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Delegation and Incentives

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Helmut Bester and Daniel Krähmer

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Abstract

This paper analyses the relation between authority and incentives. It extends the standard principal-agent model by a project selection stage in which the principal can either delegate the choice of project to the agent or keep the authority. The agent’s subsequent choice of effort depends both on monetary incentives and the selected project. We find that the consideration of effort incentives makes the principal less likely to delegate the authority over projects to the agent. In fact, if the agent is protected by limited liability, delegation is never optimal.

Keywords: Authority, delegation, incentives, moral hazard, principal-agent problem, limited liability; JEL Classification No.: D82, D86.
1 Introduction

How are decision rights and effort incentives related in the design of an organization? By specifying a structure of authority an organization determines which of its members have the right to select certain decisions. Its overall efficiency depends on how closely the individual decision makers’ interests are aligned with the organization’s objective. The structure of authority, however, also determines to what extent the organization’s members are affected by decisions that are taken by other members (see Simon (1951)). This in turn influences their incentives to provide effort for the organization’s success. The optimal allocation of authority and the provision of effort incentives are therefore interdependent. [1]

As an example, consider investment decisions within a firm. If the management derives private benefits from ‘empire building’, it favours projects that increase the firm’s size. It tends to undertake inefficiently large investments; but it is also willing to invest more effort on such projects as they generate larger private benefits. In contrast, when the firm owners take investment decisions, they are concerned with maximizing the firm’s market value rather than its size. Yet, they have to take into account that the management may show little enthusiasm to spend much effort on projects that prevent it from ‘empire building’.

To study the interaction between authority and effort incentives, we extend the standard principal–agent environment (see Holmstrom (1979), Grossmann and Hart (1983), Sappington (1983)), in which the principal provides the agent with incentives to exert a non-observable effort on a joint project. [2] The agent’s effort determines the likelihood that the project succeeds. Whereas in the standard model the project is taken as given, we add a project selection stage where one out of a number of feasible projects is chosen. To create a role for decision rights, we follow the literature (e.g. Aghion and Tirole (1997), Aghion, Dewatripont and Rey (2002), Holmstrom and Hart (2002)) by assuming that only the authority over project selection is contractible, because the selection of a particular project is neither ex ante nor ex post verifiable. Thus, in addition to a wage schedule that is contingent on the project’s outcome, the contract between the principal and the agent specifies which party has the right to select a project. The principal can either maintain the decision right over project selection or he can delegate this right to the

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[1] This point is noted already by Mirrlees (1976), who studies the optimal structure of incentives and authority in a hierarchical structure to explain the distribution of incomes within the firm.

agent. Since the agent’s private benefits vary with the type of project, his effort incentives are determined jointly by the wage schedule and the allocation of authority.

Our main finding is that the consideration of effort incentives makes the principal less likely to delegate the authority over projects to the agent. This surprising observation contrasts with the incentive view of delegation developed by Aghion and Tirole (1997). In their model the principal delegates authority in order to induce the agent to acquire information about the benefits of different projects. Because the transfer of formal authority allows the agent to select his favourite project, he will invest more effort in information acquisition.

In contrast, in our model the principal tends to keep the authority over project selection especially when he wants the agent to invest high effort. There are two main differences with the model of Aghion and Tirole (1997) that explain why the principal refrains from using delegation as an incentive device: First, in Aghion and Tirole (1997) the principal has no other means to provide effort incentives because he cannot use monetary incentives. In our model, also monetary incentives are available because the agent’s wage can be conditioned on the project outcome. Thus, instead of delegating authority to the agent, the principal may use bonus payments to induce the agent to exert effort.

The second and more significant difference is that in our model the agent’s effort choice occurs after a project has been determined; the agent’s task thus consists of completing a project. In contrast, in Aghion and Tirole (1997) the agent invests effort before the selection stage to screen the set of potential projects. This difference in timing has an important consequence for the selection of projects if the principal maintains the decision right over projects. While in Aghion and Tirole (1997) effort is sunk at the project selection stage, in our model the principal anticipates that – for a given bonus system – effort incentives increase with the agent’s private benefits from the project. Therefore, the agent’s preferences affect the choice of project even when the principal keeps authority. Since the principal’s choice takes into account that the agent is motivated by his private benefits, delegating authority for incentive reasons becomes less attractive.

We consider two types of environments in our analysis. We first assume that there are no restrictions on monetary transfers between the principal and the agent. In this case, the agent is not protected by limited liability and the principal can extract the entire surplus from the relation. Therefore, the optimal contract maximizes the expected

\footnote{In Section V.B of their paper Aghion and Tirole (1997) briefly discuss monetary incentives in an extension of their basic model.}
joint benefits subject to the agent’s moral hazard constraint and the restriction that only
decision rights are contractible. In the absence of incentive effects, control rights would
therefore be given to the party whose favourite project generates higher joint benefits.
Indeed, this is what happens if the agent’s cost is rather high so that inducing effort
is suboptimal. In this case, delegation of authority occurs if the agent’s ideal project
generates more surplus than the principal’s ideal project. In contrast, when providing
incentives becomes optimal with lower effort costs, the range of parameter constellations
where the principal refrains from delegation expands. Actually, for sufficiently low costs
the principal always keeps authority. As explained above, the reason is that the principal
takes into account that the agent’s effort is positively related to his private benefits from
the selected project. Because the principal at least partially internalizes the externality
of his choice on the agent’s preference, the joint benefits under his authority are higher
than under delegation.

