Kranich, Jan

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by
Jan Kranich

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R&D and the Agglomeration of Industries

Jan Kranich*
University of Lueneburg

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Abstract

This paper discusses a model of the New Economic Geography, in which the seminal core-periphery model of Krugman (1991) is extended by endogenous research activities. Beyond the common 'anonymous' consideration of R&D expenditures within fixed costs, this model introduces vertical product differentiation, which requires services provided by an additional R&D sector. In the context of international factor mobility, the destabilizing effects of a mobile scientific workforce are analyzed. In combination with a welfare analysis and a consideration of R&D promoting policy instruments and their spatial implications, this paper makes a contribution to the so-called brain drain debate.

Keywords: R&D, New Economic Geography, Vertical Differentiation

JEL classifications: F12, F14, F17

*Jan Kranich, Institute of Economics, D-21 332 Lueneburg, Germany, email: kranich@uni-lueneburg.de
1 Introduction

The Lisbon Strategy, constituted by the European Council in 2000 and revised in 2005, was targeted "to make the European Union (EU) the leading competitive economy in the world and to achieve full employment by 2010". A central part of this ambitious objective is to establish a sustainable growth based upon innovation and a knowledge-based economy. The Lisbon Strategy is closely connected with the European structural and cohesion policy. Under this directive, the EU provides overall €347 billion for the period 2007–2013 for national and regional development programmes, from which €84 billion will be made available for innovation investments. The main priorities are assigned to improve the economic performance of European regions and cities by promoting innovation, research capabilities and entrepreneurship with the objective of economic convergence.\(^1\) National and regional programmes accompany the supranational efforts following the key-note that industrial dispersion, as well as spacious growth, can be realized by a competition of regions.

In their annual progress report, the European Commission draws a positive interim conclusion about the Lisbon Strategy.\(^2\) The economic growth in the EU expected to rise at 2.9% in 2007, the employment rate of 66% was much closer to the target of 70%, and the productivity growth reached 1.5% in 2006. As also the report admits, the progress can only partly be ascribed to the Lisbon Strategy in the face of the global economic growth and increasing international trade. Furthermore, with regard to the political aim of cohesion, regional and national disparities are still present with respect to the recent economic advances. Likewise, the high and often cited target mark of 3% gross domestic expenditures on R&D is at a current value of 1.91% (Eurostat, estimated for 2006) still far from being achieved, which also concerns the aspired global leading position. The comparative empirical study of Crescenzi et al. (2007) finds evidence for a persisting technological gap of the EU in comparison with the United States. The authors conclude that two reasons might be responsible for this development: one is what they referred to as "national bias," which means the diversity of national innovation systems, and the second is the "European concern with cohesion, even in the genesis of innovation".\(^3\)

In this regard, a dispersive regional policy inevitably implies a waiving of spatial efficiency due to lower external economies of scale. Furthermore, a subsidization and redistribution contrary to agglomeration forces is associated with possibly high budgetary efforts due to thresholds, nonlinearities and discontinuities.\(^4\) Finally, con-

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4See Baldwin et al. (2003) Chp. 9, for instance.
Considering a regional and national policy competition, it might be questionable if the way paved by the EU is efficient and will really meet supranational objectives. Considering the linkage between regional and innovation policy as it is accelerated by the EU, the question arises if research promoting programmes are an appropriate instrument to foster economic agglomeration also in the European periphery. The leading thought of this approach is that a local research development is not only limited to high-tech sectors, but may also induce multiplicative employment and growth effects in linked sectors. However, these endeavours go along with a couple of economic and political aspects. First, R&D activities are spatially concentrated, where the proximity to research institutions, as well as technology adapting downstream sectors, is relevant, which has been demonstrated by a multitude of empirical and theoretical publications.\footnote{See, e.g., Feldman (1999) for a survey.} In this context, knowledge and knowledge production as an essential attribute of high-tech clusters may be tacit, non-tradable and featuring a strong localization due to (technological) spillover effects. Second, the geographical concentration of research capacities and linked industries is critically influenced by the mobility of highly skilled labor. And third, the growth, evolution, and the macroeconomic relevance of emerging industries may be exposed to immense technological, political or social uncertainties. Against the background of these problems, this paper addresses the following questions: 1) Which interdependencies determine the agglomeration of the research and manufacturing industry? 2) Which impact has the proceeding trade integration upon the spread and extent of R&D? 3) With respect to the conflict of agglomeration vs. dispersion, which spatial formation implies a welfare improvement? 4) How does a unilateral or regional R&D and innovation policy affect the locational competition? 5) Do bilateral competing regional or national policies really lead to spatial efficiency? The paper approaches these leading questions by means of an extended model of the New Economic Geography (NEG). The NEG is an analytical framework primarily established by Krugman (1991), Krugman and Venables (1995), and Fujita (1988), which considers geographical concentration based upon increasing returns, monopolistic competition and (iceberg) trade costs. As Fujita and Mori (2005) point out, the "NEG remains to be the only general equilibrium framework in which the location of agglomerations is determined explicitly through a microfounded mechanism".\footnote{See Fujita and Mori (2005), p. 379.} In their survey article, the authors classify agglomeration forces into E-(economic) and K-(knowledge) linkages. While the first category includes traditional mechanisms induced by production and transactions of goods and services, the second involves ideas and information creating local and global spillover effects. Models of the first generation (see above) consider R&D activities only rudimentarily within fixed costs. The footloose entrepreneur model of Forslid and Ottaviano...
extended this approach by an implementation of a skilled workforce as a (fixed) human capital. Simultaneously, Martin and Ottaviano (1999) and Baldwin et al. (2001), later on Fujita and Thisse (2003) introduce technological spillover effects within an endogenous growth environment, where the capital is accounted to be a knowledge stock produced by an innovation sector with a private and a public output. The corresponding models orientate at renowned publications of Grossman and Helpman (1991a,1991b), as well as Segerstrom et al. (1990) and Flam and Helpman (1987). Recent works incorporating knowledge production and heterogeneity of agents are published by Berliant et al. (2006).

For providing answers to the initial subject, this paper picks up the seminal core-periphery model of Krugman (1991) and recombines it with endogenous R&D activities of firms. We focus on the destabilizing effects of highly-skilled migration, commonly referred to as the brain drain. Furthermore, we concentrate on quality improving R&D, which implies vertical product differentiation. For keeping the model tractable and simple as possible, we neglect public good characteristics of knowledge and knowledge creation, as well as endogenous spillover effects. The policy part is based upon the welfare implications derived from simple Pareto criteria. In addition, the economic policy is simplifying assumed as a tax-subsidy income transfer between factor groups, which finally sidesteps the modeling of a public sector. The major advantage of this approach is that it does not require to assume either a (utilitarian) welfare function or a government objective function because governmental action can directly be derived from the welfare propositions.

In order to examine the key questions above, this paper is structured as follows. In the next section we introduce the model assumptions and basic functionalities. In Section 3, we analyze the equilibrium states in terms of existence and stability. At this, we also consider the impact of exogenous asymmetries in country size deviating from the standard symmetric constellation. Section 4 focuses on the welfare analysis, which provides the formal legitimization for policy statements in Section 5. Finally, in Section 6, we return to the initial motivation of this paper, and derive conclusions for the European R&D and innovation policy.

## 2 The Model

### Private Demand

Preferences of private households in both locations follow a nested utility function of the form:

\[
U = M^\mu A^{1-\mu},
\]

where \( A \) denotes the amount of a homogenous good produced by a traditional constant return sector, henceforth considered to be an outside industry, which repre-
sents all industries not in the focus of this model. $M$ represents a subutility from the consumption of a continuum of differentiated consumer goods:\footnote{See also Anderson et al. (1992) for details.}

$$\begin{equation}
M = \left[ \sum_{i=1}^{n} (u_i)^{1/\sigma} (x_i)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1, \quad u_i > 0.
\end{equation}$$

The subutility depends upon the amount $x$ consumed of a particular product sort $i$ out of the mass of (potential) varieties $n$. The parameter $\sigma$ can easily be shown to be equal to the constant elasticity of substitution. The parameter $u$ characterizes a further (vertical) dimension in the differentiation space, which can be interpreted as the quality of a particular variety.

The manufactures are internationally tradable involving ad valorem trade costs (Samuelson iceberg costs), $t > 1$, for goods shipped from location $r$ to location $s$. From household optimization, we obtain the corresponding demand function:

$$\begin{equation}
x_r = \mu \sum_{s=1}^{S} Y_s u_s p_r^{-\alpha} t^{1-\sigma} P_s^{\sigma-1}, \quad s = 1, ..., S
\end{equation}$$

The utility parameter $\mu$ can be derived as the share in income, $Y$, spent for manufactures. The price index $P$ summarizes information about quality and prices, of substitutes, whereat $p_r$ is the price of a particular variety. From two-stage budgeting, the price-quality index for symmetric varieties is defined to be:

$$\begin{equation}
P_s^{1-\sigma} = \sum_{r=1}^{S} n_r u_r (p_r t)^{1-\sigma}
\end{equation}$$

The Manufacturing Sector

Firms in the manufacturing sector use an increasing return technology for the production of a particular variety, which involves labor as the only input factor. The corresponding factor requirement of a single manufacturing firm in location $s$ is characterized by a fixed and variable cost:

$$\begin{equation}
l_s^M = F + ax_s.
\end{equation}$$

Due to economies of scale and consumer preference for product diversity, each firm produces only one differentiated variety, so that the firm number is equal to the number of available product sorts. The firms are in position to increase the willingness to pay of consumers by improving the quality of products, which, in turn,
requires R&D investments. Following Sutton (1991), the level of quality is concave with respect to R&D expenditures, \( R \):

\[
(6) \quad u_s (R_s) = \left[ \frac{\gamma R_s}{r_s} \right]^{1/\gamma}, \quad \gamma > 1,
\]

where \( r \) represents the price of one unit R&D, and \( \gamma \) the corresponding research cost elasticity. Because manufacturing firms finance their R&D by sales revenues, the profit function is given by:

\[
(7) \quad \pi_s = p_s x_s - R_s (u_s) - w_s F - aw_s x_s,
\]

where \( w \) denotes the wage rate for labor. Maximization leads to the standard monopolistic mark-up pricing. By normalizing \( a \) to \((\sigma - 1)/\sigma\), the profit maximizing price is equal to marginal cost: \( p = w \). Furthermore, the optimum quality and R&D expenditures are:

\[
(8) \quad u_s^* = \left( \frac{p_s x_s}{\sigma r_s} \right)^{1/\gamma} \Rightarrow R_s^* = \frac{w_s x_s}{\sigma \gamma}.
\]

From (8) it follows that firms tend to improve the quality of their products with: i) increasing sales revenues; ii) an increasing monopolistic scope, given by a lower substitution elasticity; and iii) decreasing research costs as a result of a decreasing research price, or a lower research cost elasticity.

