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WIDER Working Paper 2023/149

How should an optimal tax system react to a crisis?

Simulation results for Zambia

Dingquan Miao,¹ Ravi Kanbur,² and Jukka Pirttilä³

December 2023

Abstract: The COVID-19 pandemic increased public debt and changed the income distribution in many countries. We use a numerical simulation approach to derive optimal nonlinear marginal tax rates for the pre-crisis and crisis periods. We contribute to the literature by examining optimal tax rates numerically for a developing country and by investigating how the tax rates should be changed as a response to a crisis. Our results indicate that the actual extent of redistribution, especially via direct transfers to low-income individuals, should be considerably higher than what the present system offers. Because the crisis increased pre-tax inequality, the tax system should become more redistributive as a response to a crisis. We also demonstrate how a combination of a higher revenue requirement before the crisis and a lower revenue requirement after the crisis increases social welfare.

Key words: COVID-19 pandemic, optimal income taxation, numerical simulation, welfare analysis

JEL classification: H21

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1 Background

Optimal tax analysis is the study to examine how the government can achieve certain distributional objectives while at the same time keeping distortions created by taxation at a minimum. The analytical results stemming from optimal income tax analysis are fairly general (see, e.g., Mirrlees 1971). Much of the work has proceeded to provide simulation results, while few focus on developing countries. Simulation results are discussed, e.g., in Saez (2001) and Tuomala (2010) for income distributions calibrated to data from developed countries.

How should redistributive tax and transfer payment systems respond when income distribution in developing countries changes as a result of the crisis? The COVID-19 pandemic clearly increased public debt and changed the income distribution in many countries. Lastunen et al. (2021) analysed the effect of the COVID-19 pandemic on income distributions for five sub-Saharan African countries, and it has been shown that Zambia suffered the largest losses in disposable household incomes. The optimal tax rates are expected to differ for different income distributions. This paper analyses how optimal tax rates change due to COVID-19-related shocks in Zambia.

We use a numerical simulation approach to derive optimal nonlinear marginal tax rates for the pre-crisis and crisis periods. The simulation draws on a weighted and adjusted dataset in 2019–20, which was created by Lastunen et al. (2021).¹ Our analysis focuses on the formal sector of the economy, as only formal sector workers are liable for tax.

Our work contributes to the literature in two main ways. First, our results are informative about the desirable extent of redistribution in lower-middle-income countries in normal times. The analysis takes into account relevant features of a developing country, such as informality. Second, we assess the need for changes in tax and benefit policies in the wake of shocks.

The paper is structured as follows. Section 2 explains the theoretic framework, while Section 3 discusses our simulations approach. Section 4 presents the results, and Section 5 concludes.

2 A discrete two-period income tax model with exogenous revenue requirement

In this section we outline a model framework. Consider a model economy in which agents have the same preference ordering over consumption c and labour supply l . We assume that agents differ with respect to skills n , and the government cannot observe individuals' earning abilities. The before-tax income is defined as $z = nl$, and the tax-transfer schedule is a function of earning $T(z) = z - c$. The concave utility function is separable and specified as $u(c, l)$, with $u_c > 0 > u_l$ and $u_{ll} < 0$. Agents decide on their labour supply (or realized earning) to maximize the after-tax utility based on the tax scheme set by the government.

Suppose a population consists of N distinct skill types of agents indexed by the set $\mathbb{S} = \{1, 2, 3, \dots, N\}$. Moreover, we assume that the government faces an inter-temporal budget constraint in the pre-crisis year ($t = 1$) and the crisis year ($t = 2$). The type i agent endowed with skill n_{it} , and $n_{it} < n_{jt}$ if $i < j$ for all $i, j \in \mathbb{S}$ and $t = 1, 2$. The fraction of type i agents in the population in year t is p_{it} , and $\sum_i p_{it} = 1$. The type i agent chooses before-tax income z_{it} and consumes after-tax income c_{it} and gains pay-off $u(c_{it}, z_{it}/n_{it})$. Let $u^{it}(c_{it}, z_{it}) \equiv u(c_{it}, z_{it}/n_{it})$. The maximized value function for the type i agent is

$$v(n_{it}) = \max_{c_{it}, z_{it}} \{u^{it}(c_{it}, z_{it}) | T(z_{it}) = z_{it} - c_{it}\}. \quad (1)$$

¹ Lastunen et al. (2021) employ micro-level data that underpin the MicroZAMOD microsimulation model.

The government chooses a nonlinear tax-transfer scheme $T(z)$ to maximize the following social welfare function

$$W^t = \sum_i u^{it}(c_{it}, z_{it}) g_{it} p_{it}, \quad (2)$$

subject to the incentive compatibility constraints

$$\forall i, j \in S : u^{it}(c_{it}, z_{it}) \geq u^{it}(c_{jt}, z_{jt}), \quad (3)$$

and the government budget constraint

$$\sum_i (z_{it} - c_{it}) p_{it} \geq R_t, \quad (4)$$

where g_{it} is the Pareto weight that is attached to the welfare of individual i , and R_t is the exogenous revenue requirement. The incentive compatibility constraints are imposed to achieve a second-best equilibrium so that no one deviates from the income-consumption pair (z, c) designed for them, thereby the distortions created by taxation remain at minimum. The government collects positive revenue for public spending such as transfer programmes in order to achieve redistributive goals. The condition for pure income redistribution is $R_t = 0$.

In a two-period income tax model, the government collects revenue in the pre-crisis year and saves it for the crisis year such that the net government budget for the two periods is zero, i.e. $R_1 = -R_2$. The government maximizes social welfare for each period, and the total social welfare in two periods is

$$\widehat{W}^{total} = \widehat{W}^1 + \widehat{W}^2, \quad (5)$$

and

$$\widehat{W}^t = \sum_i v(n_{it}) g_{it} p_{it} \text{ for } t = 1, 2. \quad (6)$$

We can write the marginal tax rates formula from the first-order optimality condition:

$$T'(z_{it}) = \underbrace{\frac{\lambda_{it} u_{c_{it}}^{i+1,t}(c_{it}, z_{it})}{\gamma_t p_{it}}}_I \underbrace{(\theta(c_{it}, y_{it}, n_{it}) - \theta(c_{it}, y_{it}, n_{i+1,t}))}_{II}, \quad (7)$$

where λ_{it} is the multiplier on the i^{th} downward incentive compatibility constraint and γ_t is the multiplier on the government budget constraint. The marginal rate of substitution of income for consumption for type i agent at the point (c_{it}, z_{it}) of an indifference curve in year t is expressed as $\theta(c_{it}, z_{it}, n_{it})$. The implicit function theorem gives $MRS = -\frac{\partial u/\partial z}{\partial u/\partial c} = -\frac{\partial u/\partial l}{\partial u/\partial c} \frac{1}{n}$, which implies that the indifference curve is steeper for low-skill agents than high-skill agents, and the term II in Equation (7) is positive. Hence, marginal tax rates should be positive throughout the income distribution. In addition, Equation (7) indicates that the optimal tax rate for type i agent increases in the marginal utility of consumption and decreases in the population mass of that agent.

3 Numerical simulation

3.1 Model calibration

Our simulations apply several most commonly used model specifications to study change in progressivity of optimal tax rates in pre-crisis and crisis periods. We consider two types of utility functions. In the first, utility is quasi-linear in consumption

$$u(c, \frac{z}{n}) = c - \frac{\alpha}{1 + 1/e} \left(\frac{z}{n}\right)^{1+1/e}. \quad (8)$$

The second utility function is strictly concave and is expressed as:

$$u(c, \frac{z}{n}) = \log c - \frac{\alpha}{1 + 1/e} (\frac{z}{n})^{1+1/e}. \quad (9)$$

The compensated elasticity of labour supply is $e = 0.2$, and α is the scale parameter for the calibration of labour supply such that the median worker works full time. Equation (8) is used only to calibrate the skill distribution in the pre-crisis and crisis years, which will be elaborated in the next section. We use a strict concave utility function in the simulation (i.e. Equation (9)).

The social welfare function reflects the government's attitude towards inequality. We employ two of the most commonly used social welfare functions in the literature, namely utilitarian and Rawlsian social welfare functions. Specifically, for the utilitarian social welfare function, the government aims to maximize the sum of individual utilities and assigns equal welfare weights to the utilities of individuals. For the Rawlsian social welfare function, the government seeks to maximize the welfare of the worst-off agent.

In the simulation, we examine different revenue requirement scenarios.

Case I: In this case, the revenue requirements remain strictly positive and equal to the observed revenue requirements (approximately 10% of total income) both before and during the crisis.²

Case II: In this scenario, the revenue requirements are zero before and during the crisis.

Case III: In this scenario, the government's overall net revenue is zero before and during the crisis, indicating that the government does not generate any net revenue during this combined period.

In Cases I and III, there is no income transfer between the two periods, and the government faces the same required revenue before and during the crisis. The difference lies in Case I, where we use tax requirements similar to real-world conditions, which is 10% of total income, while in Case II, the government relaxes its fiscal demands, resulting in a total required revenue of 0. In Case III, the revenue requirement in the pre-crisis year is positive, but this revenue becomes a government subsidy in the crisis year to alleviate financial pressure. Our simulation allows for different levels of revenue requirements to capture various scenarios and their impacts.

