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**Stability under Learning of
Equilibria in Financial Markets
with Supply Information**

by

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Stability under Learning of Equilibria in Financial Markets with Supply Information

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Abstract

In a recent paper Ganguli and Yang [2009] demonstrate, that there can exist multiple equilibria in a financial market model á la Grossman and Stiglitz [1980] if traders possess private information regarding the supply of the risky asset. The additional equilibria differ in some important respects from the usual equilibrium of the Grossman–Stiglitz type which still exists in this model. This note shows that these additional equilibria are always unstable under learning. This is true for both educative learning following Guesnerie [2002] and adaptive learning via least-squares estimation (cf. Marcet and Sargent [1988] or Evans and Honkapohja [2001]). Regarding the original Grossman–Stiglitz type equilibrium, the stability results are less clear cut, since this equilibrium might be unstable under educative learning while it is always stable under adaptive learning.

Keywords: Recursive Least Squares Learning, Educative Stability, Rational Expectations, Private Information

JEL-Classification: D82, D83, C62

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1 Introduction

In a recent paper Ganguli and Yang [2009] demonstrate, that there can exist multiple equilibria in a financial market model á la Grossman and Stiglitz [1980] if traders possess private information regarding the supply of the risky asset. The informational properties of the additional equilibria differ from the usual Grossman–Stiglitz like equilibrium which still exists in this model.

As usual in case of multiple equilibria, the question arises whether or not there exists a plausible selection device which implies that traders indeed coordinate on these additional equilibria. One important selection device asks whether or not a specific equilibrium is stable under learning. Discussing this briefly, Ganguli and Yang [2009] note that the static setup of their model doesn't allow for such an analysis as learning processes are inherently dynamic.

This note argues against this view. Not only do there exist concepts of learning that are applicable to static models. It is moreover possible to put the model of Ganguli and Yang [2009] into a framework which makes it possible to analyse real-time adaptive learning processes. Using the concepts of 'eductive learning' tracing back to Guesnerie [2002] and adaptive learning via least-squares estimation following Marcet and Sargent [1988] or Evans and Honkapohja [2001] it is shown that the additional equilibria described by Ganguli and Yang [2009] are always unstable under learning. Thus, using stability under learning as a selection device, we should disregard these additional equilibria, because it is unlikely that traders will coordinate on them. Regarding the original Grossman–Stiglitz type equilibrium, we get no clear cut stability results, since this equilibrium might be unstable under eductive learning while it is always stable under adaptive learning.

2 A financial market model with supply information

There is a continuum of traders $i \in I = [0, 1]$ and each trader is endowed with \bar{x} units of the riskless asset and $\bar{z}(i)$ units of a risky asset. The riskless asset yields 1 unit, the risky asset β units of a single consumption good, where β is unknown and drawn from a normal distribution with mean $\bar{\beta}$ and precision τ . Traders possess private information regarding the return of the risky asset, but since aggregate supply of the stock is stochastic too, the REE price of the asset will not be fully revealing. Each trader observes a private signal $s(i) = \beta + u(i)$ that informs about β . Here $u(i)$ is for all i an independent and normally distributed random variable with zero mean and precision τ_u . The endowment of a trader with the risky asset is given by $\bar{z}(i) = \bar{z} + \varepsilon + \eta(i)$, where $\eta(i)$ is an idiosyncratic shock, which is normally distributed with zero mean and precision τ_η . The common shock ε to the aggregate supply of the stock is also normally distributed with zero mean and precision τ_ε . Using the riskless asset as numeraire and with p denoting the price of the risky asset as well

as z_i denoting the demand of the risky asset of trader i , his final wealth $W_{1,i}$ is:

$$W(i) = \bar{x} + p\bar{z}(i) + z(i)[\beta - p]$$

Each trader maximizes the expected utility of his final wealth $W(i)$, where the utility function exhibits constant absolute risk aversion $0 < \gamma < \infty$ for all $i \in I$. A trader's asset demand $z(i)$ in this model is conditioned on his private signal $s(i)$ regarding the asset return, his information regarding the aggregate supply of the stock contained in $\bar{z}(i)$ as well as the current asset price p . Optimal asset demand of trader i then results as:

$$z(i)^* = \frac{1}{\gamma \text{Var}[\beta | s(i), p, \bar{z}(i)]} [E[\beta | s(i), p, \bar{z}(i)] - p]$$