Assuming zero profits, the long run equilibrium output of one manufacturing firm can be derived as:

\[
(9) \quad x_s^* = \sigma F \left( \frac{\gamma}{\gamma - 1} \right).
\]

Equation (9) implies that the firm size is by the term in brackets higher than in the original Dixit-Stiglitz settings.

The R&D Sector
We introduce a separate research sector receiving the R&D expenditures of the manufacturing industry, and in turn providing R&D services. We assume a linear constant-return technology, where one unit of R&D requires one unit of scientific labor input. Furthermore, the R&D industry features a strong localization, meaning that a research facility in location \( s \) supplies its services for the manufacturing sector in the same location only. These presumptions lead to the following implications: i) R&D is not internationally tradable; ii) the size of the research sector determines the quality and thus the demand for manufactures in the corresponding location; and iii) in consequence of i) and ii), we implement strong vertical linkages between
manufacturing and the research sector.\(^8\)

The equilibrium research price, \(r\), results from the market clearing condition: \(r_s L_s^R = n_s R_s\), where the turnover of the R&D sector on the left hand side is equal to the amount of research expenditures of the whole manufacturing industry on the right hand side. \(L\) indicates the amount of R&D output, while simultaneously representing the input of research personnel, \(n\) is the number of firms in the manufacturing sector. From this expression, the corresponding market clearing price for one unit of research is:

\[
(10) \quad r_s = \frac{n_s R_s}{L_s^R},
\]

which simultaneously represents the wage rate for scientists. For the short run, we assume a fixed and price-inelastic supply of R&D and research personnel, respectively. In the long run, researchers are internationally mobile responding to real wage differentials. The migration of R&D personnel (and of R&D firms) follows the NEG models’ commonly used ad hoc dynamics.\(^9\)

\[
(11) \quad \dot{s} = (\rho_s - \rho_r) s (1 - s).
\]

The parameter \(s\) denotes the share of global researchers in one particular location: \((L_s^R/L^R)\), which is henceforward country 1 in the two-location version of this model. The real research price, \(\rho_s\), is defined to be: \(r_s P_s^{-\mu}\). For analytical simplicity, the global number of scientists, \(L^R\), is set equal to 1 so that \(s\) denotes the number of researchers in location 1, and \((1 - s)\) in location 2.

**Production Wages and Household Income**

Wages for workers in the manufacturing sector come from the so-called wage equation, which represents the break-even rates a firm is willing to pay. The wage equation can be derived from solving (3) for the price, \(p\). With the monopolistic price setting rule, we obtain:

\[
(12) \quad (w_r)^\sigma = \frac{\mu \sum_{s=1}^S Y_s u_r t^{1-\sigma} P_s^{\sigma-1}}{x_r^s}.
\]

The total supply of production labor is allocated between the traditional and the manufacturing sector, where workers are inter-sectorally mobile. For simplification,\(^8\) the localization assumption of innovative activities has been comprehensively proven by a couple of empirical studies, basically Audretsch and Feldman (1996), Jaffe et al. (1993), and Feldman (1994), which strongly provide evidence that although the costs of transmitting information are independent from spatial distance, the costs of transferring (tacit) knowledge determine local spillover effects.

\(^9\)See Baldwin et al. (2003), Chapter 2 for a detailed discussion of the ad hoc dynamics.
we set the total number of workers in both locations equal to one. In this way, the number of workers in the manufacturing industry is conform with the corresponding share $\lambda_s$, and the employment in the traditional sector corresponds with $1 - \lambda_s$. Furthermore, we assume for the traditional outside industry a linear 1:1 technology, as we did for the R&D sector. This leads to an output, which quantity is the same as the sectoral labor input. If we treat the traditional industry as numeraire and set the price for its output equal to 1, the income of workers in this sector is $1 - \lambda$. Via inter-sectoral mobility, the normalized wages in the traditional sector are equalized with the wages in the manufacturing sector. Finally, due to the (normalized) price setting of the competitive monopolists, the prices for manufactures are likewise equal to 1, which simplifies the algebra again.

In the long run, implying zero firm profits, the income of private households consists of the production wage bill, as well as the revenues from the R&D sector:

$$ Y_s = w\lambda_s + w(1 - \lambda_s) + n_s R_s = 1 + n_s R_s. \quad (13) $$

With these assumptions, the equilibrium R&D expenditures from equation (8) become with (9):

$$ R^* = \frac{F}{\gamma - 1}. \quad (14) $$

Equation (14) implies that the equilibrium research investments are the same for each firm and each location. In consequence, the income of scientists in equation (13) depends only upon the firm number in the local manufacturing industry.

In equilibrium, the total employment is equal to the total supply of manufacturing workers: $n_s J^M_s = \lambda_s$. From this market clearing condition and with the use of equations (5) and (9), the number of manufacturing firms can be derived:

$$ n_s = \frac{\lambda_s (\gamma - 1)}{F (\sigma \gamma - 1)}. \quad (15) $$

Thus, the income of private households is:

$$ Y_s = 1 + \frac{\lambda_s}{\sigma \gamma - 1}. \quad (16) $$

In the context of the two-location version, the research price (10) becomes with (14) and (15):

$$ r_1 = \frac{\lambda_1}{s (\sigma \gamma - 1)}. \quad (17a) $$

For the two-location version, the normalization implies that the global supply of production workers is twice the global supply of scientists. Though this setting is arbitrary, the qualitative results of the model are not affected. See Section 5 for details.
(17b) \[ r_2 = \frac{\lambda_2}{(1-s)(\sigma\gamma - 1)}. \]

Substituting equations (17) and (9) into (8), the product quality can be expressed as:

(18a) \[ u_1 = \left[ \frac{F\gamma(\sigma\gamma - 1)s}{(\gamma - 1)\lambda_1} \right]^{1/\gamma} \]

(18b) \[ u_2 = \left[ \frac{F\gamma(\sigma\gamma - 1)(1-s)}{(\gamma - 1)\lambda_2} \right]^{1/\gamma}. \]

In combination with firm number (15) and product quality (18), the price indices (4) can be rearranged to:

(19a) \[ P_1^{1-\sigma} = \gamma^{1/\gamma} \left[ \frac{\gamma - 1}{F(\sigma\gamma - 1)} \right]^{\gamma-1} \left[ (\lambda_1)^{\gamma-1} s^{1/\gamma} + (\lambda_2)^{\gamma-1} (1-s)^{1/\gamma} \right] \]

(19b) \[ P_2^{1-\sigma} = \gamma^{1/\gamma} \left[ \frac{\gamma - 1}{F(\sigma\gamma - 1)} \right]^{\gamma-1} \left[ (\lambda_1)^{\gamma-1} s^{1/\gamma} + (\lambda_2)^{\gamma-1} (1-s)^{1/\gamma} \right]. \]

Finally, the wage equations become:

(20a) \[ \frac{\sigma\gamma F}{\mu(\gamma - 1)} = u_1 \left[ Y_1(P_1)^{\sigma-1} + Y_2(P_2/t)^{\sigma-1} \right] \]

(20b) \[ \frac{\sigma\gamma F}{\mu(\gamma - 1)} = u_2 \left[ Y_1(P_1/t)^{\sigma-1} + Y_2(P_2)^{\sigma-1} \right]. \]

**Basic Mechanisms**

To understand the modeling results and to relate this paper to the standard NEG literature, it is useful to consider the main effects controlling the spatial allocation. For these purposes we determine the total differentials at the symmetric equilibrium so that, in the case of two symmetric locations, a change of a variable in location 1 goes along with an identical, but opposite, change in location 2. Considering a change in income, for instance, \( dY_1 \) is accompanied by \( -dY_2 \). Using the expressions given in the Appendix, we obtain from equations (16), (18), (19), and (20):

(21) \[ dY = \left( \frac{1}{\sigma\gamma - 1} \right) d\lambda \]

(22) \[ \frac{du}{u} = \left( \frac{2}{\gamma} \right) ds - \left[ \frac{\sigma\gamma - \mu}{2\mu(\sigma\gamma - 1)} \right] d\lambda \]
\[
\frac{dP}{P} = \frac{Z}{\gamma (1 - \sigma)} \left[ \left( \frac{(\sigma \gamma - \mu)(\gamma - 1)}{\mu (\sigma \gamma - 1)} \right) d\lambda + 2ds \right]
\]

(24) \[ \frac{du}{u} = \left[ \frac{Z (\mu - \sigma \gamma)}{\sigma \gamma} \right] dY + Z(\sigma - 1) \frac{dP}{P}. \]

From equations (21) – (24) follows:

**Result 2.1** Totally differentiating at the symmetric equilibrium reveals: i) a positive home-market effect; ii) a negative price-index effect with respect to both factor groups; and iii) a positive research and manufacturing employment relationship.

Equation (21) reveals the so-called **home-market effect**. The positive correlation implies that an increase in income corresponds with an increase in manufacturing employment. In contrast to the core-periphery model, this relationship is not necessarily more than proportional. In fact, this result occurs only for small values for \(\sigma\) and \(\gamma\). The dependency becomes more transparent by rearranging the term in brackets. Solving equation (15) for this expression and using (14), we obtain: \(\frac{(n_s R_s)}{\lambda_s}\). Keeping in mind that manufacturing wages are normalized by 1, the term in brackets in equation (21) can be interpreted as the ratio of total R&D expenditures and the manufacturing wage bill. This means if the R&D expenditures are higher (lower) than the labor costs, we have a more (less) than proportional employment effect due to an increase in market size.

