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ABSTRACT

Conditional value at risk (CoVaR) and marginal expected shortfall (MES) have been proposed as stock return based measures of the systemic risk created by individual financial institutions even though the literature provides no formal hypothesis test for detecting systemic risk. We address this shortcoming by constructing hypothesis test statistics for CoVaR and MES that can be used to detect systemic risk at the institution level. We apply our tests to daily stock returns data for over 3500 firms during 2006-2007. CoVaR (MES) tests identify almost 500 (1000) firms as systemically important. Both tests identify many more real-side firms than financial firms, and they often disagree about which firms are systemic. Analysis of the hypothesis tests' performance for plausible alternative hypotheses finds that return skewness can cause test rejections and, even when systemic risk imparts a strong signal in stock return distributions, hypothesis tests based on CoVaR and MES may fail to detect it. Our overall conclusion is that CoVaR and MES are not reliable measures of systemic risk.

Key Words: systemic risk, conditional value at risk, CoVaR, marginal expected shortfall, MES, systemically important financial institutions, SIFIs

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Testing for Systemic Risk using Stock Returns

I. Introduction

A number of economists have proposed using econometric measures of the tail dependence in stock returns as indicators of the "systemic risk" created by large financial institutions.[2] In this paper, we focus on two of these proposed measures: conditional value at risk (CoVaR) and marginal expected shortfall (MES). Proponents have argued that CoVaR and MES should be used as a basis to tax large complex financial institutions to penalize them for the systemic risk that they create [Acharya, Engle and Richardson (2012), Acharya, Pedersen, Philippon and Richardson (2010)] or to indirectly tax these institutions by requiring enhanced regulatory capital and liquidity requirements calibrated using these measures [Adrian and Brunnermeier (2011)].

The argument for using CoVaR and MES to measure systemic risk is simple enough. Should a systemically important financial institution teeter on the brink of default, its elevated risk of failure will negatively impact the lower tail of the stock return distributions of many other firms in the economy. The loss-tail of firms' return distributions will undergo a negative shift if a financial institution's failure will spread losses throughout the financial sector and choke off credit intermediation to the real economy. The implication is that stock returns will have asymptotic left-tail dependence with the stock returns of systemically important financial institutions. Because CoVaR and MES measure the thickness of the loss-tail of stock return distributions under a negative conditioning event, they hold promise as stock return-based measures of the systemic risk that would be created by the failure of individual financial institutions.

The CoVaR measure is the difference between two 1 percent value-at-risk (VaR)[3] measures. The 1 percent VaR of a reference portfolio is calculated using returns conditioned on the event that a single large financial institution experiences a return equal to the 1 percent quantile of its unconditional return distribution. The second step is to subtract the VaR of the same reference portfolio conditioned on the event that the large financial institution in question experiences a

[2] These papers include: Acharya, Engle and Richardson (2012), Acharya, Pedersen, Philippon and Richardson (2010), Adrian and Brunnermeier (2011) and Brownlees and Engle (2012). While these studies define firm-level systemic risk measures, Allen, Bali, and Tang (2012) introduce an aggregate systemic risk measure for the financial sector. See Flood, Lo and Valavanis (2012) for a recent survey of this literature. Kupiec (2012) or Benoit, Colletaz, Hurlin and Perignon (2012) provide a critical assessment.

[3] In this literature, a 1 percent VaR measure is defined as the 1 percent quantile of the underlying return distribution.

median return realization. The literature suggest that CoVaR be estimated using quantile regression on the grounds that such estimates are non-parametric and free from biases that may be introduced by inappropriately restrictive parametric distributional assumptions.

MES is the expected shortfall calculated from a conditional return distribution for an individual financial institution where the conditioning event is a large negative market return realization.[4] The literature suggests using a nonparametric MES estimator: the institution's average stock return on the selected sample of days when the market portfolio experiences a realization in the five percent lower tail of its sample return distribution. This measure requires no maintained hypothesis about the probability density that generates observed stock return data.[5]

The existing literature argues that should a financial firm have a CoVaR or MES estimate that is large relative to other financial firm estimates, the financial institution with the large CoVaR or MES estimate is a source of systemic risk. The "validity" of these systemic risk measures is established by showing that virtually all of the large financial institutions that required government assistance or failed during the recent financial crisis exhibited large CoVaR or MES measures immediately prior to the crisis.

An important shortcoming in the CoVaR and MES literatures is that systemic risk is identified on the basis of *ad hoc* reasoning without the use of formal statistical hypothesis tests. This issue has important policy implications. Beginning in 2010, the Dodd-Frank Wall Street Reform and Consumer Protection Act requires regulators to assess the significance of the systemic risk that would be created by the failure of large non-bank financial institutions and decide which institutions, if any, should be designated a "systemically important financial institution" and thereby be subjected to enhanced supervision and heighted prudential standards.

Our objective in this paper is to improve the technology for identifying systemic risk and move it beyond simple rank ordering of individual financial institution CoVaR and MES estimates. To do this, we develop classical hypothesis test statistics that can detect systemic risk. Our approach

[4] Two additional measures of systemic risk proposed in the literature, Systemic Expected Shortfall (SES) [Acharya, Pedersen, Phillipon, and Richardson (2010)] and the Systemic Risk Index (SRISK) [Acharya, Engle, and Richardson (2012)] are simple transformations of MES.

[5] The results and conclusions in this paper are about the nonparametric versions of MES and CoVaR statistics which uses stock returns and the equally weighted market return as the reference portfolio. Girardi and Ergun (2013) and Jiang (2012) discuss alternative parametric estimation approaches. Mainik and Schaanning (2014) identify inconsistencies in CoVaR and MES statistics' ability to rank order firms by systemic risk.

requires us to adopt a joint null hypothesis. We can only test for the presence of systemic risk by assuming a specific null hypothesis for the distribution of stock returns that precludes systemic risk.[6] The null hypothesis we adopt is that stock returns are Gaussian, a distribution with asymptotically independent tails, but there are many other possible return distributions that could have been selected as the null hypothesis. Using the Gaussian null hypothesis, we use the Durbin (1954)-Wu (1973)-Hausman (1978) technique of exploiting the difference between two consistent estimators to develop a hypothesis test statistic that can detect asymptotic tail dependence in stock returns data. We use Monte Carlo simulations to construct the critical values of the small sample distributions of our CoVaR and MES hypothesis test statistics.

We apply out test statistics to daily returns data on 3518 firms and the CRSP equally-weighted market portfolio over the sample period 2006-2007. The CoVaR test identifies roughly 500 firms as sources of systemic risk; the MES test identifies nearly 1000 firms. Both tests identify many more real-side than financial firms, and the tests often disagree about which firms are a source of systemic risk.

To better understand our hypothesis test results, we perform simulations to evaluate the performance of our hypothesis tests for nested alternative return distributions, some with asymptotic tail dependence, and others that are asymptotically tail independent. This analysis shows that our hypothesis tests may reject the null hypothesis when stock return distributions have skewness patterns that are commonly observed in the data, even when the return distributions are asymptotically tail independent.[7] While the test statistics are performing as they should (they are detecting non-Gaussian returns), the test rejection could be interpreted (falsely) as evidence of asymptotic tail dependence in returns. While this finding points to the value of additional research to refine the choice of the test statistic null hypothesis, our overall view is that, regardless of further generalizations, CoVaR and MES based tests are likely to have weak power characteristics.

[6] Many problems in finance face a similar joint hypothesis problem. See for example, the discussion in Fama (1970) or Jarrow and Larsson (2012).

[7] A distribution's skewness can cause the hypothesis tests to under- or over-reject, depending on the pattern of skewness in the data. When the market return distribution is strongly negatively skewed, and individual returns are weakly positively skewed, the tests over-reject the null hypothesis. This pattern is particularly common in our sample of stock returns.

The analysis of several plausible nested alternative stock return generating processes suggests that stock returns with weak asymptotic tail dependence will produce CoVaR and MES hypothesis test statistic distributions that significantly overlap the sampling distributions of test statistics calculated from Gaussian returns. Moreover, even when asymptotic tail dependence is strong, our analysis suggest that the CoVaR and MES test statistics have relatively weak power.

The power issue arises because the variation in the nonparametric CoVaR and MES estimators increases as the asymptotic tail dependence in stock returns strengthens. While the nonparametric estimators are consistent under asymptotic tail dependence (systemic risk), there is no guarantee that these estimators will have small standard errors in a sample size of 500 observations. This increase in variability of the nonparametric estimators increases the likelihood rejecting a false null hypothesis, which limits the power of the test statistics.

The remainder of the paper is organized as follows. Section II provides a brief overview of the existing literature. Section III estimates CoVaR and MES statistics for a wide cross section of stocks to illustrate the pitfalls of identifying systemic risk from ranking the magnitude of CoVaR and MES sample estimates. Section IV develops hypothesis test statistics that can identify systemic risk from stock returns using CoVaR and MES sample estimates under the null hypothesis that returns are Gaussian. Section V develops a simplified formulation of the hypothesis test statistics that reduce computational burden. Section VI applies our hypothesis tests to a sample of 3518 individual daily stock returns over the pre-crisis sample period 2006-2007. Section VII investigates the properties of our hypothesis test statistics under nested alternative return distribution assumptions that both include and exclude asymptotic tail dependence. Section VIII provides our summary and conclusions.

II. Literature Review

1. Conditional Value at Risk (CoVaR)

Let $\tilde{R}_P$ represent the return on any reference portfolio of stocks, $\tilde{R}_j$ represent the return on an individual stock, $\tilde{R}_P|R_j = r$ be the return on the reference portfolio conditional on a specific realized value for $\tilde{R}_j$, $\tilde{R}_j = r$, and $VaR(\tilde{R}_j,\text{p})$ represent the p-percent value at risk for stock j, or the critical value of the return distribution for $\tilde{R}_j$ below which, there is at most p-percent probability of a smaller return realization. The CoVaR measure can be defined for any reference

portfolio, $\tilde{R}_P$. For purposes of the discussion in this paper, we define the reference portfolio to be the equally-weighted CRSP market portfolio, $\tilde{R}_P = \tilde{R}_M$.

Adrian and Brunnermeier (2012) define the q-percent CoVaR measure for firm j to be the q-percent quantile of a reference portfolio's conditional return distribution, where the reference portfolio's distribution is conditioned on a return realization for stock j equal to its q-percent VaR value. Using our notation, firm j's q-percent CoVaR is defined as:

$$CoVaR\left(\tilde{R}_{M|j}, q\ \right) = VaR[\tilde{R}_M | R_j = VaR\left(\tilde{R}_{j,}\mathrm{q}\right), \mathrm{q}] \quad (1)$$

Adrian and Brunnermeier measure systemic risk using ΔCoVaR, which they define as the difference between stock j's q-percent CoVaR and the stock's CoVaR conditional on a median return of stock j,

$$\Delta CoVaR\left(\tilde{R}_{M|j}, q\right) = CoVaR\left(\tilde{R}_{M|j}, q\right) - CoVaR\left(\tilde{R}_{M|j}, 50\%\right) \quad (2)$$

Adrian and Brunnermeier (2011) set "q" equal to 1-percent and estimate ΔCoVaR using quantile regressions. They argue that ΔCoVaR measures how an institution contributes to the systemic risk of the overall financial system.[8]

2. Marginal Expected Shortfall (MES)

Acharya, Pedersen, Phillipon, and Richardson (2010) define MES for firm j is the expected value of the stock return $\tilde{R}_j$ conditional on the market portfolio return $\tilde{R}_M$ being at or below the sample q-percent quantile.

$$MES(\tilde{R}_j, q) = E\left(\tilde{R}_j | R_M < VaR(\tilde{R}_M, q)\right) \quad (3)$$

Higher levels of MES imply that firm j is more likely to be undercapitalized in the bad states of the economy and thus contribute systemic risk to the broader financial system. They use $q = 0.05$ as the left-tail market return threshold and estimate MES by taking a selected-sample

[8] Adrian and Brunnermeier (2011) also estimate a measure they call "forward-ΔCoVaR" by projecting estimates of ΔCoVaR on bank-level characteristics.

average. They argue that that MES calculated over the 2006-2007 period can predict stock returns during the crisis.[9]

III. Do ΔCoVaR and MES Statistics Measure Systemic Risk?

The existing ΔCoVaR and MES literature focuses on the stock returns of large financial instructions just prior to the financial crisis. It argues that ΔCoVaR and MES are measures of systemic risk because there is a high correspondence between large financial institutions with large MES and ΔCoVaR estimates immediately prior to the crisis, and institutions that subsequently required extensive government capital injections or failed.

While this justification can seem convincing in the context of the financial crisis bailouts and a selected sample of financial firms, the argument becomes less convincing when it is viewed in a broader context. ΔCoVaR and MES statistics can be calculated for any firms with stock return data. It is unclear whether the large financial firms identified in these studies actually have large ΔCoVaR or MES estimates relative to the entire population of firms. It is informative to compare the magnitude of ΔCoVaR and MES estimates for firms outside of the financial services industry to firms that specialize in banking and financial services before making any judgment that specific ΔCoVaR or MES estimates are "abnormally" large.

To illustrate the shortcomings of identifying systemically risky firms as the financial firms with the largest ΔCoVaR or MES estimates, we calculate the MES and ΔCoVaR measures for all CRSP stocks with about 500 days of daily return data in the sample period 2006-2007 using the nonparametric methods used in the literature.

To construct our sample, we start with all firms identified in CRSP database between 2006 and 2007 (3.5 million daily returns by 7481 firms). We calculate daily log returns on days where reported closing prices are based on actual transactions. We exclude security issues by non-US issuers and issues other than common stock such as ADRs and REITs. We eliminate firms if the market capitalization is less than $100 million. These filters result in 3518 individual stock return series.

[9] Acharya, Pedersen, Phillipon, and Richardson (2010) define an additional systemic risk measure, Systemic Expected Shortfall (SES), which is a simple transformation of an MES estimate from stock returns data. Acharya, Engle, and Richardson (2012) define a similar measure, but call it SRISK. Brownlees and Engle (2012) and Acharya, Engle and Richardson (2012) use more complex volatility and correlation models to estimate MES from stock returns.

For each firm in the sample, we estimate 1 percent ΔCoVaR statistics using quantile regression. We measure MES for each stock as the average stock returns on days when the equally-weighted market portfolio experiences a return in the 5 percent lower tail of its sampling distribution. Our reference portfolio is the CRSP equally-weighted market portfolio for both measures.[10]

Table 1 describes the sample. It lists the number of firms in the sample by industry and the average ΔCoVaR and MES estimates for each industry. The ΔCoVaR estimates indicate that, on average, the construction industry has the largest (negative) ΔCoVaR estimates. Using the identification approach applied in the prior literature, this would suggest that, on average, the construction industry is the largest source of systemic risk. Following the construction industry, according to the ΔCoVaR measure, on average, broker-dealers and mining firms are the next most important industry sources of systemic risk. According to the MES statistic, on average broker-dealers are the largest contributors to systemic risk, followed by mining companies and then the construction industry.

Table 2 lists the fifty companies that exhibit the largest ΔCoVaR measures in descending order of "systemic risk" importance as indicated by the magnitude of their 1-percent ΔCoVaR statistics. Most of the firms listed in Table 1 are part of the "real-side" of the economy and have nothing to do with the financial services sector. Among the firms listed in Table 2 are 7 financial firms. It is very unlikely that anyone would label any of these 7 firms as "systemically important." The depository institution with the largest ΔCoVaR statistic, Citizens First Bancorp, has less than half the indicated systemic risk of Proquest, the company with the largest ΔCoVaR systemic risk measure among traded firms.

Table 3 lists, in descending order, the fifty companies with the largest (negative) MES systemic risk measures. While the top-fifty MES firms are still dominated by the real sector, the MES statistic does generate a list of "high risk" financial firms that did subsequently flounder during the crisis. CompuCredit, E-trade, Countrywide Financial, IndyMac, BankUnited Financial, Net Bank, Accredited Home Lenders and Fremont General Corporation all experienced serious distress or failed subsequent to the onset of the financial crisis. Still, none of these firms are

[10] We obtain similar estimates of ΔCoVaR and MES statistics when we use the CRSP value-weighted index as the reference portfolio. The correlation between statistics based on equally-weighted and value-weighted references portfolios is 0.85 for ΔCoVaR and 0.93 for MES. When we rank the firms based on the magnitude of ΔCoVaR and MES measured against the CRSP value-weighted portfolio return, similar firms appear in the top 50 lists.

exceptionally large and none was considered to be systemically important or "too-big-to-fail" during the financial crisis. The firm with the largest MES, China Precision Steel, is not a financial firm. It is not particularly important for the U.S. domestic economy as more than 60 percent of its operations are in China.

The results in Table 1 through 3 demonstrate that when ΔCoVaR and MES statistics are calculated for all traded stocks in the years immediately preceding the crisis (2006-2007), the largest financial institutions that are alleged to be the primary sources of systemic risk for the economy do not even appear among the list of firms with the fifty most negative ΔCoVaR or MES estimates. The absence of the largest financial institutions from either list casts serious doubt on previous claims that ΔCoVaR and MES measures accurately identify a firm's potential to cause systemic risk.

A troubling feature of this simple rank-ordering approach for identifying systemic risk is that it lacks of any formal statistical hypothesis test to identify firms that create systemic risk. Our objective in writing this paper is to advance systemic risk measurement beyond the simple rank ordering of ΔCoVaR and MES estimates and introduce formal statistical hypothesis tests that can detect systemic risk in stock returns data.

