relative values using a 3-industry example.

5 Data and Variables

5.1 Circulating Capital Model

5.1.1 Sources of Data

We use data from the 2016 release of the World Input Output Database (Timmer et al., 2015) to estimate labor values and prices of production according to the methods outlined in section 3 and then test whether relative labor values and relative prices of production are 'close' to each other using the measures presented in section 4. The 2016 release of the World Input Output Database (WIOD) has data on 28 EU countries and 15 other major countries in the world for the period from 2000 to 2014.[11]

We use two sets of data from the WIOD: (a) national input-output tables (NIOT) in current prices, denoted in US dollars, and (b) socio economic accounts (SEA). From the NIOT, we extract data on: (a) input-output, or inter-industry transactions flow, matrices, all entries measured in US dollars, and (b) consumption of households, measured in US dollars. From the SEA we extract industry-level data on: (a) employment (in hours); (b) compensation of employees (in national currency units), (c) total compensation, i.e. sum of compensation of employees and self-employed persons (in national currency units), (d) value added and gross output (in units of local currencies). We use data on nominal exchange rates from the OECD to convert local currency magnitudes into US dollar magnitudes.

5.1.2 Construction of Variables

We construct the key variables for the analysis as follows:

- Gross output vector, X: For each country and year, this is the $n \times 1$ vector of industry-level gross output (in millions of US dollars);
- Net output vector, Y: For each country and year, this is the $n \times 1$ vector of industry-level net output (in millions of US dollars);

[11] See https://www.rug.nl/ggdc/valuechain/wiod/wiod-2016-release

- Input-output matrix, $\hat{A}$: For each country and year, we define $\hat{A} = Z < X >^{-1}$, where Z is the $n \times n$ transactions flow matrix and $< X >^{-1}$ is the inverse of the $n \times n$ diagonal matrix of gross outputs; entries of Z and X are each measured in millions of US dollars;

- Direct labor input vector, $\hat{l}$: For each country and year, we define $\hat{l} = L < X >^{-1}$, where L is the $1 \times n$ vector of direct labor input, measured in millions of hours;

- Average worker's consumption bundle, $\hat{b}$: For each country and year, we define $\hat{b} = C_h/L$, where C_h is the $n \times 1$ vector of household consumption, measured in millions of US dollars, and $L = \sum_i L_i$ is the (scalar) sum of direct labor inputs (measured in millions of hours) across all industries;

- Nominal wage vector, w: For each country and year, w is the $1 \times n$ industry-level nominal wage rate vector. It is computed by dividing compensation of employees (in millions of US dollars) with total hours worked by employees (in millions of hours);

- Uniform wage rate, $\bar{w}$: For each country and year, this is the average of the vector w;

- Profit share, π: For each country and year, this is defined as 1 minus the ratio of total compensation (employees + self employed) and total value added, both measured in millions of US dollars.

5.1.3 Adjustment of Industries

The 2016 release of WIOD has data on 53 industries for each country. We make two types of adjustments to the list of industries for our calculations.

For the basic circulating capital model that does not take account of unproductive industries, we drop all industries with zero gross output, and also drop non-private sector industries that do not participate in the formation of the price of production at the national level. The latter are: (a) O84: Public administration and defense; compulsory social

security; (b) T: Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use; and (c) U: Activities of extraterritorial organizations and bodies.

For the extended circulating model that differentiates between productive and unproductive industries, we take an additional step. We identify the following industries as unproductive in the sense of Marx: (a) K64: Financial service activities, except insurance and pension funding; (b) K65: Insurance, reinsurance and pension funding, except compulsory social security; (c) K66: Activities auxiliary to financial services and insurance activities; (d) L68: Real estate services; (e) M69_M70 : Legal and accounting services; activities of head offices; management consultancy services; and (f) M73: Advertising and marketing research. We drop them from value computations but retain them for price of production computations.

5.2 Model with Capital Stock

The model with capital stock (see section 3.3 for details of the model) has been estimated with input-output and related data for the U.S. economy for the year 2020. We used data from two sources: the U.S. Bureau of Economic Analysis (BEA) and the U.S. Bureau of Labor Statistics (BLS). We download the following tables from the website of BEA: (a) Use table, (b) Make table, (c) Import table, (d) full-time equivalent employees by industry; (e) current-cost depreciation of private fixed assets by industry; and (f) real inventory-sales ratios for manufacturing and trade (seasonally adjusted). From the website of the BLS, we downloaded data on industry-level nominal wage rates.

5.2.1 Construction of Variables

We construct the key variables for the analysis as follows:[12]

- Gross output vector, X: This is the $n \times 1$ vector of industry-level gross output (in millions of US dollars) that comes from the sum of each row of the Make table;

[12] In this section, we provide the basic outlines of variable construction. For full details, see Appendix A.

- Input-output matrix, $\hat{A}$ and $\hat{A}_p$: We subtract imports from the use table to generate the domestic inputs matrix, and then divide each element by the corresponding industry-level gross output. Finally, we pre-multiply with the transformation matrix to arrive at the input-output matrix, $\hat{A}$, in an industry by industry format; we select the subset of productive industries to get $\hat{A}_p$.

- Direct complex labor input vector, $\hat{l}$ and $\hat{l}_p$: For each industry, we define $\hat{l} = L < X >^{-1}$, where L is the $1 \times n$ vector of direct labor input. The latter is arrived at by multiplying the full-time equivalent employees in an industry by 2080×1000, to convert it into hours of labor.[13] We adjust for productive labor within each industry and then select the data for the subset of productive industries to get $\hat{l}_p$.

- Vector of wages, w and w_p: We take data on the nominal wage rates from the BLS and match BLS industries with BEA industries to get w. The data for the subset of productive industries give us w_p.

- Uniform wage rate, $\bar{w}$: This is the average of the vector w.

- Direct simple labor input vector, $\hat{l}_s$ and $\hat{l}_{sp}$: We multiply each element of l and l_p with the ratio of industry-level average wage rate and the minimum of the industry-level average wage rates to get $\hat{l}_s$ and $\hat{l}_{sp}$, respectively.

- Average worker's consumption bundle, $\hat{b}$: The real wage bundle, $\hat{b}$, is obtained by dividing each element of the vector of personal consumption expenditure from the Use table by the total labor input.

- Depreciation matrix, $\hat{D}$ and $\hat{D}_p$: We use information from the Use table to create an 'intermediary capital matrix', and then combine this with data on current-cost depreciation of private fixed assets by industry to generate $\hat{D}$. The matrix corresponding to the subset of productive industries gives us $\hat{D}_p$.

[13] We assume that each full-time employee works, on average, for 52 weeks in a year and 40 hours per week. Thus, on average, they work for 2080 hours per week. Data on full-time equivalent employees are in thousands; hence, we multiply by 1000.

- Capital stock coefficient matrix, $\hat{K}$: We use information from the Use table to create an 'intermediary capital matrix', and then combine this with data on current-cost net stock of private fixed assets by industry to generate $\hat{K}$.
- Turnover ratio matrix, t: We use data on the real inventory-sales ratios for manufacturing and trade to create the t matrix.

6 Results

6.1 Circulating Capital Model

6.1.1 Sample for Analysis

The 2016 release of WIOD has data on 53 industries for 43 countries for the years between 2000 and 2014. We dropped countries for which some key variables are missing or for which one of the methods of computing prices of production did not work. Table 1 provides details of countries used for of the three approaches: SI, SR and NI. For the SI approach, our sample has data on 37 countries for the years between 2000 and 2014, i.e. our sample has industry-level data on 555 country-year pairs. For the SR approach too, our sample has 555 country-year pairs. For the NI approach, our sample has between 465 and 480 country-year pairs.