Equation (23) reveals a negative **price-index effect** for both the manufacturing and the researching population, where, according to Fujita et al. (1999), the trade cost index, \(Z\), is defined to be:

(25) \[ Z \equiv \frac{1 - t^{1-\sigma}}{1 + t^{1-\sigma}}. \]

In this regard, a larger manufacturing sector implies a lower price index due to a reduction of trade costs and, thus, a higher local income and demand. An increasing R&D sector leads to an increase in quality, which finally reduces the price index, which also can be seen in equation (24).

Furthermore, by equalizing (22) and (24), we obtain:

(26) \[ d\lambda = \left[ \frac{2\sigma \mu (\sigma \gamma - 1)(1 - Z^2)}{Z(\mu - \sigma \gamma)(\mu - \sigma Z(\gamma - 1)) + \sigma (\sigma \gamma - \mu)} \right] ds. \]

Nominator and denominator in equation (26) are greater than zero implying a positive research and manufacturing employment linkage.
3 Equilibrium Analysis

Dispersion vs. Agglomeration Equilibrium
Considering the formal nature of this model, (16), (18), (19) and (20) describe a system of eight nonlinear simultaneous equations, where (11) specifies the equilibrium condition. The differential equation has three stationary points: i) at $s = 1$, where the R&D sector is totally agglomerated in 1; ii) at $s = 0$, where the R&D sector agglomerates in 2; and iii) for a zero differential: $\rho_1 = \rho_2$.

In the symmetric equilibrium, all variables are constant except from the price index that is increasing with trade costs. In consequence, the real research price is increasing with trade costs as well.

Figure 1 shows $s$, the share of researchers in location 1, with respect to trade costs. This kind of comparative-static illustration is also referred to as bifurcation or tomahawk diagram concerning the progression and structure of multiple equilibria.

[Insert Figure 1 about here.]

Usually, the spatial formation of industries is considered in respect of a decline in trade costs. Starting from a high level of trade costs, a stable equilibrium (continuous bold line) appears at the symmetry, $s = 0.5$, while the agglomeration equilibria, $s = 0, 1$, are unstable (dashed line). This implies a dispersive distribution of R&D and manufacturing industries that is unaffected by trade costs.

At a critical level, indicated by the so-called sustain point, $t^s$, the globally stable symmetric equilibrium becomes locally stable as a result of an alternating stability in the agglomeration equilibria. Later on, for trade costs lower than the break point, $t^B$, the symmetric equilibrium turns from stable to unstable, where the spatial distribution of both R&D and manufacturing industries takes form of the core-periphery outcome.

Result 3.1 In the core-periphery constellation, the whole manufacturing and R&D industries agglomerate in the core, while in the periphery the traditional sector is the only industry remaining.

This results from a complete withdrawal of researchers from the periphery, which reduces the quality of products locally manufactured. This, in turn, ceases the corresponding demand and the output of the manufacturing industry, which starts to relocate to the neighboring country with the larger sales market. In the agglomeration equilibrium, where the manufacturing industry and the research sector are entirely concentrated in one location, the earnings of private households in the core consist of the labor income plus the returns of the scientific workforce. In the peripheral location, the income comes only from labor totally employed in the constant return sector.
In between of break and sustain points, also commonly classified as medium trade costs, we obtain three locally stable equilibria in symmetry and total agglomeration, as well as two additional unstable equilibria starting from total agglomeration at \( t^S \) and converging to the symmetric equilibrium at the break point, \( t^B \).

In addition to Figure 1, this model features a second fundamental variable, the product quality in the manufacturing industry. In this context, Figure 2 shows the corresponding bifurcation diagram.

Based upon the stylized Figure 2 and apparent in equations (54) and (59a), it can be concluded:

**Result 3.2** In the case of initially symmetric locations, the level of product quality in dispersion as well as in the agglomerated core is constant and thus, independent from trade costs. The quality in the periphery is zero as a result of a total relocation of R&D and the manufacturing industry.

The constant level of quality constitutes a maximum that can be generated by the R&D facilities available in this economy. As given by equation (8), the quality depends only upon the market price for research services. In the agglomeration equilibrium, the number of manufacturing firms and scientists in the core is twice as in dispersion, which can easily be seen by equations (15), (53), and (58), respectively. Because the demand of a particular manufacturing firm for R&D services is fixed, equation (10) is constant in both agglomeration and dispersion equilibrium, since the ratio of supply and demand remains the same. In consequence, this level of quality is globally stable beyond break and sustain points. This implies that the quality of products available is always constant, before and after agglomeration.\(^{11}\)

In this context, the periphery, here location 2, does not produce manufactures due to a lack of the R&D sector that is totally relocated. Therefore, the corresponding quality, \( u_2 \), becomes zero for trade costs lower than \( t^B \). Although no R&D industry exists in region 2, there is still a hypothetical level of quality that would be generated if there would be any research activity. This hypothetical quality is represented by the (red) dashed line marking the lower limit of the unstable equilibrium arm.

However, between the critical trade costs values we observe two locally stable equilibrium qualities: i) location 1 is either in a dispersive equilibrium, or it becomes the core, which corresponds with the same quality level; ii) location 2 is either in a dispersive equilibrium, or it becomes the periphery implying the total loss of the manufacturing and R&D sectors and a zero quality.

\(^{11}\)These results may differ assuming spillover effects.
Stability
Because of the static nature of this model, the stability of equilibria is ascertained via the ad hoc dynamics given by equation (11). For illustrating an out of equilibrium adjustment process, it is quite common to plot the real wage differential against the share of the mobile workforce. In this context, Figure 3 shows the real research wage gap, $\rho_1 - \rho_2$, with respect to the share of scientists, $s$, in location 1 for different levels of trade costs.

[Insert Figure 3 about here.]

Due to the non-closeness, the wage differential can only numerically be determined. The diagrams in Figure 3 are plotted for a specific parameter constellation that is henceforth used as a reference case in the course of this paper.

The stability of solutions can heuristically be proven: a positive (negative) wage differential implies an increasing (decreasing) share of researchers. As apparent, this model always features one symmetric and two corner solutions, where the filled dots represent a stable and the blank dots an unstable equilibrium.

Having a closer look on the evolution of the equilibria constellation, we can observe converging differentials of the unstable corner solutions, until the sustain point, $t^S$, where both equilibria exhibit a zero-differential. From this point on, the core-periphery constellation becomes stable. For trade costs lower than $t^S$, the corner solutions are diverging again. At the break point, $t^B$, the slope of the symmetric equilibrium changes its sign turning from negative to positive. This implies an alternation of the stability from stable to unstable – the core-periphery equilibrium becomes the only outcome.

Sustain and break points are determined using the same approach as suggested by Fujita et al. (1999)\textsuperscript{12}. The sustain point can be found by identifying the trade costs level, where the wage differential of the agglomeration equilibrium becomes zero. The corresponding trade cost level solves:

\begin{equation}
 t^S \rightarrow (\sigma \gamma + \mu) t^{1-\sigma-\mu/\gamma} + (\sigma \gamma - \mu) t^{\sigma-1-\mu/\gamma} - 2\gamma\sigma = 0.
\end{equation}

By numerical inspection, equation (27) reveals the same qualitative characteristics of the comparative statics as the standard core-periphery model. An increasing substitution elasticity, $\sigma$, shifts the sustain point towards 1, because an increasing homogeneity of manufactures narrows the relevance of international trade. In addition, the larger the manufacturing sector, represented by an increasing share in household income, $\mu$, the larger is the range of trade costs, which contain a sustain core-periphery equilibrium. Furthermore, the research cost elasticity, $\gamma$, as an additional parameter within this model, reveals the same comparative statics like the

\textsuperscript{12}See Appendix for detailed derivations.
substitution elasticity, $\sigma$: the higher $\gamma$ implying an increasing costliness of R&D, the lower is the sustain point.

The break point can be determined by totally differentiating at the symmetric equilibrium. Hence, the equation system can be reduced to $d\rho/ds = 0$. Solving for trade costs, we obtain the marginal case, where the slope at the symmetric equilibrium (see Figure 3, $t = t^B$) becomes zero:

$$t^B = \left[ \frac{(\mu - \sigma \gamma)(\mu - \gamma(\sigma - 1))}{(\mu + \sigma \gamma)(\mu + \gamma(\sigma - 1))} \right]^{\frac{1}{1-\sigma}}.$$  \hspace{1cm} (28)

For positive values of $t^B$, the second term in the nominator of (28) must be negative. From this term, the so-called *no-back-hole condition* can be derived: \(^{13}\)

$$\mu < \gamma (\sigma - 1).$$  \hspace{1cm} (29)

The comparative statics of the break point qualitatively corresponds with the original core-periphery model: $t^B$ increases with an increasing $\mu$ and decreases with a rising homogeneity of manufactures, $\sigma$. Additionally, an increase in the cost intensity of R&D, expressed by the parameter $\gamma$, reduces the break point level, too.

**Asymmetric Locations**

We previously assumed that both countries feature the same number of production workers employed in the manufacturing, as well as in the traditional, sector. Deviating from this simplification, we treat location 1 as numeraire and normalize the number of production workers $L^M_1$ to 1. The size of the production labor force in location 2, $L^M_2$, is defined to be $a$, where $a > 0$. Thus, the standard symmetric case is a knife-edge version of this general setting, where $a = 1$. Henceforth, the employment in the manufacturing sector of location 2 is $a\lambda_2$, and in the traditional sector: $a(1 - \lambda_2)$.

Figure 4 shows the bifurcation diagram for the case that location 2 is 1% larger than location 1 (parameter values: $\sigma = 2$, $\gamma = 2$, $\mu = 0.5$, $F = 1$, $a = 1.01$).

[Insert Figure 4 about here.]

Compared to symmetric locations, the structure of the tomahawk diagram has lost its simplicity. Instead of one, two sustain points occur, the path of the stable equilibrium right from the break point is bent towards an agglomeration within the larger country. As a result, the break point also appears shifted away from symmetry. The reason for these distortions may be retraced by considering Figure 3 again. A smaller country size affects the real research wages via a higher price index.

