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Article

Picture fuzzy weighted distance measures and their application to investment selection

Amfiteatru Economic Journal

Provided in Cooperation with:

The Bucharest University of Economic Studies

Suggested Citation: Liu, Meiling; Zeng, Shouzhen; Baležentis, Tomas; Štreimikienė, Dalia (2019) : Picture fuzzy weighted distance measures and their application to investment selection, Amfiteatru Economic Journal, ISSN 2247-9104, The Bucharest University of Economic Studies, Bucharest, Vol. 21, Iss. 52, pp. 682-695, <https://doi.org/10.24818/EA/2019/52/682>

This Version is available at:

<https://hdl.handle.net/10419/281468>

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**PICTURE FUZZY WEIGHTED DISTANCE MEASURES
AND THEIR APPLICATION TO INVESTMENT SELECTION****Meiling Liu¹, Shouzhen Zeng^{2*}, Tomas Baležentis³ and Dalia Streimikiene⁴**¹*School of Business, Zhejiang Wanli University, Ningbo, China*²*School of Business, Ningbo University, Ningbo, China*^{3,4}*Lithuanian Institute of Agrarian Economics, Vilnius, Lithuania***Please cite this article as:**

Liu, M., Zeng, S., Baležentis, T. and Streimikiene, D., 2019. Picture Fuzzy Weighted Distance Measures and their Application to Investment Selection. *Amfiteatru Economic*, 21(52), pp. 682-695.

DOI: [10.24818/EA/2019/52/682](https://doi.org/10.24818/EA/2019/52/682)**Article History**

Received: 10 February 2019

Revised: 10 June 2019

Accepted: 15 July 2019

Abstract

The picture fuzzy set (PFS) is a powerful tool to collect and handle large amounts of uncertain assess information in a new light. In this study, we explore some distance measures for the PFSs and propose Picture fuzzy ordered weighted distance measure and Picture fuzzy hybrid weighted distance measure. Some of their properties are also mathematically explored. Moreover, we introduce a model for the aforesaid distance measures to solve multiple attribute group decision making (MAGDM) method in an updated way. And at the end of our paper a practical application of investment alternatives selection is provided to illustrate the validity and applicability of the presented work.

Keywords: Picture fuzzy set; distance measures; ordered weighted distance; multiple attribute group decision making; investment selection

JEL Classification: A12, C44, C60, D81, D89

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Introduction

The selection of an appropriate investment scheme is a complicated multi-attribute group decision-making (MAGDM) problem, because it is necessary to take into account the multiple characteristics and attributes of the scheme during decision process. This kind of problems often includes some uncertain and vague information as it needs evaluate the schemes from different aspects, while the decision makers (DMs) may not be familiar with the characteristics of all schemes because of distinctive academic background. In this case, it is impossible for DMs to give evaluations on alternatives with crisp values. Recently, the picture fuzzy set (PFS) proposed by Cuong and Kreinovich (2013) has been proved to be a powerful tool that allows DMs to handle such uncertain and vague information with ease. Compared with the existing fuzzy tools, such as intuitionistic fuzzy set (IFS) (Atanassov, 1986) and Pythagorean fuzzy set (Yager, 2014), the main advantages of the PFS is that it is distinguished by three different functions, namely the degree of membership, the degree of neutral membership and the degree of non-membership, which makes it fully consider the degree of acceptance, rejection conflicting and refusal during the analysis. Currently, some progress has been carried out in the research of the PFS condition: Cuong (2014) explored several properties of PFSs and studied a few distance measures between PFSs. Sing (2015) developed correlation coefficient of PFS and studied its application in clustering analysis. Wei (2016) studied a basic leadership method for weighted cross-entropy of PFS and applied it in ranking the choices. Wei (2017) exhibited some aggregation operators for PFS and applied them to MADM for selecting EPR problems. Garg (2017) explored a few aggregation operations for PFSs and use them to MADM problems. Zhang et al. (2018) employed PFS to collect and describe uncertain assess information for offshore wind power station selection. Wang et al. (2018) developed a normalized projection-based method to evaluate construction project. Wei (2018a) extended the conventional TODIM model to solve MADM problems with Picture fuzzy information. Wei (2018b) developed Hamacher aggregation operators for PFS and studied their application to MADM. Wei et al. (2018c) established a projection method for PFSs and developed its application in evaluating emerging technology commercialization. Jana et al. (2019) introduced the some Dombi aggregation operators for PFS and studied their usefulness in MADM problems.