[Table 1 about here]

6.1.2 What We Report

For each country-year pair in our sample, we compute the vector of labor values, the uniform rate of profit and the vector of prices of production using the methods outlined in section 3. Then we use the methods explained in section 4 to investigate whether the vector of relative labor values are 'close' to the vector of relative prices of production. We report the results of this analysis in Table 2 (Standard Interpretation with uniform wage rates and not taking account of unproductive industries), Table 3 (Standard Interpretation with

uniform wage rates and taking account of unproductive industries), 4 (Sraffian approach with uniform wage rates and not taking account of unproductive industries), 5 (New Interpretation with uniform wage rates and not taking account of unproductive industries), 6 (New Interpretation with differential wage rates and not taking account of unproductive industries), 7 (New Interpretation with uniform wage rates and taking account of unproductive industries) and 8 (New Interpretation with wage differentials and taking account of unproductive industries). In each table, we report the 10-th, 25-th, 50-th, 75-th and 90-th percentiles of the empirical distribution of each measure of 'closeness' of the vectors of relative labor values and the vector of relative prices of production.

6.1.3 Discussion of Results

Regression-Based Measures. The regression-based test of the deviation between the vector of relative labor values are 'close' to the vector of relative prices of production is an F-test (see section 4). If the p-value associated with this test is small, we can reject the null hypothesis that the vector of relative labor values are close to the vector of relative prices of production. The lower part of each table reports the distribution of the R-squared, the F-stat and the p-value for both the log-log and the level-level regressions. In all the seven tables, Table 2, 3, 4, 5, 6, 7 and 8, the distribution of the p-value is flat at zero.[14] Thus, for all the country-year pairs in our sample, we can very strongly reject the null hypothesis that the vector of relative labor values are close to the vector of relative prices of production.

[Table 2, 3, 4, 5, 6, 7 and 8 about here]

Non-Regression-Based Measures. The distribution of the various non-regression based measures vary across the tables, indicating that their distribution is impacted by the method we use to compute the price of production vector. Let us ignore the results in Table 4 because of the problems of the Sraffian method (see discussion at the end of section 3.1.2 for details). For the other six tables, we see the following: all the non-regression

[14] Please ignore the results in Table 4 as the Sraffian method has conceptual problems.

based measures have very high magnitudes when prices of production have been estimated with the SI (Table 2, 3); all the measures have lower magnitudes when we use the NI approach to estimate prices of production (Table 5, 6, 7 and 8). But even the results for the NI approach show that relative values are rather far apart from relative prices of production.

As an example, let us comment in some detail on the magnitudes of the classical distance measure (CDM) across the different SI and NI models. The median magnitude of CDM in Table 2 (Standard Interpretation with uniform wage rates and not taking account of unproductive industries) is 3.36. Thus, across the 555 country-year pairs for which the SI was used estimate labor values and prices of production, the deviation between relative prices of production and relative values is 336% of relative values. The median of the CDM in Table 3 (Standard Interpretation with uniform wage rates and taking account of unproductive industries) is 4.33, even higher. Even the 10-th percentile of the empirical distribution of CDM in Table 3 is 2.2. If we turn to the NI approach, the magnitudes of the CDM are lower – in comparison to the SI approach. For instance, the median magnitude of CDM in Table 8 is 0.33. This is the lowest median magnitude of the CDM across all models. It tells us that across the 465 country-year pairs for which the NI was used estimate labor values and prices of production in Table 8, the deviation between relative prices of production and relative values is 33% of relative values. Thus, even at its lowest magnitude, we see significant deviation between relative values and relative prices of production.

The same pattern can be observed across all the different non-regression-based measures: RMSE, MAD, MAWD, Angle and distance (computed with the angle). For each of the measures, the SI gives much higher magnitudes than the SI, i.e. the whole empirical distribution of any of these measures is shifted far to the right when we use SI than if we use the NI. Even with NI, the magnitudes of the median (or any other part of the empirical distribution) is high.

Comparison with existing results. The results reported in this paper stand in stark contrast with most of those reported in the literature. Indicatively, Işikara and Mokre (2022,

p. 8) investigate 42 countries over 15 years using the WIOD tables and report a "strong linear relationship" between values and prices, and interpret this to imply that labor values and prices are very close. They also find that the MAWD in most cases concentrates around 10% with the highest value observed for Malta (21.8%). Similar findings are reported by Tsoulfidis and Mariolis (2007) who employ a circulating capital model for the Greek economy, by Tsoulfidis (2008) for the Japanese economy, and by Fröhlich (2012) for the German economy. Our non-regression measures are higher than the ones these authors report. For example, the MAWD of the bottom 10th percentile observed for the entire WIOD sample using SI1 and SI2 is 106% and 201%, respectively. For the different NI specifications the deviations decrease, but even the lowest median of MAWD (18%) is significantly higher than the 10% value Işikara and Mokre (2022) report.

Of the existing studies that we have reviewed, only Steedman and Tomkins (1998, p. 385), who study the 1990 UK economy, come up with findings indicating a significant distance between values and prices of production, concluding that "price-value deviations can be both quite large and quite different from case to case." The angle and distance (using angle) measure they report, 21.53 degrees and 0.374, respectively, is close to the medians we find using the different specifications of the NI circulating capital model.

6.2 Model with Capital Stock

Results for the model with capital stock are reported in Table 9. Since the model with capital stock is more realistic and complete than the circulating capital model, the results in Table 9 are our preferred results. In this table, we present results from six different specifications: SIK-1 (SI implement for model with uniform wage rates and not taking account of unproductive industries); SIK-2 (SI implement for model with uniform wage rates and taking account of unproductive industries); NIK-1 (NI implement for model with uniform wage rates and not taking account of unproductive industries); NIK-2 (NI implement for model with uniform wage rates and taking account of unproductive industries); NIK-3 (NI implement for model with differential wage rates and not taking account of unproductive

industries); NIK-4 (NI implement for model with differential wage rates not taking account of unproductive industries).

[Table 9 about here]

For each model, we compute the vector of labor values, the uniform rate of profit and the vector of prices of production. We test for the closeness of relative labor values and relative prices of production, using both regression- and non-regression-based measures–in parallel with our analysis of the circulating capital model. Unlike in the case of the circulating model analysis, we have data for only one point in time. Hence, we report the magnitude of the measures directly, in contrast to the empirical distribution of the measures that we reported for the circulating capital model.

Regression-based Measures. Across all the six specifications, the regression-based measures strongly reject the mull hypothesis that the vector of relative values and the vector of relative values are close to each other. This is because, in each case, the p-value associated with the F-test is very small (very close to zero). This is true for both the log-log and the level-level regression.

Non-Regression-based Measures. Across all the six specifications, the six non-regression-based measures are high. Much as in the analysis of the circulating capital model, the magnitudes of all the six non-regression-based measures are far higher for the SI than for the NI approach. For instance, RMSE is 214 for SIK-2, while it is less than 2 for all the NI models. But even if we take the lowest magnitude of the RMSE of 1.2 for NIK-2, this implies that the deviation between relative prices of production and relative values is 120% of relative values.

If we use the CDM or the Angle, we arrive at the same conclusion. For instance, the angle between the vector of relative prices and the vector of relative values is about 80 degrees if we use the SI approach. The corresponding angle lies between 32 degrees and 41 degrees if we use the NI approach. Even at its lowest of 32 degrees, the angle between the two vectors is substantially large. The use of CDM leads to the same conclusion, as does the use of the MAD or the MAWD.