\(^{13}\) See Appendix.
This implies that the wage gap function shifts downward originating the deviations from the symmetric equilibrium.

The trade cost levels indicating the first sustain point \((s = 1)\) solves the equation:  

\[
(30) \quad \tilde{t}^S \rightarrow (a\sigma \gamma + a\mu) t^{1-\sigma-\mu/\gamma} + a (\sigma \gamma - \mu) t^{\sigma-1-\mu/\gamma} - (1 + a)\gamma \sigma = 0.
\]

Respectively, the second sustain point \((s = 0)\) can be derived from:  

\[
(31) \quad \tilde{t}^S \rightarrow (a\sigma \gamma + \mu) t^{1-\sigma-\mu/\gamma} + (\sigma \gamma - \mu) t^{\sigma-1-\mu/\gamma} - (1 + a)\gamma \sigma = 0.
\]

The second sustain point is singular and always existing for \(t > 1\). The break point can only be numerically investigated, but appears between the sustain points in the majority of cases. While the second sustain point is always given, the first sustain point as well as the break point disappear with an increasing exogenous asymmetry, \(a\), which finally determines how far the stable path is moved away from symmetry. However, for the analysis of the critical trade cost values it may be useful to consider a couple of numerical examples. Table 1 shows the comparative statics of both sustain points with respect to changes in substitution and research elasticity, \(\sigma\) and \(\gamma\), the size of the manufacturing sector, \(\mu\), and the asymmetry parameter, \(a\). The computations are based upon a fixed parameter constellation given below the table, where each column shows the ceteris paribus changes in the corresponding variable.

As apparent, the numerical results reveal the same dependencies as in the case of symmetric locations. The trade cost values of the sustain points decrease with decreasing horizontal and vertical differentiation indicated by an increasing \(\sigma\) and \(\gamma\), and increase the larger is the income share for manufactures, \(\mu\). Furthermore, increasing the asymmetry parameter, \(a\), the first sustain point moves towards 1 and the second moves in the opposite direction. This implies that the larger the disparity in country size, the sooner occurs the point of total agglomeration. Although, from the technical point of view, the assignment which location becomes the core and which one becomes the periphery is still ambiguous; the common literature states that the smaller country tends to be the periphery.

Although, from the technical point of view, the assignment which location becomes the core and which one becomes the periphery is still ambiguous; the common literature states that the smaller country tends to be the periphery. The argumentation is based upon the magnitude of exogenous shocks that must be sufficiently high to

14 The bar above (below) \(t\) denotes \(s = 1\) \((s = 0)\).

15 Proof: If the no-black-hole condition holds, the function (31) shows i) a unique minimum for \(t > 1\); ii) an intersection with the trade costs axis at \(t = 1\); and iii) a negative slope at \(t = 1\). Hence, there must be a second axis intersection for \(t > 1\) solving equation (31).

16 The function (30) intersects the trade cost axis at \(t = 1\) and shows a unique minimum for \(t > 1\) if \(\frac{\sigma \gamma}{\mu} \left(\frac{a - 1}{a}\right) < 2\). This minimum appears for positive as well as negative values so that a root for \(t > 1\) is not inevitably existent.

17 See Baldwin et al. (2003) and Forslid and Ottaviano (2003), for instance.
make smaller locations become the industrialized core. With increasing trade integration, country size asymmetry implies that the R&D capacity within the smaller country continuously diminishes until the break point is reached. At this critical trade cost level, the smaller location abruptly loses its residual R&D sector. Summing up, R&D mobility entails a destabilizing potential for smaller countries to the advantage of larger neighbors attracting scientists and the corresponding industry.

How does asymmetry affect the quality of manufactures? Figure 5 shows the corresponding bifurcation diagram for the product quality in location 1.

As apparent, for high trade costs right from the first sustain point, the quality in location 1 continuously decreases due to a migration of scientists to location 2, so that it can be stated:

**Result 3.3** With exogenous asymmetry in country size, the quality of manufactures produced in the larger country feature a higher quality. Due to the home-market effect and the research-manufacturing employment linkage, an increasing quality of local manufactures increases demand, income and, consequently, the employment of additional R&D capacities. This cumulative causation leads to agglomeration within the larger country for decreasing trade costs.

Left from the break point, the quality in the larger country is constant with respect to the trade cost level again because this location has attracted the whole research activities at fixed R&D expenditures.

**Interim Results**

Based upon the previous findings, the outcome of this model allows a couple of conclusions:

1. Via cumulative causation, the production follows the R&D and vice versa.

2. A spatial specialization of R&D and the manufacturing sector is no potential outcome due to strong vertical linkages.

3. The settings allow catastrophic agglomeration as a common feature of the core-periphery model, but the "disastrous" extent for the peripheral region depends upon the importance of the manufacturing industry in the whole economic context. If the manufacturing industry is characterized by a low share in income, $\mu$, a total relocation has a minor impact on welfare, in contrast to an industry exhibiting a dominant macroeconomic relevance.

18Indeed, this outcome has been confirmed by a couple of empirical studies, e.g., Hummels and Klenow (2005), Greenaway and Torstensson (2000).
4. In the case of symmetric locations, international trade and R&D mobility do not affect the level of product quality as a result of a fixed firm size and identical endowments of scientists.

5. A numerical analysis of break and sustain points reveals that international mobility of scientists affects the spatial formation of industries. But compared with the original core-periphery model of Krugman (1991), this impact is less destabilizing than the mobility of workers, which can be seen at lower values for the critical points, $t^S$ and $t^B$.

The immanent instability of spatial dispersion and the risk of a total loss of manufacturing and research, especially for smaller countries, raise the question if a subsidization of local R&D may counteract deindustrialization. The following sections take up this consideration and analyze political intervention against the background of social welfare.

4 Welfare and Spatial Efficiency

This section addresses the efficiency and optimality of agglomeration vs. dispersion. The considerations are based upon a global perspective incorporating the welfare of the population in both locations. With respect to external economies of scale as well as the distribution of social welfare, we examine the legitimization of location and research promoting policy instruments applied by supra-regional institutions. In the course of this paper, the findings provide the basis for the analysis of R&D and innovation policy instruments in the next section.

For these purposes, the approach of Charlot et al. (2006) and Baldwin et al. (2003) is applied. As discussed by the authors, the specification of a social welfare function is associated with an aggregation problem involving inequality aversion of two factor groups within two locations. However, instead of following this utilitarian approach, we rather focus on the analysis of Pareto-dominance combined with Kaldor-Hicks compensation criteria.

**Pareto Dominance**

Individual welfare is measured as the (maximized) consumer utility that can be derived as the real household income, $Y^P - \mu$. The nominal income, $Y = Y^P + nR$ consists of i) fixed production wages, $Y^P = 1$, that is the same in both equilibrium states; and ii) the income of scientists. As mentioned above, the ratio of R&D demand and supply is equal in both equilibrium states so that the nominal research wage is also the same.\(^{19}\)

In summation, the real income of production workers as well as scientists leads to

\(^{19}\)See equations (56) and (61a).
a comparison of price indices. In the agglomeration equilibrium, where the whole manufacturing industry gathers in the core (henceforth, location 1), the price index takes the lowest value compared with the peripheral location and the dispersion equilibrium. Because in the dispersive case the manufacturing is evenly spread across both locations, the corresponding price index is lower than in the periphery. Thus, it can be concluded:

**Result 4.1** Production workers in the core always prefer agglomeration, and production workers in the periphery always prefer dispersion due to higher real incomes. In this context and with the same argument, researchers always prefer agglomeration. In conclusion, whether agglomeration or dispersion is a Pareto dominant equilibrium state because the labor force is split in agglomeration winners (production workers in the core and researchers) and agglomeration losers (production workers in the periphery).

**Compensation Tests**

In accordance to Kaldor (1939) and Hicks (1940), potential compensations between agglomeration winners and losers are next to be verified. In the first step, we proof if the winning factor group is able to compensate the disadvantaged production workers for remaining in the periphery in the sense of Kaldor. For utility equalization of the losing factor group, a compensation, $C^K$, has to be paid that fulfills the condition:

\[
(1 + C^K) \bar{P}^{-\mu}_2 = P_s^{-\mu}.
\]

Substituting the price indices (55) and (60) yields:

\[
C^K = \left( \frac{1 + t^{1-\sigma}}{2t^{1-\sigma}} \right)^{\frac{\mu}{1-\sigma}} - 1,
\]

which corresponds with the outcome of Charlot et al. (2006). After compensation, the income of production workers in location 2 becomes $\bar{Y}^P_2 = 1 + C'$. The income of the winning factor groups, workers and scientists in 1, is given by: $\bar{Y}_1 = 1 + \bar{r}_1 - C^K$. In the dispersion equilibrium, the winners would earn: $1 + 2Y^R_s$. The net welfare in the sense of Kaldor is the difference of real income of the agglomeration winners in agglomeration and dispersion:

\[
\Delta W^K \equiv \bar{W}_1 - W^{P,R}_s = (1 + r - C^K) \bar{P}^{-\mu}_1 - (1 + r) P_s^{-\mu},
\]

where $r = \bar{r}_1 = r_s$. The sign of the net welfare (34) depends upon the level of trade costs:

\[
\Delta W^K \gtrless 0 \Rightarrow 2\sigma \gamma \left( \frac{1 + t^{1-\sigma}}{2} \right)^{\frac{\mu}{1-\sigma}} - (\sigma \gamma - \mu) t^\mu - (\sigma \gamma + \mu) \gtrless 0.
\]

See equations (55), (60a), and (60b).
From equation (35) follows:

**Result 4.2** For trade costs lower than a critical value $t^K$, agglomeration is preferred to dispersion due to a positive net welfare of agglomeration winners.$^{21}$

With respect to the Hicks compensation tests, the argumentation of Charlot et al. (2006) is based upon the allocation effects of a real redistributive transfer. Assuming that agglomeration losers compensate the winners by the amount of welfare surplus they would waive by staying in the symmetric equilibrium, such a transfer would prevent the clearing of factor and labor markets. The authors conclude that a (real) Hicks compensation is not feasible and thus agglomeration is always preferred to dispersion, taking into account the results of the Kaldor tests. However, this paper deviates from this approach and follows the argumentation of a hypothetical compensation rather than assuming a real transferring system. In this context, a total compensation in the sense of Hicks requires a transfer $C^H$ holding the condition:

$$ (1 + 2Y^R_s + C^H) P_s^{-\mu} = (1 + Y^R_1) \bar{P}_s^{-\mu}. $$ (36)

The corresponding net welfare of the losing factor group, the peripheral production workers in 2, is:

$$ \Delta W^H_{\text{H}} \equiv W_{P_2}^P - \bar{W}_2^P = (1 - C^H) P_s^{-\mu} - (\bar{P}_1 t)^{-\mu}. $$ (37)

Dispersion compared to agglomeration represents a Pareto improvement in the sense of Hicks if the welfare of production workers in the (symmetric) location 2 is positive after compensating the agglomeration winners in 1. This situation is given by:

$$ \Delta W^H_{\text{H}} \gtrless 0 \Rightarrow 2\sigma \gamma \left( 1 + \frac{t^{1-\sigma}}{2} \right)^{\frac{\mu}{\gamma t^r}} - (\sigma \gamma - \mu) t^{-\mu} - (\sigma \gamma + \mu) \gtrless 0. $$ (38)

From equation (38) follows:

**Result 4.3** For trade costs higher than a critical level $t^H$, dispersion is preferred to agglomeration in the sense of Hicks.