IV. Testing for Systemic Risk using ΔCoVaR and MES Statistics

1. Systemic Risk Specification Test

We develop a hypothesis test for the presence of systemic risk using a specification testing approach pioneered by Durbin (1954), Wu (1973) and Hausman (1978) (or DWH). The DWH approach requires the existence of two estimators, $\hat{b}_1$ and $\hat{b}_2$ for an unknown value b. The estimators $\hat{b}_1$ and $\hat{b}_2$ are both consistent under the null hypothesis, but $\hat{b}_1$ is designated to be the more efficient estimator. If, under the alternative hypothesis, $\hat{b}_1$ is no longer a consistent estimator for b, but $\hat{b}_2$ remains consistent, then it is possible to construct a test of the null hypothesis using the difference between $\hat{b}_1$ and $\hat{b}_2$. The intuition for the test statistic being that, under the null hypothesis, the difference between $\hat{b}_1$ and $\hat{b}_2$ should approximate zero whereas under the alternative, the difference will be non-zero.

The ΔCoVaR and MES estimators that have been proposed in the literature are both nonparametric and consistent under a wide range of alternative stock return generating distributions. The nonparametric estimation approach was intentionally specified to make the estimates robust to any parametric return distribution assumptions. The cost of this generalization is a reduction in the efficiencies of the ΔCoVaR and MES estimators compared to the efficiencies of estimators that utilize the parametric characterization of the stock returns distribution.

An explicit assumption underlying our testing approach is that asymptotic tail dependence in stock returns is symptomatic of systemic risk. Thus, statistics that can identify asymptotic tail dependence in the left tail of stock returns distribution are statistics that can detect of systemic risk.[11] Since the ΔCoVaR and MES estimators focus on the left tail of the return distribution, it is plausible that they might be capable of detecting asymptotic tail dependence in stock returns.[12]

The formal definition of asymptotic tail dependence follows. Let $(\widetilde{R_1}, \widetilde{R_2})$ represent a bivariate random vector with individual univariate marginal distributions defined by:

$$G_1(r) = Pr(\tilde{R}_1 \leq r), \text{ and } G_2(r) = Pr(\tilde{R}_2 \leq r). \quad (4)$$

Let $L(u)$ represent the conditional probability,

$$L(u) = Pr(G_1(R_1) < u | \, G_2(R_2) < u) \quad (5)$$

Asymptotic left tail dependence between $\widetilde{R_1}$ and $\widetilde{R_2}$ is defined as, $L = \lim_{u \to 0} L(u)$. If this limit is zero, the random variables are asymptotically independent in their left tails. If the limit is positive, then the random variables have asymptotic tail dependence in the left tail. The larger is the value of the limit, the stronger is the left-tail dependence.

[11] We assume that asymptotic left tail dependence is a necessary condition for systemic risk. If there is no such dependence then we can infer that there is no systemic risk. However, since our null hypothesis is a joint hypothesis, it may be rejected by non-Gaussian returns even if the returns do not exhibit asymptotic tail dependence. Thus the rejection of the null hypothesis cannot be interpreted as definitive evidence of systemic risk. This issue is discussed in detail in Section VII.

[12] The link between asymptotic and sub-asymptotic tail dependence, systemic risk and the ΔCoVaR and MES measures is made explicit in Adrian and Brunnermeier (2011), Acharya, Pedersen, Philippon and Richardson (2010), and Acharya, Engle and Richardson (2012). The extremal dependence of economy/market level and institution level losses/returns is also consistent with the systemic risk definitions adopted in studies by the International Monetary Fund [Blancher et.al. (2013)] and the Financial Stability Board (2011).

To use the DWH approach to construct hypothesis tests to detect systemic risk, we must adopt a return distribution that is consistent with the null hypothesis; that is, we must select a parametric return distribution that excludes the possibility of systemic risk. Thus, our null hypothesis is a joint hypothesis no systemic risk and a specific parametric return distribution that excludes systemic risk. The joint nature of our null hypothesis must be taken into account when interpreting test results. We discuss the construction of a specific null hypothesis in the next section.

2. The Null Hypothesis: Stock Returns are Gaussian

Under the null hypothesis, admissible distribution candidates for stock returns must be asymptotically independent in the left tail of the distribution to rule out systemic risk. Among the possible alternative distributions that satisfy this condition, we choose the Gaussian return distribution. The Gaussian distribution is analytically tractable and has a long history as a model of stock returns. In the penultimate section of this paper, we discuss the performance of our test statistic under alternative hypotheses that deviate from the Gaussian null hypothesis including alternatives distributions that are asymptotically tail independent and thereby exclude systemic risk.

Let $(\tilde{R}_j, \tilde{R}_M)$ be distributed bivariate normal, $\mathrm{N}\left[\begin{pmatrix}\mu_j\\ \mu_M\end{pmatrix}, \begin{pmatrix}\sigma_j^2 & \rho_{jM}\sigma_j\sigma_M\\ \rho_{jM}\sigma_j\sigma_M & \sigma_M^2\end{pmatrix}\right]$, where $\mathrm{N}(a_0, a_1)$ represents the Gaussian distribution function with mean a mean vector "a_0" and covariance matrix "a_1"; μ_j and μ_M represent the individual (univariate) return means, σ_j^2 and σ_M^2 represent the individual return variances, and ρ_{jM} represents the correlation between the returns. The conditional return distributions are also normal random variables,

$$\tilde{R}_j|\left(\tilde{R}_M = r_{Mi}\right) \sim \mathrm{N}\left[\mu_j + \rho_{jM}\frac{\sigma_j}{\sigma_M}(r_{Mi} - \mu_M), \left(1 - \rho_{jM}^2\right)\sigma_j^2\right] \quad (6a)$$

$$\tilde{R}_M|\left(\tilde{R}_j = r_{ji}\right) \sim \mathrm{N}\left[\mu_M + \rho_{jM}\frac{\sigma_M}{\sigma_j}\left(r_{ji} - \mu_j\right), \left(1 - \rho_{jM}^2\right)\sigma_M^2\right] \quad (6b)$$

3. Parametric ΔCoVaR Estimator for Gaussian Returns

We derive a closed form expression for the parametric ΔCoVaR estimator for the reference portfolio $\tilde{R}_M$, conditional on $\tilde{R}_j$ equal to its 1 percent value at risk in Section A.1 of the Appendix. The expression is given by,[13]

$$\Delta CoVaR\left(\tilde{R}_M|\left(\tilde{R}_j = \Phi^{-1}(.01,\ \tilde{R}_j)\right)\right) = -\beta_{jM}\cdot 2.32635\ \frac{\sigma_M^2}{\sigma_j} \tag{7a}$$

or,
$$= -\rho_{jM}\cdot 2.32635\sigma_M, \tag{7b}$$

where, $\beta_{jM} = \frac{Cov(\tilde{R}_j,\tilde{R}_M)}{\sigma_M^2}$.

The parametric estimator of $\Delta CoVaR$, $\hat{b}_1(\Delta CoVaR)$, is constructed by replacing the Gaussian population parameters with their maximum likelihood sample estimators. Based on a sample size of *N*, the estimator is,

$$\hat{b}_1(\Delta CoVaR) = -\hat{\rho}_{jM}\cdot 2.32635\hat{\sigma}_j, \tag{8}$$

where $\hat{\rho}_{jM} = \frac{\frac{1}{N}\sum_{i=1}^{N}(R_{ji}-\bar{R}_j)(R_{Mi}-\bar{R}_M)}{\sqrt{\frac{\sum_{i=1}^{N}(R_{ji}-\bar{R}_j)^2}{N}}\sqrt{\frac{\sum_{i=1}^{N}(R_{Mi}-\bar{R}_M)^2}{N}}}$, $\hat{\sigma}_j = \sqrt{\frac{\sum_{i=1}^{N}(R_{ji}-\bar{R}_j)^2}{N}}$, $\bar{R}_j = \frac{1}{N}\sum_{i=1}^{N}R_{ji}$,

and $\bar{R}_M = \frac{1}{N}\sum_{i=1}^{N}R_{Mi}$.

4. Parametric MES Estimator for Gaussian Returns

Under the assumption of bivariate normality, Section A.2 of the Appendix shows that parametric marginal expected shortfall measure is given by,

$$MES = E\left(\tilde{R}_j|\tilde{R}_M < VaR(\tilde{R}_M, 95\%)\right) = \mu_j - \rho_{jM}\sigma_j\left[\frac{\phi(-1.645)}{\Phi(-1.645)}\right] \tag{9a}$$

or,
$$= \mu_j - 2.062839\ \sigma_M\,\beta_{jM} \tag{9b}$$

[13] Reversing the order of the conditioning variable (i.e., the CoVaR for $\tilde{R}_j$ conditional on $\tilde{R}_M$ equal to its 1 percent VaR), it is straight-forward to show that the so-called exposure CoVaR measure is,

$$\Delta CoVaR\left(\tilde{R}_i|\left(\tilde{R}_M = \Phi^{-1}(.01,\ \tilde{R}_M)\right)\right) = -\beta_{jM}\cdot 2.32635\ \sigma_M$$

where,$\phi(x)$ represents the standard normal probability density evaluated at x, $\Phi(x)$ represents the cumulative standard normal distribution function evaluated at x, and $\frac{\phi(-1.645)}{\Phi(-1.645)} = 2.062839.$

The parametric estimator for the expected shortfall measure, $\hat{b}_1(MES)$, is constructed by replacing the Gaussian population parameters in expression (9a) by their maximum likelihood sample estimators. Based on a sample size of *N*, the parametric estimator is,

$$\hat{b}_1(MES) = \hat{\mu}_j - 2.062839\ \hat{\sigma}_j\hat{\rho}_{jM}, \tag{10}$$

where, $\hat{\mu}_j = \bar{R}_j$.

5. *Nonparametric ΔCoVaR Estimator*

We estimate the nonparametric ΔCoVaR statistic, $\hat{b}_2(\Delta CoVaR)$, following the exact approach taken in the literature:[14]

- We run a 1-percent quantile regression of the CRSP equally-weighted market return on R_j and estimate, $\hat{\beta}_{jM}^q$, the slope coefficient of the firm's stock return from the quantile regression.
- We estimate the 1-percent sample quantile of the firm's stock return, $\hat{Q}_j(.01) = \hat{F}_j^{-1}(.01)$, where $\hat{F}_j(R_j)$ is a nonparametric sample estimate of the cumulative distribution for $\tilde{R}_j$.[15]
- We estimate the median firm stock return, $\hat{Q}_j(.5) = \hat{F}_j^{-1}(.5)$.[16]
- The nonparametric ΔCoVaR estimator is defined as:

$$\hat{b}_2(\Delta CoVaR) = \hat{\beta}_{jM}^q \left(\hat{Q}_j(.01) - \hat{Q}_j(.5) \right) \tag{11}$$

6. *Nonparametric MES Estimator*

We follow the exact approach taken in the literature and define the nonparametric MES estimator, $\hat{b}_2(MES)$, to be the average of individual stock returns on days when the equally-weighted stock market index realizes a return in the 5 percent lower tail of the sample

[14] Adrian and Brunnermeier (Sept 15, 2011), p. 15, equation (9).
[15] In a sample size of 500 observations, the 1 percent quantile estimate is the 5th smallest return observation.
[16] In a sample size of 500, the median is the simple average of the 250th and 251st ordered return observation.

observations. Let $I\left(R_{Mi} \leq \widehat{\mathrm{Q}}_M(.05)\right)$ be an indicator function that equals one when $R_{Mi} \leq \widehat{\mathrm{Q}}_M(.05)$ and zero otherwise, where $\hat{Q}_M(.05) = \hat{F}_M{}^{-1}(.05)$ and $\hat{F}_M(R_M)$ is the sample estimate of the nonparametric cumulative distribution for $\tilde{R}_M$. The nonparametric MES estimator is defined as,

$$\hat{b}_2(MES) = \left(\sum_{i=1}^{N} R_{ji}\ I\left(R_{Mi} \leq \hat{Q}_M(.05)\right)\right)\left(\sum_{i=1}^{N} I\left(R_{Mi} \leq \hat{Q}_M(.05)\right)\right)^{-1} \quad (12)$$

7. Systemic Risk Test Statistics when Returns are Gaussian

Following DHW, we construct our hypothesis test statistic as the difference between two estimators of ΔCoVaR and MES that are both consistent under the null hypothesis of no systemic risk. Under the null hypothesis, the parametric ΔCoVaR and MES estimators are more efficient.[17] Under the alternative hypothesis, systemic risk will be associated with asymptotic tail dependence in return distributions. The nonparametric ΔCoVaR and MES estimators remain consistent but the parametric ΔCoVaR and MES estimators will no longer be consistent.

The DWH ΔCoVaR test statistic is,

$$DWH_{\Delta CoVaR} = \frac{\left(\hat{b}_2(\Delta CoVaR) - \hat{b}_1(\Delta CoVaR)\right)^2}{Var\left(\hat{b}_2(\Delta CoVaR)\right) - Var\left(\hat{b}_1(\Delta CoVaR)\right)}, \quad (13)$$

where, $Var\left(\hat{b}_j(\Delta CoVaR)\right)$ is a consistent estimator of the variance of the estimator $\hat{b}_j$, $\{j = 1,2\}$, under the null hypothesis. The $DWH_{\Delta CoVaR}$ test statistic has an asymptotic Chi-Square distribution with 1 degree of freedom. In the single parameter case, the test can also be formulated as a "t-test" of the difference between the two estimators,[18]

$$DWHT_{\Delta CoVaR} = \frac{\hat{b}_2(\Delta CoVaR) - \hat{b}_1(\Delta CoVaR)}{\sqrt{Var\left(\hat{b}_2(\Delta CoVaR)\right) - Var\left(\hat{b}_1(\Delta CoVaR)\right)}}. \quad (14)$$

[17] Under the null hypothesis, the parametric ΔCoVaR and MES estimators are the maximum likelihood estimators with asymptotic sample variances equal to the Cramér-Rao Lower bound. See, for example, the discussion in Kmenta (1971), p. 159-160.

[18] The test statistic is asymptotically normally distributed, it merely takes the form of a "t-test" of the difference between means where the variance of the difference between the parametric and nonparametric estimators is the difference of the estimator variances. The covariance between these estimators is 0 because, under the null hypothesis, the parametric estimate is the minimum variance efficient estimator and hence uncorrelated with the inefficient nonparametric estimator.

We prefer the "t-test" version of the statistic because it allows us to focus our hypothesis test on the lower tail of the stock returns distribution.

The DWH MES test statistic is,

$$DWH_{MES} = \frac{\left(\hat{b}_2(MES) - \hat{b}_1(MES)\right)^2}{Var\left(\hat{b}_2(MES)\right) - Var\left(\hat{b}_1(MES)\right)} \quad . \tag{15}$$

DWH_{MES} has an asymptotic Chi-Square distribution with 1 degree of freedom. In the single parameter case, the "t-test" formulation of the statistic is,

$$DWHT_{MES} = \frac{\hat{b}_2(MES) - \hat{b}_1(MES)}{\sqrt{Var\left(\hat{b}_2(MES)\right) - Var\left(\hat{b}_1(MES)\right)}} \quad . \tag{16}$$

In expressions (13) and (15), in vary large samples, under the null hypothesis, the numerator difference between the two estimators (nonparametric and parametric) will converge to zero. Still, as a consequence of sample variation, the test statistic will have a sampling distribution.

Under the alternative hypothesis, stock returns display characteristics consistent with systemic risk (asymptotic tail dependence) and yet the nonparametric estimators remain consistent. In particular, they will produce larger negative estimates of ΔCoVaR and MES as a reflection of asymptotic left tail dependence in the return data. This intuition is confirmed by the results of a Monte Carlo simulation analysis that is discussed in Section A.3 of the Appendix.

Under the null hypothesis, the $DWHT_{\Delta CoVaR}$ and $DWHT_{MES}$ estimators will be asymptotically normally distributed, but the speed of convergence to asymptotic normality is unknown. We use bootstrap resampling to consistently estimate $Var\left(\hat{b}_j(\Delta CoVaR)\right)$ and $Var\left(\hat{b}_j(MES)\right)$, $\{j = 1,2\}$, and use Monte Carlo simulation to estimate the small sample distributions of the hypothesis test statistics. Because our subsequent empirical tests focus on a sample sizes of 500 observations (about two years of daily return data), we estimate the sampling distribution of the $DWHT_{\Delta CoVaR}$ and $DWHT_{MES}$ for a sample size of N=500. The specific details of the Monte Carlo simulation are reported in Section A.4 of the Appendix.

The properties of sampling distributions of the $DWHT_{\Delta CoVaR}$ and $DWHT_{MES}$ test statistics for a sample size of N=500 are summarized in Table 4. The critical values from our small sample

simulated Monte Carlo distribution are close to the critical values of the standard normal distribution, and so the results suggest that relatively little error is introduced by using the asymptotic normal approximation in a sample size of 500.

