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Inflation Expectation Formation of German Consumers: Rational or Adaptive?

by

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Inflation Expectation Formation of German Consumers: Rational or Adaptive?

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Abstract

This paper analyzes the inflation expectation formation empirically for German consumers. The expectation formation process is analyzed for a representative consumer and for different demographic groups. The results indicate that German consumers are a relatively homogeneous group. There are nevertheless quantitative differences among the groups: Inflation expectations and perceived inflation tend to fall with rising income and unemployed individuals are outliers. Rational inflation expectation is not present for any group. Consumer and expert expectations have short and long run relationships. Evidence for a positive constant gain in the adaptive learning algorithm is given for almost all groups.

Keywords: Inflation expectations; conversion method; survey data; rationality tests

JEL Classification: C42, D83, D84, E31,

*University of Lüneburg, Institute of Economics, 21335 Lüneburg, Germany, e-mail: sabrowski@leuphana.de. The author would like to thank Maik Heinemann and the participants of the 12th ZEI International Summer School for helpful comments and suggestions. All computations were performed by using Stata SE 10.0 and Mathematica 5.0.
1 Introduction

The primary aim of the European Central Bank and the Bundesbank previously, is to pursue price stability. Important parameters for the price level are inflation expectations. They are an essential part of economic models. To make political advice out of these models an assumption about the inflation expectation has to be made. In accordance with the rational expectations hypothesis, inflation expectations in economic models are often assumed to be rational. But there is a significant amount of evidence that expectations are not rational in the mathematical sense. They appear to be made in an adaptive manner. One direction of research compatible with this is the growing literature in adaptive learning (Evans and Honkapohja (2001)). Another school of thought that also indicates adaptive behavior is the direction of rational inattention (Mankiw and Reis (2002), Carroll (2003), Sims (2003)). Orphanides and Williams (2005) show that the optimal monetary policy is different depending on the assumption of rational expectation or of the adaptive learning algorithm. Therefore knowledge about the formation of inflation expectations is crucial for macroeconomic analysis and especially for the conduct of monetary policy. Is the expectation formation process rational, adaptive or both? This paper gives an answer for German consumers empirically.

A measure of inflation expectations for German consumers is derived from the Business and Consumer Survey by the European Commission. This survey has been conducted every month since January 1985. The respondent selection is representative of German consumer. The respondents are also categorized into sex, age, wealth, occupations and education. Thus the analysis can be done for the representative consumer and for different demographic groups. The consumers are asked about their expected change of the price level. Therefore the survey provides qualitative data. Unfortunately an economic interpretation of the qualitative data as inflation rates is not possible. So that the qualitative data has to be converted into quantitative data for the analysis. The conversion of qualitative data into quantitative data is done by the probability method proposed by Batchelor and Orr (1988). This
method uses the answers of the respondents concerning their expected and perceived inflation and a probability assumption to calculate quantitative inflation forecasts.

This paper firstly presents differences among demographic groups concerning inflation expectations and perceived inflation. To my knowledge, differences between socioeconomic groups of German consumers have not yet been investigated. Following this rationality, the expected inflation expectation formation process in a static and an error correction model, and an adaptive learning algorithm with constant gains for the total sample and the demographic groups are examined.

The results show that the German consumers are relatively homogeneous. Nevertheless a few differences between demographic groups exist. The higher the income of an individual the lower its inflation expectation. Furthermore unemployed consumers are outliers. Their inflation expectation is much higher compared to all other groups. The rationality tests show that the inflation expectation of German consumers are not totally rational in the mathematical interpretation. The analysis of the formation of the inflation expectation shows instead that all groups form their expectations partly based on expert forecasts and partly adaptively on their own past expectation. The error correction model shows that a short and long run relationship between consumer and expert expectation exists. In an adaptive learning framework a positive constant gain is needed to produce the lowest mean squared forecast error of consumers forecast. This evidence on the formation of inflation expectations by German consumers provides a benchmark for the analysis of alternative theoretical models as it is possible to assess how well these different models are able to explain actual inflation expectations.

The paper is organized as follows. Section 2 describes the data of the Business and Consumer survey. The first evidence of the qualitative data for the different demographic groups is presented in section 3. Section 4 describes the conversion methods used in this paper. Differences among the demo-

1 Only Linden (2005) takes Germany indirectly into account. Linden deals with the European Union. Thus the results of his work need not hold for Germany. What will be proved to be true in the results of this paper.

2 The group of unemployed individuals includes students, jobless and retired persons.
graphic groups based on the converted data is shown in section 5. Section 6 illustrates the results for rationality tests. Evidence for the expectation formation is presented in section 7. Section 8 gives the description and results of the adaptive learning algorithm. Section 9 provides the conclusion.

2 Data

This empirical analysis is based on data from the Joint Harmonised EU Program of Business and Consumer Surveys conducted by the European Commission (hereafter EC). In the case of Germany the Gesellschaft für Konsumforschung (hereafter GfK) performs the survey on behalf of the EC. Approximately 2,500 consumers have been interviewed every month since 1985. From January 1985 to the end of 1996 only residents of West Germany completed the survey, however since January 1997 the GfK has also queried 500 respondents from East Germany.3

The composition of the respondents is chosen in a way that the aggregate answers of the total sample can be interpreted as answers of a representative German consumer. The response data is available for the total sample and also on a more disaggregate level. The answer probabilities are differentiated in the following categories: sex, education (primary; secondary, further), age (16-29; 30-49; 50-64; 65+), income (1st; 2nd, 3rd and 4th quartile) and occupation (ten classifications).4 This disaggregation will be the foundation of the analysis.

Questions number five and six of the survey are of interest for the analysis.5 These questions deal with the perceived and the expected inflation. In the survey the respondents are asked for their tendency. Therefore the resulting survey data is qualitative. Table (1) shows the exact wording of both questions and the possible answers for the respondents.

The EC calculates for each question an index value \( B \) based on the first

---

4The different occupations are: Self employed and professional, self employed farmers, clerical and office employees, skilled manual workers, other manual workers, total workers, other occupations, work full-time, work part-time and unemployed.
Question five
“How do you think that consumer prices have developed over the last 12 months? They have…”

Question six
“By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will…”

<table>
<thead>
<tr>
<th>Answer</th>
<th>Question five</th>
<th>Question six</th>
</tr>
</thead>
<tbody>
<tr>
<td>risen a lot</td>
<td>increase more rapidly</td>
<td></td>
</tr>
<tr>
<td>risen moderately</td>
<td>increase at the same rate</td>
<td></td>
</tr>
<tr>
<td>risen slightly</td>
<td>increase at a slower rate</td>
<td></td>
</tr>
<tr>
<td>stayed about the same</td>
<td>stay about the same</td>
<td></td>
</tr>
<tr>
<td>fallen</td>
<td>fall</td>
<td></td>
</tr>
<tr>
<td>don’t know</td>
<td>don’t know</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Questions five and six of the consumer survey

Five possible answers in the following way. The total percentage value of the first, second, third, forth, and fifth answer categories are named $PP$, $P$, $E$, $M$, and $MM$ respectively. The EC calculates the balance index $B$ by:

$$B = (PP + 0.5 \times P) - (MM + 0.5 \times M)$$

The indices that result from this calculation are published monthly by the EC and are the starting point of the following analysis. To get an overview of the data, figure (1) depicts exemplary the aggregate answer percentages given by all respondents of question five ($PP$, $P$, $E$, $M$, $MM$, and don’t know) between January 1985 and December 2007. The horizontal axes show the time horizon and the vertical axes show the percentage of the respondents of each possible answer. Figure (2) presents the results of all respondents in the same way as figure one, but with regards to inflation expectations.