3.2 Data and skill distribution

We utilize micro-level data during the COVID-19 pandemic period from the tax-benefit microsimulation model for Zambia (MicroZAMOD) (Nakamba-Kabaso et al. 2020).³ The primary data that underpin MicroZAMOD are from Zambia's 2015 Living Conditions Monitoring Survey (henceforth, LCMS). The key variable of interest is the income distribution. Our simulations employ two income distributions that were created by Lastunen et al. (2021), namely the pre-crisis income distribution from 2019 (before the COVID-19 pandemic) and the crisis income distribution from 2020. The income distribution is the sum of the employment and self-employment income distributions. According to Lastunen et al. (2021), the pre-crisis dataset was derived by weighting the original data with structural demographic changes before the pandemic. The crisis income distribution was obtained by randomly distributing the (negative) industry-level GDP shocks to individual earnings, i.e. adjustments for the COVID-19 crisis based on the pre-crisis income distribution.

² According to the summary statistics of the SOUTHMOD data, in Zambia during the years 2019 and 2020, the government's share of GDP in terms of revenue is approximately 10%. The revenue comes from different sources, including direct taxes, indirect taxes, and social security contributions.

³ MicroZAMOD is part of the SOUTHMOD project, initiated by UNU-WIDER, the EUROMOD team at the Institute for Social and Economic Research (ISER) at the University of Essex, and Southern African Social Policy Research Institute (SASPRI).

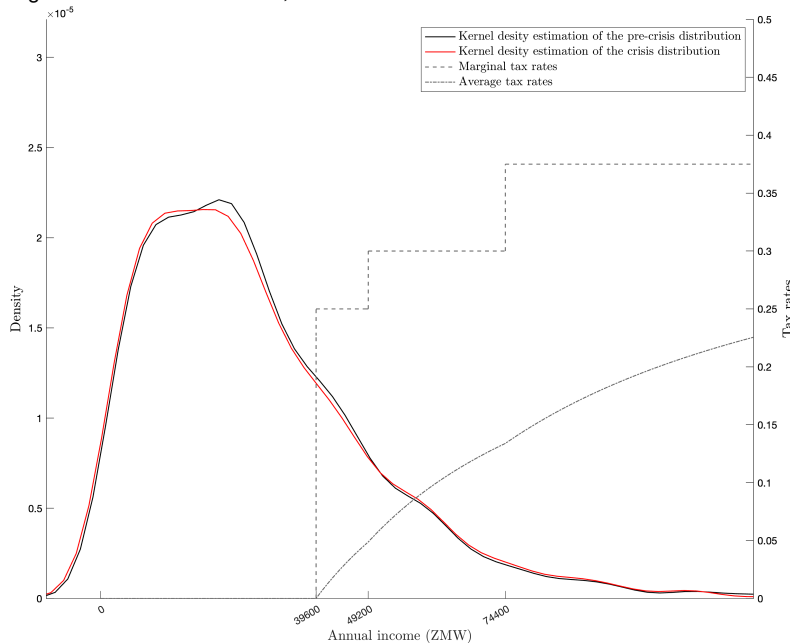
Table 1: Estimates for the fitted log-normal distribution

	Parameters (μ , σ)	Difference in mean	Difference in variance
Pre-crisis annual income distribution	(10.293, 1.281)	—	—
Crisis annual income distribution	(10.234, 1.319)	−0.9%	11%
Pre-crisis potential income distribution	(10.329, 1.310)	—	—
Crisis potential income distribution	(10.269, 1.347)	−1.0%	10%

Note: the potential income distribution (or skill distribution) is derived optimally using the actual incomes and empirical piece-wise linear tax rates. The third column measures the dispersion of the distributions. For example, the crisis income distribution is about 11% more dispersed than the pre-crisis income distribution. Source: authors' estimates based on LCMS data.

The data includes relevant labour market information for tax policy analysis such as formality of the employment. Our study solely examines the income of individuals employed in formal sectors, as they are legally required to fulfill their tax obligations. In addition, we take zero/low-income populations into account when simulating optimal tax rates. Individuals with zero income before the crisis and during the crisis account for 9.3% and 11% of the total population, respectively. We can then analyse how income should be optimally redistributed in the formal sector. To provide a more thorough evaluation of welfare, we incorporate data at the household level and adjust for equivalence scales. Furthermore, sample weights are employed to ensure the robustness and representativeness of the data construction process. The parameter estimates of the fitted log-normal income distributions are summarized in Table 1. When comparing the income distribution before and during the crisis, it is clear that there is a higher level of dispersion during the crisis, approximately 11% greater. In addition, Figure 1 depicts the income distributions through kernel density estimates, making it easier to see the contrasts between the two distributions before and during the crisis. To further compare the differences between the two income distributions, we report statistics on income inequality in Table 2. During the COVID-19 pandemic period, there was a noticeable increase in income inequality between the wealthiest individuals and those with the lowest incomes. Despite this, the overall level of income inequality did not show a significant difference. We do not expect significant changes in the optimal income tax rate pattern between the two periods.

Figure 1: Income distribution, Zambia



Source: authors' construction based on LCMS data.

Table 2: Income inequality

	$\frac{p_{90}}{p_{10}}$	$\frac{p_{90}}{p_{50}}$	$\frac{p_{10}}{p_{50}}$	$\frac{p_{75}}{p_{25}}$	Gini	A(0.5)	A(1)	A(2)
Pre-crisis income distribution	12.652	2.834	0.224	3.723	0.46749	0.18911	0.40339	0.93029
Crisis income distribution	14.018	2.916	0.208	3.979	0.47504	0.19576	0.41729	0.93671
Pre-crisis potential income distribution	13.899	3.113	0.224	3.998	0.48288	0.20031	0.42166	0.93475
Crisis potential income distribution	15.400	3.204	0.208	4.273	0.49065	0.20725	0.43557	0.94071

Source: authors' estimates based on LCMS data.

Once the income distributions are obtained, we can determine the skill (potential income) distribution for both the pre-crisis and crisis periods. The potential income distribution is derived using empirical piecewise linear tax rates and income distribution by inverting the labour supply first-order condition, i.e. $n = z/(1-t)^e$.⁴ As mentioned in Section 3.1, quasi-linear preference is only used to derive individual's skill distribution. We simulate optimal tax rates based on derived skill distribution and concave preference.⁵ We use the log-normal distribution to fit potential income distributions, and the estimated parameters are reported in Table 1. From the table, it becomes evident that the distribution of potential incomes exhibits a higher dispersion level during the crisis than the pre-crisis distribution. As expected, the difference in potential income distribution before and during the crisis is larger than in income distribution before and during the crisis. To create a set of discrete skill levels, we divide the potential income distribution into a set of equal proportional bins (i.e. each bin contains the same share of the population). The median value of each bin represents the potential income level of that bin, and we choose 200 skill types of agents. This potential income/skill distribution is then used as a primitive to simulate optimal tax rates.

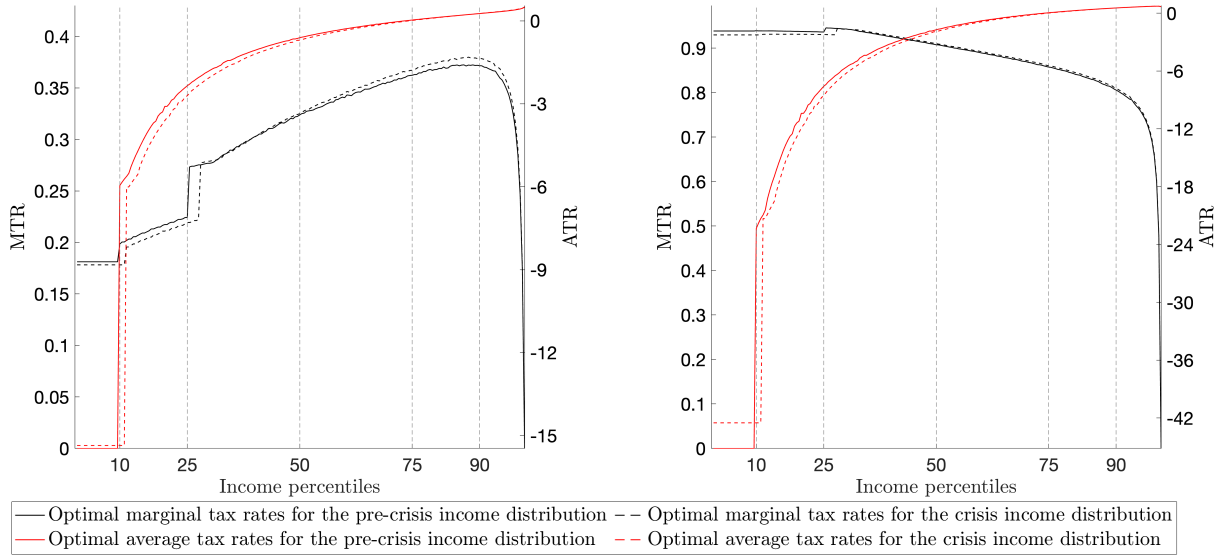
4 Results

Figure 2 illustrates the optimal marginal tax rates and average tax rates during the pre-crisis and crisis periods of the COVID-19 crisis in Zambia, considering both utilitarian and Rawlsian social welfare functions with a government tax revenue requirement of 10% (i.e. Case I). Under the utilitarian social welfare function, we obtain an optimal progressive tax system, where the average tax rates increase with income. In general, there is no significant difference in the average tax rates before and during the crisis. The graph illustrates that when we include individuals with zero income in the population, they face an optimal average tax rate that is considerably low. Specifically, the average tax rate for those with zero income is lower than for individuals with slightly higher skills (the next skill type of the agent) by approximately -140% to -130% . For instance, the optimal average tax rate for around 9% of the population before the crisis is substantially low. During the crisis, the optimal average tax rate for approximately 11% of the population is also remarkably low, precisely matching the proportion of individuals with no income in both periods. A similar pattern emerges when examining the optimal marginal tax rates. Additionally, we notice that during the crisis period, the optimal marginal tax rates for low-income individuals should be lower than the pre-crisis period, while high-income individuals should face higher optimal marginal tax rates. The optimal marginal tax rate for high-income individuals can reach approximately 37.5% (the difference between the top marginal tax rates before the crisis and the crisis period is less than 1%), which aligns remarkably well with the actual tax rates.

