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**The Strength of Vertical Linkages**

by  
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# The Strength of Vertical Linkages

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## Abstract

This paper discusses the interdependencies that exist between vertically-linked industries in the (Spence-)Dixit-Stiglitz model of monopolistic competition. The main objective is to develop a concept for quantifying the magnitude of sectoral coherence in models of the New Economic Geography. It is motivated by the suggestion, by Venables (1996), that 'strategic industries' be identified in terms of their agglomeration potential.

Using a partial-analytic approach, we focus on inter-industrial relations in a closed economy to draw conclusions regarding international trade. We ascertain that two factors have an impact upon the strength of industrial linkages: 1) the monopolistic scope of intermediate suppliers, in terms of (technical) substitution elasticity; and the share in downstream costs for intermediates. Within a simulation study, this paper applies this new theoretical concept to eight basic industries across ten European countries.

**Keywords:** New Economic Geography, Vertical Linkages

**JEL classifications:** F12, F14, F17

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# 1 Introduction

The *New Economic Geography* (from now on NEG), initially introduced by Krugman (1991), provides explanations for industrial agglomeration based upon increasing returns and imperfect competition. As summarized by Baldwin et al. (2003), three effects determine the spread of industries: i) the *market-access effect*, that reflects the tendency of firms to locate their production in a larger market and export to the one that is smaller; ii) the *cost-of-living effect*, that describes how the local firm number influences the consumer price index (therefore, also called the *price-index effect*); and iii) the *market-crowding effect*, that is the preference of firms for locations with low competition. While the market-crowding effect counteracts industrial clustering, the market-access and cost-of-living effects imply agglomeration and, combined with labor migration, so-called *cumulative causality* – also known as forward and backward linkages. This means that the firm number responds positively to market size, and market size positively to firm number, due to lower costs of living, rising wages and implicit labor migration.

The observation that agglomeration also is present in regions with relatively low labor migration has challenged the application of forward and backward linkages to inter-industrial trade, which is referred to as vertical linkages. In their analysis of European industries, Midelfart-Knarvik et al. (2000) point out that forward and backward linkages have become increasingly significant since 1980, in terms of inter-industrial trade. Hummels et al. (2001) estimate that about 30% of world exports account for inter-industrial trade.<sup>1</sup> This share has grown by 40% since 1970, which emphasizes the increasing role of what the authors call *vertical specialization*. These results consistent with those Yeats (1998), who considers the exports of the OECD countries within the classification group SITC-7 (key machinery and transportation equipment). In 1995, the share of components and parts was about 30%, which approximates 132 billion US-\$. Characterizing the relevance of vertical linkages in expanding international trade, Hummels et al. (1998) come to the conclusion that the nature of international trade 'has changed to the point where countries increasingly specialize in producing particular stages of goods, rather than making a complete good from start to finish'.

Based upon the seminal works of Ethier (1982), Rivera-Batiz (1988) and Markusen (1989), Krugman and Venables (1995) implement vertical linkages into the *core-periphery model* from 1991, where the upstream industry provides differentiated intermediate products to the downstream industry, that produces differentiated consumer goods once again. For simplification, both sectors are integrated into one so that one firm is producing its own intermediates. Venables (1996) separates the sectoral structure and considers the particular spatial distribution of both, the upstream and the downstream industry. Further work has been done by Ottaviano

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<sup>1</sup>Estimation for 1995.

and Robert-Nicoud (2006), regarding free mobility of capital and human capital.<sup>2</sup> In the context of existing NEG literature considering vertical linkages, the dimension of industrial agglomeration depends upon three categories of factors: i) trade costs; ii) the conditions of production and consumption at each location (factor cost, productivity, income); and iii) the strength of vertical linkages. The impact of trade costs has been fairly intensely discussed as the main concern of NEG models. The characteristics of the second category imply further dispersal or agglomeration forces, and can be quantified without major problems. Although the strength of vertical linkages is attributed to be an important factor for industrial clustering, it only is discussed casually. For quantification, a frequently used reference is the share of downstream costs for intermediate products. This approach raises certain questions: Is the strength of linkages an endogenous or exogenous factor? Can the sectoral coherence be described as one pooled measure, or does it require a separate analyses dealing with forward and backward linkages? What are the main factors controlling industrial interdependencies, and is the strength of linkages fixed or variable?

Against this background, the objective of this paper is to suggest a concept for quantifying the strength of vertical linkages in models of the NEG, and to make a contribution for a classification of industries in terms of their implicit potential for agglomeration, as demanded by Venables (1996). To avoid the analytical problems of NEG models (non-closed solutions, bifurcations and ambiguous equilibria), we consider a separated input-output structure, in accordance with work by Venables, in a closed economy. Using this approach, we can analyze the interaction between vertically-linked industries intensely, and develop an adequate measuring concept for sectoral coherence, also with respect to international trade.

The basic idea is to identify a set of inter-sectoral allocation functions and an associated equilibrium. We arrive at the conclusion that, with respect to the sectoral firm numbers, the zero-profit isoclines can be interpreted as such forward and backward linkages. The corresponding elasticities, evaluated at equilibrium, provide information about the strength of these linkages. We ascertain that these values only depend upon two exogenous factors: 1) technical substitution elasticity (or the monopolistic mark-up of intermediate producers); and 2) the share in downstream costs for intermediates. Based upon these results, we apply the theoretical concept to ten European countries, and compute the strength of linkages for several exemplary industries. From the simulation outcomes, we can derive the major implications for economic policy and its efficiency: for the majority of industries, the forward linkage is much stronger than the backward linkage. This means that, for regional growth strategies based upon vertical linkages, promotional activities primarily should be initiated at downstream industries. Furthermore, a projected industrial development should incorporate not only bilateral input-output relationships, but also the

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<sup>2</sup>Their work is based on earlier papers as discussed in Baldwin et al. (2003), Chapter 8.

interaction of multi-industrial supply networks and their role in international trade. The paper is structured as follows: Section 2 introduces the basic model of monopolistic competition in a closed one-sector economy. This framework is extended by vertical linkages in Section 3, including equilibrium analysis and comparative statics. Based upon these results, we develop the measuring concept and discuss its properties. Section 4 represents the application in simulation; and Section 5 summarizes the main findings and implications for international trade and economic policy.

## 2 The Basic Model of Monopolistic Competition

In this section, we reconstruct the basic model of monopolistic competition, as developed by Dixit and Stiglitz (1977), later labeled by Fujita et al. (1999) as being the 'workhorse model' of the NEG. In the first step, we consider the demand side, represented by private households that consume differentiated manufactures and a homogenous agricultural good.<sup>3</sup> Further on, in the second step, we inspect the supply of consumer goods and the resulting market equilibrium.

Starting from consumer preferences, private households face a linear-homogenous utility function in the form of:

$$(1) \quad U = M^\mu A^{1-\mu}, \quad 0 < \mu < 1,$$

where  $M$  represents a sub-utility from the consumption of manufactures,  $A$  is the quantity of the homogenous agricultural good, and  $\mu$  the share in private expenditures for manufactures. The sub-utility,  $M$ , is given by:

$$(2) \quad M = \left[ \sum_{i=1}^n (x_i)^\rho \right]^{1/\rho}, \quad 0 < \rho < 1.$$

$x_i$  is the quantity of a particular variety,  $i$ , out of all available varieties,  $n$ , that are produced by the consumer good industry. For concavity, the preference parameter,  $\rho$ , needs to be smaller than '1'. If we set  $\sigma = 1/(1 - \rho)$ , the sub-utility is characterized by a constant elasticity of substitution, and Equation (2) becomes:

$$(3) \quad M = \left[ \sum_{i=1}^n (x_i)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1.$$

Equations (2) and (3) reflect consumer preference for a wide range of differentiated products. This is due to the increasing marginal sub-utility from a rising number

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<sup>3</sup>For definition, we refer to the differentiated manufactures as *consumer goods* – excluding the (traditional) agricultural good.

of available product sorts.<sup>4</sup> The love of variety becomes weaker, as varieties become more homogenous in consumers' eyes (going along with an increase in  $\sigma$ ).

The demand for manufactures can be derived by applying the *two-stage budgeting procedure*:<sup>5</sup>

$$(4) \quad x = \mu Y (p)^{-\sigma} P^{\sigma-1},$$

where  $p$  is the price of manufactures, and  $\mu Y$  denotes the share in income of the private households spent on consumer goods.  $P$  is the consumer price index, defined as:

$$(5) \quad P \equiv \left[ \sum_{i=1}^n (p_i)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

Under the assumption that all varieties again have the same price, the index (5) becomes:

$$(6) \quad P = pn^{\frac{1}{1-\sigma}}.$$

Because the first derivative, with respect to  $n$ , is negative, an increasing number of available varieties implies a reduction in the consumer price index. This results from the diminishing marginal (sub-)utility, so that a given level of utility can be achieved with a lower quantity of each product sort.<sup>6</sup>

Via differentiation, the most important characteristics of the demand function (4) can be described.<sup>7</sup> The demand curve decreases monotonically, as indicated by the negative derivation with respect to price (see Equation (67)). Because of the specific assumptions of the sub-utility function, the substitution elasticity is constant at  $\sigma$  (see Equation (71)). The first derivative, with respect to the price index  $P$ , exhibits a positive sign, signifying that an increase in the price of all alternative product varieties goes along with a rising demand for one particular sort (see Equation (69)). In terms of changes in  $Y$ , the demand rises with an expanding income spent on consumer goods ( $\mu Y$ ) (see Equation (70)). Finally, an increase in the elasticity of substitution reduces the demand, as a consequence of rising substitutability and,

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<sup>4</sup>This easily can be seen, assuming identical demand for all varieties. Consequently, what follows from (3) is:  $M = xn^{\sigma/(\sigma-1)}$ .

<sup>5</sup>See Appendix 6.1.

<sup>6</sup>A higher substitution elasticity makes the price index rise, which can be explained by interpreting the price index as the minimum expenditure for gaining one unit of sub-utility  $M$ . An increasing substitutability requires a higher product variety for the same sub-utility level. This, in turn, increases the associated consumer expenditure.

<sup>7</sup>See Appendix 6.2.

thus, decreasing monopolistic scope (apparent at the negative exponent of  $p$ ). However, this effect is constrained by a simultaneous change in the price index. This can be seen by substituting (6) into (4). Hence, private demand becomes:

$$(7) \quad x = \mu Y p^{-\sigma} [pn^{1/(1-\sigma)}]^{\sigma-1} = \mu Y p^{-\sigma} [p^{\sigma-1} n^{-1}].$$

An increase in  $\sigma$  reduces demand by the negative exponent of  $p$ . The terms in brackets represent the price index, which rises with  $\sigma$ . In the case of identical prices, these effects compensate for each other, such that (7) ultimately is independent of  $\sigma$ .