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The graphs show the effects of the second Gulf War at the beginning of the 1990’s, the introduction of the Euro in January 2002, and the sales tax increase in Germany of three percentage points in January 2007 for the expected inflation and also for the perceived inflation. The events Gulf War and tax increase cause the answer probabilities ”risen a lot / increase more rapidly” to rise. The introduction of the Euro in 2002 induces the answer probabilities for a rising inflation expectation to fall and for a higher perceived inflation to rise. The increase in the perceived inflation rate throughout the course of the introduction of the euro in Germany is a common phenomenon.\footnote{Cf. Brachinger (2006) and Hoffmann et al. (2005).}
3 Results of empirical investigation

Plenty of effort is undertaken in the literature stated below to analyze heterogeneity in inflation expectations and perceived inflation among different demographic groups (Jonung (1981), Bryan and Venkatu (2001a), Bryan and Venkatu (2001b), Palmqvist and Strömberg (2004), Linden (2005), and Pfajfar and Santoro (2007)). This paper is the first one that deals with heterogeneity among demographic groups for Germany. This section presents results for differences among the groups based on the index values calculated by the EC and compares these results with the findings in existing literature.

Table (2) presents the outcome for the balance index for the expected inflation and the perceived inflation for different groups from the first quarter of 1990 to the fourth quarter of 2007. Since January 1990 the data has been available for all different groups. Therefore the time horizon of the
analysis starts in January 1990. For clarity only three of the ten different occupations are used throughout the whole paper. These are "clerical and office employees", "Work full-time" and "unemployed". This can be justified by the fact that the first two occupations are representative for all excluded occupations. All numbers are rounded to one decimal place. The results for the mean and the standard deviation (SD) of each group are presented.

<table>
<thead>
<tr>
<th>Group</th>
<th>Inflation expectation</th>
<th>Perceived Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>Full sample</td>
<td>31.8</td>
<td>13.1</td>
</tr>
<tr>
<td>Male</td>
<td>32.0</td>
<td>13.0</td>
</tr>
<tr>
<td>Female</td>
<td>31.6</td>
<td>13.1</td>
</tr>
<tr>
<td>Primary education</td>
<td>32.3</td>
<td>12.9</td>
</tr>
<tr>
<td>Secondary education</td>
<td>31.0</td>
<td>13.2</td>
</tr>
<tr>
<td>Further education</td>
<td>33.7</td>
<td>14.4</td>
</tr>
<tr>
<td>16-29 years old</td>
<td>29.2</td>
<td>13.2</td>
</tr>
<tr>
<td>30-49 years old</td>
<td>32.3</td>
<td>14.0</td>
</tr>
<tr>
<td>50-64 years old</td>
<td>32.9</td>
<td>12.6</td>
</tr>
<tr>
<td>above 65 years</td>
<td>32.6</td>
<td>12.9</td>
</tr>
<tr>
<td>1st Quartile Income</td>
<td>33.5</td>
<td>12.2</td>
</tr>
<tr>
<td>2nd Quartile Income</td>
<td>32.7</td>
<td>13.6</td>
</tr>
<tr>
<td>3rd Quartile Income</td>
<td>32.5</td>
<td>12.7</td>
</tr>
<tr>
<td>4th Quartile Income</td>
<td>30.9</td>
<td>13.6</td>
</tr>
<tr>
<td>Clerical and office employees</td>
<td>31.3</td>
<td>13.8</td>
</tr>
<tr>
<td>Work full-time</td>
<td>31.6</td>
<td>13.4</td>
</tr>
<tr>
<td>Unemployed</td>
<td>35.9</td>
<td>12.2</td>
</tr>
</tbody>
</table>

Table 2: Index values of expected and perceived inflation on a quarterly base

The data shows that between the first quarter of 1990 and the fourth quarter of 2007 no difference exists in the inflation expectation and the perceived inflation among men and women. In contrast to this, the majority

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8The results of the excluded groups are available by the author on request.

9In the following analysis data is used that is only available on a quarterly base. For uniformity and comparability the whole analysis is done on a quarterly base. Nevertheless all results are qualitatively the same for quarterly and monthly data. The monthly data is aggregated to quarterly values by using a simple mean value transformation (Cf. Reckwerth (1997)).
of the literature for other countries obtains the result that males have lower inflation expected and perceived inflation than females (Cf. Jonung (1981), Bryan and Venkatu (2001a), Bryan and Venkatu (2001b), Palmqvist and Strömberg (2004), Linden (2005) and Pfajfar and Santoro (2007)). Therefore it seems that the reasons for the differences between males and females in other countries don’t exist in Germany (for example buyer behavior).

Another result is that the inflation expectations and the perceived inflation fall with a rising income. This result is also found by the majority of the literature mentioned above.

With respect to different ages, no systematic difference among the categories for expected and perceived inflation are found. Nor does the literature ascertain a well defined outcome.

Results from the education group offer one quandary. It would be expected that the results for income and education would be similar because of the high correlation of these two variables. Instead the results for the education group are hump shaped for expected inflation and only the perceived inflation fall with a higher education. For a more detailed investigation concerning the reasons, micro data would be necessary.

Other remarkable outcomes are shown by the group of unemployed respondents. This group is always more "negative" when compared to all other groups. This means that their expected and perceived inflation is much higher when compared to all other groups.

4 Conversion of the data

A problem with the data described above is that up to this point it is only possible to compare index values of the different groups. Unfortunately it is not possible with the index values to give economic interpretations. The index values cannot be interpreted as inflation rates. Therefore the qualitative data from the EC have to be converted into quantitative data. For the computation I use the probability approach of Batchelor and Orr (1988) and Berk (1999). The starting point for both approaches is the work of Carlson and Parkin (1975). Carlson and Parkin designed a probability method to

In the EC survey the consumers have to choose between six answers. The answer "don’t know" is ignored for the computation from qualitative to quantitative data because it does not change the results. The answer fractions of "don’t know" are divided into the other response categories in same parts what presents the usually approach in the literature.\textsuperscript{10} Thus the business and consumer survey can be interpreted as a survey with five response possibilities and the probability method of Batchelor and Orr can be applied.

The conversion methods of Carlson/Parkin, Batchelor/Orr, and Berk are common accepted in the literature. These conversion methods have been applied by many researchers (including Berk (1999), Mankiw et al. (2003), Berk and Hebbink (2006), Döpke et al. (2008)) and institutions as the European Central Bank, the Bundesbank, and the Centre of European Economic Research (ZEW).