In 2008, Xu and Chen introduced a new distance measure known as the order weighted distance (OWD) measure, which can underestimate or overestimate the influence of extremely large/small deviations on the aggregation results by adjusting the corresponding weights. For this advantage, the researches on the OWD has become a hot research issue and have achieved numerous accomplishments, for example, Zeng and Su (2011) studied the application of the OWD in IFS situation, and developed the IFOWD measure. Zeng et al. (2012) studied the fuzzy OWD and explored its application in MADM. Zhou et al. (2013) developed a MADGM model based on the continuous OWD measure. Cai et al. (2014) introduced the linguistic OWD (LOWD) measure, and they proposed a pattern recognition method based on the presented LOWD measure. Merigó et al. (2017) introduced a new extension of the OWD by using weighted averages and Bonferroni means. Qin et al. (2017) developed the Pythagorean fuzzy OWD measure and applied it to assess the service quality of airlines. Sahin et al. (2018) explored the usefulness of the OWD measure in simplified neutrosophic situation and used it to select investment company. Silva et al. (2018) used the OWD to aid the logistics problems. Zeng et al. (2016) explored the usefulness of OWD

measure in Pythagorean fuzzy MAGDM. Zeng and Xiao (2018) studied the application of the OWD and TOPSIS method in hesitant fuzzy situations.

The previous instances and discussions shows that PFS is a powerful and effective tool to demonstrate uncertain and vague information in real-world issues. However, none of researches mentioned previously focuses on the application of the OWD measure in PFS environment. Therefore, the primary focus of this study is to explore the usefulness of the OWD measure mentioned earlier in PFS situation. With this aim in mind, we first present a Picture fuzzy ordered weighted distance (PFOWD) measure. The PFOWD is very suitable to handle the deviations between the PFSs. Moreover, it can reduce the influence of extremely large (or small) values on the final results by giving them low (or high) weights. Furthermore, it is a generalized model that includes diversity cases, such as the Picture fuzzy weighted distance (PFWD) measure, the Picture fuzzy ordered weighted Hamming distance (PFOWHD) measure, and so on. Then, based on the PFOWD measure, a Picture fuzzy hybrid weighted distance (PFHWD) measure is presented, whose merit is that it not only includes the ordered weights of individual distances, but also considers their importance. A MAGDM approach is further established based on the PFHWD measure. Finally, an illustrative example concerning investment selection problem is provided to demonstrate the usefulness of the developed method, and a comparative analysis with existing relative methods is also presented.

1. Preliminaries

In this section, some essential concepts concerning PFS and OWD measure are reviewed, which will be used in the rest of this work.

1.1. Picture fuzzy set (PFS)

Definition 1 (Atanassov, 1986). An IFS A in $Z = \{z_1, z_2, \dots, z_n\}$ is defined by :

$$A = \left\{ \left\langle z, (E_A(z), N_A(z)) \right\rangle \mid z \in Z \right\} \tag{1}$$

where the function $0 \leq E_A(z) \leq 1$ and $0 \leq N_A(z) \leq 1$ are named as the degree of membership and the non-membership, respectively, and satisfy $0 \leq E_A(z) + N_A(z) \leq 1$.