V. Systemic Risk Hypothesis Tests without Bootstrap Variance Estimates

1. Bootstrap Variance Estimates Increase Computational Burden

The $DWHT_{\Delta CoVaR}$ and $DWHT_{MES}$ test statistics require bootstrap resampling to estimate the sample variances of the nonparametric and parametric estimators. These bootstrap simulations are computationally time intensive. To avoid the bootstrap, we create an alternative, more user-friendly test statistic that uses a different normalization to construct the hypothesis test statistic. The "cost" of this computational simplification is the modified test statistic will depend on a nuisance parameter, the correlation coefficient, and so we have to calculate critical test statistic values conditional on the observed estimate of the stock return correlation when we estimate small sample distributions of this test statistic using Monte Carlo simulation.

2. A Simplified and More Feasible $\Delta CoVaR$ Statistic

Expression (7b) shows that the parametric $\Delta CoVaR$ estimator is proportional to the bivariate correlation ρ_{jM} parameter and the reference portfolio standard deviation, σ_M. Under the null hypothesis, the standard error of the non-parametric quantile regression estimator for ΔCoVaR, $\hat{\beta}_{jM}^{q}$, is also proportional σ_M. Consequently, we normalize by the difference between the nonparametric and parametric $\Delta CoVaR$ estimators by $\hat{\sigma}_M$, and our reformulated $\Delta CoVaR$ systemic risk test statistic is,

$$= -\frac{\hat{b}_2(\Delta CoVaR) - \hat{b}_1(\Delta CoVaR)}{\widehat{\sigma_M}}. \qquad (17)$$

Under the Gaussian null hypothesis, the sampling distribution of κ_{CoVaR} still depends on the estimated correlation between the stock returns and the returns on the CRSP equally-weighted market portfolio and we explicitly account for this dependence when we estimate the small sample critical values for the test statistic.

We provide simulation evidence that the $\hat{\sigma}_M$ normalization results in a test statistic with a distribution that is independent of the market return volatility. We perform a Monte Carlo exercise in which we estimate the sampling distributions for expression (17) under the null

hypothesis of Gaussian returns for different values for the market return volatility parameter, σ_M. We use the Cramer von Mises two sample test to test the hypothesis that each sample is drawn from the same underlying population. We calculate the test statistic using pairwise sample comparisons, and test the individual samples against the pooled sample. The test results, reported in Table 5, do not reject the hypothesis that all of the samples are drawn from an identical underlying population.

Under the alternative hypothesis of systemic risk, we expect $\hat{b}_2(\Delta CoVaR) - \hat{b}_1(\Delta CoVaR)$ is to be negative, and since $\hat{\sigma}_M$ is positive, systemic risk may be present when the test statistic produces a large positive value. Statistical significance is determined by comparing the estimated value for κ_{CoVaR} with its sampling distribution under the null.[19] When the test value of κ_{CoVaR} is in the far right-hand tail of its sampling distribution, we can reject the null hypothesis of no systemic risk. For example, rejecting the null hypothesis for test values at or above the 95 percent quantile of the sampling distribution for κ_{CoVaR} is consistent with a 5 percent type 1 error meaning there is at most 5 percent chance of rejecting a true null hypothesis.

3. A Simplified and More Feasible MES Test Statistic

Expression (9a) indicates that, under the null hypothesis, the parametric MES estimator is proportional to both ρ_{jM} and σ_j. Expression (12) also shows that, under the null hypothesis, the nonparametric estimator will also be proportional to σ_j through the term. As a consequence, we scale the difference between the nonparametric and parametric MES estimators by σ_j, and define our MES systemic risk test statistic as,

$$\kappa_{MES} = -\frac{\hat{b}_2(\text{MES}) - \hat{b}_1(\text{MES})}{\hat{\sigma}_i} \tag{18}$$

When comparing κ_{MES} test statistics across stocks with different mean parameters, the variation in the location parameter μ_j in expression (9a) is exactly offset by a shift in the mean of the distribution $\tilde{R}_j$ in the nonparametric estimator (12), and so the variation in μ_j does not impact the distribution of the κ_{MES} test statistic.

[19] Monte Carlo simulations demonstrate that $\sqrt{N}\kappa_{CoVaR}$ converges in distribution. Our analysis suggests that the limiting distribution is leptokurtotic relative to the normal distribution. For modest correlation values we cannot reject normality, but for correlations above 0.75 our simulation results reject standard tests for normality. The exact parametric form of the limiting distribution is not critical for our purposes. In our empirical tests, we compare sample tests statistics to the exact small sample distribution critical value estimates reported in Table 7.

We provide simulation evidence that the normalization produces a hypothesis test statistic with a distribution that is independent of $\hat{\sigma}_i$. We estimate the sampling distribution for expression (18) under the null hypothesis of Gaussian returns using Monte Carlo simulations for different values for $\hat{\sigma}_i$. We use the Cramer von Mises two sample test to test the hypothesis that each sample is drawn from the same underlying population. The test is employed both pairwise and comparing each sample against the pooled sample. The test results, reported in Table 6, do not reject the null hypothesis that all of the samples are drawn from an identical underlying population.

Under the alternative hypothesis of systemic risk, $\hat{b}_2(\text{MES})$ is expected to produce a larger negative number compared to $\hat{b}_1(\text{MES})$, and since $\hat{\sigma}_i$ is positive, systemic risk should generate a test statistic with a large positive value. As in the ΔCoVaR case, expression (18) still depends on the correlation between the stock and the reference portfolio and so we condition for the correlation parameter estimate when we estimate the critical values of the test statistic's small sample distribution.[20]

4. Small Sample Critical Values for the κ_{CoVaR} and κ_{MES} Test Statistics

We calculate the small sample critical values for the κ_{CoVaR} and κ_{MES} test statistics under the null hypothesis of no systemic risk using Monte Carlo simulation. To account for estimation risk in the test static correlation parameter, we use the Fisher z-transformation to randomize the correlation coefficient estimate in the Monte Carlo simulations.

The shape of the sampling distribution for $\hat{\rho}_{jM}$ depends the true correlation parameter, ρ_{jM}, and the sample size *N*. Fisher (1915, 1921) developed a transformation for $\hat{\rho}_{jM}$, called the ρ to z transformation,

$$\hat{z} = \frac{1}{2}\, Ln\left[\frac{1+\hat{\rho}}{1-\hat{\rho}}\right]. \tag{19}$$

For large N, the distribution for $\hat{z}$ approaches normality with a variance approximately equal to, $(N-3)^{-1}$, which is independent of ρ.

[20] In a Monte Carlo analysis of the distribution of the κ_{MES} test statistic for alternative Gaussian parameter values, we cannot reject the hypothesis that as N increases, the sampling distribution of $\sqrt{N}\,\kappa_{MES}$ converges to a normal distribution with a mean of 0 and a standard deviation approximately equal to $3.92 - 1.96\,\hat{\rho}_{jM}^2$. In our empirical tests, we use critical values from the small sample distribution estimates reported in Table 7.

We incorporate the estimation uncertainty associated with the correlation parameter using the following algorithm:

- For each correlation value, $\hat{\rho}_{jM}$.we construct $\hat{z}_{jM} = \frac{1}{2} Ln\left[\frac{1+\hat{\rho}_{jM}}{1-\hat{\rho}_{jM}}\right]$.
- The first Monte Carlo draw for the Fisher z statistic is constructed as $\hat{z}_{jM}(1) = \hat{z}_{jM} + e_1\sqrt{\frac{1}{N-3}}$ where e_1 is a random draw from a standard normal distribution.
- We invert the Fisher z-statistic transformation to construct the correlation parameter used in the first Monte Carlo replication, $\hat{\rho}_{jM}(1) = \frac{exp(2\cdot\hat{z}_{jM}(1))-1}{exp(2\cdot\hat{z}_{jM}(1))+1}$
- The randomization process for the correlation parameter is repeated with new random draws, e_i, for each of the remaining Monte Carlo Replications i=2,3,…,50,000.

Table 7 reports the small sample distribution estimates for the 1, 5 and 10 percent critical values for the κ_{CoVaR} and κ_{MES} test statistics for 12 different portfolio-stock return correlation assumptions between -0.2 and 0.9.[21] The critical values are calculated for a sample size of 500 observations. [22] We focus on 500 trading days, a two-year estimation window using daily returns, because the characteristics of institutions, especially large financial institutions, change very quickly over time as mergers and acquisitions change both their size and their business activities. The critical value statistics we report in Table 7 are based on 50,000 Monte Carlo replications.

VI. Systemic Risk Test Application to 2006-2007 Stock Returns Data

We use daily CRSP stock return data from the period 2006-2007 to calculate κ_{CoVaR} and κ_{MES} and compare individual test statistics to Monte Carlo critical value estimates calculated for $\hat{\rho}_{jM}$ correlation estimates measured to two decimal places. Table 8 summarizes κ_{CoVaR} and κ_{MES} hypothesis test results. Evaluated at the 5 percent level of the test, the κ_{CoVaR} test identifies 496 of 3518 firms as systemically important, or 14 percent of all sample firms. Ranked by industry, the mining industry has the largest share of firms identified as systemically important (36.4 percent), followed by "other financial" (25.7 percent) and wholesale trade (22.1 percent).

[21] We calculated critical test values for correlation values between -0.2 and 0.9 in increments of 0.01 to use in our empirical tests. However we only report the critical values for 12 correlation setting.
[22] We use the quantile regression package QUANTREG in R written by Roger Koenker.

Among the remaining financial services industries, broker dealers have the largest share of firms identified as systemically important (14.5 percent), followed by insurance (13 percent) and depository institutions (9.4 percent).

The κ_{MES} test identifies a much larger number of firms as systemically important. At the 5 percent level, the MES test identifies 979 firms as potential sources of systemic risk, or 28 percent of the firms in the sample. Among specific industries, public administration has the largest share of firms with systemic risk potential (54.5 percent), followed by broker dealers (49.1 percent), and transportation, communications and utilities (46.9 percent). Among the remaining financial services industries, insurance had the largest share of firms identified as systemically risky (43.5 percent), followed by depository institutions (37.7 percent) and other financial (28.4 percent).

These hypothesis tests identify many more firms as sources of systemic compared to previous papers that examined only a small subset of financial institutions. And, unlike prior papers, because we examine all firms with actively traded equity, we identify many real-side firms as systemically important. If systemic risk is manifested as asymptotic tail dependence in stock returns, and if our MES and ΔCoVaR tests reliably identify tail dependence, then a lot of firms are sources of systemic risk. But before jumping to conclusions, it is important to assess the reliability of these test statistics.

Our hypothesis test is the test of a joint hypothesis: asymptotic tail independent returns and Gaussian distributed returns. Violations of either of these conditions could trigger a test rejection. Since the Gaussian null hypothesis is a restrictive model for stock returns, it is important to understand when non-Gaussian stock return characteristics can lead to a test rejection even when returns are asymptotically tail independent.

VII. Do κ_{CoVaR} and κ_{MES} Rejections Detect Asymptotic Tail Dependence?

Many distributions can be used to model stock returns, and among these, any distribution that exhibits asymptotic left-tail independence is a candidate for use as the null hypothesis in the construction of our κ_{CoVaR} and κ_{MES} test statistics. In this section, we consider the properties of the Gaussian null hypothesis test statistic against a family of alternative distributions nested within the bivariate skew-t distribution. Depending on parameter values, the bivariate skew-t-distribution nests the bivariate skewed Gaussian distribution and the bivariate symmetric Student-t and Gaussian distributions.

Let d be the dimension of the multivariate distribution. Let y be a $(d \times 1)$ random vector, and β and α $(d \times 1)$ vectors of constants. Ω is a $(d \times d)$ positive definite matrix. $\nu \in (0, \infty)$ be a scalar degrees of freedom parameter. The multivariate skew-t density $y \sim f_T(y, \beta, \Omega, \alpha, \nu)$ is defined in Azzalini (2005):

$$f_T(y, \beta, \Omega, \alpha, \nu) = 2t_d(y; \beta, \Omega, \nu) T_1\left(\alpha^T \omega^{-1}(y - \beta)\left(\frac{\nu + d}{Q_y + \nu}\right)^{0.5}; \nu + d\right) \tag{20}$$

where, $$t_d(y; \beta, \Omega, \nu) = \frac{\Gamma\left(0.5(\nu + d)\right)}{|\Omega|^{0.5}(\pi\nu)^{d/2}\Gamma(0.5\nu)\left(1 + Q_y/\nu\right)^{(\nu+d)/2}}$$

is the density function of a d-dimensional student t random variate with ν degrees of freedom. $T_1(x;\nu+d)$ denotes the scalar t distribution with $\nu+d$ degrees of freedom; β is the location parameter which controls the distribution means; α is the parameter which controls the skewness of the distribution;[23] and, Ω is a generalized covariance matrix.[24] The remaining parameters are, $\omega = diag(\Omega)^{0.5}$ and $Q_y = (y - \beta)^T \Omega^{-1}(y - \beta)$.

The skew-t distribution nests the symmetric t-distribution, $(\alpha = 0, 0 < \nu < \infty)$, the skewed Gaussian distribution, $(\alpha \neq 0, \nu = \infty)$, and the symmetric Gaussian distribution, $(\alpha = 0, \nu = \infty)$. The flexibility of the skew-t distribution is ideal for evaluating the behavior of the κ_{CoVaR} and κ_{MES} test statistics under a number of alternative hypothesis that generate plausible representations of stock return data.

[23] α is not equal to but monotonically related to skewness. $\alpha=0$ implies a symmetric non-skewed distribution, whereas $\alpha > 0$ ($\alpha < 0$) implies positive (negative) skewness. It is also called the "shape" or the "slant" parameter.
[24] Ω is equal to the covariance matrix of y only when the skewness and kurtosis are 0 ($\alpha=0$, $\nu=\infty$).

Like the Gaussian distribution, the skewed-Gaussian distribution is asymptotically tail independent [Bortot (2010)]. Both the skewed and symmetric t-distributions have asymptotic tail dependence where the tail dependence depends on the degrees of freedom parameter, the generalized correlation parameter, and the skewness parameters.

The expression for the asymptotic left tail dependence of a bivariate symmetric t distribution is given by,

$$L_{sym\,t} = 2T_1\left(-\frac{\sqrt{(\nu+1)(1-\rho)}}{\sqrt{1+\rho}}, \upsilon+1\right) \tag{21}$$

where $T_1(x,\nu)$ is the distribution function for a univariate t distribution with ν degrees of freedom. Figure 1 illustrates the relationship between the asymptotic tail dependence in the symmetric bivariate t distribution and the distribution's correlation and degrees of freedom parameters. The higher the bivariate correlation, ρ and smaller the degrees of freedom, ν, the stronger is the distribution's asymptotic tail dependence. More specifically, as $\nu \to \infty$, the t distribution converges to the Gaussian distribution. Figure 1 shows, as $\nu \to \infty$, the distribution becomes asymptotically tail independent.

In most cases, there is no closed-form expression for the asymptotic tail dependence of a bivariate skew-t distribution. In the special case where the skew is identical across the individual random variables, $\alpha_1 = \alpha_2 = \alpha$, the distribution's asymptotic tail dependence is given by

$$L_{skew-t} = K(\alpha,\nu,\rho)\;\; 2T_1\left(-\frac{\sqrt{(\nu+1)(1-\rho)}}{\sqrt{1+\rho}}, \upsilon+1\right), \tag{22}$$

where,
$$K(\alpha,\nu,\rho) = \frac{T_1\left(2\alpha\sqrt{\frac{(\nu+2)(1+\rho)}{2}};\nu+2\right)}{T_1\left(\frac{\alpha(1+\rho)\sqrt{\nu+1}}{\sqrt{1+\alpha^2(1-\rho^2)}};\nu+1\right)}$$

Figure 2 illustrates the relationship between strength of the asymptotic tail dependence for the skew-t distribution and the degrees of freedom parameter, ν, correlation parameter ρ, and skewness parameters $\alpha_1 = \alpha_2 = \alpha$. We observe that negative skewness increases tail dependence, but only when degrees of freedom are small, that is when tails of the distribution are fat.

We estimated the parameters for the bivariate skew-t distribution using maximum likelihood for each of the 3518 stocks in our sample paired with the CRSP equally-weighted market return.[25] The subscript j (M) indicates a parameter estimate that is associated with the individual stock return (equally-weighted CRSP portfolio return). The distribution of resulting parameter estimates is reported in Table 9, where key parameters of interest are $\alpha_j, \alpha_M,\ \upsilon$, and ρ_{jM}.

Table 9 shows that, for most of the stocks in the sample over the 2006-2007 sample period, $\hat{\alpha}_j > 0$, indicating that most individual stock's return distributions are positively skewed. This finding is consistent with the literature [e.g., Singleton and Wingender (2006), or Carr et al. (2002)].

α_M is the parameter that determines the skewness of the market return distribution. While its magnitude varies among the 3518 bivariate maximum likelihood estimations, Table 9 reports that in all cases, $\hat{\alpha}_M < 0$, indicating that the market return distribution is negatively skewed.[26] The finding that the market return distribution is negatively skewed is also consistent with the literature [e.g., Fama (1965), Duffee (1995), Carr et al. (2002)), or Adrian and Rosenberg (2008)].

The degrees of freedom parameter, υ, is the most important determinant of the asymptotic tail dependence for this bivariate distribution. The results in Table 9 show that in more than 75 percent of the sample, $\upsilon < 4.5$.

Our κ_{CoVaR} and κ_{MES} test statistics are derived under the null hypothesis that stock returns Gaussian. The results in Table 9 suggest that the Gaussian null hypothesis is too restrictive in a large number of cases as indicated by non-zero skewness parameter estimates and a predominance of small values for the estimated degrees of freedom. It is important to understand how alternative return distributions affect the sampling distribution of our test statistics.