For completing the demand side of this model, the marginal revenue function is given by:

$$(8) \quad MR = \left( \frac{\sigma - 1}{\sigma} \right) p.$$

It can be shown that the marginal revenue curve is the (inverse) demand function, compressed by the factor  $(\sigma - 1)/\sigma$ ; thus, it is constant-elastic, as well.<sup>8</sup>

Turning to the supply side of the model, the technology for consumer good production is given by:

$$(9) \quad l = F + ax,$$

where  $l$  is the amount of labor as the only input factor necessary to produce the quantity,  $x$ , of one variety, whereas  $a$  denotes the corresponding production coefficient.  $F$  is a fixed factor requirement that consequently causes diminishing average cost. The resulting (internal) economies of scale and consumer preferences for differentiated products imply that one firm produces only one variety. With (9), (71) and the wage rate,  $w$ , the profit of such a firm is given by:

$$(10) \quad \pi = px - wl = [\mu Y P^{\sigma-1}]^{1/\sigma} x^{\frac{\sigma-1}{\sigma}} - w(F + ax),$$

Profit maximization yields:

$$(11) \quad \left( \frac{\sigma - 1}{\sigma} \right) p = wa,$$

which means that, at maximum profit, marginal revenue is equal to marginal cost. Because of free market entry, the equilibrium profit is zero. Using (11), the equilibrium quantity supplied is:

$$(12) \quad x^* = (\sigma - 1) \frac{F}{a}.$$

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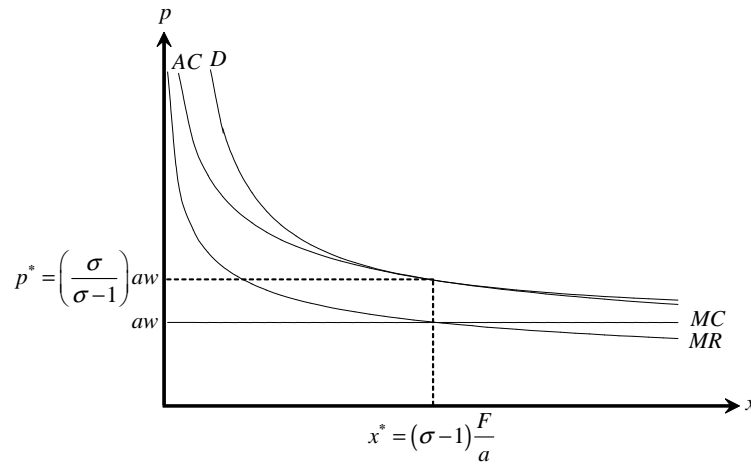
<sup>8</sup>See Equation (71) and (72).



At equilibrium, the average cost is:

$$(13) \quad AC = \frac{wF}{x^*} + wa = \frac{wa}{(\sigma - 1)} + aw = aw \left( \frac{\sigma}{\sigma - 1} \right),$$

which is equal to the profit-maximizing price. This implies that, at equilibrium, the average cost curve intersects the demand curve. Equation (12) shows that, at equilibrium, firm size is determined by the technological parameters,  $F$  and  $a$ , as well as by substitution elasticity. If they are assumed to be fixed, the level of output  $x^*$  is totally independent of income, wage rate and price index.<sup>9</sup> For summary, Figure 1 shows the equilibrium according to Neary (2001). In accordance with Neary (2001),



Source: based on Neary (2001)

**Figure 1:** Equilibrium in the Dixit-Stiglitz Model

the stability of equilibrium can be proven by the total differential of the profit,  $\pi$ :<sup>10</sup>

$$(14) \quad d\pi = \frac{\partial\pi}{\partial p}dp + \frac{\partial\pi}{\partial x}dx = \frac{p}{\sigma}dx.$$

Equation (14) shows that firm profit responds only to changing demand. Taking an adjustment process into account, the interdependence between firm number and profit can be expressed generally as:

$$(15) \quad \dot{n} = f(\pi), \quad f(0) = 0, \quad \partial f/\partial\pi > 0.$$

An increase in the firm number reduces the price index and, as demonstrated above, the consumer demand for manufactures. As shown in Equation (14), the decrease

<sup>9</sup>In addition, the wage rate where firms break even is left unstated. The pricing rule (11) solved for  $w$  and supplemented by the demand function (4) yields:  $w^* = \frac{(\sigma-1)}{\sigma a} \left[ \frac{\mu Y P^{\sigma-1}}{x^*} \right]^{1/\sigma}$ . As can be seen, the wage rate increases with rising income and price index.

<sup>10</sup>See Appendix 6.2, Equations (73) and (74).

in demand accompanies a decline in firm profits and, due to (15), with a market exit of firms. Hence, the equilibrium is prove to be stable.

The attendant question is: what is firm number at equilibrium? It is obvious that total expenditures for manufactures must be equal to the total turnover of the consumer good industry:

$$(16) \quad \mu Y = n^* p^* x^*.$$

Using (11) and (12), the equilibrium number of firms can be expressed as:

$$(17) \quad n^* = \frac{\mu Y}{w F \sigma},$$

where  $F\sigma$  is equal to the equilibrium labor input of each firm,  $l^*$ , which can be derived from (9) and (12). The term  $\mu Y/w$  represents the ratio of total expenditures on consumer goods (and, thus, the total costs of the whole industry at equilibrium) to the wage rate. This ratio reflects the total labor input of the economy, and may be defined as  $L^*$ .<sup>11</sup> Hence, (17) becomes:

$$(18) \quad n^* = \frac{L^*}{F\sigma} = \frac{L^*}{l^*}.$$

To complete the picture, Table 1 shows the comparative statics for the equilibrium values of firm number, input and output of each firm and the whole industry, as well as the price for manufactures. In the first row, it is shown that an increase in the

**Table 1:** *Comparative Statics of the Basic Model*

respect to	$\partial n^*$	$\partial l^*$	$\partial L^*$	$\partial x^*$	$\partial (n^* x^*)$	$\partial p^*$
$\partial F$	-	+	0	+	0	0
$\partial a$	0	0	0	-	-	+
$\partial c$	-	0	-	0	-	+
$\partial \sigma$	-	+	0	+	+	-
$\partial (\mu Y)$	+	0	+	0	+	0

fixed factor requirement affects (besides the factor usage of each firm) the number of firms and the output of each firm only, while overall industrial output remains unchanged. This results from higher procurement costs, thus reducing profits and market exits. The residual firms expand their output at constant prices (because of unchanged marginal costs). This implies that higher procurement costs are compensated by higher economies of scale.

<sup>11</sup>The labor supply is assumed to be wage-inelastic and equal to labor demand.

An increase in productivity, expressed as a lower production coefficient,  $a$ , decreases marginal costs and price. This raises consumer demand and the output of each firm. The output of the whole industry rises but without increasing the number of firms. A rise in wages leads to higher marginal costs and manufacture prices. This, in turn, likewise reduces demand and total manufacturing output. Because the production of each firm is independent of factor costs, the only possibility for supply adjustment is a decrease in the number of firms.

Rising substitution elasticity requires a higher consumption of manufactures to attain the same sub-utility level, which increases demand and, hence, the output of each firm. At the same time, the price index declines and restricts the growing demand. In the case of identical prices, these effects compensate for each other, such that the demand is independent of the substitution elasticity.<sup>12</sup> As can be seen from Equation (8), marginal revenue declines and forces the producers to cut their prices. This step must go along with higher firm output to compensate losses by sinking average cost. Overall, the firm number decreases, because the quantity effect exceeds the price effect.<sup>13</sup>

Finally, a larger market, by increasing consumer expenditures, leads to the entry of new firms as a consequence of resulting profits, until the old output level of each firm (and zero-profit) again is achieved. The outcome is a higher quantity of firms and proportionally-higher industrial output.

In the next section, we extend the basic model by implementing a second industry as an intermediate supplier. The emerging linkages between sectors and their basic characteristics are the object of further investigation.

## 3 Vertical Linkages

### 3.1 Extensions of the Basic Model

First shown by Ethier (1982), intermediate inputs included in the Dixit-Stiglitz framework induce additional agglomerative force. At this stage, we maintain the assumptions of a closed economy and an exogenous labor market, with the aim of focusing on inter-sectoral dependencies. The supply chain is structured as follows: an upstream sector producing intermediate goods for the downstream sector (the consumer good industry from the previous section), and private households as the final consumers. The upstream industry produces differentiated products using

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<sup>12</sup>The price index becomes, as with (6):  $x = \mu Y p^{-\sigma} (pn^{1/(1-\sigma)})^{\sigma-1} = \mu Y \frac{p}{n}$ .

<sup>13</sup>The effects become apparent via differentiating Equation (17), with respect to  $\sigma$ :  $\frac{\partial n^*}{\partial \sigma} = -\frac{\mu Y}{(p^*)^2 x^*} \frac{\partial p^*}{\partial \sigma} - \frac{\mu Y}{p^* (x^*)^2} \frac{\partial x^*}{\partial \sigma}$ . Expressed by exogenous parameters and factored out, the derivative is given by:  $\frac{\partial n^*}{\partial \sigma} = \frac{n^*}{(\sigma-1)} \left[ \frac{1}{\sigma} - 1 \right] < 0$ , where the term in brackets is the price change on the left and the output change on the right hand side.

economies of scale, as does the downstream industry. The only input requirement for intermediate production is labor, so that the technology is the analogue of (9):

$$(19) \quad l^u = F^u + a^u x^u,$$

where the superscripts  $u$  and  $d$  denote *upstream* and *downstream*, respectively. Demand for intermediates comes from the downstream industry (and from consumers indirectly). As opposed to the previous section, consumer good producers combine labor and intermediates, both as essential factors. The representing production function is of the Cobb-Douglas type, and given by:

$$(20) \quad F^d + a^d x^d = Z (l^d)^{1-\alpha} I^\alpha.$$

The right hand side of Equation (20) represents the input composite of labor and intermediates in order to produce one unit of downstream output,  $x^d$ , which involves a fixed cost,  $F^d$ , and a variable cost,  $a^d$ , on the left hand side.  $Z$  controls the output level; and  $\alpha$  is the partial substitution elasticity of the intermediate aggregate,  $I$ , that is:

$$(21) \quad I = \left[ \sum_{i=1}^n (x_i^u)^{(\varsigma-1)/\varsigma} \right]^{\varsigma/(\varsigma-1)}.$$

Because the downstream and upstream industries, are in a 1:1 supply relationship, Krugman and Venables (1995) relegate them to one sector. This approach enables analysis of input-output-structures without introducing an extra industry. The basic principle behind this simplification is that the aggregated industry produces varieties for consumption, while a certain proportion of output again is used as an (intermediate) input. As opposed to this approach, we follow the example by Venables (1996) and analyze both sectors separately, with the aim of considering forward and backward linkages explicitly.

Production function and intermediate aggregate are structurally the same as utility and sub-utility functions in which  $\varsigma$  corresponds with  $\sigma$ .<sup>14</sup> The common pattern involving downstream and consumer preferences implies a price index for intermediates that is similar to the one for consumer goods:

$$(22) \quad P^u \equiv \left[ \sum_{i=1}^n (p_i^u)^{1-\varsigma} \right]^{1/(1-\varsigma)}.$$

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<sup>14</sup>An issue of particular importance is the economic motivation for the love of intermediate variety. Fujita et al. (1999) consider it to be a matter of technology, but assume, for simplification, identical substitution elasticities in downstream industry and consumer preferences. Alternatively, permutations of intermediates may be a source of final product variety. From this point of view,  $\varsigma$  tends to be a consumer-driven parameter which, in turn, argues for the simplification described above. For now, we refrain from equating the substitution elasticities with the objective to distinguish the different interdependencies of variables.