Comparison with existing results. The above results indicate a much higher deviation between values and prices than can be found in the existing literature. Ochoa (1989) using data for the US economy for the years 1947, 1958, 1961, 1963, 1967-1970, 1972 finds an average MAWD of 17.4%. Shaikh (1998) finds an even lower average MAWD (4.4%) for a selection of years for the US economy. Similar results are reported for the Yugoslavian economy (Petrovic, 1987), and the Greek economy (Tsoulfidis and Maniatis, 2002), with each study using different non-regression-based statistical measures. Our estimates of MAWD for the 2020 US economy, for instance, range from 53%-80% for the NI specifications and from 487% to 568% for the SI ones. These magnitudes are far higher than those reported in existing studies.

7 Conclusion

In this paper, we have revisited the question of whether the deviation between the vector of labor values and the vector of prices of production is small. In contrast to a large and distinguished literature on this issue, we find that the deviation between the vector of relative labor values and the vector of relative prices of production is large.

We estimate prices of production, the uniform rate of profit and labor values for circulating capital models for 555 country-year pairs using 53-industry input-output and other data from the 2016 release of the WIOD. We estimate prices of production, the uniform rate of profit and labor values for the model with capital stock using 65-industry input-output and other relevant data for the US economy in 2020. In computing prices of production, we implement the SI, the SR and the NI approaches, with an without the extensions of differential wage rates and taking account of unproductive industries in labor value calculations.

For all our models and using a large number of measures of deviation, we find that the deviation between the vector of relative prices of production and the vector of relative labor values is large. Our preferred model is the model with capital stock implemented with

the NI approach that takes account of unproductive industries and that allows for wage differentials across industries (last column in Table 9). For this model, the regression-based measures strongly reject the null hypothesis that relative prices of production and relative labor values are close. For the non-regression-based measures, we find: RMSE of 163%, MAD of 99%, MAWD of 53%, CDM of 59% and the angle between the vector of relative labor values and relative prices of production being 38 degrees.

Our results are in stark contrast to the existing literature that has found values and prices to be close to each other. However, it is important to clarify that our findings are not meant to and do not pose a challenge to Marx's Labour Theory of Value, but rather to a particular interpretation of it, which is mainly associated with scholars within the SI tradition. This group of scholars consider the prediction that values and prices are close to each other to be at the core of Marx's Labour Theory of Value. It seems fair to say that this choice was influenced by the need to provide an answer to the Sraffian critique with respect to the redundancy of values as a significant category of analysis (Steedman, 1977); this is supported by the fact that several of the papers in this literature make explicit reference to this issue and consider the findings to be a response to that.[15]

We contend that this line of argumentation is faulty on several grounds: First, even if there is an empirically verifiable 'close' relationship between values and prices, this is not a convincing argument about the value category being essential—or even useful-–in understanding prices of production, if the latter can still be computed without any reference to a value system. Moreover, given the impressive results the SI has been able to produce–summarized by the Fundamental Marxist Theorem–which stand independent of how close values and prices are, it is a rather odd choice to wager the relevance of the theory on such a narrow claim. Third, and perhaps less importantly, there is no textual evidence to support such a claim: Marx himself, in his (admittedly flawed) examples in volume III, derives values that are very far from the prices of production and asserts that the law of value is maintained in the aggregate of the economy. Thus, this line of argument suffers in theory,

[15]In fact, the paper that started this literature Shaikh (1984) is written with the explicit purpose to reply to that critique and separate the Marxian from the Sraffian price system.

presents a break with Marx's understanding of what the labour theory of value implies in principle, and—if our results are correct-–is not verified empirically. We propose that our findings should be understood as an opportunity for Marxist scholars to turn to alternative answers about why Marx's Labour Theory of Value remains useful and relevant[16], and free the theory from an unnecessary empirical burden.

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Table 1: Countries used for Analysis of the Circulating Capital Model

	Country	SI (Table 2, 3)	SR (Table 4)	NI1 (Table 5)	NI2 (Table 6)	NI3 (Table 7)	NI4 (Table 8)
1	AUS	✓	✓	✓	✓	✓	✓
2	AUT	✓	✓	✓	✓	✓	✓
3	BEL	✓	✓	✓	✓	✓	✓
4	BGR	✓	✓	✓	✓	✓	✓
5	BRA	✓	✓	✓	✓	✓	✓
6	CAN	✓	✓	✓	✓	✓	✓
7	CHE	✓	✓	✓	✓	✓	✓
8	CYP	✓	✓				
9	CZE	✓	✓	✓	✓	✓	✓
10	DEU	✓	✓	✓	✓	✓	✓
11	DNK	✓	✓	✓	✓	✓	✓
12	ESP	✓	✓	✓	✓	✓	✓
13	EST	✓	✓	✓	✓	✓	✓
14	FIN	✓	✓	✓	✓	✓	✓
15	FRA	✓	✓	✓	✓	✓	✓
16	GBR	✓	✓	✓	✓	✓	✓
17	GRC	✓	✓	✓	✓	✓	✓
18	HRV	✓	✓	✓	✓	✓	
19	HUN	✓	✓	✓	✓	✓	✓
20	IDN	✓	✓	✓	✓	✓	✓
21	IRL	✓	✓	✓	✓		
22	ITA	✓	✓	✓	✓	✓	✓
23	JPN	✓	✓	✓	✓		
24	KOR	✓	✓	✓	✓	✓	✓
25	LTU	✓	✓	✓	✓	✓	✓
26	LUX	✓	✓	✓	✓		
27	NLD	✓	✓	✓	✓	✓	✓
28	NOR	✓	✓	✓	✓	✓	✓
29	POL	✓	✓	✓	✓	✓	✓
30	PRT	✓	✓	✓	✓	✓	✓
31	ROU	✓	✓	✓	✓	✓	✓
32	RUS	✓	✓	✓	✓	✓	✓
33	SVK	✓	✓				
34	SVN	✓	✓	✓	✓	✓	✓
35	SWE	✓	✓	✓	✓	✓	✓
36	TUR	✓	✓	✓	✓	✓	✓
37	USA	✓	✓	✓	✓	✓	✓

Notes: The ✓ indicates that the country was used for the analysis. For each country, we had 15 years in the sample, 2000–2014. Each country-year pair has data on a maximum of 53 industries.

Table 2: Deviation between labor values and prices of production for a circulating capital model with uniform wage rates and no unproductive industries: The Standard Interpretation (SI1)

	N	Pctile10	Pctile25	Median	Pctile75	Pctile90
Non Regression-Based Measures						
RMSE	555	8.48	14.95	55.3	216.51	1032.89
MAD	555	2.57	3.55	8.52	24.05	69.27
MAWD	555	1.06	1.61	2.69	4.43	7.47
CDM	555	1.41	2.04	3.36	5.54	13.78
Angle (degrees)	555	69.42	74.65	80.71	83.89	85.73
Distance (using angle)	555	1.14	1.21	1.3	1.34	1.36
Regression-Based Measures						
Log-Log Regression						
R-Squared	555	0	0.01	0.03	0.07	0.11
F-Stat	555	56.6	113.24	178.42	288.28	413.22
P-Value	555	0	0	0	0	0
Level-Level Regression						
R-Squared	555	0	0	0	0.01	0.01
F-Stat	555	6.64	11.44	23.63	48.64	85.01
P-Value	555	0	0	0	0	0
Memo: Uniform Rate of Profit	555	0.17	0.33	0.85	9.25	138.03

Notes: This table presents empirical distributions of various measures of deviation between labor values and prices of production for a circulating capital model with uniform wage rates across industries and counting all industries as productive. The prices of production have been computed according to the Standard Interpretation of Marx's labor theory of value; for details, see section 3.1.1. The data comes from the harmonized input-output tables and the socio-economic accounts (SEA) of the World Input Output Database for 37 countries for the period 2000–2014. Each country has up to 53 industries. RMSE = root mean squared error; MAD = mean absolute distance; MAWD = mean absolute weighted distance; CDM = classical distance measure. N refers to the number of country-year pairs for which labor values and prices of production have been computed. Empirical distributions reported in this table have been computed over N.