[25] We used the "sn" package in "R" written by Adelchi Azzalini

[26] The market skewness parameter is estimated simultaneously with the individual stock's skewness parameter and the single degrees of freedom parameter for the bivariate distribution. Ideally we would prefer to estimate the entire multivariate (3518 dimensions) skew-t distribution simultaneously and thereby get one set of parameters for the market portfolio. However, with only 500 daily observations, the joint return system not be identified unless additional structural simplification are imposed to reduce the number of independent parameters that must be estimated.

In the remainder of this section, we generate simulated stock return data from alternative parameterizations of the skew-t distributions and estimate the sampling distribution for our hypothesis test statistics using Monte Carlo simulation and kernel density estimation. For alternative parameterizations, we calculate our hypothesis test statistics for 10,000 samples of 500 observations. We consider alternative parameterizations of the skew-t distribution that use parameter values representative of the underlying distribution of the actual stock returns.

We first investigate whether κ_{CoVaR} and κ_{MES} have statistical power against alternative hypothesis that include negative asymptotic tail dependence. Figure 3 plots the sampling distribution for the κ_{CoVaR} test statistic under the Gaussian null hypothesis distribution and two symmetric t distributions with degrees of freedom that are characteristic of the estimates produced in our cross section of stock returns.

Across the panels in Figure 3, as ρ increases, the underlying stock return distributions have stronger and stronger asymptotic tail dependence. For most of the distributions, the sampling distribution for the κ_{CoVaR} calculated under the null hypothesis substantially overlaps the sampling distributions for κ_{CoVaR} calculated for returns generated by the alternative hypothesis. Moreover, the sampling distributions under the alternative hypothesis have relatively little cumulative probability to the right of the 5 percent critical values of the test statistic. These relationships indicate that is highly probable that, should the alternative hypothesis be true, the calculated test statistics may not reject the null. In other words, the κ_{CoVaR} test has lower power for detecting asymptotic tail dependence. This is especially true in the panels for which $\rho = 0$ and $\rho = .2$. For these panels the asymptotic tail dependence of the alternative t distributions is under 30 percent.

Figure 4 repeats the symmetric t distribution power simulations for the κ_{MES} test statistic and the results are similar. While the κ_{MES} test has slightly better power characteristics compared to κ_{CoVaR}, it still has poor power unless the returns have strong asymptotic tail dependence. For example, the panel with ρ=.7 shows that, even when asymptotic tail dependence is 0.489, the power of the κ_{MES} test is still only about 50 percent meaning that, if the alternative hypothesis of tail dependence is true, the test will not reject the null hypothesis about half the time.

Another notable feature evident in these Figures is that, as asymptotic tail dependency increases, the variance of κ_{CoVaR} and κ_{MES} test statistics grow significantly. Hence, almost paradoxically, these test statistics become noisier and less efficient when the distribution has greater systemic risk. We return to this issue at the end of this section and explain why the test statistics become less precise as asymptotic tail dependence becomes stronger.

Figures 5 and 6 analyze the behavior of the κ_{CoVaR} and κ_{MES} test statistics under alternative return distributions in which market returns are symmetric Gaussian, but individual firm returns are positively skewed Gaussian. For these alternatives, returns are asymptotically independent. Figures 5 and 6 show that the sampling distributions for κ_{CoVaR} and κ_{MES} test statistics under these alternative hypothesis are very similar to the sampling distribution under the null, and there is little risk that positively skewed Gaussian individual returns (with symmetric Gaussian market returns) will generate rejections of the null hypothesis.

Figures 7 and 8 analyze the behavior of the κ_{CoVaR} and κ_{MES} test statistics under alternative return distributions in which market returns are negatively skewed Gaussian, but individual firm returns are symmetric Gaussian. Returns are asymptotically independent. These figures show that negatively skewed market returns can cause false rejections of the null hypothesis. This is especially true when the individual stocks and the market are highly positively correlated. The κ_{CoVaR} test seems to be more sensitive to these characteristics, but both statistics frequently reject the null hypothesis. The null hypothesis should be rejected because returns are not Gaussian. However, the rejection is not caused by asymptotic tail dependence, but by return skewness. The issue is one of interpretation. Unless the joint nature of the hypothesis test is taken into account, these rejections could easily be mistaken as a false indications of systemic risk.

Figures 9 and 10 analyze the behavior of the κ_{CoVaR} and κ_{MES} test statistics under alternative return distributions in which market returns are negatively skewed Gaussian, and individual firm returns are positively-skewed Gaussian. Negative market skewness with positive individual stock skewness is the most common pattern observed in the data. Here again, returns are asymptotically independent. The pattern in Figures 9 and 10 is similar. Mild positive individual stock skewness with relatively strong negative market skewness and strong positive return correlation is a pattern that is likely to generate rejections of the null hypothesis. This particular skewness and correlation pattern is predominant in the 2006-2007 sample data suggesting that

the large number of rejections in our sample may owe in part to an overly restrictive null hypothesis. Generalizing the test statistic to incorporate a skewed Gaussian distribution under the null would be a step in the right direction, but again the poor ΔCoVaR and MES test power characteristics show there may be limited benefits for further hypothesis test refinements.

Our final two figures speak directly to the power issue. Figure 11 plots the sampling distribution for the nonparametric ΔCoVaR estimator applied to stock returns generated by a Student t distribution with varying degrees of freedom. The chart shows that, as the degrees of freedom parameter declines and the asymptotic tail dependence in returns increases, the variance of the nonparametric ΔCoVaR estimator increases dramatically. While the nonparametric ΔCoVaR estimator remains consistent when applied to asymptotically tail dependent stock return distributions, it becomes increasingly noisy as the tail dependence increases. Figure 12 shows that the nonparametric MES estimator shares this characteristic. These results suggest that the nonparametric ΔCoVaR and the nonparametric MES estimators are unlikely to provide a basis for constructing powerful hypothesis tests for systemic risk (asymptotic tail dependence) in a sample size of 500. [27]

VIII. Summary and Conclusions

We contribute to the systemic risk measurement literature by developing formal hypothesis test statistics that can be used to detect systemic risk using ΔCoVaR and MES estimates based on daily stock return data. Our test statistics are based on the null hypothesis of Gaussian stock returns, however our methodology can be generalized other distributions that exhibit asymptotic tail independence. We use Monte Carlo simulation to estimate the critical values of the sampling distributions of our proposed test statistics and use these critical values to test for evidence of systemic risk in a broad cross section of stocks using daily return data over the period 2006-2007.

While we are the first to introduce formal hypothesis tests for detecting systemic risk using ΔCoVaR and MES estimates, our hypothesis tests are composite tests, and they do not provide

[27] A way to mitigate the power issue could be to employ intraday returns in the estimation of systemic risk proxies. As a result, the number of return series used in the estimation will increase and the nonparametric estimators can become more efficient.. For instance, Zhang, Zhou, Zhu (2009) show that volatility measures based on high-frequency tick data can better explain the likelihood of tail events, such as default probabilities. A caveat of this approach is that trading noise in tick data might adversely bias the systemic risk estimates.

an ideal solution to the testing problem. Our hypothesis tests are joint test because they require the adoption of a specific return distribution under the null hypothesis. The joint nature of the null hypothesis is problematic because there is a wide range of distributions that exhibit asymptotic tail independence. The joint nature of the test complicates the interpretation of test rejections. For example, we show that, depending on the return generating process, our tests may reject the null hypothesis of no systemic risk when returns are generated by skewed, but tail-independent distributions. Thus, the choice of a specific return distribution to characterize returns under the null hypothesis is a crucial aspect of systemic risk test design. While this finding suggests the need for additional research focused on clarifying the null hypothesis used to characterize stock return distributions, our findings suggest that ΔCoVaR and MES tests are likely to have only weak power, so further efforts to refine this test may provide only limited improvements in test performance.

In particular, our simulation results suggest that nonparametric ΔCoVaR and MES statistics are unlikely to detect asymptotic tail dependence unless the tail dependence is strong. And even then, the power of these test statistics is limited by large variation in sampling distribution of nonparametric estimators. If systemic risk is truly manifested as asymptotic left-tail dependence in stock returns, our analysis suggests that ΔCoVaR and MES measures may be incapable of reliably detecting a firm's systemic risk potential.

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Appendix

A.1 Parametric Estimator for ΔCoVaR when Returns are Gaussian

In this section, we derive the CoVaR measure for the returns on a market portfolio, $\tilde{R}_M$, conditional on a specific stock return, $\tilde{R}_j$, equal to its 1 percent value at risk. The market portfolio conditional return distribution is given by,

$$\tilde{R}_M \left| \left(\tilde{R}_j = \mathrm{G}^{-1}(.01,\ \tilde{R}_j) \right) \sim N\left[\mu_M + \rho \frac{\sigma_M}{\sigma_j} \left(\Phi^{-1}(.01,\ \tilde{R}_j) - \mu_j \right), \left(1 - \rho_{jM}^2\right)\sigma_M^2 \right], \right. \tag{A1}$$

where $\mathrm{G}^{-1}(.01,\ \tilde{R}_j)$ represents the inverse cumulative normal distribution of the unconditional return $\tilde{R}_j$ evaluated at the 0.01 cumulative probability. Using, $\mathrm{G}^{-1}(.01, \tilde{R}_j) = \mu_j - 2.32635\ \sigma_j$, the 1 percent CoVaR for the market conditional on $\tilde{R}_j$ equal to its 1 percent VaR is,

$$CoVaR\left(\tilde{R}_M \left| \left(\tilde{R}_j = \Phi^{-1}(.01,\ \tilde{R}_j) \right) \right.\right) = \mu_M - \rho_{jM} \frac{\sigma_M}{\sigma_j} \left(2.32635\ \sigma_j\right) - 2.32635\ \sigma_M \sqrt{1 - \rho_{jM}^2}\ . \tag{A2}$$

Similarly, the return distribution for $\tilde{R}_M$, conditional on $\tilde{R}_j$ equal to its median is,

$$\tilde{R}_M \left| \left(\tilde{R}_j = \mathrm{G}^{-1}(.50,\ \tilde{R}_j) \right) \sim N\left[\mu_M, \left(1 - \rho_{jM}^2\right)\sigma_M^2\right]. \right. \tag{A3}$$

Consequently, the CoVaR for the portfolio with $\tilde{R}_j$ evaluated at its median return is,

$$CoVaR\left(\tilde{R}_M \left| \left(\tilde{R}_j = \mathrm{G}^{-1}(.50,\ \tilde{R}_j) \right) \right.\right) = \mu_M - 2.32635\sigma_M \sqrt{1 - \rho_{jM}^2}\ , \tag{A4}$$

Subtracting (A4) from (A2) and defining $\beta_{jM} = \frac{Cov(\tilde{R}_j, \tilde{R}_M)}{\sigma_M^2}$, the so-called contribution CoVaR measure, ΔCoVaR is[28],

$$\Delta CoVaR\left(\tilde{R}_M \left| \left(\tilde{R}_j = \mathrm{G}^{-1}(.01,\ \tilde{R}_j) \right) \right.\right) = -\beta_{jM} \cdot 2.32635\ \frac{\sigma_M^2}{\sigma_j} \tag{A5a}$$

$$= -\rho_{jM} \cdot 2.32635\sigma_M \tag{A5b}$$

[28] Reversing the order of the conditioning variable (i.e., the CoVaR for $\tilde{R}_j$ conditional on $\tilde{R}_M$ equal to its 1 percent VaR), it is straight-forward to show that the so-called exposure CoVaR measure is,

$$\Delta CoVaR\left(\tilde{R}_i \left| \left(\tilde{R}_M = \mathrm{G}^{-1}(.01,\ \tilde{R}_M) \right) \right.\right) = -\beta_{jM} \cdot 2.32635\ \sigma_M$$

A.2 Parametric MES Estimator for Gaussian Returns

The marginal expected shortfall measure is the expected shortfall calculated from a conditional return distribution. The conditioning event is the return on a market portfolio, $\tilde{R}_M$, less than or equal to its 5 percent VaR value.

Under the assumption of bivariate normality, the conditional stock return is normally distributed, and consequently,

$$E\left(\tilde{R}_j|\tilde{R}_M = r_M\right) = \mu_j - \rho\frac{\sigma_j}{\sigma_M}\,\mu_M + \rho\frac{\sigma_j}{\sigma_M}r_M. \tag{A6}$$

Now, if $\tilde{R}_M$ is normally distributed with mean μ_M and standard deviation σ_M, then the expected value of the market return truncated above the value "b" is,

$$E\left(\tilde{R}_M|\tilde{R}_M < b\right) = \mu_M - \sigma_M\left[\frac{\phi\left(\frac{b-\mu_M}{\sigma_P}\right)}{\Phi\left(\frac{b-\mu_M}{\sigma_M}\right)}\right], \tag{A7}$$

If b is the lower 5 percent tail value, $b = \mu_M - 1.645\sigma_M$, and the expected shortfall measure is,

$$E\left(\tilde{R}_j|\tilde{R}_M < VaR(\tilde{R}_M, 95\%)\right) = \mu_j - \rho\sigma_j\left[\frac{\phi(-1.645)}{\Phi(-1.645)}\right] \tag{A8a}$$

$$= \mu_j - 2.062839\,\sigma_M\,\beta_{jM} \tag{A8b}$$

where the constant (2.062839) is a consequence of the 5 percent tail conditioning on the market return, i.e., $\frac{\phi(-1.645)}{\Phi(-1.645)} = 2.062839$.

A.3 Can $\hat{b}_2(\Delta CoVaR) - \hat{b}_1(\Delta CoVaR)$ and $\hat{b}_2(MES) - \hat{b}_1(MES)$ Detect Systemic Risk?

We use Monte Carlo simulation to confirm the intuition that the difference between the nonparametric and the parametric ΔCoVaR and MES estimators can detect systemic risk. In particular, under the alternative hypothesis that stock returns have asymptotic tail dependence, we expect the nonparametric ΔCoVaR and MES estimators to produce larger negative values relative to their parametric estimator counterparts.

Table A1 reports the results of a Monte Carlo exercise in which the sampling distribution of the difference in the nonparametric and parametric ΔCoVaR (and MES) estimators is constructed for a sample size of 500 for bivariate Gaussian and Student-t return generating processes. While the

numerical estimates of the sampling distribution quantiles in Table A1 are specific to the population parameter values used in the simulation, the qualitative results generalize. When returns are generated by the Student-t distribution, a distribution with asymptotic tail dependence, the nonparametric ΔCoVaR and MES estimators remain consistent and tend to produce larger negative values compared to their parametric counterparts. As a consequence, the quantiles of sampling distribution of the difference between the nonparametric and parametric estimators are shifted to the left under alternative hypothesis that include asymptotic tail dependence.

Table A1: The Sampling Distribution of the Difference Between the Nonparameteric and Parametric ΔCoVaR and MES Estimators under Gaussian and Alternative Student t Return Generating Process

Monte Carlo Sample Quantile	ΔCoVaR Bivariate Gaussian Returns	ΔCoVaR Bivariate Student t with 2 Degrees of Freedom	MES Bivariate Gaussian Returns	MES Bivariate Student t with 2 Degrees of Freedom
0.01	-0.173	-1.640	-0.072	-0.337
0.05	-0.125	-1.070	-0.049	-0.221
0.10	-0.097	-0.827	-0.038	-0.179
0.25	-0.051	-0.456	-0.019	-0.117
0.50	-0.001	-0.030	0.002	-0.043

Monte Carlo Estimates of the sample distributions of the difference between the nonparametric and the parametric ΔCoVaR and MES estimators under two alternative return generating provcess: a bivariate Gaussian (the null hypothesis) and a bivariate Student-t (an alternative hypothesis with asymptotic tail dependence). Both return generating process have zero mean, return standard deviations of 0.20 for both components, and returns correlation of 0.5. The Monte Carlo quantile estimates are based on a sample size of 500, with 2000 replications and the output smoothed using a Gaussian Kernel density estimator.

A.4 Small Sample Properties of the $DWHT_{\Delta CoVaR}$ and $DWHT_{MES}$ Test Statistics

To better understand the small sample properties of the $DWHT_{\Delta CoVaR}$ and $DWHT_{MES}$ test statistics, we performed a Monte Carlo simulation to estimate the distributions of these test statistics in a sample size of 500 daily stock returns.

Our Monte Carlo estimates were constructed as follows. For both $\Delta CoVaR$ and MES simulations, we set the reference portfolios equal to the equally-weighted CRSP market return portfolio, $\tilde{R}_P = \tilde{R}_M$. We draw a set of three bivariate Gaussian parameter values $(\rho_{jM}, \sigma_M, \sigma_j)$

from a set of three possible values that are characteristic of 10 percentile, median, and 90 percentile values of the parameters observed in the population of stock returns: $\rho_{jM} = \{.17, .42, .58\}$; $\sigma_M = \{.014, .023, .037\}$; and $\sigma_j = \{.005, .008, .03\}$. [29] On each Monte Carlo replication, we randomly select one parameter value from each set, where the probability of selecting any one of the three possible parameters is 1/3.

