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# Progressivity, government size, and redistribution in an optimal tax model

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# **Progressivity, Government Size, and Redistribution in an Optimal Tax Model**

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## Abstract

Although the U.S. has a more progressive tax system than many other high-income countries, its overall fiscal system does less to reduce inequality because its tax system is smaller (OECD 2008). What societal preferences can explain these cross-country differences? We address this question by computationally solving a two-bracket optimal tax problem with lump sum redistribution. In this framework, a social planner can achieve redistributive goals by selecting progressive tax rates or scaling up the entire tax system to fund a larger transfer. We find that planners who exhibit stronger inequality aversion prefer tax systems that are similarly or slightly less progressive, but substantially larger, than the tax systems chosen by planners who exhibit weaker inequality aversion. These larger tax systems, like those in other high-income countries, result in greater reduction in inequality. In addition, planners who favor middle-income individuals or both low- and middle-income individuals (e.g., the bottom 99 percent of the distribution) prefer smaller, more progressive tax systems than planners who favor the lowest-income individuals. These smaller tax systems, like that of the U.S., result in less reduction in overall inequality.

We thank Alan Viard for helpful comments.

## I. Introduction

In 2008, the Organisation for Economic Cooperation and Development (OECD) released a report suggesting that the U.S. has one of the most progressive tax systems – defined as the greatest concentration of tax payments, relative to income, at the high end of the income distribution – among member nations. However, the U.S. fiscal system induces less redistribution (reduction in income inequality) than most other OECD countries (OECD 2008). The intuition behind these results is that the degree of redistribution induced by a tax system depends on its size as well as its progressivity. All else equal, more progressive tax systems induce more redistribution by concentrating tax payments at the upper end of the income distribution. All else equal, larger tax systems can induce more redistribution for two reasons. First, if they are at least mildly progressive, scaling them up directly reduces income inequality. Second, even if they are proportional or regressive, larger tax systems raise more revenue to fund transfer payments on the spending side of the budget. Compared to other OECD nations, the U.S. has a more progressive but smaller tax system. Overall, other countries impose less progressive taxes, but these taxes are higher and fund larger welfare states.<sup>1</sup>

Motivated by these stylized facts, we use an optimal tax model to investigate how alternative societal preferences can give rise to different degrees of tax progressivity, tax system size, and overall redistribution. Although optimal tax models are often used to derive normative results, they can also be used for positive analysis – that is, to understand the societal preferences that can give rise to observed tax and transfer systems (see Saez 2001 and Saez and Stantcheva 2016). Our analysis is in that vein. We aim to show how variation in societal preferences can explain stylized facts about the progressivity, size, and degree of redistribution of tax and transfer systems. Related literature includes work by Christiansen and Jansen (1978) and Ahmad and Stern (1984), who examine the societal preferences

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<sup>1</sup> See Slavov and Viard (2016a,b) for further discussion. Lindert (2003) also argues that countries with large welfare states tend to impose relatively efficient taxes, featuring lower rates on capital compared to labor, consumption, and goods that generate negative externalities.

behind indirect taxes in Norway and India, respectively. More recently, Jacobs, Jongen, and Zoutman (2017) examine the societal preferences behind the tax policy proposals of Dutch political parties, concluding that all parties appear to give greater welfare weight to middle-income earners than low-income earners. Bourguignon and Spadaro (2012) examine the societal preferences that rationalize the French tax and transfer system, finding that these preferences may involve negative welfare weights on the highest-income individuals.

To capture the general structure of real-world tax and transfer systems in a computationally tractable manner, we consider a two-bracket optimal tax model with a lump-sum rebate.<sup>2</sup> The two tax brackets represent the tax system. The lump sum rebate represents the transfer system. A two-bracket system is more restrictive than many optimal tax models, which allow a different marginal tax rate at each level of income. However, the two-bracket model is not a bad representation of real-world tax systems, which (presumably due to administrative costs) generally feature piecewise linear, rather than continuous, tax schedules. Because allowing for additional brackets provides modest social welfare gains (Andrienko et al. 2016), we would not expect additional brackets to fundamentally change our results. The lump-sum transfer captures the fact that real-world transfer programs tend to be highly progressive.<sup>3</sup>

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<sup>2</sup> Other papers that use this approach include Sheshinski (1989); Slemrod, Yitzhaki, Mayshar, and Lumsdahl (1994); Hsu and Yang (2013); Apps, Long, and Rees (2014); Apps and Rees (2016); Andrienko, Apps, and Rees (2016); and Bastani, Blomquist, and Micheletto (2019).

<sup>3</sup> Under a lump-sum transfer, all individuals receive the same absolute transfer payment. However, the transfer system is progressive because the transfer as a *share* of income is decreasing in income. For example, according to estimates by the Tax Policy Center (2020), the bottom 20 percent of households in the U.S. received 3.7 percent of total before-tax income in 2017. Those households would receive a much larger share – 20 percent – of the benefits of a lump-sum transfer. Distributional estimates for real-world spending programs show that they are highly progressive, although the degree of progressivity varies depending on what spending is counted, whether all households or non-elderly households are considered, and other assumptions. For example, Prante and Hodge (2013) estimate a distribution of U.S. federal spending (including traditional transfer programs and public goods) across all households and find results that are roughly consistent with a lump-sum transfer. They show that, in 2012, households in the bottom 20 percent received an average of \$24,125 of federal spending, and those in the top 20 percent received an average of \$21,402 of federal spending. In contrast, the Congressional Budget Office (2013) estimates that, in 2006, the bottom 20 percent of U.S. non-elderly households received roughly half of all federal transfer spending, while the top 20 percent received less than 7 percent. Those estimates suggest a

In standard optimal tax models, there is no distinction between taxes and transfers, as transfers are simply negative taxes.<sup>4</sup> However, taxes and transfers are often considered separately in policy discussions, as policymakers often want to understand their separate roles.<sup>5</sup> For example, OECD (2008) distinguishes between the progressivity of taxes and the progressivity of transfer payments and computes the degree of redistribution induced by each of the two parts of the fiscal system. In the two-bracket model, redistribution can be increased either by increasing tax progressivity or by scaling up the entire tax system to fund the highly progressive transfer system.

Slemrod et al. (1994) and Apps et al. (2014) also employ the two-bracket model. Slemrod et al. (1994) find that optimal marginal tax rates are mildly regressive under the assumption of a lognormal wage distribution with parameters that reflect data from Stern (1976). Apps et al. (2014) argue that a Pareto distribution for wages provides a more accurate representation of reality, particularly given the recent rise in inequality towards the top of the distribution. They find strongly progressive tax rates under that assumption.<sup>6</sup> We consider the basic model of Slemrod et al. (1994) with both Pareto and lognormal wage distributions, as well as a broader range of preference parameters. Our results confirm these two papers' findings that, for a standard social welfare function, the Pareto distribution results in strongly progressive rates while the lognormal distribution results in mildly regressive rates.

Relative to Slemrod et al. (1994) and Apps et al. (2014), our contribution is to examine how alternative societal preferences affect tax progressivity, government (tax system) size, and overall redistribution. We consider not only planners with different degrees of inequality aversion (as do those

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transfer system that is even more progressive than the lump-sum transfer in our model. An even more progressive transfer system would likely strengthen our findings by making transfers (supported by a large tax system) an even more effective way to target the lowest-income households.

<sup>4</sup> Kaplow (2007) makes this point and examines the optimal design of transfer programs, including features like benefit levels and phase-outs.

<sup>5</sup> In practice, researchers make judgments calls about whether to classify a program as a tax or a transfer. For example, the Earned Income Tax Credit is often treated as a tax because it is administered through the tax code even though it results in negative net taxes (transfers) for many of its beneficiaries.