4.1 Probability method (three answer categories)

In their seminal work Carlson and Parkin (1975) describe how to calculate quantitative data out of qualitative answers with a probability method. Their approach was constructed for a trichotomous survey, i.e. the respondents have three answer categories.\textsuperscript{11} The answer categories are fall/decrease, no change/stay the same, and rise/increase. Carlson and Parkin assume that every individual forms his own subjective probability distribution function of the variable $x$ for period $t+1$. The subjective distribution function can be aggregated to the expectations distribution $f(x_{t+1})$. Furthermore they assume that an interval around zero exists, in that the individual will answer "no change". The boundaries of the interval are given by the values $\delta_t$ and


−δ_t. Above the limit δ_t the individual will expect the variable to rise. If the percentage is below the limit −δ_t, the individual will expect the variable to fall. If the question corresponds to inflation development, the individual will answer:

- "Inflation will rise" if \( x_{t+1} \geq \delta_t \)
- "Inflation will fall" if \( x_{t+1} \leq -\delta_t \)
- "Inflation will stay the same" if \( -\delta_t < x_{t+1} < \delta_t \)

where \( x_{t+1} \) is the median of the subjective probability distribution function.

The expected inflation rate of the whole population is assumed to be the mean of the median of each individual’s subjective distribution function. Carlson and Parkin assume that the distribution of the expected inflation is normally distributed. Furthermore they assume that the just noticeable difference \( \delta_t \) between the inflation rate and an inflation of zero is common to all individuals and equally for positive and negative deviation of the inflation rate from zero.

The answer proportions from the survey are called "\( A_t \)" for the answer category "Inflation will fall", "\( B_t \)" for the category "Inflation will stay the same", and "\( C_t \)" for "Inflation will rise". Thus it follows:

\[
P(x_{t+1} < -\delta_t) = \int_{-\infty}^{-\delta_t} f(x_{t+1})dx_{t+1} = F(-\delta_t) = A_t
\]

\[
P(-\delta_t < x_{t+1} < \delta_t) = \int_{-\delta_t}^{\delta_t} f(x_{t+1})dx_{t+1} = F(\delta_t) - F(-\delta_t) = B_t
\]

\[
P(x_{t+1} > \delta_t) = \int_{\delta_t}^{\infty} f(x_{t+1})dx_{t+1} = 1 - F(\delta_t) = C_t
\]

Under the assumption of a normal distribution, \( x_{t+1} \) can be standardized by:
\[ a_t = F^{-1}(A_t) = \frac{-\delta_t - \mu_{t+1}}{\sigma_{t+1}} \] (1)

and

\[ b_t = F^{-1}(A_t + B_t) = \frac{\delta_t - \mu_{t+1}}{\sigma_{t+1}} \] (2)

Where \( \sigma_{t+1} \) is the standard deviation and \( F^{-1}(\cdot) \) is the inverse of the normal distribution. Combining these equations provide finally: \(^{12}\)

\[ \mu_{t+1} = \frac{\delta_t (a_t + b_t)}{(a_t - b_t)} \] (3)

Where \( \mu_{t+1} \) is the mean of the distribution function in period \( t \) of the future expected inflation in period \( t + 1 \) and can therefore be interpreted as the expected future inflation. The parameter \( a_t \) is the standardized value of the percentage answer of "fall" and the parameter \( b_t \) of the answers "fall" plus "stay the same". Whereas both parameters \( a_t \) and \( b_t \) can be calculated with the percentage values of the different answers. \(^{13}\) In Equation (3) the just noticeable difference is only a scale parameter. Carlson and Parkin assume that expectations are on average unbiased. Furthermore they assume that the just noticeable difference of the inflation around zero does not change over time. Therefore they make the average value of the inflation over the whole sample equal to the average value of the expected inflation over the same period to calculate \( \delta \):

\[
\sum_{t=1}^{T} \frac{\pi_t}{T} = \delta \sum_{t=1}^{T} \frac{(a_t + b_t)}{(a_t - b_t)}/T
\]

\[
\delta = \sum_{t=1}^{T} \frac{\pi_t}{T} \sum_{t=1}^{T} \frac{(a_t + b_t)}{(a_t - b_t)}
\] (4)

\(^{12}\)An extensive derivation is given in appendix A.

\(^{13}\)At this point a disadvantage of the Carlson Parkin method occurs that is also exists for the Batchelor and Orr method presented in the next section. It is not always possible to calculate a value for \( a_t \) or \( b_t \). Applied procedures for these cases are presented in Appendix B.
where $\pi_t$ is the inflation in period $t$. This method is used inter alia by the ZEW for their monthly published ZEW Financial Market report.\textsuperscript{14}

4.2 Probability method (five answer categories)

This section is based on the work by Batchelor and Orr (1988) who expands the Carlson-Parkin method to surveys with five response possibilities (pentachotomous survey).\textsuperscript{15} The argumentation will be done with respect to inflation.

The basic ideas are the same as described in section 4.1. The only difference is that Batchelor and Orr assume that $f$ is logistic distributed. What kind of distribution will be used in this paper will be explained in section 5.

With regards to inflation expectation it is assumed that the individuals will answer, for example to question six of the consumer survey:

- ”fall”, if $x_{t+1} < -\delta_t$
- ”stay about the same”, if $-\delta_t < x_{t+1} < \delta_t$
- ”increase at a slower rate”, if $\delta_t < x_{t+1} < \mu'_t - \epsilon_t$
- ”increase at the same rate”, if $\mu'_t - \epsilon_t < x_{t+1} < \mu'_t + \epsilon_t$
- ”increase more rapidly”, if $\mu'_t + \epsilon_t < x_{t+1}$

Where $\pm \delta_t$ is the just noticeable difference of the inflation around zero and $\pm \epsilon_t$ is the just noticeable difference of the inflation around the perceived inflation $\mu'_t$.

Thus the individual will answer ”fall” if the mean of the expected future inflation is less than the negative deviation of the inflation from zero, which is just noticeable by the individual. The answer will be ”stay about the same”, if the expected mean inflation is not distinguish from zero inflation. The respondents will answer ”increase at a slower rate”, if the mean expected

inflation is distinguishable from zero inflation and is less than the mean perceived inflation minus the just noticeable difference of the inflation around the mean perceived inflation. If the expected inflation is not distinguishable from the mean perceived inflation, the respondent will answer "increase at the same rate". It will be answered "increase more rapidly", if the mean expected inflation is noticeable greater than the mean perceived inflation.

The subjective distribution function can be aggregated to the expectations distribution \( f(x_{t+1}) \). Writing the proportions of the response "fall" as \( A_t \), of "stay about the same" as \( B_t \), of "increase at a slower rate" as \( C_t \), of "increase at the same rate" as \( D_t \) and "increase more rapidly" as \( E_t \), it can be seen in figure (3) that the proportions of \( A_t \) estimate the area under \( f \) in the range between \(-\infty \) and \(-\delta_t \). Proportions of \( A_t \) and \( B_t \) estimate the area between \(-\infty \) and \( \delta_t \) and so forth.\(^{16}\)