Once the Gaussian parameters are selected, a sample of 500 daily bivariate Gaussian returns is simulated and the sample parametric and nonparametric $\Delta CoVaR$ and MES estimators are calculated. The 500 simulated returns are then resampled 500 times using the bootstrap technique to calculate sample estimates for $Var\left(\hat{b}_j(\Delta CoVaR)\right)$ and $Var\left(\hat{b}_j(MES)\right)$, for $j = 1,2$. The $DWHT_{\Delta CoVaR}$ and $DWHT_{MES}$ test statistics are calculated. The process then repeats 25,000 times with parameter selection, simulation, estimation, and the bootstrap to generate a Monte Carlo sampling distribution for the $DWHT_{\Delta CoVaR}$ and $DWHT_{MES}$ test statistics.

[29] Percentile distribution for the market volatility is based on a historical distribution between 1990 and 2007.

Table 1: Sample Characteristics by Industrial Classification

Industry	Identifying Information	Number of Firms	Mean ΔCoVaR	Mean MES
Broker Dealers	SIC 62	55	-0.0321	-0.0102
Construction	SIC 15,16,17	41	-0.0345	-0.0095
Depository Institutions	SIC 60; NAICS 5221, 551111; FRBNY *	426	-0.0196	-0.0067
Insurance	SIC 63,64	138	-0.0206	-0.0075
Manufacturing	SIC 20-39	1336	-0.0231	-0.0076
Mining	SIC 10-14	151	-0.0272	-0.0100
Other Financial	SIC 61,65,67: NAICS 52	74	-0.0267	-0.0087
Public Administration	SIC 91-99	11	-0.0085	-0.0033
Retail Trade	SIC 52-59	227	-0.0231	-0.0075
Services	SIC 70,72,73,75,76,78,79,80-89	626	-0.0213	-0.0070
Transportation, Communications	SIC 40-49	320	-0.0211	-0.0086
Wholesale Trade	SIC 50-51	113	-0.0234	-0.0084

ΔCoVaR is estimated as the difference between two linear quantile regressions estimates of the market portfolio return on individual stock returns: the 1 percent quantile estimate less the 50 percent quantile estimate. The reference portfolio is the CRSP equally-weighted market index. Quantile regressions are estimated using the R package Quantreg. MES is estimated as the average of individual stock returns on sample subset of days that correspond with the 5 percent worst days of the equally-weighted CRSP stock market index. SIC is the standard industrical classification. NAICS is the North American Industry Classicification System.

*Depository institutions are identified using the Bank Holding Company (BHC) dataset prepared by the Federal Reserve Bank of New York (FRBNY, March 18, 2008). http://www.newyorkfed.org/research/banking_research/datasets.html. We supplement this definition adding institutions that either have a 2-digit SIC of "60", or have a 4-digit NAICS of "5221" or a 6-digit NAICS or "551111" in the depository definition.

Table 2. Fifty Firms with the Largest ΔCoVaR Estimates over 2006-2007

Rank	Company Name	Median Market Capitalization ($thous)	Industry	CoVaR
1	PROQUEST CO	370,338	Manufacturing	-0.0400
2	TETRA TECHNOLOGIES INC	1,739,641	Manufacturing	-0.0246
3	NEWSTAR FINANCIAL INC	516,271	Other Financial	-0.0245
4	IBASIS INC	324,017	Transportation, Communications	-0.0226
5	ISILON SYSTEMS INC	852,653	Manufacturing	-0.0215
6	NEOWARE SYSTEMS INC	265,074	Manufacturing	-0.0207
7	MATTSON TECHNOLOGY INC	493,509	Manufacturing	-0.0207
8	TITAN INTERNATIONAL INC ILL	397,960	Manufacturing	-0.0203
9	VALERO ENERGY CORP NEW	37,153,498	Manufacturing	-0.0200
10	ROBERT HALF INTERNATIONAL INC	6,167,046	Services	-0.0200
11	DOVER CORP	9,885,112	Manufacturing	-0.0196
12	MURPHY OIL CORP	9,977,096	Manufacturing	-0.0194
13	VENOCO INC	814,278	Mining	-0.0192
14	CONSTELLATION BRANDS INC	4,953,574	Manufacturing	-0.0188
15	CITIZENS FIRST BANCORP INC	203,931	Depository Institutions	-0.0187
16	M S C INDUSTRIAL DIRECT INC	2,149,047	Manufacturing	-0.0186
17	SMURFIT STONE CONTAINER CORP	2,928,624	Manufacturing	-0.0184
18	SOUTHERN COPPER CORP	16,450,064	Mining	-0.0180
19	STILLWATER MINING CO	1,095,439	Mining	-0.0180
20	A M R CORP DEL	5,753,965	Transportation, Communications	-0.0179
21	AMERIPRISE FINANCIAL INC	13,095,030	Broker Dealers	-0.0178
22	NASDAQ STOCK MARKET INC	3,617,941	Broker Dealers	-0.0178
23	U A L CORP	4,264,734	Transportation, Communications	-0.0177
24	PARKER HANNIFIN CORP	9,950,826	Manufacturing	-0.0176
25	TECH DATA CORP	2,065,805	Wholesale Trade	-0.0176
26	ORMAT TECHNOLOGIES INC	1,368,098	Transportation, Communications	-0.0175
27	WESTERN REFINING INC	1,830,752	Manufacturing	-0.0175
28	AMERICA SERVICE GROUP INC	152,270	Services	-0.0174
29	CIRCOR INTERNATIONAL INC	568,005	Manufacturing	-0.0174
30	CATERPILLAR INC	46,739,598	Manufacturing	-0.0174
31	RADIO ONE INC	595,811	Transportation, Communications	-0.0174
32	CIMAREX ENERGY CO	3,213,883	Mining	-0.0173
33	EMPIRE RESOURCES INC DEL	104,041	Wholesale Trade	-0.0173
34	CLEVELAND CLIFFS INC	2,149,713	Mining	-0.0173
35	R P C INC	1,463,770	Mining	-0.0173
36	EVERCORE PARTNERS INC	175,262	Other Financial	-0.0172
37	PROCENTURY CORP	194,361	Insurance	-0.0172
38	KOMAG INC	1,111,125	Manufacturing	-0.0172
39	LEGG MASON INC	12,749,954	Broker Dealers	-0.0171
40	PULTE HOMES INC	7,395,542	Construction	-0.0171
41	DYNAMEX INC	244,848	Transportation, Communications	-0.0170
42	GATEHOUSE MEDIA INC	745,417	Manufacturing	-0.0170
43	FREEPORT MCMORAN COPPER & GOLD	11,781,130	Mining	-0.0170
44	ROWAN COMPANIES INC	4,092,873	Mining	-0.0170
45	MOSAIC COMPANY	9,378,757	Manufacturing	-0.0170
46	HOME DIAGNOSTICS INC	198,165	Manufacturing	-0.0169
47	OIL STATES INTERNATIONAL INC	1,680,397	Manufacturing	-0.0169
48	COOPER CAMERON CORP	6,046,972	Manufacturing	-0.0168
49	HANMI FINANCIAL CORP	910,360	Depository Institutions	-0.0168
50	MARINE PRODUCTS CORP	358,240	Manufacturing	-0.0168

ΔCoVaR is estimated as the difference between two linear quantile regressions estimates of the market portfolio return on individual stock returns: the 1 percent quantile estimate less the 50 percent quantile estimate. The reference portfolio is the CRSP equally-weighted market index. Quantile regressions are estimated using the R package Quantreg. The median market capitalization is the median value of the closing share price times the number of shares outstanding.

Table 3. Fifty Firms with the Largest MES Estimates over 2006-2007

Rank	Company Name	Median Market Capitalization ($thous)	Industry	MES
1	CHINA PRECISION STEEL INC	191,188	Manufacturing	-0.0780
2	FREMONT GENERAL CORP	1,083,450	Depository Institutions	-0.0771
3	ESCALA GROUP INC	177,063	Services	-0.0742
4	ACCREDITED HOME LENDERS HLDG CO	697,385	Other Financial	-0.0697
5	URANERZ ENERGY CORP	133,479	Mining	-0.0688
6	CRAWFORD & CO	160,407	Insurance	-0.0629
7	ORBCOMM INC	360,348	Transportation, Communications	-0.0591
8	TRIAD GUARANTY INC	674,462	Insurance	-0.0590
9	STANDARD PACIFIC CORP NEW	1,525,954	Construction	-0.0589
10	DELTA FINANCIAL CORP	211,543	Other Financial	-0.0587
11	W C I COMMUNITIES INC	792,183	Construction	-0.0583
12	COMPUCREDIT CORP	1,786,092	Other Financial	-0.0582
13	BEAZER HOMES USA INC	1,601,873	Construction	-0.0573
14	EMPIRE RESOURCES INC DEL	104,041	Wholesale Trade	-0.0570
15	HOUSTON AMERICAN ENERGY CORP	120,614	Mining	-0.0566
16	RADIAN GROUP INC	4,673,175	Insurance	-0.0557
17	I C O GLOBAL COMMS HLDGS LTD DE	590,548	Transportation, Communi	-0.0555
18	E TRADE FINANCIAL CORP	9,718,507	Broker Dealers	-0.0552
19	BUCKEYE TECHNOLOGIES INC	451,391	Manufacturing	-0.0548
20	BADGER METER INC	383,436	Manufacturing	-0.0545
21	CHESAPEAKE CORP VA	280,014	Manufacturing	-0.0543
22	AVANEX CORP	376,476	Manufacturing	-0.0543
23	BANKUNITED FINANCIAL CORP	945,137	Depository Institutions	-0.0538
24	NASTECH PHARMACEUTICAL CO INC	328,747	Services	-0.0533
25	IDAHO GENERAL MINES INC	254,642	Mining	-0.0533
26	GEORGIA GULF CORP	694,153	Manufacturing	-0.0524
27	WINN DIXIE STORES INC	1,001,750	Retail Trade	-0.0523
28	PANACOS PHARMACEUTICALS INC	241,728	Manufacturing	-0.0520
29	CROCS INC	1,864,861	Manufacturing	-0.0517
30	GRUBB & ELLIS CO	266,193	Other Financial	-0.0514
31	K B W INC	877,814	Broker Dealers	-0.0513
32	FRANKLIN BANK CORP	425,906	Depository Institutions	-0.0511
33	ASIAINFO HOLDINGS INC	289,401	Services	-0.0508
34	FIRST AVENUE NETWORKS INC	613,862	Transportation, Communications	-0.0508
35	WHEELING PITTSBURGH CORP	293,215	Manufacturing	-0.0505
36	STILLWATER MINING CO	1,095,439	Mining	-0.0504
37	TOREADOR RESOURCES CORP	353,326	Mining	-0.0500
38	COUNTRYWIDE FINANCIAL CORP	21,663,474	Depository Institutions	-0.0497
39	CHAMPION ENTERPRISES INC	781,824	Manufacturing	-0.0495
40	REVLON INC	554,749	Manufacturing	-0.0494
41	TIENS BIOTECH GROUP USA INC	285,336	Services	-0.0493
42	AIRSPAN NETWORKS INC	142,206	Transportation, Communications	-0.0492
43	FREEPORT MCMORAN COPPER & GOLD	11,781,130	Mining	-0.0490
44	MERITAGE HOMES CORP	1,069,891	Construction	-0.0486
45	EDDIE BAUER HOLDINGS INC	273,345	Retail Trade	-0.0485
46	A Z Z INC	235,577	Manufacturing	-0.0483
47	GRAFTECH INTERNATIONAL LTD	722,492	Manufacturing	-0.0483
48	P M I GROUP INC	3,817,586	Insurance	-0.0483
49	ASYST TECHNOLOGIES INC	341,391	Manufacturing	-0.0480
50	NEUROGEN CORP	213,849	Manufacturing	-0.0480

MES is estimated as the average of individual stock returns on sample subset of days that correspond with the 5 percent worst days of the equally-weighted CRSP stock market index. The median market capitalization is the median value of the closing share price times the number of shares outstanding.

Table 4: Monte Carlo Small Sample Distribution Estimates for the Critical Values of the DWHT ΔCoVaR and DWHT MES Test Statistics

	Critical Test Statitic Value		
Type I error rate	Normal	DWHT ΔCoVaR	DWHT MES
0.01	-2.33	-2.36	-2.28
0.05	-1.64	-1.60	-1.58
0.10	-1.28	-1.23	-1.22

Monte Carlo estimates of the hypothesis tests critical values are for a sample size 500 observations. Critical values are estimated as follows. For each replication, the Gauassian parmeters are choosen with equal probability from a set of three representative population values ρ_{jM}={.17,.42,.58}; σ_M={.014,.023,.037}; and σ_j={.005,.008,.03}. The DHWT ΔCoVaR and DHWT MES hypothesis test statistics are calulated using bootstrap estimates for the nonparametric and parametric estimator variances using 500 bootstrap samples. The critical value estimates are based on 25 thousand Monte Carlo replications.

Table 5: ΔCoVaR Cramer-von Mises Test for Equality of Two Distributions

	Against $\sigma_M = 0.40$ sample		Against pooled sample	
σ_M	test value	probability	test value	probability
0.10	0.061	0.81	0.059	0.82
0.20	0.046	0.90	0.028	0.98
0.30	0.045	0.91	0.035	0.96
0.40	0.000	1.00	0.024	0.99

The underlying systemic risk test statistic is the difference between the nonparameteric and parametric ΔCoVaR estimators scaled by an estimate of the market portfolio return standard deviation in a sample size of 500 observations when the stock and market return portfolio are distriburted bivariate Gaussian. The test statistic sampling distribution is simulated using 5000 replications for each value of the market return standard deviation holding all other distribution parameters constant. The Cramer-von Mises test statistic tests whether two specific samples are drawn from the same underlying distribution (e.g., a market volatility of .10 versus a market volatility of .40) or whether the specific samples are drawn from the same distribution that generated the pooled sample. The reported probability value indicates the probability that the samples are drawn from the same underlying distribution.

Table 6: MES Cramer-von Mises Test for Equality of Two Distributions

	Against σ_j=.40 sample		Against pooled sample	
σ_j	test value	probability	test value	probability
0.10	0.122	0.49	0.074	0.725
0.20	0.072	0.74	0.041	0.926
0.30	0.083	0.67	0.033	0.966
0.40	0.000	1.00	0.059	0.82

The underlying systemic risk test statistic is the difference between the nonparameteric and parametric MES estimators scaled by an estimate of the individual stock return standard deviation in a sample size of 500 observations when the stock and market return portfolio are distriburted bivariate Gaussian. The test statistic sampling distribution is simulated using 20000 replications for each value of the market return standard deviation holding all other distribution parameters constant. The Cramer-von Mises test statitic tests whether two specific samples are drawn from the same underlying distribution (e.g., a stock volatility of .10 versus a stock volatility of .40) or whether the specific samples are drawn from the same distribution that generated the pooled sample. The reported probability value indicates the probability that the samples are drawn from the same underlying distribution.

Table 7: Critical Values for κ_{CoVaR} and κ_{MES} Test Statistics

	Critical Value of ΔCoVaR Test Statistic K_{CoVaR}			Critical Value of MES Test Statistic, K_{MES}		
Correlation	Type I error=10%	Type I error=5%	Type I error=1%	Type I error=10%	Type I error=5%	Type I error=1%
-0.2	47.6	60.7	87.0	21.7	27.8	39.6
-0.1	47.8	62.3	88.5	21.9	28.1	39.8
0.0	47.4	61.3	88.4	21.9	28.2	39.9
0.1	47.1	61.5	88.9	21.7	28.0	39.7
0.2	46.7	60.7	87.2	21.3	27.6	39.2
0.3	44.6	58.1	83.8	20.8	26.9	38.4
0.4	43.0	56.2	81.9	20.1	25.9	37.1
0.5	40.8	53.4	79.8	19.1	24.8	35.3
0.6	38.1	50.2	74.0	18.0	23.3	33.1
0.7	34.4	45.7	68.2	16.4	21.4	30.4
0.8	30.0	40.3	59.5	14.7	19.0	27.0
0.9	23.7	32.1	48.6	12.3	16.0	22.9

In each replication, we draw 500 observations drawn randomly from a bivariate Gaussian distribution with zero mean and the indicated correlation and we estimate the κ_{CoVaR} and κ_{MES} test statistics. The 10%, 5%, and 1% critical values estimates are based on 50,000 Monte Carlo replications.

Table 8: Summary of κ_{CoVaR} and κ_{MES} Hypothesis Test Results

Industry	Number of Firms	ΔCoVaR Statistically Significant at 1% level	ΔCoVaR Statistically Significant at 5% level	Percentage of Firms Sig. at 5% Level under ΔCoVaR Test	MES Statistically Significant at 1% level	MES Statistically Significant at 5% level	Percentage of Firms Sig. at 5% Level under MES Test
Broker Dealers	55	3	8	14.5%	22	27	49.1%
Construction	41	1	4	9.8%	4	11	26.8%
Depository Institutions	426	16	40	9.4%	88	158	37.1%
Insurance	138	5	18	13.0%	36	60	43.5%
Manufacturing	1336	75	159	11.9%	132	326	24.4%
Mining	151	22	55	36.4%	11	28	18.5%
Other Financial	74	9	19	25.7%	13	21	28.4%
Public Administration	11	1	2	18.2%	0	6	54.5%
Retail Trade	227	10	27	11.9%	27	63	27.8%
Services	626	38	80	12.8%	50	129	20.6%
Transportation, Communications	320	21	59	18.4%	80	150	46.9%
Wholesale Trade	113	9	25	22.1%	8	24	21.2%

The κ_{CoVaR} statistic is statistically significant at 1% (5%) level if the sample estimated value exceeds its critical value for a given correlation level and Type I error of 1% (5%) reported in Table 7. Similarly, the κ_{MES} statistic is statististically significant if the sample estimated value exceeds the critical value reported in Table 7 for a given correlation and Type I error level .