Table 3: Deviation between labor values and prices of production for a circulating capital model with uniform wage rates and allowing for unproductive industries: The Standard Interpretation (SI2)

	N	Pctile10	Pctile25	Median	Pctile75	Pctile90
Non Regression-Based Measures						
RMSE	555	8.15	15.32	30.82	78.29	677.11
MAD	555	2.8	4.41	7.3	12.99	40.94
MAWD	555	2.01	2.82	3.94	6.01	10.95
CDM	555	2.2	3.24	4.33	7.25	16.82
Angle (degrees)	555	67.32	71.09	76.28	80.9	84.98
Distance (using angle)	555	1.11	1.16	1.24	1.3	1.35
Regression-Based Measures						
Log-Log Regression						
R-Squared	555	0	0	0.02	0.05	0.12
F-Stat	555	21.13	56.22	94.85	182.64	269.27
P-Value	555	0	0	0	0	0
Level-Level Regression						
R-Squared	555	0	0	0	0.01	0.03
F-Stat	555	6.37	13.96	28.37	48.29	66.18
P-Value	555	0	0	0	0	0
Memo: Uniform Rate of Profit	555	0.08	0.27	0.74	8.75	131.1

Notes: This table presents empirical distributions of various measures of deviation between labor values and prices of production for a circulating capital model with uniform wage rates across industries and allowing for unproductive industries. The prices of production have been computed according to the Standard Interpretation of Marx's labor theory of value; for details, see the very end of section 3.2.2. The data comes from the harmonized input-output tables and the socio-economic accounts (SEA) of the World Input Output Database for 37 countries for the period 2000–2014. Each country has up to 53 industries. When testing for deviations between relative prices and relative values, we restrict the analysis to the subset of productive industries – because elements of the value vector are, by definition, zero for unproductive industries. RMSE = root mean squared error; MAD = mean absolute distance; MAWD = mean absolute weighted distance; CDM = classical distance measure. N refers to the number of country-year pairs for which labor values and prices of production have been computed. Empirical distributions reported in this table have been computed over N.

Table 4: Deviation between labor values and prices of production for a circulating capital model with uniform wage rates and no unproductive industries: The Sraffian Approach (SR)

	N	Pctile10	Pctile25	Median	Pctile75	Pctile90
Non Regression-Based Measures						
RMSE	555	0.18	0.22	0.3	0.47	0.56
MAD	555	0.14	0.16	0.23	0.33	0.39
MAWD	555	0.08	0.09	0.13	0.19	0.21
CDM	555	0.18	0.22	0.31	0.46	0.71
Angle (degrees)	555	8.86	10.73	14.44	20.02	22.81
Distance (using angle)	555	0.15	0.19	0.25	0.35	0.4
Regression-Based Measures						
Log-Log Regression						
R-Squared	555	0.68	0.75	0.82	0.88	0.92
F-Stat	555	183.49	282.58	537.25	1267.34	2496.01
P-Value	555	0	0	0	0	0
Level-Level Regression						
R-Squared	555	0.69	0.76	0.84	0.9	0.94
F-Stat	555	141.62	270.85	720.23	2908.57	5777.21
P-Value	555	0	0	0	0	0
Memo: Uniform Rate of Profit	555	0.26	0.4	0.7	4.66	79.07

Notes: This table presents empirical distributions of various measures of deviation between labor values and prices of production for a circulating capital model with uniform wage rates across industries and counting all industries as productive. The prices of production have been computed according to the Sraffian approach to prices of production; for details, see section 3.1.2. The data comes from the harmonized input-output tables and the socio-economic accounts (SEA) of the World Input Output Database for 37 countries for the period 2000–2014. Each country has up to 53 industries. RMSE = root mean squared error; MAD = mean absolute distance; MAWD = mean absolute weighted distance; CDM = classical distance measure. N refers to the number of country-year pairs for which labor values and prices of production have been computed. Empirical distributions reported in this table have been computed over N. Please note that the Sraffian approach has a serious conceptual problem, as we have pointed out at the end of section 3.1.2; hence, the results in this table can be ignored. We report these results only for completeness.

Table 5: Deviation between labor values and prices of production for a circulating capital model with uniform wage rates and not taking account of unproductive industries: The New Interpretation (NI1)

	N	Pctile10	Pctile25	Median	Pctile75	Pctile90
Non Regression-Based Measures						
RMSE	525	0.35	0.46	0.69	1.09	1.82
MAD	525	0.25	0.33	0.46	0.65	0.98
MAWD	525	0.13	0.17	0.24	0.35	0.57
CDM	525	0.26	0.34	0.55	0.88	1.96
Angle (degrees)	525	15.5	18.58	25.4	34.91	46.56
Distance (using angle)	525	0.27	0.32	0.44	0.6	0.79
Regression-Based Measures						
Log-Log Regression						
R-Squared	525	0.13	0.34	0.5	0.69	0.83
F-Stat	525	41.36	108.24	244.32	386.55	517.66
P-Value	525	0	0	0	0	0
Level-Level Regression						
R-Squared	525	0.14	0.32	0.56	0.71	0.84
F-Stat	525	82.43	136.42	219.32	364.3	691.36
P-Value	525	0	0	0	0	0
Memo: Uniform Rate of Profit	525	0.44	0.65	1.09	2.23	4.27

Notes: This table presents empirical distributions of various measures of deviation between labor values and prices of production for a circulating capital model with uniform wage rates across industries and counting all industries as productive. The prices of production have been computed according to the New Interpretation of Marx's labor theory of value; for details, see section 3.1.3. The data comes from the harmonized input-output tables and the socio-economic accounts (SEA) of the World Input Output Database for 35 countries for the period 2000–2014. Each country has up to 53 industries. RMSE = root mean squared error; MAD = mean absolute distance; MAWD = mean absolute weighted distance; CDM = classical distance measure. N refers to the number of country-year pairs for which labor values and prices of production have been computed. Empirical distributions reported in this table have been computed over N.

Table 6: Deviation between labor values and prices of production for a circulating capital model with differential wage rates across industries but not taking account of unproductive industries: The New Interpretation (NI2)

	N	Pctile10	Pctile25	Median	Pctile75	Pctile90
Non Regression-Based Measures						
RMSE	525	0.03	0.16	0.39	0.59	1.04
MAD	525	0.02	0.11	0.29	0.41	0.63
MAWD	525	0.01	0.07	0.17	0.25	0.4
CDM	525	0.17	0.22	0.35	0.56	0.87
Angle (degrees)	525	1.69	8.38	16.84	22.72	33.49
Distance (using angle)	525	0.03	0.15	0.29	0.39	0.58
Regression-Based Measures						
Log-Log Regression						
R-Squared	525	0.2	0.45	0.65	0.98	1
F-Stat	525	186.56	277.44	553.71	1302.21	2389.99
P-Value	525	0	0	0	0	0
Level-Level Regression						
R-Squared	525	0.24	0.5	0.75	0.97	1
F-Stat	525	129.3	229.12	550.02	2371.47	4193.8
P-Value	525	0	0	0	0	0
Memo: Uniform Rate of Profit	525	0.44	0.65	1.08	2.2	4.27

Notes: This table presents empirical distributions of various measures of deviation between labor values and prices of production for a circulating capital model with wage rate differentials across industries and counting all industries as productive. The prices of production have been computed according to the New Interpretation of Marx's labor theory of value; for details, see section 3.2.1. The data comes from the harmonized input-output tables and the socio-economic accounts (SEA) of the World Input Output Database for 35 countries for the period 2000–2014. Each country has up to 53 industries. RMSE = root mean squared error; MAD = mean absolute distance; MAWD = mean absolute weighted distance; CDM = classical distance measure. N refers to the number of country-year pairs for which labor values and prices of production have been computed. Empirical distributions reported in this table have been computed over N.