From the assumptions made above it then follows that the rational expectations equilibrium of the model is linear. In particular this means:

Proposition 1 *If $\frac{\tau_u \tau_\eta}{\gamma^2} < \frac{1}{4}$ then there exist two rational expectations equilibria in which asset demand $z^*(i)$ of trader i observing the signal $s(i)$, his endowment $\bar{z}(i)$ and the current price p is given by the linear function $z(i)^* = \delta_0^* + \delta_1^* s(i) + \delta_2^* p + \delta_3^* \bar{z}(i)$, where*

$$\delta_0^* = \frac{(1 - \delta_3^*) [(1 - \delta_3^*) \bar{\beta} \tau + \delta_1^* \tau_\epsilon \bar{z}]}{(1 - \delta_3^*)^2 \gamma + \delta_1^* (\tau_\eta + \tau_\epsilon)} \quad (1a)$$

$$\delta_1^* = \frac{\tau_u}{\gamma} \quad (1b)$$

$$\delta_2^* = -\frac{\delta_1^{*2} (\tau_\epsilon + \tau_\eta) + (1 - \delta_3^*)^2 (\tau + \tau_u)}{(1 - \delta_3^*)^2 \gamma + \delta_1^* (\tau_\eta + \tau_\epsilon)} \quad (1c)$$

$$\delta_3^* = \frac{\delta_1^* \tau_\eta}{\gamma(1 - \delta_3^*)} \quad (1d)$$

Proof. See Proposition 1 of Ganguli and Yang [2009]. \square

Multiple equilibria arise from the quadratic equation (1d). If $\frac{\tau_u \tau_\eta}{\gamma^2} < \frac{1}{4}$ this equation exhibits two real solutions, henceforth denoted $\delta_{3,I}^*$ and $\delta_{3,II}^*$:

$$\delta_{3,I}^* = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{\tau_u \tau_\eta}{\gamma^2}}, \quad \delta_{3,II}^* = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\tau_u \tau_\eta}{\gamma^2}} \quad (2a)$$

As (1b) reveals, δ_1^* is unique across these equilibria, while δ_0^* and δ_2^* are not. Thus, if $\frac{\tau_u \tau_\eta}{\gamma^2} < \frac{1}{4}$ we end up with two rational expectations equilibria characterized by $\Delta_I = (\delta_{0,I}^*, \delta_1^*, \delta_{2,I}^*, \delta_{3,I}^*)$ and $\Delta_{II} = (\delta_{0,II}^*, \delta_1^*, \delta_{2,II}^*, \delta_{3,II}^*)$.

2.1 The T-map

In what follows, the analysis of learning processes, either educative or adaptive, will be conducted with the help of the so called T-map. This T-map describes how parameters of a linear decision rule followed by the agents change with the passage of

(virtual or real) time due to learning. This T-map is extensively used in the analysis of adaptive learning processes following the approaches of Marcet and Sargent [1988] and Evans and Honkapohja [2001]. In the present context, this T-map turns out to coincide with the best response mapping defined in the following Proposition.

Proposition 2 *If asset demand $z(i)$ of all traders i is linear in $s(i)$, p and $\bar{z}(i)$ and given by $z(i) = \delta_0 + \delta_1 s(i) + \delta_2 p + \delta_3 \bar{z}(i)$, the best response of any trader $j \in I$ is also a linear function $z(j)^* = \delta'_0 + \delta'_1 s(j) + \delta'_2 p + \delta'_3 \bar{z}(j)$, where*

$$\delta'_0 = \frac{(1 - \delta_3)^2 \bar{\beta} \tau - \delta_1 [\delta_0 (\tau_\epsilon + \tau_\eta) - (1 - \delta_3) \tau_\epsilon \bar{z}]}{\gamma(1 - \delta_3)^2} \quad (3a)$$