Table 9: Maximum Likelihood Estimates of Bivariate Skew-t Distribution for 3518 firms

Distribution Quantiles	β_j	β_M	Ω_{jj}	Ω_{jM}	Ω_{MM}	α_j	α_M	ν	ρ_{jM}
1%	-1.050	0.239	0.400	0.002	0.243	-0.304	-1.611	2.306	0.002
5%	-0.408	0.332	0.705	0.079	0.311	-0.165	-1.301	2.805	0.100
25%	0.010	0.398	1.501	0.293	0.387	0.062	-1.056	3.426	0.313
50%	0.197	0.435	2.608	0.445	0.426	0.213	-0.925	3.875	0.454
75%	0.389	0.469	4.234	0.601	0.470	0.382	-0.815	4.480	0.549
95%	0.776	0.522	7.767	0.908	0.552	0.698	-0.629	5.737	0.651
99%	1.189	0.574	11.369	1.137	0.663	0.987	-0.468	7.617	0.713

This table summarizes the distribution quantiles of the bivariate skew-t parameter estimates for 3518 daily stock and market return pairs between January 2006 and December 2007. The parameters are estimated for each firm by maximum likelihood using the Adelchi Azzalini's "SN" package in "R". The subscript j (M) indicates a parameter estimate that is associated with the individual stock return (equally weighted CRSP portfolio return). Parameter ρ_{jM} is not estimated but calculated as $\rho_{jM} = \sigma_{jM}(\sigma_{jj}\sigma_{MM})^{-0.5}$. Estimates are for 100 times daily log returns.

Figure 1: Symmetric T-Distribution Asymptotic Tail Dependence as a Function of Correlation and Degrees of Freedom

The figure plots the asymptotic tail dependence for various levels of degrees of freedom and correlation for the symmetric bivariate t distribution. Asymptotic tail dependence is given by:

$$L_{sym\ t} = 2T_1\left(-\frac{\sqrt{(\nu+1)(1-\rho)}}{\sqrt{1+\rho}}, \upsilon+1\right)$$

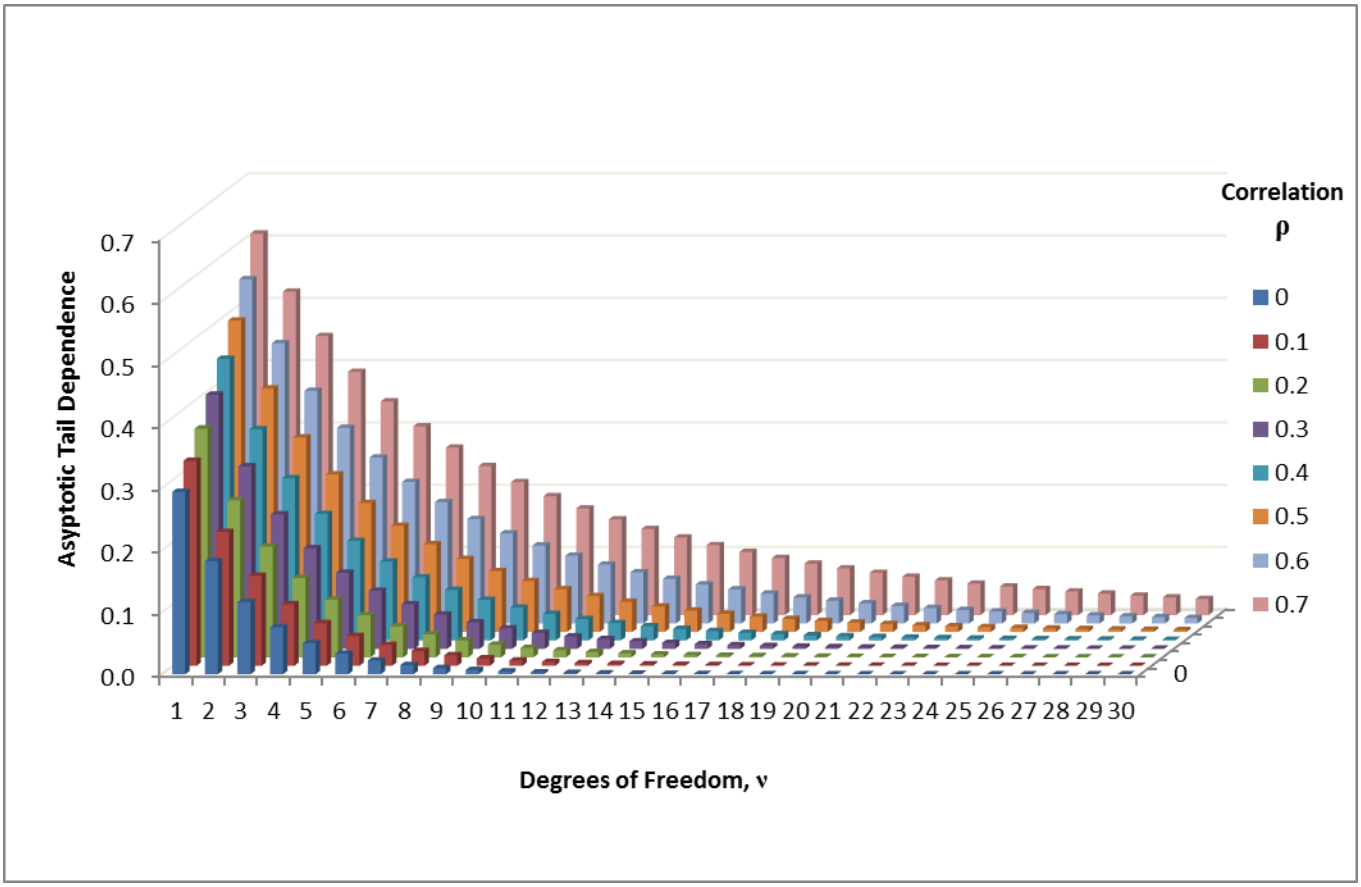


Figure 2: Magnitude of the Asymptotic Left Tail Dependence in a Skew-t Distribution

The figure illustrates the relationship between strength of the asymptotic left tail dependence for the skew-t distribution and the degrees of freedom parameter, ν, correlation parameter, ρ and skewness parameters, $\alpha_1 = \alpha_2 = \alpha$. The distribution's asymptotic tail dependence, L_{skew-t} is given by:

$$L_{skew-t} = \frac{T_1\left(-2\alpha\sqrt{\frac{(\nu+2)(1+\rho)}{2}}; \nu+2\right) * 2T_1\left(-\frac{\sqrt{(\nu+1)(1-\rho)}}{\sqrt{1+\rho}}, \nu+1\right)}{T_1\left(\frac{-\alpha(1+\rho)\sqrt{\nu+1}}{\sqrt{1+\alpha^2(1-\rho^2)}};\ \nu+1\right)}$$

where T_1 represents the cumulative distribution function of univariate student t.

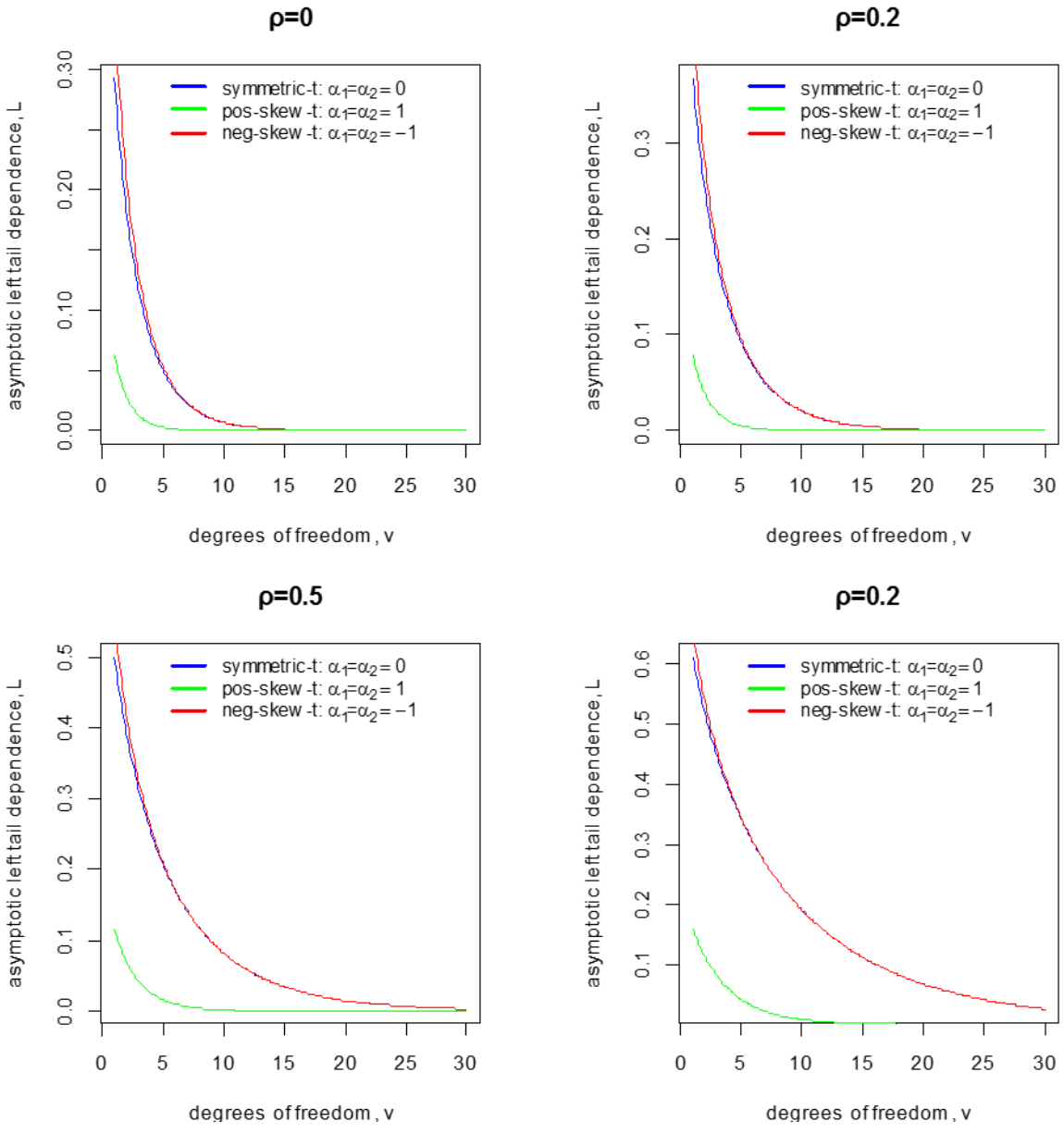


Figure 3: κ_{CoVaR} Power against Symmetric T-Distribution Alternatives

The figure shows the simulated sampling distributions κ_{CoVaR} based on a sample size of 500 and 10 thousand Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When ρ varies Ω_{12} varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the κ_{CoVaR} test statistic under the null hypothesis. The red and light green distributions are symmetric t distributions $(\alpha_1 = \alpha_2 = 0)$ with degrees of freedom $\nu = 3.6\ (light\ green)$, $\nu = 2.5\ (red)$ and median parameter values from Table 6 for the remaining parameters. "atd" is an abbreviation for the asymptotic tail dependence of the t-distribution.

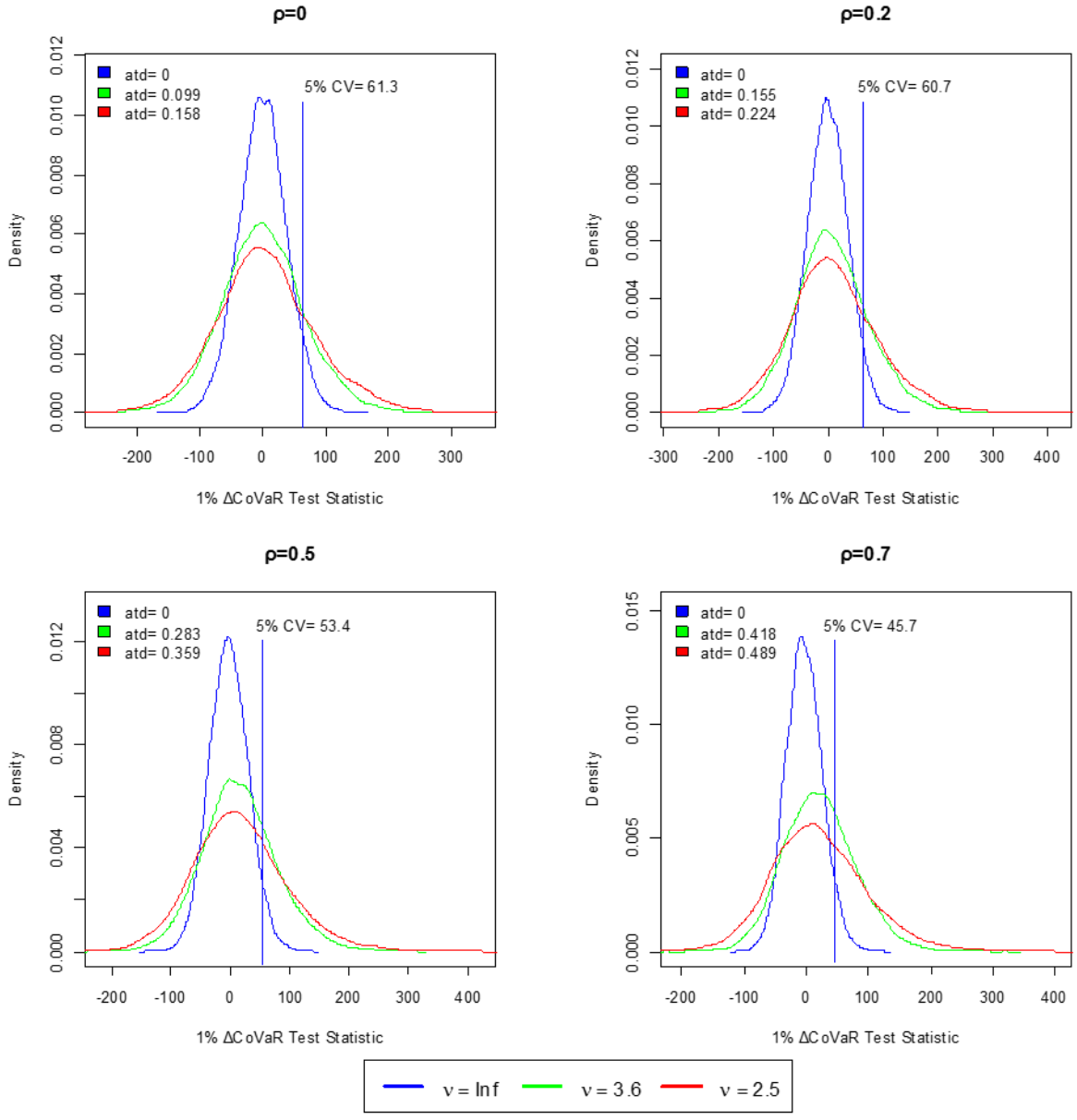


Figure 4: κ_{MES} Power against Symmetric T-Distribution Alternatives

The figure shows the simulated sampling distributions for κ_{MES} based on a sample size of 500 with 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When ρ varies Ω_{12} varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the κ_{MES} test statistic under the null hypothesis. The red and light green distributions are symmetric t distributions $(\alpha_1 = \alpha_2 = 0)$ with degrees of freedom $\nu = 3.6$ $(light\ green)$, $\nu = 2.5$ (red) and median parameter values from Table 6 for the remaining parameters. "atd" is an abbreviation for the asymptotic tail dependence of the t-distribution.