This can be written as:

\[ P(x_{t+1} < -\delta_t) = \int_{-\delta_t}^{-\delta_t} f(x_{t+1}) dx_{t+1} = F(-\delta_t) = A_t \]

\[ P(-\delta_t < x_{t+1} < \delta_t) = \int_{-\delta_t}^{\delta_t} f(x_{t+1}) dx_{t+1} = F(\delta_t) - F(-\delta_t) = B_t \]

\[ P(\delta_t < x_{t+1} < \mu'_t - \epsilon_t) = \int_{\delta_t}^{\mu'_t - \epsilon_t} f(x_{t+1}) dx_{t+1} = F(\mu'_t - \epsilon_t) - F(\delta_t) = C_t \]

\[ P(\mu'_t - \epsilon_t < x_{t+1} < \mu'_t + \epsilon_t) = \int_{\mu'_t - \epsilon_t}^{\mu'_t + \epsilon_t} f(x_{t+1}) dx_{t+1} = F(\mu'_t + \epsilon_t) - F(\mu'_t - \epsilon_t) = D_t \]

\[ P(x_{t+1} > \mu'_t + \epsilon_t) = \int_{\mu'_t + \epsilon_t}^{\infty} f(x_{t+1}) dx_{t+1} = 1 - F(\mu'_t + \epsilon_t) = E_t \]

Where \( F(\cdot) \) is the cumulative distribution function of \( f(\cdot) \). This can be standardized by:

\[ \frac{-\delta_t - \mu_{t+1}}{\sigma_{t+1}} = F^{-1}(A_t) = a_t \]  \hspace{1cm} (5)

\[ \frac{\delta_t - \mu_{t+1}}{\sigma_{t+1}} = F^{-1}(A_t + B_t) = b_t \]  \hspace{1cm} (6)

\[ \frac{\mu'_t - \epsilon_t - \mu_{t+1}}{\sigma_{t+1}} = F^{-1}(A_t + B_t + C_t) = c_t \]  \hspace{1cm} (7)

\[ \frac{\mu'_t + \epsilon_t - \mu_{t+1}}{\sigma_{t+1}} = F^{-1}(A_t + B_t + C_t + D_t) = d_t \]  \hspace{1cm} (8)

The parameter \( \epsilon_t \) and \( \delta_t \) are the just noticeable limens, \( \mu_t \) is the mean expected inflation and \( \mu'_t \) is the mean perceived inflation. Combining these equations provides finally:\textsuperscript{17}

\textsuperscript{17} A comprehensive derivation is given in appendix A.
\[ \mu_t = \frac{\mu'_t(a_t + b_t)}{(a_t + b_t - c_t - d_t)} \]  

(9)

With the same approach it is possible to express the mean perceived inflation as (based on the fifth question of the Consumer Survey):

\[ \mu'_t = \frac{\hat{\pi}_t(a'_t + b'_t)}{(a'_t + b'_t - c'_t - d'_t)} \]  

(10)

Where \( a'_t, b'_t, c'_t, d'_t \) represent the standardized values of the answer categories of question five of the survey. The variable \( \hat{\pi}_t \) is the perceived past inflation and represents a scaling factor of the perceived inflation. If equation (10) is substituted in equation (9), the mean expected inflation \( \mu_t \) can be computed by:

\[ \mu_t = \frac{\hat{\pi}_t(a'_t + b'_t)(a_t + b_t)}{(a'_t + b'_t - c'_t - d'_t)(a_t + b_t - c_t - d_t)} \]  

(11)

The only still unanswered question for the calculation of the expected inflation is, what variable should be used for the perceived past inflation.

### 4.3 Combination of the probability approaches

One way to determine the perceived past inflation is described by Berk (1999). His approach is applied to pentachotomous surveys. The basic idea is to use data out of survey question five that deals with the perceived inflation rate to determine the perceived past inflation of the respondents. Corresponding to these questions the respondent has five respond possibilities (Consumer prices in the last 12 month have: risen a lot; risen moderately; risen slightly; stayed about the same; fallen). Berk’s idea is to translate the pentachotomous surveys of question five in a survey with only three respond possibilities and then to use the Carlson-Parkin approach to compute the perceived past inflation.

Therefore Berk sums up the first three answer categories (risen a lot; risen moderately; risen slightly) to a single category called “up”. After this transformation he uses the Carlson-Parkin method as described in section
4.1. The technical procedure is the same. The only difference concerns the computation of the difference limen around zero. On the one hand Carlson and Parkin calculate the difference limen in the way that they equate the mean of the expected inflation with the mean of the actual rate of inflation over the same period (equation (4)). They assume by construction that the estimate of the inflation rate is unbiased. On the other hand Berk calculates the difference limen in a way that he equates the mean of the perceived past inflation of the respondent with the mean of the actual past inflation rate. Berk uses the indifference limen to calculate the perceived inflation rate as Carlson and Parkin by:

$$\mu'_t = \delta \left( \frac{a_t + b_t}{a_t - b_t} \right)$$

The result is used as the scaling factor in equation (9) to calculate the expected inflation.

5 Results of the conversion

The conversion methods of Batchelor and Orr and Berk described in section 4 are used in this analysis. For the probability method of Batchelor and Orr I follow the majority of the literature and use the past inflation over the last year, lagged by one period as the scaling factor (Cf. Deutsche Bundesbank (2001), Döpke et al. (2008)). Furthermore I assume a normal distribution for the probability distribution function that is also commonly accepted in literature (Cf. Batchelor and Orr (1988), Berk (1999), Deutsche Bundesbank (2001), Berk (2002), Lyziak (2003), Mankiw et al. (2003), Henzel and Wollmershäuser (2005), Berk and Hebbink (2006), Döpke et al. (2008)). A second argument in favor of the normal distribution in addition to the popularity is that Berk (2002) shows that the normal distribution outperforms other distribution assumptions.

As a first step the data is converted using the conversion method of Batchelor and Orr and the means and the standard deviations for the different groups are calculated.\textsuperscript{18} As a graphically example figure (4) shows exclu-

\textsuperscript{18}All calculations were also done with data converted by the method of Berk. The
Figure 4: Actual and expected inflation

sively the results for the total sample.

The results after the conversion should be similar to the results based on the index data presented in section 3. Table (3) presents the results for Germany in the period comprising the first quarter of 1990 to the fourth quarter of 2007.

After the transformation the results in table (3) are still the same as described in section 3. During the time between the first quarter of 1990 and the fourth quarter of 2007 there is no difference in the inflation expectation and the perceived inflation among men and women. The inflation expectation and the perceived inflation fall with a rising income. The groups divided into different ages do not show a systematic difference among the categories. The unemployed respondents are still "outliers". With quantitative data, the mean squared forecast error of the inflation expectation for the different groups can be calculated. The results show that unemployed individuals results are qualitatively the same as the results of the data converted by the method of Batchelor and Orr. For simplicity, only the results based on the conversion by Batchelor and Orr are presented in this paper. All results are available by the author on request.
<table>
<thead>
<tr>
<th>Inflation expectation</th>
<th>Mean</th>
<th>SD</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realized inflation</td>
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<td>1.3</td>
<td>0</td>
</tr>
<tr>
<td>Full sample</td>
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<td>1.44</td>
<td>100</td>
</tr>
<tr>
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<tr>
<td>Female</td>
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<td>1.45</td>
<td>102</td>
</tr>
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<td>1.47</td>
<td>102</td>
</tr>
<tr>
<td>Secondary education</td>
<td>1.62</td>
<td>1.37</td>
<td>98</td>
</tr>
<tr>
<td>Further education</td>
<td>1.65</td>
<td>1.43</td>
<td>98</td>
</tr>
<tr>
<td>16-29 years old</td>
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<td>1.36</td>
<td>101</td>
</tr>
<tr>
<td>30-49 years old</td>
<td>1.72</td>
<td>1.48</td>
<td>103</td>
</tr>
<tr>
<td>50-64 years old</td>
<td>1.73</td>
<td>1.45</td>
<td>100</td>
</tr>
<tr>
<td>above 65 years old</td>
<td>1.74</td>
<td>1.48</td>
<td>100</td>
</tr>
<tr>
<td>1st Quartile Income</td>
<td>1.80</td>
<td>1.53</td>
<td>111</td>
</tr>
<tr>
<td>2nd Quartile Income</td>
<td>1.73</td>
<td>1.47</td>
<td>101</td>
</tr>
<tr>
<td>3rd Quartile Income</td>
<td>1.68</td>
<td>1.43</td>
<td>98</td>
</tr>
<tr>
<td>4th Quartile Income</td>
<td>1.60</td>
<td>1.41</td>
<td>99</td>
</tr>
<tr>
<td>Clerical and office employees</td>
<td>1.63</td>
<td>1.41</td>
<td>100</td>
</tr>
<tr>
<td>Work full-time</td>
<td>1.67</td>
<td>1.43</td>
<td>101</td>
</tr>
<tr>
<td>Unemployed</td>
<td>1.95</td>
<td>1.63</td>
<td>116</td>
</tr>
</tbody>
</table>