Definition 2 (Cuong and Kreinovich, 2013). A picture fuzzy set (PFS) P in $Z = \{z_1, z_2, \dots, z_n\}$ is defined as in (2):

$$P = \left\{ \left\langle z, (E_p(z), I_p(z), N_p(z)) \right\rangle \mid z \in Z \right\} \tag{2}$$

where $0 \leq E_p(z) \leq 1$ is called the degree of positive membership, $0 \leq I_p(z) \leq 1$ is called the degree of neutral membership and $0 \leq N_p(z) \leq 1$ is called the degree of negative membership of Z , and they satisfy the following

condition: $0 \leq E_p(z) + I_p(z) + N_p(z) \leq 1$, $\forall z \in Z$. Then for $z \in Z$, $T_p(z) = 1 - (E_p(z) + I_p(z) + N_p(z))$ is called the degree of refusal membership of Z .

Obviously, If $I_p(z) = 0$, then the PFS reduces to the Atanassov's IFS (Atanassov, 1989), which shows that the IFS is a special case of the PFS. A picture fuzzy number (PFN), is simply denoted as $\alpha = (E, I, N)$ for calculation convenience.

Definition 3. Let $\alpha_1 = (E_1, I_1, N_1)$ and $\alpha_2 = (E_2, I_2, N_2)$ be two PFNs, then some of their operational laws are defined as follows (Wei, 2017):

- (1) $\alpha_1 \oplus \alpha_2 = (E_1 + E_2 - E_1 * E_2, I_1 * I_2, N_1 * N_2)$;
- (2) $\lambda \alpha_1 = (1 - (1 - E_1)^\lambda, (I_1)^\lambda, (N_1)^\lambda), \lambda \geq 0$.

Definition 4. Let $\alpha_1 = (E_1, I_1, N_1)$ and $\alpha_2 = (E_2, I_2, N_2)$ be two PFNs, then their distance is (Wei, 2018a):

$$d(\alpha_1, \alpha_2) = \frac{1}{2} (|E_1 - E_2| + |I_1 - I_2| + |N_1 - N_2|) \tag{3}$$

1.2 The OWD measure

On the basis of the ordered weighted averaging (OWA) operator (Yager, 1988), Xu and Chen (2008) introduced the ordered weighted distance (OWD) measure:

Definition 5. Let $X = \{x_1, x_2, \dots, x_n\}$ and $Y = \{y_1, y_2, \dots, y_n\}$ be two collections of real numbers, and $d(x_j, y_j) = |x_j - y_j|$ be the distance between x_j and y_j , then

$$OWD(X, Y) = \left(\sum_{j=1}^n w_j (d(x_{\sigma(j)}, y_{\sigma(j)}))^\lambda \right)^{1/\lambda} \tag{4}$$

is called the ordered weighted distance (OWD) between X and Y , where $\lambda > 0$, $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$, such that

$$d(x_{\sigma(j-1)}, y_{\sigma(j-1)}) \geq d(x_{\sigma(j)}, y_{\sigma(j)}) \tag{5}$$

and $w = (w_1, w_2, \dots, w_n)$ is the weight vector of OWD measure, satisfying

$$w_j \in [0, 1] \text{ and } \sum_{j=1}^n w_j = 1.$$

Especially, if $\lambda = 1$, then the OWD measure reduces to the ordered weighted Hamming

distance (OWHD) measure:

$$OWHD(X, Y) = \sum_{j=1}^n w_j d(x_{\sigma(j)}, y_{\sigma(j)}) \tag{6}$$

And if $\lambda = 2$, then the ordered weighted Euclidean distance (OWED) measure:

$$OWED(X, Y) = \sqrt{\sum_{j=1}^n w_j (d(x_{\sigma(j)}, y_{\sigma(j)}))^2} \tag{7}$$

The OWD measure has been extended to a variety of uncertain environments, except for PFS. In this study, we explore the application of OWD measure in PFS situation.

2. Picture fuzzy OWD measure

We first give the weighted distance measure for PFSs.