<sup>6</sup> Andrienko et al. (2016) further show that increases in income inequality generally result in more progressive optimal rate structures.

authors), but also planners who place explicit weights on individuals in different parts of the wage distribution, in line with the preferences expressed by some policy makers.<sup>7</sup> In the U.S., for example, some Democratic policy makers have emphasized the welfare of “the 99 percent” – the bottom 99 percent of income earners – relative to the top 1 percent (Stelter 2011), and some Republican policy makers have emphasized the impact of taxes on small business owners, who represent a larger share of income in higher tax brackets (Congressional Research Service 2010). Both Democrats and Republicans commonly express particular concern for the welfare of the “middle class.” We calculate summary measures of progressivity, government size, and redistribution for each planner’s optimal policy to understand how they use these aspects of the tax-and-transfer system to achieve their distributive goals.

We find that social planners who exhibit greater inequality aversion choose either similarly or slightly less progressive tax systems than planners who exhibit less inequality aversion. However, the more inequality-averse planners choose a much larger tax system, resulting in a fiscal system that induces more overall redistribution. We further show that social planners who care only about the welfare of the bottom 99 percent of the wage distribution tend to choose more progressive, but lower, taxes – resulting in less overall redistribution – than social planners who care only about the welfare of the bottom 10 percent of the wage distribution. A similar result holds for planners who care only about the welfare of individuals in the middle of the wage distribution relative to planners who emphasize the welfare of individuals at the bottom of the wage distribution. Under certain assumptions, planners who care only about the welfare of the top 50 percent of the wage distribution also choose small, highly progressive tax systems compared to planners who apply other weights.

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<sup>7</sup> This extension is related to Saez and Stantcheva’s (2016) analysis of optimal tax models with generalized welfare weights, which they show can capture a broader range of societal objectives than the utilitarianism that is implicit in standard models.

The intuition behind these results is as follows. Selecting more progressive tax rates is the most effective way to increase the welfare of those in the middle of the wage distribution. In contrast, raising taxes to increase the size of the highly progressive transfer system is the most effective way to increase the welfare of those towards the bottom of the wage distribution. However, both approaches to redistribution result in efficiency losses. To limit the distortive effect of taxes, a planner who chooses a more progressive system reduces the size of the tax system, and a planner who chooses a larger system reduces progressivity.

## II. Model

### a. Optimal Tax Problem

We begin with the optimal tax problem presented in Slemrod et al. (1994). We denote by  $T(y_i)$  the net tax paid by an individual with a pre-tax income of  $y_i$ . Each individual,  $i = 1, \dots, N$ , chooses consumption,  $c_i \geq 0$ , leisure,  $z_i \geq 0$ , and labor supply,  $l_i \geq 0$ , to maximize a constant elasticity of substitution (CES) utility function

$$U_i(c_i, l_i) = \left[ \alpha c_i^{\left(\frac{\epsilon-1}{\epsilon}\right)} + (1-\alpha) z_i^{\left(\frac{\epsilon-1}{\epsilon}\right)} \right]^{\left(\frac{\epsilon}{\epsilon-1}\right)} \quad (1)$$

subject to the time constraint  $z_i + l_i = 1$  and the budget constraint  $c_i = w_i l_i - T(w_i l_i)$ , where  $w_i$  is the individual's exogenous wage (reflecting their ability to earn income), and  $\epsilon$  and  $\alpha$  are preference parameters. We assume individuals are ordered from lowest to highest ability-to-earn; that is,  $w_1 \leq w_2 \leq \dots \leq w_N$ . Net taxes are determined by a two-bracket system with a lump-sum transfer as follows:

$$T(w_i l_i) = \begin{cases} -g + t_1 w_i l_i & \text{if } w_i l_i \leq \bar{y} \\ -g + t_1 \bar{y} + t_2 (w_i l_i - \bar{y}) & \text{if } w_i l_i > \bar{y} \end{cases} \quad (2)$$

In this equation,  $t_1$  is the marginal tax rate that applies to income in the lower bracket,  $t_2$  is the marginal tax rate that applies to income in the upper bracket,  $\bar{y}$  is the threshold between the brackets, and  $g$  is the lump sum transfer. We denote the optimal values of consumption and labor supply for individual  $i$



by  $c_i^*(t_1, t_2, \bar{y}, g)$  and  $l_i^*(t_1, t_2, \bar{y}, g)$  and the maximized utility function as  $V(t_1, t_2, \bar{y}, g) \equiv U(c_i^*(t_1, t_2, \bar{y}, g), l_i^*(t_1, t_2, \bar{y}, g))$ .

The planner chooses  $t_1 \in [0,1]$ ,  $t_2 \in [0,1]$ ,  $\bar{y} \in [0, w_N]$ , and  $g \geq 0$  to maximize a social welfare function

$$\sum_{i=1}^N \theta_i \left( \frac{1}{v} \right) U_i[c_i^*(t_1, t_2, \bar{y}, g), l_i^*(t_1, t_2, \bar{y}, g)]^v \quad (3)$$

subject to the government budget constraint  $\sum_{i=1}^N T_i[w_i l_i^*(t_1, t_2, \bar{y}, g)] = 0$ . In this equation,  $v$  is a parameter that captures the planner's aversion to inequality, and  $\theta_i$  is the explicit welfare weight that the planner places on individual  $i$ 's utility, potentially reflecting a political preference for individuals in a particular part of the wage distribution. Because before-tax income is weakly increasing in wage for all tax systems we consider, the  $\theta_i$ 's can be interpreted as welfare weights that vary based on an individual's rank in both the wage and before-tax income distributions. The nonnegativity constraints on the tax rates and the transfer payment ensure that after-tax income (consumption) is always nonnegative. Later, in Section III, we discuss some of the implications of relaxing these nonnegativity constraints.

#### *b. Calibration*

Following Slemrod et al. (1994), we consider 1,000 individuals such that the  $i$ -th individual's wage corresponds to a cumulative probability of  $0.0005 + .0001(i - 1)$  in the wage distribution. That is, the first individual's wage corresponds to a cumulative probability of 0.0005, the second individual's wage corresponds to a cumulative probability of 0.1500, and so on. The 1,000<sup>th</sup> individual's wage corresponds to a cumulative probability of 0.9995. We first consider Stern's (1976) and Slemrod et al.'s (1994) lognormal distribution of wages with  $\mu = -1$  and  $\sigma = 0.39$ , representing the mean and standard deviation, respectively, of log wages. The mean of the (continuous) wage distribution is  $\exp\left(\mu + \frac{\sigma^2}{2}\right) = 0.3969$ . Following Apps et al. (2014), we also consider a Pareto distribution for wages.

We set the shape parameter for this distribution to 1.6 and calibrate the scale parameter so that it has the same mean wage as the lognormal distribution. The Gini coefficient for the lognormal wage distribution (which is the same as the Gini coefficient for the before-tax income distribution under the assumption that all individuals supply the same amount of labor) is 0.22. The Gini coefficient for the Pareto wage distribution is 0.44.

In the individual utility function (1), we consider elasticities of substitution,  $\epsilon = 1.4, 1.2, 1.0, 0.4$ , and 0.2. In each case, we choose  $\alpha$  such that the individual with the mean wage has a labor supply of 0.67 in the absence of taxes. Table 1 shows the values of  $\alpha$  for each of the five values of  $\epsilon$ . The values of 1.0, 0.4, and 0.2 correspond to the cases considered in Slemrod et al. (1994). However, as Apps et al. (2014) note, these values do not generate realistic labor supply elasticities. We therefore also consider  $\epsilon = 1.4$  and 1.2, which result in more realistic labor supply elasticities. For each value of  $\epsilon$ , Table 1 also shows the uncompensated labor supply elasticity for the mean-wage individual in the absence of taxes. Labor supply elasticities are negative for  $\epsilon = 0.2$  and 0.4. For other values of  $\epsilon$ , elasticities are either zero or small and positive, which is more in line with empirical estimates (see McClelland and Mok 2012).