⁴ In this standard expression, t is the actual tax rate and e is the (compensated) elasticity of taxable income (see, e.g., Saez 2010). Moreover, we did not observe any significant bunching around kink points in Figure 1.

⁵ If quasi-linear preferences are employed, there will be a lack of income redistribution.

Figure 2: Optimal tax rates for the pre-crisis and crisis income distributions (with 10% of the revenue requirement)



Note: left—utilitarian social welfare function; right—Rawlsian social welfare function.

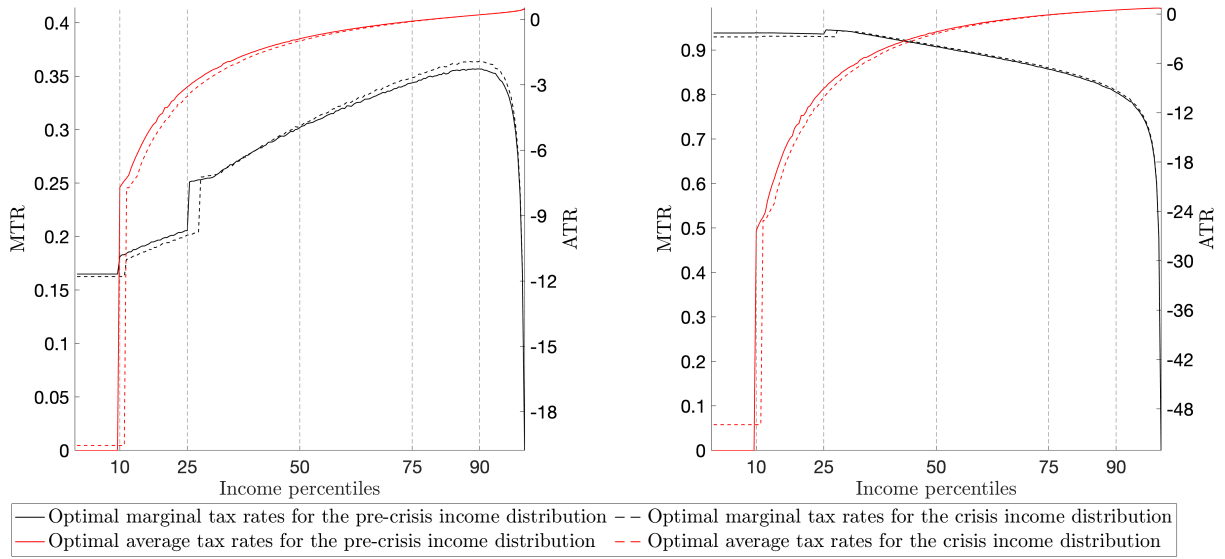
Source: authors' construction based on LCMS data.

When using the Rawlsian social welfare function, the general pattern of marginal tax rates tends to be downward sloping. Optimal (marginal and average) tax rates before and during crises do not exhibit significant differences, especially for individuals with medium to high incomes. The average tax rate for individuals with zero income is currently around -90% to -70% , as compared to those with slightly higher skills (the next skill type of the agent). Compared to the utilitarian social welfare function, the overall marginal tax rates are higher under the Rawlsian social welfare function, due to the government's extreme focus on the welfare of the worst-off individuals in such circumstances. Meanwhile, we observe the highest tax rates for the lowest income individuals. High marginal tax rates at lower income levels are designed to allow only low-income individuals to access various types of social assistance while discouraging high-income individuals from reducing their work hours and applying for these transfer payments. The high tax rates at the lower end, as obtained in the simulation, correspond to the phased-out guarantee income levels. Simulation results indicate that the government should impose high tax rates at the lower end to provide welfare benefits only to the low-income population. Similar to the results of Saez (2001), as in many countries, once welfare programmes are phased out, tax rates gradually decrease. The optimal marginal tax rate for high-income earners (the top 10% of income earners) is approximately 80%, close to the revenue-maximizing tax rate.⁶

Figure 3 illustrates the optimal tax rates under the scenario where all tax revenues are used for income redistribution (i.e. Case II: tax revenue equals zero). We observe that there is not a significant difference in the optimal tax rates between the utilitarian and Rawlsian social welfare function cases compared to situations with tax requirements. Aside from a minor parallel decline in the optimal marginal tax rates, for instance, the highest marginal tax rate only slightly decreases from $37\% \sim 38\%$ down to $36\% \sim 37\%$ when using the utilitarian social welfare function. This pattern also holds for Rawlsian social welfare functions. A more pronounced observation is that the optimal average tax rates for low-income individuals are lower when there is no revenue requirement, reflecting that reducing government's tax revenue more effectively benefits the poor based on Zambia's income distribution.

⁶ If an unbounded distribution at the top were assumed, the revenue-maximizing tax rates, $\tau^* = 1/(1 + a \cdot e)$, would amount to 83% for both the pre-crisis and crisis income distributions.

Figure 3: Optimal tax rates for the pre-crisis and crisis income distributions (with zero revenue requirement)



Note: left—utilitarian social welfare function; right—Rawlsian social welfare function.

Source: authors' construction based on LCMS data.

4.1 Inter-temporal revenue transfers: implications for social welfare

Does adjusting the government's revenue requirements across different periods to alleviate fiscal pressure during times of crisis enhance overall inter-temporal social welfare? Specifically, the government can save money through taxation during normal periods, which allows it to reduce financial demands or offer subsidies in times of crisis. This exercise analyses the usefulness of revenue adjustments in pre-crisis and crisis periods. It highlights the importance of considering how these adjustments affect inter-temporal social well-being.

Table 3 reports how the government's required revenue affects social welfare in pre-crisis and crisis periods, and the revenue requirement ranges from 0 to 20%. Changes in welfare are benchmarked against social welfare without transfers between two periods and are expressed as relative changes. Again, the simulations employ a concave utility function.

When using fitted parametric estimation to estimate income distribution, there is a limitation in preserving the difference in mean between pre-crisis and crisis income distributions. The mean of crisis incomes created from the fitted log-normal income distribution is approximately 1.6% smaller than the mean of the fitted pre-crisis income distribution, while the mean of the actual crisis distribution is about 5% smaller than the mean of the pre-crisis distribution. To mitigate this problem, we use an adjusted (or synthetic) crisis income distribution in computing the welfare change for different transfer amounts. This adjusted crisis income distribution is created using log-normal estimation with the level of dispersion as the fitted parametric distribution from Table 1, but with a smaller mean, i.e. 5% smaller than the mean of the pre-crisis distribution that is the same as the actual difference. In addition, we also compute the welfare change using the fitted crisis distribution from Table 1, and we elaborate more in Appendix A1.

The upper segment of the table shows equal fiscal transfers conducted from the pre-crisis to the crisis period, where the net revenue remains zero for the two periods combined (i.e. Case III). The result shows that an appropriate inter-temporal transfer would enhance overall social welfare in two periods. The maximum increase in social welfare occurs when the transfers amount to 1% of the total income prior to the crisis.

Table 3: Adjusted crisis skill distribution with a smaller mean

Revenue requirement		Relative change in total social welfare in two periods	
Pre-crisis period, crisis period	Net revenue requirement	Utilitarian social welfare function	Rawlsian social welfare function
0%, 0%	0%	-	-
0.5%, -0.5%	0%	0.00076%	0.00073%
1%, -1%	0%	0.00113%	0.00105%
2%, -2%	0%	0.00074%	0.00042%
3%, -3%	0%	-0.00117%	-0.00197%
5%, -5%	0%	-0.00958%	-0.01201%
10%, -10%	0%	-0.05723%	-0.06759%
20%, -20%	0%	-0.26869%	-0.31081%
----- 5%, 0% -----	5%	-2.692%	-3.290%
0%, -5%	-5%	2.682%	3.278%
10%, -5%	5%	-2.713%	-3.314%
5%, -10%	-5%	2.646%	3.235%
10%, 0%	10%	-5.395%	-6.592%
0%, -10%	-10%	5.338%	6.525%
20%, -10%	10%	-5.509%	-6.720%
10%, -20%	-10%	5.183%	6.342%
20%, 0%	20%	-10.8466%	-13.2447%
0%, -20%	-20%	10.5779%	12.9339%

Note: the revenue requirement is expressed as a percentage of the total pre-crisis income. The symbols (+) and (-) in the first column signify the government's tax collection prior to the crisis and subsequent transfers to households during the crisis. The final two columns depict variations in social welfare, presented as percentages, in comparison to the scenario without government revenue requirements (Case II). The model utilizes a concave utility function.