This result differs from the outcome of the referenced study, where the authors state that only agglomeration is a Pareto improvement until the critical trade cost level $t^K$ is reached. Due to diverging concepts of compensation, this paper finds that a range of trade costs exists where dispersion might be a preferred equilibrium outcome. In this context, Figure 6 shows the net welfare functions with respect to trade costs for the standard numerical example.

$^{21}$K is mnemonic for Kaldor.
For the numerical case considered, there are two ranges of trade costs where either agglomeration is preferred in the sense of Kaldor ($1 < t < t^K$), or dispersion is preferred in the sense of Hicks ($t^H < t$). Between the critical levels, any statements about potential Pareto improvements are not possible. In this regard, the break and sustain points remarkably appear in between. This implies that with decreasing trade costs, dispersion loses its welfare dominance sooner than the economy reaches the core-periphery outcome. Furthermore, agglomeration later appears to be a clear Pareto improvement than the core-periphery formation actually becomes established.

**Varying Research Potential**

With regard to the arbitrary setting of the global number of scientists ($L^R = 1$), the question may arise if the results of the welfare analysis crucially depend upon the size of this factor group, which benefits from agglomeration. Based upon this consideration, an increasing number of internationally mobile researchers may imply an increasing potential for compensating agglomeration losers. In turn, in the sense of Hicks, this may imply a higher claim for compensation to be borne by production workers in the (potential) periphery, so that agglomeration becomes increasingly a Pareto improvement the larger the size of the researcher population. However, this outcome appears not to be imminent considering the competitive research market. The higher the supply of researchers and of R&D, respectively, the lower is the corresponding market price. This leads to an increase in the product quality and simultaneously to a reduction of the price-quality index. Because this affects each individual manufacturing firm, the demand for manufactures finally remains unchanged.\(^{22}\) At a constant demand, firm size remains constant as well, and for this reason the R&D expenditures, too. This finally results in unaffected research investments of the total industry, which implies lower per-capita income of scientists. All in all, the nominal household income remains the same, while the price index declines. In consequence, the welfare is higher in the case of an increasing researcher population. Having a look on the corresponding price indices for both equilibria, agglomeration and dispersion, they reveal the impact of an increasing number of scientists on the critical trade cost value, $t^K$ and $t^H$:

\[ P_{s}^{1-\sigma} = \left(\frac{\gamma L^R}{2}\right)^{1/\gamma} \left[ \frac{\mu (\gamma - 1)}{F(\sigma \gamma - \mu)} \right]^{\frac{\gamma - 1}{\gamma}} (1 + t^{1-\sigma}) \]

\[ \bar{P}_{1}^{1-\sigma} = (\gamma L^R)^{1/\gamma} \left[ \frac{2\mu (\gamma - 1)}{F(\sigma \gamma - \mu)} \right]^{\frac{\gamma - 1}{\gamma}}, \bar{P}_2 = \bar{P}_1 t. \]

\(^{22}\)This can easily be seen by considering simplifying a closed economy. The demand is $x = \mu Y p^{-\sigma} P^{\sigma-1}$. For symmetric varieties, the price index becomes: $P^{1-\sigma} = n^{-1} u^{-1} p^{\sigma-1}$. Substitution yields a demand that is independent from quality, $u$, eventually.
These price indices differ from the basic price indices, (55) and (60), only in terms of the first expression in brackets. This implies that the trade cost values, where the net welfare functions, (34) and (37), become zero, must be identical to the values solving (35) and (38). In consequence, it can be stated:

**Result 4.4** The critical trade cost values, $t^K$ and $t^H$, are independent from the total number of scientists. Hence, a political intervention in terms of an increase in R&D capacities leads to a higher social welfare, but it does not affect the evaluation of equilibria in terms of Kaldor and Hicks.

**Numerical Calibration**

General results beyond this numerical example, especially with regard to the existence of both critical trade cost values, $t^K$ and $t^H$, are not possible due to non-closeness of equations (35) and (38). Following Charlot et al. (2006), a numerical analysis of parameter values in real economic domains draws a rough picture about the welfare situation within this model. In this context, Table 2 shows the calibration results, which are structured in the same way as demonstrated for Table 1.23

[Insert Table 2 about here.]

Although this sample lacks generality, some observations can still be made: i) both values are always separated, while $t^K < t^H$; ii) the critical trade cost value of the Hicks compensation test disappears for large values of the income share, $\mu$; iii) varying the substitution elasticity, $\sigma$, reveals that $t^H$ falls below the sustain point, for values larger than 5, even below the break point, while $t^K$ is always smaller than break and sustain points; iv) the critical values show the same comparative static behavior like the break and sustain points (increasing with decreasing $\sigma$ and $\gamma$, and increasing with $\mu$). Summarizing, the numerical investigation demonstrates the complexity of welfare statements with respect to industrial agglomeration. If an equilibrium represents a Pareto improvement, it critically depends upon the parameter constellation, which is nonetheless evidently sensitive with respect to exogenous changes.

Complementing these results from a supra-national perspective, the next section considers first the impact of a unilateral R&D policy, and second the case of conflicting bilateral policies.

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23The parameters are oriented to the range of values given by Charlot et al. (2006), following Head and Mayer (2004).
5 R&D and Innovation Policy

In the course of the policy analysis during this section, the paper focuses on the impact of a public R&D policy within one country, which henceforward will be location 2. Considering the outcomes of the previous sections, the social welfare of the population in the core is higher than in the periphery. Starting from dispersion but threatening agglomeration (the economy is close to the break point), an individual national policy would most likely take actions to avoid the situation in which the domestic location (2) becomes the periphery. A political goal may be that the dispersion is maintained, or, better yet, the location 2 becomes the industrialized core. In this model, a policy meant to promote local R&D involves the subsidization of private R&D activities. With regard to real economic policy, a large repertory of instruments is utilized ranging from public funding of research projects, start-up promotion for high-tech firms, and tax abatements for private R&D expenditures, for instance.

Based upon the previous considerations, this section concerns three major questions: 1) Does a unilateral subsidization of R&D lead to a reallocation of industrial activities to the advantage of the intervening location? 2) In the case that country 2 is smaller: Which amount of subsidization has to be transferred to balance out the corresponding local disadvantage? 3) In contrast to a centrally planned or cooperative solution between both countries, as described in Section 4, what is the outcome of conflicting bilateral R&D policies? 4) Does this locational competition lead to a socially preferred outcome?

R&D Subsidies in the Symmetric Case

Based upon the findings in the previous section, the government in country 2 aims its own location to become the industrialized core knowing that all its inhabitants would benefit due to higher real incomes. Starting from a situation, in which both locations are in the dispersion equilibrium, the government in 2 decides to introduce a system of income transfer between both factor groups. The simple idea is that a lump-sum subsidy for the mobile scientific workforce may imply a sufficient incentive to migrate towards location 2. The subsidy is financed by a (non-distorting) lump-sum tax, \(0 < \tau < 1\), paid by the immobile production workers in 2. Because the transfer is only realized between the inhabitants of one location, the nominal income of households remains the same as in the model without distributive intervention. Thus, the equations (16), (18), (19), and (20) describing the system do not change, contrary to the equilibrium condition given by (11), where the real research price in location 2 becomes: \(\rho_2 = (r_2 + \tau) P_2^\mu\). Because the real research earnings of scientists in 2 are higher after subsidization, the real wage differential curve must be shifted downwards. This results in a distortion of the tomahawk symmetry generating a bifurcation that is the same as for exogenous asymmetry. Indeed, the subsidization produces an allocation as if location 2 would be larger in terms of
country size. The appropriate sustain points for agglomeration in location 1 \((s = 1)\) solve:

\[
\tilde{t}_s^s \rightarrow t^\mu - \left[ \left( \frac{\sigma \gamma + \mu}{2\sigma \gamma} \right) t^{1-\sigma} + \left( \frac{\sigma \gamma - \mu}{2\sigma \gamma} \right) t^{\sigma-1} \right] - \left( \frac{\sigma \gamma - \mu}{2\mu} \right) \tau = 0,
\]

and for agglomeration location 2 \((s = 0)\):

\[
\tilde{t}_s^s \rightarrow t^\mu - \left[ \left( \frac{\sigma \gamma + \mu}{2\sigma \gamma} \right) t^{1-\sigma} + \left( \frac{\sigma \gamma - \mu}{2\sigma \gamma} \right) t^{\sigma-1} \right] + \left( \frac{\sigma \gamma - \mu}{2\mu} \right) \tau t^\mu = 0.
\]

Against the background of these possibilities the question arises: What is the optimum R&D policy to affect industrial agglomeration for location 2? First of all, it must be constituted if the government follows a strategy of instantaneous agglomeration at given trade costs. However, this policy may go along with a high burden for tax payers depending upon the degree of trade integration. In contrast, policymakers could also aim to achieve total agglomeration in the long run, while trade costs decrease. This approach is based upon the tendency of location 2 to agglomerate as a result of a quasi-exogenous asymmetry induced by a subsidy. Which strategy is chosen, depends upon the time preference of the economic agents, as well as the tax burden the production workers are willing to accept.