In the social welfare function (3), we consider  $\nu = 1$  and  $\nu = -2$ ; the latter case corresponds to greater inequality aversion. As is common in optimal tax models, both Slemrod et al. (1994) and Apps et al. (2014) assume that  $\theta_i = 1$  for all  $i$ . In addition to this standard assumption, we consider several alternative sets of weights. First, we consider cases where  $\theta_i = 1$  for  $i \leq k$  and  $\theta_i = 0$  for  $i > k$ . That is, the planner cares only about the bottom  $k$  individuals. We set  $k = 990, 500$ , and 100. When  $k = 990$ , the planner cares only about the welfare of the bottom 99 percent of the wage distribution. When  $k = 500$ , the planner cares only about the bottom 50 percent of the wage distribution. When  $k = 100$ , the planner cares only about the bottom 10 percent of the wage distribution. Second, we consider cases in which the planner sets  $\theta_i = 1$  for  $(1000 - k) \geq i \geq k$ , and 0 for all other  $i$ . We choose  $k = 100, 200$ ,

and 300, corresponding to planners who care about individuals in the middle 80 percent, 60 percent, and 40 percent of the wage distribution. Finally, we consider cases in which the planner sets  $\theta_i = 1$  for  $i \geq k$  and  $\theta_i = 0$  for  $i < k$ . That is, the planner only cares about higher wage earners. We choose  $k = 10, 100$ , and 500, corresponding to planners who care only about the top 1 percent, 10 percent, and 50 percent, respectively.

When  $\theta_i$  varies across individuals, the planner's preferences reflect two redistributive goals. First, due to concavity of the utility function, the planner wants to redistribute income downward within the favored group. Second, the planner wants to redistribute income away from any non-favored groups. These goals push in the same direction for a planner who favors lower or middle-and-lower income individuals. Such a planner wants to redistribute income downwards within the favored group and downwards from higher-income individuals to the favored group. A planner who favors middle-income individuals wants to redistribute income downwards within this group and downwards from higher-income to middle-income individuals; however, the planner also wants to redistribute income upwards from lower-income individuals. A planner who favors high-income individuals wants to redistribute income downwards within this group but upwards from the non-favored groups.

To solve the model, we perform an adaptive grid search over the tax system parameters  $t_1$ ,  $t_2$ , and  $\bar{y}$ . For each value of these tax parameters that we consider in the search, we solve numerically for the value of  $g$  that balances the government budget under the assumption that individuals maximize utility (considering both interior and corner solutions for labor supply), given the tax parameters.

### *c. Progressivity, Redistribution, and Efficiency Measures*

Tax progressivity in the model depends on the two marginal rates,  $t_1$  and  $t_2$ , as well as the share of individuals who are subject to each rate (which is determined by  $\bar{y}$ ). To summarize the overall

progressivity of the tax system, we use the Kakwani (1976) index.<sup>8</sup> Let  $y_i$  denote the pre-tax income of individual  $i$ , and let  $T_i$  denote the tax burden of individual  $i$ . Individuals are assumed to be ranked from lowest to highest by income. Define  $\mu_i \equiv \frac{y_i}{\sum_{j=1}^N y_j}$  and  $\lambda_i \equiv \frac{T_i}{\sum_{j=1}^N T_j}$ . Thus,  $\lambda_i$  represents the share of society's total taxes paid by individual  $i$ , and  $\mu_i$  represents the share of society's total income earned by individual  $i$ . The Kakwani index is defined as follows:

$$2 \sum_{k=1}^N \frac{1}{N} \left( \frac{k}{N} - \sum_{i=1}^k \lambda_i \right) - 2 \sum_{k=1}^N \frac{1}{N} \left( \frac{k}{N} - \sum_{i=1}^k \mu_i \right) \quad (4)$$

The second term in this equation is the Gini coefficient for pre-tax income, with each term in parenthesis representing the deviation of the income share of the bottom  $k$  individuals from the share those individuals would earn if income were distributed perfectly equally ( $\frac{k}{N}$ ). The first term in the equation is the concentration coefficient for taxes, with each term in parenthesis representing the deviation of the tax share of the individuals with the  $k$  lowest incomes from the share those individuals would pay if taxes were distributed perfectly equally. It is analogous to a Gini coefficient for taxes, except that individuals are ranked by pre-tax income rather than taxes paid.

Intuitively, the Kakwani index measures the degree to which tax payments are concentrated at the top of the income distribution relative to the degree to which income is concentrated at the top of the income distribution. A higher Kakwani index implies greater tax progressivity. A proportional tax system – in which each person's tax share is the same as his or her income share – would result in a Kakwani index of zero. A progressive tax system – in which tax payments are more concentrated at the top than incomes – would have a positive Kakwani index. A regressive tax system – in which tax payments are less concentrated at the top than incomes – would have a negative Kakwani index. One

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<sup>8</sup> See Slavov and Viard (2016a) for additional discussion, examples, and references related to the concepts in this section.

property of the Kakwani index is that scaling up any given tax system by increasing tax burdens across-the-board leaves individuals' tax shares, and therefore the index, unchanged.

We summarize the degree of redistribution in the tax, transfer, or overall fiscal system with the Reynolds-Smolensky index (Reynolds and Smolensky 1977). The Reynolds-Smolensky index is defined as the difference between the Gini coefficient for before-tax income and the Gini coefficient for (as appropriate) after-tax, after-transfer, or after-tax-and-transfer income.

Define the size of government (or the tax system) as follows:

$$S = \left( \frac{\sum_{i=1}^N T_i}{\sum_{i=1}^N y_i - \sum_{i=1}^N T_i} \right) \quad (5)$$

That is,  $S$  represents total taxes as a share of total after-tax income. Kakwani (1984) shows that the Kakwani index ( $K$ ) and Reynolds-Smolensky index ( $R$ ) for a tax system are related as follows:

$$R = S \cdot K \quad (6)$$

Thus, the amount of redistribution induced by a tax system (the Reynolds-Smolensky index) is equal to the product of its progressivity (the Kakwani index) and its size.<sup>9</sup> According to this equation, even though scaling up a progressive tax system does not increase the Kakwani index (i.e., it leaves progressivity unchanged), it does induce more income redistribution (i.e., it results in a larger reduction in the Gini coefficient). Thus, all else equal, larger tax systems – if they are even slightly progressive – induce more redistribution than smaller tax systems. For example, if the entire tax burden was placed on the person with the highest income, the system would be extremely progressive. However, if only one dollar of tax revenue was raised, there would be almost no reduction in inequality. Intuitively, the progressivity of

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<sup>9</sup> See also Creedy (1999) and Creedy, Enright, Gemmell, and McNabb (2008) for further discussion. This formulation assumes that the tax system does not result in a reranking of individuals, whereby one individual may have higher before-tax income but lower after-tax income than another individual. Reranking does not occur in the two-bracket system considered here. If reranking occurs, a reranking term (Atkinson 1980; Plotnick 1981) must be subtracted from the right-hand-side of the equation.

the tax system measures the amount of redistribution achieved per dollar of tax collected. Multiplying by the size of the tax system then yields the total amount of redistribution.

The relationship between progressivity and size is depicted in Figure 1 for progressive tax systems (i.e.,  $K > 0$ ). Because the level of redistribution is determined jointly by progressivity and size, each point in the  $K - S$  plane lies on an iso-redistribution curve, and the level of redistribution increases as we move further away from the origin. The arrow shows that planners may choose a larger tax system to achieve a particular distributional objective and move up to a higher iso-redistribution curve. As they do this, they may need to mitigate the distortive effect by lowering the progressivity of the tax system.

Equation (6) and Figure 1, which link a tax system's Kakwani and Reynolds-Smolensky indices, do not consider any transfer payments that are funded by the taxes. All else equal, a larger tax system allows for larger transfer payments, which can further reduce income inequality. Given that transfers tend to be highly progressive, even regressive taxes (such as the value added taxes that are common outside the U.S.) can result in more redistribution than progressive taxes if they are large enough.