Source: authors' estimates based on LCMS data.

The result from Appendix Table A1 indicates that when the difference in the mean of the income distribution is very small (1.6%, as mentioned above), any fiscal transfers from the pre-crisis to the crisis period would reduce total social welfare. We also conducted simulations with increased shocks during the crisis and found that a larger transfer amount is necessary in such cases, for instance, a 3% inter-group transfer for a mean difference larger than 15% between the pre-crisis and crisis distributions. This highlights the necessity of inter-temporal fiscal transfers in alleviating the fiscal pressures associated with more severe crises.

The lower segment of the table presents non-equal fiscal transfers conducted from the pre-crisis to the crisis period. The impact of inter-temporal tax revenue on social welfare can be further decomposed into two components: reduced social welfare due to pre-crisis taxation and increased social welfare due to subsidies during the crisis. The results indicate that, from a social welfare perspective, the (larger) subsidies provided during the crisis, at an equivalent level, are insufficient to offset individuals' tax burden before the crisis. Lastly, the simulation results remain consistent, regardless of choice between utilitarian and Rawlsian social welfare functions.

4.2 Inter-temporal revenue transfers: implications for optimal tax rates

From the analysis in the previous section, we observe that inter-temporal transfers can, to some extent, enhance overall social welfare in two periods, especially when the impact during the crisis is substantial. Larger fiscal transfers are then needed to increase social welfare. However, how does such fiscal transfer affect the optimal redistribution of income for each period individually, and which section of the population is more significantly influenced by the fiscal transfer? In this section, we illustrate the impact of inter-temporal transfers on the optimal tax rates for each respective period.

Figure 4 presents optimal marginal and average tax rates for the case with 1% of the inter-temporal revenue transfer from the pre-crisis to the crisis period. The left panel employs a utilitarian function,

from which we observe that a 1% fiscal transfer by the government from the pre-crisis period to the crisis period does not alter the general forms of the optimal (marginal and average) tax rates, both in terms of their patterns and numerical values. In other words, increasing welfare through fiscal transfers does not impact the shape of the optimal redistributive tax rates within each respective period. However, this result may not hold under larger fiscal transfer scenarios. As discussed in the previous section, we know that larger crises require greater fiscal transfers to maximize social welfare, and we present in the appendix how the optimal tax rates change with different inter-temporal transfers.

Furthermore, despite observing from the graph that the optimal tax rates for the majority of individuals do not change significantly between the two periods, we can still observe that fiscal transfers are more prevalent among the low-income population (lowest 10%). For instance, fiscal transfers would raise (lower) the minimum average tax rates for the low-income group before the crisis (during the crisis). Similar conclusions are drawn when using a Rawlsian function: a noticeable impact on the tax rates of the low-income group is observed when the inter-temporal transfer is 1%.

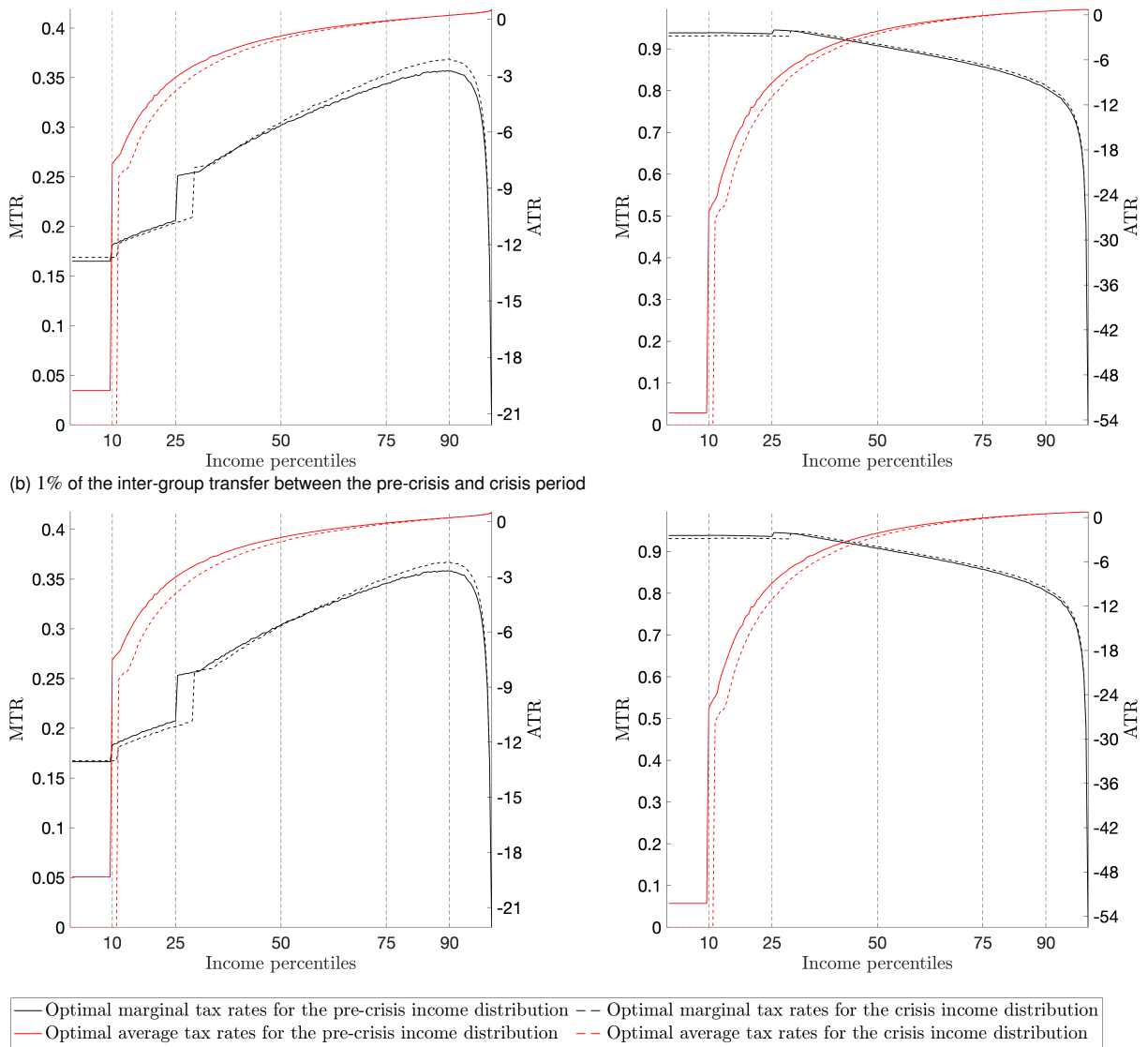
The optimal marginal and average tax rates for different required revenues are plotted in Appendix Figure B1. For utilitarian social welfare functions, we observe that with an increase in inter-temporal transfer payments from pre-crisis to crisis periods, the overall optimal marginal tax rates during the crisis period move downward in parallel. This is more pronounced in reducing average tax rates for low-income and middle-income groups during the crisis period as transfer payments increase. However, of concern is that the tax burden resulting from transfers is primarily borne by the low-income population from the pre-crisis period.

In the case of the Rawlsian social function, the impact of inter-temporal transfer payments on the optimal marginal tax rate can be neglected. The overall average tax rate before the crisis (during the crisis) increases (decreases) with an increase in inter-temporal transfer, with this difference being primarily evident among low-income and middle-income groups.

Regardless of the government's attitude towards inequality (i.e. utilitarian or Rawlsian social welfare function), inter-temporal transfers do not significantly affect the general shape of the optimal marginal tax rate; instead, more variations occur in the average tax rate. Inter-temporal transfers do not substantially change income redistribution within one period; instead, they mainly impact the average tax rates for low-income and middle-income individuals.

In summary, certain fiscal transfers can enhance overall social welfare across both periods. Specifically, a larger proportion of these fiscal transfers occurs within the low-income population. In other words, individuals with low skills before the crisis need to assist those with similar challenges during the crisis.

Figure 4: Optimal tax rates for the pre-crisis and (adjusted) crisis income distributions
(a) No inter-group transfer between the pre-crisis and crisis period



Note: left—utilitarian social welfare function; right—Rawlsian social welfare function.

Source: authors' construction based on LCMS data.

5 Conclusion

This paper investigated numerically, using the methods in the optimal income tax literature, the desirable level and pattern of income taxes and direct transfers using data from a low-income country, Zambia. The simulations were conducted for two distributions of income: a pre-crisis distribution and a crisis distribution, where the latter depicts the income distribution in the country during the coronavirus crisis.

Our research offers two contributions to the existing literature. Firstly, it highlights the need for a nuanced approach to income redistribution within lower-middle-income nations during stable periods, taking into account complex factors such as informal economic activity. Secondly, we show the importance of adaptable tax and benefit policies in confronting unforeseen crises, exemplified by the COVID-19 pandemic.

Our findings demonstrate that optimal tax rates should be dynamically adjusted in response to evolving income distributions. During crises, our analysis supports the reduction of the tax rates on low-income individuals while imposing higher rates on similar individuals in better economic times. This approach aligns with the principles of progressive taxation and the goal of mitigating economic disparities.

In summary, our study highlights the importance of flexible tax policies that can adapt to the economic circumstances typical of developing countries that experience significant changes in income distribution during crises. The insights we generate can be a valuable resource for policy-makers seeking to develop resilient and equitable tax and benefit systems that promote economic stability and social equity in the face of uncertainty.