Definition 6. Let $P = (\alpha_1, \alpha_2, \dots, \alpha_n)$ and $Q = (\beta_1, \beta_2, \dots, \beta_n)$ be two sets of PFNs, then

$$PFWD(P, Q) = \left(\sum_{j=1}^n w_j (d(\alpha_j, \beta_j))^\lambda \right)^{1/\lambda}, \lambda > 0. \tag{8}$$

is called the Picture fuzzy weighted distance (PFWD) measure between P and Q , where $d(\alpha_j, \beta_j)$ is the distance between the PFNs α_j and β_j , and w_j is the weight of the individual distance $d(\alpha_j, \beta_j)$, satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Specially, if $\lambda = 1$, the PFWD measure is degenerated the Picture fuzzy weighted Hamming distance (PFWHD) measure:

$$PFWHD(P, Q) = \sum_{j=1}^n w_j d(\alpha_j, \beta_j). \tag{9}$$

And we get the Picture fuzzy weighted Euclidean distance (PFWED) measure if $\lambda = 2$:

$$PFWED(P, Q) = \left(\sum_{j=1}^n w_j (d(\alpha_j, \beta_j))^2 \right)^{1/2}. \tag{10}$$

The PFWD measures considers the importance of the individual distances, and then aggregates these distances together with their weights. Next, we define the Picture fuzzy OWD (PFOWD) measure, which takes the order weights into consideration.

Definition 7. Let $P = (\alpha_1, \alpha_2, \dots, \alpha_n)$ and $Q = (\beta_1, \beta_2, \dots, \beta_n)$ be two sets of PFNs, then

$$PFOWD(P, Q) = \left(\sum_{j=1}^n w_j \left(d(\alpha_{\sigma(j)}, \beta_{\sigma(j)}) \right)^\lambda \right)^{1/\lambda} \tag{11}$$

is called Picture fuzzy OWD (PFOWD) measure between P and Q , where $\lambda > 0$, $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$, such that

$$d(\alpha_{\sigma(j-1)}, \beta_{\sigma(j-1)}) \geq d(\alpha_{\sigma(j)}, \beta_{\sigma(j)}) \tag{12}$$

and $w = (w_1, w_2, \dots, w_n)$ is the weight vector with $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Specially, if $\lambda = 1$ and $\lambda = 2$, then the PFOWD measure will generate the Picture fuzzy ordered weighted Hamming distance (PFOWHD) measure and Picture fuzzy ordered weighted Euclidean distance (PFOWED) measure, respectively:

$$PFOWHD(P, Q) = \sum_{j=1}^n w_j d(\alpha_{\sigma(j)}, \beta_{\sigma(j)}), \tag{13}$$

$$PFOWED(P, Q) = \sqrt{\sum_{j=1}^n w_j \left(d(\alpha_{\sigma(j)}, \beta_{\sigma(j)}) \right)^2}. \tag{14}$$

Example 1.

Let $P = \{(0.6, 0.1, 0.2), (0.5, 0.2, 0.2), (0.1, 0.5, 0.4), (0.3, 0.1, 0.5), (0.3, 0.3, 0.3)\}$ and $Q = \{(0.5, 0.3, 0.1), (0.1, 0.6, 0.2), (0.7, 0.1, 0.1), (0.3, 0.2, 0.4), (0.5, 0, 0.5)\}$ be two sets of PFNs, then

$$d(\alpha_1, \beta_1) = \frac{1}{2} (|0.6 - 0.5| + |0.1 - 0.3| + |0.2 - 0.1|) = 0.2,$$

Similarly, we get

$$d(\alpha_2, \beta_2) = 0.3, \quad d(\alpha_3, \beta_3) = 0.65, \quad d(\alpha_4, \beta_4) = 0.1, \quad d_{IFD}(\alpha_5, \beta_5) = 0.35.$$

