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Working Paper

Transboundary Pollution Control with Both Production and Consumption Emissions

CESifo Working Paper, No. 10667

Provided in Cooperation with:
Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Haruna, Shoji; Goel, Rajeev K. (2023) : Transboundary Pollution Control with Both Production and Consumption Emissions, CESifo Working Paper, No. 10667, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:
https://hdl.handle.net/10419/282355

CESifo WORKING PAPERS

10667
2023

September 2023

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 10667*

Transboundary Pollution Control with Both Production and Consumption Emissions

Abstract

This paper adds to the literature on transboundary pollution by considering pollution related to both production and consumption activities. In particular, we consider a symmetric strategic two firm-two country game model with bilateral trade and transboundary pollution to analyze the effects of trade liberalization on economic performance under two types of pollution. Our game-theoretic results with two trading countries find significant differences compared to the case where only production pollution is considered. When trade liberalization policy is mutually implemented, consumer surplus and social environmental damage in the Home and Foreign countries are both increased under production pollution, while they are both decreased under consumption pollution. Furthermore, when the two countries face either production or consumption pollution composed of transboundary pollution and local pollution, consumer surplus, producer surplus, and social environmental damage are larger in the presence of consumption pollution than in the presence of production pollution; and, under certain conditions, social welfare can be larger or smaller in the presence of production pollution than in the presence of consumption pollution. It is uniquely shown that in the three-stage game model trade policy may lose its effectiveness as a policy under consumption pollution. Policy implications are discussed.

JEL-Codes: D430, F130, L130, Q560.

Keywords: environment, transboundary pollution, consumption pollution, production pollution, trade liberalization, environment tax, oligopoly, tariff.

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Shoji Haruna would like to acknowledge for financial support from the Japan Society for the Promotion of Science under the Grant-in-Aid for Scientific Research (B), Nos. JP20H01507 and JP22H00846.

1. Introduction

The Industrial Revolution that began in England in the 18th century and caused a dramatic change in production methods and transformations of production organization has created a rapid expansion in products, so people's lives have changed dramatically. On the other hand, global warming has been generated through the emissions of greenhouse gases such as carbon dioxide, methane gas, nitrous oxide, etc. As a result, the average temperature of the earth has now risen by more than one degree, compared to pre-industrial times. The warming has been causing serious problems with temperature, sea level, and crops. Nowadays, it has been taken to suppress the temperature within 1.5 degrees measures against global warming by lots of countries led industrial countries[1]. This is a big trend that will continue forever.

Both consumption and production sectors emit pollutants worldwide. For instance, the household sector in Japan is estimated to emit 14.7% of total emissions of CO_2 in 2021[2]. It is expected that this percentage will further increase in countries with a higher share of the consumption sector in GDP. Therefore, we cannot disregard the amount of CO_2 emissions from the household sector. Furthermore, lots of countries suffer environmental damage due to border crossing of some pollution caused by the manufacturing process of foreign firms and/or the consumption activities of the foreign consumer except for local environmental damage. As environmental issues that transcend national borders, there are, for example, hazardous air pollutants, acid precipitation, contamination of international rivers, global sea water pollution, and deforestation. Acid precipitation generated by fossil fuel use and including sulfur and nitrogen oxides may descend to regions thousands of kilometers away from its generation source. In addition, it may be very difficult to stop completely generation of pollutants and confine them within our own country, so that neighboring countries may have issues of cross-border (transboundary) pollution. As well-known cross-border pollutants there are, for example, Carbon dioxide, Methane, Sulphur dioxide, acid rain, and microplastics[3]. The cross-border pollution will easily happen among neighboring countries in the EU, continental North American and East Asian region.

As a relatively recent trend, many Regional Trade Agreements (RTAs) have been rapidly concluded among countries since the early 1990s.[4] Although there are many objectives of RTAs, their main purpose is to reduce/eliminate tariffs among participants. On the other hand, some movements against RTAs are happening, like the decoupling of the U.S. and China from each other, as typified by "Chip War" on semiconductors. However, the importance of trade

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[1] For example, see Nordhaus (2013) and Ministry of Environment (2022).

[2] Total amount of CO_2 emissions in Japan was 1,064million tons in 2021 (https://www.mlit.go.jp/sogoseisaku/environment/sosei_environment_tk_000007.htmll).

[3] Missfeldt (1999) describes emitted pollutants and environmental problem in detail for each of three geographical characteristics of "Local", "Regional", and "Global".

[4] Free Trade Agreement (FTA), Economic Partnership Agreement (EPA), Preferential Trade Agreement (PTA), and Customs Union (CU) are collectively referred to as an RTA. See, for example, Jinji, Zhang, and Haruna (2021) for RTAs and globalization of firms.

liberalization will not likely decrease in the future.

The purpose of this paper is to make a comparison between economic performance in free-trading economies when there are production pollution and consumption pollution. Pollution on both the production side and the consumption is prevalent in many goods. A common example is the production/exploration of petroleum products and then their consumption. The crux of the difference is that a government can impact foreign production through tariffs but can't impact foreign consumption. With both consumption and production emitting pollutants and there is the presence of cross-border pollution spillovers, ignoring consumption pollution, as most of the related literature has done, would undermine recommendations for an effective environmental policy to control global pollution.

For our analysis, we make use of a simple multistage game model to prioritize ease of handling. Our analysis is divided into two parts. In the first part, by invoking the model of Lai and Hu (2005), we examine the effects of trade liberalization on economic performance under production and consumption pollutions and compare the results obtained to see what impacts a difference in pollution sources has on such performance. In this analysis tariffs are taken as an exogeneous variable.

In the other part, by using a three-stage game model, we investigate the characteristics of Subgame-Perfect Nash Equilibria in the presence of both pollutions where tariffs are an endogenous variable, then comparing them.

A lot of papers have analyzed the strategic behavior of oligopolistic firms and governments under international trade and environmental externalities. There are two approaches for these analyses: One is to investigate their strategic behavior under production pollution, and the other is to investigate the behavior under consumption pollution. The former analysis has a long history, but the latter has relatively a short formal investigation span. Furthermore, an analysis taking transboundary (cross-border) pollution into account has been conducted sometimes.

As a paper dealing with both the environment with the emission of pollutants generated by production activities and international trade, there are the following. For example, Kennedy (1994) investigates strategic incentives (a rent-capture effect and a pollution-shifting effect) under production pollution and free trade by using a two-stage game model. As the rent-capture effect, he shows that a unilateral change in the environmental tax has a larger impact on domestic production than on domestic consumption, and, as the pollution-shifting effect, that a unilateral tax increase shifts domestic production and associated pollution to the foreign country. Copeland and Taylor (1995) analyze the relationship between trade and welfare and pollution levels by employing a general equilibrium model for analysis[5]. Burguet and Sempere (2003)

[5] Haruna and Goel (2019) consider optimal pollution abatement and environmental policy under mixed oligopoly with emissions-reducing R&D and imperfect appropriation by using a three-stage game model. Barrett (1994) examines the relationship between marginal damage of pollution and the marginal cost of abatement when firms and governments simultaneously or intertemporally determine outputs and environmental pollution standards, respectively and shows that the marginal damage of pollution is equal to the marginal cost of abatement under the simultaneous decisions, but the former

take a two-stage game model of two firms and two countries facing production pollution and consider the impact of a bilateral tariff reduction on environmental policy. They find that a reduction in tariffs increases welfare, but the effect on environmental standards is ambiguous. Tsai, Tu, and Chiou (2015) find that by using a three-stage game model where a domestic firm only discharges pollutants and a foreign firm exports products to the domestic market, tariffs and environmental taxes do not monotonically increase in the case of corner solutions, different from the case of interior solutions. Nimubona (2012) considers the relationship between pollution policy and trade liberalization in a developing country by using a three-stage game model with production pollution but no transboundary pollution. He shows that trade liberalization of environmental goods (EGs) imported from a developed country may cause emissions taxes decrease, so that the environmental damage in the developing country will increase. It is Burguet and Sempere (2003) who consider the effects of a mutual reduction in tariffs on environmental policies and welfare in detail when there is production pollution but no transboundary pollution. The direction of their analysis is generally similar to our paper, albeit they consider no pollution spillovers across borders.

Incidentally, there are many papers that empirically examine the environmental impacts of trade liberalization. For example, Cherniwchan (2017) performs empirical analysis between trade liberalization and environment by using data on NAFTA and U.S. manufacturing and demonstrates that the amount of pollution emissions and their emissions densities (PM_{10} and SO_2) of the U.S. manufacturing are decreased. Next, Gutierrez and Teshima (2018) consider the impacts of tariffs on the use of plant fuel, plant's abatement expenditures and air contamination by employing plant-level data in Mexico and show that a reduction in tariffs improves plant energy efficiency, reducing both associated pollutant emissions and direct investment in environmental protection. These results demonstrate that the conclusion of RTAs may contribute to environmental improvement.

Cross-border pollution has not been taken into consideration in the papers mentioned above. However, as an early paper concerning cross-border pollution generated by production activities and international trade, there is, for example, Markusen (1975). He mainly investigates the levels of the second-best consumption taxes, production taxes, and tariffs, separately, by using a two-country game model with production pollution which affect the environment of a trading partner country and compares welfare.

As the other literature that takes cross-border pollution into consideration, there are Lumeda and Wooton (1994) and Chen and Wang (2010)[6]. By using a two-country noncooperative game of the domestic and exporting countries with unilateral cross-border pollution created from the latter, Lumeda and Wooton (1994) consider their optimal

exceeds the latter under intertemporal pollution.

[6] Benchekroun and Martin-Herran (2016) use a dynamic differential pollution game model of myopic and foresighted countries and examine the effects of switching from a myopic country to a nonmyopic one on overall emissions and welfare.

environmental and tariff policies under trade barriers and trade liberalization, and both environmental policy and a process standard under trade liberalization. Then they point out the possibility of an importing country to introduce the process standard as the third method of environmental policy against free trade. Chen and Wang (2010) examine the impacts of trade liberalization on transboundary pollution and an environmental tax by using an international mixed oligopoly model[7]. Missfeldt (1999) gives an overview of papers with game-theoretic approach in transboundary pollution and classifies it by the terms of geographical characteristics, emitted pollutants, environmental problems, and the most important abatement. Besides, Barcena-Ruiz and Campo (2012) make comparative statics on environmental taxes and damages, and welfare when there are both production pollution and transboundary pollution.

Many papers have conducted analyses of the relationship between governments' environmental and trade policies and strategic behavior of international oligopolistic firms when the environment is contaminated. The background of these analyses concerns production pollution, notwithstanding that toxic substances are, for example, discharged through consumption activities as well as production activities.

In comparison with these papers, there are a small number of papers on both optimal environmental and trade policies under pollution caused by both consumption activities and transboundary pollution. In the few, as a paper focusing on the effects of trade liberalization, there are, for example, Lai and Hu (2005, 2008), Kayalica and Kayalica (2005), and Chen and Wang (2010). Lai and Hu (2005) examine the effect of a bilateral tariff reduction on the welfare and global environment. Moreover, Lai and Hu (2008) extend their earlier analysis of (2005) to incorporate price differentiation and cooperation of the countries, and compare environmental taxes under cooperation in both taxes and tariffs with those under no cooperation. Further, Kayalica and Kayalica (2005) derive optimal environmental and trade policies in an international duopolistic two-stage game model with consumption pollution and compare them between the two countries. Chen and Wang (2010) consider the effects of bilateral trade liberalization on environmental taxes, environmental damages and welfare in transboundary pollution by focusing on an international mixed duopoly.

Besides, some papers investigate the impact of trade liberalization on economic performance when governments and firms face both global (cross-border) and local pollution in consumption. These papers employ a partial equilibrium framework, while Copeland and Taylor (1995, p.765) show that by using a simple general equilibrium model with consumption pollution "trade shifts pollution-intensive consumption to the low-income, low-regulation country".

The impacts of trade liberalization on economic performance such as welfare, consumer

[7] For example, Pal and Saha (2014) consider how environmental policy is affected by a mixed duopoly, compared with a conventional duopoly.

surplus, and environmental damage, have been considered separately under production pollution and consumption pollution in these papers, whereas there are no papers that simultaneously compare their impacts. Then, we can elucidate how sources of pollution influence the economic performance of countries in a partial equilibrium analysis. Specifically, we compare the environmental and tariff policies of countries locally and globally emitting pollutants and their welfare, consumer surplus, producer surplus, and environmental damages in response to a change in trade policy under two types of pollution, generated by production and consumption behavior.

This analysis has important policy implications. For example, if there are multiple products traded between countries, they are divided into an importer for some products mainly generating consumption pollution and an exporter for others, mainly generating production pollution. Then, by revealing the effects of trade liberalization on the economic performance of the import and the export countries evident we may provide appropriate trade policy for each government.

Next, we elucidate the characteristics of SPNEs in the noncooperative three-stage game with production or consumption pollution and trade and make a comparison. As far as we know, there are no papers investigating the noncooperative behavior of conventional oligopolistic firms and environmental and tariff policies of governments under pollution generated by production and consumption activities. We can shed light on how a difference in pollution sources influences the structure of the SPNE. In a related study, Lai and Hu (2008) derive the optimal environmental taxes and tariffs with consumption pollution and compare the optimal environmental taxes and tariffs. They show that countries may engage in trade liberalization when cooperating in both tariffs and environmental taxes.

This paper is organized as follows. First, in Section 2 we introduce the basic model of a two good-two country game to setup subsequent analysis. In Sections 3 and 4 we discuss the impacts of trade liberalization on the performance such as the welfare, consumer surplus, producer surplus, and social environmental damages of the two counties in the presence of production pollution by using a symmetric strategic two-stage game model, and its impacts in the presence of consumption pollution, respectively. We, furthermore, consider the characteristics of Subgame-Perfect Nash Equilibria in an extended three-stage game in Sections 3 and 4. In Section 5, the results obtained in Sections 3 and 4 are compared. Section 6 is assigned to conclusions.

2. The basic model

In general, in environmental externalities there are two types of pollution from an emitter's perspective: One type comes from production activities and the other from consumption

activities[8]. In other words, social environmental damage is created by emissions generated through not only production but also consumption activities. For example, when manufacturing durable goods such as automobiles or household electrical appliances, we cannot avoid producing emissions. On the other hand, when consumers use durable goods, air pollutants are also directly or indirectly generated as well as the emissions of pollutants in the production stage. The characteristics of the durable automobiles or household electrical appliances are that they continue to be used by the consumers for a long time after their manufacture and sale.

Consider a symmetric strategic two firm-two country game model with bilateral trade and transboundary pollution to analyze the effects of trade liberalization on economic performance under two types of pollution. The firms produce a homogeneous tradable good and sell the good in both markets. Each of them is located in a different country, Home (country 1) and Foreign (country 2). Both countries are identical. The firms are involved in strategic quantity competition both in the domestic and foreign markets, whose inverse demand functions are downward sloping.

Let q_i be the output of firm i $(= 1, 2)$, sold in (produced for) the home market, and q_i^* be the output of firm i sold in (produced for) the foreign market. The two markets are imperfectly competitive. Now q_1^* stands for the export of good from Home to Foreign and, on the other hand, q_2^* for the export of good from Foreign to Home. Foreign variables are denoted by an asterisk (*). Then, the inverse demand function of the home market is given by $p(Q) = A - Q$, where $p(Q)$ stands for the price of the domestic market, and $Q = q_1 + q_2$ is the total consumption in the domestic market, q_2 is the import of Home from Foreign, and A denotes the size of the market and is positive and constant. Foreign market is given by $p(Q^*) = A - Q^*$, where $p(Q^*)$ stands for the price of the foreign market, and $Q^* = q_1^* + q_2^*$ is the total consumption in the foreign market. Note that q_1^* is the export of Home to Foreign. Now it is assumed that the marginal production costs of the domestic and foreign firms are constant and the same, given by c, because we do not focus on the analysis of a difference in the marginal production costs of the two firms. For convenience, it is also assumed that the size of the two markets is the same and that the two marginal production costs are the same for simplicity. As usual, we assume that one unit of output in production generates one unit of emission as a by-product of production. Then firm i's emissions per output become $(q_i + q_i^*)$[9]. The pollution damage generated in it is not confined to the country where its output is produced, but also parts of pollutants are transmitted to the other country. That is, there is transboundary pollution through production and consumption activities. Now, production pollution is assumed to be not only local but also global in Section 3. There are similarly two types of consumption pollution, local and global, in Section 4, and emissions are generated as

[8] Anderson (1992) mentions consumption pollution in addition to Lai and Hu (2005, 2008).
[9] We assume that emissions do not have durability. However, see, for example, Goel and Hsieh (2004) for the analysis between Pigouvian tax and durable emissions.

a by-product of consumption, as in the production pollution case.