By applying two-stage-budgeting again, we obtain the cost function for a downstream firm:

$$(23) \quad C^d = (F^d + a^d x^d) w^{1-\alpha} (P^u)^\alpha.$$

The downstream cost depends positively on wage level, on fixed and variable factor requirements, and on the intermediate price index. The latter responds to changes in the number of upstream firms in the same way as the price index for consumer goods. This implies that increasing the number of suppliers will cut down the cost of the downstream industry, via a decreasing price index for intermediates. From the cost function (23), the demand for intermediates can be derived:<sup>15</sup>

$$(24) \quad x^u = \alpha C^d (p^u)^{-\varsigma} (P^u)^{\varsigma-1}.$$

One can see that the pattern of demand for intermediates is the same as for consumer goods. If we look at equation (23), the cost rate of a factor composite of labor and intermediates is:  $w^{1-\alpha} (P^u)^\alpha$  so that the downstream profit function is given by:<sup>16</sup>

$$(25) \quad \pi^d = p^d x^d - w^{1-\alpha} (P^u)^\alpha [F^d + a^d x^d].$$

Including consumer demand (4), Equation (25) becomes:

$$(26) \quad \pi^d = p^d \mu Y (p^d)^{-\sigma} P^{\sigma-1} - w^{1-\alpha} (P^u)^\alpha [F^d + a^d \mu Y (p^d)^{-\sigma} P^{\sigma-1}].$$

From profit maximization we obtain the first order condition:

$$(27) \quad \frac{\partial \pi^d}{\partial p^d} = (1 - \sigma) \mu Y (p^d)^{-\sigma} P^{\sigma-1} + \sigma w^{1-\alpha} (P^u)^\alpha a^d \mu Y (p^d)^{-\sigma-1} P^{\sigma-1} = 0.$$

After simplification, the profit maximizing consumer good price can be written as:

$$(28) \quad (p^d)^* = w^{1-\alpha} (P^u)^\alpha a^d \left( \frac{\sigma}{\sigma - 1} \right),$$

where the cost rate,  $w^{1-\alpha} (P^u)^\alpha$ , is the equivalent to the wage rate in (11), so that the monopolistic pricing rule remains valid. The equilibrium output of a downstream firm ultimately is:

$$(29) \quad (x^d)^* = (\sigma - 1) \frac{F^d}{a^d},$$

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<sup>15</sup>See Appendix 6.3 for a detailed derivation.

<sup>16</sup>The consumer good price,  $p$ , becomes  $p^d$ .

which, again, is identical to (12).

Similarly, firm profit, price and equilibrium output of the upstream industry can be derived. The corresponding sector is technologically the same as the manufacturing industry from the previous section. The only difference is that private demand for manufactures is now the intermediate demand (24), so that the upstream profit function becomes:

$$(30) \quad \pi^u = p^u x^u - w(F^u + a^u x^u).$$

The profit maximizing upstream price is:

$$(31) \quad (p^u)^* = w a^u \left( \frac{\varsigma}{\varsigma - 1} \right),$$

and from the zero profit condition the equilibrium output is:

$$(32) \quad (x^u)^* = (\varsigma - 1) \frac{F^u}{a^u}.$$

### 3.2 The Equilibrium Number of Firms

How does an input-output structure change the number of firms? At equilibrium, total turnover is equal to total cost; thus, the share  $(1 - \alpha)$  in total turnover,  $n^* p^* x^*$ , is spent on labor:

$$(33) \quad wL = (1 - \alpha) n^* p^* x^*.$$

Using (33) to solve for  $n$ , we obtain:

$$(34) \quad n^* = \frac{wL}{(1 - \alpha) p^* x^*}.$$

Alternatively, we can separate the sectors again, and the number of upstream firms is equal to the ratio of intermediate market size and turnover per firm:

$$(35) \quad n^u = \frac{n^d \alpha C^d}{(p^u)^* (x^u)^*}.$$

Combining (31), (32) and (23), Equation (35) can be expressed as:

$$(36) \quad n^u = \alpha \frac{\sigma F^d}{\varsigma F^u} \left( \frac{P^u}{w} \right)^\alpha n^d.$$

Assuming identical intermediate prices and quantities per firm, the upstream price index becomes  $p^u (n^u)^{1/(1-\varsigma)}$ . Thus, Equation (36) can be expressed, by using (31), as:

$$(37) \quad n^u = \left[ \alpha \frac{\sigma F^d}{\varsigma F^u} \left( \frac{a^u \varsigma}{\varsigma - 1} \right)^\alpha n^d \right]^{\frac{1-\varsigma}{1-\varsigma-\alpha}}.$$

It is evident from Equation (37) that the number of upstream firms depends positively upon the number of downstream firms. This interdependence is called *forward linkage*, meaning that the size of the upstream market responds to the size of the subsequent industry. Equation (37), solved for  $n^d$ , is denoted by  $N^u$ :

$$(38) \quad N^u \equiv \frac{\varsigma F^u}{\alpha \sigma F^d} \left( \frac{\varsigma - 1}{a^u \varsigma} \right)^\alpha (n^u)^{\frac{1-\varsigma-\alpha}{1-\varsigma}}.$$

As is apparent from the derivatives of (38), the progression of the curve is convex, which implies that sectoral coherence diminishes with an increasing number of downstream firms:

$$(39) \quad \begin{aligned} \frac{\partial N^u}{\partial n^u} &= \left( \frac{1-\varsigma-\alpha}{1-\varsigma} \right) N^u (n^u)^{-1} > 0 \\ \frac{\partial^2 N^u}{\partial (n^u)^2} &= \left( \frac{\alpha}{\varsigma-1} \right) \frac{\partial N^u}{\partial n^u} (n^u)^{-1} > 0. \end{aligned}$$

Similarly, the number of downstream firms can be stated as:

$$(40) \quad n^d = \frac{\mu Y}{(p^d)^* (x^d)^*}.$$

Using (28) and (29) to insert values into (40):

$$(41) \quad n^d = \frac{\mu Y}{w^{1-\alpha} (P^u)^\alpha \sigma F^d},$$

which, after substituting the cost rate,  $w^{1-\alpha} (P^u)^\alpha$ , for the wage rate,  $w$ , from Section 2, is nothing else but Equation (17). The intermediate price index can be written as above, and the number of downstream firms becomes:

$$(42) \quad n^d = \frac{\mu Y}{w \sigma F^d} \left( \frac{\varsigma - 1}{a^u \varsigma} \right)^\alpha (n^u)^{\frac{\alpha}{\varsigma-1}} \equiv N^d.$$

Equation (42) characterizes the so-called *backward linkage*. The number of downstream firms depends upon the number of upstream firms via the intermediate price index effect. The more firms produce in the preceding market, the lower the intermediate price index and, therefore, the lower the procurement cost for the consumer good industry. Because of rising profits new downstream firms enter the market,

until the zero profit condition holds. Hereby, an increasing number of upstream firms implies that there is an increasing number of downstream firms.

Via differentiation of (42), we can describe the basic characteristics of the curve progression:

$$(43) \quad \begin{aligned} \frac{\partial N^d}{\partial n^u} &= \left(\frac{\alpha}{\varsigma-1}\right) N^d (n^u)^{-1} > 0 \\ \frac{\partial^2 N^d}{\partial (n^u)^2} &= \left(\frac{\alpha+1-\varsigma}{\varsigma-1}\right) \frac{\partial N^d}{\partial n^u} (n^u)^{-1} \leq 0 \quad \forall \quad \alpha \geq (\varsigma-1) \end{aligned}$$

Because of the ambiguous sign of the second derivative, industrial dependency becomes stronger with an increasing number of upstream firms, for  $\alpha > (\varsigma - 1)$ . This case differentiation stems from the dependence of the (intermediate) price index effect on technical substitution elasticity. The lower  $\varsigma$  is, the greater the price index reduction induced by an increase in the number of upstream firms. This price index effect can become so strong that the backward linkage *escalates*.<sup>17</sup>

With (37) and (42), we obtained a system of simultaneous equations. Setting  $N^u = N^d$  yields equilibrium at:<sup>18</sup>

$$(44) \quad (n^u)^* = \frac{\alpha\mu Y}{w\varsigma F^u} \quad , \quad (n^d)^* = \frac{\varsigma F^u}{\alpha\sigma F^d} \left(\frac{\varsigma-1}{a^u\varsigma}\right)^\alpha \left(\frac{\alpha\mu Y}{\varsigma w F^u}\right)^{\frac{1-\varsigma-\alpha}{1-\varsigma}} .$$

Because of parameter settings the existence of a positive equilibrium firm number always is given, which explicitly excludes equilibrium at the origin. Table 2 shows the comparative statics for the equilibrium number of firms. Because an increase in fixed cost of downstream firms results in a change in equilibrium output only, the total factor requirement of the downstream industry remains unchanged, while the number of downstream firms declines.<sup>19</sup> If fixed cost rises during the upstream production, the same mechanism works except for the impact on the downstream industry. Via the intermediate price index effect, the production cost for downstream firms increases, which creates losses and accompanying exit of firms.

A change in productivity causes an adjustment in the output of both sectors, while firm number is unaffected – in contrast to the inter-industrial effect. An increasing upstream production coefficient implies higher intermediate prices and a reduction of downstream profits, and, hence, an exit of firms.

A rise in wages leads to higher marginal cost and a diminishing firm number. In addition, the increase in downstream cost is strengthened, via the price index effect that is generated by the shrinking intermediate product range.

<sup>17</sup>This marginal case may be excluded, like the so-called *no-black-hole* condition; see Fujita et al. (1999).

<sup>18</sup>Similar to (18),  $(n^u)^*$  can be expressed as  $(n^u)^* = L^u / (l^u)^*$ , where  $L^u$  is the number of workers employed in the upstream industry as a whole, resulting from the proportion  $\alpha$  of total downstream turnover spent on intermediates and the wage rate.  $(l^u)^*$  is the equilibrium labour requirement of one upstream firm.

<sup>19</sup>See Table 1 for the intra-sectoral changes.



The effect of the forward linkage becomes apparent in the derivatives, with respect to consumer expenditures for downstream products,  $\mu Y$ . Because of a greater consumer market, the number of downstream firms increases, which, in turn, increases the number of upstream firms.

**Table 2: Comparative Statics of the Extended Model**

Respect to	$\partial (n^u)^*$	$\partial (n^d)^*$
$\partial (F^d)$	0	–
$\partial a^u$	0	–
$\partial w$	–	–
$\partial (\mu Y)$	+	+
$\partial F^u$	–	–
$\partial \alpha$	+	$(n^u)^* \geq \Omega_1 \Leftrightarrow \frac{\partial (n^d)^*}{\partial \alpha} \geq 0$
$\partial \varsigma$	–	$(n^u)^* \geq \Omega_2 \Leftrightarrow \frac{\partial (n^d)^*}{\partial \varsigma} \leq 0^{20}$

The influence of the intermediate intensity in downstream production,  $\alpha$ , and the technical substitution elasticity,  $\varsigma$ , on the equilibrium number of upstream firms again can be traced back to the forward linkage. While an increasing  $\alpha$  enlarges the upstream market and, thus, the number of intermediate suppliers, higher intermediate differentiation leads to a higher equilibrium output for each firm, and *ceteris paribus* to a firm reduction in the upstream industry.

The response of the downstream sector to changes in the preceding industry via the forward linkage is a bit more complex. The direction of change among downstream firms follows the exponential functions  $\Omega_1$  and  $\Omega_2$ . An increasing intermediate share in downstream cost results in a higher number of upstream firms, as shown above. This lowers the intermediate price index consequently. Finally, the number of downstream firm rises, if the price index effect exceeds the higher cost for intermediate products. This can be shown with the derivatives of the intermediate price index, with respect to  $\alpha$ .<sup>21</sup> They describe a progressively declining function, meaning that the higher the share of upstream products, the weaker the intermediate price index effect. Along with high marginal upstream cost and consequent high intermediate prices, the price index effect is compensated by higher procurement cost within the downstream sector.