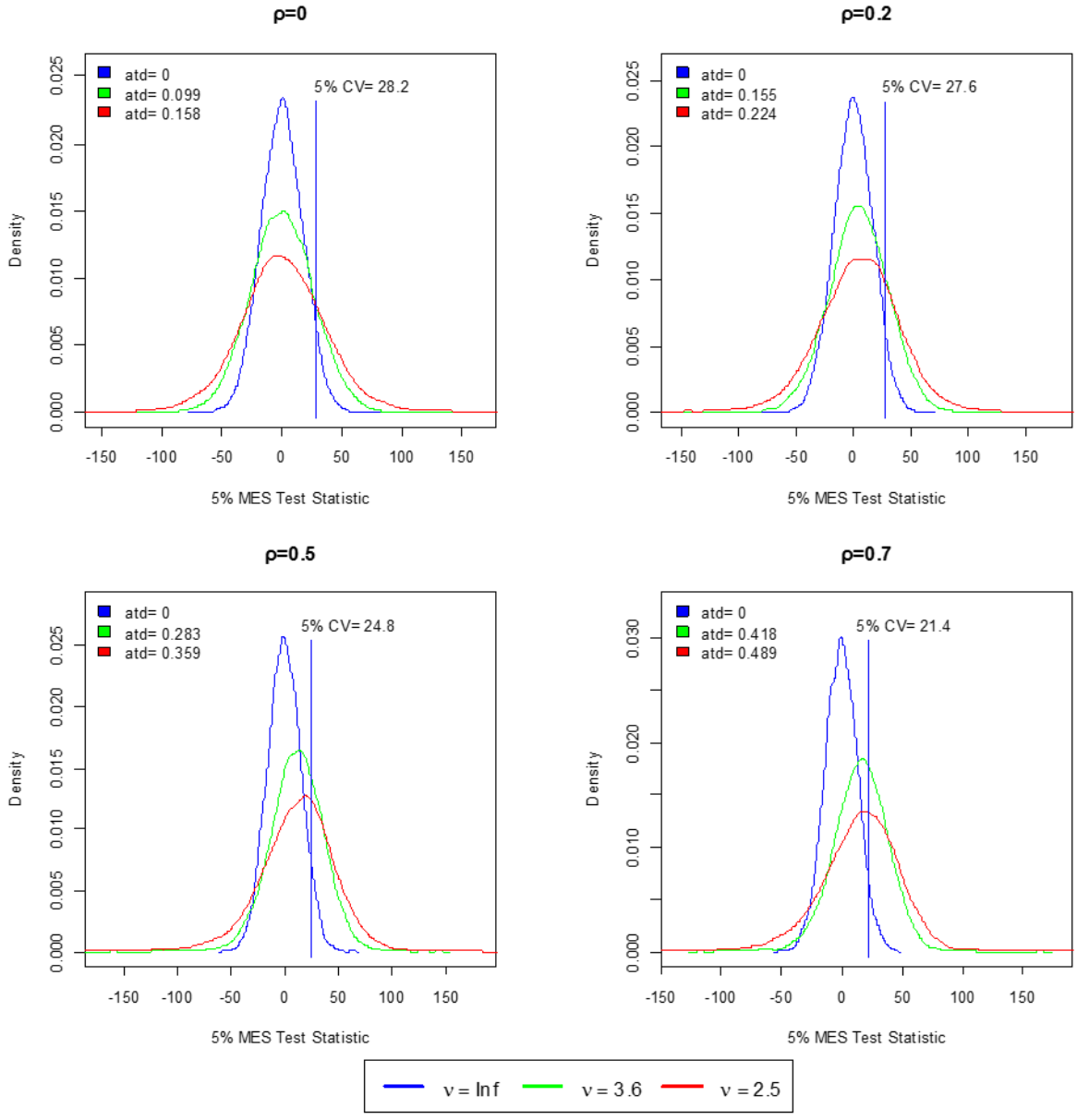


Figure 5: κ_{CoVar} Tests Statistics when Individual Stocks are Positive-Skewed Gaussian

The figure shows the simulated sampling distributions for κ_{CoVaR} based on a sample size of 500, with 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When ρ varies Ω_{12} varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the κ_{CoVaR} test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is symmetric and the individual stock return is positively skewed. The light green distribution has mild positive skewness $(\alpha_1 = .2\,, \alpha_2 = 0, \nu = \infty\,)$ while the red distribution has a stronger positive skew $(\alpha_1 = .8\,, \alpha_2 = 0, \nu = \infty\,)$. Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.

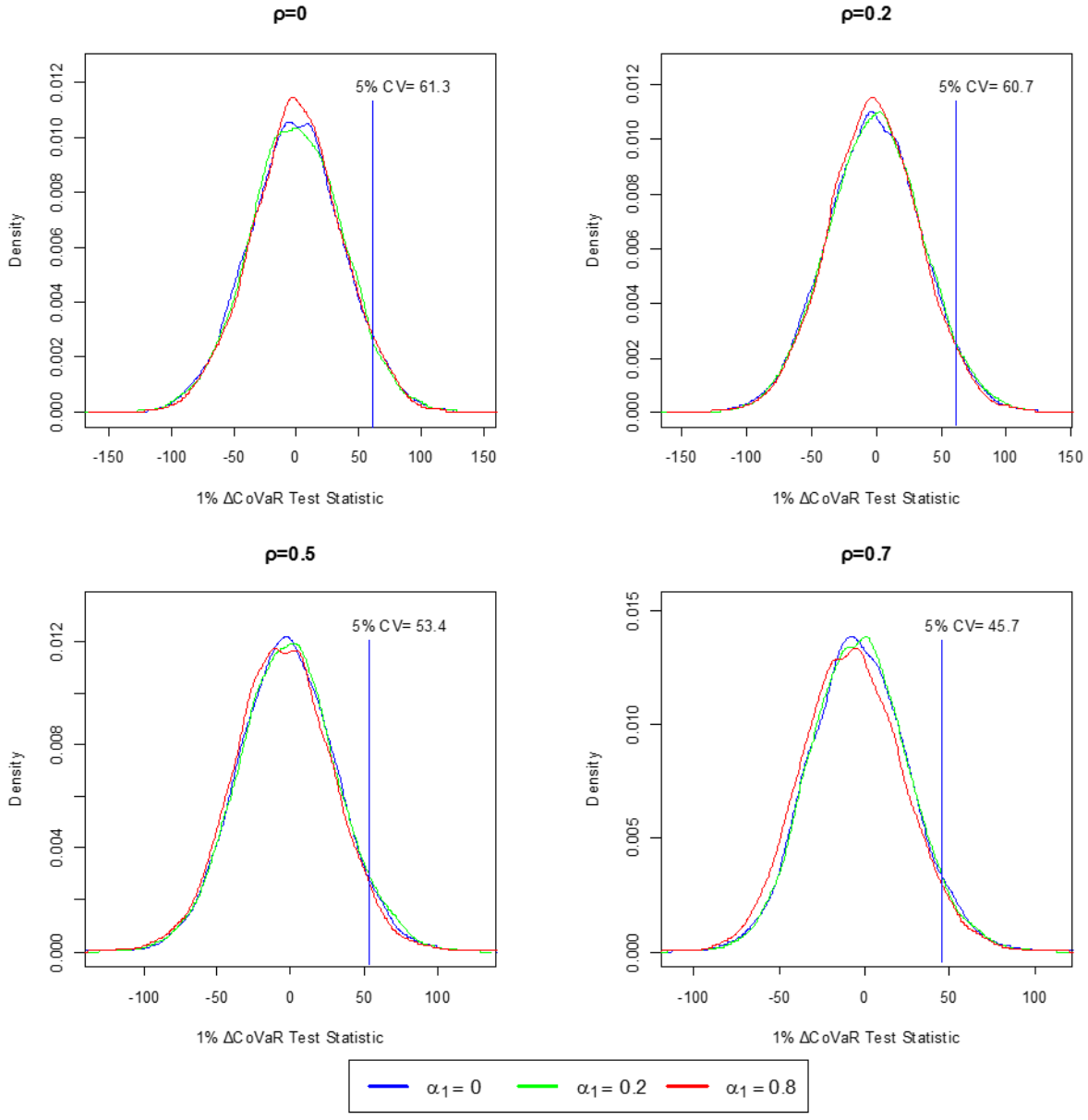


Figure 6: κ_{MES} Tests Statistics when Individual Stocks are Positive-Skewed Gaussian

The figure shows the simulated sampling distributions for κ_{MES} based on a sample size of 500 with 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When ρ varies Ω_{12} varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the κ_{MES} test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is symmetric and the individual stock return is positively skewed. The light green distribution has mild positive skewness $(\alpha_1 = .2\,, \alpha_2 = 0, \nu = \infty\,)$ while the red distribution has a stronger positive skew $(\alpha_1 = .8\,, \alpha_2 = 0, \nu = \infty\,)$. The remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.

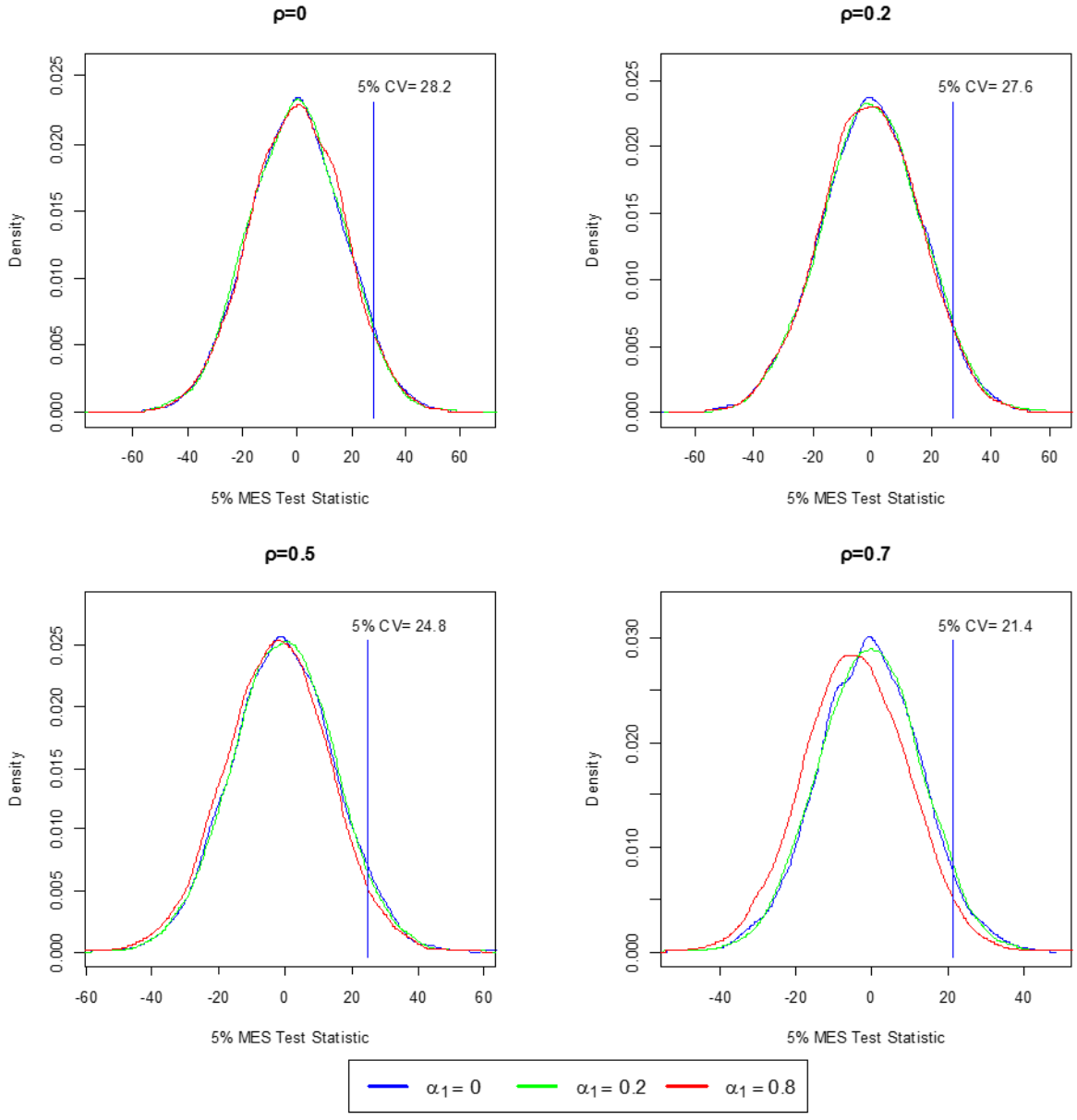


Figure 7: κ_{CoVar} Tests Statistics when Market Returns are Negative-Skewed Gaussian

The figure shows the simulated sampling distributions for κ_{CoVaR} based on a sample size of 500 with 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When ρ varies Ω_{12} varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the κ_{CoVaR} test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the individual stock return is symmetric and the market return is negatively skewed. The light green distribution has mild negative skewness $(\alpha_1 = 0, \alpha_2 = -1, \nu = \infty)$ while the red distribution has a stronger negative skew $(\alpha_1 = 0, \alpha_2 = -1.6, \nu = \infty)$. Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.

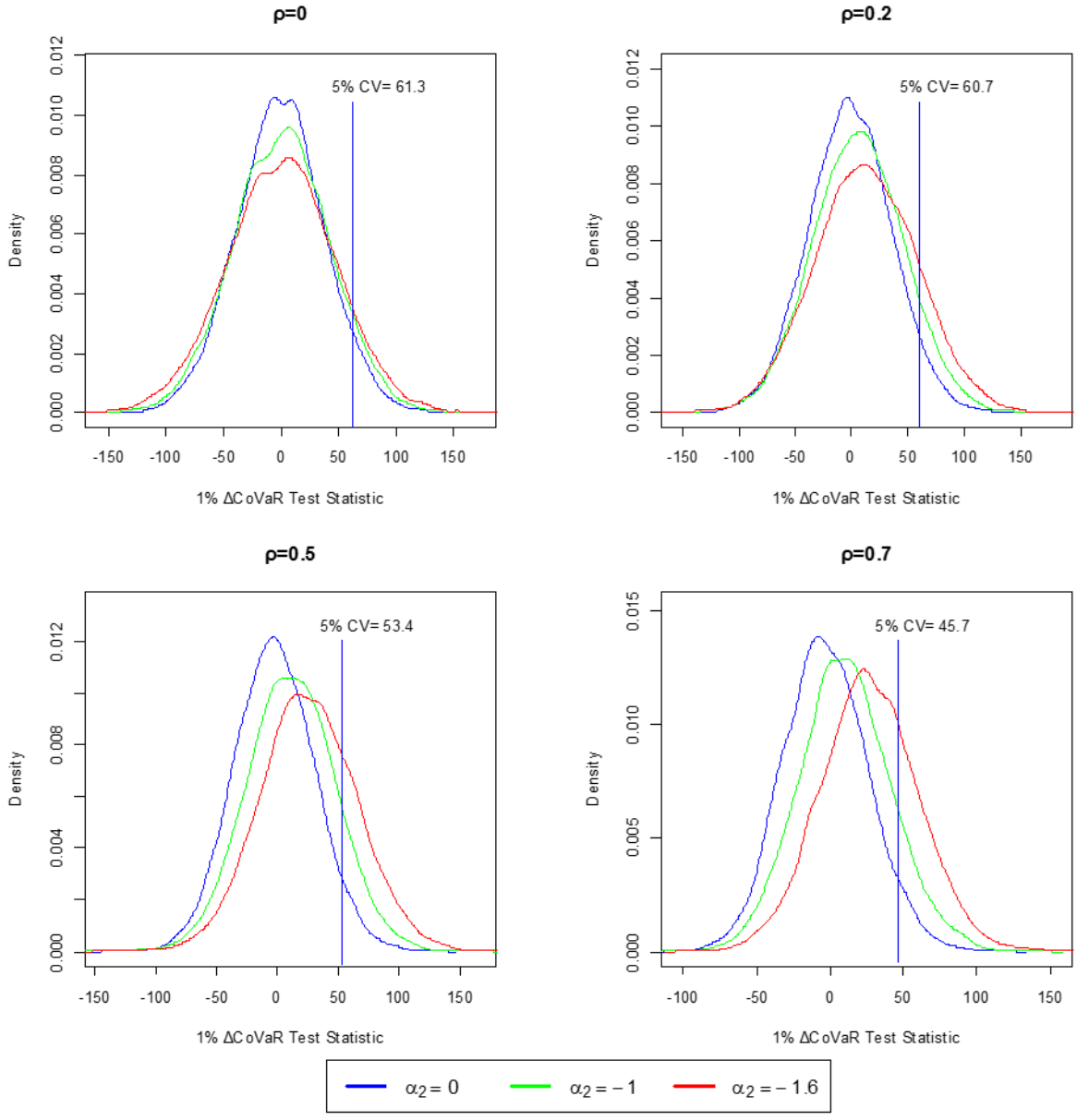


Figure 8: κ_{MES} Tests Statistics when Market Returns are Negative-Skewed Gaussian

The figure shows the simulated sampling distributions for κ_{MES} based on a sample size of 500, 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When ρ varies Ω_{12} varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the κ_{MES} test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is symmetric and the individual stock return is positively skewed. The light green distribution has mild negative skewness $(\alpha_1 = 0\,, \alpha_2 = -1, \nu = \infty\,)$ while the red distribution has a stronger negative skew $(\alpha_1 = 0\,, \alpha_2 = -1.6, \nu = \infty\,)$. Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.