$$\delta'_1 = \frac{\tau_u}{\gamma} \quad (3b)$$

$$\delta'_2 = \frac{(1 - \delta_3)^2 (\tau + \tau_u) + \delta_1 (\delta_1 + \delta_2) (\tau_\epsilon + \tau_\eta)}{\gamma(1 - \delta_3)^2} \quad (3c)$$

$$\delta'_3 = \frac{\delta_1 \tau_\eta}{\gamma(1 - \delta_3)} \quad (3d)$$

Proof. See Appendix. \square

With $\delta' = (\delta'_0, \dots, \delta'_3)'$ and $\delta = (\delta_0, \dots, \delta_3)'$ equations (3a)–(3d) give rise to the so called T-map which is central to the analysis of learning processes:

$$\delta' = T_\delta(\delta) \quad (4)$$

Obviously, the above described REE Δ_I and Δ_{II} are fixed points of this T-map.

2.2 Eductive learning

The concept of a strongly rational expectations equilibrium (SREE) asks, whether a specific REE can be 'educed' by agents assuming nothing more than individual rationality and common knowledge.¹ The idea is that agents will not follow strategies that are not best responses to other agent's strategies. Thus, in a way analogous to the concept of a rationalizable Nash-equilibrium, non-best responses can be eliminated from the agent's strategy sets. A REE is eductively stable or a SREE, whenever the REE is the unique outcome of this process. Guesnerie [2002] provides a comprehensive description of this concept and the reader is referred to this reference for details.

Regarding the proof of eductive stability, the essential point is that this proof obviously depends on the properties of the best response mapping. A REE is eductively stable if and only if this REE turns out to be a locally stable stationary point of the

¹The terms 'strongly rational expectations equilibrium' and 'eductively stable equilibrium' can be used interchangeable.

best response mapping. As this best response mapping coincides with the T-map, eductive stability requires that all eigenvalues of the Jacobian of the T-map (4) evaluated at the specific REE are less than one in absolute value. Now, from (3a)–(3d) and using (1b) we get that the eigenvalues $\lambda_1, \dots, \lambda_4$ of the T-map are given by:

$$\lambda_1 = 0, \quad \lambda_2 = \frac{\tau_u \tau_\eta}{(1 - \delta_3^*)^2 \gamma^2}, \quad \lambda_3 = \lambda_4 = -\lambda_2 - \frac{\tau_u \tau_\varepsilon}{(1 - \gamma_3^*)^2 \gamma^2} \quad (5)$$

(1d) implies $(1 - \delta_3^*) = \frac{\tau_u \tau_\eta}{\gamma^2 \delta_3^{*2}}$ and so λ_2 becomes $\lambda_2 = \frac{\gamma^2 \delta_3^{*2}}{\tau_u \tau_\eta}$. Together with (2a) we then get that λ_2 is non negative and always greater than one in case of the Δ_{II} -REE and always smaller than one in case of the Δ_I -REE. While this implies that the Δ_{II} -REE is never a SREE, it does not imply that the Δ_I -REE is always eductively stable. As (5) reveals, $0 < \lambda_2 < 1$ doesn't rule out that the remaining two eigenvalues λ_3 and λ_4 are smaller than -1 . This simply repeats an already known result (cf. Desgranges and Heinemann [2003], Heinemann [2004]) according to which the unique REE of the original Grossman–Stiglitz model isn't always a SREE.

The papers by Desgranges and Heinemann [2003] and Heinemann [2004] show that REE with private information in which agents try to extract information from current market prices are eductively stable if and only if the informational content of the market price is less than the informational content of the private signals, the agents receive. In the present context, this condition requires that:

$$\text{Var}[\beta | s(i)] < \text{Var}[\beta | p, \bar{z}(i)]$$

Now, simple computations show that $\text{Var}[\beta | s(i)] = \frac{1}{\tau + \tau_u}$, while $\text{Var}[\beta | p, \bar{z}(i)] = \frac{(1 - \delta_3^*)^2}{(1 - \delta_3^*)^2 \tau + \delta_1^{*2} (\tau_\varepsilon + \tau_\eta)}$. Using (1b) and (1d), the condition for an SREE therefore becomes:

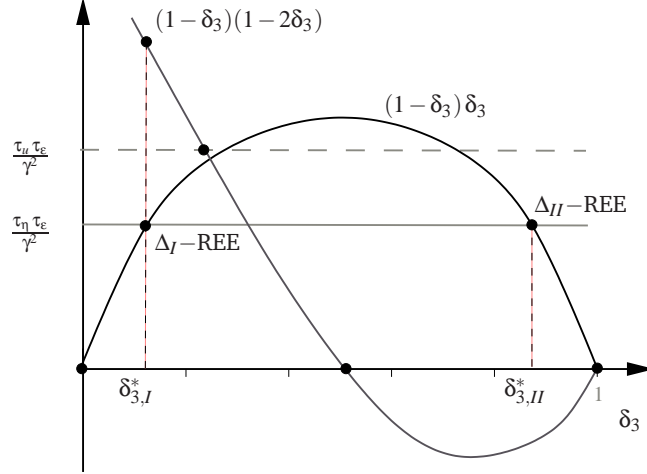
$$(1 - \delta_3^*)^2 > \frac{\tau_u (\tau_\varepsilon + \tau_\eta)}{\gamma^2} \quad \Leftrightarrow \quad (1 - \delta_3^*) (1 - 2\delta_3^*) > \frac{\tau_u \tau_\varepsilon}{\gamma^2} \quad (6)$$

Now, according to (1d) the Δ_{II} -REE necessarily implies $1 - 2\delta_3^* < 0$ such that condition (6) can never be satisfied for this equilibrium. On the other hand, the Δ_I -REE always implies $1 - 2\delta_3^* > 0$. Thus, it might well be that condition (6) is satisfied in case of a Δ_I -REE. This depends on the parameters τ_u , τ_ε , τ_η and γ . Figure 1 depicts a case, where the Δ_I -REE is eductively stable. The figure also reveals that a sufficient condition for eductive stability of the Grossman–Stiglitz like Δ_I -REE is $\tau_u < \tau_\eta$

3 Stability under adaptive learning

In order to analyze the stability of the two above described REE under adaptive learning it is necessary to embed the hitherto static model into a dynamic framework

Figure 1: *Multiple REE and SREE-condition*



such it is at all possible to analyze real time learning processes. Thus, from now on it is assumed that the just described static model is repeated over a long horizon. In each period t , two ex ante unobserved random variables \bar{z}_t and β_t realize and traders observe their private signals $s(i)_t = \beta_t + u(i)_t$ as well as $\bar{z}(i)_t = \bar{z}_t + \varepsilon_t + \eta(i)_t$. Individual asset demand depends on an estimator $\hat{\beta}(i)_t$ of the unknown asset as well as an estimator for its variance $\text{Var}[\hat{\beta}](i)_t$ based on data available up to time t . At the end of every period, agents then revise their estimates $\hat{\beta}(i)$ and $\text{Var}[\hat{\beta}](i)$ in the light of new data, consisting of the endogenous variable p_t and their private signals $s(i)_t$ and $\bar{z}(i)_t$ as well as the ex post observed realizations \bar{z}_t and β_t . The recursive estimation is done using recursive least squares.

Estimation of the equation

$$\beta = \alpha_0 + \alpha_1 s(i) + \alpha_2 p + \alpha_3 \bar{z}(i),$$

by trader i using data up to time t then leads to an estimator $\hat{\beta}(i)_{t+1}$ for β given by $\hat{\beta}(i)_{t+1} = y(i)'_{t+1} \hat{\alpha}(i)_{t+1}$, where $y(i)_t = (1, s(i)_t, p_t, \bar{z}(i)_t)'$, $\alpha(i)_t = (\alpha(i)_{0,t}, \dots, \alpha(i)_{3,t})'$ and

$$\hat{\alpha}(i)_{t+1} = \hat{\alpha}(i)_t + \frac{1}{t} R(i)_t^{-1} y(i)_t (\beta_t - y(i)'_t \hat{\alpha}(i)_t) \quad (7a)$$

$$R(i)_{t+1} = R(i)_t + \frac{1}{t} (y(i)_t y(i)'_t - R(i)_t) \quad (7b)$$

An estimator $\hat{v}(i)$ for the variance results as

$$\hat{v}(i)_{t+1} = \hat{v}(i)_t + \frac{1}{t} ([\beta_t - y(i)'_t \hat{\alpha}(i)_t]^2 - v(i)_t) \quad (7c)$$