Each country employs pollution (emissions) taxes as part of its environmental policy. We distinguish an emission tax on production pollution from that on consumption pollution for convenience. Specifically, the Home (Foreign) government uses an emissions tax (specific tax) t_0 (t_0^*) per unit of output against the emissions per unit of production of firm i[10]. On the other hand, pollutants are also discharged via consumption activities, so that an environmental tax t_1 (t_1^*) per unit of consumption (Pigouvian tax) is also levied against related emissions. However, we do not simultaneously consider these two pollutions in the following analysis. Firm i pays environmental taxes $t_0 \cdot (q_i + q_i^*)$, $i = 1, 2,$ to its government. As the two countries trade, each country imposes a tariff on imported goods. Although the role of tariffs is generally to control imports and to shift rents from the foreign country to the domestic country by using its terms-of-trade effect, governments may employ tariffs to avoid social environmental damage from foreign emissions when there is transboundary pollution[11].

To solve the model via backward induction, we take the final stage in two firm-two country multistage model with trade and pollution. In this stage, one domestic firm and one foreign firm are involved in quantity competition a la Cournot, and uncooperatively and simultaneously determine their outputs to maximize respective profits. The prototype of the home firm's profit function is given by

$$\pi = [p(Q) - c - t_1] \cdot q_1 + [p(Q^*) - c - t_1^* - \tau^*] \cdot q_1^* - t_0 \cdot (q_1 + q_1^*), \tag{1-1}$$

where $\tau^*(\geq 0)$ is a (specific) tariff per unit of imported goods imposed by the foreign government[12]. On the other hand, the prototype of the foreign firm's profit function is given by

$$\pi^* = [p(Q^*) - c - t_1^*] \cdot q_2^* + [p(Q) - c - t_1 - \tau] \cdot q_2 - t_0^* \cdot (q_2 + q_2^*), \tag{1-2}$$

where τ (≥ 0) is a tariff per unit on imported goods imposed by the home government. It is assumed $p(Q) - c - \tau > 0$ and $p(Q^*) - c - \tau^* > 0$ in order to guarantee trade between the two countries. The governments employ a tariff as customs policy to lower emissions in their countries. We assume that both firms produce goods with constant-return-to-scale technologies which are identical and, moreover, that there are no fixed costs. Then, the profit of the domestic (foreign) firm equals the producer surplus of the domestic (foreign) country. However, the presence or absence of fixed costs does not affect the results derived later.

As mentioned earlier, it is assumed that transboundary (cross-border) pollution is generated.

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[10] See, for example, Lai and Hu (2005, 2008), and Chen and Wang (2010) for consumption-type externalities (pollution emission generated by the activities of the consumer).

[11] Incidentally, Beghin, Roland-Holst and der Mensbrugghe (1994) state that it is inefficient to address polluters causing negative externalities with output taxes, and, moreover, that the taxes are not better than tariffs.

[12] There are two kinds of tariffs: specific tariffs and ad valorem tariffs. We use the former.

When such pollution occurs, the government of each country uses an import tariff to curb pollution of foreign origin. The government attempts to control the production of pollution-generating goods with terms-of-trade effects, and finally to reduce cross-border pollution. The channels through which the domestic government can influence the production of a foreign country are limited to import tariffs, process standards and pollution-content tariffs[13].

We turn to the social welfare. We must take into consideration negative externalities and one of the well-known negative externalities is pollution. We treat two types of pollution. In the analysis in Section 3, the products of the two countries are assumed to cause social environmental damages to themselves through their production processes. Simultaneously, each country bilaterally suffers from parts of the emissions of the other country by cross-border pollution. Namely, some parts are transferred (spill over) out of the emissions-producing country onto the other country. Alternatively, the consumption of goods 1 and 2 causes social environmental damage in each country where they are consumed. Namely, pollution in each country is composed of two types of pollution from the viewpoint of the place of origin, i.e., local pollution and global one. That is, global pollution implies inflow of pollutants from the Foreign country. We take the following linear damage function as to production pollution in the Home and the Foreign countries to express the scale of social environmental damage by the two pollutions, respectively[14]:

$$D_0 = d[(q_1 + q_1^*) + \delta(q_2 + q_2^*)]$$
$$D_0^* = d[(q_2 + q_2^*) + \delta(q_1 + q_1^*)],$$

where d stands for the density (degree) of the social environmental damage, i.e., the marginal environmental damage, caused by pollution emissions through production or consumption, independent of whether such damage comes from either the home or the foreign countries, and δ $(\in [0, 1])$ stands for the extent of the social environmental damage of a country generated by transboundary (global) pollution[15, 16]. Therefore, the larger the value of δ, the greater transboundary pollution. It is assumed that these two parameters are the same for the countries and $d > \delta$. Zero density of the marginal environmental damage, $d = 0$, corresponds to the case that there are no local and transboundary pollutions, or the governments do not become

[13] As for the interventions to control foreign pollution, see, for example, Markusen (1975), Ludema and Wooton (1994), Kennedy (1994), and Copeland (1996).
[14] For example, Nimubona (2012), Xu and Lee (2015), and Ye and Zhao (2016) employ a linear function of the social damage due to pollutant emissions generated through the production activities (production process) of firms. On the other hand, Tanguay (2001), Kayalica and Kayalica (2005), and Lai and Hu (2008) also employ a linear social damage function when pollution is generated from consumption activities.
[15] The amount of pollution from abroad cannot be directly, but indirectly controlled through the import of final goods. See, for example, Copeland (1996) for the control of cross-border pollution.
[16] Although Tanguay (2001) introduces transboundary pollution into consideration, he does not consider the case of full transboundary pollution. Besides, see, for example, Goel and Saunoris (2020) for empirical research for transboundary pollution.

conscious of pollutions[17]. The product δd denotes the marginal environmental damage of cross-border pollution. On the other hand, zero δ corresponds the case that there is no transboundary pollution, or the domestic government does not become conscious of transboundary pollution. Now, we assume $A - c > d$ to assure the operation of firms.

Furthermore, it is assumed that one unit of consumption generates one unit of emission as a by-product. Then, the social environmental damage generated in consumption does not only confine itself to the country where products are consumed, but some pollutants are moved/transmitted to the other country. That is, there exists transboundary pollution (global) in addition to local pollution. We take the following linear damage function as to consumption pollution in the domestic and foreign countries, respectively:

$$D_1 = d[(q_1 + q_2) + \delta(q_1^* + q_2^*)]$$
$$D_1^* = d[(q_1^* + q_2^*) + \delta(q_1 + q_2)],$$

where $q_1 + q_2$ and $q_1^* + q_2^*$ in the first parts of D_1 and D_1^*stand for local pollutant emissions in the domestic and foreign countries, respectively. For analytical convenience, we assume that the density of the social environmental damage and the transboundary rate of pollution generated by consumption in the foreign country are the same as those in the production case.

The social welfare of a country is defined as the total of consumer surplus, the firm's profits, tax and tariff revenues, and social environmental damage costs. Then, the prototypes of the social welfare functions of the Home and the Foreign countries are given by, respectively:

$$W_i = \int_0^Q p(\beta)d\beta - p(Q)Q + \pi_i + t_1 Q + \tau q_2 - D_0 - D_1, \quad i = 0, 1, \tag{2-1}$$

$$W_i^* = \int_0^{Q^*} p(\beta^*)d\beta^* - p(Q^*)Q^* + \pi_i^* + t_1^* Q^* + \tau^* q_1^* - D_0^* - D_1^*. \tag{2-2}$$

First, we explain the two-stage game employed in Sections 3 and 4. In the first stage, the governments of the two countries simultaneously and noncooperatively choose their environmental taxes to maximize own social welfare, respectively. In the second stage, after observing taxes, firms of the home and foreign countries simultaneously and noncooperatively determine outputs for the domestic and the foreign markets to maximize profits. We derive Subgame-Perfect Nash Equilibrium (SPNE) by using backward induction.

3. Government behavior in the presence of production pollution only: Case I

[17] See, for example, Barcena-Ruiz and Garzon (2006), and Pal and Saha (2015) for an analysis including the government's unconsciousness of environmental damage.

We derive two Subgame-Perfect Nash Equilibria:
Case (I) where there is only production pollution (environmental damages brought by production activities); and
Case (II) where there is only consumption pollution (brought by consumption activities).

This section is composed of two parts: One is Case I-(1) Trade liberalization and its effects on production pollution; and the other is Case I-(2) Extension of the basic model with production pollution to a three-stage game model.

3.1. Case I-(1) Trade liberalization and its effects under production pollution

When firms' operations only pollute the environment, both $t_1 = t_1^* = 0$ and $D_1 = D_1^* = 0$ hold. Then, the profit functions (1-1) and (1-2) of the home and foreign firms become, respectively[18]:

$$\pi_0 = [p(Q) - c] \cdot q_1 + [p(Q^*) - c - \tau^*] \cdot q_1^* - t_0 \cdot (q_1 + q_1^*) \quad (1\text{-}1)'$$
$$\pi_0^* = [p(Q^*) - c] \cdot q_2^* + [p(Q) - c - \tau] \cdot q_2 - t_0^* \cdot (q_2 + q_2^*). \quad (1\text{-}2)'$$

These profits are the same as the producer surplus of the domestic and the foreign firms, respectively.

To derive the first-order conditions for profit maximization in the second stage, differentiating (1-1)' with respect to the outputs for the domestic and the foreign markets of the home firm yields

$$\frac{d\pi_0}{dq_1} = A - c - t_0 - 2q_1 - q_2 = 0 \quad (3\text{-}1)$$

$$\frac{d\pi_0}{dq_1^*} = A - c - t_0 - \tau^* - 2q_1^* - q_2^* = 0, \quad (3\text{-}2)$$

and differentiating (1-2)' with respect to those of the foreign firm also yields

$$\frac{d\pi_0^*}{dq_2} = A - c - t_0^* - \tau - q_1 - 2q_2 = 0 \quad (4\text{-}1)$$

$$\frac{d\pi_0^*}{dq_2^*} = A - c - t_0^* - q_1^* - 2q_2^* = 0. \quad (4\text{-}2)$$

We assume that there are interior solutions in the second stage. When solving the first-order conditions from (3-1) to (4-2) with respect to the decision variables, we have:

[18] Our model is on the extension line of Brander and Spencer (1983).

$$q_1 = \frac{A - c - 2t_0 + t_0^* + \tau}{3},\ q_2 = \frac{A - c + t_0 - 2t_0^* - 2\tau}{3} \qquad \text{for the domestic market}$$

$$q_1^* = \frac{A - c - 2t_0 + t_0^* - 2\tau^*}{3},\ q_2^* = \frac{A - c + t_0 - 2t_0^* + \tau^*}{3} \qquad \text{for the foreign market.} \tag{5}$$

In consequence, the outputs of the two firms become functions of the emissions taxes and tariffs of the Home and Foreign governments, i.e., $q_i = q_i(t_0,\ t_0^*;\ \tau,\ \tau^*)$ and $q_i^* = q_i^*(t_0,\ t_0^*;\ \tau,\ \tau^*),\ i = 1, 2.$

Let us derive comparative statics with respect to environmental taxes by using the results obtained. Then we obtain the following[19]:

$$\frac{dq_1}{dt_0} = -\frac{2}{3} < 0,\ \frac{dq_1}{dt_0^*} = \frac{1}{3} > 0, \qquad \frac{dq_2}{dt_0} = \frac{1}{3} > 0, \qquad \frac{dq_2}{dt_0^*} = -\frac{2}{3} < 0,$$

$$\frac{dq_1^*}{dt_0} = -\frac{2}{3} < 0, \quad \frac{dq_1^*}{dt_0^*} = \frac{1}{3} > 0, \quad \frac{dq_2^*}{dt_0} = \frac{1}{3} > 0, \quad \frac{dq_2^*}{dt_0^*} = -\frac{2}{3} < 0.$$

The comparative static results are symmetric given the model's character. For example, they show that a unilateral increase in the domestic emissions tax t_0 (t_0^*) reduces (increases) q_1 and q_1^*, because such an increase causes increased marginal production costs and then increases (reduces) q_2 and q_2^*, so that its associated pollution is shifted to the foreign country. That is, the increase gives rise to "the pollution-shifting effect" (Kennedy (1994, p.58)). In addition, "the rent-capturing effect" (Kennedy (1994, p.57)) also holds. Further, note that a unilateral increase in the tariff τ increases (reduces) q_1 (q_2), while that in τ^* reduces (increases) q_1^* (q_2^*). The terms-of-trade effect acts on the exports of the countries. The direct effect of a change in the tariff of one country is limited to its own output. Finally, an increased environmental tax in each country leads to a reduction in its own consumption and then to an increased output price.

We move to the first stage of the two-stage game in which the governments choose environmental taxes. The goods 1 and 2 cause social environmental damage to their producing countries, respectively, and each country simultaneously suffers from parts of the social environmental damage (transboundary pollution) caused by the production activities of the other country. Then, the social welfare of the country is the sum of consumer surplus,

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[19] For reference, we express the direct effect of tariffs on the outputs when environmental taxes are not a decision variable or are constant:

$$\frac{dq_1}{d\tau} = \frac{1}{3} > 0,\ \frac{dq_2}{d\tau} = -\frac{2}{3} < 0,\ \frac{dq_1^*}{d\tau^*} = -\frac{2}{3} < 0,\ \frac{dq_2^*}{d\tau^*} = \frac{1}{3} > 0.$$

producer surplus, and the environmental tax and tariff revenues of the government net of its environmental damage, so that the welfare functions (2-1) and (2-2) of the Home and Foreign under production pollution are reduced to, respectively:

$$W_0 = \frac{1}{2}Q^2 + \pi_0 + t_0 \cdot (q_1 + q_1^*) + \tau q_2 - D_0$$

$$= \frac{1}{2}Q^2 + [p(Q) - c] \cdot q_1 + [p(Q^*) - c - \tau^*] \cdot q_1^* + \tau q_2 - D_0, \quad (2\text{-}1)'$$

$$W_0^* = \frac{1}{2}(Q^*)^2 + [p(Q^*) - c] \cdot q_2^* + [p(Q) - c - \tau] \cdot q_2 + \tau^* q_1^* - D_0^*, \quad (2\text{-}2)'$$

where the social environmental damage generated by the consumption activities of the two countries and the environmental taxes is supposed to be zero, i.e., $D_1 = D_1^* = 0$ and $t_1 = t_1^* = 0$.[20] When there are environmental taxes and tariffs, the welfare of the domestic country is obtained by deducing the social environmental damage from the sum of consumer surplus, producer surplus, the environmental tax and tariff revenues of the government. Furthermore, arranging the welfare function, we have the function (2-1)', where the first, second-third, fourth and fifth terms stand for consumer surplus, quasi-producer surplus (QPS_0), tariff revenues, and social environmental damage, respectively[21]. The producer surplus is equal to the profits, $\pi_0 = (q_1)^2 + (q_1^*)^2$, of the domestic firm, and QPS_0 is derived by deducing the governmental revenues of the environmental tax, i.e., $t_0 \cdot (q_1 + q_1^*)$, from the producer surplus.

Let us turn to the first stage. The government in each country simultaneously and noncooperatively chooses its environmental tax to maximize its social welfare. Given the tariffs of the two countries, i.e., τ and τ^*, when the Home and the Foreign governments differentiate (2-1)' and (2-2)' with respect to the emissions taxes and we use the results of the comparative statics, we have the first-order conditions for the social welfare maximization of their governments, respectively:

$$\frac{dW_0}{dt_0} = Q\frac{dQ}{dt_0} - q_1\frac{dQ}{dt_0} + [p(Q) - c]\frac{dq_1}{dt_0} - q_1^*\frac{dQ^*}{dt_0} + [p(Q^*) - c - \tau^*]\frac{dq_1^*}{dt_0}$$

$$+ \tau\frac{dq_2}{dt_0} - d\left[\frac{dq_1}{dt_0} + \frac{dq_1^*}{dt_0} + \delta\left(\frac{dq_2}{dt_0} + \frac{dq_2^*}{dt_0}\right)\right]$$

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[20] The contents of the welfare function (2-1)' are as follows: The domestic country's consumer surplus is $\frac{Q^2}{2}$, the profits of the domestic firm are $[p(Q) - c] \cdot q_1 + [p(Q^*) - c - \tau^*] \cdot q_1^* - t_0 \cdot (q_1 + q_1^*) = (q_1)^2 + (q_1^*)^2$, the domestic government's revenues are $t_0 \cdot (q_1 + q_1^*) + \tau q_2$, and the domestic environmental damage is $D_0 = d[(q_1 + q_1^*) + \delta(q_2 + q_2^*)]$. Now, each government's revenues from its emissions tax are canceled out in its welfare function.