Similarly, the downstream firms respond to changes in technical substitution elastic-

<sup>20</sup>  $\Omega_1 = \frac{1}{e} \left( \frac{a^u \varsigma}{\varsigma - 1} \right)^{\varsigma - 1}$ ,  $\Omega_2 = e^{\frac{(\varsigma - 1) \left( \frac{(n^u)^*}{\varsigma F^u} \right)^{\frac{1}{1 - \varsigma}}}{\varsigma} + \frac{1 - \varsigma}{\varsigma}}$   
<sup>21</sup>  $\frac{\partial P^u}{\partial \alpha} = \frac{1}{1 - \varsigma} \alpha^{\frac{\varsigma}{1 - \varsigma}} \left( \frac{w a^u \varsigma}{\varsigma - 1} \right) \left( \frac{\mu Y}{\varsigma w F^u} \right)^{\frac{1}{1 - \varsigma}} < 0$ ,  $\frac{\partial^2 P^u}{\partial \alpha^2} = \frac{\varsigma}{(1 - \varsigma)^2} \alpha^{\frac{2\varsigma - 1}{1 - \varsigma}} \left( \frac{w a^u \varsigma}{\varsigma - 1} \right) \left( \frac{\mu Y}{\varsigma w F^u} \right)^{\frac{1}{1 - \varsigma}} > 0$

ity. With increasing  $\varsigma$ , the number of upstream firms declines; thus, the intermediate price index rises. Otherwise, the marginal upstream cost decreases with increasing output of each firm, and so do upstream prices. These contrary forces compensate for each other at  $\Omega_2$ .

For stability analysis of the equilibrium, it is important to note that  $N^d$  intersects  $N^u$  always from above. This may be demonstrated by differentiating Equations (42) and (38) at equilibrium. The resulting derivative for  $N^u$  always is greater than the one for  $N^d$ .<sup>22</sup> The stability of the equilibrium can be analyzed by the same approach as above. We assume an out-of-equilibrium adjustment process with the following characteristics:

$$(45) \quad \begin{aligned} \dot{n}^u &= f(\pi^u) \ , \ \partial f/\partial \pi^u > 0 \\ \dot{n}^d &= f(\pi^d) \ , \ \partial f/\partial \pi^d > 0 \ , \ f(0) = 0. \end{aligned}$$

The relative profit functions subject to the number of upstream and downstream firms can be derived from (30) and (26), using profit maximizing prices (31) and (28), as well as downstream cost (23) and intermediate demand (24):

$$(46a) \quad \pi^u = \alpha w F^d \frac{\sigma}{\varsigma} \left( \frac{a^u \varsigma}{\varsigma - 1} \right)^\alpha (n^u)^{\frac{\alpha + \varsigma - 1}{1 - \varsigma}} n^d - w F^u \equiv K_1 (n^u)^{\frac{\alpha + \varsigma - 1}{1 - \varsigma}} n^d - w F^u$$

$$(46b) \quad \pi^d = \frac{\mu Y}{\sigma n^d} - w F^d \left( \frac{a^u \varsigma}{\varsigma - 1} \right)^\alpha (n^u)^{\frac{\alpha}{1 - \varsigma}} \equiv K_2 (n^d)^{-1} - K_3 (n^u)^{\frac{\alpha}{1 - \varsigma}} \ ,$$

where  $K_1, K_2$  and  $K_3 > 0$ . Totally differentiating profit functions (46a) and (46b) yields:

$$(47a) \quad d\pi^u = \underbrace{\left[ \left( \frac{\alpha + \varsigma - 1}{1 - \varsigma} \right) K_1 (n^u)^{\frac{\alpha + \varsigma - 1}{1 - \varsigma} - 1} n^d \right]}_{<0} dn^u + \underbrace{\left[ K_1 (n^u)^{\frac{\alpha + \varsigma - 1}{1 - \varsigma}} \right]}_{>0} dn^d$$

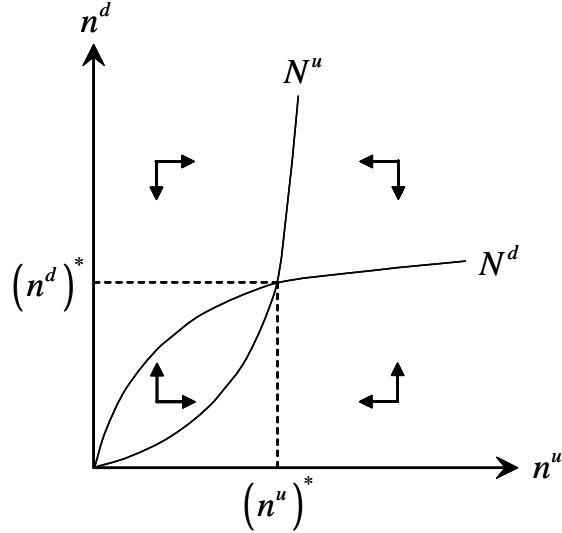
$$(47b) \quad d\pi^d = \underbrace{\left[ -K_2 (n^d)^{-2} \right]}_{<0} dn^d + \underbrace{\left[ \left( \frac{\alpha}{\varsigma - 1} \right) K_3 (n^u)^{\frac{\alpha}{1 - \varsigma} - 1} \right]}_{>0} dn^u \ .$$

As apparent as the sign of the partial derivative in (47a), an increase in the number of upstream firms out of the zero-profit isocline,  $N^u$ , generates losses in this industry that are caused by the intermediate price index effect. Via the assumed adjustment process given by (45), the number of upstream firms decreases again, until they break even. A secondary effect works in the downstream sector. The decreasing

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<sup>22</sup>Evaluation of the first derivatives with the equilibrium number of upstream firms and comparison of the resulting values leads to  $\varsigma > 1$  fundamentally being assumed.

intermediate price index reduces procurement cost for downstream firms, and make them realize profits, which, in turn, attract more downstream firms. The entry of new firms into the downstream industry reduces their profits, again, via the price index effect (see Equation (46b)), which retracts the number of downstream firms back to the zero-profit isocline (42). The overall result is a globally stable equilibrium. In Figure 2, the mechanism behind these interactions between forward and backward linkages is illustrated for the case of  $\alpha < (\zeta - 1)$ .



Source: author

**Figure 2:** *Equilibrium Number of Upstream and Downstream Firms*

### 3.3 The Strength of Vertical Linkages

Considering the zero-profit isoclines,  $N^u$  and  $N^d$ , as forward and backward linkages provides information about the mutual coherence between the upstream and downstream sectors. The basic idea of this paper is that the slope of the isoclines represents the strength of the relative linkages. Assuming an infinitely fast adjustment process, the derivatives evaluated at equilibrium are:

$$(48a) \quad S^u = \frac{\partial n^u}{\partial n^d} \Big|_{(n^d)^*} = \sigma F^d \left( \frac{1 - \zeta}{1 - \zeta - \alpha} \right) \left( \frac{a^u \zeta}{\zeta - 1} \right)^\alpha \left( \frac{\zeta F^u}{\alpha} \right)^{\frac{\alpha + 1 - \zeta}{\zeta - 1}} \left( \frac{\mu Y}{w} \right)^{\frac{\alpha}{1 - \zeta}}$$

$$(48b) \quad S^d = \frac{\partial N^d}{\partial n^u} \Big|_{(n^u)^*} = \frac{\left( \frac{\zeta - 1}{a^u \zeta} \right)^\alpha \left( \frac{\alpha \mu Y}{w} \right)^{\frac{\alpha}{\zeta - 1}} (\zeta F^u)^{\frac{\alpha - \zeta + 1}{1 - \zeta}}}{(\zeta - 1) \sigma F^d}.$$

The derivatives quantify the change in the number of firms in one sector, in response to changes in the quantity of firms in the other sector. The concept of derivatives

raises some fundamental problems. First, derivatives are quite abstract and difficult for economic interpretation. Second, these values are scaled according to firm number. Third, the comparative statics overflow in the context of changes in sign and associated nonlinear functions. Finally, empirical values for some parameters (fixed cost, for instance) are difficult, if not impossible to estimate.

The average change in firm numbers may provide a superior solution. If we choose the point elasticities at equilibrium, we obtain:

$$(49a) \ \varepsilon^u = \frac{1 - \varsigma}{1 - \varsigma - \alpha} \ , \ 0 < \varepsilon^u < 1$$

$$(49b) \ \varepsilon^d = \frac{\alpha}{\varsigma - 1} > 0.$$

The only parameters affecting sectoral coherence are the intermediate differentiation and the intermediate intensity in downstream production, inasmuch as the elasticities are positive, constant and independent from exogenous parameters as market size or technology.<sup>23</sup> In this context, the strength of vertical linkages can be measured as the percentage change in the quantity of firms in one industry, due to a one percent change in the number of firms in the other industry. The major advantages of this approach are the availability of the parameters from official statistics and econometric estimations respectively, the potential to compare industrial linkages beyond particular supply chains, a dimensionless measure, and an ultimately intuitive economic interpretation.

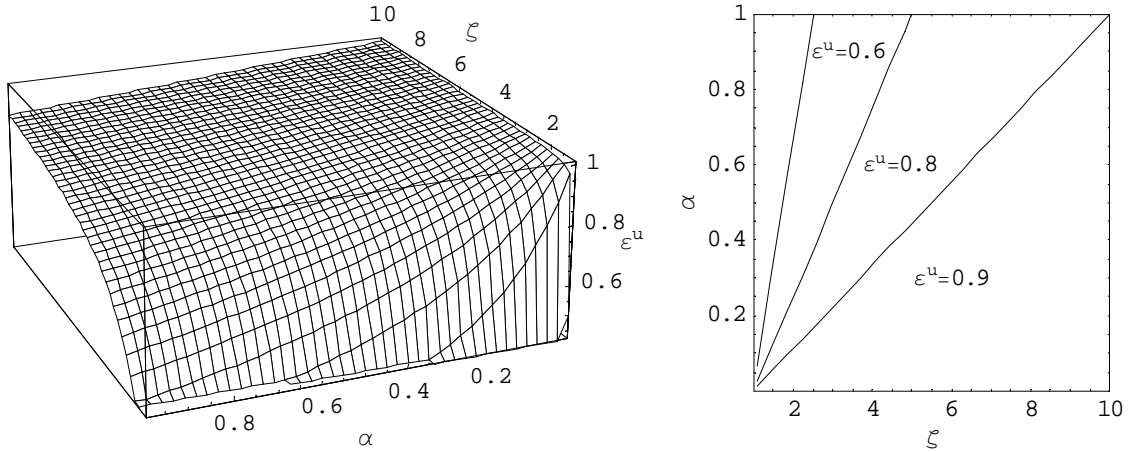
Figures 3 and 4 represent the graphs of Equation (49a) and (49b), wherein the right hand side illustrates the respective isoclines. We see that the dependence of upstream firms on the downstream industry increases, the lower that the intermediate differentiation and the intermediate share in downstream cost. The backward linkage and, in this context, the dependence of downstream firms on their suppliers, intensify with increasing intermediate differentiation and expanding intermediate share. The isoclines for a given elasticity are linear, with a slope,  $(1 - \bar{\varepsilon}^u) / \bar{\varepsilon}^u$ , for the forward linkage and  $\bar{\varepsilon}^d$  for the backward linkage. This means that, to maintain a certain level of linkage strength, an advance of  $\varsigma$  must go along with an  $\alpha$  that is adjusted using the appropriate slopes.