Given these estimates, asset demand of trader i in period t is given by:

$$z(i)_t = \frac{\hat{\beta}(i)_t - p}{\gamma \text{Var}[\hat{\beta}](i)_t} = \frac{1}{\gamma \hat{v}(i)_t} (\hat{\alpha}(i)_{0,t} + \hat{\alpha}(i)_{1,t} s(i)_t + (\hat{\alpha}(i)_{2,t} - 1) p_t + \hat{\alpha}(i)_{3,t} \bar{z}(i)_t), \quad (8)$$

Equation (8) is again linear in $s(i)$, p and $\bar{z}(i)$ and the question now is, whether adaptive learning implies that the coefficients of this linear demand function converge against their REE counterparts Δ_I or Δ_{II} . With respect to this, it turns out that the asymptotic properties of the adaptive learning process are again characterized by the properties of the above described T-map. Using the stochastic approximation tools described by Evans and Honkapohja [2001], it can be shown (see Appendix A.2 for details) that the asymptotic dynamics of the learning algorithm are governed by a system of ordinary differential equations, which is given by:

$$\begin{pmatrix} \dot{\alpha} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} T_\alpha(\alpha, v) - \alpha \\ T_v(\alpha, v) - v \end{pmatrix} \quad (9)$$

Thus, a REE of the model is stable under adaptive learning whenever the eigenvalues of Jacobian of (T_α, T_v) evaluated at an REE are smaller than one (implying that the eigenvalues of the map (9) are negative).

Now, the eigenvalues of the Jacobian of $(T_\alpha(\alpha, v), T_v(\alpha, v))$ evaluated at an REE coincide with the respective eigenvalues of the Jacobian of $T_\delta(\delta)$ (see again Appendix A.2 for details). Therefore, as the above discussion of educative stability revealed, the Δ_{II} -REE cannot be stable under adaptive learning as this equilibrium implies that one eigenvalue (λ_2 from (5)) is greater than one. On the other hand, the above described results imply that the Δ_I -REE is always stable under adaptive learning.

4 Summary

The aim of the paper was to show that it is possible to analyze the properties of multiple equilibria existing in the financial market model of Ganguli and Yang [2009] under learning. This analysis revealed that the additional equilibria which arise in their model due the existence of supply shocks are unstable under educative as well as adaptive learning. If ever, the original Grossman–Stiglitz type REE turns out to be stable under learning as this equilibrium is always stable under adaptive learning and potentially stable under educative learning.

As the model analyzed in the present paper is one where private information is exogenously given, it remains to discuss, whether the endogenization of the decision to acquire information can lead to any changes of the stability results. However, as any decision of a trader to acquire information will be based on the expected benefits of private information acquisition, this decision will be based on the expectation of a specific REE. Therefore, an REE which is unstable under learning with exogenous information must be also unstable under learning when the decision to acquire

information is endogenous. With respect to educative learning, this is demonstrated by Desgranges and Heinemann [2003] in a model similar to the financial market model of Grossman and Stiglitz [1980]. They show that that educative stability with exogenous information is a necessary condition for educative stability with endogenous acquisition of information as the latter leads to additional and possibly stronger conditions for educative stability.