[21] QPS_0 is $[p(Q) - c] \cdot q_1 + [p(Q^*) - c - \tau^*] \cdot q_1^*$.

$$= \frac{-4(A-c)+3\tau+2\tau^*+6(2-\delta)d-7t_0-t_0^*}{9} = 0 \tag{6-1}$$

$$\frac{dW_0^*}{dt_0^*} = Q^*\frac{dQ^*}{dt_0^*} - q_2^*\frac{dQ^*}{dt_0^*} + [p(Q^*)-c]\frac{dq_2^*}{dt_0^*} - q_2\frac{dQ}{dt_0^*} + [p(Q)-c-\tau]\frac{dq_2}{dt_0^*}$$
$$+ \tau^*\frac{dq_1^*}{dt_0^*} - d[\frac{dq_2}{dt_0^*} + \frac{dq_2^*}{dt_0^*} + \delta\left(\frac{dq_1}{dt_0^*} + \frac{dq_1^*}{dt_0^*}\right)]$$
$$= \frac{-4(A-c)+2\tau+3\tau^*+6(2-\delta)d-t_0-7t_0^*}{9} = 0. \tag{6-2}$$

The second-order conditions for welfare maximization with respect to taxes in the two countries are all satisfied[22]. Now we can derive the best-reaction functions for the two countries in terms of environmental taxes[23]:

$$t_0 = \frac{-t_0^*-4(A-c)+6(2-\delta)d+3\tau+2\tau^*}{7} = f_1(t_0^*)$$
$$t_0^* = \frac{-t_0-4(A-c)+6(2-\delta)d+2\tau+3\tau^*}{7} = f_1^*(t_0). \tag{7}$$

The reaction curves of the taxes are downward-sloping, and their intersection becomes equilibrium emissions taxes in the first stage, as shown in Figure 1 (see Appendix I-(i)), where $f_1(t_0^*)$ and $f_1^*(t_0)$ are the reaction curves on the environmental taxes of the Home and the Foreign countries, respectively. These functions show how the Home (Foreign) country's government reacts to a change in the other country' tax. There is interdependency between the taxes (due to the existence of pollution spillovers across borders).

Solving the two reaction functions in (7), we obtain the following equilibrium emissions taxes ($\dot{t}_0$, $\dot{t}_0^*$):

$$\dot{t}_0 = \frac{-24(A-c)+36(2-\delta)d+19\tau+11\tau^*}{48}$$
$$\dot{t}_0^* = \frac{-24(A-c)+36(2-\delta)d+11\tau+19\tau^*}{48}. \tag{8}$$

Let us see the properties of the equilibrium in the first stage. One country's emissions tax

[22] These conditions become $d^2W_0/(dt_0)^2 = d^2W_0^*/(dt_0^*)^2 = -\frac{7}{9} < 0$.

[23] It is assumed for the time being that both $[-4(A-c)+6(2-\delta)d+3\tau+2\tau^*]$ and $[-4(A-c)+6(2-\delta)d+2\tau+3\tau^*]$ in the numerators in the second part of (7) are positive.

depends on both countries' tariffs. Moreover, the domestic country's tariff has a greater impact on its tax than the foreign country's tariff. This result is apparently different from the results of Lai and Hu (2005, 2008), derived under consumption pollution.

We examine the effects of the marginal environmental damage on the taxes and outputs. By differentiating (8) with respect to d, we have $\frac{dt_0}{dd} = \frac{dt_0^*}{dd} = \frac{3(2-\delta)}{4} > 0$: The emissions taxes are an increasing function of the marginal environmental damage. Increased marginal environmental damage leads to an increase in each of the environmental taxes in order to prevent further environmental degradation.

Next, when examining the effects of the degree of transboundary pollution on taxes, we have $dt_0/d\delta = dt_0^*/d\delta = -3d/4 < 0$. Moreover, we find from (7) that the tax responses of both governments to a change in the degrees of the social environmental damage and the transboundary pollution are unambiguously different, although both increases finally have the impacts which make the social environmental damage greater. This difference is explained as follows. When the marginal environmental damage increases, the domestic government raises its environmental tax to prevent the damage from increasing, while when pollution is increased by an increased degree of transboundary pollution, the government reduces its tax because it gives domestic production priority to reduce transboundary pollution. In addition, it follows from (8) that the effects of the tariffs on the emissions taxes of the Home and the Foreign countries are positive, and their own effects are larger than their cross effects, i.e., $\frac{dt_0}{d\tau} = \frac{dt_0^*}{d\tau^*} = \frac{19}{48} > 0$ and $\frac{dt_0}{d\tau^*} = \frac{dt_0^*}{d\tau} = \frac{11}{48} > 0$. The emissions taxes, t_0 and t_0^*, are an increasing function of both tariffs. On the other hand, increased tariffs cause their reaction curves to shift outward, so that the equilibrium in the first stage moves upward, as shown in Figure 2 (Appendix I-(i)). As an increase in the tariffs leads to both a reduction in imports and a rise in domestic production, each government raises the emissions tax as a remedy for the social environmental damage. Now we find that the emissions tax does not equal the Pigouvian tax even if free trade between the two countries holds.

By substituting (8) into (5), we obtain the following outputs:

$$\dot{q}_1 = \frac{24[(A-c)-d] + 12\delta d + 7\tau - \tau^*}{48}, \quad \dot{q}_2 = \frac{24[(A-c)-d] + 12\delta d - 33\tau - 9\tau^*}{48}$$

$$\dot{q}_1^* = \frac{24[(A-c)-d] + 12\delta d - 9\tau - 33\tau^*}{48}, \quad \dot{q}_2^* = \frac{24[(A-c)-d] + 12\delta d - \tau + 7\tau^*}{48} \tag{9}$$

$$\dot{Q} = \frac{48[(A-c)-d] + 24\delta d - 26\tau - 10\tau^*}{48}, \quad \dot{Q}^* = \frac{48[(A-c)-d] + 24\delta d - 10\tau - 26\tau^*}{48}.$$

$$\dot{Q}_{PP}^1 = \frac{48[(A-c)-d] + 24\delta d - 2\tau - 34\tau^*}{48}, \quad \dot{Q}_{PP}^2 = \frac{48[(A-c)-d] + 24\delta d - 34\tau - 2\tau^*}{48},$$

where $\dot{Q}_{PP}^1 = q_1 + q_1^*$ and $\dot{Q}_{PP}^2 = q_2 + q_2^*$ stand for the total outputs in the countries 1 and 2, respectively, and subscript "PP" stands for production pollution. When taking the effects of tariffs on the environmental taxes, the sum of the direct and indirect effects is obtained as follows:

$$\frac{dq_1}{d\tau} = \frac{dq_2^*}{d\tau^*} = \frac{7}{48} > 0, \quad \frac{dq_1}{d\tau^*} = \frac{dq_2^*}{d\tau} = -\frac{1}{48} < 0$$

$$\frac{dq_2}{d\tau} = \frac{dq_1^*}{d\tau^*} = -\frac{33}{48} < 0, \quad \frac{dq_2}{d\tau^*} = \frac{dq_1^*}{d\tau} = -\frac{9}{48} < 0. \tag{10}$$

We notice that a unilateral reduction in a tariff leads to an increase in the goods supplied for the domestic and the foreign markets in each country, except for the first two results in (10). Moreover, such a reduction in the domestic (foreign) tariff leads to an increase in the total consumption in the domestic (foreign) market, i.e., $dQ/d\tau < 0$ $(dQ^*/d\tau^* < 0)$. Further, we have $\frac{dQ^*}{d\tau}(\frac{dQ}{d\tau^*}) = -\frac{5}{24} < 0$: The cross-effect of a domestic (foreign) tariff on foreign (domestic) total output becomes negative. This implies that an increase in the domestic tariff leads to a decrease in transboundary pollution through a reduction in the foreign production. That is, tariffs play a beneficial role in reducing transboundary pollution. Besides, we realize the interesting fact from (9) that a raised δ leads to increased outputs, although such a rise implies an increase in the domestic environmental damage. Emissions taxes are largely involved in this result. As explained previously, this is because tariff increase reduces the emissions taxes, which in turn increase outputs.

RTAs are one of international methods to reduce tariffs and nontariff barriers, except for spontaneous reductions in them. For example, the conclusion of an RTA between two countries is the driving force behind trade liberalization. It seems that the trend of global trade liberalization cannot be stopped nowadays[24]. Then, we turn to the consideration of a bilateral reduction in tariffs on welfare and social environmental damage in both countries with production pollution[25].

Suppose that both countries reach an RTA and then engage in mutual tariff reduction[26]. That is, the two countries reduce tariffs by the same extent, i.e., $d\tau = d\tau^*$, by mutual

[24] See https://rtais.wto.org/UI/PublicMaintainRTAHome.aspx for RTAs. According to WTO, the cumulative number of RTAs in force is 355 in February 2023.

[25] Both countries have consultations on tariff reduction, not on other topics such as a reduction in transboundary pollution and cooperative tax decisions.

[26] RTAs include Free Trade Agreements (FTAs) and Customs Unions (CUs).

agreement. Before this consideration, let us examine the relationship between trade liberalization and the outputs of the firms. Totally differentiating q_i and q_i^* with respect to $d\tau$ and $d\tau^*$, using the results of (10) and setting $d\tau = d\tau^*$, we obtain

$$\left.\frac{dQ}{d\tau}\right|_{d\tau = d\tau^*} = \left.\frac{dQ^*}{d\tau^*}\right|_{d\tau = d\tau^*} = -\frac{3}{4} < 0. \tag{11}$$

This shows that trade liberalization leads to an increase in supply for every market and consequently reduces the two market prices.

We take an interest in the relationship between the economy of the country suffering pollution and trade liberalization. Specifically, we would like to investigate how the governments' bilateral actions about trade liberalization influence social environmental damage, welfare and so on. To see the effect of trade liberalization on environmental pollution generated by production behavior, totally differentiating D_0 with respect to $d\tau$ and $d\tau^*$ by using (10) and setting $d\tau = d\tau^*$, we have

$$\left.\frac{dD_0}{d\tau}\right|_{d\tau = d\tau^*} = \frac{\partial D_0}{\partial \tau} + \frac{\partial D_0}{\partial \tau^*} = -\frac{(1+\delta)d}{3} - \frac{10(1+\delta)d}{24} = -\frac{3(1+\delta)d}{4} < 0. \tag{12}$$

The effect of a bilateral tariff reduction on the social environmental damage to the domestic country is divided into two effects, i.e., the direct and the indirect effects. Specifically, the first term in the third part of (12) is the direct effect, which is negative, and the second term is the indirect effect, which is also negative (see Appendix I-(iii)). The direct effect is the one that a change in the bilateral tariff directly affects social environmental damage, keeping the emissions taxes constant, and the indirect effect is the one that such a change affects the damage only through the emissions taxes[27]. That is, this effect is a strategic effect which reflects environmental policy of each government. In this effect, such a tariff reduction causes the emissions taxes to go down, so that consumption in both markets increases, as shown in (11), leading to increased local and global pollution. Since the two effects work in the same direction, trade liberalization has a detrimental effect on environmental improvement[28]. To improve the environment under trade liberalization we need to use some other environmental policy, e.g., process standards. From this result we have the following proposition.

Proposition 1. In the presence of production pollution, a bilateral tariff reduction leads to deterioration in the social environmental damage of the Home country.

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[27] Lai and Hu (2005) call the indirect effect "the linkage effect".

[28] Forslid, Okubo and Sanctuary (2017) show that trade liberalization does not affect global emissions when environmental taxes are symmetric by employing a monopolistic competitive model, but otherwise it affects them.

The same result for the foreign country holds by symmetry. This proposition is the opposite of Lai and Hu's Proposition 1 obtained under consumption pollution, which is also shown in Section 4. The effect on the environmental damage becomes negative in the presence of production pollution by an increase of an emissions tax in response to a tariff reduction, but positive in the presence of consumption pollution. The cause of this lies in the indirect effect. Although trade liberalization may be beneficial for welfare without pollution, it may be detrimental to welfare with it[29]. Therefore, if the countries pursue lowering tariff barriers together, they must simultaneously tighten environmental regulations through policies other than tariff reduction policy to keep the level of pollution emissions constant. Even in the case without transboundary pollution, lowering tariffs causes increased environmental damage, while the presence of its pollution magnifies the damage. Although tariff reduction can achieve its original purpose, it obviously brings a side effect at the same time. Interestingly, by the introduction of an emissions tax, trade liberalization increases the amounts of pollutants more when there is an emissions tax. This is because the existence of an emissions tax degrades the environment, as shown previously.

Before we investigate the effect of a bilateral tariff reduction on welfare, let us see the effects on consumer surplus, and producer surplus (profits), in turn.

1) Consumer surplus

We examine the effect of a tariff reduction on domestic consumer surplus. It is divided into the direct and indirect effects, as shown previously. Then, the direct and the indirect effects of the tariff reduction on its consumer surplus both become negative (see Appendix I-(iv)). The former shows that tariff reduction increases consumer surplus, *ceteris paribus*. Namely, when pollution taxes are kept constant, mutual tariff reduction leads to an expansion in imports, raising domestic consumption. The latter shows that domestic consumption increases in response to mutually reduced tariffs when the domestic government can adjust its emissions tax on that reduction. Consequently, the total of the two effects become negative. We conclude that trade liberalization policy is beneficial for the Home country from a viewpoint of consumer surplus.

2) Producer surplus (profits)

We consider the effect on producer surplus. Now the producer surplus of the domestic country is shown by $PS_0 = (q_1)^2 + (q_1^*)^2$ when we use (3-1) and (3-2). We totally differentiate the PS_0 function with respect to tariffs, τ and τ^*, and then obtain

$$dPS_0 = \frac{\partial PS_0}{\partial \tau} d\tau + \frac{\partial PS_0}{\partial \tau^*} d\tau^*.$$

[29] As a paper proceeding Lai and Hu (2005), there is Burguet and Sempere (2003). They point out that a bilateral tariff reduction increases welfare in the absence of transboundary pollution.

We have (see Appendix I-(iv)):

$$\left.\frac{dPS_0}{d\tau}\right|_{d\tau = d\tau^*} = \frac{\partial PS_0}{\partial \tau} + \frac{\partial PS_0}{\partial \tau^*} = -\frac{216[(A-c)-d] + 108\delta d - 105\tau - 345\tau^*}{288}.$$

As the direct and the indirect effects are of either sign, so that the total effect also becomes ambiguous. Thus, it is not obvious whether a mutual reduction in tariffs increases or decreases producer surplus (the profits of the domestic firm). This result is of interest, compared with the result in the presence of consumption pollution (see the following section).

3) Welfare

We turn to the effect on domestic welfare. Totally differentiating the welfare function W_0 with respect to tariffs, τ and τ^*, we obtain:

$$dW_0 = \frac{\partial W_0}{\partial \tau} d\tau + \frac{\partial W_0}{\partial \tau^*} d\tau^*.$$

Then the total effect is given by setting $d\tau = d\tau^*$ as follows (see Appendix I-(iv)):

$$\left.\frac{dW_0}{d\tau}\right|_{d\tau = d\tau^*} = \frac{-642\tau + 318\tau^* + 648\delta d}{(24)^2}. \tag{13}$$

It is ambiguous whether the lowering of bilateral tariffs increases Home country welfare, i.e., $\frac{dW_0}{d\tau} \gtreqless 0$. This is the same as in the Foreign country. What plays an important role in examining the effect is the values of the parameters δ and d. Next, we divide that effect into the direct and the indirect effects on the welfare (see Appendix I-(iv)):

$$\text{The direct effect} = \frac{96(A-c) - 588\tau - 492\tau^* - 48(2+5\delta)d}{(24)^2},$$

$$\text{The indirect effect} = \frac{-96(A-c) - 54\tau + 810\tau^* + 8(12+111\delta)d}{(24)^2}.$$

The direct and the indirect effects are both ambiguous. Even if there is no transboundary pollution, the two effects become ambiguous. Now, we establish the following proposition from the result (13).