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Appendix

A Different inter-temporal transfers between pre-crisis and crisis periods

A1 Impact of inter-temporal government transfers on social welfare: pre-crisis and crisis perspectives

Table A1 reports how social welfare evolves in response to the government's required revenue in pre-crisis and crisis periods. Changes in welfare are benchmarked against social welfare without transfers between two periods and are expressed as relative changes. Again, the simulations employ a concave utility function. Recall that Case II involves no income requirements in pre-crisis and crisis periods. The upper segment of the table shows equal fiscal transfers conducted from the pre-crisis to the crisis period, where the net revenue remains zero for the two periods combined. The overall social welfare (spanning both periods) decreases as transfers increase. This implies that individuals are not inclined to contribute more taxes for future precautionary purposes.

Table A1: Actual crisis skill distribution

Revenue requirement		Relative change in total social welfare in two periods	
Pre-crisis period, crisis period	Net revenue requirement	Utilitarian social welfare function	Rawlsian social welfare function
0%, 0%	0%	-	-
0.5%, -0.5%	0%	0.000%	0.000%
1%, -1%	0%	-0.001%	-0.001%
2%, -2%	0%	-0.003%	-0.004%
3%, -3%	0%	-0.006%	-0.008%
5%, -5%	0%	-0.018%	-0.022%
10%, -10%	0%	-0.073%	-0.085%
20%, -20%	0%	-0.295%	-0.342%
----- 5%, 0% -----	5% -----	-2.654% -----	-3.246% -----
0%, -5%	-5%	2.636%	3.98%
10%, -5%	5%	-3.00%	-3.98%
5%, -10%	-5%	3.00%	3.98%
10%, 0%	10%	-6.01%	-7.95%
0%, -10%	-10%	6.01%	7.95%
20%, -10%	10%	-5.89%	-7.76%
10%, -20%	-10%	5.86%	7.76%
20%, 0%	20%	-11.89%	-15.71%
0%, -20%	-20%	11.86%	15.71%

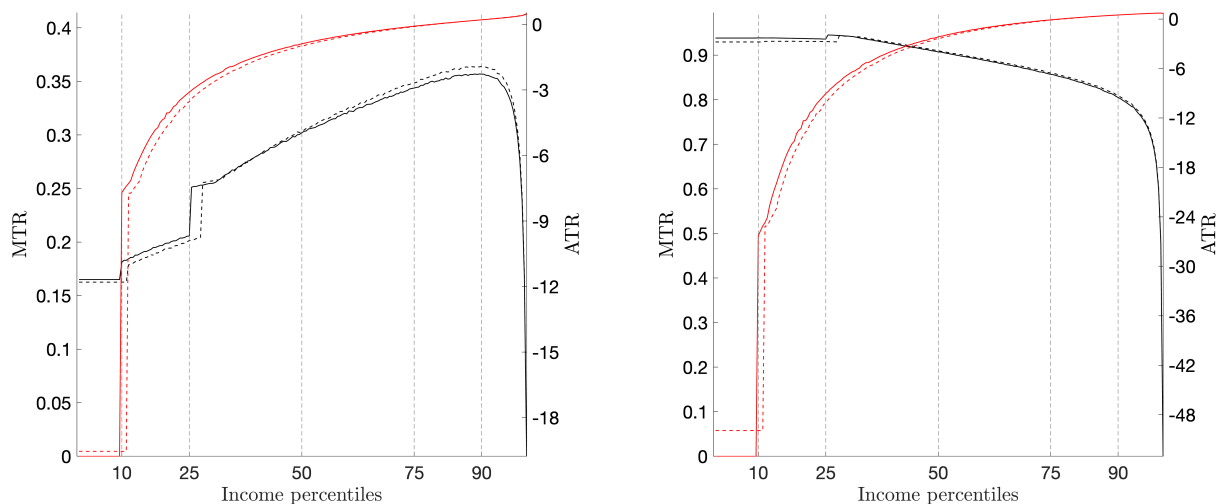
Note: the revenue requirement is expressed as a percentage of the total pre-crisis income. The symbols (+) and (-) in the first column signify the government's tax collection prior to the crisis and subsequent transfers to households during the crisis. The final two columns depict variations in social welfare, presented as percentages, in comparison to the scenario without government revenue requirements (Case II). The model utilizes a concave utility function.

Source: authors' estimates based on LCMS data.

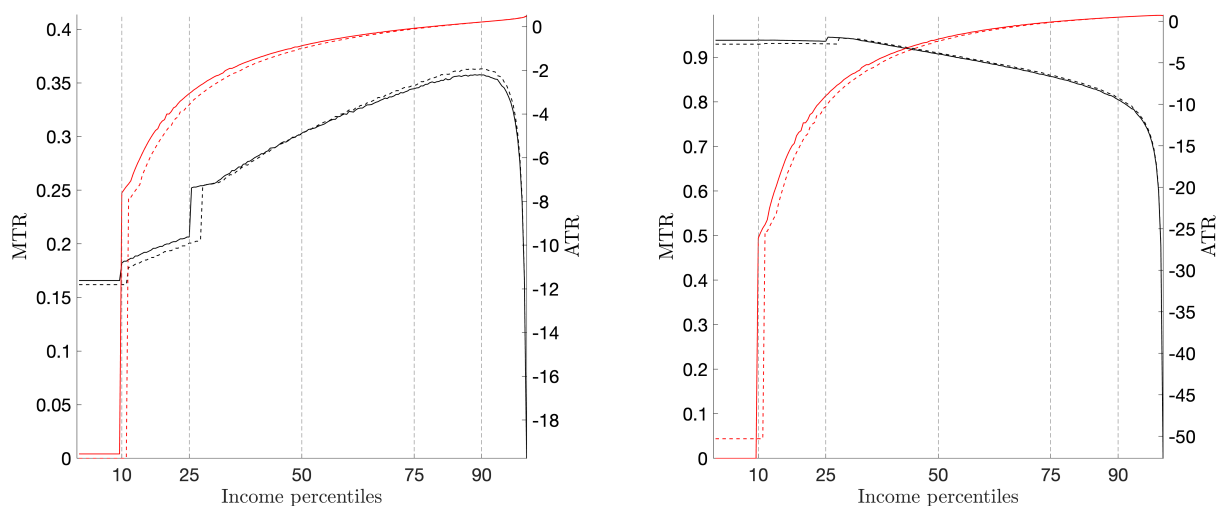
A2 Optimal marginal and average tax rates for different inter-temporal transfers between pre-crisis and crisis periods

Figure A1: Optimal tax rates for different inter-temporal government revenue requirements

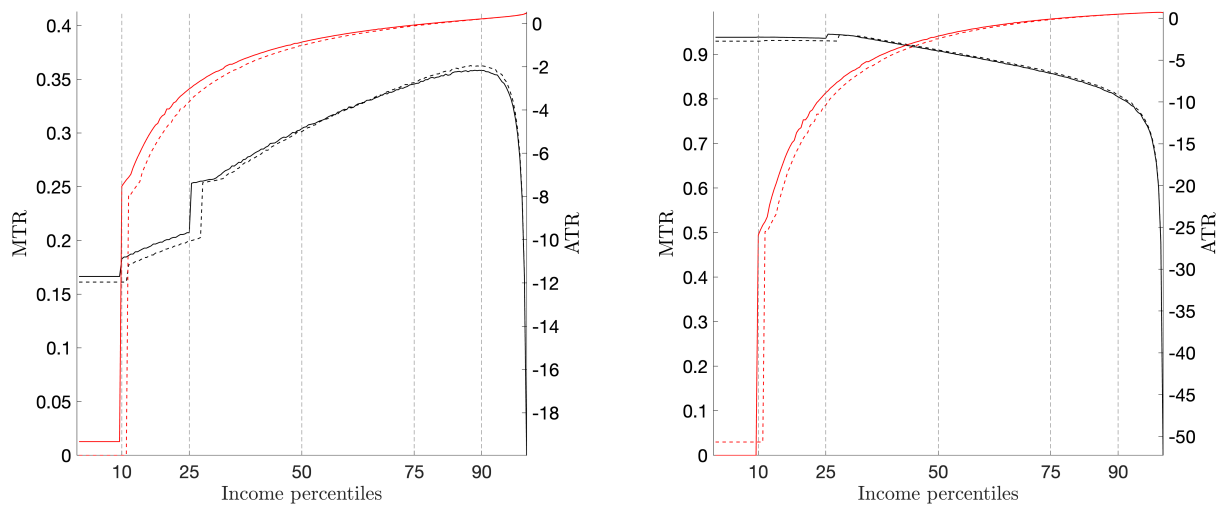
(a) 0% of total income in the pre-crisis period is transferred to the crisis period