Ranking the $d(\alpha_j, \beta_j)$ according to decreasing order, we have

$$d(\alpha_{\sigma(1)}, \beta_{\sigma(1)}) = 0.65, \quad d(\alpha_{\sigma(2)}, \beta_{\sigma(2)}) = 0.35, \quad d(\alpha_{\sigma(3)}, \beta_{\sigma(3)}) = 0.3, \\ d(\alpha_{\sigma(4)}, \beta_{\sigma(4)}) = 0.2, \quad d(\alpha_{\sigma(5)}, \beta_{\sigma(5)}) = 0.1.$$

Suppose the weight vector of the PFOWD is $w = (0.2, 0.25, 0.3, 0.15, 0.1)$, and without loss of generality, let $\lambda=2$, then we can perform the PFOWD to calculate distance between P and Q :

$$PFOWD(P, Q) = (0.2 \times 0.65^2 + 0.25 \times 0.35^2 + 0.30 \times 0.3^2 + 0.15 \times 0.2^2 + 0.1 \times 0.1^2)^{1/2} = 0.3862$$

It can be seen from Example 1 that the PFOWD measure focuses on the importance of the ordered position of the individual distances instead of the arguments themselves, while the PFWD measure only considers the importance of given individual distances. Therefore, the weights vector in the PFWD and PFOWD measures denote different means. Next we develop a Picture fuzzy hybrid weighted distance (PFHWD) measure to combine the advantages of both the PFWD and PFOWD measures.

Definition 8. Let $P = (\alpha_1, \alpha_2, \dots, \alpha_n)$ and $Q = (\beta_1, \beta_2, \dots, \beta_n)$ be two sets of PFNs, then

$$PFHWD(P, Q) = \left(\sum_{j=1}^n w_j \dot{d}(\alpha_{\sigma(j)}, \beta_{\sigma(j)}) \right)^{1/\lambda}, \quad \lambda > 0 \quad (15)$$

is called the Picture fuzzy hybrid weighted distance (PFHWD) measure between P and Q , where $\dot{d}(\alpha_{\sigma(j)}, \beta_{\sigma(j)})$ is the j th largest of the weighted distance $\dot{d}(\alpha_j, \beta_j)$ (here $\dot{d}(\alpha_j, \beta_j) = n\omega_j (d(\alpha_j, \beta_j))^2$, $j = 1, 2, \dots, n$), $w = (w_1, w_2, \dots, w_n)$ is the weight vector related to the PFHWD measure, and ω_j is the weight of the individual distance $d(\alpha_j, \beta_j)$, satisfying $\omega_j \in [0, 1]$ and the sum of the weights is 1. n is the balancing coefficient, playing a role of balance.

Example 2. (Example 1 continuation). In order to utilize the PFHWD, let $\omega = (0.2, 0.15, 0.15, 0.1, 0.4)$ and $\lambda=2$, then

$$\begin{aligned} \dot{d}(\alpha_1, \beta_1) &= 5 \times 0.2 \times 0.2^2 = 0.04, \quad \dot{d}(\alpha_2, \beta_2) = 5 \times 0.15 \times 0.3^2 = 0.0675, \\ \dot{d}(\alpha_3, \beta_3) &= 5 \times 0.15 \times 0.65^2 = 0.317, \quad \dot{d}(\alpha_4, \beta_4) = 5 \times 0.1 \times 0.1^2 = 0.005, \\ \dot{d}(\alpha_5, \beta_5) &= 5 \times 0.4 \times 0.35^2 = 0.245. \end{aligned}$$

Reorder the weighted distances $\dot{d}(\alpha_j, \beta_j)$ ($j = 1, 2, \dots, n$), we have

$$\begin{aligned} \dot{d}(\alpha_{\sigma(1)}, \beta_{\sigma(1)}) &= 0.317, \quad \dot{d}(\alpha_{\sigma(2)}, \beta_{\sigma(2)}) = 0.245, \quad \dot{d}(\alpha_{\sigma(3)}, \beta_{\sigma(3)}) = 0.0675, \\ \dot{d}(\alpha_{\sigma(4)}, \beta_{\sigma(4)}) &= 0.04, \quad \dot{d}(\alpha_{\sigma(5)}, \beta_{\sigma(5)}) = 0.005. \end{aligned}$$