Proposition 2. Suppose that there is no transboundary pollution. Then, in the presence of production pollution, a bilateral reduction in both tariffs leads to an increase in the welfare of

the domestic country as far as the difference between the rates of domestic and foreign tariff is limited to a small range.

This proposition is explained as follows. A bilateral reduction in the tariffs revitalizes exports, so that the market prices decrease through increased supply, and then the consumer surplus (CS_0) of the domestic country increases (see Appendix I-(iv)), while the effect on the social environmental damage (D_0) increases through expanded production, as shown above. Although the effect conflicts with the effect on consumer surplus, the latter effect seems to be greater than the former under no transboundary pollution. Burguet and Sempere (2003) also show that trade liberalization increases welfare when there is no cross-border pollution.

3.2. Case I-(2) Extension of the basic model with production pollution (endogenous tariffs)

In order to deepen our analysis, we extend the basic (two-stage game) model to a three-stage game model where the governments determine their tariffs in the first stage, and the governmental decision on an environmental tax and the firms' outputs are moved down in order[30]. Newly, we make tariffs endogenous which are noncooperatively chosen by each government to maximize its welfare. To obtain optimal tariffs, differentiating the welfare functions of the Home and Foreign countries yields

$$\frac{dW_0}{d\tau} = \frac{192[(A-c)-d] + 516\delta d \ - 577\tau - 65\tau^*}{(24)^2} = 0 \tag{14-1}$$

$$\frac{dW_0^*}{d\tau^*} = \frac{192[(A-c)-d] + 516\delta d \ - 65\tau - 577\tau^*}{(24)^2} = 0^{31}. \tag{14-2}$$

Now the welfare reaction curves are both downward sloping in a two-dimensional plane with respect to (τ, τ^*). Whether these curves are shifted inward or outward against a change in the marginal environmental damage depends on transboundary pollution, as well as market size and marginal production costs. Their intersection denotes an equilibrium in the third stage. At the equilibrium we have the optimal tariffs:

$$\dot{\tau} = \dot{\tau}^* = \frac{32[(A-c)-d] + 86\delta d}{107} > 0. \tag{15}$$

[30] For example, Youssef (2009) compares pollution, social welfare and a subsidy rate for R&D under autarky and the international trade economies by using a three-stage game model of two country-two polluting firms, where there is transboundary pollution, and the firms undertake R&D investment. Tanguay (2001) considers the relationship between emissions taxes and tariffs when these are simultaneously determined by the governments. In our analysis they are asynchronously determined. Further, Ulph and Ulph (1996) investigate the environmental policies of the governments and R&D strategies of the polluting domestic and foreign firms by employing a strategic three-stage game model with transboundary pollution.

[31] The second-order conditions for maximization with respect to tariffs are satisfied.

The result (15) shows that both tariffs are raised in response to an expansion in the market size, to prevent the environment from becoming worse. Transboundary pollution also has the effect of raising tariffs. Simultaneously, the magnitude of the effects of the marginal environmental damage on tariffs is affected by the amount of transboundary pollution. We establish the following proposition.

Proposition 3. In the production pollution, (i) an expansion in the market size increases the tariff rates, (ii) increased marginal environmental damage raises tariffs, and (iii) an increased degree of the transboundary pollution also raises tariffs.

The results (i), (ii), and (iii) are explained as follows. As for (i), the expansion of the market size leads to an increase in the consumption in each market, which in turn increases the environmental damage. To prevent the damage spreading the government raises its own tariff. As for (ii), the government needs to raise its tariff against increased marginal environmental damage in order to protect degradation in welfare. As for (iii), an increased intrusion rate of foreign pollution means an increase in the domestic environmental damage, so the domestic government raises tariff on imported goods in order to avoid further increases in the pollution of foreign origin.

Incidentally, in the three-stage game, the noncooperative optimal environmental taxes of both governments are obtained by substituting (15) into (8):

$$\dot{t_0} = \dot{t_0^*} = \frac{-67[(A-c)-d] + (214-53\delta)d}{214}. \qquad (8)'$$

The coefficient of the marginal environmental damage d in the numerator in (8)' is positive, as $\delta \in [0,1]$. The relationship between the environmental tax and the parameters, $(A-c)$, d, and δ, is indicated as follows. (i) An expansion in the market size reduces emissions taxes of both countries. This is because the government prefers increasing welfare through reducing the taxes. Conversely, (ii) an increase in the marginal environmental damage raises emissions taxes. (iii) An increase in transboundary pollution reduces emissions taxes, inversely.

Roughly, as for the results (i) and (iii), the governments' responses to a change in the parameters on their taxes are the opposite of their responses to the tariffs. If both pollutions are absent or the marginal environmental damage is considerably small, then the emissions taxes may become negative, namely, may be transformed to subsidies. This is the reason why governments need not impose an environmental tax under modest environmental damage from production activities.

We obtain the outputs for the domestic and the foreign markets from (9) and (15):

$$\dot{q}_1 = \dot{q}_2^* = \frac{115[(A-c)-d]+75\delta d}{214} \quad \text{and} \quad \dot{q}_1^* = \dot{q}_2 = \frac{51[(A-c)-d]-97\delta d}{214}. \tag{9)'}$$

We find that the supply for the domestic market of each country, $q_i + q_j$ or $q_i^* + q_j^*,\ i,\ j = 1, 2,\ \ i \neq j$, becomes less in the presence of transboundary pollution, and the same result holds with respect to the sum of the exports. A rise in transboundary pollution leads to a reduction in the supply for the domestic market of each country. This is explained as follows. As such a rise incurs more environmental damage through cross-boundary pollution, each government attempts to curb emissions by raising tariffs on imported goods, whereas the import substitution works in the domestic market, then the reduction effect of a tariff becomes larger. Further, total output of each country, $Q_{pp}^i = q_i + q_i^*,\ i = 1, 2$, is reduced by the presence of cross-border pollution.

Consequently, the SPNE in the presence of production pollution is represented from (15), (8)', and (9)' as follows:

$$\begin{aligned}
&\dot{\tau} = \dot{\tau}^* = \frac{32[(A-c)-d]+86\delta d}{107} \\
&\dot{t_0} = \dot{t}_0^* = \frac{-67[(A-c)-d]+(214-53\delta)d}{214} \\
&\dot{q}_1 = \dot{q}_2^* = \frac{115[(A-c)-d]+75\delta d}{214} \quad \text{and} \quad \dot{q}_1^* = \dot{q}_2 = \frac{51[(A-c)-d]-97\delta d}{214} \\
&\dot{Q} = \dot{Q}^* = \frac{83[(A-c)-d]-11\delta d}{214}.
\end{aligned} \tag{16}$$

We find that the equilibrium tariffs are positive. Next, we consider pollution associated with consumption.

4. Government behavior in the presence of consumer pollution: Case II

This section is composed of two parts: One is Case II-(1) Trade liberalization and its effects under consumption pollution; and the other is Case II-(2) Extension of the basic model with consumption pollution to a three-stage game model.

4.1. Case II-(1) Trade liberalization and its effects under consumption pollution

The model in Case II-(1) fundamentally invokes Lai and Hu (2005), who focus only on the two-stage game model. Particularly, our analysis in the subsection II-(1) overlaps with theirs. We extend their model to the three-stage game model later and then elucidate the properties of the SPNE by comparing with Case I.

We derive the SPNE when consumers emit pollution instead. First, $t_0 = t_0^* = 0$ and $D_0 = D_0^* = 0$ hold in (1-1), (1-2), (2-1) and (22). However, some pollution generated through consumption behavior in the foreign country cross into the domestic country, just like production pollution. That is, there occurs mutual transboundary pollution. To repeat, t_1 and t_1^* denote the environmental taxes of the Home and Foreign, respectively. Alternatively, we can interpret them as consumption taxes. The social environmental damage in the Home and Foreign due to emissions generated by consumption activities is given by D_1 and D_1^*, as mentioned before. Then, the profit functions of the home and foreign firms are reduced to:

$$\pi_1 = [p(Q) - c - t_1] \cdot q_1 + [p(Q^*) - c - t_1^* - \tau^*] \cdot q_1^*, \qquad (1\text{-}1)''$$

and

$$\pi_1^* = [p(Q^*) - c - t_1^*] \cdot q_2^* + [p(Q) - c - t_1 - \tau] \cdot q_2. \qquad (1\text{-}2)''$$

The profits also denote the respective producer surpluses.

First, let us solve the two-stage model where the governments choose environmental taxes in the first stage, and firms choose outputs for the domestic and the foreign markets in the second stage. We derive the first-order conditions for profit maximization of the firms. Differentiating (1-1)" with respect to the products for both markets of the domestic firm yields

$$\frac{d\pi_1}{dq_1} = A - c - t_1 - 2q_1 - q_2 = 0 \qquad (17\text{-}1)$$

$$\frac{d\pi_1}{dq_1^*} = A - c - t_1^* - \tau^* - 2q_1^* - q_2^* = 0, \qquad (17\text{-}2)$$

and differentiating (1-2)" with respect to those of the foreign firm also yields

$$\frac{d\pi_1^*}{dq_2} = A - c - t_1 - \tau - q_1 - 2q_2 = 0 \qquad (18\text{-}1)$$

$$\frac{d\pi_1^*}{dq_2^*} = A - c - t_1^* - q_1^* - 2q_2^* = 0. \qquad (18\text{-}2)$$

The second-order conditions for maximization are satisfied. When solving the first-order conditions (17-1) to (18-2) with respect to outputs, given emissions taxes and tariffs, we have

$$q_1 = \frac{A - c - t_1 + \tau}{3},\ q_2 = \frac{A - c - t_1 - 2\tau}{3} \qquad \text{for the domestic market}$$

$$q_1^* = \frac{A - c - t_1^* - 2\tau^*}{3},\ q_2^* = \frac{A - c - t_1^* + \tau^*}{3} \qquad \text{for the foreign market.} \qquad (19)$$

In consequence, the products of the domestic and foreign firms become decreasing functions of emissions taxes, i.e., $q_i = q_i(t_i;\ \tau)$ and $q_i^* = q_i^*(t_i^*;\ \tau^*), i = 1, 2,$ for given tariffs, respectively. Because domestic consumption $(Q = q_1 + q_2)$ is affected only by its own environmental policy, except for the marginal production cost of the domestic firm, the domestic and foreign markets are segmented, different from the case of production pollution shown in (5).

Determining comparative statics with respect to environmental taxes by using (19), we obtain the following:

$$\frac{dq_1}{dt_1} = \frac{dq_2}{dt_1} = -\frac{1}{3} < 0, \text{ and } \frac{dq_1^*}{dt_1^*} = \frac{dq_2^*}{dt_1^*} = -\frac{1}{3} < 0. \tag{20}$$

This shows that a unilateral increase in the environmental tax causes a decrease in outputs of each firm and then shifts its associated pollution to the foreign country (the pollution-shifting effect). The rent-shifting effect does not hold, but rather the anti-rent-shifting effect holds.

Incidentally, goods 1 and 2 cause social environmental damages in domestic and foreign countries along with consumption in each country, respectively. Moreover, since the foreign country cannot confine all of pollution caused by her consumption behavior to her territory, parts of the social environmental damage (transboundary pollution) from the territory are carried into the domestic country. Since cross-border spillovers of emissions occur globally, the governments must take transboundary pollution into consideration when framing environmental policy.

We move on to the first stage in which the governments choose environmental taxes. The social welfare of the country is the sum of domestic consumer surplus, domestic producer surplus (the profits of the domestic firm), and the government tax and tariff revenues, net of environmental damage. Then, the welfare functions (2-1) and (2-2) of the Home and the Foreign countries are reduced to:

$$W_1 = \frac{1}{2}Q^2 + [p(Q) - c] \cdot q_1 + [p(Q^*) - c - t_1^* - \tau^*] \cdot q_1^* + (t_1 + \tau) \cdot q_2 - D_1, \tag{2-1}"$$

and

$$W_1^* = \frac{1}{2}(Q^*)^2 + [p(Q^*) - c] \cdot q_2^* + [p(Q) - c - t_1 - \tau] \cdot q_2 + (t_1^* + \tau^*) \cdot q_1^* - D_1^*, \tag{2-2}"$$

where the social environmental damage generated by the production activities of the two countries are supposed to be zero, i.e., $D_0 = D_0^* = 0$. The governmental revenues of the Home are given by $(t_1 + \tau) \cdot q_2$ in (2-1)" where $t_1 q_2$ and τq_2 are the environmental tax and the tariff (GTTR), respectively. Interestingly, the environmental tax revenues appear in the welfare functions of (2-1)" and (2-2)", whereas the government revenues are canceled out and

generally disappear from the welfare functions of (2-1)' and (2-2)' in the presence of production pollution. The producer surplus, or profit, in the presence of consumption pollution is shown by $PS_1 = (q_1)^2 + (q_1^*)^2$. The sum of the second and third terms in (2-1)" is $QPS_1 = PS_1 + (t_1 q_1 + t_1^* q_1^*)$.

The government of each country simultaneously and noncooperatively maximizes its social welfare with respect to its own environmental tax. Given the tariffs of the two countries, i. e., τ and τ^*, when differentiating (2-1)" and (2-2)" with respect to the emission taxes of the domestic and foreign countries and using the results (20), we have the following first-order conditions for their social welfare maximization:

$$\frac{dW_1}{dt_1} = \frac{1}{3}[q_1 + 2q_2 - (A - c) - (t_1 + \tau) + 2d] = \frac{2}{3}(-t_1 - \tau + d) = 0$$

$$\frac{dW_1^*}{dt_1^*} = \frac{1}{3}[2q_1^* + q_2^* - (A - c) - (t_1^* + \tau^*) + 2d] = \frac{2}{3}(-t_1^* - \tau^* + d) = 0.$$

Solving these conditions with respect to the emissions taxes yields

$$\ddot{t}_1 = d - \tau \text{ and } \ddot{t}_1^* = d - \tau^*, \tag{21}$$

where superscript " $\ddot{}$ " stands for the equilibrium values under consumption pollution[32]. This shows that the emissions tax is less than the Pigouvian tax by the size of the tariff. If there are no tariffs or free trade is held, then both taxes become equal. Alternatively, we realize that the environmental policies are also segmented by a reflection of market segmentation, respectively. That is, the emissions tax of the domestic government is influenced only by its tariff.

The occurrence of market segmentation has been pointed out by Kayalica and Kayalica (2005), and Lai and Hu (2005, 2008). In particular, Lai and Hu (2008) conclude that market segmentation is due to the linearity of the environmental damage functions. However, their conclusion is imperfect. As shown in Case I, even if the functions are linear, such market segmentation does not appear under production pollution.

Two factors are needed for market segmentation, i.e., consumption pollution, the linearity of the social environmental damage function and its structure. In fact, market segmentation is closely related with consumption pollution. This lies in a unique characteristic of consumption pollution - the domestic government cannot control emissions generated by foreign consumers by its tariff and environmental policies. If you turn your back, then market

[32] The optimal emissions taxes (21) derived under the two firms having the same cost function correspond to the degenerated results (11) and (12) of Lai and Hu (2005). They also derive the optimal emissions taxes under a quadratic environmental damage function in their Appendix.

segmentation will not be established when environmental damage functions are non-linear. This is the reason that mutual interdependency in the final product markets exists. Therefore, the domestic government can indirectly influence consumption (and then its transboundary pollution) in the foreign country through the mutual interdependency.

On top of that, we find that there is a reverse relationship between emissions taxes and tariffs, whereas such a reverse relationship does not appear in the presence of production pollution, as shown by (8). This relationship is explained as follows. A reduction in the tariff increases domestic consumption through a reduction in market price, then increasing welfare losses. On the contrary, the government raises its emissions tax to decrease domestic emissions. We realize that there is a possibility that an emissions tax may become an emissions subsidy as a special case when either the degree of the marginal environmental damage is small, or the rates of tariffs are high.