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A Appendix

A.1 Best response mapping

Given $z(i) = \delta_0 + \delta_1 s(i) + \delta_2 p + \delta_3 \bar{z}(i)$ for all $i \in I$, we have $p = \frac{(\bar{z} + \varepsilon)(1 - \delta_3) - \delta_0 - \delta_1 \beta}{\delta_2}$. With $y(j) = (s(j), p, \bar{z}(j))'$ and $\bar{y} = (\bar{\beta}, \bar{p}, \bar{z})'$ it then follows:

$$\begin{aligned} E[\beta | p, s(j), \bar{z}(j)] &= \bar{\beta} - M_{yy}^{-1} M'_{\beta y} \bar{y} + M_{yy}^{-1} M'_{\beta y} y(j) \\ \text{Var}[\beta | p, s(j), \bar{z}(j)] &= \frac{1}{\tau} - M'_{\beta y} M_{yy}^{-1} M_{\beta y} \end{aligned}$$

Here $M_{yy} = E[y(j)y(j)']$ and $M_{\beta y} = E[y(j)\beta]$ and the respective moments appearing in the matrix M_{yy} and vector $M_{\beta y}$ are functions of $\delta_0, \dots, \delta_3$. It then follows that optimal asset demand $z^*(j) = \frac{E[\beta | p, s(j), \bar{z}(j)] - p}{\gamma \text{Var}[\beta | p, s(j), \bar{z}(j)]}$ of a trader j is a linear function of $s(j)$, p and $\bar{z}(j)$ the coefficients of which depend on $\delta_0, \dots, \delta_3$ too. Computing the respective moments substituting these into the asset demand function then gives the best response mapping.

A.2 Asymptotic Properties of Least-Squares Learning

Using stochastic approximation tools described by Evans and Honkapohja [2001], it follows that with respect to $\hat{\alpha}(i)$ and $\hat{v}(i)$ the asymptotic dynamics of the learning process (7a)–(7c) are governed by ordinary differential equations which in the present context are given as follows:

$$\begin{aligned}\dot{\alpha}(i) &= E \left[R(i)^{-1} y(i) (\beta - y(i)' \alpha(i)) \right] = (E[y(i)y(i)'])^{-1} E[y(i)\beta] - \alpha(i) \\ &= M_{yy}^{-1} M_{\beta y} - \alpha(i)\end{aligned}\tag{A.10a}$$

$$\begin{aligned}\dot{v}(i) &= E \left[(\beta - y'(i) \alpha_i)^2 - v(i) \right] = E[\beta^2] - E[y(i)\beta]' (E[y(i)y'(i)])^{-1} E[y(i)\beta] - v(i) \\ &= \frac{1}{\tau} - M'_{\beta y} M_{yy}^{-1} M_{\beta y} - v(i)\end{aligned}\tag{A.10b}$$

The moments appearing in the matrix M_{yy} and the vector $M_{\beta y}$ are functions of the parameters $\alpha_0, \dots, \alpha_3$ and v of the other traders' demand functions. Thus, (A.10a) and (A.10b) define two dynamic equations $\dot{\alpha}(i) = T_\alpha(\alpha, v) - \alpha(i)$ and $\dot{v}(i) = T_v(\alpha, v) - v(i)$. Now, all traders learn in an identical way from individual data which is drawn from identical distributions. Due to this symmetry, we can drop the individual subscripts when studying the asymptotic behavior of the learning process such that we end up with the following dynamic system:

$$\begin{pmatrix} \dot{\alpha} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} T_\alpha(\alpha, v) - \alpha \\ T_v(\alpha, v) - v \end{pmatrix}\tag{A.11}$$

A REE is a stable stationary point of this system, whenever the eigenvalues of the Jacobian of (T_α, T_v) evaluated at the RRE are smaller than one. Computing the respective derivatives and using the fact that in a REE we must have $\frac{\alpha_0^*}{\gamma v^*} = \delta_0^*$, $\frac{\alpha_2^* - 1}{\gamma v^*} = \delta_2^*$ and $\frac{\alpha_3^*}{\gamma v^*} = \delta_3^*$ as well as

$$v^* = \text{Var}[\beta | p, s(i), \bar{z}(i)] = \frac{(\delta_3^* - 1)^2}{(\delta_3^* - 1)^2(\tau + \tau_u) + \delta_1^{*2}(\tau_\epsilon + \tau_\eta)}$$

then reveals after some manipulation that the eigenvalues of the Jacobian of (T_α, T_v) coincide with the eigenvalues of T_δ

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