Table 7: Deviation between labor values and prices of production for a circulating capital model with uniform wage rates but allowing for unproductive industries: The New Interpretation (NI3)

	N	Pctile10	Pctile25	Median	Pctile75	Pctile90
Non Regression-Based Measures						
RMSE	480	0.31	0.47	0.67	0.99	1.62
MAD	480	0.23	0.33	0.43	0.62	0.93
MAWD	480	0.12	0.17	0.23	0.33	0.51
CDM	480	0.25	0.35	0.49	0.76	1.44
Angle (degrees)	480	14.52	18.01	24.79	34.3	43.28
Distance (using angle)	480	0.25	0.31	0.43	0.59	0.74
Regression-Based Measures						
Log-Log Regression						
R-Squared	480	0.04	0.21	0.43	0.63	0.83
F-Stat	480	30.66	81.68	222.26	381.12	592.69
P-Value	480	0	0	0	0	0
Level-Level Regression						
R-Squared	480	0.03	0.18	0.39	0.61	0.82
F-Stat	480	65.8	99.2	170.31	266.56	397.91
P-Value	480	0	0	0	0	0
Memo: Uniform Rate of Profit	480	0.41	0.57	1.03	2.02	3.42

Notes: This table presents empirical distributions of various measures of deviation between labor values and prices of production for a circulating capital model with uniform wage rates across industries and allowing for unproductive industries. The prices of production have been computed according to the New Interpretation of Marx's labor theory of value; for details, see section 3.2.2. The data comes from the harmonized input-output tables and the socio-economic accounts (SEA) of the World Input Output Database for 32 countries for the period 2000–2014. Each country has up to 53 industries. When testing for deviations between relative prices and relative values, we restrict the analysis to the subset of productive industries – because elements of the value vector are, by definition, zero for unproductive industries. RMSE = root mean squared error; MAD = mean absolute distance; MAWD = mean absolute weighted distance; CDM = classical distance measure. N refers to the number of country-year pairs for which labor values and prices of production have been computed. Empirical distributions reported in this table have been computed over N.

Table 8: Deviation between labor values and prices of production for a circulating capital model allowing for different wage rates across industries and allowing for unproductive industries: The New Interpretation (NI4)

	N	Pctile10	Pctile25	Median	Pctile75	Pctile90
Non Regression-Based Measures						
RMSE	465	0.04	0.17	0.4	0.58	0.92
MAD	465	0.02	0.11	0.3	0.41	0.56
MAWD	465	0.01	0.06	0.17	0.24	0.37
CDM	465	0.15	0.22	0.32	0.47	0.66
Angle (degrees)	465	1.7	8.61	16.35	23.14	31.87
Distance (using angle)	465	0.03	0.15	0.28	0.4	0.55
Regression-Based Measures						
Log-Log Regression						
R-Squared	465	0.1	0.25	0.57	0.98	1
F-Stat	465	206.1	311.61	618.62	1125.68	1991.73
P-Value	465	0	0	0	0	0
Level-Level Regression						
R-Squared	465	0.07	0.24	0.56	0.97	1
F-Stat	465	148.5	244.54	456.34	1052.13	2456.17
P-Value	465	0	0	0	0	0
Memo: Uniform Rate of Profit	465	0.41	0.54	0.97	1.94	3.03

Notes: This table presents empirical distributions of various measures of deviation between labor values and prices of production for a circulating capital model with differential wage rates across industries and allowing for unproductive industries. The prices of production have been computed according to the New Interpretation of Marx's labor theory of value; for details, see section 3.2.2. The data comes from the harmonized input-output tables and the socio-economic accounts (SEA) of the World Input Output Database for 31 countries for the period 2000–2014. Each country has up to 53 industries. When testing for deviations between relative prices and relative values, we restrict the analysis to the subset of productive industries – because elements of the value vector are, by definition, zero for unproductive industries. RMSE = root mean squared error; MAD = mean absolute distance; MAWD = mean absolute weighted distance; CDM = classical distance measure. N refers to the number of country-year pairs for which labor values and prices of production have been computed. Empirical distributions reported in this table have been computed over N.

Table 9: Deviation between labor values and prices of production for the model with capital stock

	SIK-1	SIK-2	NIK-1	NIK-2	NIK-3	NIK-4
Non Regression-Based Measures						
RMSE	181.77	214.99	3.04	2.81	1.75	1.63
MAD	31.27	34.54	1.54	1.45	1.04	0.99
MAWD	5.68	4.87	0.80	0.73	0.59	0.53
CDM	4.60	4.24	0.97	0.83	0.72	0.59
Angle	79.96	80.62	48.70	47.38	39.28	37.97
Distance (using angle)	1.29	1.29	0.82	0.80	0.67	0.65
Regression-Based Measures						
Log-Log Regression						
R-Squared	0.02	0.00	0.00	0.00	0.00	0.01
F-Stat	138.94	85.18	603.77	729.51	692.79	938.95
P-value	0.00	0.00	0.00	0.00	0.00	0.00
Level-Level Regression						
R-Squared	0.01	0.02	0.00	0.00	0.00	0.01
F-Stat	28.86	24.72	241.63	267.54	299.17	386.04
P-value	0.00	0.00	0.00	0.00	0.00	0.00
Memo: Uniform Rate of Profit	0.08	0.08	0.21	0.20	0.16	0.15

Notes: This table presents measures of deviation between labor values and prices of production for a model with capital stock. The prices of production have been computed according to either the standard or the new interpretation of Marx's labor theory of value. The data comes from the US input-output tables and other relevant sources for the year 2020 (for details see section 5.2). RMSE = root mean squared error; MAD = mean absolute distance; MAWD = mean absolute weighted distance; CDM = classical distance measure. SIK refers to the model with capital stock implemented with the Standard Interpretation: SIK-1 (uniform wage rates + do not allow for unproductive industries); SIK-2 (uniform wage rates + allow for unproductive industries); for details see section 3.3.1. NIK refers to the model with capital stock implemented with the New Interpretations: NIK-1 (uniform wage rates + do not allow for unproductive industries); NIK-2 (uniform wage rates + allow for unproductive industries); NIK-3 (allow for wage differential + do not allow for unproductive industries); NIK-4 (allow for wage differential + allow for unproductive industries); for details see section 3.3.2. When testing for deviations between relative prices and relative values in specifications that take account of unproductive industries, we restrict the analysis to the subset of productive industries – because elements of the value vector are, by definition, zero for unproductive industries.

Appendix A Data Construction for Capital Stock Model

A.1 Create input-output coefficients matrix ($\hat{A}$)

1. Download the following from BEA
   (a) "The Use of commodities by Industries – Summary" We will call this as Use from now on.
   (b) "The Domestic Supply of Commodities by Industries – Summary". We will call this $Make$ from now on.
   (c) "ImportMatrices_Before_Redefinitions_SUM_1997-2021". We will call this $Import$ from now on. Note that all three are in a commodity-by-industry format.
2. Transpose the $Make$ table.
3. Exclude
   (a) From all tables the commodity and industry 524, HS and ORE.
   (b) From $Make$ all columns after sector 81 and the last row
   (c) From Use and $Import$ all columns and rows after sector 81
4. Build $Use.Domestic$ by subtracting the $Import$ table from the Use table.
5. Compute the industry output using the $Make$ table by summing over each row.
6. Divide the elements of the $Use.Domestic$ table by the appropriate industry output.
7. Pre-multiply with the transformation matrix (W) (see below "Create transformation matrix") to $\hat{A}$ in an industry-by-industry format.