We obtain the equilibrium outputs by substituting the emissions taxes (21) into (19) as follows:

$$\begin{aligned} \ddot{q}_1 &= \frac{[(A-c)-d]+2\tau}{3}, \quad \ddot{q}_2 = \frac{[(A-c)-d]-\tau}{3} \\ \ddot{q}_1^* &= \frac{[(A-c)-d]-\tau^*}{3}, \quad \ddot{q}_2^* = \frac{[(A-c)-d]+2\tau^*}{3} \\ \ddot{Q} &= \frac{2[(A-c)-d]+\tau}{3}, \quad \ddot{Q}^* = \frac{2[(A-c)-d]+\tau^*}{3}. \end{aligned} \tag{22}$$

It is interesting that the equilibrium outputs are determined free from transboundary pollution, different from the outputs (9) in the presence of production pollution. From the results above the impacts of each tariff are given by

$$\frac{dq_1}{d\tau} = \frac{dq_2^*}{d\tau^*} = \frac{2}{3}, \quad \frac{dq_2}{d\tau} = \frac{dq_1^*}{d\tau^*} = -\frac{1}{3}, \quad \frac{dQ}{d\tau} = \frac{dQ^*}{d\tau^*} = \frac{1}{3}. \tag{23}$$

A unilateral reduction in the tariff in each country leads to a reduction in domestic consumption. This is because such a reduction causes production for the domestic market to decrease and imports to increase, and, moreover, the decrease in the former is greater than the increase in the latter. We find from (22) that there is no cross-effect of domestic tariffs and foreign production. Therefore, the domestic government cannot control emissions, i.e., transboundary pollution, generated by consumption in the foreign country even with customs and environmental policies[33]. This is apparently different from the result under production pollution. Thus, our

[33] See, for example, Copeland (1996) for uncontrollability over foreign consumption by the domestic government.

consideration of consumption pollution provides new and different insights.

Let us see the effect of trade liberalization on the social environmental damage generated by the consumption activities of the Home and Foreign countries. Totally differentiating D_1 with respect to $d\tau$ and $d\tau^*$ and setting $d\tau = d\tau^*$, we have

$$\left.\frac{dD_1}{d\tau}\right|^{CP}_{d\tau = d\tau^*} = -\frac{(1+\delta)d}{3} + \frac{2(1+\delta)d}{3} = \frac{(1+\delta)d}{3} > 0, \tag{24}$$

where superscript "CP" stands for a variable in the case of consumption pollution. This result shows that a bilateral tariff reduction decreases the social environmental damage[34]. Now the effect of such a reduction is divided into two effects, i.e., the direct and indirect effects. The direct effect catches the impact of the reduction on the social environmental damage, keeping emissions taxes constant, and the indirect effect catches the impact on the social environmental damage through the emissions taxes, keeping the tariffs constant in exchange for them. The first term in the second part of (24) is the direct effect, which is negative, and the second term is the indirect effect, which is positive. If taxes do not change with tariffs, the direct effect implies that the bilateral tariff reduction will lead to increased outputs and exacerbate the environmental damage. On the other hand, if the reduction causes the emissions taxes to rise, consumption in the domestic market decreases, then simultaneously incurring both a reduction in local and global pollution. Although the two effects in (24) work in the opposite directions, the indirect effect becomes larger than the direct effect, so that the trade liberalization policy has a beneficial impact on environmental improvement. We need not use another environmental policy at the same time. From this result we have the following proposition[35].

Proposition 4. In the presence of consumption pollution, a bilateral tariff reduction in the Home and the Foreign countries leads to a reduction in the social environmental damage of the Home country.

This result also holds in the case of the Foreign country from the nature of the model. The result of Proposition 4 is obviously opposite to that of Proposition 1 in the presence of production pollution. Namely, there is a difference between the effects of trade liberalization on the environment, depending on the source of pollution.

Secondly, let us take the effect of a bilateral tariff reduction on consumer surplus. Its direct and indirect effects become negative and positive, respectively. That is, the direct effect shows that consumer surplus increases with tariff reduction, *ceteris paribus*, whereas the indirect

[34] Chen and Wang (2010) show that trade liberalization leads to a reduction in the social environmental damage in an international mixed duopoly with consumption pollution and its transboundary spillovers.
[35] This result is also the same as Proposition 1 derived by Lai and Hu (2005).

effect shows that consumer surplus decreases with that (see Appendix II-(iii)). When the emissions taxes are kept constant, a reduction in tariffs leads to an expansion in imports, so that consumption in the domestic country is increased. By contrast, when the government can adjust emissions tax to reduce the social environmental damage, the raised tariffs lead to a decrease in consumption, decreasing consumer surplus. The total of the two effects becomes positive because the indirect effect is greater than the direct effect. Then, as trade liberalization invites a reduction in consumer surplus, this policy may be rather detrimental to the consumer. In contrast, this policy leads to an increase in the consumer surplus in the presence of production pollution, as shown previously. Trade liberalization policy is beneficial to the consumer when contaminants are emitted through production activities, but detrimental when they are emitted through consumption activities. This may be viewed as a novel insights from this research.

Thirdly, let us consider the effect on producer surplus. This effect is also divided into the direct and the indirect effects, which become ambiguous. However, the total of these effects becomes positive. That is, a bilateral reduction in tariffs decreases producer surplus (see Appendix II-(iii)). This implies that trade liberalization is detrimental to each firm. We realize that the direct and the indirect effects of tariffs on producer surplus are derived, perfectly independent of transboundary pollution, different from the case of production pollution. Only the magnitude of D_1 is influenced by pollution from the neighboring country. Therefore, the firms need not take transboundary pollution into consideration in their decision making. Then we have the following proposition:

Proposition 5. In the presence of consumption pollution, both consumer surplus and producer surplus of the Home country decrease with a bilateral reduction in tariffs and are determined independently of transboundary pollution from the Foreign country.

Comparing the results of Proposition 5 with the results obtained under production pollution, we notice the following two points. One, the effects of such a reduction on consumer surplus and producer surplus under consumption pollution are different from these under production pollution. Concretely, the result as to consumer surplus under consumption pollution is the inverse of that under production pollution. Two, transboundary pollution from the foreign country has no impact on consumer surplus, whereas this result does not hold under production pollution. Whether its pollution is included in the effects on both consumer surplus and producer surplus depends crucially on the source of emissions[36]. It is unclear whether

[36] Whether transboundary pollution influences the outputs and emissions taxes depends crucially on the functional form of the social environmental damage. Put it differently, this result will not hold when we assume a quadratic environment damage function.

such distinction is made in environmental policymaking in practice.[37]

Lastly, we turn to the effect of trade liberalization on the welfare of the domestic country. We totally differentiate the welfare function W_1 with respect to tariffs, τ and τ^*, as follows:

$$dW_1 = \frac{\partial W_1}{\partial \tau} d\tau + \frac{\partial W_1}{\partial \tau^*} d\tau^*.$$

Then the effect on the welfare is given by (see Appendix II-(iii))[38] :

$$\left.\frac{dW_1}{d\tau}\right|^{CP}_{d\tau = d\tau^*} = \frac{[(A-c) - d - \tau^*] - 3(\tau - \tau^* + \delta d)}{9}. \tag{25}$$

This result is the same as in the Foreign country. In order to investigate the effect of trade liberalization on the welfare, let us examine its direct and indirect effects on the welfare. The two effects on welfare are given as follows, respectively (see Appendix II-(iii)):

$$\text{The direct effect on welfare} = -\frac{[(A-c) - d - \tau^*] - 3\delta d}{9},$$

$$\text{The indirect effect on welfare} = \frac{2[(A-c) - d - \tau] - \tau + \tau^* - 6\delta d}{9}.$$

The equation (25) is the sum of the two effects, which are ambiguous. The direct effect becomes negative if there is no transboundary pollution. On the other hand, the indirect effect becomes positive if there is no transboundary pollution and if the tariffs are the same. Although the signs of these effects are different, the total effect becomes positive under the assumptions of no transboundary pollution and the same tariff rate (see (25))[39]. A bilateral tariff reduction decreases welfare. In contrast, trade liberalization increases welfare under the same conditions in the presence of production pollution (see (13)). Furthermore, such a reduction makes domestic consumption lower, as shown by (22). Secondly, the social environmental damage (D_1) is also decreased by a bilateral reduction in tariffs, because such a reduction decreases consumption in each of the segmented markets. This has a little explanation. The bilateral reduction acts to increase welfare by a decrease in the related social environmental damage. Both consumer and producer surplus are decreased by a bilateral tariff

[37] In practice, there may also be practical issues with separating the source of emissions, especially when the sources are quite geographically dispersed even with the same nation.

[38] This result is also the same as the result (14), derived by Lai and Hu (2005). They consider the effect of a bilateral tariff reduction on the welfare of each country in the case where the social environmental damage function is quadratic (see their Appendix).

[39] Kayalica and Kayalica (2005) show that bilateral reductions in both consumption taxes and tariffs of two countries with only local consumption pollution are Pareto-improving.

reduction. Eventually, since the responses of the main three components to a bilateral change of tariffs vary, we cannot generally specify the benefit of trade liberalization on welfare. Then, the following proposition holds.

Proposition 6. Suppose that there is no transboundary pollution. Then, in the presence of consumption pollution, a bilateral reduction in both tariffs leads to a decrease in the welfare of the Home country when the domestic and foreign tariffs are the same.

This result is in conflict with the result in the presence of production pollution.

4.2. Case II- (2) Extension of the basic model

Let us turn to the extension of the basic model with endogenized tariffs. Namely, both governments determine their tariffs in the first stage, as in Case I-(2). In this extended model emissions taxes and outputs come to be determined in the second and third stages, respectively.

The decisions in the third and second stages have already been made. So, when differentiating the welfare function (2-1)" and (2-2)" with respect to tariffs, i.e., τ and τ^*, and arranging it, we obtain the first-order conditions for welfare maximization, respectively:

$$\frac{dW_1}{d\tau} = \frac{[(A-c)-d]-\tau}{3} = 0 \tag{26-1}$$

$$\frac{dW_1^*}{d\tau^*} = \frac{[(A-c)-d]-\tau^*}{3} = 0. \tag{26-2}$$

There are no interdependencies between domestic tariff and foreign tariff, so that each government independently chooses its tariff. From (26-1) and (26-2) we provisionally have the following domestic and foreign tariffs:

$$\tau = \tau^* = (A-c)-d. \tag{27}$$

This is not the final solution. Further, it follows from (21) and (27) that the environmental taxes are derived as follows:

$$\ddot{t}_1 = \ddot{t}_1^* = -[(A-c)-d]+d. \tag{28}$$

When substituting (27) into exports $(\ddot{q}_1^*, \ddot{q}_2)$ in (22), we find that the two countries do not trade with each other, i.e., $(\ddot{q}_1^*, \ddot{q}_2) = 0$. That is, each firm switches from overseas exports to sales to the domestic market. Therefore, each government does not employ tariffs as trade policy. Consequently, the equilibrium in the first stage turns out to be a corner solution. Taking account

of this result, we have the following SPNE in the three-stage game under consumption pollution:

$$\begin{aligned} &\ddot{\tau} = \ddot{\tau}^* = 0 \\ &\ddot{t}_1 = \ddot{t}_1^* = -\left[(A-c)-d\right]+d \\ &\ddot{q}_1 = \ddot{q}_2^* = (A-c)\ -d, \text{ and } \ddot{q}_1^* = \ddot{q}_2 = 0. \end{aligned} \tag{29}$$

In deriving the SPNE, we should remember that we do not assume $(A-c)=d$. We find from (29) that when the marginal environmental damage is raised, the emissions taxes are raised, but the outputs are reduced. This is explained as follows. The reason why governments raise emissions taxes is to reduce emissions, and a reduction in the outputs occurs because the marginal production costs increase. It is interesting to note that cross-border pollution from the foreign country never influences the decisions of the domestic government and the domestic firm under consumption pollution, different from the results under production pollution. We come to the following proposition from these results:

Proposition 7. Suppose that pollution is generated by consumption activities. (i) The economies of two countries are reduced to be closed economies in the three-stage game. (ii) The environmental policies of each government are narrowed down to one policy, i.e., environmental policy.

We realize that there is no trade between the two countries in the three-stage game under consumption pollution, and market segmentation entirely establishes. Namely, there is no interdependency between emissions taxes of the Home and Foreign countries. An anomalous result like this may be limited to the three-stage game with both consumption pollution and a linear environmental damage function, because it does not occur in the two-stage game with them (see, for example, Tanguay (2001), Kayalica and Kayakica (2005), and Lai and Hu (2008)). Since the economies of the two countries become closed, the policy choice of each government is limited to an environmental policy. That is, governments lose trade policy as a policy tool. This outcome is obviously different from that under production pollution. On the other hand, Lai and Hu (2008) show that the domestic and the foreign governments are engaged in free trade when they cooperate in environmental taxes and tariffs.

What does it mean that there are no terms standing for transboundary pollution in the domestic equilibrium emissions taxes and equilibrium outputs in (29) even if foreign pollution emissions generate and cross the border into the domestic country? In other words, why are such decision variables determined as if emissions do not cross the border? An answer to this question may lie in market segmentation based on uncontrollability of the domestic government over them. Consequently, the emissions taxes and the outputs may be distorted by unawareness

of transboundary pollution and then become an inappropriate value from viewpoints of welfare and profit maximization, respectively. We note that in making its decisions the domestic government does not consider global emissions generated by consumption activities, but global emissions are correctly reflected in its decisions under production pollution *per se* (see the SPNE in Case I-(2))[40].

Then, we conclude that governments may not choose the optimal environmental taxes and the firms not the optimal outputs, because they do not take account of the existence of global emissions. This means that environmental policy to attain the maximization of welfare is not fully functional. This is called government policy failure. Thus, it follows that in addition to the deadweight losses in market imperfection and negative externalities due to local pollution, the government failure occurs in the presence of cross-border emissions based on consumption activities. One way to avoid the problem of such policy failure is that both governments need to keep in close contact on consumption pollution.

The environmental and trade policies of both countries are not affected by the other country's emissions as far as their governments take optimal behavior on the choices of an emissions tax and a tariff, different from the case of production pollution. The occurrence of this anomality is due to market segmentation. That is, the demand for good of each consumer in the domestic country is affected by the marginal production cost of its firm and its emissions tax and its tariff, but not by the marginal production cost of the foreign firm, and the environmental policy of the foreign country. Furthermore, the more interesting result is the result (i) of Proposition 7. Namely, the domestic firm's profits depend only on domestic market, by market segmentation, so that it supplies the product only to the home market. However, the conversion of the economy from an open to a closed economies like this never happens when there is production pollution. Consequently, the domestic government loses its trade method to control imports, which in turn affects foreign emissions.

5. Comparisons of the results

It is meaningful that we clarify how a difference in the generation of production pollution and consumption pollution creates a difference in the performance, and environmental and trade policies of countries. First, compare the comparative static results with respect to consumer surplus, producer surplus, social environmental damage, and welfare obtained under trade liberalization in the presence of production pollution and consumption pollution in the two-stage game (see Appendix III (i)). Second, we compare the SPNE values with production and consumption pollution in the three-stage game (with endogenous tariffs).

[40] For example, Barcena-Ruiz, Carlos and Garzon (2006), and Pal and Saha (2015) examine the relationship between firm privatization and social environmental damage by using a mixed oligopoly under production pollution where a public firm does not take its damage into consideration.

5-1. Comparison of the effects of trade liberalization

We compare the effects of a mutual tariff reduction on consumer surplus (CS), producer surplus (PS), social environmental damage, and welfare under production pollution and consumption one.

As for the effects of trade liberalization on consumer surplus we obtain

$$\left.\frac{dCS_0}{d\tau}\right|^{PP}_{d\tau = d\tau^*} < 0 \quad \text{and} \quad \left.\frac{dCS_1}{d\tau}\right|^{CP}_{d\tau = d\tau^*} > 0,$$

where superscript "PP" stands for production pollution. When governments implement trade liberalization, consumer surplus increases with trade liberalization under production pollution, whereas it decreases with its liberalization under consumption pollution. Therefore, government policy is beneficial for the consumer in the presence of production pollution, but not in the presence of consumption pollution. Conversely, trade protection policy will be beneficial in the presence of consumption pollution. Thus, when implementing trade policy targeting consumer surplus, the governments have to ascertain whether pollutants are emitted through production or consumption behavior.

Secondly, comparing the effects on producer surplus, we have

$$\left.\frac{dPS_0}{d\tau}\right|^{PP}_{d\tau = d\tau^*} \gtreqless \left.\frac{dPS_1}{d\tau}\right|^{CP}_{d\tau = d\tau^*}.$$

It is ambiguous whether the effect of trade liberalization on producer surplus is larger in the presence of production pollution than in the presence of consumption pollution.

Thirdly, as for social environmental damage (D), the following relationship is derived:

$$\left.\frac{dD_0}{d\tau}\right|^{PP}_{d\tau = d\tau^*} < 0 \quad \text{and} \quad \left.\frac{dD_1}{d\tau}\right|^{CP}_{d\tau = d\tau^*} > 0.$$

Trade liberalization policy reduces social environmental damage in the presence of consumption pollution and raises it in the presence of production pollution. This indicates that whether the policy makes the environment worse relies significantly on the source of pollution. It follows from the results above that the effects of trade liberalization on consumer surplus and social environmental damage are conflicting. This tension between the impacts on the consumers versus the environment can be challenging for policymaking, especially in democracies.