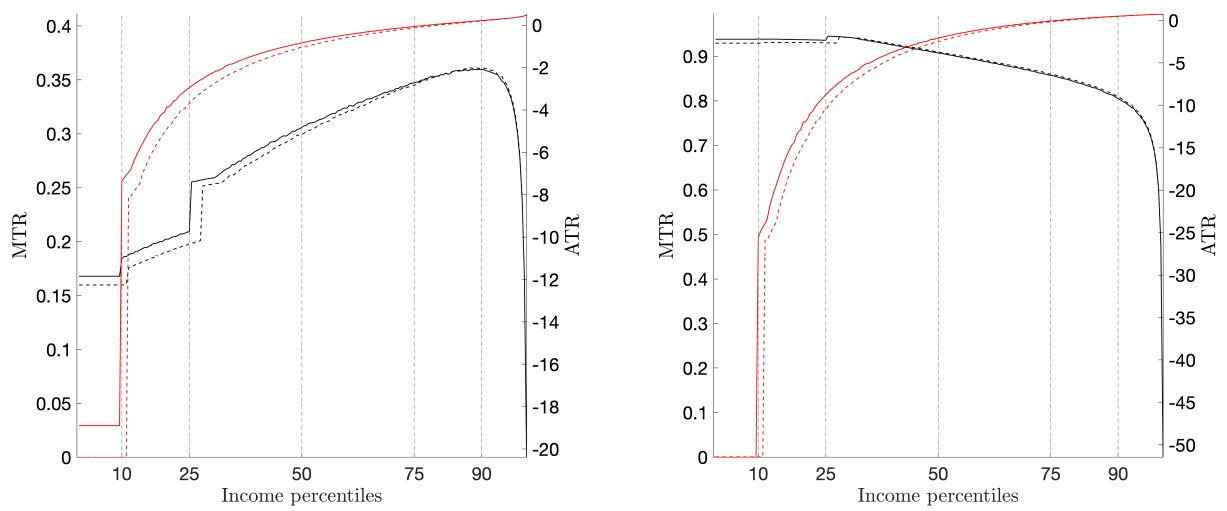
(b) 0.5% of total income in the pre-crisis period is transferred to the crisis period



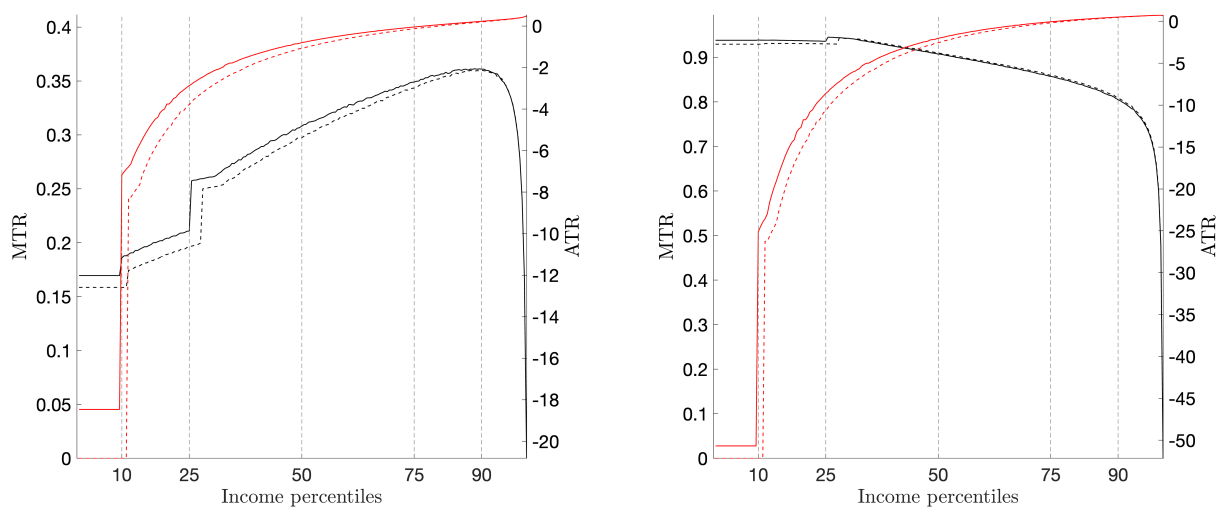
(c) 1% of total income in the pre-crisis period is transferred to the crisis period



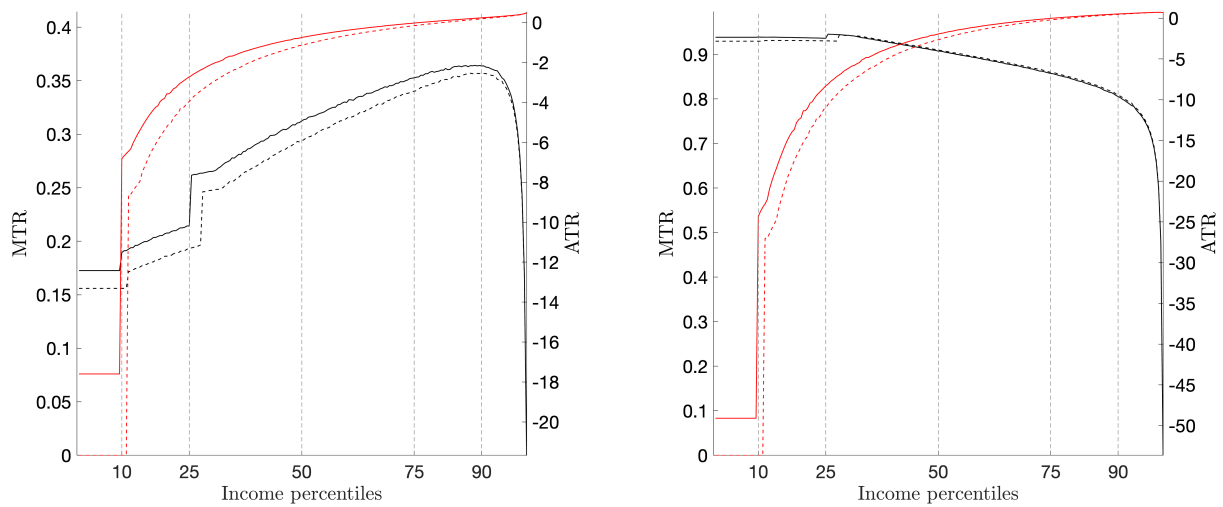
(d) 2% of total income in the pre-crisis period is transferred to the crisis period



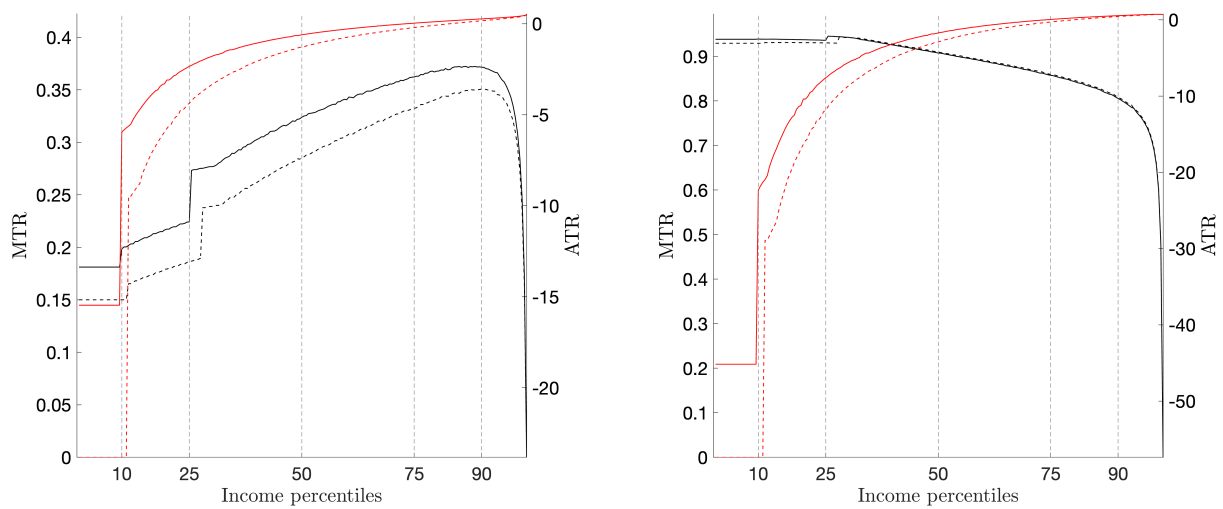
(e) 3% of total income in the pre-crisis period is transferred to the crisis period



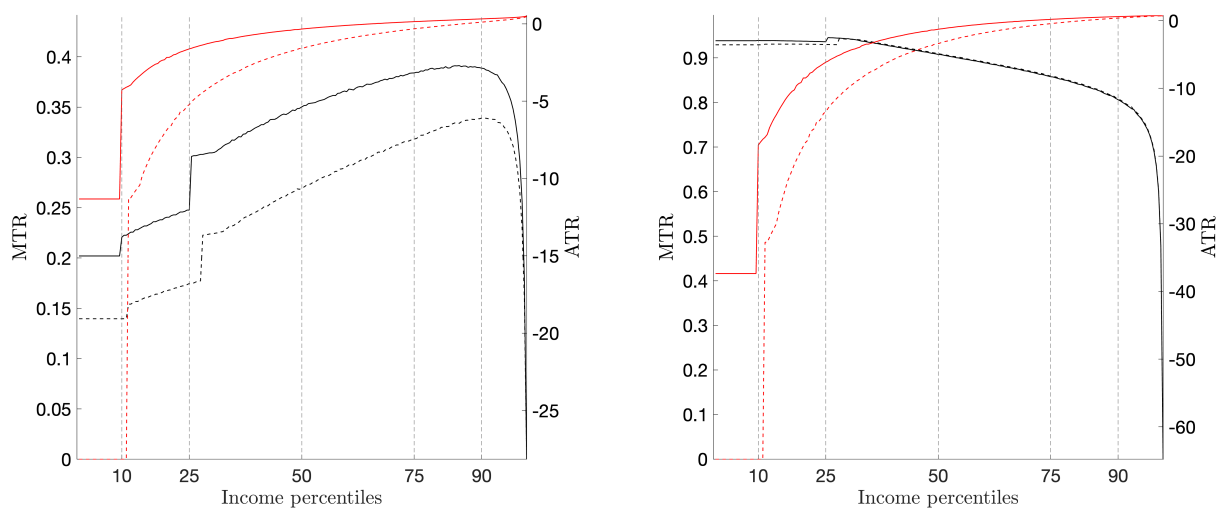
(f) 5% of total income in the pre-crisis period is transferred to the crisis period