Table 3: Inflation expectations of different groups on a quarterly base

produce the highest mean squared forecast error. Beside of these differences, the German consumers represent a quite homogeneous group.

6 Rationality tests

After the overview of the mean inflation expectations of the different groups, this section performs tests for the rationality of the different groups following inter alia Mankiw et al. (2003). The simplest test of efficiency is done by a regression of the forecast error on a constant:

\[ \pi_t - \pi_t^e = \alpha + \varepsilon_t \]  

(12)

Where \( \pi_t \) is the inflation at time \( t \) and \( \pi_t^e \) is the expected inflation for time \( t \). This test shows if the inflation forecast is on average equal to the
realized inflation. The constant $\alpha$ must be insignificant for rationality. The results for the OLS regression are shown in the second column of table (4).

Another test for rationality is given by the equation:

$$\pi_t = \alpha + \beta \pi_t^e + \epsilon_t$$ (13)

If the inflation expectations are rational, $\alpha$ must be equal to zero and $\beta$ must be equal to one. Equation (13) can also be rewritten as:

$$\pi_t - \pi_t^e = \alpha + \beta \pi_t^e - \pi_t^e + \epsilon_t$$
$$\pi_t - \pi_t^e = \alpha + (\beta - 1)\pi_t^e + \epsilon_t$$ (14)

Equation (14) analyzes if the forecasts themselves have information for the forecast error. If the expectations are rational, both estimated parameters $\alpha$ and $\beta$ must be equal to zero. The test is equal to the one described by equation (13). If the expectations of the respondents are rational, the coefficients should be insignificant. The results for this test are also presented in table (4).

The regressions for both equations show that the estimated parameters $\alpha$ and $\beta$ are both significant for all different groups with the exception of the unemployed group for the first rationality test. Therefore the null hypothesis of rationality for German consumers has to be rejected.

Persistence in forecasting errors is also checked and a regression on the equation $\pi_t - \pi_t^e = \alpha + \beta(\pi_{t-1} - \pi_{t-1}^e) + \epsilon_t$ is run. If the actual forecast error is based on the forecast error for the previous period, the parameter $\beta$ would be significant. A significant $\beta$ shows that the past forecast error has explanatory power for the actual forecast. Therefore the hypothesis of rationality has to be rejected if $\beta$ is statistically significant. The results for the total sample are $\alpha = 0.08$ with a standard error equal to 0.09 and $\beta = 0.83$ with standard errors equal to 0.07. Thus $\beta$ is significant. The results for all other groups are similar to these results.\(^{19}\) Therefore the rationality hypothesis has to be

---

\(^{19}\)Results for all groups are not shown here. They are available from the author on request.
Rationality test  \( \pi_t - \pi^e_t = \alpha + \epsilon_t \)  \( \pi_t - \pi^e_t = \alpha + (\beta - 1)\pi^e_t + \epsilon_t \)

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \alpha )</th>
<th>( \beta - 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>0.38***</td>
<td>1.11***</td>
<td>-0.43***</td>
</tr>
<tr>
<td>Male</td>
<td>0.39***</td>
<td>1.10***</td>
<td>-0.42***</td>
</tr>
<tr>
<td>Female</td>
<td>0.37***</td>
<td>1.11***</td>
<td>-0.43***</td>
</tr>
<tr>
<td>Primary education</td>
<td>0.33**</td>
<td>1.10***</td>
<td>-0.44***</td>
</tr>
<tr>
<td>Secondary education</td>
<td>0.46***</td>
<td>1.10***</td>
<td>0.39***</td>
</tr>
<tr>
<td>Further education</td>
<td>0.42***</td>
<td>1.10***</td>
<td>0.41***</td>
</tr>
<tr>
<td>16-29 years old</td>
<td>0.49***</td>
<td>1.12***</td>
<td>-0.39***</td>
</tr>
<tr>
<td>30-49 years old</td>
<td>0.34**</td>
<td>1.12***</td>
<td>-0.43***</td>
</tr>
<tr>
<td>50-64 years old</td>
<td>0.34**</td>
<td>1.10***</td>
<td>-0.43***</td>
</tr>
<tr>
<td>above 65 years</td>
<td>0.34**</td>
<td>1.10***</td>
<td>-0.43***</td>
</tr>
<tr>
<td>1st Quartile Income</td>
<td>0.28*</td>
<td>1.14***</td>
<td>-0.48***</td>
</tr>
<tr>
<td>2nd Quartile Income</td>
<td>0.34**</td>
<td>1.11***</td>
<td>-0.44***</td>
</tr>
<tr>
<td>3rd Quartile Income</td>
<td>0.39***</td>
<td>1.09***</td>
<td>-0.42***</td>
</tr>
<tr>
<td>4th Quartile Income</td>
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<td>1.11***</td>
<td>-0.40***</td>
</tr>
<tr>
<td>Clerical and office employees</td>
<td>0.44***</td>
<td>1.11***</td>
<td>-0.41***</td>
</tr>
<tr>
<td>Work full-time</td>
<td>0.40***</td>
<td>1.11***</td>
<td>-0.42***</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.12</td>
<td>1.11***</td>
<td>-0.51***</td>
</tr>
</tbody>
</table>

Table 4: Tests of rationality
* *, **, *** indicate respectively statistically significance to the 10, 5, and 1 percent level.

rejected once again for all groups.

The analysis to this point shows that there are small differences among German consumers but altogether they are a quiet homogenous group and that their inflation expectations are not rational in a mathematical sense. It is still unanswered how expectations are formed. Section 7 deals with this issue.

### 7 The formation of consumer expectations

This section analyzes how the inflation expectations of consumers are formed and which information is taken into account. Following Carroll (2003) and his derivation of a micro foundation for the sticky information model by Mankiw and Reis (2002), it is analyzed what information is taken into ac-
count for consumers in their inflation expectation formation. The variables of interest are the inflation forecasts of professionals and past consumer inflation forecasts. The expert expectations are given by data from the company Consensus Economics. Consensus Economics interviews about 30 firms and institutions in Germany inter alia about their quantitative inflation expectations.\(^{20}\) The mean of all answers is used as expert expectations.