## 4 Simulation

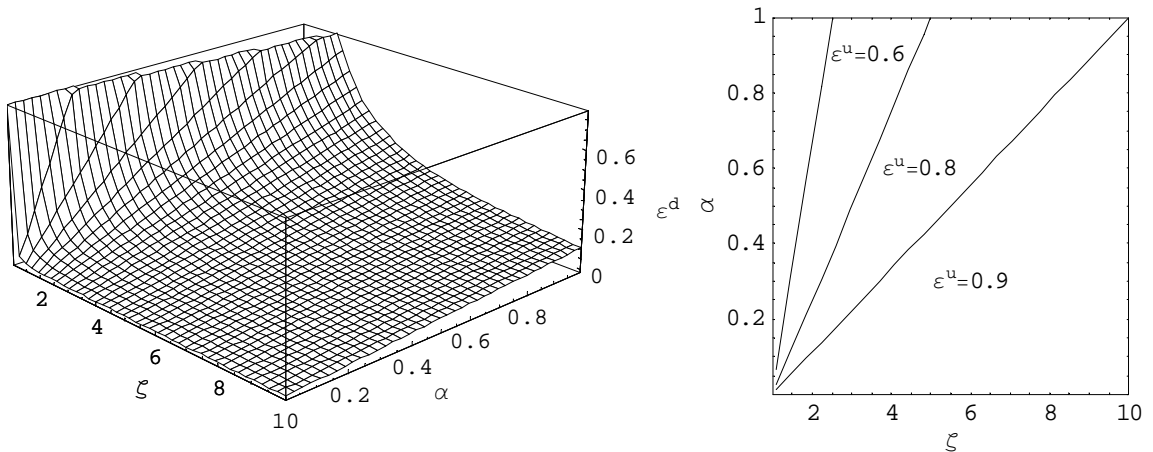
In this section, we adapt the theoretical concept to real economic data. For this purpose, we need to set parameters for substitution elasticity,  $\varsigma$  and for the cost share,

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<sup>23</sup>This outcome can be attributed to the specific CES-typed factor demand and increasing returns. This, once again, illustrates the convenient characteristics of the Dixit-Stiglitz framework.



*Figure 3: Strength of Forward Linkage*



*Figure 4: Strength of Backward Linkage*

$\alpha$ . While the latter can be ascertained from official input-output-tables, substitution elasticity is left to be appraised by means of econometric estimations. In this context, a commonly-used analysis has been made by Hummels (1999), who estimated trade cost and substitution elasticities for 61 2-digit SITC-classified commodities, by using data from seven countries in 1994, including the U.S., New Zealand, and five Latin American nations. This approach indirectly assumes that technical substitution elasticity,  $\zeta$ , is independent of location and, hence, is an industry-specific parameter.

A connected issue considers the nature of the cost share,  $\alpha$ . Although assumed to be exogenous, the parameter could be differentiated not only with respect to industry, but also with respect to location. This might be a result of industries with different national product foci, which ultimately depend upon the respective aggregation level. Consequently, the German chemicals industry, for instance, might be aligned to pharmaceuticals stronger than the Hungarian chemicals sector, and this eventually could lead to different intermediate cost shares. One may ask if the

same reasoning could hold for substitution elasticity that is considered to be independent of location. Although this assumption challenges criticism, it is the lack of econometric estimations that reduces national differentiation of each industry to only one degree of freedom – the cost share,  $\alpha$ . The assumption of a closed economy provides another dilemma. Contrary to the increasing role of international trade in intermediates, we neglect foreign upstream supplies and, therefore, assume that the cost structure for imported intermediates is the same as for domestic.<sup>24</sup>

Tables 4 – 11 in Appendix 6.4 show the simulation results for several European countries, using EUROSTAT data from 2000. Some of the main findings shall be summarized here:

- To a relatively large extent, the cost share,  $\alpha$ , differs from country to country, and this cannot be attributed to measurement errors only; in fact, it may be evidence of country-specific technologies.
- For nearly every case, forward linkages generally are much stronger than backward linkages. This result is not really surprising, if we compare the effectiveness of market size and price index effect. The only exception is the coherence of Metal Production and Ores in Table 8. In this case, except from Slovenia (the relationships for Netherlands and Poland being non-existent), the downstream strength exceeds the upstream strength, which can be attributed to extremely low substitution elasticity.<sup>25</sup>
- A further noticeable issue is that most values for upstream and downstream strength are relatively close to the overall averages of 0.9690 and 0.0454, respectively. Against this homogenous industrial picture, Metal Production and Ores form an example of definite outliers, followed by Foods and Agriculture, and then by Metal Manufactures and Metal Production. For illustration, Figure 5 shows the distribution of the average values (cross-national) for upstream and downstream strength. For almost all cases, the summed upstream and downstream strength approximately yields 1.0.<sup>26</sup> This might be utilized for attribution of total sectoral coherence (sum function) to the separate linkage strengths in terms of percentage.

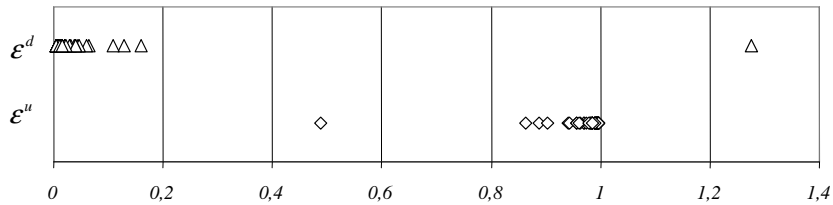
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<sup>24</sup>Critical country-oriented values are indicated by an asterisk. See Appendix 6.4 for further explanations.

<sup>25</sup>The OLS-estimated value by Hummels (1999) is not significant, as opposed to the nonlinear estimation value 0.8, which is out of the parameter domain of this actual model.

<sup>26</sup>This appearance can be ascribed to the attributes of the sum function, which decreases monotonically with rising  $\zeta$  and falling  $\alpha$ ; the asymptotic limiting value is 1; for the majority of real observations, the sum function, as well as (49a) and (49b), are relatively inelastic.

*Figure 5: Average Values of Sectoral Strength*



## 5 Concluding Remarks

As determined in the previous section, the most *real* industries create sectoral strength that is in a comparatively inelastic domain. This requires that many branches are relatively homogenous in terms of sectoral linkages, and respond to changes in substitution elasticity and intermediate cost share to a minor degree. Furthermore, the forward linkage is much stronger than the backward linkage and close to its upper limit. This implies that economic policy, in terms of location development, should promote downstream industries utilizing sectoral linkages for agglomeration. Although downstream elasticity has a (positive) infinite codomain and, hence, greater inherent potential than upstream elasticity, the simulation outcomes for downstream strength are close to the lower bound. Due to the response duality of sectoral linkages, with respect to substitution elasticity and cost share, a strong intermediate industry implies a weak subsequent industry, and vice versa. Another issue is the evaluation of supply networks rather than of supply chains. Regarding the industries in Section 4, the input-output inter-relationships are not 1:1, as with n:m. This implies that any sensitivity analysis at a macroeconomic level requires computations within an input-output matrix, instead of single supply chain vectors, as performed above. In this way, the impact of changes in an exogenous parameter can be traced through all sectors. For determination of sectoral coherence at the macro-level, an adequate output variable is needed. This could be the change in the whole economy relative to changes in firm number in a particular industry, for instance.

If we extend the model to refer to international trade, the convenience of the quantifying concept for sectoral coherence elaborated in this paper is lost. In the first instance, the complexity increases by means of location-based differences in consumer preferences, income, technology and factor prices. Furthermore, spatial distance is implemented by trade costs that are incurred for goods shipped between locations. The problem now involves not only the distribution of firms with respect to different sectors, but also to different locations. Venables (1996) shows that the spatial distribution of an industry, quantified as the ratio of the total production values at each location, depends positively on the distribution of the upstream and downstream sectors. Similar to upstream and downstream elasticity in this paper,

the sectoral coherence can be quantified using the elasticities of those distribution functions evaluated in the equilibrium state. Following this principle, the potential to have multiple equilibria (as one of the major investigation objectives of the NEG-literature) comes into play. Consequently, the existence of multiple equilibria implies multiple elasticities for the same industry – dependent upon output level. For the determination of elasticities, we must access numerical methods, due to non-closed solutions, which lead away from the simple measuring concept for sectoral strength in a closed economy.

But is this approach of evaluating open-economy industries really necessary? In the context of agglomeration, the main concern related to identifying 'strategic' industries, as required by Venables, may be the derivation of adequate criteria. The concept discussed in this paper provides information about the potential impact of the implicit agglomerative force. Combined with data on the macroeconomic importance of particular industries and supply chains, it enables one to define sector classification in the context of an industry's applicability within location development strategies.

## 6 Appendix

### 6.1 Derivation of Private Demand

The optimization problem of the private households is characterized by the nested utility function (1) and (2). Applying the two-stage-budgeting procedure, the utility maximization, subject to the budget constraint, has to be solved stepwise. Starting on the *lower* stage (the sub-utility), the minimum cost for a given level of  $M$  are to determine:

$$(50) \quad \min. \quad \sum_{i=1}^n p_i x_i \quad \text{s.t.} \quad \bar{M} = \left[ \sum_{i=1}^n (x_i)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} .$$

The resulting Lagrangian is:

$$(51) \quad \mathcal{L} = \sum_{i=1}^n p_i x_i + \lambda \left( \bar{M} - \left[ \sum_{i=1}^n (x_i)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \right) .$$

Differentiating (51), with respect to  $x_i$ , and solving for its corresponding price, yields:

$$(52) \quad p_i = \lambda \left[ \sum_{i=1}^n (x_i)^{(\sigma-1)/\sigma} \right]^{-\sigma} (x_i)^{-1/\sigma} .$$



Similarly, it follows for a particular variety  $j$ :

$$(53) \quad p_j = \lambda \left[ \sum_{i=1}^n (x_j)^{(\sigma-1)/\sigma} \right]^{-\sigma} (x_j)^{-1/\sigma}.$$

Dividing (52) by (53) and solving for the quantity of variety  $i$ , results in:

$$(54) \quad x_i = x_j \left( \frac{p_j}{p_i} \right)^\sigma.$$

After substitution of (54) into constraint (2) and solving for  $x_j$ , we obtain the compensated demand for variety  $j$ :

$$(55) \quad x_j = \left[ \sum_{i=1}^n (p_i)^{\sigma-1} \right]^{\sigma/(1-\sigma)} (p_j)^{-\sigma} \bar{M}.$$

The minimum cost for a given sub-utility,  $\bar{M}$ , follows from (55):

$$(56) \quad \sum_{i=1}^n p_i x_i = \bar{M} P,$$

where  $P$  is the consumer price index (5). Via substitution of (5), we obtain for (55):

$$(57) \quad x_j = \left( \frac{p_j}{P} \right)^{\sigma/(\sigma-1)} \bar{M}.$$

At the second stage of the optimization, we turn to the maximization of the utility function (1), subject to the budget constraint of private households. With  $p^A$  for a given price of the agricultural good, the income of the households,  $Y$ , is given by:

$$(58) \quad Y = p^A A + PM.$$

With (58) the optimization problem is:

$$(59) \quad \max. \quad U = M^\mu A^{1-\mu} \quad \text{s.t.} \quad Y = p^A A + PM,$$

and the associated Lagrangian:

$$(60) \quad \mathcal{L} = M^\mu A^{1-\mu} + \lambda (Y - p^A A - PM).$$

From (60), first order conditions are:

$$(61) \quad (1 - \mu) M^\mu A^{-\mu} = \lambda p^A$$

$$(62) \quad \mu M^{\mu-1} A^{1-\mu} = \lambda P.$$

From the proportion of (61) and (62), we derive:

$$(63) \quad M = \frac{\mu}{(1-\mu)} \frac{p^A A}{P}$$

$$(64) \quad A = \frac{(1-\mu)}{\mu} \frac{MP}{p^A}.$$

Substitution of (63) and (64) into the budget constraint (58) yields the optimum levels of  $A$  and  $M$ :

$$(65) \quad A^* = \frac{Y(1-\mu)}{p^A}$$

$$(66) \quad M^* = \frac{\mu Y}{P}.$$

Equation (66) delivers the general optimum level of  $M$  and, herewith, the value for the placeholder  $\bar{M}$  at the lower stage of the optimization. By inserting into (57) and assuming identical prices for all varieties, we obtain the demand function (4), finally.