A.2 Create input-output coefficients matrix for productive industries ($\hat{A}_p$)

1. Follow steps 1,2 from "Create input-output coefficients matrix"

2. Refer to table $\hat{A}$ to exclude the unproductive sectors by removing both columns and rows from Use, $Make$ and $Import$ to get Use_p, $Make_p$ and $Import_p$.

3. Follow steps 4 to 7 to derive $\hat{A}_p$ using W_p instead of W.

Table A.1 gives the division of industry/activity BEA codes into productive and unproductive industries.

A.3 Create transformation matrix (W)

1. Use $Make$ table to compute total output of industries without scrap – captured by the "Used" commodity – and total industry output.

2. Divide non-scrap output with total output to get the non-scrap ratio.

3. Create market share matrix by dividing each element of the matrix with the sum of each respective column.

4. Divide each element of the market share matrix with the respective industry's non-scrap ratio, i.e., divide each row with the appropriate non-scrap ratio.

5. Exclude columns "Used" and "Other".

A.4 Create transformation matrix for productive industries (W_p)

Follow the same process as the one for W but use the $Make_p$ matrix instead of the $Make$ matrix.

A.5 Create labor input vectors

1. Download from BEA "Table 6.5D. Full-Time Equivalent Employees by Industry".

2. Exclude sectors 524, HS, ORE

3. Multiply the series in step 1 by 2080×1000, where 2080 is 52 weeks times 40 hours a week to get the figures in a per hour basis. Call this vector as L.

4. Divide by gross output of each industry to get $\hat{l}$.

5. Perform a matching exercise with data from BLS to get data on productive and unproductive labour (the latter being computed as total minus productive) (see table A.2 for the codes of each BLS series and how it was matched to the BEA series) and compute the productive ratio showing how many of the workers in an industry are productive.

6. Multiply the productive ratio with the $\hat{l}$ vector to remove all unproductive labor. Then, use Table A.1 to exclude all unproductive sectors to get $\hat{l}_p$.

7. Use the wage series from BLS (see table A.2) to compute reduction coefficients by dividing each wage with the minimum observed wage.

8. Multiply each element of $\hat{l}$ and $\hat{l}_p$ with the respective reduction coefficients to get $\hat{l}_s$ and $\hat{l}_{sp}$, respectively.

A.6 Wage (w) and wages in the productive industries (w_p)

Refer to table A.2 for the series retrieved from BLS and the matching that was done with BEA data. To compute w_p exclude the unproductive sectors as identified in table A.1.

A.7 Create turnover ratio matrix (t)

1. Download "Table 3BU. Real Inventory-Sales Ratios for Manufacturing and Trade, Seasonally Adjusted [Based on chained 2012 dollars, 1997 forward, NAICS]".

2. Isolate the industries included in the input output table and fill the industries not present in the Inventory-Sales ratios with 0.

3. Create diagonal matrix with the newly created series from step 2 in an order that matches the order appearing in the A matrix to get t.

A.8 Create depreciation matrix ($\hat{D}$)

1. Create a binary index vector series to capture which industries produce fixed capital goods. Closest approximation on an IO table can be found in the 1997 capital flow data, where BEA identifies the capital producing industries. A matching exercise is possible based on the provided codes in the document found here. Refer to table A.3 for the vector of capital forming commodities.

2. Create an "intermediary capital matrix" from the table, as follows:

   (a) Multiplying with 0 every row of the Use table that contains a commodity which does not create capital using the series from step 1 – this ensures that only capital augmenting inputs are considered.

   (b) Compute the sum of every column to get the total input of each industry.

   (c) Divide each element by the respective column total computed in step 2 to create a weights matrix.

3. Download the depreciation vector found in BEA as "Table 3.4ESI. Current-Cost Depreciation of Private Fixed Assets by Industry".

4. Divide each element of the series in step 3 by the respective industry's total output.

5. Create a diagonal matrix out of the vector resulting in step 4.

6. Post-multiply the diagonal matrix to the matrix created in step 2. (The diagonal matrix comes second)

7. Multiply the matrix from step 6 with the transformation matrix – pre-multiply the W matrix to get an industry-by-industry format matrix $\hat{D}$.

A.9 Create depreciation matrix for productive industries ($\hat{D}_p$)

Follow the same procedure as was used for creating $\hat{D}$, but use Use_p and W_p in place of Use and W, respectively.

A.10 Create capital stock coefficient matrix ($\hat{K}$)

The process is identical in structure to that for constructing the depreciation matrix, $\hat{D}$. So, follow the same steps as the ones to create $\hat{D}$ with the difference that the source series is called: "Table 3.1ESI. Current-Cost Net Stock of Private Fixed Assets by Industry".

A.11 Create real wage bundle ($\hat{b}$)

1. Take the PCE from the Use table.
2. Exclude sectors 524, HS, ORE.
3. Divide the PCE vector by the total labour input, $\sum_i L_i$.

Productive	11CA, 113FF, 211, 212, 213, 22, 23, 321, 327, 331, 332, 333, 334, 335, 3361MV, 3364OT, 337, 339, 311FT, 313TT, 315AL, 322, 323, 324, 325, 326, 481, 482, 483, 484, 485, 486, 487OS, 493, 511, 512, 513, 514, 5415, 5412OP, 562, 61, 621, 622, 623, 711AS, 713, 721, 722, 81
Unproductive	42, 441, 445, 452, 4A0, 521CI, 523, 525, HS, ORE, 523RL, 5411, 55, 561, 624

Table A.1: Productive and unproductive industry/activity BEA codes

BEA industry code	**BLS Wage ID**	**BLS Productive employees ID**	**BLS Total employment ID**

111CA	Mean annual hourly wage rate from Farm Labor Survey (FLS)	See Farm Labor Survey (FLS)	See Farm Labor Survey (FLS)
113FF	Mean hourly wage from BLS Occupational Employment and Wage Statistics (OEWS) survey, sector 11 (agriculture, forestry, fishing, and hunting)	CES1011330006	CES1011330001
211	CES1021100003	CES1021100006	CES1021100001
212	CES1021200003	CES1021200006	CES1021200001
213	CES1021300003	CES1021300006	CES1021300001
22	CES4422000003	CES4422000006	CES4422000001
23	CES2000000003	CES2000000006	CES2000000001
311FT	CES3231100003	CES3231100006	CES3231100001
313TT	Average: CES3231300003, CES3231400003	Average: CES3231300006, CES3231400006	Average: CES3231300001, CES3231400001
315AL	CES3231500003	CES3231500006	CES3231500001
321	CES3132100003	CES3132100006	CES3132100001
322	CES3232200003	CES3232200006	CES3232200001
323	CES3232300003	CES3232300006	CES3232300001