(g) 10% of total income in the pre-crisis period is transferred to the crisis period



(h) 20% of total income in the pre-crisis period is transferred to the crisis period



— Optimal marginal tax rates for the pre-crisis income distribution -- Optimal marginal tax rates for the crisis income distribution
 — Optimal average tax rates for the pre-crisis income distribution -- Optimal average tax rates for the crisis income distribution

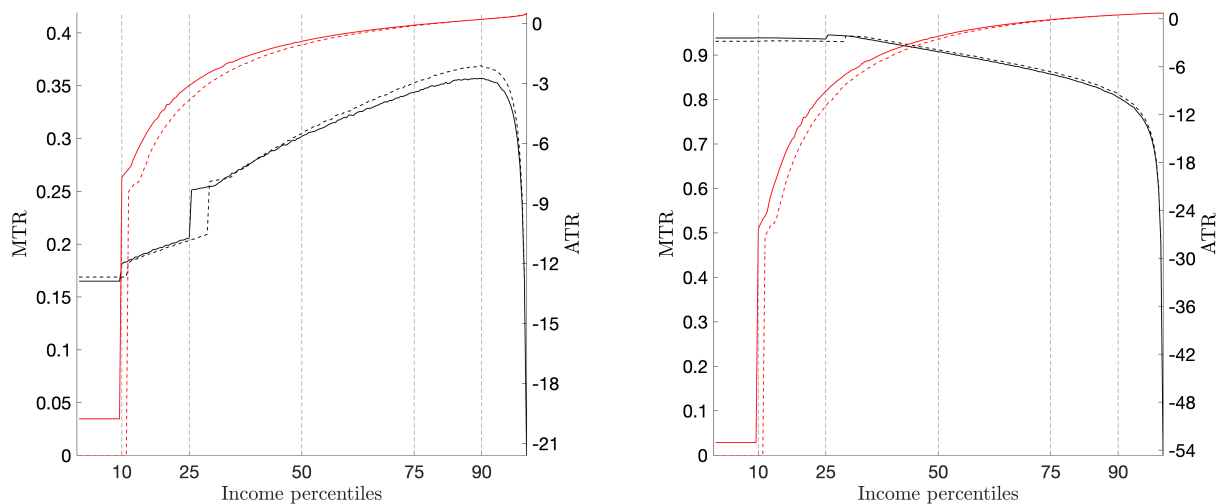
Note: left—utilitarian social welfare function; right—Rawlsian social welfare function.

Source: authors' construction based on LCMS data.

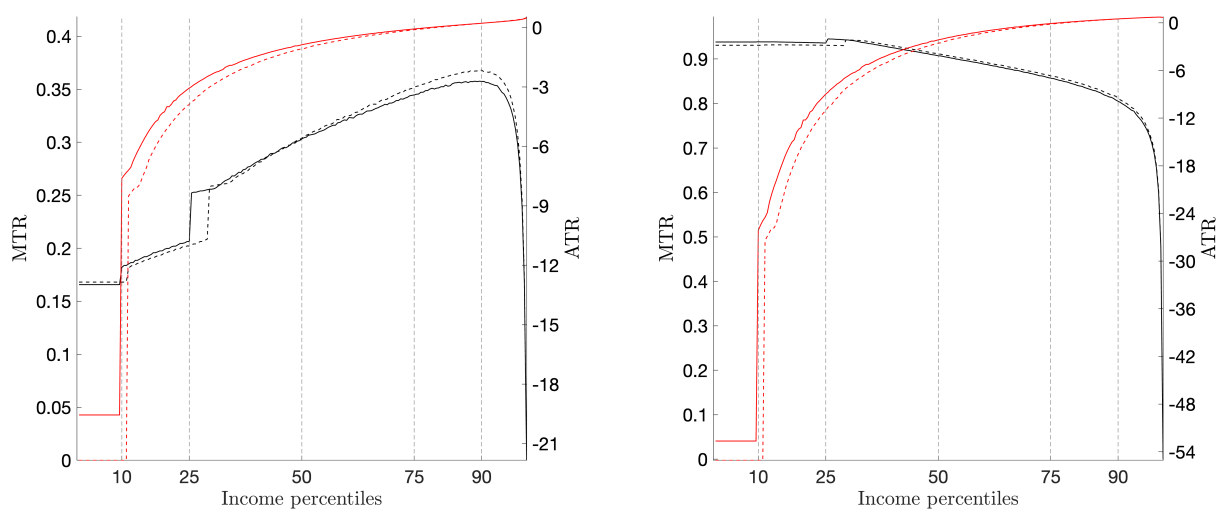
B Optimal marginal and average tax rates for different inter-temporal transfers between pre-crisis and (adjusted) crisis periods

Figure B1: Optimal tax rates for different inter-temporal government revenue requirements

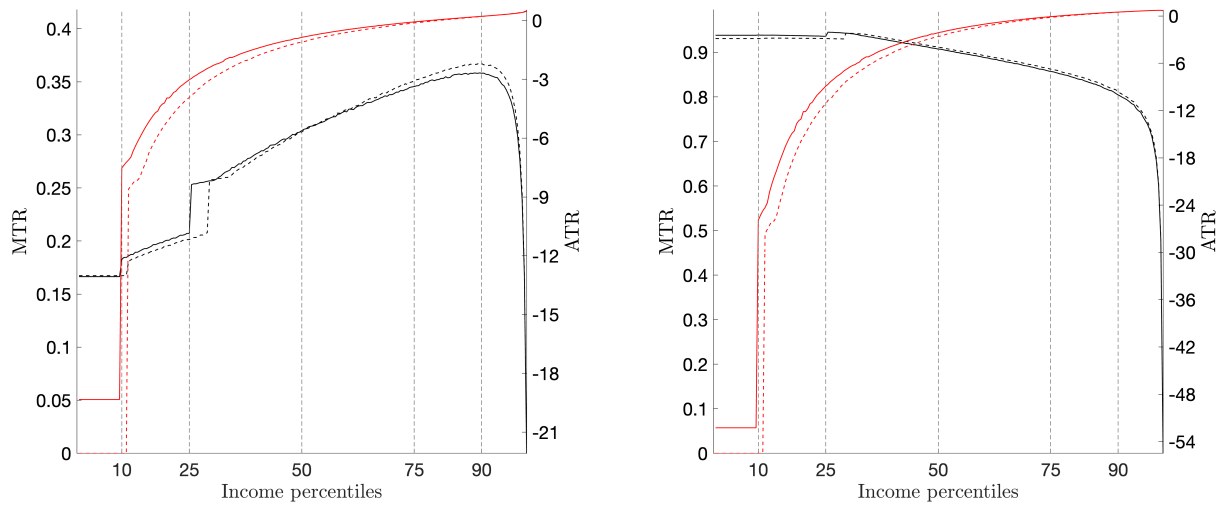
(a) 0% of total income in the pre-crisis period is transferred to the crisis period



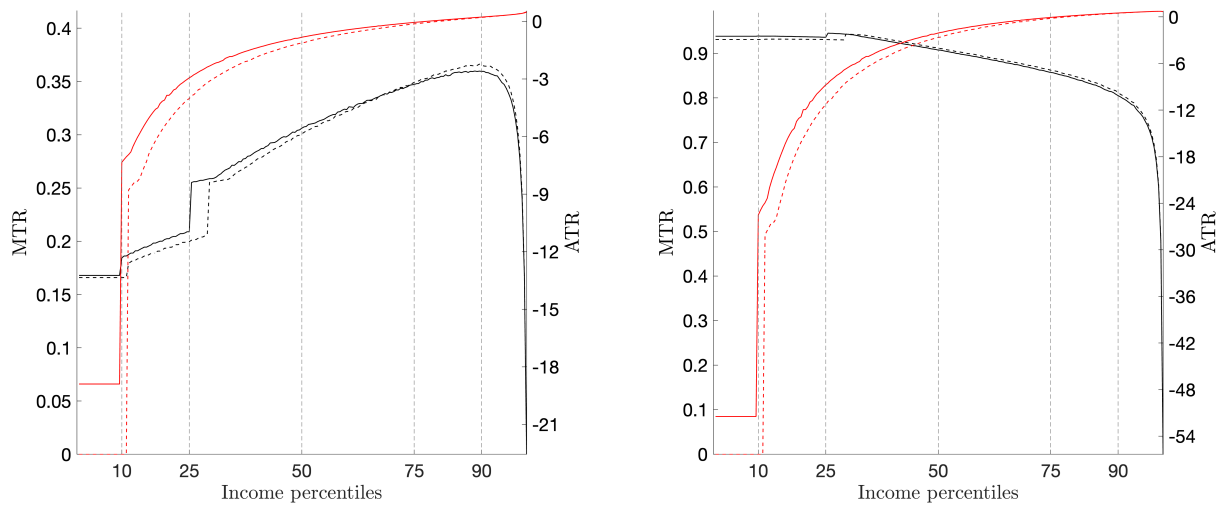
(b) 0.5% of total income in the pre-crisis period is transferred to the crisis period