As a proxy for the efficiency loss from the redistribution, we calculate the reduction in GDP for each tax system relative to its value in the no-tax economy. While GDP loss is not the standard excess burden measure used by economists, policy makers often care about the impact of taxes on economic growth. To provide a more standard measure of excess burden, for each individual,  $i$ , we also calculate the compensating variation ( $CV_i$ ) of a tax system with parameters  $(t_1, t_2, \bar{y}, g)$  by solving

$$V(t_1, t_2, \bar{y}, g + CV_i) = V(0, 0, \bar{y}, -T_i^*), \quad (7)$$

where,  $T_i^* \equiv T(w_i l_i^*(t_1, t_2, \bar{y}, g))$ . Intuitively,  $CV_i$  is the additional lump-sum transfer that must be provided to equate utility under the tax system represented by  $(t_1, t_2, \bar{y}, g)$  with utility in a system in which the same amount of net tax is collected as a lump sum. We represent the aggregate efficiency loss by summing the compensating variation amounts across individuals and dividing it by GDP.

We calculate the measures discussed in this section for each optimal tax solution in our model. We note that these measures are not the only way (and not necessarily even the best way) to capture the underlying concepts of progressivity, government size, redistribution, and economic efficiency. However, they are representative of measures that policy analysts use to evaluate tax systems and make cross country comparisons (e.g., OECD 2008). Thus, they allow us to connect theory and practice.

### III. Results

In this section, we present and discuss detailed results for the Pareto income distribution with  $\epsilon = 1.2$ . Results for the lognormal distribution and other values of  $\epsilon$  are summarized only briefly. Focusing primarily on one set of assumptions aids exposition, and results for other sets of assumptions generally go in the same direction. We also focus on GDP loss as the main inefficiency measure, noting that compensating variation leads to the same conclusions. Full results for all parameter values and income distribution assumptions, as well as compensating variation amounts, are provided in the appendix.

Table 2 presents our main results for the Pareto income distribution with  $\epsilon = 1.2$ . Each row represents an alternative social welfare objective. The first four columns of results represent the tax parameters – the two rates ( $t_1$  and  $t_2$ ) the lump sum transfer ( $g$ ) as a share of per capita GDP, and the fraction of individuals in the bottom bracket (which is determined by  $\bar{y}$  and realized incomes). The next four columns show each system’s progressivity (Kakwani Index), size, and redistribution (Reynolds-Smolensky indices for taxes alone and for the entire fiscal system). The last column shows one measure of efficiency loss – the percentage reduction in GDP relative to an economy with no taxes.

Rows (1) and (2) of Table 2 show the optimal tax and transfer policies for planners with equal weights. Marginal rates are progressive, in line with Apps et al. (2014).<sup>10</sup> The progressivity of the tax system is confirmed by the positive Kakwani indices and the positive Reynolds-Smolensky indices for the tax system. The more inequality-averse planner (row (2)) chooses a slightly less progressive but significantly larger tax system. Because the tax system is progressive, simply scaling it up increases the amount of redistribution done by the tax system. That is why the Reynolds-Smolensky index for the tax system is substantially larger in the second row than in the first. The planner in row (2) induces further redistribution through a larger transfer payment, funded by higher taxes across the board, and is more willing to sacrifice GDP for distributional objectives.

There is an ongoing debate about the optimal top tax rate (see, e.g., Piketty, Saez, and Stantcheva 2014; Diamond and Saez 2011; Mankiw, Weinzierl, and Ferris 2009). Andrienko et al. (2016) consider this question in the context of a piecewise linear tax system. Given the limitations of the two-bracket model, we do not directly address this question. However, it is worth noting that even in the two-bracket model, a higher top tax rate does not map directly into greater progressivity. The top tax rate depends on both the size and the progressivity of the tax system. Progressivity depends upon the relative levels of the two tax rates, as well as the dividing line between the two brackets. For example, the top tax rate is higher in row (2) of Table 2 than in row (1). However, the bottom tax rate increases by a greater proportion in row (2) compared to row (1), and fewer people are in the bottom bracket, causing the tax system in row (2) to be less progressive.

Rows (3), (4), and (5) of Table 2 consider planners who set  $\theta_i = 1$  for only the bottom 99 percent (row (3)), 50 percent (row (4)), and 10 percent (row (5)) of the income distribution. While these

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<sup>10</sup> Our results are not the same as Apps et al. (2014) because our utility function and wage distribution parameters are chosen to make the Pareto case comparable to the lognormal case of Slemrod et al. (1994). Apps et al. (2014) assume quasilinear utility and calibrate their wage distribution to match recent survey data from Australia. However, our results are qualitatively similar to theirs.



planners always choose progressive taxes, progressivity decreases – and the size of government, overall redistribution, and GDP loss increase – as the planner’s weights shift from the bottom 99 percent to the bottom 10 percent. Rows (6), (7), and (8) show results for planners who set  $\theta_i = 1$  for only the middle 80 percent (row (6)), middle 60 percent (row (7)), and middle 40 percent (row (8)). These “middle class” planners choose more progressive taxes than planners who apply equal weights (rows (1) and (2)) and planners who favor lower or middle-and-lower income individuals (rows (3)-(5)). They choose less overall redistribution than planners with high inequality aversion (row (2)) and planners who favor lower income individuals (rows (4) and (5)). However, they choose more redistribution overall than planners with low inequality aversion (row (1)) and planners who favor the bottom 99 percent (row (3)).

Panels (9), (10), and (11) show results for planners favoring the top 1 percent (row (9)), 10 percent (row (10)), and 50 percent (row (11)) of wage earners. Planners favoring the top 1 percent or 10 percent choose zero tax rates. A planner favoring the top 50 percent of wage earners selects a very small, progressive tax system. Thus, the Pareto distribution has enough inequality at the high end to make some degree of redistribution appealing even when societal preferences favor relatively high earners. However, compared to the results for other types of societal preferences, that redistribution takes place primarily through progressive taxes rather than government size, as one would expect given that the desired redistribution is within the favored group of high earners.

The nonnegativity constraints on the two tax rates and the lump sum transfer become important for a planner favoring high-income individuals. As discussed earlier, such a planner’s preferences reflect two redistributive goals. First, due to concavity of the utility function, the planner wants to redistribute income downward within the favored group. Second, the planner wants to redistribute income upwards from the non-favored groups. Positive tax rates achieve the former goal, while negative tax rates achieve the latter. The direction of redistribution and the sign of the unconstrained tax rates are therefore ambiguous. Allowing for negative tax rates complicates the

solution to the model. As negative tax rates represent a wage subsidy, which is financed by lump-sum taxes (negative  $g$ ), additional constraints are needed to rule out negative consumption. However, our exploration suggests that for many parameter values, unconstrained tax rates for planners favoring high-income individuals are negative.

Under a lognormal income distribution, planners with equal weights favor regressive taxes, in line with Slemrod et al. (1994). When  $\epsilon = 1.0, 0.4$ , and  $0.2$  – the values considered by Slemrod et al. (1994) – our results replicate theirs, with slight differences that are likely due to differences in computational methods. Just as in the Pareto case, when societal preferences exhibit greater inequality aversion, the regressivity of the system remains similar. However, the planner increases the size of government and uses the revenue to fund larger transfers, resulting in more overall redistribution. Planners who favor the bottom 99 percent or the “middle class” choose progressive taxes. Planners who favor the bottom 50 percent or bottom 10 percent choose regressive taxes (and regressivity is greater in the latter case). However, as in the Pareto case, these tax systems are substantially larger, resulting in greater overall redistribution and larger reductions in GDP, particularly for the planner who favors the bottom 10 percent. In the lognormal case, planners who favor high earners always choose zero tax rates.