324	CES3232400003	CES3200000006	CES3200000001
325	CES3232500003	CES3232500006	CES3232500001
326	CES3232600003	CES3232600006	CES3232600001
327	CES3132700003	CES3132700006	CES3132700001
331	CES3133100003	CES3133100006	CES3133100001
332	CES3133200003	CES3133200006	CES3133200001
333	CES3133300003	CES3133300006	CES3133300001
334	CES3133400003	CES3133400006	CES3133400001
335	CES3133500003	CES3133500006	CES3133500001
3361MV	Average:	Average:	Average:
	CES3133610003,	CES3133610006,	CES3133610001,
	CES3133620003,	CES3133620006,	CES3133620001,
	CES3133630003	CES3133630006	CES3133630001
337	CES3133700003	CES3133700006	CES3133640001
339	CES3133900003	CES3133900006	CES3133900001
42	CES4142000003	CES4142000006	CES4142000001
441	CES4244100003	CES4244100006	CES4244100001
445	CES4244500003	CES4244500006	CES4244500001
452	CES4245200003	CES4245200006	CES4245200001
481	CES4348100003	CES4348100006	CES4348100001
482	CES4300000003	CES4300000006	CES4300000001
483	CES4300000003	CES4300000006	CES4300000001
484	CES4348400003	CES4348400006	CES4348400001
485	CES4348500003	CES4348500006	CES4348500001
486	CES4300000003	CES4300000006	CES4300000001
487OS	Average:	Average:	Average:
	CES4348800003,	CES4348800006,	CES4348800001,
	CES4349200003	CES4349200006	CES4349200001

493	CES4349300003	CES4349300006	CES4349300001
4AO	Average:	Average:	Average:
	CES4244200003	CES4244200006	CES4244200001
	CES4244300003	CES4244300006	CES4244300001
	CES4244400003	CES4244400006	CES4244400001
	CES4244600003	CES4244600006	CES4244600001
	CES4244700003	CES4244700006	CES4244700001
	CES4244800003	CES4244800006	CES4244800001
	CES4245100003	CES4245100006	CES4245100001
	CES4245300003	CES4245300006	CES4245300001
	CES4245400003	CES4245400006	CES4245400001
521CI	CES5552200003	CES5552200006	CES5552200001
523	CES5552300003	CES5552300006	CES5552300001
525	CES5500000003	CES5500000006	CES5500000001
532RL	CES5553200003	CES5553200006	CES5553200001
5411	CES6054110003	CES6054110006	CES6054110001
5412OP	Average:	Average:	Average:
	CES6054120003	CES6054120006	CES6054120001
	CES6054130003	CES6054130006	CES6054130001
	CES6054140003	CES6054140006	CES6054140001
	CES6054160003	CES6054160006	CES6054160001
	CES6054170003	CES6054170006	CES6054170001
	CES6054180003	CES6054180006	CES6054180001
	CES6054190003	CES6054190006	CES6054190001
5415	CES6054150003	CES6054150006	CES6054150001
55	CES6055000003	CES6055000006	CES6055000001
561	CES6056100003	CES6056100006	CES6056100001
562	CES6056200003	CES6056200006	CES6056200001

61	CES6500000003	CES6500000006	CES6500000001
621	CES6562100003	CES6562100006	CES6562100001
622	CES6562200003	CES6562200006	CES6562200001
623	CES6562300003	CES6562300006	CES6562300001
624	CES6562400003	CES6562400006	CES6562400001
711AS	Average:	Average:	Average:
	CES7071100003	CES7071100006	CES7071100001
	CES7071200003	CES7071200006	CES7071200001
713	CES7071300003	CES7071300006	CES7071300001
721	CES7072100003	CES7072100006	CES7072100001
722	CES7072200003	CES7072200006	CES7072200001
81	CES8000000003	CES8000000006	CES8000000001

Table A.2: Sources for wages, and for productive and total workers by industry

Capital-augmenting	211, 212, 213, 23, 321, 327, 331, 332, 333, 334, 335, 3361MV, 3364OT, 337, 339, 42, 4A0, 481, 482, 483, 484, 511, 513, 514, 532RL, 5415, 5412OP
Not capital-augmenting	11CA, 113FF, 22, 311FT, 313FT, 315AL, 322, 323, 324, 325, 326, 441, 445, 452, 485, 486, 487OS, 493, 512, 521CI, 523, 525, 5411, 55, 561, 562, 61, 621, 622, 623, 624, 711AS, 713, 721, 722, 81

Table A.3: Capital-augmenting and not capital-augmenting commodities

Appendix B Two Examples

In this section, we illustrate the ideas developed in this paper with two simple 3-industry examples that were used by Ochoa (1984). The first example uses physical input-output data and the second uses market-price input-output data.[17]

[17]It might be pertinent to point out that the computations about these two examples that are reported in Ochoa (1984) are full of mistakes. In both example 1 and 2, the eigenvectors, and hence the prices of

B.1 Example 1: Physical IO Data

In this example, we follow Ochoa (1984, pp. 46-57) and use physical IO data. The basic matrices and vectors for this 3-industry example is as follows:

$$A = \begin{bmatrix} 0.265 & 0.968 & 0.00681 \\ 0.0121 & 0.391 & 0.0169 \\ 0.0408 & 0.808 & 0.165 \end{bmatrix}; D = \begin{bmatrix} 0 & 0 & 0 \\ 0.00568 & 0.0267 & 0.0028 \\ 0.00265 & 0.0147 & 0.00246 \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & 0 & 0 \\ 0.120 & 0.791 & 0.096 \\ 0.037 & 0.251 & 0.043 \end{bmatrix}; l = \begin{bmatrix} 0.193 & 3.562 & 0.616 \end{bmatrix};$$

$$t = \begin{bmatrix} 0.317 & 0 & 0 \\ 0 & 0.099 & 0 \\ 0 & 0 & 0.187 \end{bmatrix}; Q = \begin{bmatrix} 26530 \\ 18168 \\ 73840 \end{bmatrix}; b = \begin{bmatrix} 0.0109 \\ 0.0275 \\ 0.296 \end{bmatrix}; m = \begin{bmatrix} 4 & 60 & 7 \end{bmatrix};$$

where A, D, K, l, t, Q, b and m, are the input-output matrix, the depreciation matrix, the capital stock coefficient matrix, the direct labor input vector, the diagonal matrix of turnover rates, the gross output vector, the real wage bundle and the vector of market prices.

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production, are all incorrect. If Ochoa (1984) used the same computational methods for the data analysis as for the examples, there is likely to be errors in the main results of the paper too. Of course, there is no way to verify this because we do not have access to the actual IO data used by Ochoa (1984).

B.2 Example 2: Market-price IO Data

In this example, we follow Ochoa (1984, pp. 61-67) and use market-price IO data. The basic matrices and vectors for this 3-industry example is as follows:

$$\hat{A} =< m > A < m^{-1} >= \begin{bmatrix} 0.265 & 0.645 & 0.004 \\ 0.182 & 0.391 & 0.145 \\ 0.071 & 0.094 & 0.165 \end{bmatrix};$$

$$\hat{D} =< m > D < m^{-1} >= \begin{bmatrix} 0 & 0 & 0 \\ 0.085 & 0.027 & 0.024 \\ 0.005 & 0.002 & 0.003 \end{bmatrix}$$

$$\hat{K} =< m > K < m^{-1} >= \begin{bmatrix} 0 & 0 & 0 \\ 1.80 & 0.791 & 0.819 \\ 0.065 & 0.029 & 0.043 \end{bmatrix};$$

$$\hat{l} = l < m^{-1} >= \begin{bmatrix} 0.048 & 0.059 & 0.088 \end{bmatrix};$$

$$\hat{b} =< m > b = \begin{bmatrix} 0.437 \\ 1.65 \\ 2.07 \end{bmatrix},$$

where the 'hat'-matrices contain data that are expressed in terms of market prices.[18] The gross output vector and the matrix of turnover times remains the same between the two examples.

[18] For the relationship between the physical and market-prices matrices and vectors, refer to section 2.2.1.

B.3 Results

For both example 1 and 2, we estimate the basic circulating capital model–with a uniform wage rate and without any unproductive industries–with SI (section 3.1.1), with the Sraffian approach (section 3.1.2) and with NI (section 3.1.3). We estimate the capital stock model–with a uniform wage rate and without any unproductive industries–with SI (section 3.3.1), and with NI (section 3.3.2).