## 6.2 Further Supplements to Section 2

$$(67) \quad \frac{\partial x_j}{\partial p_j} = -\sigma \mu Y (p_j)^{-\sigma-1} P^{\sigma-1} < 0$$

From (67), elasticity of substitution can be determined:

$$(68) \quad \sigma = \left| \frac{\partial x_j}{\partial p_j} \frac{p_j}{x_j} \right| = |-\sigma \mu Y (p_j)^{-\sigma} P^{\sigma-1} (x_j)^{-1}| = |-\sigma x_j (x_j)^{-1}| = \sigma$$

$$(69) \quad \frac{\partial x_j}{\partial P} = (\sigma - 1) \mu Y (p_j)^{-\sigma} P^{\sigma-2} > 0$$

$$(70) \quad \frac{\partial x_j}{\partial Y} = \mu (p_j)^{-\sigma} P^{\sigma-1} > 0$$

For determination of the marginal revenue, we obtain the revenue function from the inverse demand:

$$(71) \quad p = \left[ \frac{x}{\mu Y P^{\sigma-1}} \right]^{-1/\sigma} \longrightarrow px = [\mu Y P^{\sigma-1}]^{1/\sigma} x^{\frac{\sigma-1}{\sigma}}$$

From (71), marginal revenue can be written as:

$$(72) \quad \frac{\partial (px)}{\partial x} = \left( \frac{\sigma - 1}{\sigma} \right) [\mu Y P^{\sigma-1}]^{1/\sigma} x^{-1/\sigma} = \left( \frac{\sigma - 1}{\sigma} \right) p.$$

Totally differentiating profit  $\pi$  and using pricing rule (11) yields:

$$(73) \quad d\pi = \frac{\partial \pi}{\partial p} dp + \frac{\partial \pi}{\partial x} dx = \left( x - \frac{(\sigma - 1) F}{\sigma a} - \frac{(\sigma - 1) x}{\sigma} \right) dp + \left( p - \frac{(\sigma - 1) p}{\sigma} \right) dx.$$

By factoring out, (73) becomes:

$$(74) \quad d\pi = \frac{x}{\sigma} dp + \frac{p}{\sigma} dx - \frac{x}{\sigma} dp = \frac{p}{\sigma} dx.$$

### 6.3 Cost and Demand Function of a Downstream Firm

Similar to the utility maximization of households, the cost function and demand for intermediates can be determined in two steps. Again starting from the lower stage, the cost for a given intermediate aggregate  $\bar{I}$  is to minimize:

$$(75) \quad \min. \sum_{i=1}^n p_i^u x_i^u \quad s.t. \quad \bar{I} = \left[ \sum_{i=1}^n (x_i^u)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}.$$

The associated Lagrangian is:

$$(76) \quad \mathcal{L} = \sum_{i=1}^n p_i^u x_i^u + \lambda \left( \bar{I} - \left[ \sum_{i=1}^n (x_i^u)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \right).$$

From the first order conditions, we obtain :

$$(77) \quad p_i^u = \lambda \left[ \sum_{i=1}^n (x_i^u)^{(\sigma-1)/\sigma} \right]^{-\sigma} (x_i^u)^{-1/\sigma}$$

$$(78) \quad p_j^u = \lambda \left[ \sum_{i=1}^n (x_j^u)^{(\sigma-1)/\sigma} \right]^{-\sigma} (x_j^u)^{-1/\sigma}.$$

Setting (77) in proportion to (78) and solving for the quantity of intermediate sort  $i$  yields:

$$(79) \quad x_i^u = x_j^u \left( \frac{p_j^u}{p_i^u} \right)^\sigma.$$

From substitution of (79) into the constraint, we obtain the compensated demand for intermediate variety  $j$ :

$$(80) \quad x_j^u = \frac{(p_j^u)^{-\varsigma}}{\left[\sum_{i=1}^n (p_i^u)^{1-\varsigma}\right]^{\varsigma/(\varsigma-1)}} \bar{I}.$$

The minimum cost for intermediates results from (80):

$$(81) \quad \sum_{i=1}^n p_i^u x_i^u = \bar{I} \left[ \sum_{i=1}^n (p_i^u)^{1-\varsigma} \right]^{1/(1-\varsigma)} \equiv \bar{I} P^u,$$

where  $P^u$  is the intermediate price index. The compensated demand for intermediates (80) can be written with (22) as:

$$(82) \quad x_j^u = \left( \frac{p_j^u}{P^u} \right)^{-\varsigma} \bar{I}.$$

On the upper stage of optimization, the minimum total cost is to determine. In this context we minimize cost, constrained by the production function:

$$(83) \quad \min. \quad C^d = P^u I + w l^d \quad s.t. \quad F^d + a^d x^d = Z (l^d)^{1-\alpha} I^\alpha,$$

with the Lagrangian:

$$(84) \quad \mathcal{L} = P^u I + w l^d + \lambda \left[ F^d + a^d x^d - Z (l^d)^{1-\alpha} I^\alpha \right].$$

From the first order conditions, we obtain:

$$(85) \quad P^u = \lambda \alpha Z (l^d)^{1-\alpha} I^{\alpha-1}$$

$$(86) \quad w = \lambda (1 - \alpha) Z (l^d)^{-\alpha} I^\alpha.$$

From the ratio of (85) and (86), the quantities of labor and intermediates can be derived:

$$(87) \quad l^d = \frac{(1 - \alpha) P^u I}{\alpha w}$$

$$(88) \quad I = \frac{\alpha l^d w}{(1 - \alpha) P^u}.$$

Inserting (87) and (88) in the production function, delivers the optimum quantity of labor and intermediates:

$$(89) \quad (l^d)^* = (F^d + a^d x^d) Z^{-1} w^{-\alpha} (P^u)^\alpha \left[ \frac{\alpha}{1-\alpha} \right]^{-\alpha}$$

$$(90) \quad I^* = (F^d + a^d x^d) Z^{-1} w^{1-\alpha} (P^u)^{\alpha-1} \left[ \frac{1-\alpha}{\alpha} \right]^{\alpha-1}.$$

With (89) and (90) total cost is:

$$(91) \quad C^d = (F^d + a^d x^d) Z^{-1} w^{1-\alpha} (P^u)^\alpha \left[ \left( \frac{1-\alpha}{\alpha} \right)^{\alpha-1} + \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \right].$$

Equation (23) results from normalizing  $Z$  by  $(1-\alpha)^{\alpha-1} \alpha^{-\alpha}$ . For determining the demand function, labor has to be replaced by (87) in the general cost function. Solving for the intermediate aggregate yields:

$$(92) \quad I^* = \frac{\alpha C^d}{P^u}.$$

Equation (92) delivers the value for the placeholder  $\bar{I}$ , so that the uncompensated demand function (90) becomes (24).

## 6.4 Simulation Results

The substitution elasticities,  $\varsigma$ , in Table 3 are taken from Hummels (1999), Table 4 (OLS-estimates). Because the simulation aims to quantify sectoral coherence in European industries, substitution elasticities, estimated by Hummels, are combined with the cost share of several industries derived from the input-output-tables of EUROSTAT for 2000. This procedure raises the problem of different categorization systems: i) the product classification system SITC (United Nations Statistics Division), used by Hummels; and ii) the activity classification NACE, used by the European Community. Table 3 shows the proposed translation of classification codes for several industries.

The Tables 4 – 11 provide simulation results for the respective downstream industry named in the title of the tables. The columns reflect the computed cost share,  $\alpha$ , the upstream and downstream strength,  $\varepsilon^u$  and  $\varepsilon^d$ , as well as the sums of the sectoral strengths arranged for the relevant upstream industries (the average cost share,  $\alpha$ , is larger than 0.02). The last column displays the import share of the respective downstream industry, where the asterisk behind the country name indicates an associated value larger than 0.5. The rows show the outcomes for ten European

*Table 3: Parameter Settings*

<b>Industry</b>	<b><math>\zeta</math></b>	<b>NACE</b>	<b>SITC</b>	<b>Note</b>
Agriculture	5.23	A-01	00 - 11	Average Value
Chemicals	5.28	DG-24	51 - 56	Average Value
Coke / Coal	4.40	DF-23	32	
Foods	3.40	DA-15	06 - 11	Average Value
Machines	6.98	DK-29	74	
Metal Manufactures	4.85	DJ-28	69	
Metal Production	3.53	DJ-27	67	
Nonmetallic Manufactures	2.65	DI-26	66	
Ores	1.10	CB-13	28	
Paper	4.25	DE-21	64	
Petrol	5.61	CA-11	33	
Plastics	4.82	DH-25	57, 62	Average Value
Power Generating Machines	7.87	DL-31	71	
Printing	4.88	DE-22	89	Subordinated to SITC-89
Road Vehicles	7.11	DM-34	78	
Telecommunication	9.44	DL-32	76	
Textile Fibers	5.12	DB-17	26	

countries, the overall average and standard deviation. For computation of the cost share,  $\alpha$ , some necessary adjustments of the official data have to be made. First, we limit the activities to NACE A 01 – DN 36 for an improved model adaptation. This excludes services, trade, transportation and the homogenous products energy and water. Furthermore, intra-industrial trade is ruled out by model specification, so internal activities of the accounted downstream sector are also subtracted. The denominator for the appropriate cost share of a downstream industry are the inputs described above in terms of their acquisition costs and the sectoral wage bill finally. Another issue for confusion are potential bilateral input-output relationships, which inevitably arise from highly aggregated sectors (see chemicals and plastics, for instance). We ignore this appearance and compute twofold values, according to the direction of relations.