#### **IV. Discussion and Conclusion**

In this paper, we have presented computational solutions to an optimal tax model with two brackets and lump sum redistribution. This model captures important features of real-world fiscal systems, including piecewise linear tax schedules, a highly progressive transfer system, and a separation of the tax system from the transfer system. We have examined how different social welfare functions can give rise to differing degrees of tax progressivity, government size, and overall income redistribution. Our results suggest that smaller, more progressive tax systems are consistent with less

societal aversion to inequality and more emphasis on the well-being of either the middle class, the bottom 99 percent, or the top 50 percent. Larger, less progressive tax systems – which also carry out more overall redistribution – are consistent with more societal aversion to inequality and more emphasis on poverty reduction (the well-being of the bottom 10 percent). These differences arise because scaling up the transfer system (while limiting efficiency losses by making taxes less progressive) is the most effective way to direct resources towards the lowest-income individuals. In contrast, making the tax system more progressive (while limiting efficiency losses by keeping taxes low) is the most effective way to direct resources towards middle-income individuals. These results could help to explain the varying levels of tax progressivity and redistribution observed across countries.

Is there any evidence that societal preferences across countries are in line with those suggested by our model? Compared to European countries, the United States does seem to have societal preferences that are more conducive to progressive but small tax systems. According to the Pew Research Center (Table 3), when asked what issues they feel pessimistic about, Americans express less concern than Europeans about income inequality and the availability of well-paying jobs relative to the way their political system works. While not definitive, concern about income inequality can be interpreted as a greater societal aversion to inequality, while concern about the availability of well-paying jobs can be interpreted as greater weight on the welfare of the bottom 10 percent of the wage distribution. Further empirical work on cross-country differences in attitudes towards income redistribution could provide additional insight into this question.

It is also worth noting that our findings are related to an argument made by Slavov and Viard (2016b), who analyze cross-country measures of progressivity and redistribution based on data in OECD (2008). They suggest that the U.S. fiscal system may be the result of Democrats' emphasis on high-end inequality (e.g., the bottom 99 percent versus the top 1 percent) and high-income individuals paying their "fair share" of taxes, combined with Republicans' focus on low taxes (versus more efficient but

higher taxes). A compromise between those two positions may have resulted in a small, progressive tax system. Consistent with that argument, we have formally shown that an emphasis on high-end inequality (the welfare of the bottom 99 percent) does predict a more progressive but smaller tax system than an emphasis on low-end inequality (the welfare of the bottom 10 percent). Favoring the top 50 percent of the wage distribution could also result in a small, progressive tax system. However, we have not modeled the political compromises that give rise to any particular set of social preferences. In follow-up research, we will explore this question by adding a political economy dimension to the model.

## References

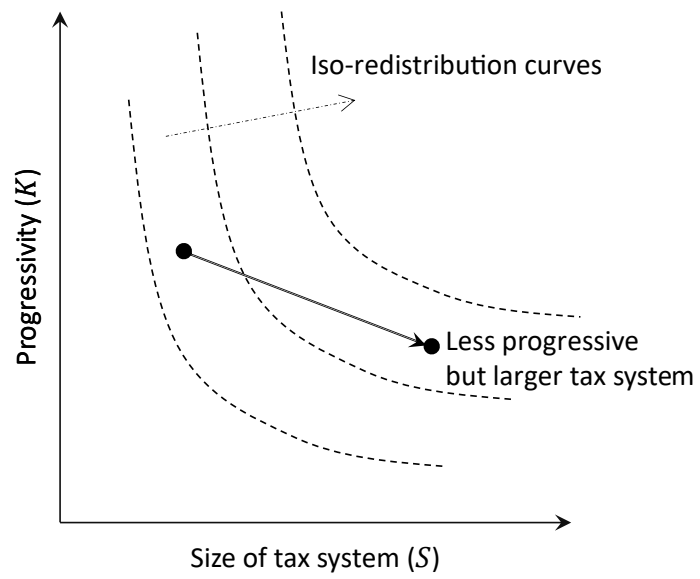
- Ahmad, Ehtisham and Nicholas Stern. The Theory of Reform and Indian Indirect Taxes. *Journal of Public Economics* 25: 259-298,
- Andrienko, Yuri, Patricia Apps, and Ray Rees. Optimal Taxation and Top Incomes. 2016. *International Tax and Public Finance* 23: 981-1003.
- Apps, Patricia, Ngo Van Long, and Ray Rees. 2014. Optimal Piecewise Linear Income Taxation. *Journal of Public Economic Theory* 16(4): 523-545.
- Apps, Patricia and Ray Rees. 2016. Optimal Taxation and the Household. CESifo Working Paper No. 5845 (April).
- Atkinson, Anthony B. 1980. Horizontal Equity and the Distribution of the Tax Burden. In *The Economics of Taxation* (ed. Henry Aaron and Michael Boskin). Brookings Institution Press.
- Bastani, Spencer, Sören Blomquist, and Luca Micheletto. 2019. Nonlinear and Piecewise Linear Income Taxation and the Subsidization of Work-Related Goods. *International Tax and Public Finance* 26: 806-834.

- Bourguignon, Francois and Amedeo Spadaro. 2012. Tax-benefit Revealed Social Preferences. *Journal of Economic Inequality* 10(1): 75-108.
- Christiansen, Vidar and Eilev S. Jansen. 1978. Implicit Social Preferences in the Norwegian System of Indirect Taxation. *Journal of Public Economics* 10(2): 217-245.
- Congressional Budget Office. 2013. The Distribution of Federal Spending and Taxes in 2006. November 7.
- Congressional Research Service. 2010. Distribution of Small Business Ownership and Income by Individual Tax Rates and Selected Policy Issues. February 26.
- Creedy, John. 1999. Taxation, Redistribution, and Progressivity: An Introduction. *Australian Economic Review* 32(4): 410-422.
- Creedy, John, Jamas Enright, Norman Gemmell, and Nick McNabb. 2008. Equity and Efficiency Measures of Tax-Transfer Systems: Some Evidence for New Zealand. New Zealand Treasury Working Paper No. 08/04. December.
- Devlin, Kat and J.J. Moncus. 2020. Many Around the World Were Pessimistic about Inequality Even Before Pandemic. Pew Research Center. August 6.
- Diamond, Peter and Emmanuel Saez. 2011. The Case for a Progressive Tax: From Basic Research to Policy Recommendations. *Journal of Economic Perspectives* 25(4): 165-190.
- Hsu, Minchung and C.C. Yang. 2013. Optimal Linear and Two-Bracket Income Taxes with Idiosyncratic Earnings Risk. *Journal of Public Economics* 105: 58-71.
- Jacobs, Bas, Egbert L.W. Jongen, and Floris T. Zoutman. 2017. Revealed Social Preferences of Dutch Political Parties. *Journal of Public Economics* 156(2017): 81-100.
- Kakwani, Nanak C. 1976. Measurement of Tax Progressivity: An International Comparison. *Economic Journal* 87 (345): 71-80.

- Kakwani, Nanak C. 1984. On the Measurement of Tax Progressivity and Redistributive Effect of Taxes with Applications to Horizontal and Vertical Equity. *Advances in Econometrics* 3: 149-168.
- Kaplow, Louis. 2007. Optimal Income Transfers. *International Tax and Public Finance* 14: 295-325.
- Lindert, Peter H. 2003. Why the Welfare State Looks Like a Free Lunch. NBER Working Paper No. 9869. July.
- Mankiw, N. Gregory, Matthew Charles Weinzierl, and Danny Ferris Yagan. 2009. Optimal taxation in theory and practice. *Journal of Economic Perspectives* 23(4): 147-174.
- McClelland, Robert and Shannon Mok. 2012. A Review of Recent Research on Labor Supply Elasticities. Congressional Budget Office Working Paper 2012-12. October.
- Organisation for Economic Cooperation and Development. 2008. Growing Unequal? Income Distribution and Poverty in OECD Countries.
- Piketty, Thomas, Emmanuel Saez, and Stefanie Stantcheva. 2014. Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities. *American Economic Journal: Economic Policy* 6(1): 230-271.
- Plotnick, Robert. 1981. A Measure of Horizontal Inequity. *Review of Economics and Statistics* 63(2): 283-288.
- Prante, Gerald and Scott A. Hodge. 2013. The Distribution of Tax and Spending Policies in the United States. Tax Foundation Special Report No. 211. November 8.
- Reynolds, Morgan and Eugene Smolensky. 1977. *Public Expenditures, Taxes, and the Distribution of Income: The United States 1950, 1961, 1970*. Academic Press.
- Saez, Emmanuel. 2001. Using Elasticities to Derive Optimal Income Tax Rates. *Review of Economic Studies* 68(1): 205-229.
- Saez, Emmanuel and Stefanie Stantcheva. 2016. Generalized Social Marginal Welfare Weights for Optimal Tax Theory. *American Economic Review* 106(1): 24-45.