Fourthly, as for the comparison on welfare we get

$$\left.\frac{dW_0}{d\tau}\right|^{PP}_{d\tau = d\tau^*} \gtreqless \left.\frac{dW_1}{d\tau}\right|^{CP}_{d\tau = d\tau^*}.$$

It is generally indeterminate whether either of the effects on welfare in the Home and Foreign countries is large or small. However, trade liberalization leads to an increase in welfare in the presence of production pollution and to a decrease in it in the presence of consumption pollution, as long as there is no transboundary pollution, and the two tariffs are the same. The following proposition holds from the results above.

Proposition 8. In the two-stage game model, when trade liberalization policy is mutually implemented, consumer surplus and social environmental damage in the Home and Foreign countries are increased under production pollution, while they are decreased under consumption pollution.

It is of interest that trade liberalization policy derives the conflicting results on welfare under production pollution and consumption pollution, even under limited conditions. This shows that the government should pay attention to which pollution is prevalent, otherwise unintended consequences could occur.

5-2. Comparison of the SPNEs in the three-stage game

An interior equilibrium solution in the first stage of the extended model with production pollution exists, while a corner solution exists in the model with consumption pollution exists. Namely, in the latter case each country does not engage in international trade when consumption activities create contaminants. Each firm produces output only for the domestic market. In addition, the countries and the firms make decisions without taking account of global environmental damage due to transboundary pollution, different from the case with production pollution. Consequently, there may be a tendency to increase mutual contaminants because foreign checks on pollutant emissions do not work. One method to prevent further deterioration of the environment is that countries and firms make a cooperative decision on their environmental policies and outputs, respectively. However, there is a concern that such cooperative behavior of the firms may infringe upon antitrust laws.

Next, let us compare the SPNE values of tariffs, environmental taxes, and outputs obtained in the three-stage game with production pollution and consumption pollution, in turn (see Appendix III (ii)).

Firstly, each country trades products with the other country and simultaneously sets tariffs to adjust imports in the presence of production pollution, while both countries do not trade and then do not set tariffs in the presence of consumption pollution. Namely, they do not implement trade policy to reduce social environmental damage. In contrast, tariffs are set under

production pollution. We compare the emissions taxes. If either each country is free from transboundary pollution (or its degree is zero or small), then an emissions tax is larger in the presence of production pollution than in the presence of consumption pollution.

Secondly, when considering that both countries do not trade products in the presence of consumption pollution, we have the following outcome:

(i)	$\ddot{q}_1 > \dot{q}_1$ and $\ddot{q}_2^* > \dot{q}_2^*$	for the domestic market	
(ii)	$\dot{q}_1^* > \ddot{q}_1^* (= 0)$ and $\dot{q}_2 > \ddot{q}_2 (= 0)$	for the foreign market	(30)
(iii)	$\dot{Q} < \ddot{Q}$ and $\dot{Q}^* < \ddot{Q}^*$	for total output.	

Even if the two countries limit production to the domestic market and further there are no tariffs in the presence of consumption pollution, the total output of each firm becomes larger in comparison with that under production pollution, because its marginal production costs are reduced in the absence of tariffs.

Pollution creates negative externalities, regardless of whether it is generated by production or consumption activities. It is of interest to know which pollution imposes greater negative externalities on the society. By the two SPNE outcomes (16) and (29) under production pollution and consumption pollution, we can know the impacts of a difference in the resources generating pollution on the welfare in both cases. Then, parts of characteristics of the SPNEs will be revealed through this comparison. The following proposition is derived (see Appendix III (iii)):

Proposition 9. Suppose that the two countries face either production or consumption pollution composed of transboundary pollution and local pollution. Then, in equilibrium,

(i) consumer surplus is larger in the presence of consumption pollution than in the presence of production pollution,

(ii) producer surplus (the profits of the domestic firm) is (are) larger in the presence of consumption pollution than in the presence of production pollution,

(iii) social environmental damage is also larger in the presence of consumption pollution than in the presence of production pollution,

(iv) 1) given $0 < A - c < \frac{d(24 + 203\delta)}{24}$ and $\delta > 0$, welfare is larger in the presence of production pollution than in the presence of consumption pollution, and 2) given $\frac{d(24 + 203\delta)}{24} < A - c$ and $\delta > 0$, the reverse holds.

As for the result (i), it is derived because the equilibrium price in the domestic market is less in the presence of consumption pollution than in the presence of production pollution. As for

the result (ii), it holds because there is no tariff and, further, total output becomes larger in the case with consumption pollution relative to production pollution. We remember that the profit function of each firm is an increasing function of output. As for the result (iii), the social environmental damage becomes larger when there is consumption pollution than when there is production pollution since it is an increasing function of total output. As for the result (iv), whether the welfare of each country is greater or smaller in the presence of consumption pollution than in the presence of production pollution is dependent on the magnitude of the difference between two values, i.e., market size (A) and the sum of marginal production costs and marginal environmental damage $(c + d)$, i.e., $[A - (c + d)]$. Specifically, the comparison in the size of welfare is dependent on whether the magnitude of $[A - (c + d)]$ is larger or smaller than $203\delta d/24$. That is, when it is larger (or smaller) than $203\delta d/24$, welfare is greater (smaller) in the presence of consumption pollution than in the presence of production pollution.

Incidentally, if there is no transboundary pollution, i.e., $\delta = 0$, as a special case, then we have $W_0 < W_1$. This findings has important political economy implications. While the control of production pollution seems to receive a majority of the attention of policymakers, it seems that consumption-related pollution might often be a largely neglected culprit. As our results show, under certain conditions, the welfare loss could be greater from consumption pollution than from production pollution. Thus, effective environmental policy should take account of both production and consumption pollution, and related cross-boundary transmissions.

6. Conclusion

The continuing concern about the environment and the general inability of policies/governments to effectively combat climate change continues to challenge researchers. To further deepen our understanding of the related issues, this paper considers transboundary pollution, when emissions are generated through both production and consumption activities. While consumption and production pollutants, whether generated at home or abroad, might be practically indistinguishable, there is a crucial difference in that a government can impact foreign production through tariffs but can't impact foreign consumption.

In the context of the literature, either transboundary emissions have not been considered much or only production or consumption pollution has been considered. Besides, adding to the literature, our consideration of both consumption and production pollution spillovers makes the analysis more realistic, albeit we have to resort to a stylistic model to obtain tractable results. The effects of trade liberalization on economic performance under two types of pollution are made clear.

In the multi-stage framework considered, in the first stage the governments of the two countries simultaneously and noncooperatively choose their environmental taxes to maximize

their social welfare, respectively. After observing the taxes, the firms of the home and foreign countries simultaneously and noncooperatively determine their outputs for the domestic and the foreign markets to maximize profits in the second stage. Our game-theoretic results with two trading countries find significant differences compared to the case where only production pollution is considered. It is not possible to infer the results obtained on the trade and environmental policies of governments facing consumption pollution compared to the policies facing production pollution. Namely, the latter cannot be applied to the former. The same can be said about the equilibrium results of the three-stage game.

Specifically, the following different outcomes are derived under production pollution and consumption pollution. Our Subgame-Perfect Nash Equilibrium findings show that, in the two-stage game model, when trade liberalization policy is mutually implemented, consumer surplus and social environmental damage in the Home and Foreign countries are both increased under production pollution, while they are both decreased under consumption pollution. In addition, under certain conditions, trade liberalization policy increases social welfare in both countries under production pollution but decreases welfare under consumption pollution. These results in the presence of production pollution are evidently in conflict with those in the presence of consumption pollution. When we consider the fact that pollution simultaneously occurs in each country, we can infer that the results obtained only in the presence of production pollution are impaired by the co-presence of the two types of pollution. Incidentally, the coexistence leads to a decrease in the effectiveness of trade liberalization policy.

Furthermore, when the two countries face either production or consumption pollution composed of transboundary pollution and local pollution, then, in equilibrium consumer surplus, producer surplus, and social environmental damage are larger in the presence of consumption pollution than in the presence of production pollution; and, under certain conditions, social welfare in the presence of production pollution can be larger or smaller than in the presence of consumption pollution.[41] In addition, when we investigate trade and environmental policies of governments in a three-stage game, the countries with production pollution trade goods between them, while those with consumption pollution do not, i.e., both economies become closed. Thus, the type of pollution can have implications for a nation's trade openness.

From a policy perspective, as cross-border emissions become a more pronounced issue in an increasingly globalized world facing climate challenges, the main lesson is that policymakers should consider both emissions on the production and consumption sides in framing policies. Policies to liberalize trade between countries can have adverse welfare consequences and are sensitive to which type of pollution (i.e., consumption or production pollution) is considered. Besides, in the three-stage game trade policy may lose its effectiveness as a policy under consumption pollution. That is, when governments attempt to implement environmental and

[41] We do want to, however, note that the findings are based on a stylized model and that more sophisticated setups, if analytically tractable, could provide additional insights.

tariff policies, the optimal tariffs are reduced to be zero, because the corner solutions hold in the SPNE. As a result, the two available policies will be limited to an environmental policy. Tis result is novel, with obvious policy importance.

7. References

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Appendix

Appendix I on Case I

(i) Figures on equilibrium emissions taxes

Figure 1　Equilibrium emissions taxes

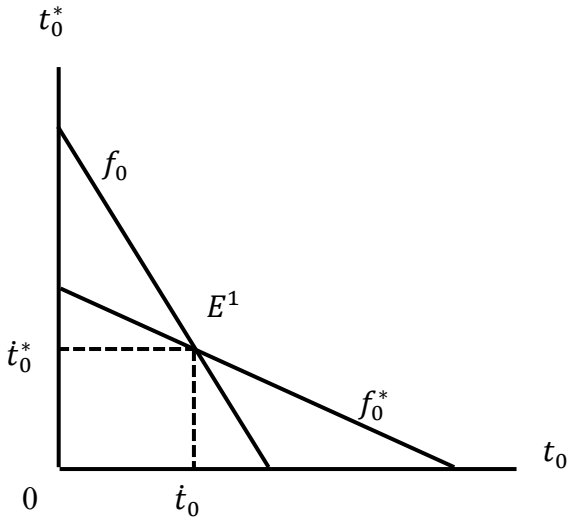
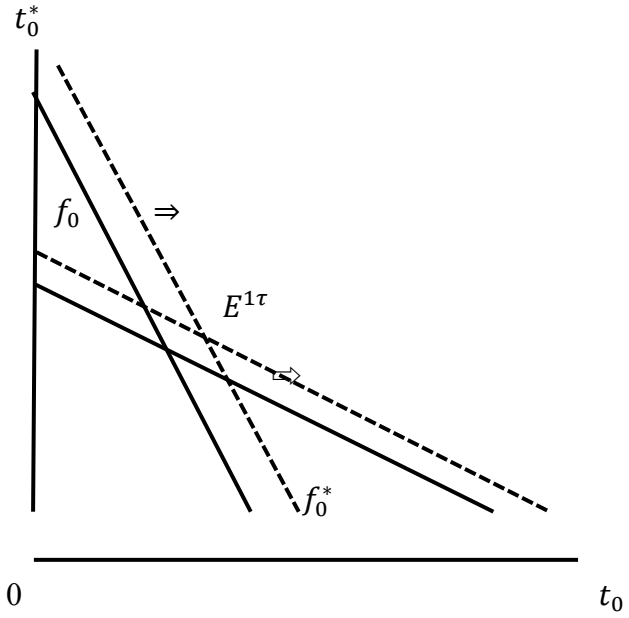


Figure 2　Effects of a tariff on the emissions taxes



(ii) The derivation of the emissions taxes in the second stage

$$\frac{dW_0}{dt_0} = Q\frac{dQ}{dt_0} - q_1\frac{dQ}{dt_0} + [p(Q) - c]\frac{dq_1}{dt_0} - q_1^*\frac{dQ^*}{dt_0} + [p(Q^*) - c - \tau^*]\frac{dq_1^*}{dt_0} + \tau\frac{dq_2}{dt_0} -$$

$$d\left[\frac{dq_1}{dt_0} + \frac{dq_1^*}{dt_0} + \delta\left(\frac{dq_2}{dt_0} + \frac{dq_2^*}{dt_0}\right)\right]$$

$$= q_2\frac{dQ}{dt_0} + [(A - c) - Q]\frac{dq_1}{dt_0} - q_1^*\frac{dQ^*}{dt_0} + [(A - c) - Q^* - \tau^*]\frac{dq_1^*}{dt_0} + \tau\frac{dq_2}{dt_0} - d[\frac{dq_1}{dt_0}\frac{dq_1^*}{dt_0} +$$

$$\delta\left(\frac{dq_2}{dt_0} + \frac{dq_2^*}{dt_0}\right)]$$

$$= \frac{-4(A - c) + 6(2 - \delta)d + 3\tau + 2\tau^* - 7t_0 - t_0^*}{9} = 0,$$

and

$$\frac{dW_0^*}{dt_0^*} = Q^*\frac{dQ^*}{dt_0^*} - q_2^*\frac{dQ^*}{dt_0^*} + [p(Q^*) - c]\frac{dq_2^*}{dt_0^*} - q_2\frac{dQ}{dt_0^*} + [p(Q) - c - \tau]\frac{dq_2}{dt_0^*}$$

$$+ \tau^*\frac{dq_1^*}{dt_0^*} - d[\frac{dq_2}{dt_0^*} + \frac{dq_2^*}{dt_0^*} + \delta\left(\frac{dq_1}{dt_0^*} + \frac{dq_1^*}{dt_0^*}\right)]$$

$$= \frac{-4(A - c) + 6(2 - \delta)d + 2\tau + 3\tau^* - t_0 - 7t_0^*}{9} = 0.$$

(iii) The effects of trade liberalization

Totally differentiating the social environmental damage D_0 caused through production pollution (PP) and using $d\tau = d\tau^*$ yields the following by using (10):

$$dD_0 = \frac{\partial D_0}{\partial \tau}d\tau + \frac{\partial D_0}{\partial \tau^*}d\tau^* = -\frac{(1 + \delta)d}{3}d\tau - \frac{5(1 + \delta)d}{12}d\tau = \frac{-3(1 + \delta)d}{4}d\tau, \quad (12A)$$

where

$$\frac{\partial D_0}{\partial \tau} = \left[\frac{\partial D_0}{\partial \tau} + \left(\frac{\partial D_0}{\partial t_0}\frac{\partial t_0}{\partial \tau} + \frac{\partial D_0}{\partial t_0^*}\frac{\partial t_0^*}{\partial \tau}\right)\right]d\tau$$

$$\frac{\partial D_0}{\partial \tau^*} = \left[\frac{\partial D_0}{\partial \tau^*} + \left(\frac{\partial D_0}{\partial t_0}\frac{\partial t_0}{\partial \tau^*} + \frac{\partial D_0}{\partial t_0^*}\frac{\partial t_0^*}{\partial \tau^*}\right)\right]d\tau^*,$$

and the first and the second terms in the third part of (12A) stand for the direct and the indirect effects, respectively.