A. Instantaneous Agglomeration

For the case that policymakers in location 2 aim to achieve instantaneous agglomeration within their home country, they set a subsidy that completely shifts the wage gap function below zero. By means of Figure 3, it becomes apparent that the (real) subsidy, necessary to push location 2 into the core, depends upon the level of trade costs. Again, it is useful to differentiate between three cases: 1) For high trade costs \((e.g., t = 2.5)\), the corresponding real subsidy is equal to the wage differential at \(s = 0\). 2) If the trade integration continues, the maximum of the wage gap curve separates from the corner solution at a certain level of trade costs (approximately \(t = 1.9\) in terms of Figure 3). 3) In the third stage, the wage differential of the corner solution \(s = 1\) outruns the wage gap maximum \((t = 1.85\) in Figure 3).

1. For high trade costs, where the corner solution \(s = 0\) gives the highest wage gap value, the corresponding subsidy \(\tau'\) can be derived from equation (42) by solving for \(\tau\):  

\[
\tau' (t \in T') = \left( \frac{2\mu}{\sigma \gamma - \mu} \right) \left\{ \left[ \left( \frac{\sigma \gamma + \mu}{2\sigma \gamma} \right) t^{1-\sigma} + \left( \frac{\sigma \gamma - \mu}{2\sigma \gamma} \right) t^{\sigma-1} \right] t^{-\mu} - 1 \right\},
\]

The subscripts denote subsidy – the situation after introducing income transfers; the superscripts still stand for sustain point.

Furthermore, it can easily be shown that production workers in the subsidized core feature a higher welfare due to agglomeration, in spite of a lump-sum tax, until a critical level of trade costs, \(t > 1\), has passed. For trade costs higher than this level, the welfare loss in the course of taxation is higher than the welfare gain due to a lower price index.
where $T'$ is the domain of trade costs in which the corner solution $s = 0$ is relevant.

2. In the second stage, the critical subsidy given by the inner maximum of the wage gap curve can only numerically be determined. At a given level of trade costs, $T''$, this subsidy fulfills:

$$\tau''(t \in T'') \rightarrow t = t_{S}^{B},$$

where $T''$ is the domain of trade costs in which the inner maximum of the wage differential curve, $\partial \Omega/\partial s = 0$, is relevant.

3. Finally, for achieving instantaneous agglomeration, a policy concerning the corner solution $s = 1$ has to be applied in the third stage. The corresponding critical subsidy can be derived from equation (41):

$$\tau'''(t \in T''') = \left( \frac{2\mu}{\sigma\gamma - \mu} \right) \left\{ t^\mu - \left[ \left( \frac{\sigma\gamma + \mu}{2\sigma\gamma} \right) t^{1-\sigma} + \left( \frac{\sigma\gamma - \mu}{2\sigma\gamma} \right) t^{\sigma-1} \right]^\gamma \right\},$$

where $T'''$ is analogically the domain of trade costs in which the relevant target value is represented by the wage differential at the corner solution $s = 1$.

For illustration, Figure 7 shows the critical subsidy that ensures instantaneous agglomeration with respect to trade costs.

As apparent, the curve decreases for high trade costs ($\tau'$) following the corner solution $s = 0$. From the point, where the policy alternates from equation (43) to (44), the curve ($\tau''$) is kinked and decreases with a lower slope. Finally, where the wage differential at the corner solution $s = 1$ exceeds the inner maximum, the subsidy ($\tau'''$) increases again due to stronger agglomeration forces. After a unique maximum, the curve declines towards zero for $t \rightarrow 1$. Furthermore, while the sustain point level of trade costs is always element of $T''$, the trade costs indicating the break point are always in the domain of $T'''$.

### B. Long Run Agglomeration

In the case that policymakers decide to achieve agglomeration in the long run presuming decreasing trade costs, it is useful to distinguish two initial situations: 1) trade costs are higher; and 2) trade costs are lower than the sustain point level. The optimum R&D policy is illustrated by a numerical example given by the parameters: $\sigma = 2$, $\gamma = 2$, and $\mu = 0.4$. The break point occurs at: $t_{B}^{S} = 1.8333$, the sustain point at: $t_{S}^{S} = 1.8567$. Introducing a subsidy, $\tau = 0.001$, yields two sustain points for $s = 1$ at $t_{S1}^{S} = 1.0076$ and $t_{S2}^{S} = 1.8448$, one sustain point for $s = 0$ at $t_{S}^{S} = 1.8717$. A break point does not exist. For cases i) and ii), the following policy statements can be formulated:
1. For trade costs higher than the sustain point, a subsidization leads to a migration of scientists towards location 2, which makes $s$ decrease. Potentially, the new break and sustain points, $t^B_S$ and $t^B_S$, are immediately reached depending upon the level of subsidization. As long as the trade costs are above the (uncontrolled) sustain point level, $t^S$, a cancelation of the transfer system would lead back to dispersion again. Accordingly, the subsidization must be maintained until i) location 2 becomes the core ($t < t^B_S$), and ii) the trade costs become sufficiently low so that the (symmetric) sustain point, $t < t^S$ has passed. This finally ensures that location 2 becomes the (locally) stable core after stopping subsidization. Considering the parameterized example, trade costs are $t = 2$, for instance, the research sector is almost totally agglomerated in location 2. After reaching the sustain point, $t^S$, a subsidization is not necessary anymore because the core-periphery equilibrium is (locally) stable.

2. For trade costs lower than the sustain point level, and agglomeration in the competing location, the subsidy should be set in order to move the (asymmetric) sustain point $s = 1$ leftwards. In terms of the wiggle diagram, this policy implies a downward shift of the wage gap function, until the corner solution $s = 1$ becomes negative. This finally destabilizes the core in location 1, the economy alternates to the decreasing arm in the tomahawk diagram generating an increasing advantage for location 2. In the case of the numerical example, we assume trade costs at $t = 1.84$ again. Increasing the subsidy to $\tau = 0.002$ makes the opposite sustain points diverge from each other. The critical sustain points become: $\bar{t}^S_{S,2} = 1.8326$ and $\underline{t}^S_{S} = 1.8864$. In consequence, location 2 attracts the whole R&D and manufacturing industry. However, the subsidization may be limited for very low trade costs. Assuming a very large manufacturing sector with $\mu = 0.8$ and a very low costliness of R&D with $\gamma = 1.3$, the corresponding targeted sustain point is $\bar{t}^S_{S,2} = 10.2567$ that can be just realized by subsidy of $\tau = 1$. For trade costs below this level, a way out of the periphery is impossible for location 2. In this context, the maximum tax burden, $\tau = 1$, which is the whole income of production workers, is totally exhausted. With regard to a more realistic picture, the maximum reasonable tax rate would be much lower, so that the domain of parameter values restricting this agglomeration trap becomes much larger.

**R&D Subsidies and Exogenous Asymmetry**

In the next step, we pursue the question: If location 2 is smaller in terms of country size, how can its government implement a subsidization policy to alleviate the disadvantageous effects of agglomeration?

As shown in Section 3, an exogenous difference in country size shifts the wage gap function. For the situation now considered, where location 2 is smaller, this curve moves upwards implying a stable arm in the tomahawk diagram that is bent towards
s = 1. In contrast, an R&D subsidy works like an artificial country enlargement, because the wage gap function is shifted downwards. Hence, a subsidy level exists where the disadvantage of country size is totally compensated by the subsidization effect. Based upon this consideration, the government in location 2 should implement a subsidy (and tax) that is larger than this critical level. In consequence, the smaller country would gain a migration tendency directed to total agglomeration for trade costs lower than the break point level.

For determining the critical level of subsidization, it is necessary to equate the symmetric with the asymmetric differential and solve for the subsidy, \( \tau \). Following this approach, two problems occur: First, due to non-closeness, the wage gap can only numerically computed, except from the symmetric and corner solutions. Second, while a subsidy implies a parallel shift of the wage differential curve, an exogenous difference in country size also changes the shape of this curve. Hence, the critical level of subsidization pushing the smaller country to agglomeration can only numerically be determined. The critical subsidy solves:

\[
\tau^* = \left( \frac{2P_2^\mu}{\sigma \gamma - 1} \right) \left( \lambda_1 P_1^{-\mu} - a \lambda_2 P_2^{-\mu} \right)
\]

Figure 8 plots the subsidy, \( \tau^* \), with respect to trade costs for the standard numerical example.

The curve features three essential attributes: 1) The subsidy totally compensating a disadvantage in terms of country size is unique and positive for trade costs, \( t > 1 \). 2) For \( a = 1 \) and \( t = 1 \) the subsidy is zero. 3) The subsidy varies with the degree of trade openness, where the subsidization increases with increasing trade costs.\(^{26}\) This results from higher price indices implying larger real wage differentials, which have to be overcome by the subsidy.

In addition to the considerations about the optimum R&D policy in the symmetric case, as given in the previous subsection, the government in 2 is in the position to route its country to agglomeration in spite of an initial disadvantage in terms of country size. According to the instantaneous agglomeration strategy as described above, the critical subsidy given by equation (46) has to be added on top the values of (42), (44), and (45), respectively. For the case that political decision-makers aim to achieve long run agglomeration, the corresponding subsidy has to exceed \( \tau^* \).

### Conflicting Bilateral Policies

In the case of opposite policies in two symmetric countries, the considerations above

\(^{26}\)For large values of \( \sigma \) and \( \gamma \), the subsidy features a unique maximum and a moderate decline right from this point.
may lead to an escalation of the R&D subsidy competition. Because agglomeration implies a welfare improvement for researchers as well as production workers in the core, both governments aim to direct their own locations into agglomeration. Contemplating the situation like a sequential game, one country would exceed the subsidy of the other country to finally gain an agglomeration advantage. To formally treat this situation, we refer to the results above and assume by reason of formal simplicity that both countries follow an instantaneous agglomeration strategy. The basic principle of an R&D and innovation policy is still the same: a subsidy introduced by country 2 leads to a downward shift of the wage gap function. With regard to the policy implications above, one country would, given a policy of the rivalling country, consequently choose a subsidy according to equations (43), (44), and (45), respectively. To that above, we distinguish between three cases according to the degree of trade integration: 1) high trade costs ($t \in T'$); 2) medium trade costs ($t \in T''$); and 3) low trade costs ($t \in T'''$).