- Sheshinski, Eytan. 1989. Note on the Shape of the Optimum Income Tax Schedule. *Journal of Public Economics* 40(2): 201-215.
- Slavov, Sita and Alan Viard. 2016a. Taxes, Transfers, Progressivity, and Redistribution: Part 1. *Tax Notes* (September 5): 1437-1450.
- Slavov, Sita and Alan Viard. 2016b. Taxes, Transfers, Progressivity, and Redistribution: Part 2. *Tax Notes* (September 26): 1437-1450.
- Slemrod, Joel, Shlomo Yitzhaki, Joram Mayshar, and Michael Lundholm. 1994. The Optimal Two-Bracket Linear Income Tax. *Journal of Public Economics* 53(2): 269-290.
- Stelter, Brian. 2011. Camps Are Cleared, but '99 Percent' Still Occupies the Lexicon. *New York Times*. November 30.
- Stern N. H. 1976. On the Specification of Models of Optimum Income Taxation. *Journal of Public Economics* 6(1-2): 123-162.
- Tax Policy Center. 2020. Historical Income Distribution for All Households. November 13.

**Figure 1: Relationship Between Progressivity and Size of Tax System**





**Table 1: Preference Parameters**

$\varepsilon$	$\alpha$	Labor Supply Elasticity
1.4	0.68	0.13
1.2	0.68	0.07
1	0.67	0.00
0.4	0.59	-0.20
0.2	0.46	-0.26

**Table 2: Progressivity and Redistribution Under Alternative Social Welfare Objectives**

Social Welfare Objective	$t_1$	$t_2$	g / Per Capita GDP	Fraction in Bottom Bracket	Kakwani Index	Size of Gov't	RS Index for		
							Taxes	Fiscal System	GDP Loss / No-Tax GDP
(1) Equal Weights ( $v = 1$ )	0.17	0.51	0.28	0.89	0.18	0.39	0.07	0.19	0.17
(2) Equal Weights ( $v = 2$ )	0.25	0.64	0.43	0.82	0.17	0.75	0.13	0.30	0.29
(3) Bottom 99%	0.22	0.64	0.32	0.94	0.16	0.47	0.07	0.21	0.20
(4) Bottom 50%	0.30	0.65	0.53	0.68	0.13	1.11	0.15	0.39	0.36
(5) Bottom 10%	0.52	0.68	0.60	0.85	0.03	1.51	0.05	0.44	0.40
(6) Middle 80% (10-90th percentile)	0.08	0.67	0.30	0.87	0.36	0.42	0.15	0.25	0.24
(7) Middle 60% (20-80th percentile)	0.00	0.66	0.29	0.83	0.49	0.40	0.20	0.27	0.26
(8) Middle 40% (30th-70th percentile)	0.05	0.66	0.34	0.78	0.39	0.51	0.20	0.29	0.28
(9) Top 1%	0.00	0.00	0.00	0.00	NaN	0.00	0.00	0.00	0.00
(10) Top 10%	0.00	0.00	0.00	0.00	NaN	0.00	0.00	0.00	0.00
(11) Top 50%	0.00	0.27	0.02	1.00	0.53	0.02	0.01	0.02	0.01

Notes: Authors' calculation of optimal tax rates, and associated measures of progressivity and redistribution, under alternative social welfare objectives.

Calculations assume a Pareto wage distribution and elasticity of substitution  $\epsilon = 1.2$ .

$t_1$  = bottom bracket rate,  $t_2$  = top bracket rate, g = lump sum transfer, RS = Reynolds-Smolensky.

**Table 3: Percentage of Survey Respondents Reporting Pessimism on Given Issue**

	<i>Income Inequality</i>	<i>Political System</i>	<i>Availability of Well-Paying Jobs</i>	<i>Education System</i>
U.S.	61%	63%	36%	58%
France	86%	76%	69%	62%
Germany	79%	46%	44%	43%
Greece	82%	75%	80%	64%
Italy	65%	68%	68%	36%
Netherlands	64%	43%	30%	41%
Spain	84%	74%	76%	57%
Sweden	55%	42%	28%	39%
U.K.	76%	71%	57%	52%
Bulgaria	74%	69%	59%	58%
Czech Republic	60%	51%	36%	27%
Hungary	70%	51%	55%	49%
Lithuania	61%	44%	52%	34%
Poland	41%	31%	35%	32%
Slovakia	65%	55%	53%	32%

Source: Devlin and Moncus (2020), <https://www.pewresearch.org/fact-tank/2020/08/06/many-around-the-world-were-pessimistic-about-inequality-even-before-pandemic/>

## Appendix: Full Results

**Table A1: Progressivity and Redistribution with Equal Weights - Lognormal Wage Distribution**

$\epsilon$	$t_1$	$t_2$	$\bar{y}$	$g$ / (GDP Per Capita)	Fraction in Bottom Bracket	Pre-Tax Gini	Kakwani Index	Size of Government	Reynolds-Smolensky Index for Taxes	Smolensky Index for Fiscal System	GDP Loss / No-Tax GDP	CV / No-Tax GDP
<i>Equal Weights, <math>\nu = 1</math></i>												
1.4	0.105	0.000	1.019	0.105	0.999	0.258	0.000	0.118	0.000	0.027	0.053	0.003
1.2	0.115	0.113	0.156	0.114	0.186	0.243	-0.003	0.129	0.000	0.027	0.049	0.003
1	0.130	0.124	0.123	0.127	0.062	0.227	-0.005	0.145	-0.001	0.028	0.045	0.003
0.4	0.234	0.203	0.308	0.232	0.769	0.187	-0.006	0.303	-0.002	0.042	0.031	0.004
0.2	0.372	0.323	0.313	0.370	0.784	0.180	-0.005	0.587	-0.003	0.065	0.022	0.006
<i>Equal Weights, <math>\nu = -2</math></i>												
1.4	0.305	0.001	1.023	0.305	0.999	0.297	0.000	0.439	0.000	0.091	0.181	0.028
1.2	0.331	0.287	0.401	0.330	0.921	0.280	-0.003	0.492	-0.002	0.091	0.167	0.028
1	0.366	0.322	0.290	0.362	0.767	0.263	-0.007	0.567	-0.004	0.093	0.152	0.028
0.4	0.542	0.469	0.296	0.537	0.769	0.214	-0.007	1.158	-0.008	0.111	0.093	0.028
0.2	0.669	0.609	0.331	0.666	0.839	0.203	-0.003	1.994	-0.006	0.133	0.058	0.026

**Table A2: Progressivity and Redistribution with Equal Weights - Pareto Wage Distribution**

$\epsilon$	$t_1$	$t_2$	$\bar{y}$	$g$	Fraction in Bottom Bracket	Pre-Tax Gini	Kakwani Index	Size of Government	Reynolds-Smolensky Index for Taxes	Smolensky Index for Fiscal System	GDP Loss / No-Tax GDP	CV / No-Tax GDP
<i>Equal Weights, <math>\nu = 1</math></i>												
1.4	0.163	0.492	0.310	0.276	0.903	0.550	0.172	0.380	0.065	0.199	0.180	0.030
1.2	0.174	0.508	0.287	0.283	0.894	0.508	0.176	0.394	0.069	0.193	0.167	0.032
1	0.192	0.532	0.268	0.295	0.880	0.466	0.174	0.418	0.073	0.189	0.149	0.033
0.4	0.367	0.728	0.234	0.455	0.820	0.361	0.112	0.834	0.093	0.215	0.067	0.038
0.2	0.607	0.903	0.257	0.677	0.851	0.370	0.060	2.099	0.126	0.291	0.007	0.044
<i>Equal Weights, <math>\nu = -2</math></i>												
1.4	0.232	0.628	0.138	0.424	0.844	0.578	0.170	0.736	0.125	0.317	0.312	0.076
1.2	0.252	0.639	0.138	0.429	0.823	0.532	0.172	0.752	0.129	0.302	0.285	0.076
1	0.283	0.659	0.141	0.444	0.801	0.488	0.164	0.798	0.131	0.289	0.254	0.076
0.4	0.526	0.825	0.172	0.624	0.757	0.388	0.083	1.657	0.138	0.294	0.113	0.072
0.2	0.731	0.934	0.227	0.788	0.816	0.392	0.040	3.711	0.147	0.340	0.022	0.064