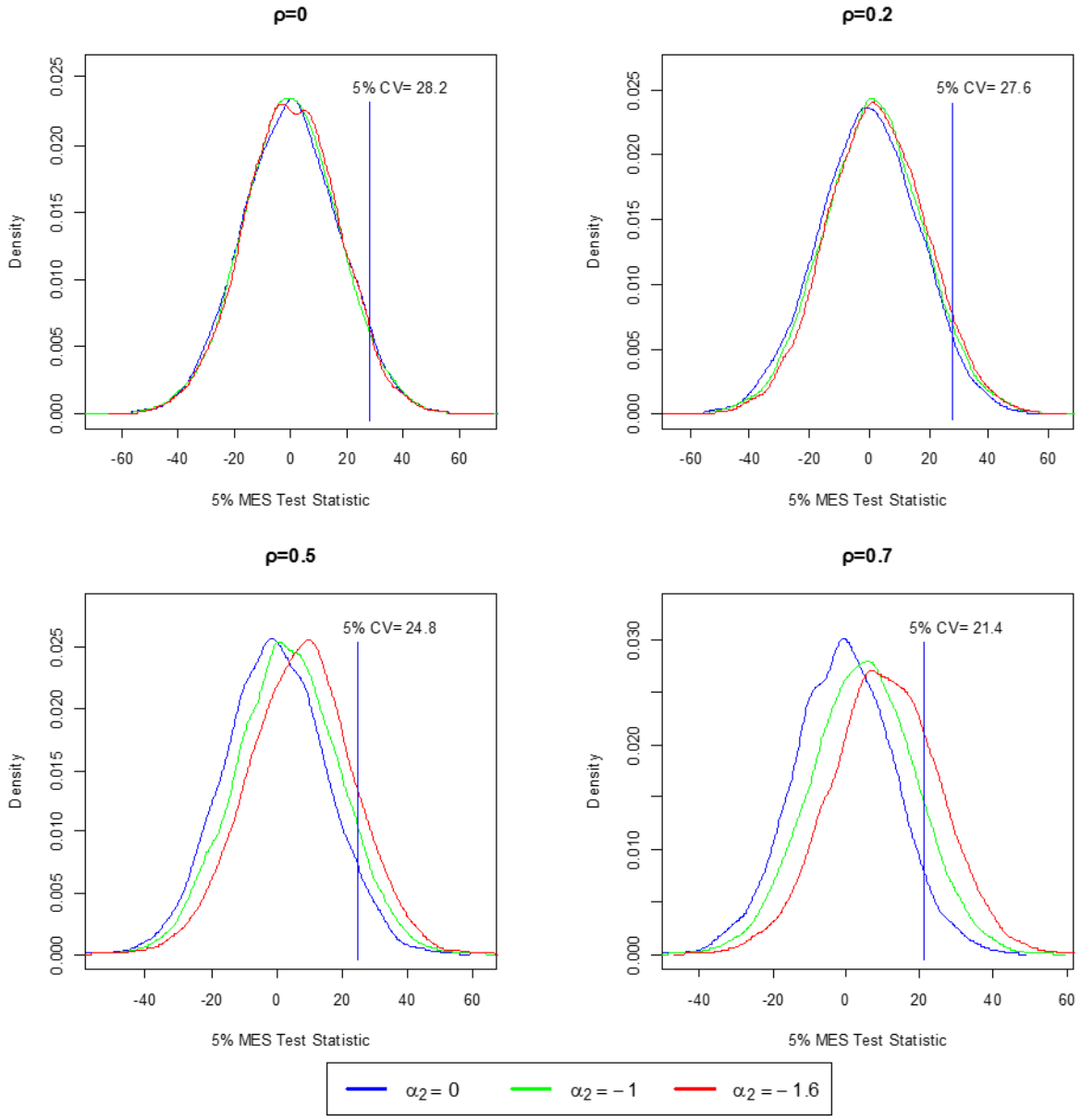


Figure 9: κ_{CoVar} Tests Statistics when Individual Stocks are Positive-Skewed and Market Returns are Negative-Skewed Gaussian

The figure shows the simulated sampling distributions for κ_{CoVaR} based on a sample size of 500, 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When ρ varies Ω_{12} varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the κ_{CoVaR} test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is negatively skewed and the individual stock return is positively skewed. The light green distribution has strong positive skewness $(\alpha_1 = .8\,, \alpha_2 = -1, \nu = \infty\,)$ while the red distribution has a lower positive skew $(\alpha_1 = .2\,, \alpha_2 = -1.6, \nu = \infty\,)$. Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.

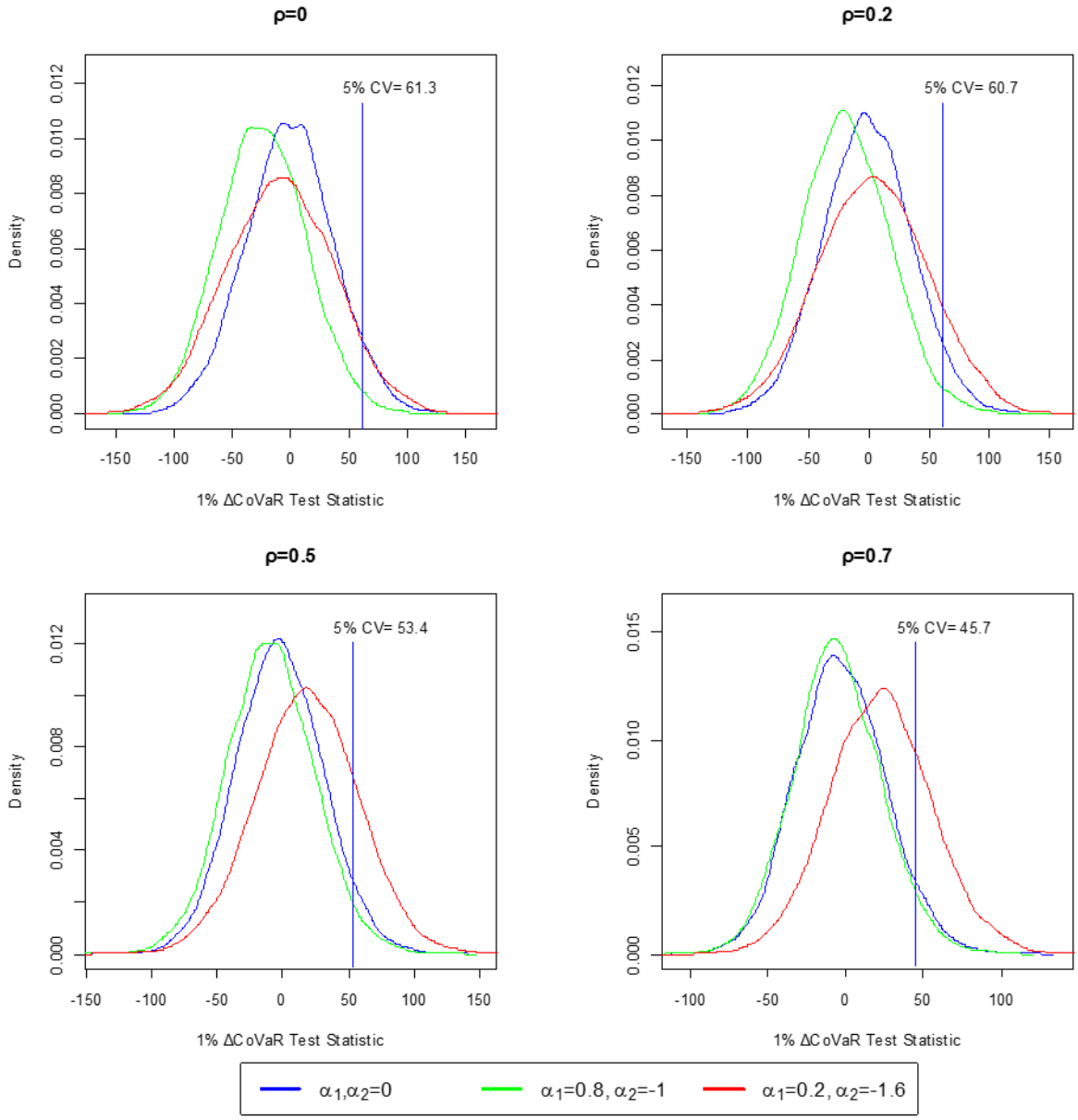


Figure 10: κ_{MES} Tests Statistics when Individual Stocks are Positive-Skewed and Market Returns are Negative-Skewed Gaussian

This figure plots the simulated sampling distributions for κ_{MES} based on a sample size of 500, 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When ρ varies Ω_{12} varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the κ_{MES} test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is symmetric and the individual stock return is positively skewed. The light green distribution has strong positive skewness $(\alpha_1 = .8\,, \alpha_2 = -1, \nu = \infty\,)$ while the red distribution has lower positive skew $(\alpha_1 = .2\,, \alpha_2 = -1.6, \nu = \infty\,)$. Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.

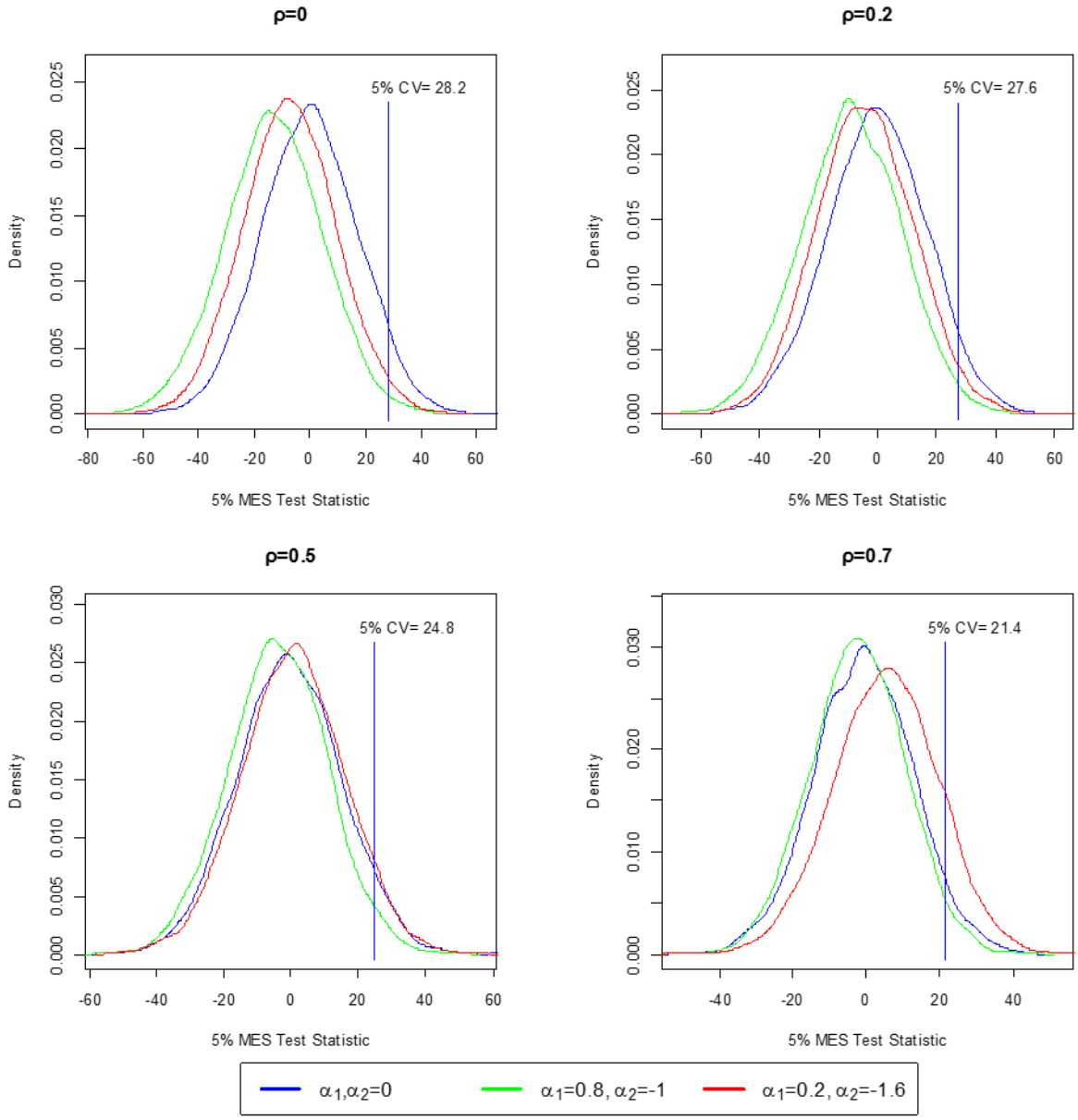
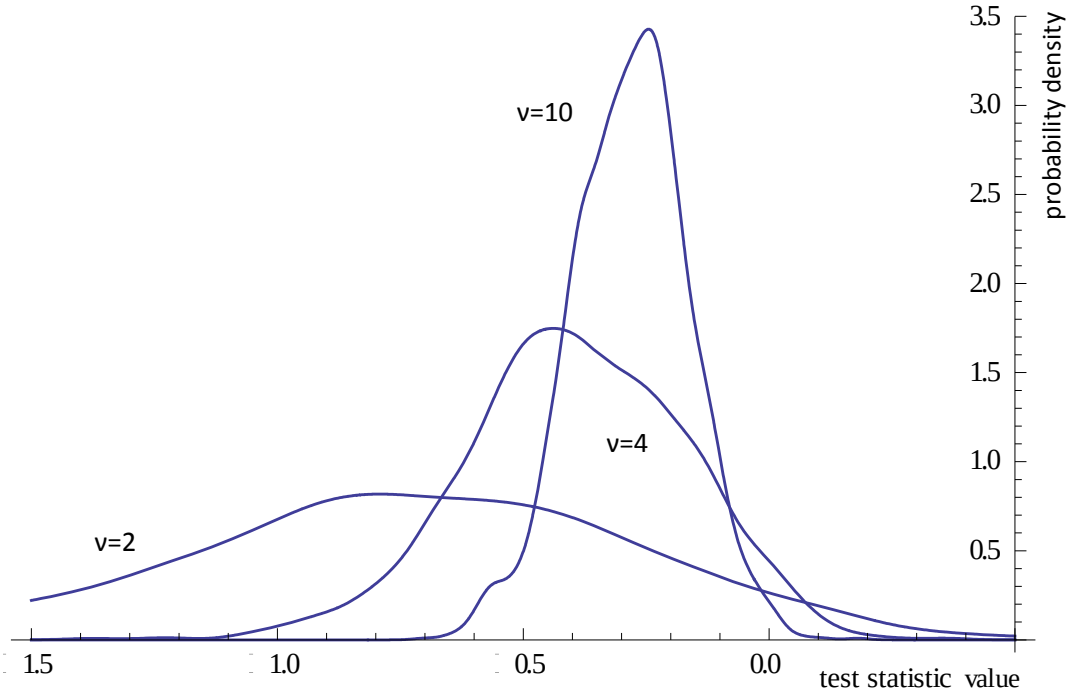
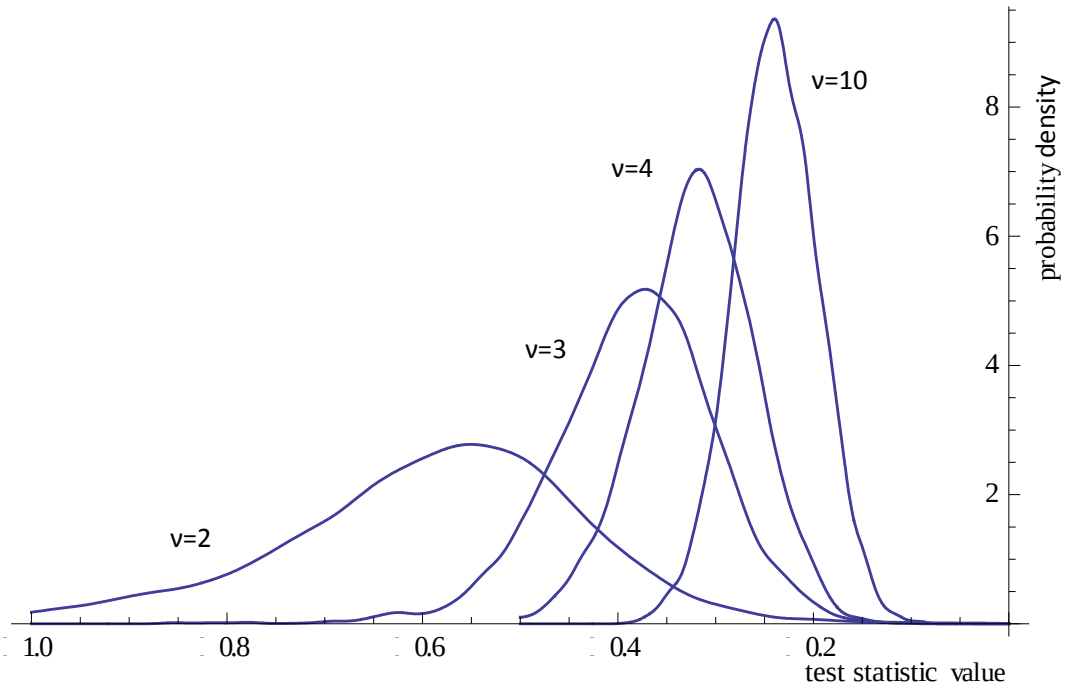


Figure 11: The Distribution of the Nonparameteric ΔCoVaR Estimator and Asymptotic Tail Dependence



The sampling distribution of the nonparmetric ΔCoVaR estimator is estimated for different levels of asymptotic tail dependence based on a Monte Carlo simultaion of 2000 samples of size 500 generated from a bivariate t distribution with $\sigma_M=\sigma_j=.20$, and $\rho_{jM}=.50$. A Gaussian Kernel density estimator is used to smooth the Monte Carlo estimates. ν indicates the Student t degrees of freedom parameter.

Figure 12: The Distribution of the Nonparamteric MES Estimator and Asymptotic Tail Dependence



The sampling distribution of the nonparametric MES estimator for different levels of asymptotic tail dependence based on a Monte Carlo simultaion of 5000 samples of size 500 generated from a bivariate t distribution with $\sigma_M=\sigma_j=.20$, and $\rho_{jM}=.50$. A Gaussian Kernel density estimator is used to smooth the Monte Carlo estimates. ν indicates the Student t degrees of freedom parameter.