Let the weight vector $w = (0.2, 0.25, 0.3, 0.15, 0.1)$, we can get

$$\begin{aligned} PFHWD(P, Q) &= (0.2 \times 0.317 + 0.25 \times 0.245 + 0.3 \times 0.0675 + 0.15 \times 0.04 + 0.1 \times 0.005)^{1/2} \\ &= 0.3891. \end{aligned}$$

From the Example 2 and Definition 8, we know that the operational rules of the PFHWD is that it first weights the input individual distances, and then re-ranks these weighted individual distances in descending order, and finally converts them into a collective one by the its weights and the parameter λ . Therefore, The PFHWD measure unifies both advantages of the PFWD and PFOWD measure, and reflects the importance degrees of both the given individual distances and their ordered positions.

Theorem 1. The PFWD measure is a special case of the the PFHWD measure.

Proof Let $w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)$, then

$$\begin{aligned} PFHWD(P, Q) &= \left(\sum_{j=1}^n w_j \dot{d}_{IFD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)}) \right)^{1/\lambda} = \left(\frac{1}{n} \sum_{j=1}^n \dot{d}(\alpha_j, \beta_j) \right)^{1/\lambda} \\ &= \left(\frac{1}{n} \sum_{j=1}^n n \omega_j (d(\alpha_j, \beta_j))^\lambda \right)^{1/\lambda} = \left(\sum_{j=1}^n \omega_j (d(\alpha_j, \beta_j))^\lambda \right)^{1/\lambda} \\ &= PFWD(P, Q) \end{aligned}$$

which completes the proof of Theorem 1.

Theorem 2. The PFOWD measure is a special case of the PFHWD measure.

Proof Let $\omega = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)$, then

$$\dot{d}(\alpha_{\sigma(j)}, \beta_{\sigma(j)}) = n \omega_j (d(\alpha_{\sigma(j)}, \beta_{\sigma(j)}))^\lambda = (d(\alpha_{\sigma(j)}, \beta_{\sigma(j)}))^\lambda$$

which completes the proof of Theorem 2.

3. An approach based on the PFHWD measure for MAGDM under Picture fuzzy environment

In this section, we present a new MAGDM model based on the proposed PFHWD measure, whose process can be summarized as follows.

Step 1. Let $D = \{d_1, d_2, \dots, d_t\}$ be the set of DMs (whose weight vector is $L = (l_1, l_2, \dots, l_t)$, $l_k \geq 0$, $\sum_{k=1}^t l_k = 1$). After evaluating the alternatives $A = \{A_1, A_2, \dots, A_n\}$ based on the attributes $G = \{G_1, G_2, \dots, G_n\}$, each DM gives his/her evaluation represented by PFNs, and then constructs the individual decision matrix $R^k = (r_{ij}^{(k)})_{m \times n}$.

Step 2. Aggregate all individual decision information into a collective one and then form the group decision matrix:

$$R = (r_{ij})_{m \times n} = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix}, \tag{16}$$

where the PFN $r_{ij} = \sum_{k=1}^t l_k r_{ij}^{(k)}$.

Step 3: Compute the ideal levels for each attributes to form the ideal scheme (see Table no. 1):

Table no. 1: Ideal scheme

	G_1	G_2	\dots	G_n
I	\tilde{y}_1	\tilde{y}_2	\dots	\tilde{y}_n

Step 4: Utilize the PFHWD to calculate the distances between the ideal scheme I and each alternative $A_i (i = 1, 2, \dots, m)$.

Step 5: Rank the alternatives and identify the best one(s) according to the results obtained from Step 4.