Table 4: Chemicals

Country	Petrol			Foods			Paper			Printing			Coke / Coal			
	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	
Belgium	0.0004	0.9999	0.0001	0.0277	0.9886	0.0115	0.0301	0.9908	0.0092	1.0001	0.0133	0.9966	0.0034	0.2214	0.9389	0.0651
Finland	0.0099	0.9979	0.0021	0.0562	0.9771	0.0234	0.0822	0.9753	0.0253	1.0006	0.0321	0.9918	0.0083	0.1246	0.9647	0.0366
France	0.0075	0.9984	0.0016	0.0449	0.9816	0.0187	0.0482	0.9854	0.0148	1.0002	0.0200	0.9949	0.0051	0.0793	0.9772	0.0233
Germany	0.0171	0.9963	0.0037	0.0605	0.9754	0.0232	0.0492	0.9851	0.0152	1.0002	0.0088	0.9977	0.0023	0.0927	0.9734	0.0273
Hungary*	0.0721	0.9846	0.0156	0.0113	0.9953	0.0047	0.0201	0.9939	0.0062	1.0000	0.0050	0.9987	0.0013	0.3131	0.9157	0.0921
Italy	0.0066	0.9986	0.0014	0.1439	0.9434	0.0600	0.1055	0.9685	0.0325	1.0010	0.0729	0.9816	0.0188	0.1282	0.9637	0.0377
Slovenia*	0.0150	0.9967	0.0033	0.0000	-	-	0.0869	0.9740	0.0267	1.0007	0.0179	0.9954	0.0046	0.0220	0.9936	0.0065
Netherlands	0.1465	0.9692	0.0318	0.0324	0.9867	0.0135	0.0187	0.9943	0.0058	1.0000	0.0506	0.9871	0.0131	0.2386	0.9344	0.0702
Poland	0.1492	0.9686	0.0324	0.0273	0.9887	0.0114	0.0303	0.9908	0.0093	1.0001	0.0152	0.9961	0.0039	0.0583	0.9831	0.0172
Sweden	0.0034	0.9993	0.0007	0.0235	0.9903	0.0098	0.0353	0.9893	0.0109	1.0001	0.0183	0.9953	0.0047	0.1100	0.9687	0.0324
Average	<b>0.0428</b>	<b>0.9909</b>	<b>0.0093</b>	<b>0.0428</b>	<b>0.9808</b>	<b>0.0198</b>	<b>0.0506</b>	<b>0.9847</b>	<b>0.0156</b>	<b>1.0003</b>	<b>0.0254</b>	<b>0.9935</b>	<b>0.0065</b>	<b>0.1388</b>	<b>0.9613</b>	<b>0.0408</b>
S.D.	<b>0.0590</b>	<b>0.0124</b>	<b>0.0128</b>	<b>0.0401</b>	<b>0.0154</b>	<b>0.0164</b>	<b>0.0305</b>	<b>0.0090</b>	<b>0.0094</b>	<b>0.0003</b>	<b>0.0211</b>	<b>0.0053</b>	<b>0.0054</b>	<b>0.0907</b>	<b>0.0243</b>	<b>0.0267</b>

Country	Plastics			Nonmetallic Manufactures			Metal Manufactures			Machines			Import		
	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$
Belgium	0.0355	0.9908	0.0093	0.0131	0.9921	0.0079	0.0235	0.9939	0.0061	1.0000	0.0021	0.9996	0.0004	0.4534	1.0000
Finland	0.0302	0.9922	0.0079	0.0158	0.9905	0.0095	0.0335	0.9914	0.0087	1.0001	0.0325	0.9946	0.0054	0.4203	1.0000
France	0.0822	0.9789	0.0215	0.0273	0.9837	0.0166	0.0691	0.9824	0.0180	1.0003	0.0178	0.9970	0.0030	0.3184	1.0000
Germany	0.0387	0.9900	0.0101	0.0183	0.9890	0.0111	0.0578	0.9852	0.0150	1.0002	0.0311	0.9948	0.0052	0.3727	1.0000
Hungary*	0.0516	0.9867	0.0135	0.0114	0.9932	0.0069	0.0332	0.9915	0.0086	1.0001	0.0469	0.9922	0.0079	0.5034	1.0000
Italy	0.1200	0.9696	0.0314	0.1671	0.9080	0.1013	0.0341	0.9912	0.0089	1.0001	0.0371	0.9938	0.0062	0.3530	1.0000
Slovenia*	0.0472	0.9878	0.0124	0.0165	0.9901	0.0100	0.0322	0.9917	0.0084	1.0001	0.0106	0.9982	0.0018	0.5109	1.0000
Netherlands	0.0265	0.9931	0.0069	0.0042	0.9974	0.0026	0.0200	0.9948	0.0052	1.0000	0.0197	0.9967	0.0033	0.4104	1.0000
Poland	0.0926	0.9763	0.0242	0.0260	0.9845	0.0158	0.0156	0.9960	0.0040	1.0000	0.0412	0.9931	0.0069	0.4841	1.0000
Sweden	0.0449	0.9884	0.0118	0.0175	0.9895	0.0106	0.0199	0.9949	0.0052	1.0000	0.0301	0.9950	0.0050	0.4128	1.0000
Average	<b>0.0569</b>	<b>0.9854</b>	<b>0.0149</b>	<b>0.0317</b>	<b>0.9818</b>	<b>0.0192</b>	<b>0.0339</b>	<b>0.9913</b>	<b>0.0088</b>	<b>1.0001</b>	<b>0.0269</b>	<b>0.9955</b>	<b>0.0045</b>	<b>0.4239</b>	<b>1.0000</b>
S.D.	<b>0.0309</b>	<b>0.0078</b>	<b>0.0081</b>	<b>0.0480</b>	<b>0.0262</b>	<b>0.0291</b>	<b>0.0172</b>	<b>0.0044</b>	<b>0.0045</b>	<b>0.0001</b>	<b>0.0141</b>	<b>0.0023</b>	<b>0.0024</b>	<b>0.0645</b>	<b>0.0000</b>

Table 5: Foods

Country	Agriculture			Paper			Chemicals			Metal Manufactures			Plastics			Import		
	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$
Belgium	0.5623	0.8827	0.1329	0.0312	0.9905	0.0096	0.0358	0.9917	0.0084	1.0001	0.0183	0.9953	0.0047	0.0383	0.9901	0.0100	0.2968	
Finland	0.5290	0.8888	0.1251	0.0383	0.9884	0.0118	0.0231	0.9946	0.0054	1.0000	0.0147	0.9962	0.0038	0.0302	0.9922	0.0079	0.1567	
France	0.5034	0.8937	0.1190	0.0270	0.9918	0.0083	0.0184	0.9957	0.0043	1.0000	0.0245	0.9937	0.0064	0.0504	0.9870	0.0132	0.1373	
Germany	0.5181	0.8909	0.1225	0.0573	0.9887	0.0115	0.0116	0.9973	0.0027	1.0000	0.0233	0.9940	0.0060	0.0255	0.9934	0.0067	0.1772	
Hungary	0.5511	0.8847	0.1303	0.0454	0.9862	0.0140	0.0570	0.9869	0.0133	1.0002	0.0248	0.9936	0.0064	0.0515	0.9867	0.0135	0.1102	
Italy	0.7865	0.8432	0.1859	0.0430	0.9869	0.0132	0.0132	0.9969	0.0031	1.0000	0.0217	0.9944	0.0056	0.0371	0.9904	0.0097	0.1537	
Slovenia	0.3970	0.9142	0.0938	0.0454	0.9862	0.0140	0.0210	0.9951	0.0049	1.0000	0.0201	0.9948	0.0052	0.0553	0.9857	0.0145	0.2127	
Netherlands	0.5821	0.8790	0.1376	0.0372	0.9887	0.0114	0.0182	0.9958	0.0042	1.0000	0.0267	0.9931	0.0069	0.0253	0.9934	0.0066	0.2481	
Poland	0.5267	0.8893	0.1245	0.0316	0.9904	0.0097	0.0169	0.9961	0.0039	1.0000	0.0103	0.9973	0.0027	0.0244	0.9937	0.0064	0.0935	
Sweden	0.4754	0.8990	0.1124	0.0393	0.9880	0.0121	0.0192	0.9955	0.0045	1.0000	0.0258	0.9933	0.0067	0.0367	0.9905	0.0096	0.2225	
Average	<b>0.5432</b>	<b>0.8865</b>	<b>0.1284</b>	<b>0.0376</b>	<b>0.9886</b>	<b>0.0116</b>	<b>0.0234</b>	<b>0.9946</b>	<b>0.0055</b>	<b>1.0000</b>	<b>0.0210</b>	<b>0.9946</b>	<b>0.0055</b>	<b>0.0375</b>	<b>0.9903</b>	<b>0.0098</b>	<b>0.1809</b>	
S.D.	<b>0.0999</b>	<b>0.0181</b>	<b>0.0236</b>	<b>0.0062</b>	<b>0.0019</b>	<b>0.0019</b>	<b>0.0135</b>	<b>0.0031</b>	<b>0.0032</b>	<b>0.0001</b>	<b>0.0053</b>	<b>0.0014</b>	<b>0.0014</b>	<b>0.0115</b>	<b>0.0030</b>	<b>0.0001</b>	<b>0.0637</b>	

Table 6: Machines

Country	Chemicals			Plastics			Metal Production			Metal Manufactures			Power Gen. Mach.					
	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$			
Belgium*	0.0254	0.9546	0.0055	0.0421	0.9891	0.0110	0.0493	0.9809	0.0195	0.0004	0.2425	0.9407	0.0630	1.0037	0.1115	0.9840	0.0162	1.0003
Finland	0.0088	0.9979	0.0021	0.0222	0.9942	0.0058	0.2032	0.9257	0.0803	1.0000	0.0898	0.9772	0.0233	1.0005	0.0762	0.9890	0.0111	1.0001
France	0.0192	0.9955	0.0045	0.0399	0.9897	0.0105	0.1413	0.9471	0.0558	1.0000	0.1965	0.9515	0.0510	1.0025	0.0389	0.9944	0.0057	1.0000
Germany	0.0321	0.9926	0.0075	0.0372	0.9904	0.0097	0.0826	0.9684	0.0327	1.0010	0.1236	0.9689	0.0321	1.0010	0.0763	0.9890	0.0111	1.0001
Hungary*	0.0455	0.9895	0.0106	0.0359	0.9907	0.0094	0.2621	0.9061	0.1036	1.0097	0.0643	0.9836	0.0167	1.0003	0.0952	0.9863	0.0139	1.0002
Italy	0.0297	0.9931	0.0069	0.0761	0.9805	0.0199	0.2561	0.9081	0.1012	1.0093	0.4232	0.9010	0.1099	1.0109	0.0897	0.9871	0.0131	1.0002
Slovenia	0.0431	0.9900	0.0101	0.0553	0.9857	0.0145	0.1230	0.9536	0.0486	1.0023	0.2071	0.9490	0.0538	1.0027	0.1092	0.9844	0.0159	1.0002
Netherlands	0.0093	0.9978	0.0022	0.0215	0.9944	0.0056	0.1095	0.9585	0.0433	1.0018	0.1516	0.9621	0.0394	1.0015	0.0396	0.9943	0.0058	1.0000
Poland*	0.0261	0.9939	0.0061	0.0326	0.9915	0.0085	0.2123	0.9226	0.0839	1.0065	0.1411	0.9646	0.0367	1.0013	0.0793	0.9886	0.0115	1.0001
Sweden	0.0164	0.9962	0.0038	0.0282	0.9927	0.0074	0.0984	0.9626	0.0589	1.0015	0.1265	0.9682	0.0328	1.0010	0.0666	0.9904	0.0097	1.0001
Average	0.0254	0.9941	0.0059	0.0391	0.9899	0.0102	0.1538	0.9434	0.0608	1.0041	0.1766	0.9567	0.0459	1.0025	0.0782	0.9888	0.0114	1.0001
S.D.	0.0126	0.0029	0.0030	0.0164	0.0042	0.0043	0.0747	0.0261	0.0295	0.0035	0.1021	0.0235	0.0265	0.0031	0.0251	0.0036	0.0037	0.0001