The derivation of equation (13)

We totally differentiate the welfare function W_0 with respect to tariffs, τ and τ^*, as follows:

$$dW_0 = \frac{\partial W_0}{\partial \tau}d\tau + \frac{\partial W_0}{\partial \tau^*}d\tau^*.$$

The following results are obtained through calculation:

$$\frac{\partial W_0}{\partial \tau} = \frac{1}{2}\frac{\partial (Q)^2}{\partial \tau} + \frac{\partial [p(Q) - c]\cdot q_1}{\partial \tau} + \frac{\partial [p(Q^*) - c - \tau^*]\cdot q_1^*}{\partial \tau} + \frac{\partial \tau\cdot q_2}{\partial \tau} - \frac{\partial D_0}{\partial \tau}$$

$$\frac{1}{2}\frac{\partial (Q)^2}{\partial \tau} = \frac{-312(A - c) + 156(2 - \delta)d + 169\tau + 65\tau^*}{(24)^2}$$

$$\frac{\partial[p(Q)-c]\cdot q_1}{\partial\tau}=\frac{156(A-c)-36(2-\delta)d+91\tau+11\tau^*}{(24)^2}$$

$$\frac{\partial[p(Q^*)-c-\tau^*]\cdot q_1^*}{\partial\tau}=\frac{60(A-c)-84(2-\delta)d-45\tau-33\tau^*}{(24)^2}$$

$$\frac{\partial\tau\cdot q_2}{\partial\tau}=\frac{288(A-c)-144(2-\delta)d\ -780\tau-108\tau^*}{(24)^2}$$

$$\frac{\partial D_0}{\partial\tau}=\frac{-(24+408\delta)d}{(24)^2}.$$

As the subtotal above, we get

$$\frac{\partial W_0}{\partial\tau}=\frac{192(A-c)-108(2-\delta)d\ -565\tau-65\tau^*+(24+408\delta)d}{(24)^2}. \qquad 1\text{-}①$$

Similarly, we have the following results:

$$\frac{\partial W_0}{\partial\tau^*}=\frac{1}{2}\frac{\partial(Q)^2}{\partial\tau^*}+\frac{\partial[p(Q)-c]\cdot q_1}{\partial\tau^*}+\frac{\partial[p(Q^*)-c-\tau^*]\cdot q_1^*}{\partial\tau^*}+\tau\frac{\partial q_2}{\partial\tau^*}-\frac{\partial D_0}{\partial\tau^*}$$

$$\frac{1}{2}\frac{\partial(Q)^2}{\partial\tau^*}=\frac{-120(A-c)+60(2-\delta)d+65\tau+25\tau^*}{(24)^2}$$

$$\frac{\partial[p(Q)-c]\cdot q_1}{\partial\tau^*}=\frac{60(A-c)-36(2-\delta)d+11\tau-5\tau^*}{(24)^2}$$

$$\frac{\partial[p(Q^*)-c-\tau^*]\cdot q_1^*}{\partial\tau^*}=\frac{-132(A-c)-132(2-\delta)d-33\tau+363\tau^*}{(24)^2}$$

$$\frac{\partial\tau q_2}{\partial\tau^*}=\frac{-120\tau}{(24)^2}$$

$$\frac{\partial D_0}{\partial\tau^*}=\frac{-(408+24\delta)d}{(24)^2}.$$

From the results above we obtain

$$\frac{\partial W_0}{\partial\tau^*}=\frac{-192(A-c)-108(2-\delta)d-77\tau+383\tau^*-(408+24\delta)d}{(24)^2}. \qquad 1\text{-}②$$

Making the sum of 1-① and 1-② and using $d\tau=d\tau^*$ yields

$$dW_0=\frac{\partial W_0}{\partial\tau}d\tau+\frac{\partial W_0}{\partial\tau^*}d\tau^*=\frac{192(A-c)-108(2-\delta)d\ -565\tau-65\tau^*+(24+408\delta)d}{(24)^2}d\tau+$$

$$\frac{-192(A-c)-108(2-\delta)d-77\tau+383\tau^*+(408+24\delta)d}{(24)^2}d\tau^*$$

$$=\frac{-216(2-\delta)d\ -642\tau+318\tau^*+432(1+\delta)d}{(24)^2}d\tau.$$

Finally, from the equation above we have

$$\left.\frac{dW_0}{d\tau}\right|_{d\tau=d\tau^*}=\frac{-642\tau+318\tau^*+648\delta d}{(24)^2}. \qquad 13A\ (I\text{-}①)$$

Next, we totally differentiate the welfare function W_0^* with respect to tariffs, τ and τ^*, as follows:

$$dW_0^* = \frac{\partial W_0^*}{\partial \tau} d\tau + \frac{\partial W_0^*}{\partial \tau^*} d\tau^*.$$

The following results with respect to $\partial W_0^*/\partial \tau$ on the right-hand side are obtained through calculation:

$$\frac{\partial W_0^*}{\partial \tau} = \frac{1}{2}\frac{\partial (Q^*)^2}{\partial \tau} + \frac{\partial [p(Q^*) - c]\cdot q_2^*}{\partial \tau} + \frac{\partial [p(Q) - c - \tau]\cdot q_2}{\partial \tau} + \frac{\partial \tau^* \cdot q_1^*}{\partial \tau} - \frac{\partial D_0^*}{\partial \tau}$$

$$\frac{1}{2}\frac{\partial (Q^*)^2}{\partial \tau} = \frac{-120(A-c) + 60(2-\delta)d + 25\tau + 65\tau^*}{(24)^2}$$

$$\frac{\partial [p(Q^*) - c]\cdot q_2^*}{\partial \tau} = \frac{60(A-c) - 36(2-\delta)d - 5\tau + 11\tau^*}{(24)^2}$$

$$\frac{\partial [p(Q) - c - \tau]\cdot q_2}{\partial \tau} = \frac{-132(A-c) - 132(2-\delta)d + 363\tau - 33\tau^*}{(24)^2}$$

$$\frac{\partial \tau^* q_1^*}{\partial \tau} = \frac{-120\tau^*}{(24)^2}$$

$$\frac{\partial D_0^*}{\partial \tau} = \frac{-(408 + 24\delta)d}{(24)^2}.$$

As the subtotal above, we get

$$\frac{\partial W_0^*}{\partial \tau} = \frac{-192(A-c) - 108(2-\delta)d + 383\tau - 77\tau^* + (408 + 24\delta)d}{(24)^2}. \qquad 1\text{-}③$$

Similarly, we have the following results with respect to $\partial W_0^*/\partial \tau^*$:

$$\frac{\partial W_0^*}{\partial \tau^*} = \frac{1}{2}\frac{\partial (Q^*)^2}{\partial \tau^*} + \frac{\partial [p(Q^*) - c]\cdot q_2^*}{\partial \tau^*} + \frac{\partial [p(Q) - c - \tau]\cdot q_2}{\partial \tau^*} + \frac{\partial \tau^* \cdot q_1^*}{\partial \tau^*} - \frac{\partial D_0^*}{\partial \tau^*}$$

$$\frac{1}{2}\frac{\partial (Q^*)^2}{\partial \tau^*} = \frac{-312(A-c) + 156(2-\delta)d + 65\tau + 169\tau^*}{(24)^2}$$

$$\frac{\partial [p(Q^*) - c]\cdot q_2^*}{\partial \tau^*} = \frac{156(A-c) - 36(2-\delta)d + 11\tau + 91\tau^*}{(24)^2}$$

$$\frac{\partial [p(Q) - c - \tau]\cdot q_2}{\partial \tau^*} = \frac{60(A-c) - 84(2-\delta)d - 33\tau - 45\tau^*}{(24)^2}$$

$$\frac{\partial \tau^* \cdot q_1^*}{\partial \tau^*} = \frac{288(A-c) - 144(2-\delta)d\ - 120\tau - 780\tau^*}{(24)^2}$$

$$\frac{\partial D_0^*}{\partial \tau^*} = \frac{-(24 + 408\delta)d}{(24)^2}.$$

From the results above we obtain

$$\frac{\partial W_0^*}{\partial \tau^*} = \frac{192(A-c) - 108(2-\delta)d\ - 65\tau - 565\tau^* + (24 + 408\delta)d}{(24)^2}. \qquad 1\text{-}④$$

Making the sum of 1-③ and 1-④ and using $d\tau = d\tau^*$ yields

$$dW_0^* = \frac{\partial W_0^*}{\partial \tau} d\tau + \frac{\partial W_0^*}{\partial \tau^*} d\tau^* = \frac{-192(A-c) - 108(2-\delta)d + 383\tau - 77\tau^* + (408 + 24\delta)d}{(24)^2} d\tau +$$

$$\frac{192(A-c)-108(2-\delta)d\ -65\tau-565\tau^*+(24+408\delta)d}{(24)^2}d\tau^*$$

$$=\frac{-216(2-\delta)d\ +318\tau-642\tau^*+432(1+\delta)d}{(24)^2}d\tau^*.$$

Finally, we have

$$\left.\frac{dW_0^*}{d\tau^*}\right|_{d\tau=d\tau^*}=\frac{318\tau-642\tau^*+648\delta d}{(24)^2}. \qquad \text{I-②}$$

(iv) The separation of the effects of trade liberalization

The effect (13A) of a bilateral tariff reduction on the welfare W_0 of the Home country is divided into the direct and the indirect effects. That is, the latter is a strategic effect brought about by environmental policy. The two effects are as follows, respectively:

The direct effect: $\left.\frac{dW_0}{d\tau}\right|_{d\tau=d\tau^*}=\frac{96(A-c)-588\tau-492\tau^*-48(2+5\delta)d}{(24)^2}$,

The indirect effect: $\left.\frac{dW_0}{d\tau}\right|_{d\tau=d\tau^*}=\frac{-96(A-c)-54\tau+810\tau^*+8(12+111\delta)d}{(24)^2}$.

These effects are not determinate, so the total effect on the welfare becomes also ambiguous.

Consumer surplus $(CS_0)=\frac{Q^2}{2}$:

$$\left.\frac{dCS_0}{d\tau}\right|_{d\tau=d\tau^*}=\frac{1}{2}\frac{\partial(Q)^2}{\partial\tau}+\frac{1}{2}\frac{\partial(Q)^2}{\partial\tau^*}=-\frac{Q}{3}-\frac{5Q}{24}=\frac{-432[(A-c)-d]-216\delta d+234\tau+90\tau^*}{(24)^2}<0,$$

where the first and the second terms in the third part are the direct and the indirect effects, respectively. These effects both become negative, so that the total of these becomes negative.

When producer surplus in the presence of PP is given by $(PS_0)=(q_1)^2+(q_1^*)^2$, the effect of trade liberalization is as follows:

$$\left.\frac{dPS_0}{d\tau}\right|_{d\tau=d\tau^*}=\left[\frac{\partial(q_1)^2}{\partial\tau}+\frac{\partial(q_1^*)^2}{\partial\tau}\right]+\left[\frac{\partial(q_1)^2}{\partial\tau^*}+\frac{\partial(q_1^*)^2}{\partial\tau^*}\right]$$

$$=-\frac{96[(A-c)-d]+48\delta d-100\tau-260\tau^*}{288}-\frac{120[(A-c)-d]+60\delta d-5\tau-85\tau^*}{288}$$

$$=-\frac{216[(A-c)-d]+108\delta d-105\tau-345\tau^*}{288},$$

where the first and the second terms in the penultimate equation stand for the direct and the indirect effects, respectively, and $PS_0=[p(Q)-c]\cdot q_1+[p(Q^*)-c-\tau^*]\cdot q_1^*-t_0\cdot(q_1+q_1^*)=\pi_0$. The signs of these effects become ambiguous.

Government tariff revenues $(GTR)=\tau q_2$:

$$\left.\frac{dGTR}{d\tau}\right|_{d\tau=d\tau^*}=\frac{\partial(\tau q_2)}{\partial\tau}+\frac{\partial(\tau q_2)}{\partial\tau^*}=\left(q_2-\frac{2\tau}{3}\right)-\frac{5\tau}{24}=\frac{8[(A-c)-d]+4\delta d-25\tau-3\tau^*}{16},$$

where the first and the second terms in the third part stand for the direct and the indirect effects,

respectively. The former becomes ambiguous, but the latter becomes negative, so that the total effect is ambiguous.

Social environmental damage (D_0):

See (12A).

The two effects are both negative, so that the total effect becomes negative.

Total outputs (Q and Q*):

$$\left.\frac{dQ}{d\tau}\right|_{d\tau = d\tau^*} = -\frac{1}{3} - \frac{5}{51} = -\frac{22}{51} < 0,$$

where the first and the second terms in the third part stand for the direct and the indirect effects, respectively.

Substituting (15) into (8), we have the optimal environmental taxes:

$$\dot{t}_0 = \dot{t}_0^* = \frac{-67[(A-c)-d] + (214 - 53\delta)d}{214}.$$

Appendix II on Case II

(i) The derivation of the optimal environmental taxes (20)

Firstly, differentiating W_1 and arranging it with respect to t_1, we have the following first-order condition for maximization:

$$\frac{dW_1}{dt_1} = \frac{q_1 + 2q_2 - (A-c) - (t_1+\tau) + 2d}{3} = \frac{2(-t_1 - \tau + d)}{3} = 0.$$

Then, the optimal environmental tax of the Home country is $\ddot{t}_1 = d - \tau$. The second-order condition for maximization is satisfied, i.e., $d^2W_1/(dt_1)^2 = -\frac{2}{3} < 0$.

Secondly, we derive the following equation as the first-order condition with respect to the Foreign country:

$$\frac{dW_1^*}{dt_1^*} = \frac{2q_1^* + q_2^* - (A-c) - (t_1^* + \tau^*) + 2d}{3} = \frac{2(-t_1^* - \tau^* + d)}{3} = 0.$$

Then, the optimal environmental tax of the Foreign country is $\ddot{t}_1^* = d - \tau^*$. The second-order condition for maximization is satisfied.

(ii) The effects of trade liberalization

<u>The derivation of equation (24)</u>

Totally differentiate the social environmental damage D_1 due to consumption pollution (CP) and using $d\tau = d\tau^*$ yields the following:

$$dD_1 = \frac{\partial D_1}{\partial \tau}d\tau + \frac{\partial D_1}{\partial \tau^*}d\tau^* = -\frac{(1+\delta)d}{3} + \frac{2(1+\delta)d}{3} = \frac{(1+\delta)d}{3}d\tau > 0, \qquad (24A)$$

where

$$\frac{\partial D_1}{\partial \tau} = d\left[\frac{\partial (q_1 + q_1^*)}{\partial \tau} + \delta \frac{\partial (q_2 + q_2^*)}{\partial \tau}\right] = \frac{1}{3}d$$

$$\frac{\partial D_1}{\partial \tau^*} = d\left[\frac{\partial (q_1 + q_1^*)}{\partial \tau^*} + \delta \frac{\partial (q_2 + q_2^*)}{\partial \tau^*}\right] = \frac{\delta}{3}d,$$

and the first and the second terms in the third part of (24A) stand for the direct and the indirect effects, respectively.

Next, we turn to the effect of trade liberalization on the welfare of the domestic country. We totally differentiate the welfare function W_1 with respect to tariffs, τ and τ^*, and obtain:

$dW_1 = \frac{\partial W_1}{\partial \tau} d\tau + \frac{\partial W_1}{\partial \tau^*} d\tau^*.$

The following results are obtained through calculation:

$$\frac{\partial W_1}{\partial \tau} = \frac{1}{2}\frac{\partial (Q)^2}{\partial \tau} + \frac{\partial [p(Q) - c]\cdot q_1}{\partial \tau} + \frac{\partial [p(Q^*) - c - t_1^* - \tau^*]\cdot q_1^*}{\partial \tau} + \frac{\partial (t_1 + \tau)\cdot q_2}{\partial \tau} - \frac{\partial D_1}{\partial \tau}$$

$$\frac{1}{2}\frac{\partial (Q)^2}{\partial \tau} = \frac{2(A - c) - 2d + \tau}{9}$$

$$\frac{\partial [p(Q) - c]\cdot q_1}{\partial \tau} = \frac{(A - c) + 5d - 4\tau}{9}$$

$$\frac{\partial [p(Q^*) - c - t_1^* - \tau^*]\cdot q_1^*}{\partial \tau} = 0$$

$$\frac{\partial (t_1 + \tau)\cdot q_2}{\partial \tau} = -\frac{d}{3}$$

$$\frac{\partial D_1}{\partial \tau} = \frac{d}{3}.$$

As a subtotal above, we get

$$\frac{\partial W_1}{\partial \tau} = \frac{3(A - c) - 3d - 3\tau}{9}. \qquad 2\text{-}①$$

Similarly, we have the following results:

$$\frac{\partial W_1}{\partial \tau^*} = \frac{1}{2}\frac{\partial (Q)^2}{\partial \tau^*} + \frac{\partial [p(Q) - c]\cdot q_1}{\partial \tau^*} + \frac{\partial [p(Q^*) - c - t_1^* - \tau^*]\cdot q_1^*}{\partial \tau^*} + \frac{\partial (t_1 + \tau)\cdot q_2}{\partial \tau^*} - \frac{\partial D_1}{\partial \tau^*}$$

$$\frac{1}{2}\frac{\partial (Q)^2}{\partial \tau^*} = 0$$

$$\frac{\partial [p(Q) - c]\cdot q_1}{\partial \tau^*} = 0$$

$$\frac{\partial [p(Q^*) - c - t_1^* - \tau^*]\cdot q_1^*}{\partial \tau^*} = \frac{-2(A - c) + 2d + 2\tau^*}{9}$$

$$\frac{\partial (t_1 + \tau)\cdot q_2}{\partial \tau^*} = 0$$

$$\frac{\partial D_1}{\partial \tau^*} = \frac{\delta d}{3}.$$

From the results above we obtain

$$\frac{\partial W_1}{\partial \tau^*} = \frac{-2(A-c)+2d+2\tau^*-3\delta d}{9}. \qquad 2\text{-}②$$