**Table A3: Progressivity and Redistribution with Weights on Lower/Middle Income - Lognormal Wage Distribution**

$\epsilon$	$t_1$	$t_2$	$\bar{y}$	$g$ / (GDP Per Capita)	Fraction in Bottom Bracket	Pre-Tax Gini	Kakwani Index	Size of Government	Reynolds- Smolensky Index for Taxes	Smolensky Index for Fiscal System	GDP Loss / No- Tax GDP	CV / No-Tax GDP
<i>Bottom 99%</i>												
1.4	0.119	0.242	0.517	0.120	0.974	0.257	0.008	0.136	0.001	0.032	0.065	0.004
1.2	0.128	0.249	0.500	0.129	0.974	0.241	0.007	0.148	0.001	0.032	0.060	0.004
1	0.140	0.258	0.485	0.142	0.972	0.226	0.006	0.165	0.001	0.033	0.055	0.004
0.4	0.251	0.365	0.438	0.251	0.974	0.186	0.002	0.336	0.001	0.047	0.037	0.005
0.2	0.396	0.523	0.428	0.397	0.973	0.180	0.001	0.657	0.001	0.072	0.027	0.008
<i>Bottom 50%</i>												
1.4	0.419	0.344	0.309	0.412	0.829	0.344	-0.010	0.700	-0.007	0.137	0.260	0.053
1.2	0.443	0.360	0.312	0.436	0.835	0.318	-0.010	0.773	-0.008	0.134	0.238	0.052
1	0.476	0.385	0.313	0.469	0.836	0.293	-0.010	0.882	-0.008	0.133	0.216	0.052
0.4	0.682	0.569	0.316	0.675	0.834	0.235	-0.007	2.079	-0.014	0.154	0.140	0.055
0.2	0.821	0.736	0.327	0.816	0.838	0.223	-0.004	4.443	-0.017	0.179	0.094	0.053
<i>Bottom 10%</i>												
1.4	0.632	0.431	0.119	0.527	0.396	0.459	-0.031	1.113	-0.035	0.225	0.351	0.090
1.2	0.669	0.461	0.121	0.564	0.371	0.429	-0.032	1.292	-0.041	0.224	0.330	0.092
1	0.700	0.494	0.131	0.605	0.368	0.393	-0.032	1.533	-0.049	0.218	0.301	0.092
0.4	0.836	0.679	0.183	0.792	0.430	0.277	-0.021	3.799	-0.080	0.203	0.182	0.084
0.2	0.919	0.816	0.199	0.893	0.437	0.246	-0.012	8.369	-0.101	0.209	0.117	0.073

**Table A4: Progressivity and Redistribution with Weights on Lower/Middle Income - Pareto Wage Distribution**

$\epsilon$	$t_1$	$t_2$	$\bar{y}$	$g$	Fraction in Bottom Bracket	Pre-Tax Gini	Kakwani Index	Size of Government	Reynolds- Smolensky Index for Taxes	Smolensky Index for Fiscal System	GDP Loss / No- Tax GDP	CV / No-Tax GDP
<i>Bottom 99%</i>												
1.4	0.206	0.637	0.466	0.316	0.950	0.555	0.150	0.462	0.069	0.223	0.217	0.045
1.2	0.215	0.636	0.400	0.320	0.941	0.510	0.156	0.471	0.073	0.213	0.197	0.045
1	0.233	0.644	0.352	0.331	0.929	0.466	0.153	0.496	0.076	0.205	0.175	0.045
0.4	0.416	0.793	0.269	0.496	0.882	0.364	0.095	0.982	0.094	0.228	0.076	0.045
0.2	0.649	0.929	0.281	0.710	0.881	0.375	0.051	2.451	0.124	0.302	0.011	0.050
<i>Bottom 50%</i>												
1.4	0.275	0.632	0.050	0.529	0.676	0.665	0.120	1.122	0.135	0.415	0.401	0.107
1.2	0.300	0.649	0.063	0.526	0.681	0.602	0.131	1.108	0.146	0.385	0.364	0.106
1	0.325	0.674	0.074	0.531	0.653	0.542	0.138	1.132	0.156	0.361	0.322	0.106
0.4	0.550	0.844	0.106	0.688	0.533	0.418	0.088	2.207	0.195	0.349	0.153	0.102
0.2	0.917	0.140	6.852	0.916	0.999	0.447	-0.001	10.941	-0.006	0.409	0.055	0.100
<i>Bottom 10%</i>												
1.4	0.543	0.681	0.300	0.605	0.922	0.792	0.019	1.533	0.029	0.491	0.446	0.123
1.2	0.525	0.677	0.163	0.602	0.848	0.689	0.034	1.513	0.052	0.435	0.396	0.118
1	0.558	0.699	0.155	0.625	0.822	0.613	0.035	1.663	0.059	0.405	0.352	0.117
0.4	0.783	0.874	0.234	0.809	0.850	0.454	0.016	4.249	0.069	0.381	0.171	0.115
0.2	0.925	0.209	7.109	0.924	0.999	0.452	-0.001	12.109	-0.006	0.417	0.060	0.106

**Table A5: Progressivity and Redistribution with Weights on Middle Income - Lognormal Wage Distribution**

$\epsilon$	$t_1$	$t_2$	$\bar{y}$	$g / (\text{GDP Per Capita})$	Fraction in Bottom Bracket	Pre-Tax Gini	Kakwani Index	Size of Government	Reynolds-Smolensky Index for Taxes	Smolensky Index for Fiscal System	GDP Loss / No-Tax GDP	CV / No-Tax GDP
<i>Middle 80% (10-90th percentile)</i>												
1.4	0.000	0.282	0.268	0.032	0.736	0.218	0.648	0.033	0.022	0.028	0.075	0.010
1.2	0.000	0.294	0.266	0.033	0.725	0.207	0.653	0.034	0.022	0.028	0.069	0.010
1	0.000	0.311	0.263	0.034	0.711	0.195	0.657	0.035	0.023	0.029	0.064	0.010
0.4	0.000	0.453	0.253	0.048	0.645	0.165	0.647	0.051	0.033	0.039	0.044	0.011
0.2	0.000	0.617	0.246	0.074	0.585	0.163	0.623	0.080	0.050	0.058	0.034	0.013
<i>Middle 60% (20-80th percentile)</i>												
1.4	0.000	0.336	0.246	0.044	0.677	0.212	0.642	0.046	0.029	0.037	0.103	0.017
1.2	0.000	0.352	0.246	0.043	0.663	0.200	0.650	0.045	0.029	0.037	0.096	0.016
1	0.000	0.375	0.247	0.044	0.663	0.189	0.656	0.046	0.030	0.037	0.088	0.016
0.4	0.000	0.545	0.247	0.057	0.628	0.161	0.660	0.061	0.040	0.047	0.059	0.018
0.2	0.000	0.707	0.244	0.081	0.603	0.160	0.642	0.088	0.057	0.065	0.043	0.019
<i>Middle 40% (30-70th percentile)</i>												
1.4	0.000	0.359	0.229	0.053	0.646	0.210	0.627	0.056	0.035	0.044	0.123	0.021
1.2	0.000	0.379	0.231	0.052	0.639	0.198	0.637	0.055	0.035	0.044	0.114	0.021
1	0.000	0.405	0.234	0.053	0.637	0.187	0.644	0.056	0.036	0.044	0.104	0.021
0.4	0.000	0.588	0.239	0.066	0.611	0.160	0.653	0.071	0.046	0.054	0.070	0.022
0.2	0.000	0.746	0.239	0.089	0.590	0.160	0.639	0.098	0.063	0.071	0.050	0.023