Table 7: Metal Manufactures

Country	Telecommunication			Road Vehicles			Metal Production			Machines			Import					
	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$						
Belgium*	0.0107	0.9987	0.0013	0.0486	0.9921	0.0080	0.4966	0.7687	0.3010	1.0696	0.3531	0.8775	0.1396	1.0171	0.0021	0.9997	0.0003	1.0000
Finland	0.0096	0.9989	0.0011	0.0863	0.9861	0.0141	0.0181	0.9892	0.0109	1.0001	0.4070	0.8614	0.1609	1.0223	0.0534	0.9911	0.0089	1.0001
France	0.0349	0.9959	0.0041	0.0020	0.9997	0.0003	0.0012	0.9993	0.0007	1.0000	0.2455	0.9116	0.0970	1.0086	0.0470	0.9922	0.0079	1.0001
Germany	0.0276	0.9967	0.0033	0.0035	0.9994	0.0006	0.0132	0.9921	0.0080	1.0001	0.2739	0.9023	0.1083	1.0106	0.0488	0.9919	0.0082	1.0001
Hungary*	0.0295	0.9965	0.0035	0.0141	0.9977	0.0023	0.0056	0.9975	0.0025	1.0000	0.5662	0.0033	0.1000	1.0000	0.0437	0.9927	0.0073	1.0001
Italy	0.0606	0.9929	0.0072	0.0080	0.9987	0.0013	0.0058	0.9965	0.0035	1.0000	0.1957	0.8311	0.2032	1.0343	0.0437	0.9927	0.0073	1.0001
Slovenia	0.0429	0.9949	0.0051	0.0000	0.9999	0.0000	0.7419	0.9812	0.0192	1.0004	0.4096	0.7419	0.7732	1.0665	0.0344	0.9943	0.0058	1.0000
Netherlands	0.0687	0.9919	0.0081	0.0153	0.9975	0.0025	0.0153	0.9975	0.0025	1.0000	0.4377	0.4850	0.8391	1.0308	0.0063	0.9989	0.0011	1.0000
Poland*	0.0038	0.9996	0.0004	0.0238	0.9961	0.0039	0.0072	0.9957	0.0044	1.0000	0.5044	0.8760	0.1415	1.0175	0.0293	0.9951	0.0049	1.0000
Sweden	0.0635	0.9925	0.0075	0.0438	0.9929	0.0072	0.0373	0.9779	0.0226	1.0005	0.3280	0.3829	0.8686	1.0133	0.0561	0.9907	0.0094	1.0001
Average	0.0352	0.9959	0.0042	0.0245	0.9956	0.0045	0.4046	0.9690	0.0380	1.0071	0.3831	0.8640	0.1599	1.0239	0.0357	0.9941	0.0060	1.0000
S.D.	0.0235	0.0028	0.0028	0.0275	0.0045	0.0001	0.1470	0.0412	0.0581	0.0171	0.1335	0.0412	0.0581	0.0171	0.0187	0.0031	0.0000	0.0000

Table 7: Metal Manufactures

Country	Chemicals			Nonmetallic Manufactures			Metal Production			Machines			Import		
	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$	$\alpha$	$\epsilon^u$	$\epsilon^d$			
Belgium*	0.0371	0.9914	0.0087	0.0371	0.9914	0.0087	0.3531	0.8775	0.1396	1.0171	0.0021	0.9997	0.0003	1.0000	0.7994
Finland	0.0354	0.9918	0.0083	0.0181	0.9892	0.0109	0.4070	0.8614	0.1609	1.0223	0.0534	0.9911	0.0089	1.0001	0.1489
France	0.0619	0.9857	0.0145	0.0012	0.9993	0.0007	0.2455	0.9116	0.0970	1.0086	0.0470	0.9922	0.0079	1.0001	0.1265
Germany	0.0276	0.9936	0.0065	0.0132	0.9921	0.0080	0.2739	0.9023	0.1083	1.0106	0.0488	0.9919	0.0082	1.0001	0.1350
Hungary	0.0272	0.9937	0.0064	0.0058	0.9965	0.0035	0.5141	0.8311	0.2032	1.0343	0.0437	0.9927	0.0073	1.0001	0.3820
Italy	0.0501	0.9884	0.0117	0.0317	0.9812	0.0192	0.7419	0.9812	0.0192	1.0004	0.0344	0.9943	0.0058	1.0000	0.0630
Slovenia	0.0143	0.9967	0.0033	0.0073	0.9956	0.0044	0.4850	0.8391	1.0308	1.0000	0.0063	0.9989	0.0011	1.0000	0.2421
Netherlands	0.0193	0.9855	0.0045	0.0072	0.9957	0.0044	0.3581	0.8760	0.1415	1.0175	0.0293	0.9951	0.0049	1.0000	0.2573
Poland	0.0500	0.9885	0.0117	0.0373	0.9779	0.0226	0.3829	0.8686	1.0133	1.0199	0.0561	0.9907	0.0094	1.0001	0.2326
Sweden	0.0335	0.9922	0.0078	0.0091	0.9945	0.0055	0.2842	0.8990	1.1124	1.0113	0.0354	0.9941	0.0059	1.0000	0.1757
Average	0.0357	0.9917	0.0083	0.0628	0.9690	0.0380	0.4046	0.8640	0.1599	1.0239	0.0357	0.9941	0.0060	1.0000	0.2563
S.D.	0.0148	0.0034	0.0035	0.0152	0.0707	0.0927	0.1470	0.0412	0.0581	0.0171	0.0187	0.0031	0.0000	0.0000	0.2104



Table 8: Metal Production

Country	Ores			Coke / Coal			Chemicals			Nonmetallic Manufactures			Metal Manufactures		
	$\alpha$	$\varepsilon^u$	$\varepsilon^d$	$\alpha$	$\varepsilon^u$	$\varepsilon^d$	$\alpha$	$\varepsilon^u$	$\varepsilon^d$	$\alpha$	$\varepsilon^u$	$\varepsilon^d$	$\alpha$	$\varepsilon^u$	$\varepsilon^d$
Belgium	0.1254	0.4436	1.2544	0.0457	0.9867	0.0134	0.0398	0.9908	0.0093	0.0356	0.9789	0.0215	0.1622	0.9596	0.0421
Finland	0.2710	0.2695	2.7105	0.0428	0.9876	0.0126	0.1480	0.9666	0.0346	0.0159	0.9904	0.0097	0.1054	0.9734	0.0274
France	0.0686	0.5930	0.6865	0.0631	0.9818	0.0186	0.0722	0.9834	0.0169	0.0434	0.9744	0.0263	0.0759	0.9807	0.0197
Germany	0.1119	0.4719	1.1191	0.0823	0.9764	0.0242	0.0690	0.9841	0.0161	0.0398	0.9764	0.0241	0.0360	0.9907	0.0094
Hungary	0.1680	0.3731	1.6800	0.1479	0.9583	0.0435	0.0473	0.9891	0.0110	0.0521	0.9694	0.0316	0.0630	0.9839	0.0164
Italy	0.1029	0.4928	1.0292	0.1513	0.9574	0.0445	0.2173	0.9517	0.0508	0.1763	0.9035	0.1069	0.1253	0.9685	0.0325
Slovenia	0.0157	0.8642	0.1571	0.0274	0.9920	0.0080	0.0008	0.9998	0.0002	0.0330	0.9804	0.0200	0.0611	0.9844	0.0159
Netherlands*	0.0000	-	-	0.0109	0.9968	0.0032	0.0226	0.9948	0.0053	0.0070	0.9958	0.0042	0.0692	0.9823	0.0180
Poland	0.0000	-	-	0.0503	0.9854	0.0148	0.0517	0.9881	0.0121	0.0456	0.9731	0.0276	0.0487	0.9875	0.0126
Sweden	0.1574	0.3885	1.5738	0.0405	0.9882	0.0119	0.0408	0.9906	0.0095	0.0194	0.9884	0.0118	0.0842	0.9786	0.0219
Average	0.1021	0.4871	1.2763	0.0662	0.9811	0.0195	0.0709	0.9839	0.0166	0.0468	0.9731	0.0284	0.0831	0.9789	0.0216
S.D.	0.0856	0.1795	0.7555	0.0479	0.0134	0.0141	0.0645	0.0143	0.0151	0.0477	0.0258	0.0289	0.0381	0.0094	0.0099

Country	Machines			Power Gen. Mach.			Import
	$\alpha$	$\varepsilon^u$	$\varepsilon^d$	$\alpha$	$\varepsilon^u$	$\varepsilon^d$	
Belgium	0.0057	0.9990	0.0010	0.0128	0.9981	0.0019	0.3914
Finland	0.0560	0.9907	0.0094	0.0048	0.9993	0.0007	0.2356
France	0.0412	0.9932	0.0069	0.0235	0.9966	0.0034	0.3298
Germany	0.0634	0.9895	0.0106	0.0175	0.9975	0.0025	0.3699
Hungary	0.0495	0.9918	0.0083	0.0125	0.9982	0.0018	0.4798
Italy	0.0608	0.9899	0.0102	0.0402	0.9942	0.0059	0.3734
Slovenia	0.1234	0.9798	0.0206	0.0000	-	-	0.4888
Netherlands*	0.0673	0.9889	0.0112	0.0058	0.9992	0.0008	0.5768
Poland	0.0741	0.9878	0.0124	0.1128	0.9838	0.0164	0.2916
Sweden	0.0607	0.9900	0.0101	0.0123	0.9982	0.0018	0.3418
Average	0.0602	0.9901	0.0101	0.0242	0.9961	0.0039	0.3879
S.D.	0.0293	0.0048	0.0049	0.0331	0.0049	0.0001	0.1016

Table 9: Plastics

Country	Textile Fibers			Paper			Chemicals			Metal Manufactures			Import		
	$\alpha$	$\varepsilon^u$	$\varepsilon^d$	$\alpha$	$\varepsilon^u$	$\varepsilon^d$	$\alpha$	$\varepsilon^u$	$\varepsilon^d$	$\alpha$	$\varepsilon^u$	$\varepsilon^d$			
Belgium	0.0160	0.9961	0.0039	0.0677	0.9796	0.0208	0.5033	0.8948	0.1176	0.0264	0.9932	0.0069	0.0021	0.9997	0.0003
Finland	0.0174	0.9958	0.0042	0.0384	0.9883	0.0118	0.4041	0.9137	0.0944	0.0239	0.9938	0.0062	0.0188	0.9969	0.0031
France	0.0215	0.9948	0.0052	0.0037	0.9989	0.0011	0.4383	0.9071	0.1024	0.0164	0.9958	0.0043	0.0342	0.9943	0.0057
Germany	0.0136	0.9967	0.0033	0.0101	0.9969	0.0031	0.4295	0.9088	0.1004	0.0092	0.9965	0.0035	0.0309	0.9949	0.0052
Hungary	0.0464	0.9889	0.0113	0.0130	0.9960	0.0040	0.5537	0.8854	0.1294	0.0360	0.9907	0.0093	0.0222	0.9963	0.0037
Italy	0.0327	0.9921	0.0079	0.0450	0.9863	0.0138	0.7385	0.8528	0.1725	0.0519	0.9867	0.0135	0.0154	0.9974	0.0026
Slovenia	0.0787	0.9813	0.0191	0.0050	0.9985	0.0015	0.4083	0.9129	0.0954	0.0045	0.9886	0.0115	0.0344	0.9943	0.0057
Netherlands	0.0149	0.9964	0.0036	0.0238	0.9927	0.0073	0.4837	0.8985	0.1130	0.0115	0.9973	0.0028	0.0143	0.9976	0.0024
Poland	0.0397	0.9905	0.0096	0.0166	0.9949	0.0051	0.4075	0.9131	0.0952	0.0083	0.9862	0.0140	0.0142	0.9976	0.0024
Sweden	0.0037	0.9991	0.0009	0.0454	0.9862	0.0140	0.3122	0.9320	0.0730	0.0050	0.9937	0.0063	0.0333	0.9945	0.0056
Average	0.0285	0.9932	0.0069	0.0269	0.9918	0.0083	0.4679	0.9019	0.1093	0.0301	0.9922	0.0078	0.0220	0.9963	0.0037
S.D.	0.0219	0.0052	0.0053	0.0213	0.0064	0.0066	0.1154	0.0214	0.0270	0.0156	0.0440	0.0041	0.0110	0.0018	0.0000



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