1. At a given level of high trade costs, $t \in T'$, both locations follow a subsidization policy according to equation (43). Since both countries implement a transfer system, the subsidy in one location is set with respect to trade costs, and a given level of subsidy in the competing location. The corresponding symmetric reaction functions in the terms of a Cournot competition are:

\[
(47a) \quad \tau_1 (\tau_2, t) = \tau' + \tau_2 t^{-\mu} \\
(47b) \quad \tau_2 (\tau_1, t) = \tau' + \tau_1 t^{-\mu}
\]

Equations (47) are illustrated in Figure 9 by means of a specific numerical example.

[Insert Figure 9 about here.]

Generally, both functions are linearly increasing. Concluding from equation (42), the critical subsidy, $\tau'$, is always positive. Furthermore, the reaction function $\tau_2$ intersects the antagonistic curve $\tau_1$ always from above because $t > 1$. These results imply a unique, globally stable and positive Nash equilibrium at:

\[
(48) \quad \tau^* = \tau' \left[ \frac{1 + t^\mu}{t^\mu - t^{-\mu}} \right] \quad \forall \ t \in T'.
\]

In spite of a lack of generality, we can again derive a couple of results from numerical investigations. Table 3 shows the equilibrium subsidy with respect to trade costs for several parameter constellations in the same way, as illustrated in Tables 1 and 2, where fields without values are out of the domain $t \in T'$.
As reproduced, the equilibrium subsidy increases with i) increasing trade costs; ii) decreasing horizontal and increasing vertical differentiation (high $\sigma$ and $\gamma$); and iii) decreasing income share, $\mu$. In general, the calibration reveals that the equilibrium subsidy is relatively high; a couple of parameters show values even beyond the maximum tax base, $Y^P = 1$.

In summary, the subsidy race does not promote agglomeration; in fact, it preserves spatial dispersion. The picture changes if exogenous asymmetry is included, because the larger country has access to a larger tax base and is able to finally exceed the subsidy of the smaller country.

2. For medium trade costs, $t \in T''$, the regional governments orientate at the general policy given by equation (44). Similar to the reaction functions (47), the subsidy in one location should exceed the maximum wage gap plus the increase of real research wages due to the subsidization of the competing country:

$$(49a) \tau_1 (\tau_2, t) = \tau'' + \tau_2 P_2^{-\mu}$$

$$(49b) \tau_2 (\tau_1, t) = \tau'' + \tau_1 P_1^{-\mu}.$$  

The resultant Nash equilibrium occurs at:

$$(50) \tau^{**} = \tau'' \left( \frac{1 + P_s^{-\mu}}{P_s^{-\mu} - P_s^{-\mu}} \right) \forall t \in T''.$$  

The equilibrium subsidy in this stage is also positive, globally stable, and increasing in trade costs.

3. For low trade costs, $t \in T'''$, the reaction functions become according to (45):

$$(51a) \tau_1 (\tau_2, t) = \tau''' + \tau_2 t^\mu$$

$$(51b) \tau_2 (\tau_1, t) = \tau''' + \tau_1 t^\mu.$$  

Because the corner solution $s = 1$ ($s = 0$) is relevant for location 2 (1), the slopes of both functions are inverse compared to equations (47). In consequence, a positive equilibrium is not existent, and the subsidy race escalates.

Summarizing the results of a policy competition:
Result 5.1 For the case of a bilateral R&D subsidy competition, a Nash equilibrium subsidy exists for high trade costs, but is not realizable due to a high tax burden for critically high trade costs. As trade costs decrease, the equilibrium subsidy decreases as well; the symmetric distribution across both locations remains unchanged also after passing the sustain point. After a critical value, where \( t \in T''' \), the Nash equilibrium disappears. In comparison with the laissez-faire outcome, the political competition preserves industrial dispersion also for trade costs below the sustain point. However, as soon as the trade costs fall below the domain \( T'' \), the subsidy race escalates until the maximum viability of the tax base is reached. Even at this point, the symmetric outcome remains because both countries subsidize local R&D in the same (maximum) extent.

Herewith, the symmetry perpetuates so that both the break and sustain points lose their central relevance. In the end, both countries are situated in a regional-political prisoners’ dilemma. Due to mutual compensation of subsidy effects, both locations bear an income transfer between production workers and scientists that does not have an impact either on product quality and private R&D expenditures or on the spatial distribution of industries.

In the case of exogenous asymmetries, the smaller country has to apply a higher subsidy for compensating the initial disadvantage. Therewith, its already lower maximum tax burden is sooner exhausted so that the smaller country finally loses the locational competition.

Combining these results with the outcome of Section 4, an answer can be found for the question, if a locational competition leads to a welfare improvement in contrast to a centrally planned or cooperative solution:

Result 5.2 For high trade costs \( t > t^H \), dispersion implies a Hicksian welfare improvement despite unavailing policy efforts, if the net welfare of production workers, \( \Delta W^H \), is higher than the lump sum tax to be paid for R&D subsidies.

If the trade costs decrease, until agglomeration is socially preferred, the outcome of the political game allows the following conclusion:

Result 5.3 For low trade costs \( t < t^K \), the preservation of symmetry as a result of an (escalating) political R&D competition precludes not only a welfare improving agglomeration in the sense of Kaldor, but also burdens the tax paying production workers at their maximum capacity.

Concerning governmental R&D strategies, the standard trade policy literature follows the seminal work of Spencer and Brander (1983). In terms of product R&D, prominent contributions are provided by Zhou et al. (2002), Park (2001), and Jinji (2003), who consider vertically differentiating oligopolies. The structure of these
models basically differs from the underlying paper in trade flows: two firms in two countries, either developed and less developed or symmetric, supply the market of a third country. The authors show that in the unilateral case, a government has an incentive to subsidize the firm in the home country. In the bilateral constellation, the optimum policy depends upon which oligopolistic competition, Bertrand or Cournot, is assumed for the market, the technological endowment, as well as political objectives. The latter implies the main difference to policymakers in the present model. Instead of affecting product quality to maximize local welfare, which is simply firm profits minus subsidy, the grant in this paper is used as a migration incentive aiming to achieve agglomeration within the home country. In spite of the complexity of model assumptions, especially with respect to the general equilibrium framework, the implications of the political game derived here are much simpler. Primarily, this is due to the monopolistic competitive setting of the manufacturing sector, where firms lack the opportunity for strategic behavior.

6 Conclusions

Summing up and coming back to the European case, the model results demonstrate a strong destabilizing impact of R&D and highly skilled migration on the spatial formation of industries. For an increasing trade integration, the mechanisms inevitably lead to a total relocation of R&D and the corresponding downstream sector. As shown in Section 3, larger countries reveal a stronger agglomeration tendency due to a larger market potential and, thus, stronger home market and price index effects.

Turning to the recent policy efforts, from a unilateral perspective an R&D and innovation policy via subsidization is viable to achieve agglomeration in the home country. In particular, a realization is not inevitably possible due to a limited tax base, and depending upon the level of trade costs, not a welfare improvement. Further on, as the numerical calibrations reveal, outcomes and thus policy implications are very sensitive to the choice of parameters so that general yes/no recommendations are not reasonable.

Before the constitution of the Lisbon strategy, each European state has followed its own R&D policy and still does. Since direct export subsidizations are explicitly prohibited in the course of the common market, subsidizing R&D is partly used as indirect promotion of the domestic industry. However, as the non-cooperative policy game results of this paper show, a conflicting subsidy race may lead to a stable Nash equilibrium for high and medium trade costs, where the equilibrium subsidy decreases with increasing integration. At a critical level the equilibrium disappears implying an escalating race. Between similar countries, the policy interventions do not have an impact at all. The tax-subsidy income transfer is inefficient with respect to the political objectives. Nonetheless, a unilateral exit would lead to a total relo-
cation so that both countries are forced to maintain the subsidy competition. For large differences in country size, the smaller country has to pay a higher subsidy to compensate its initial disadvantage. For decreasing trade costs, this country would finally lose the R&D race – a result that occurs also for the laissez-faire case. In summary, these outcomes clearly legitimate a harmonized R&D policy, which raises the conflict between European dispersion and agglomeration.

In spite of European and national political interests, spatial dispersion is not necessarily a Pareto superior state, as shown in Section 4. In the course of a further European integration going along with decreasing trade costs, agglomeration increasingly tends to be a welfare improvement due to an increasing compensation potential in the sense of Kaldor. In addition, an agglomeration of research and manufacturing sectors implies a higher spatial efficiency as a result of external economies of scale. The model demonstrates that in the agglomeration equilibrium, the periphery entirely focuses on the constant return sector, while the production labor force in the core is mainly employed in the monopolistic competitive sector. On one hand, this spatial specialization implies a distinguished industrial pattern that is also verified for Europe by the much noticed empirical studies of Midelfart-Knarvik et al. (2000) and Combes and Overman (2004). On the other hand, this outcome concludes that primarily low skilled and mature industries tend to relocate towards peripheral countries. In fact, this restricts the technological potential of these regions but ensures a division of labor according to comparative advantages and, thus, an increasing employment in these industries.

Midelfart-Knarvik et al. (2000) and Combes and Overman (2004) provide a couple of stylized results relevant for this paper: 1) The location of R&D intensive industries is increasingly responsive to the local endowments of researchers. 2) High-tech and increasing returns industries tend to be more spatially concentrated. 3) The dissimilarities between peripheral and core countries become more and more obvious involving an increasing degree of geographical specialization. Furthermore, Midelfart-Knarvik and Overman (2002) show that countries with an increasing endowment of high-skilled labor succeed in attracting R&D intensive industries. What can be concluded from the model results presented in this paper?