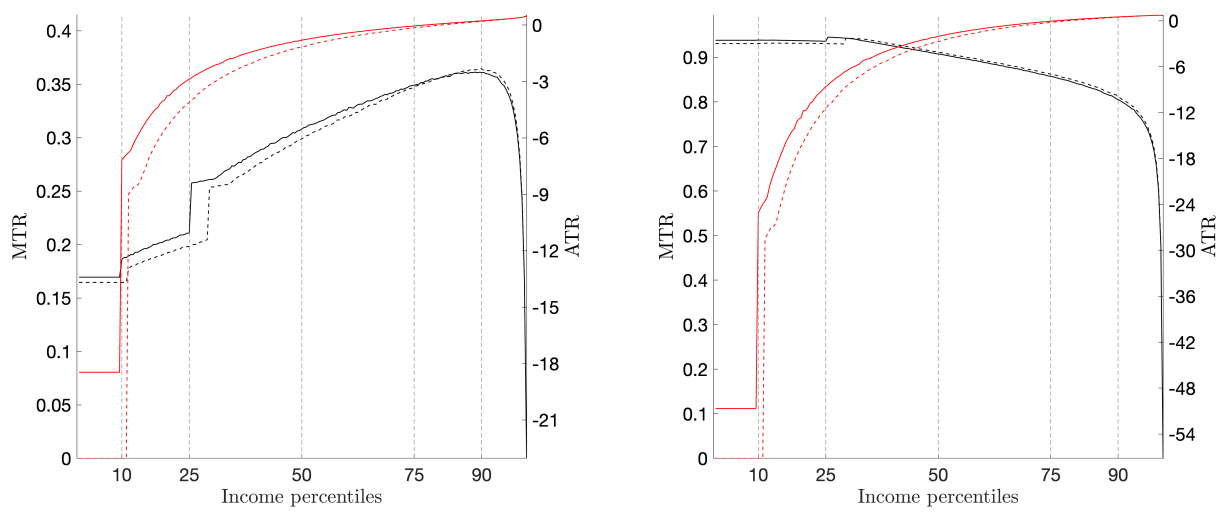
(c) 1% of total income in the pre-crisis period is transferred to the crisis period



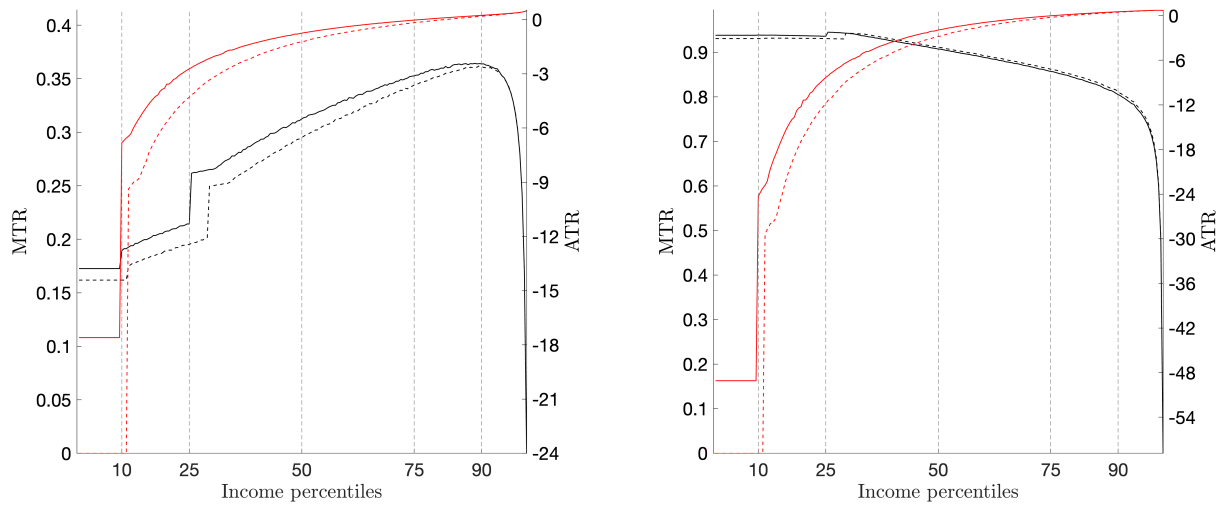
(d) 2% of total income in the pre-crisis period is transferred to the crisis period



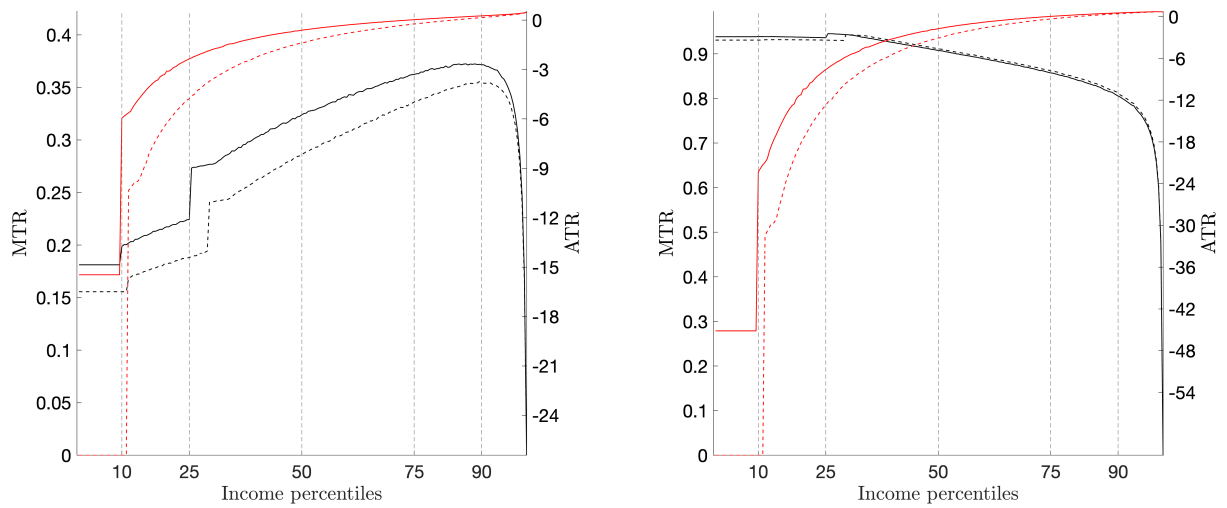
(e) 3% of total income in the pre-crisis period is transferred to the crisis period



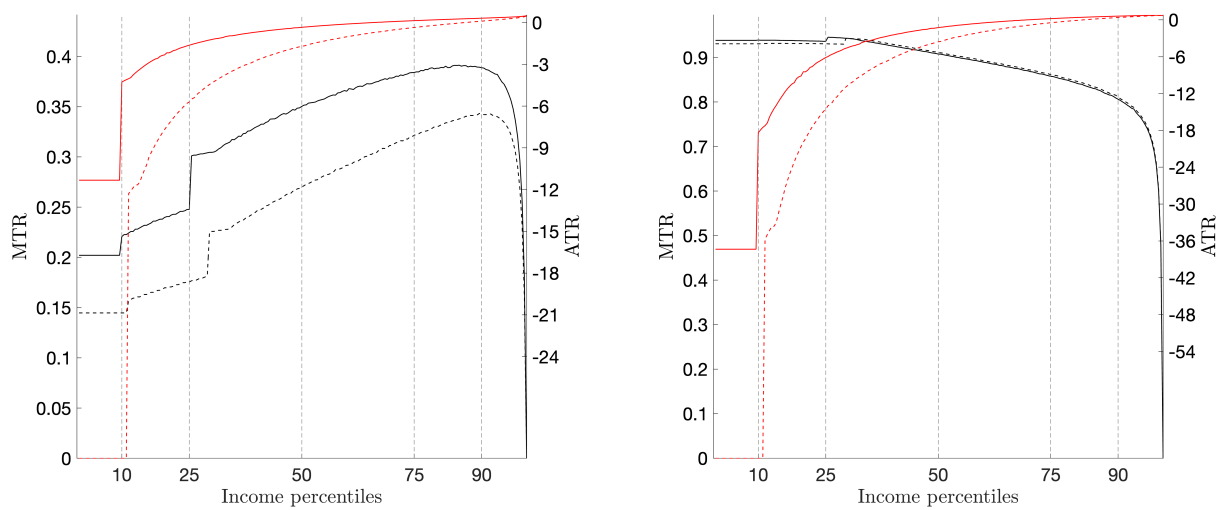
(f) 5% of total income in the pre-crisis period is transferred to the crisis period



(g) 10% of total income in the pre-crisis period is transferred to the crisis period



(h) 20% of total income in the pre-crisis period is transferred to the crisis period



— Optimal marginal tax rates for the pre-crisis income distribution -- Optimal marginal tax rates for the crisis income distribution
— Optimal average tax rates for the pre-crisis income distribution -- Optimal average tax rates for the crisis income distribution

Note: left—utilitarian social welfare function; right—Rawlsian social welfare function.

Source: authors' construction based on LCMS data.