**Table A6: Progressivity and Redistribution with Weights on Middle Income - Pareto Wage Distribution**

$\epsilon$	$t_1$	$t_2$	$\bar{y}$	$g$	Fraction in Bottom Bracket	Pre-Tax Gini	Kakwani Index	Size of Government	Reynolds- Smolensky Index for Taxes	Smolensky Index for Fiscal System	GDP Loss / No- Tax GDP	CV / No-Tax GDP
<i>Middle 80% (10-90th percentile)</i>												
1.4	0.072	0.654	0.171	0.303	0.881	0.505	0.352	0.435	0.153	0.260	0.259	0.069
1.2	0.079	0.666	0.175	0.295	0.867	0.467	0.364	0.419	0.153	0.245	0.235	0.068
1	0.088	0.687	0.178	0.291	0.858	0.429	0.371	0.411	0.152	0.233	0.207	0.067
0.4	0.300	0.843	0.188	0.452	0.806	0.355	0.197	0.825	0.162	0.249	0.093	0.062
0.2	0.525	0.938	0.192	0.650	0.773	0.370	0.107	1.855	0.199	0.310	0.015	0.060
<i>Middle 60% (20-80th percentile)</i>												
1.4	0.000	0.648	0.120	0.302	0.793	0.503	0.452	0.433	0.196	0.288	0.285	0.079
1.2	0.000	0.663	0.128	0.286	0.832	0.464	0.486	0.402	0.195	0.272	0.258	0.078
1	0.000	0.686	0.136	0.272	0.818	0.426	0.519	0.374	0.194	0.258	0.228	0.077
0.4	0.250	0.849	0.159	0.444	0.766	0.358	0.247	0.799	0.197	0.269	0.104	0.072
0.2	0.450	0.938	0.164	0.619	0.712	0.373	0.147	1.622	0.238	0.321	0.020	0.067
<i>Middle 40% (30-70th percentile)</i>												
1.4	0.000	0.646	0.104	0.323	0.798	0.514	0.434	0.478	0.207	0.307	0.302	0.084
1.2	0.050	0.661	0.112	0.337	0.779	0.484	0.391	0.508	0.199	0.295	0.281	0.083
1	0.050	0.685	0.122	0.322	0.753	0.442	0.419	0.475	0.199	0.277	0.246	0.082
0.4	0.350	0.849	0.146	0.528	0.725	0.373	0.182	1.119	0.203	0.293	0.118	0.080
0.2	0.500	0.939	0.153	0.662	0.686	0.380	0.127	1.958	0.249	0.336	0.025	0.072



**Table A7: Progressivity and Redistribution with Weights on High Income - Lognormal Wage Distribution**

$\epsilon$	$t_1$	$t_2$	$\bar{y}$	g / (GDP Per Capita)	Fraction in Bottom Bracket	Pre-Tax Gini	Kakwani Index	Size of Government	Reynolds- Smolensky Index for Taxes	Smolensky Index for Fiscal System	GDP Loss / No- Tax GDP	CV / No-Tax GDP
<i>Top 1%</i>												
1.4	0.000	0.000	0.006	0.000	0.000	0.245	-0.245	0.000	0.000	0.000	0.000	0.000
1.2	0.012	0.000	0.013	0.001	0.000	0.231	-0.231	0.001	0.000	0.000	0.000	0.000
1	0.032	0.000	0.030	0.004	0.000	0.217	-0.217	0.004	-0.001	0.000	0.000	0.000
0.4	0.000	0.000	0.032	0.000	0.000	0.175	-0.175	0.000	0.000	0.000	0.000	0.000
0.2	0.000	0.000	0.032	0.000	0.000	0.160	-0.160	0.000	0.000	0.000	0.000	0.000
<i>Top 10%</i>												
1.4	0.000	0.000	0.038	0.000	0.000	0.217	-0.217	0.000	0.000	0.000	0.000	0.000
1.2	0.000	0.000	0.030	0.000	0.000	0.175	-0.175	0.000	0.000	0.000	0.000	0.000
1	0.000	0.000	0.046	0.000	0.000	0.160	-0.160	0.000	0.000	0.000	0.000	0.000
0.4	0.000	0.000	0.000	0.000	0.000	0.245	-0.245	0.000	0.000	0.000	0.000	0.000
0.2	0.001	0.000	0.057	0.000	0.000	0.231	-0.231	0.000	0.000	0.000	0.000	0.000
<i>Top 50%</i>												
1.4	0.000	0.000	0.000	0.000	0.000	0.245	-0.245	0.000	0.000	0.000	0.000	0.000
1.2	0.001	0.000	0.057	0.000	0.000	0.231	-0.231	0.000	0.000	0.000	0.000	0.000
1	0.000	0.000	0.046	0.000	0.000	0.217	-0.217	0.000	0.000	0.000	0.000	0.000
0.4	0.000	0.000	0.046	0.000	0.000	0.175	-0.175	0.000	0.000	0.000	0.000	0.000
0.2	0.000	0.000	0.046	0.000	0.000	0.160	-0.160	0.000	0.000	0.000	0.000	0.000

**Table A8: Progressivity and Redistribution with Weights on High Income - Pareto Wage Distribution**

$\epsilon$	$t_1$	$t_2$	$\bar{y}$	$g$	Fraction in Bottom Bracket	Pre-Tax Gini	Kakwani Index	Size of Government	Reynolds- Smolensky Index for Taxes	Smolensky Index for Fiscal System	GDP Loss / No- Tax GDP	CV / No-Tax GDP
<i>Top 1%</i>												
1.4	0.000	0.000	0.000	0.000	0.000	0.500	NaN	0.000	0.000	0.000	0.000	0.000
1.2	0.000	0.000	0.000	0.000	0.000	0.471	NaN	0.000	0.000	0.000	0.000	0.000
1	0.000	0.000	0.000	0.000	0.000	0.436	NaN	0.000	0.000	0.000	0.000	0.000
0.4	0.000	0.000	0.000	0.000	0.000	0.309	NaN	0.000	0.000	0.000	0.000	0.000
0.2	0.000	0.000	0.000	0.000	0.000	0.268	NaN	0.000	0.000	0.000	0.000	0.000
<i>Top 10%</i>												
1.4	0.000	0.000	0.000	0.000	0.000	0.436	NaN	0.000	0.000	0.000	0.000	0.000
1.2	0.000	0.000	0.000	0.000	0.000	0.309	NaN	0.000	0.000	0.000	0.000	0.000
1	0.000	0.000	0.001	0.000	0.000	0.268	-0.268	0.000	0.000	0.000	0.000	0.000
0.4	0.000	0.150	7.565	0.004	0.999	0.501	0.498	0.004	0.002	0.004	0.003	0.000
0.2	0.000	0.266	2.818	0.018	0.995	0.471	0.527	0.018	0.009	0.018	0.012	0.001
<i>Top 50%</i>												
1.4	0.000	0.150	7.565	0.004	0.999	0.501	0.498	0.004	0.002	0.004	0.003	0.000
1.2	0.000	0.266	2.818	0.018	0.995	0.471	0.527	0.018	0.009	0.018	0.012	0.001
1	0.000	0.349	1.719	0.026	0.992	0.434	0.563	0.027	0.015	0.026	0.019	0.003
0.4	0.000	0.548	0.675	0.026	0.982	0.308	0.684	0.027	0.019	0.026	0.010	0.005
0.2	0.000	0.708	0.532	0.031	0.976	0.275	0.715	0.032	0.023	0.031	0.001	0.005