The spatial concentration on the national aggregation level can also be continued on the regional level. Midelfart-Knarvik and Overman (2002) show that on the international level the industrial pattern in Europe did not change in a major degree. In contrast, on the regional level, the trend of increasing concentration has accelerated. This development can be traced back to the decreasing relevance of trade costs the lower the level of spatial aggregation. Combined with a higher regional labor mobility, this leads to a weakening of dispersive forces and an enhancement of agglomeration forces between regions. In consequence, growing high-tech clusters are located in already established metropolitan areas. In the case of biotechnology,
these are mainly in the direct proximity of London, Munich, and Paris.\textsuperscript{27}

A proceeding integration process, especially in terms of the common market, and a further liberalization of national labor markets remove still persisting mobility barriers for workers also with respect to the Eastern European acceding countries, which exhibit a considerably westward brain drain tendency.\textsuperscript{28} All in all, an increasing importance of R&D intensive industries and highly-skilled migration will foster industrial disparities across the EU on a national, as well as on a regional, level.

Midelfart-Knarvik and Overman (2002) find that two scenarios might be possible for the European future depending upon the degree of factor mobility and agglomeration gains. The authors state that at low labor and high capital mobility, as well as small agglomeration gains, a possible outcome is specialization. Furthermore, the study finds evidence for strong agglomeration forces within rather than across industries implying "industry black holes," which connotes agglomeration for particular industries. Between those possibilities, the model supports the latter. Although low-skilled migration is relatively low in Europe, the mobility of high-skilled workers, especially of scientists, is higher and increasing.\textsuperscript{29} Although the home market effect due to high skilled migration is presumably low, the migration effect is strengthened by the accompanying gain of R&D advantages.

In consequence, industry black hole agglomeration will occur primarily for R&D intensive industries and this most likely in the European core. Since the welfare implications argue for agglomeration rather than dispersion at increasing trade integration, not only a geographical specialization in terms of vertically linked sectors has to be promoted, but also in terms of R&D intensity. To put the policy implication more strikingly, the model combined with the empirical results suggest a European structural policy that promotes high-tech industries within the core, and low- and medium-tech within the periphery.

7 Technical Appendix

Symmetric Equilibrium

Because the equation system is symmetric in terms of location, there exists a symmetric equilibrium, which can be determined by simply dropping the subscripts and substitution. Thus, the symmetric equilibrium is:\textsuperscript{30}

\begin{equation}
Y_s = \frac{\sigma \gamma}{\sigma \gamma - \mu}
\end{equation}

\textsuperscript{27}See, e.g., Allansdottir et al. (2000).
\textsuperscript{28}See, e.g., Straubhaar (2000).
\textsuperscript{29}This aspect is also part of the EU program European Research Area (ERA). See "Second Implementation Report on A Mobility Strategy for the European Research Area", Commission Staff Working Paper, SEC(2004) 412.
\textsuperscript{30}In this context, the subscript \textit{s} is mnemonic for \textit{symmetry}. 

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\[ \lambda_s = \frac{\mu (\sigma \gamma - 1)}{\sigma \gamma - \mu} \]  

\[ u_s = \left[ \frac{F \gamma (\sigma \gamma - \mu)}{2 \mu (\gamma - 1)} \right]^{1/\gamma} \]  

\[ P_s^{1-\sigma} = \left( \frac{\gamma}{2} \right)^{1/\gamma} \left[ \frac{\mu (\gamma - 1)}{F (\sigma \gamma - \mu)} \right]^{\frac{\gamma - 1}{\gamma}} (1 + t^{1-\sigma}) \]  

\[ r_s = \frac{2\mu}{\sigma \gamma - \mu}. \]

**Corner Solutions**

Assuming that the whole R&D sector is totally agglomerated in location 1, so that \( s = 1 \) and \( \lambda_2 = 0 \) (analogously, for the inverse relation: \( s = 0 \) and \( \lambda_1 = 0 \) ), we obtain the agglomeration equilibrium characterized by the following derivations:

\[ \tilde{Y}_1 = \frac{\sigma \gamma + \mu}{\sigma \gamma - \mu} \]  

\[ \tilde{Y}_2 = 1 \]  

\[ \tilde{\lambda}_1 = \frac{2\mu (\sigma \gamma - 1)}{\sigma \gamma - \mu} \]  

\[ \tilde{u}_1 = \left[ \frac{F \gamma (\sigma \gamma - \mu)}{2 \mu (\gamma - 1)} \right]^{1/\gamma} \]  

\[ \tilde{u}_2 = \frac{\sigma \gamma (\sigma \gamma - \mu)^{1-\gamma} \left( \frac{\gamma F}{\mu (\gamma - 1)} \right)^{1/\gamma}}{(\sigma \gamma + \mu)(\sigma \gamma - \mu)^{1-\gamma} + t^{1-\sigma} + t^{\sigma-1}} \]  

\[ \tilde{P}_1^{1-\sigma} = \gamma^{1/\gamma} \left[ \frac{2\mu (\gamma - 1)}{F (\sigma \gamma - \mu)} \right]^{\frac{\gamma - 1}{\gamma}} \]  

\[ \tilde{P}_2 = \tilde{P}_1 t \]  

\[ \tilde{r}_1 = \frac{2\mu}{\sigma \gamma - \mu} = r_s \]
(61b) \( \bar{r}_2 = \bar{r}_1 \left[ \frac{(\sigma \gamma + \mu) t^{1-\sigma} + (\sigma \gamma - \mu) t^{\sigma-1}}{2\sigma \gamma} \right]^\gamma \)

Sustain and Break Points
Considering the agglomeration equilibrium, the sustain point occurs at the zero-differential: \( \bar{\rho}_1 - \bar{\rho}_2 = 0 \). Using equation (60b), this expression can be rearranged to: \( \bar{r}_1 = \bar{r}_2 t^{-\mu} \). Substituting (61b) yields equation (27) for the sustain point, finally.

For the break point, we determine the trade costs level, where the slope of the symmetric wage differential with respect to \( s \) becomes zero. Totally differentiating the real research price yields:

\[
(62) \quad d\rho = 2P^{-\mu} = \left[ \frac{d\lambda}{(\sigma \gamma - 1)} - \left( \frac{2\mu}{\sigma \gamma - \mu} \right) ds - \left( \frac{\mu^2}{\sigma \gamma - \mu} \right) \frac{dP}{P} \right].
\]

From substitution of equations (23) and (26) in (62) and solving for \( Z \), we obtain:

\[
(63) \quad Z = \frac{1 - t^{1-\sigma}}{1 + t^{1-\sigma}} = \frac{\mu \gamma (2\sigma - 1)(\sigma \gamma - \mu)}{\gamma^3 \sigma^2 (\sigma - 1) - \mu \left[ \mu (\mu - \sigma \gamma) - \gamma^2 \sigma (1 - \sigma) \right]}.
\]

Solving for trade costs, equation (63) becomes (28).

No-Black-Hole Condition
The validity of the no-black-hole condition can be seen at the requirement for a positive nominator of equation (28). The condition implies that both break and sustain points occur for \( t > 1 \). This restriction can also be derived by following the approach of Fujita et al. (1999). Starting from the total differential of the real research wage, we obtain at the symmetric equilibrium:

\[
(64) \quad \frac{dp}{\rho} = \frac{dr}{r} - \mu \frac{dP}{P}.
\]

From equation (17) in combination with (53) and (56), the relative change in the research wage is:

\[
(65) \quad \frac{dr}{r} = \left[ \frac{\sigma \gamma - \mu}{\mu (\sigma \gamma - 1)} \right] d\lambda - 2 ds.
\]

By substitution of equations (23) and (65) in (64) and considering the case of infinitely high trade costs, implying that \( Z = 1 \), leads to:

\[
(66) \quad \frac{dp}{\rho} = \left[ \frac{\sigma \gamma - \mu}{\mu (\sigma \gamma - 1)} \right] \left[ (1 - \sigma) \frac{\gamma - \mu (\gamma - 1)}{\gamma (1 - \sigma)} \right] d\lambda + 2 \left[ \frac{\mu - \gamma (\sigma - 1)}{\gamma (\sigma - 1)} \right] ds.
\]

The sign of the partial derivative with respect to \( s \) in equation (66) provides information about the slope of the wage gap function plotted in Figure 3. Considering
the upper limit of the trade costs domain, that is \( Z = 1 \), a negative slope of the wage differential in the symmetric equilibrium ensures that the break point must occur for smaller, non-infinite values of trade costs. Therefore, the nominator of the derivative must be negative, which finally requires: \( \mu < \gamma (\sigma - 1) \).
References


Figures

**Figure 1:** Bifurcation Diagram for the Share of Scientists

**Figure 2:** Bifurcation Diagram for Product Qualities
Figure 3: Wiggle Diagrams for Real Research Wage Differentials (Parameter values: \( \sigma = 2, \gamma = 2, \mu = 0.4, F = 1 \))
Figure 4: Bifurcation Diagram for Asymmetric Locations

Figure 5: Bifurcation Diagram for Product Quality in Asymmetric Locations
**Figure 6:** Net Welfare After Compensation

**Figure 7:** Critical Subsidy (Unilateral R&D Policy)
Figure 8: Critical Subsidy for Asymmetric Locations (Parameter values: $\sigma = 2$, $\gamma = 2$, $\mu = 0.4$, $a = 0.99$)

Figure 9: Equilibrium Subsidy in the Symmetric Policy Game (Parameter values: $\sigma = 2$, $\gamma = 2$, $\mu = 0.4$, $t = 3$)
Tables

Table 1: Sustain Points and Exogenous Asymmetry

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Table 2: Welfare and Critical Thresholds

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Table 3: Comparative Statics of Nash Equilibrium Subsidies

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</tr>
<tr>
<td>2.0</td>
<td>&gt;1</td>
<td>0.751615</td>
<td>-</td>
</tr>
<tr>
<td>3.0</td>
<td>&gt;1</td>
<td>&gt;1</td>
<td>0.08242</td>
</tr>
</tbody>
</table>

Reference case: $\sigma=2; \gamma=2; \mu=0.4$


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