### 7.1 Simple baseline model

The basic idea for this analysis is that consumers have information about macroeconomic variables, their own past inflation forecasts, and the forecasts from experts if they are doing their inflation forecast for the next period. The consumers should assume that the forecasts from the experts are better than their own. Therefore the consumers should adopt the expert forecasts. The information about the expert forecasts is for example available through newspapers in that the expert forecasts are printed.\(^{21}\) Carroll (2003) shows that inflation forecasts of American consumers are only influenced by the own inflation forecast of the last period and the expert forecast. Based on the findings of Carroll it is analyzed if this result also hold for German consumers by the following simple baseline model.

\[
\pi_t^e = \alpha_1 \text{Expert}_t + \alpha_2 \pi_{t-1}^e + \epsilon_t
\]

Where \(\pi_t^e\) are the inflation expectations of the consumers and \(\text{Expert}_t\) are the professional forecasts. The results of an ordinary least square regression

\(^{20}\)The survey data are available from the second quarter of 1994 to the fourth quarter of 2007. Therefore the analysis is taken for this time horizon. Based on the survey data, the expert forecasts for the next year on a quarterly based are calculated. For Germany the following institutes and firms are interviewed by Consensus Economics: IW-Cologne Institute, Bayerische LBank, Delbruck & Co, DIW- Berlin, Commerzbank, DekaBank, Dresdner Bank, DZ Bank, FAZ Institute, Helaba Frankfurt, Lehman Brothers, UBS Warburg, West LB, WZG Bank, Bank Julius Baer, Bankgesellschaft Berlin, BHF Bank, Deutsche Bank, HSBC Trinkaus, HWWA, HypoVereinsbank, Invesco Bank, JP Morgan, MM Warburg, Morgan Stanley, RWI Essen, Sal Oppenheim und SEB.

\(^{21}\)For more details see Carroll (2003)
are presented in table (5).^{22}

<table>
<thead>
<tr>
<th>Expectation formation</th>
<th>$\pi_t = \alpha_1 \text{Expert}<em>t + \alpha_2 \pi</em>{t-1}^e + \epsilon_t$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$R^2$</th>
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</thead>
<tbody>
<tr>
<td>Full sample</td>
<td></td>
<td>0.31***</td>
<td>0.50***</td>
<td>0.94</td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td>0.31***</td>
<td>0.49***</td>
<td>0.94</td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td>0.31***</td>
<td>0.51***</td>
<td>0.94</td>
</tr>
<tr>
<td>Primary education</td>
<td></td>
<td>0.33***</td>
<td>0.49***</td>
<td>0.94</td>
</tr>
<tr>
<td>Secondary education</td>
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<td>0.30***</td>
<td>0.50***</td>
<td>0.94</td>
</tr>
<tr>
<td>Further education</td>
<td></td>
<td>0.30***</td>
<td>0.50***</td>
<td>0.94</td>
</tr>
<tr>
<td>16-29 years old</td>
<td></td>
<td>0.30***</td>
<td>0.47***</td>
<td>0.94</td>
</tr>
<tr>
<td>30-49 years old</td>
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<td>0.32***</td>
<td>0.50***</td>
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</tr>
<tr>
<td>50-64 years old</td>
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<td>0.32***</td>
<td>0.51***</td>
<td>0.94</td>
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<tr>
<td>above 65 years</td>
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<td>0.31***</td>
<td>0.51***</td>
<td>0.94</td>
</tr>
<tr>
<td>1st Quartile Income</td>
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<td>0.35***</td>
<td>0.47***</td>
<td>0.94</td>
</tr>
<tr>
<td>2nd Quartile Income</td>
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<td>0.32***</td>
<td>0.50***</td>
<td>0.94</td>
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<tr>
<td>3rd Quartile Income</td>
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<td>0.94</td>
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<td>4th Quartile Income</td>
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<td>0.94</td>
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<tr>
<td>Clerical and office employees</td>
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<tr>
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<td>0.37***</td>
<td>0.50***</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 5: Formation of inflation expectation: simple model

All standard errors are corrected for heteroscedasticity and serial correlation using a Newey-West procedure with four lags. *, **, *** indicate statistically significance to the 10, 5, and 1 percent level respectively.

The results show that the adjusted $R^2$ is very high for all different demographic groups and $\alpha_1$ and $\alpha_2$ are highly significant for all groups. The findings show only quantitative differences. The estimated coefficient $\alpha_1$ is located between 0.28 and 0.37. Following the interpretation of Carroll this means that consumers updated their inflation expectation from expert expectations between every 8 to 11 months. These results are close to the results of Döpke et al. (2008): founding that the that the representative German consumer updates his expectation between once every 11 to 15 month.^{22}

The augmented dickey fuller tests shows that all consumer expectations are not stationary. This test indicates that the null hypothesis of a unit root can be refused just at a 10 percent significance level for the expert forecasts. In addition to this the Johansen test indicate cointegration between expert and consumer forecasts. Therefore a OLS regression does not provide distorted estimators.
Carroll gets to the result that the American consumer updates his inflation expectation once a year.

In summary the results show that the inflation expectations of consumers are influenced by expert expectations and their own past inflation expectations. If the expert forecasts are interpreted as rational forecast as done by Carroll, German consumers form their inflation expectations partly rational and partly adaptive.

7.2 Error correction model

It is also possible that a stable long run relationship between the consumers’ and experts’ expectation exists. If a stable long run relationship exists, an error correction model has to be applied to check for the influence of expert forecast on consumers’ inflation expectation. This is done by the following error correction model:

\[ \Delta \pi_t^e = \beta_1 \Delta \text{Expert}_t + \beta_2 \pi_{t-1}^e + \beta_3 \text{Expert}_{t-1} + \epsilon_t \]

The long run relationship between the consumer forecast \( \pi_t^e \) and the expert forecast \( \text{Expert}_t \) is given by \( \beta_3/\beta_2 \) and the short run relationship by \( \beta_1 \). A regression on the error-correction model delivers the results presented in table 6.