4. Illustrative example

In the following, we present a numerical example concerning investment selection to show the usefulness of the developed MAGDM model. Assume a investment form wants to

invest money in a company. After analyzing the market, the committee considers five possible alternatives:

- (1) Invest in a chemical company called A_1 ;
- (2) Invest in a food company called A_2 ;
- (3) Invest in a computer company called A_3 ;
- (4) Invest in a car company called A_4 ;
- (5) Invest in a furniture company called A_5 .

The committee invites three experts to evaluate these companies from the following four characteristics:

- (1) G_1 : Benefits in the short term;
- (2) G_2 : Benefits in the mid term;
- (3) G_3 : Benefits in the long term;
- (4) G_4 : Risk of the investment.

The assess results provided by the experts are shown in Tables no. 2-4.

Table no. 2: Characteristics of the investments-expert 1

	G_1	G_2	G_3	G_4
A_1	(0.5,0.1,0.3)	(0.5,0.2,0.1)	(0.2,0.3,0.5)	(0.3,0.1,0.5)
A_2	(0.7,0.1,0.1)	(0.7,0.2,0)	(0.6,0.1,0.2)	(0.4,0.5,0.1)
A_3	(0.5,0,0.4)	(0.6,0.2,0.1)	(0.6,0.1,0.2)	(0.4,0.1,0.4)
A_4	(0.7,0.1,0.2)	(0.7,0.2,0.1)	(0.4,0.3,0.2)	(0.6,0.2,0.2)
A_5	(0.6,0.2,0.2)	(0.4,0.1,0.3)	(0.7,0.1,0.1)	(0.6,0.1,0.2)

Table no. 3: Characteristics of the investments-expert 2

	G_1	G_2	G_3	G_4
A_1	(0.5,0.2,0.2)	(0.8,0,0.2)	(0.6,0.2,0.2)	(0.5,0.1,0.1)
A_2	(0.4,0,0.5)	(0.6,0.1,0.2)	(0.7,0.1,0.1)	(0.8,0.1,0.1)
A_3	(0.5,0.2,0.2)	(0.7,0.1,0.2)	(0.8,0.1,0.1)	(0.6,0.1,0.3)
A_4	(0.6,0.1,0.2)	(0.3,0.2,0.4)	(0.5,0.4,0.1)	(0.5,0.3,0.2)
A_5	(0.7,0.1,0.2)	(0.8,0.1,0.1)	(0.6,0.3,0.1)	(0.7,0.1,0.1)

Table no. 4: Characteristics of the investments-expert 3

	G_1	G_2	G_3	G_4
A_1	(0.5,0.1,0.3)	(0.7,0,0.2)	(0.5,0.2,0.2)	(0.4,0.1,0.3)
A_2	(0.6,0.1,0.1)	(0.6,0.2,0.1)	(0.7,0.2,0.1)	(0.6,0.1,0.2)
A_3	(0.7,0.2,0.1)	(0.4,0.3,0.2)	(0.6,0.1,0.1)	(0.4,0.2,0.2)
A_4	(0.4,0.3,0.1)	(0.6,0.2,0.1)	(0.4,0.1,0.3)	(0.5,0.2,0.2)
A_5	(0.5,0.4,0.1)	(0.5,0.3,0.1)	(0.8,0.1,0.1)	(0.6,0.1,0.2)

Suppose the weights of three experts are $l_1 = 0.3$, $l_2 = 0.3$ and $l_3 = 0.4$, respectively. With this information, we can aggregate the individual opinions into a collective result, which is shown in Table no. 5.

Table no. 5: Collective result

	G_1	G_2	G_3	G_4
A_1	(0.50,0.12,0.27)	(0.69,0,0.16)	(0.46,0.23,0.26)	(0.41,0.10,0.25)
A_2	(0.59,0,0.16)	(0.63,0.16,0)	(0.67,0.13,0.12)	(0.63,0.16,0.13)
A_3	(0.59,0,0.19)	(0.59,0.19,0.16)	(0.68,0,0.12)	(0.47,0.13,0.28)
A_4	(0.57,0.16,0.15)	(0.57,0.20,0.15)	(0.43,0.21,0.19)	(0.53,0.23,0.20)
A_5	(0.60,0.21,0.15)	(0.60,0.16,0.14)	(0.72,0.14,0.1)	(0.63,0.10,0.16)

In accordance with its objectives, the committee of company constructs the ideal investment scheme shown in Table no. 6.