B.3.1 Circulating Capital Model

Example 1: Physical IO Data: For basic circulating capital model estimated with the SI, the vector of values, the uniform rate of profit and the vector of (absolute) prices of production are:

$$\lambda = \begin{bmatrix} 0.440 & 7.739 & 0.898 \end{bmatrix}; r = 38.78\%; p = \begin{bmatrix} 0.570 & 0.239 & 1.341 \end{bmatrix}.$$

To estimate the same model with the Sraffian approach, we need additional data on the average nominal wage rate and the profit share. We compute the average nominal wage rate as $mb = 3.756$, i.e. the amount of money necessary to purchase the real wage bundle, b, at the market prices. Finally, we assume that the profit share is $1/3$. With this data for the Sraffian approach, the corresponding quantities are:

$$\lambda = \begin{bmatrix} 0.440 & 7.739 & 0.898 \end{bmatrix}; r = 34.16\%; p = \begin{bmatrix} 2.636 & 45.285 & 4.329 \end{bmatrix};$$

To estimate the the same model with the NI, we need additional information on the value of labor power. Assuming that the value of labor power is equal to the wage share, we take this to be $2/3$. Using this data for the NI, the corresponding quantities are:

$$\lambda = \begin{bmatrix} 0.440 & 7.739 & 0.898 \end{bmatrix}; r = 21.16\%; p = \begin{bmatrix} 2.622 & 45.475 & 4.704 \end{bmatrix}.$$

Example 2: Market-price IO Data: For basic circulating capital model estimated with

the SI, the vector of values, the uniform rate of profit and the vector of (absolute) prices of production are:

$$\lambda = \begin{bmatrix} 0.109 & 0.129 & 0.128 \end{bmatrix}; r = 38.78\%; p = \begin{bmatrix} 0.267 & 1.675 & 1.097 \end{bmatrix}.$$

To estimate the same model with the Sraffian approach, we need additional data on the average nominal wage rate and the profit share. We compute the average nominal wage rate as $m\hat{b} = 113.68$, i.e. the amount of money necessary to purchase the real wage bundle, $\hat{b}$, at the market prices. Finally, we assume that the profit share is 1/3. With this data for the Sraffian approach, the corresponding quantities are:

$$\lambda = \begin{bmatrix} 0.109 & 0.129 & 0.128 \end{bmatrix}; r = 34.16\%; p = \begin{bmatrix} 19.895 & 22.785 & 18.668 \end{bmatrix};$$

To estimate the the same model with the NI, we need additional information on the value of labor power. Assuming that the value of labor power is equal to the wage share, we take this to be 2/3. Using this data for the NI, the corresponding quantities are:

$$\lambda = \begin{bmatrix} 0.109 & 0.129 & 0.128 \end{bmatrix}; r = 24.24\%; p = \begin{bmatrix} 21.202 & 24.464 & 21.301 \end{bmatrix}.$$

B.3.2 Model with Capital Stock

Example 1: Physical IO Data: For basic model with capital stock estimated with the SI, the vector of values, the uniform rate of profit and the vector of (absolute) prices of production are:

$$\lambda = \begin{bmatrix} 0.519 & 8.309 & 0.941 \end{bmatrix}; r = 23.37\%; p = \begin{bmatrix} 0.284 & 1.661 & 1.094 \end{bmatrix};$$

the same model estimated with the NI gives,

$$\lambda = \begin{bmatrix} 0.519 & 8.309 & 0.941 \end{bmatrix}; r = 15.69\%; p = \begin{bmatrix} 3.905 & 48.328 & 5.018 \end{bmatrix}.$$

Example 2: Market-price IO Data: For basic model with capital stock estimated with

the SI, the vector of values, the uniform rate of profit and the vector of (absolute) prices of production are:

$$\lambda = \begin{bmatrix} 0.129 & 0.138 & 0.134 \end{bmatrix}; r = 23.37\%; p = \begin{bmatrix} 0.056 & 4.909 & 0.377 \end{bmatrix};$$

the same model estimated with the NI gives,

$$\lambda = \begin{bmatrix} 0.519 & 8.309 & 0.941 \end{bmatrix}; r = 14.68\%; p = \begin{bmatrix} 28.071 & 23.503 & 21.043 \end{bmatrix}.$$

B.3.3 Measures of Deviation for Both Examples

Table B.1 presents results about the deviation between *relative* values and *relative* prices of production using the non-regression-based measures (section 4.3). Since the example has only 3 industries, the number of relative values and relative prices of production are only 3. This small sample size makes the use of regression-based measures infeasible.

[Table B.1 about here]

The top panel in Table B.1 presents results for example 1 (physical IO data). Columns 1, 3 and 4 relate to the circulating capital model; columns 2 and 5 refer to the capital stock model. We see three interesting results in the top panel in Table B.1. First, comparing columns 1, 3 and 4, we see that the deviation between relative values and relative prices of production are far higher when we use SI, compared to both SR and NI, for estimating the circulating capital model. For instance, the classical distance measure for the circulating capital model shows 152% for SI, 6% for SR and 3% for NI. This is true across all measures. Second, comparing columns 2 and 5, we see the same pattern: the deviation between relative values and relative prices of production are far higher when we use SI, compared to NI, for estimating the capital stock model. Third, comparing columns 1 and 2 or comparing columns 4 and 5, we see that while deviation between relative values and relative prices of production are lower for the capital stock model than for the circulating capital model

if both are estimated with SI (columns 1 versus 2), the opposite holds if the models are estimated with NI (columns 4 versus 5).

The bottom panel in Table B.1 presents results for example 2 (market-price IO data). We see the same patterns as we noted for the top panel of the table, with one change. Now, we see that while deviation between relative values and relative prices of production are lower for the circulating capital model than for the capital stock model for both SI (columns 1 versus 2) and NI (columns 4 versus 5).[19]

[19]The measure of angle seems to be an exception to this for the NI.

Table B.1: Deviation between relative labor values and relative prices of production: 3-Industry Examples

	SI1	SI2	SR	NI1	NI2
Example 1: Physical IO Data					
RMSE	23.69	1.15	0.19	0.11	0.30
MAD	14.04	1.03	0.16	0.09	0.26
MAWD	1.52	0.21	0.06	0.03	0.08
CDM	1.52	0.21	0.06	0.03	0.08
Angle (degrees)	59.21	51.24	5.85	3.52	7.29
Distance (using angle)	0.99	0.86	0.10	0.06	0.13
Example 2: Market-price IO Data					
RMSE	0.69	6.75	0.19	0.12	0.28
MAD	0.68	4.49	0.16	0.11	0.25
MAWD	0.18	1.26	0.06	0.04	0.07
CDM	0.18	1.26	0.06	0.04	0.07
Angle	48.24	59.51	5.85	4.06	6.89
Distance (using angle)	0.82	0.99	0.10	0.07	0.12

Notes: This table presents results of implementing examples from Ochoa (1984). Example 1 uses physical IO data (Ochoa, 1984, pp. 46–57) and example 2 uses market-price IO data (Ochoa, 1984, pp. 61–67). The data can be seen in section B. SI1 = circulating capital model implemented with SI (see section 3.1.1); SI2 = capital stock model implemented with SI (see section 3.3.1); SR = circulating capital model implemented with the Sraffian approach (see section 3.1.2); NI1 = circuit of capital model implemented with NI (see section 3.1.3); SI2 = capital stock model implemented with NI (see section 3.3.2). RMSE = root mean squared error; MAD = mean absolute distance; MAWD = mean absolute weighted distance; CDM = classical distance measure. In these examples, MAWD and CDM are equal because, by construction, the nominal wage rate is the same in the three industries.