The results show that the long run relationship between the consumers’ forecasts and the experts’ forecasts are higher compared to the relationship from the simple baseline model. The long run relationship for all groups is between 0.60 and 0.76. A long run relationship of for example 0.60 means that if the experts forecast rise by 1 percent point the consumer forecast rise by 0.6 percent points. The half-life of the different demographic groups is between 4.8 and 6.1 months. What is in line with the update frequency out of the static model. Thus beside the short run dependence of the two variables, there also exists a long run relationship that is higher than the influence of the simple baseline model. It can be summarized that between the consumer inflation expectation and the expert inflation expectation exists a short and long run relationship.
Expectation formation

\[ \Delta \pi_t = \beta_1 \Delta \text{Expert}_t + \beta_2 \pi_{t-1} + \beta_3 \text{Expert}_{t-1} + \epsilon_t \]

<table>
<thead>
<tr>
<th>Expectation formation</th>
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<th>(\beta_2)</th>
<th>(\beta_3)</th>
<th>Long</th>
<th>Half-life</th>
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<td>5.6</td>
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<td>0.24***</td>
<td>0.63</td>
<td>5.5</td>
</tr>
<tr>
<td>Female</td>
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<td>0.24***</td>
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</tr>
<tr>
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<td>0.22***</td>
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<td>5.8</td>
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<tr>
<td>Further education</td>
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<td>0.24***</td>
<td>0.63</td>
<td>5.5</td>
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<td>0.25***</td>
<td>0.66</td>
<td>5.5</td>
</tr>
<tr>
<td>50-64 years old</td>
<td>0.26*</td>
<td>-0.35***</td>
<td>0.24***</td>
<td>0.69</td>
<td>5.9</td>
</tr>
<tr>
<td>above 65 years</td>
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<td>0.25***</td>
<td>0.66</td>
<td>5.5</td>
</tr>
<tr>
<td>1st Quartile Income</td>
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<td>0.26***</td>
<td>0.70</td>
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</tr>
<tr>
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<td>-0.38***</td>
<td>0.26***</td>
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</tr>
<tr>
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<td>-0.37***</td>
<td>0.24***</td>
<td>0.65</td>
<td>5.6</td>
</tr>
<tr>
<td>4th Quartile Income</td>
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<td>0.22***</td>
<td>0.61</td>
<td>5.8</td>
</tr>
<tr>
<td>Work full-time</td>
<td>0.27**</td>
<td>-0.37***</td>
<td>0.24***</td>
<td>0.65</td>
<td>5.6</td>
</tr>
<tr>
<td>Unemployed</td>
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<td>-0.38***</td>
<td>0.29***</td>
<td>0.76</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Table 6: Formation of inflation expectation: ECM

*, **, *** indicate statistically significance to the 10, 5, and 1 percent level respectively. The half life is scaled in month.

8 Learning

The previous section show that German consumers form their inflation expectation partly adaptive. For further knowledge about the adaptive behavior, this section analyzes if the inflation expectation formation can be described by an adaptive learning algorithm with constant gains. The analysis follows mainly Branch and Evans (2006).

The aim of this section is to test if the commonly used learning approach of constant gain learning in the literature is supported by the data. Data about inflation and GDP growth is needed for this analysis. Because GDP data is only available on a quarterly basis, the calculations are also made on a quarterly basis. Therefore the monthly inflation expectations are again aggregated to quarterly inflation expectations. The adaptive learning algo-
The algorithm is given by the following assumptions. It is assumed that the economic law of motion of the German consumers takes the form:

\[ \pi_t = b_t' x_t + \varepsilon_t \]

Where \( b_t \) is the parameter vector and \( x_t \) is the vector of explanatory variables. It is assumed that \( x_t = (1, \pi_{t-1}, y_{t-1})' \). The actual inflation is again given by \( \pi_t \) and \( y_t \) represents the real GDP growth over the last year. The parameter vector is estimated by:

\[ \hat{b}_t = \hat{b}_{t-1} + \gamma R_{t-1}^{-1} x_t (\pi_t - \hat{\pi}_{t-1} x_t) \]
\[ R_t = R_{t-1} + \gamma (x_t x_t' - R_{t-1}) \]

Where \( \hat{b}_t \) is the parameter estimator, \( \gamma \) is the constant gain parameter, and \( R_t \) denotes the moment matrix for \( x_t \).

Initial values for \( b_0 \) and \( R_0 \) are needed to start the calculation. Following Branch and Evans I compute these values in the first period with an ordinary least square regression on a horizon that I choose to be as long as possible. In the second period the individuals learn if new information is available. Based on the last parameters of the second period, inflation forecast is computed for the third period.

Data for inflation and GDP growth in Germany is available from the first quarter of 1971 onwards. Inflation expectations of the consumers are given from the first quarter of 1990 onwards. Because of the availability of the data the first period starts in 1971Q1 and ends in 1989Q4. The rest of the sample is divided into two periods of the same length. Thus the second period is from 1990Q1 until 1999Q1 and the third period from 1999Q2 until 2007Q4. The constant gains that produce the lowest mean squared forecast error in the second period are presented in table (7).\(^{23}\) The forecast error is the squared difference between the inflation expectation for period \( t \) and the realized inflation in period \( t \).

The results show that a constant gain bigger than zero is needed to pro-

\(^{23}\)By searching over all \( \gamma \in (0.001, 1) \)
duce the lowest mean square forecast error for all demographic groups. All groups show homogeneous results with the exception of the unemployed. This is further evidence that the inflation expectation formation of German consumers is at least partly adaptive.

<table>
<thead>
<tr>
<th>Constant gain learning</th>
<th>Gains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>0.015</td>
</tr>
<tr>
<td>Male</td>
<td>0.015</td>
</tr>
<tr>
<td>Female</td>
<td>0.015</td>
</tr>
<tr>
<td>Primary education</td>
<td>0.014</td>
</tr>
<tr>
<td>Secondary education</td>
<td>0.016</td>
</tr>
<tr>
<td>Further education</td>
<td>0.016</td>
</tr>
<tr>
<td>16-29 years old</td>
<td>0.016</td>
</tr>
<tr>
<td>30-49 years old</td>
<td>0.015</td>
</tr>
<tr>
<td>50-64 years old</td>
<td>0.014</td>
</tr>
<tr>
<td>above 65 years</td>
<td>0.014</td>
</tr>
<tr>
<td>1st Quartile Income</td>
<td>0.013</td>
</tr>
<tr>
<td>2nd Quartile Income</td>
<td>0.014</td>
</tr>
<tr>
<td>3rd Quartile Income</td>
<td>0.015</td>
</tr>
<tr>
<td>4th Quartile Income</td>
<td>0.016</td>
</tr>
<tr>
<td>Clerical and office employees</td>
<td>0.016</td>
</tr>
<tr>
<td>Work full-time</td>
<td>0.015</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 7: Gains of the adaptive learn algorithm

9 Conclusion

This paper shows that German consumers are, in contrast to consumers from other countries (USA; EU; Sweden), a quite homogeneous group concerning their inflation expectation and their perceived inflation. Therefore it seems that the reasons for the differences between the demographic groups in the other countries don’t exist in Germany. Differences in Germany among demographic characteristics are only present for the group divided by income and the group of unemployed consumers: The inflation expectation and the perceived inflation fall with a rising income and unemployed consumers are
outliers regarding inflation expectation, perceived inflation, mean squared forecast errors, and constant gains in an adaptive learning framework.

Rationality tests indicated that the inflation expectations of German consumers are not rational in a mathematical sense. The expectation formation shows adaptive behavior. It exists a short and a long run relationship between consumer and expert forecasts. The analysis of the adaptive learning algorithm shows that an algorithm with positive constant gains produces the lowest mean squared forecast errors for the inflation expectation of consumers.