Table no. 6: Ideal scheme

	G_1	G_2	G_3	G_4
I	(0.9,0,0.1)	(0.8,0.1,0)	(0.9,0,0.1)	(1,0,0)

Assume the weight vectors of the attributes and the PFHWD are $\omega = (0.2, 0.3, 0.15, 0.35)$ and $w = (0.15, 0.35, 0.3, 0.2)$, respectively. Given the previous information, it is possible to employ the PFHWD (without loss of generality, let $\lambda=2$) to derive the distances between the alternative $A_i (i = 1, 2, 3, 4)$ and the ideal scheme I :

$$PFHWD(A_1, I) = 0.419, \quad PFHWD(A_2, I) = 0.259,$$

$$PFHWD(A_3, I) = 0.327, \quad PFHWD(A_4, I) = 0.364,$$

$$PFHWD(A_5, I) = 0.258.$$

The results show that A_5 has the least distance from the ideal scheme, which indicates it is the most prefer alternative. Moreover, we can get the ranking of the alternatives:

$$A_5 \succ A_2 \succ A_3 \succ A_4 \succ A_1.$$

In order to better verify the superiority of the PFHWD, we use the PFWD and the PFOWD method calculate the deviations of all alternatives to the ideal scheme. For the PFWD measure, we get:

$$PFWD(A_1, I) = 0.397, \quad PFWD(A_2, I) = 0.250,$$

$$PFWD(A_3, I) = 0.334, \quad PFWD(A_4, I) = 0.354,$$

$$PFWD(A_5, I) = 0.247.$$

And for the PFOWD operator, we have:

$$PFOWD(A_1, I) = 0.402, \quad PFOWD(A_2, I) = 0.475,$$

$$PFOWD(A_3, I) = 0.507, \quad PFOWD(A_4, I) = 0.576,$$

$$PFOWD(A_5, I) = 0.469.$$

It is observed from the above results that that the most desirable alternative is A_5 for the PFWD measure, while A_1 for PFOWD method, which is different to the result derived by the PFHWD operator. Moreover, we can see that the final ranking of the alternatives may change based on different measures used. The main reason is that the PFWD measure only takes the importance of deviation element into consideration, while the PFOWD focuses on the importance of position during the aggregation process, but not the deviation itself. While the PFHWD combines these two merit, and is able to account for the both the importance that the PFWD and PFOWD measures consider.

Conclusions

In this study, we have explored the extension of the OWD measure in Picture fuzzy environment. We defined the Picture fuzzy OWD (PFOWD) measure, which is suitable to deal with the Picture fuzzy information. Then, by the help of the PFOWD, we developed the Picture fuzzy hybrid weighted distance (PFHWD) measure, which combines the both features of the PFOWD and the Picture fuzzy weighted distance (PFWD) measure. Furthermore, a MAGDM model based on the PFHWD is presented and a numerical example concerning investment selection is provided to show its feasibility. The

application shows that the PFHWD is able to reflect the degrees of pessimism or optimism of multiple decision makers, as well as the importance of various attributes during the process of aggregation. Additionally, it can deal with more complex decision making problems with uncertain information evaluated with the PFNs.

For future research, we expect to propose some new methods and application of the PFHWD measure, such as induced aggregation and the probability information.

Acknowledgement

We would like to thank the anonymous referees for valuable comments that have improved the quality of the paper. This paper is supported by Philosophy and social science planning subject in zhejiang province (No. 17NDJC290YB), Soft science research project in zhejiang province (No. 2017C35013) and Soft science research project in Ningbo (No. 2019A10025).

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