Making the sum of 2-① and 2-② and using $d\tau = d\tau^*$ yields

$$dW_1 = \frac{\partial W_1}{\partial \tau}d\tau + \frac{\partial W_1}{\partial \tau^*}d\tau^* = \frac{3(A-c)-3d-3\tau}{9}d\tau + \frac{-2(A-c)+2d+2\tau^*-3\delta d}{9}d\tau^*$$

$$= \frac{(A-c)\ -3\tau+2\tau^*-\ (1+3\delta)d}{9}d\tau.$$

Finally, we have

$$\left.\frac{dW_1}{d\tau}\right|_{d\tau=d\tau^*} = \frac{(A-c)\ -3\tau+2\tau^*-\ (1+3\delta)d}{9}. \qquad 25A\ (II\text{-}①)$$

Next, we totally differentiate the welfare function W_1^* with respect to tariffs, τ and τ^*:

$$dW_1^* = \frac{\partial W_1^*}{\partial \tau}d\tau + \frac{\partial W_1^*}{\partial \tau^*}d\tau^*.$$

The following results with respect to $\partial W_1^*/\partial\tau$ on the right-hand side are obtained:

$$\frac{1}{2}\frac{\partial (Q^*)^2}{\partial \tau} = 0$$

$$\frac{\partial [p(Q^*)-c]\cdot q_2^*}{\partial \tau} = 0$$

$$\frac{\partial [p(Q)-c-t_1\ -\tau]\cdot q_2}{\partial \tau} = \frac{-2(A-c)+2d\ +2\tau}{9}$$

$$\frac{\partial (t_1^*+\tau^*)\cdot q_1^*}{\partial \tau} = 0$$

$$\frac{\partial D_1^*}{\partial \tau} = \frac{\delta d}{3}.$$

As the subtotal above, we get

$$\frac{\partial W_1^*}{\partial \tau} = \frac{-2(A-c)+2d\ +2\tau-3\delta d}{9}. \qquad 2\text{-}③$$

Similarly, we have the following results with respect to $\partial W_1^*/\partial\tau^*$:

$$\frac{\partial W_1^*}{\partial \tau^*} = \frac{1}{2}\frac{\partial (Q^*)^2}{\partial \tau^*} + \frac{\partial [p(Q^*)-c]\cdot q_2^*}{\partial \tau^*} + \frac{\partial [p(Q)-c-t_1\ -\tau]\cdot q_2}{\partial \tau^*} + \frac{\partial (t_1^*+\tau^*)\cdot q_1^*}{\partial \tau^*} - \frac{\partial D_1^*}{\partial \tau^*}$$

$$\frac{1}{2}\frac{\partial (Q^*)^2}{\partial \tau^*} = \frac{2(A\ -\ c)\ -\ 2d\ +\ \tau^*}{9}$$

$$\frac{\partial [p(Q^*)-c]\cdot q_2^*}{\partial \tau^*} = \frac{(A-c)+5d-4\tau^*}{9}$$

$$\frac{\partial [p(Q)-c-t_1\ -\tau]\cdot q_2}{\partial \tau^*} = 0$$

$$\frac{\partial(t_1^* + \tau^*)\cdot q_1^*}{\partial \tau^*} = -\frac{d}{3}$$

$$\frac{\partial D_1^*}{\partial \tau^*} = \frac{d}{3}.$$

From the results above we obtain

$$\frac{\partial W_1^*}{\partial \tau^*} = \frac{3(A-c) - 3d - 3\tau^*}{9}. \qquad 2\text{-④}$$

Making the sum of 2-③ and 2-④ and using $d\tau = d\tau^*$ yields

$$dW_1^* = \frac{\partial W_1^*}{\partial \tau} d\tau + \frac{\partial W_1^*}{\partial \tau^*} d\tau^* = \frac{-2(A-c) + 2d + 2\tau - 3\delta d}{9} d\tau + \frac{3(A-c) - 3d - 3\tau^*}{9} d\tau^*$$

$$= \frac{(A-c) + 2\tau - 3\tau^* - (1+3\delta)d}{9} d\tau^*.$$

Finally, we have

$$\left.\frac{dW_1^*}{d\tau^*}\right|_{d\tau = d\tau^*} = \frac{[(A-c) - d] + 2\tau - 3\tau^* - 3\delta d}{9}. \qquad \text{II-②}$$

(iii) The separation of the effects on trade liberalization

First, we derive the effects of a bilateral tariff reduction on the welfare of the domestic country. The result (25A) is divided into the direct and the indirect effects as follows:

The direct effect: $\left.\frac{dW_1^*}{d\tau}\right|_{d\tau = d\tau^*}^{CP} = -\frac{[(A-c) - d - \tau^*] - 3\delta d}{9}$,

The indirect effect: $\left.\frac{dW_1^*}{d\tau}\right|_{d\tau = d\tau^*}^{CP} = \frac{2[(A-c) - d - \tau] - \tau + \tau^* - 6\delta d}{9}$.

The effects of bilateral tariffs on the components of welfare in the Home country

Consumer surplus (CS_1):

$$\left.\frac{dCS_1}{d\tau}\right|_{d\tau = d\tau^*}^{CP} = \frac{1}{2}\frac{\partial(Q)^2}{\partial \tau} + \frac{1}{2}\frac{\partial(Q)^2}{\partial \tau^*} = -\frac{Q}{3} + \frac{2Q}{3} = \frac{Q}{3} = \frac{2[(A-c) - d] + \tau}{9} > 0,$$

where the first and the second terms in the third part stand for the direct and the indirect effects, respectively. The former is negative, but the latter is positive. The total of the two effects is positive.

Producer surplus (PS_1) $= (q_1)^2 + (q_1^*)^2$:

$$\left.\frac{dPS_1}{d\tau}\right|_{d\tau = d\tau^*}^{CP} = \frac{-2[(A-c) - d] + 4\tau + 4\tau^*}{9} + \frac{4[(A-c) - d] + 4\tau - 2\tau^*}{9} = \frac{2\{[(A-c) - d] + 4\tau + \tau^*\}}{9} > 0,$$

where the first and the second terms in the middle part stand for the direct and the indirect effects, respectively, and $PS_1 = [p(Q) - c] \cdot q_1 + [p(Q^*) - c - \tau^*] \cdot q_1^* - (t_1 q_1 + t_1^* q_1^*) = \pi_1$. Although the former and the latter effects are ambiguous, the total effect becomes positive. That is, trade liberalization policies of the two countries rather lead to a reduction in their firms' profits (producer surplus) in the presence of CP.

Government (tax·tariff) revenues (GTTR) = $(t_1 + \tau) \cdot q_2$:

$$\left.\frac{dGTTR}{d\tau}\right|_{d\tau = d\tau^*}^{CP} = -\frac{2d}{3} + \frac{d}{3} = -\frac{d}{3} < 0,$$

where the first and second terms in the middle part stand for the direct and the indirect effects, respectively. The former effect is negative, but the latter is positive. The total becomes negative.

Social environmental damage (D_1):

$$\left.\frac{dD_1}{d\tau}\right|_{d\tau = d\tau^*}^{CP} = -\frac{(1+\delta)d}{3} + \frac{2(1+\delta)d}{3} = \frac{(1+\delta)d}{3} > 0,$$

where the first and second terms in the middle part stand for the direct and the indirect effects, respectively. The former effect becomes negative, but the latter becomes positive. The total becomes positive. Interestingly, it follows that the direct and the indirect effects on *CS*, *GTTR*, and D_1 except for *PS* alike become negative and positive, respectively.

Appendix III on comparison

(i) Comparison of the effects of trade liberalization:

1) Consumer surplus (CS):

$$\left.\frac{dCS_0}{d\tau}\right|_{d\tau = d\tau^*}^{PP} < 0 < \left.\frac{dCS_1}{d\tau}\right|_{d\tau = d\tau^*}^{CP}.$$

2) Producer surplus (PS) :

$$\left.\frac{dPS_0}{d\tau}\right|_{d\tau = d\tau^*}^{PP} - \left.\frac{dPS_1}{d\tau}\right|_{d\tau = d\tau^*}^{CP} = \frac{85(A-c) - 595d + 451\tau + 394\tau^* - 180(2-\delta)d}{3\times 255} \gtreqless 0.$$

3) Government revenues, i.e., GTR vs. GTTR:

$$\left.\frac{dGTR}{d\tau}\right|_{d\tau = d\tau^*}^{PP} - \left.\frac{dGTTR}{d\tau}\right|_{d\tau = d\tau^*}^{CP} = \frac{105[(A-c)-d] + 10(3\delta - 8)d - 3(125\tau - 7\tau^*)}{255}.$$

In general, the sign of this comparison becomes indeterminate. Now, the effect on GTR gets larger than that on GTTR for $\tau = \tau^*$ and $d > 3\tau$. This implies that if the marginal environmental damage is not quite larger, the sign is not positive.

4) Social environmental damage (D_0 and D_1):

$$\left.\frac{dD_0}{d\tau}\right|_{d\tau = d\tau^*}^{PP} < 0 < \left.\frac{dD_1}{d\tau}\right|_{d\tau = d\tau^*}^{CP}.$$

5) Welfare (W):

$$\left.\frac{dW_0}{d\tau}\right|_{d\tau = d\tau^*}^{PP} - \left.\frac{dW_1}{d\tau}\right|_{d\tau = d\tau^*}^{CP} = \frac{-1760(A-c) - 9627\tau + 6302\tau^* + (2390 + 11300\delta)d}{51\times 255} \gtreqless 0.$$

(ii) Comparison of the SPNE values:

1) Tariff:

As the domestic country does not trade with the foreign country in the presence of CP, tariffs do not exist. For reference, when we compare them, tariff becomes as follows:

$$\dot{\tau} > \ddot{\tau} = 0 \quad \text{for any } \delta,$$

where $\dot{\tau} = \dot{\tau}^*$ and $\ddot{\tau} = \ddot{\tau}^*$.

2) Emissions tax:

$$\dot{t}_0 - \ddot{t}_1 = \frac{147[(A-c)-d] - 53\delta d}{214},$$

where $\dot{t}_0 = \dot{t}_0^*$ and $\dot{t}_1 = \ddot{t}_1^*$.

3) Output and total output:

$$\dot{q}_1 - \ddot{q}_1 = \dot{q}_2^* - \ddot{q}_2^* = \frac{-99[(A-c)-d] + 75\delta d}{214},\ \dot{q}_1^* = \dot{q}_2 > \ddot{q}_1^* = \ddot{q}_2 = 0.$$

$$\dot{Q} - \ddot{Q} = \dot{Q}^* - \ddot{Q}^* = -\frac{48B + 22\delta d}{214} < 0, \text{ and } \dot{Q} < \ddot{Q} = \ddot{q}_1,\ \dot{Q}^* < \ddot{Q}^* = \ddot{q}_2^*. \quad ①$$

Now, we find the following. As $\dot{q}_1$ and $\ddot{q}_2$ are both positive, $\ddot{q}_1 > \dot{q}_1$ holds from $\dot{Q} < \ddot{Q}$ because $\dot{q}_2$ is positive. Similarly, $\ddot{q}_2^* > \dot{q}_2^*$.

(iii) Proof of Proposition 8:

1) Prices

As the inverse demand curves are downward sloping, we have

$$\dot{Q} < \ddot{Q} \ \rightarrow\ p(\ddot{Q}) = \ddot{p} < p(\dot{Q}) = \dot{p} \ \text{ and } \ \dot{Q}^* < \ddot{Q}^* \ \rightarrow\ p(\ddot{Q}^*) = \ddot{p}^* < p(\dot{Q}^*) = \dot{p}^*.$$

2) Consumer surplus (CS)

As $CS = \frac{1}{2}Q^2$, from ① we have [domestic CS under CP > domestic CS under PP], i.e., $CS_0 < CS_1$.

3) Profits (π) = producer surplus

Profits of the domestic firm are expressed as $\pi_i = (q_1)^2 + (q_1^*)^2, i = 0, 1$, in the presence of PP and CP. Let compare the profits, π_0 and π_1, under PP and CP. As shown by ①, $\dot{Q} < \ddot{Q}$ holds, so that $(\dot{Q})^2 < (\ddot{Q})^2$. When expanding this inequality, it follows that $(\dot{q}_1)^2 + (\dot{q}_2)^2 + 2\dot{q}_1\dot{q}_2 < (\ddot{q}_1)^2 + (\ddot{q}_2)^2 + 2\ddot{q}_1\ddot{q}_2$, where $\ddot{q}_2 = 0$. Now, as $\dot{q}_1\dot{q}_2 > 0$, we get $(\dot{q}_1)^2 + (\dot{q}_2)^2 < (\dot{Q})^2 < (\ddot{q}_1)^2 = (\ddot{Q})^2$. Therefore, the profits of the domestic firm are larger in the presence of CP than in the presence of PP, namely, $\pi_0 < \pi_1$.

4) Social environmental damage (D)

The social environmental damage of the domestic country under PP and CP is $D_0 = d[(q_1 + q_1^*) + \delta(q_2 + q_2^*)]$ and $D_1 = d[(q_1 + q_2) + \delta(q_1^* + q_2^*)]$, respectively. Using (16) and (29), we can derive the social environmental damage under PP and CP:

$$D_0 = \left[\frac{166B - 22\delta d}{214} + \frac{(166B - 22\delta d)\delta}{214}\right] d = \frac{(1+\delta)d(166B - 22\delta d)}{214}, \text{ and } D_1 = \frac{214(1+\delta)B}{214},$$

where $B = (A - c) - d$. It follows that $D_0 - D_1 = -\frac{(1+\delta)d(486B + 22\delta d)}{214} < 0$, namely, $D_0 < D_1$.

5) Welfare (W)

The welfare function of the domestic country under PP is

$$W_0 = \frac{1}{2}Q^2 + [p(Q) - c] \cdot q_1 + [p(Q^*) - c - \tau^*] \cdot q_1^* + \tau q_2 - D_0 = CS_0 + QPS_0 + \tau q_2 - D_0.$$

When we use (16), the consumer surplus of the domestic country is reduced to

$$CS_0 = \frac{13778B^2 - 3652\delta dB + 242(\delta d)^2}{(214)^2}.$$

QPS_0 is rewritten as $QPS_0 = (q_1)^2 + (q_1^*)^2 + t_0(q_1 + q_1^*)$. Thus, we have

$$QPS_0 = \frac{4704B^2 + 32\delta dB + 16200(\delta d)^2 + 35524dB - 4708\delta d^2}{(214)^2}.$$

Finally, we have

$$\tau q_2 - D_0 = \frac{3264B^2 - 32960\delta dB - 11976(\delta d)^2 - 35524dB + 4708\delta d^2}{(214)^2}.$$

From these results we have

$$W_0 = \frac{21746B^2 - 36580\delta dB + 4466(\delta d)^2}{(214)^2}. \quad ②$$

On the other hand, the welfare function of the domestic country under CP is

$$W_1 = \frac{1}{2}Q^2 + [p(Q) - c] \cdot q_1 + [p(Q^*) - c - t_1^* - \tau^*] \cdot q_1^* + (t_1 + \tau) \cdot q_2 - D_1$$
$$= \frac{1}{2}Q^2 + [p(Q) - c] \cdot q_1 - D_1,$$

where $q_1^* = 0$ and $q_2 = 0$. As a result of calculation, we obtain

$$W_1 = \frac{22898B^2 - 45796\delta dB}{(214)^2}. \quad ③$$

Subtracting ③ from ② and arranging, we have

$$W_0 - W_1 = \frac{-2[576B^2 - 4608\delta dB - 2233(\delta d)^2]}{(214)^2}. \quad ④$$

By using the quadratic formula, we get two solutions of B that satisfy the numerator on the right hand-side of the equation ④:

$B = \frac{\delta d(96 \pm 107)}{24}$, i.e., $B_1 = \frac{203\delta d}{24}$, and $B_2 = -\frac{11\delta d}{24}$.

When we take account of these results and the functional form of welfare, the following results are derived:

$$W_0 \geq W_1 \text{ for } 0 < A - c \leq \frac{d(24 + 203\delta)}{24}, \text{ and } W_0 < W_1 \text{ for } \frac{d(24 + 203\delta)}{24} < A - c,$$

where $W_0 = W_1$ for $A - c = \frac{d(24 + 203\delta)}{24}$.

Furthermore, we find from ④ that $W_0 < W_1$ for no transboundary pollution ($\delta = 0$).