In summary the results of this empirical investigation show that the assumption of rational inflation expectations in economic models can not be supported by the data. Instead of that an adaptive modeling of the expectation process is closer to reality.
References


A Derivations

This part of the appendix presents the derivation of the equations (3) and (11) in detail:

The starting point of the derivation of equation (3) is the definitions of \( a_t \) and \( b_t \):

\[
    a_t = F^{-1}(A_t) = \frac{-\delta_t - \mu_{t+1}}{\sigma_{t+1}}
\]

and

\[
    b_t = F^{-1}(A_t + B_t) = \frac{\delta_t - \mu_{t+1}}{\sigma_{t+1}}
\]

These equations can be written as:

\[
    a_t \sigma_{t+1} = -\delta_t - \mu_{t+1}
\]

\[
    \mu_{t+1} = -\delta_t - a_t \sigma_{t+1}
\]
and
\[ b_t \sigma_{t+1} = \delta_t - \mu_{t+1} \]
\[ \mu_{t+1} = \delta_t - \sigma_{t+1} \]

The combination of these equations and solving for \( \sigma_{t+1} \) delivers:
\[ \delta_t - b_t \sigma_{t+1} = -\delta_t - a_t \sigma_{t+1} \]
\[ -b_t \sigma_{t+1} + a_t \sigma_{t+1} = -2\delta_t \]
\[ \sigma_{t+1} (a_t - b_t) = -2\delta_t \]
\[ \sigma_{t+1} = -2\delta_t / (a_t - b_t) \]

Solving the equations for \( \mu_{t+1} \) delivers:
\[ \mu_{t+1} = -\delta_t - a_t \sigma_{t+1} \]
\[ \mu_{t+1} + \delta_t = -a_t \sigma_{t+1} \]
\[ (\mu_{t+1} + \delta_t) / -a_t = \sigma_{t+1} \]
\[ \mu_{t+1} = \delta_t - b_t \sigma_{t+1} \]
\[ \mu_{t+1} - \delta_t = -b_t \sigma_{t+1} \]
\[ (\mu_{t+1} + \delta_t) / (-b_t) = \sigma_{t+1} \]
\[ (\mu_{t+1} + \delta_t) / (-b_t) = (\mu_{t+1} + \delta_t) / (-a_t) \]
\[ (\mu_{t+1} + \delta_t) (-a_t) = (\mu_{t+1} + \delta_t) (-b_t) \]
\[ -a_t \mu_{t+1} - a_t \delta_t = -b_t \mu_{t+1} - b_t \delta_t \]
\[ b_t \mu_{t+1} - a_t \mu_{t+1} = a_t \delta_t - b_t \delta_t \]
\[ \mu_{t+1} = \delta_t (a_t - b_t) / (-a_t + b_t) \]
\[ \mu_{t+1} = -\delta_t (a_t + b_t) / (-a_t + b_t) \]
\[ \mu_{t+1} = \delta_t (a_t + b_t) / (a_t - b_t) \]

The starting point of the derivation of equation (22) is again the defini-
tions of $a_t, b_t, c_t$ and $d_t$:

\[
a_t = F^{-1}(A_t) = \frac{-\delta_t - \mu_{t+1}}{\sigma_{t+1}} \tag{16}
\]

\[
b_t = F^{-1}(A_t + B_t) = \frac{\delta_t - \mu_{t+1}}{\sigma_{t+1}} \tag{17}
\]

\[
c_t = F^{-1}(A_t + B_t + C_t) = \frac{\mu_t' - \epsilon_t - \mu_{t+1}}{\sigma_{t+1}} \tag{18}
\]

\[
d_t = F^{-1}(A_t + B_t + C_t + D_t) = \frac{\mu_t' + \epsilon_t - \mu_{t+1}}{\sigma_{t+1}} \tag{19}
\]

Solving equation (17) for $\delta_t$ gives:

\[
\delta_t = \sigma_{t+1} b_t + \mu_{t+1}
\]

Substituting this in equation (16) leads to:

\[
\frac{-\sigma_{t+1} b_t - \mu_{t+1} - \mu_{t+1}}{\sigma_{t+1}} = a_t
\]

\[
a_t = -b_t - \frac{2\mu_{t+1}}{\sigma_{t+1}} \tag{20}
\]

Solving equation (18) for $\epsilon_t$ shows:

\[
\epsilon_t = -c_t \sigma_{t+1} + \mu_t' - \mu_{t+1}
\]

After substituting this in equation (19) and solving for $\sigma_t$ it follows, that:

\[
\frac{\mu_t' - c_t \sigma_{t+1} + \mu_t' - \mu_{t+1} - \mu_{t+1}}{\sigma_{t+1}} = d_t
\]

\[
\frac{-c_t \sigma_{t+1} + 2\mu_t' - 2\mu_{t+1}}{\sigma_{t+1}} = d_t
\]

\[
\frac{-c_t \sigma_{t+1} + 2\mu_t' - 2\mu_{t+1}}{\sigma_{t+1}} = d_t \sigma_{t+1}
\]

\[
\frac{2\mu_t' - 2\mu_{t+1}}{\sigma_{t+1}} = d_t \sigma_{t+1} + c_t \sigma_{t+1}
\]

\[
\sigma_{t+1} = \frac{2\mu_t' - 2\mu_{t+1}}{d_t + c_t} \tag{21}
\]
Equation (21) can be used in equation (20) so that it is possible to express the mean expected inflation in dependence of the probabilities of the response categories and the mean expected inflation in the last period:

\[ a_t = -b_t - \frac{2\mu_{t+1}}{d_t + c_t} \]
\[ a_t + b_t = \frac{\mu_t (d_t + c_t)}{\mu_t - \mu_{t+1}} \]
\[ a_t \mu_t' + b_t \mu_t' - a_t \mu_{t+1} - b_t \mu_{t+1} = -\mu_{t+1} d_t - \mu_{t+1} c_t \]
\[ a_t \mu_t' + b_t \mu_t' = -\mu_{t+1} d_t - \mu_{t+1} c_t + a_t \mu_{t+1} + b_t \mu_{t+1} \]
\[ a_t \mu_t' + b_t \mu_t' = \mu_{t+1} (-d_t - c_t + a_t + b_t) \]
\[ \mu_{t+1} = \frac{\mu_t' (a_t + b_t)}{(a_t + b_t - c_t - d_t)} \]

(22)

Equation (15) and (22) are the final equations (3) and (11) for the conversion method of Carlson and Batchelor and Orr presented in the text.

**B  Correction of the data**

For the conversion methods described above an assumption about the aggregate distribution must be made. Based on the literature a normal distribution is assumed. Therefore there are three cases in which equation (3) and (11) cannot be calculated.\(^{24}\)

- The value of the aggregate distribution function is zero.
- The value of the aggregate distribution function is one.
- The denominator is zero.

If the value of the aggregate distribution function is zero, the value of the inverse of the normal distribution approaches minus infinity. In this case I added \(1/(2n + 1)\) to the response category that is equal to zero. Whereas

\(^{24}\)For the correction of the data I am following mainly Henzel and Wollmershäuser (2005).
$n$ is the number of respondents. This procedure can be justified by the fact that the survey only approximates the representative consumer. In addition to this the correction does not change the survey results significantly. The lowest number of respondents is in the category "further education" in which 159 individuals are interviewed. If in this group the aggregate distribution function is zero, the data are corrected by adding $1/(2n+1) = 0.003$ to this category. That is very close to the true value of zero.

If the value of the aggregate distribution function is one, the value of the inverse of the normal distribution approaches plus infinity. If this occurs $1/(2n+1)$ is subtracted from the aggregate distribution function. This procedure does not change the results significantly either. The third case is only a theoretical one. It does not appear in the data used here.

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