In the second environment we assume that limited liability on the part of the agent
precludes negative wages. We find that in this situation it is never optimal for the principal
to delegate the decision right to the agent. The reason is that limited liability prevents
the principal from extracting the agent’s surplus. Therefore, the ex ante optimal contract
no longer maximizes the joint surplus. With limited liability, the principal’s ex ante
interest at the contracting and his ex post objective at the project selection stage are
identical. This implies that he cannot gain by delegating authority to the agent. Indeed,
if delegation were optimal for incentive reasons, then it would also be optimal for the
principal to select the same project as the agent. Thus by keeping authority, the principal
can always ensure himself at least the same payoff as by delegation. Actually, we can show
that he can even do better than selecting the agent’s favourite project, which implies that
delegation is inferior.

The literature on agency provides several insights into the relation between incentives
and organizational design that are related to our analysis. For example, as shown by
Holmstrom and Milgrom (1991), in a multi-tasking environment the number of tasks that
an agent optimally performs depends on the reliability of performance measures. The
underlying problem is that increasing the incentive for one task may induce the agent
to spend less effort on other tasks. A similar effect explains why in our model it may
be suboptimal for the principal to delegate authority. Under delegation the agent faces
the dual task of selecting a project and devoting effort on its completion. Even in the
absence of limited liability restrictions, the principal cannot design a payment schedule
that induces the agent to perform both tasks efficiently.
In our analysis, the only available performance measure of the agent’s effort is the project outcome. This differs from Prendergast (2002), where the principal’s delegation decision depends on the choice between monitoring inputs or outputs. If he monitors the agent’s effort input, he restricts the set of activities that the agent is allowed to engage in. Alternatively, the principal can monitor the agent’s output and delegate the choice of action to the agent. The comparison between these alternatives shows that the principal will delegate decision-making power more in uncertain environments.

Several authors investigate the relation between private information and the allocation of authority. Riordan and Sappington (1987) consider a two-stage production process where the party that carries out production at any stage becomes privately informed about its cost. The principal and the agent are equally adept at performing the second production stage. It turns out that it depends on the correlation of costs at the two stages whether the principal prefers to delegate second stage production to the agent or not. Athey and Roberts (2001) consider performance-based incentive contracts that must be designed to balance the dual goals of effort provision and efficient investment decisions. They argue that it may be optimal to assign decision rights to someone other than the best informed party. Dessein (2002) studies delegation as an alternative to communication. Under delegation the principal grants decision rights to an agent who is better informed but has different objectives. Alternatively, the principal may keep authority and base his decision on the information reported by the agent. In this setting, the principal optimally delegates control as long as the divergence in objectives is not too large. To focus on the relation between effort incentives and authority, our analysis abstracts from private information. Yet, as we point out in the concluding remarks of this paper, it may be interesting to extend our model by studying the allocation of decision rights when both incentives for information revelation and effort incentives play a role.

The remainder of this paper is organized as follows. Section 2 extends the standard principal–agent framework by introducing decision rights over projects as part of the contracting problem. In Section 3, we consider the relation between authority and incentives in the absence of limited liability restrictions. Section 4 analyses the optimal allocation of decision rights when the agent is protected by limited liability. Section 5 contains concluding remarks. The proofs of all formal results are relegated to an appendix in Section 6.
2 The Model

We consider a principal and an agent who can jointly undertake a project \( d \in D \), where \( D = [0, 1] \) is set of feasible projects. The selection of a particular project is not verifiable to outsiders and, hence, not contractible. Only the decision right over \( D \) can be assigned contractually either to the principal or to the agent. If the principal keeps authority, he maintains control over the critical resources to initiate a project. Otherwise, if he delegates the decision right, he transfers the control over these resources to the agent.

Whether the selected project succeeds or fails depends on the agent’s effort \( e \in \{e_L, e_H\} \). The agent chooses his effort after a project \( d \) has been determined. Even though the choice of \( d \) is not publicly verifiable, we assume that it is internally observable for the principal and the agent. Thus, at the stage where the agent chooses his effort, he is informed about the project \( d \) also when the principal has the decision right. If the agent selects effort \( e \), he incurs the effort cost \( c(e) \) and the project succeeds with probability \( p(e) \). Let \( p_H \equiv p(e_H) > p_L \equiv p(e_L) > 0 \) and \( c \equiv c(e_H) > c(e_L) \equiv 0 \). As in the standard principal-agent model with moral hazard, the agent’s effort choice is not observable.

In the event of project failure the private benefits of the principal and the agent are zero. If the project succeeds, the principal and the agent receive the private and non-verifiable benefits \( u_P \) and \( u_A \), respectively. These benefits depend on the selected project as:

\[
\begin{align*}
 u_P(d|k_P) &= r_P - k_P \ell(|d_P - d|), \\
 u_A(d|k_A) &= r_A - k_A \ell(|d_A - d|).
\end{align*}
\]

with \( 0 < k_P < r_P, 0 < k_A < r_A \) and \( \ell(0) = \ell'(0) = 0, \ell(1) = 1 \), and \( \ell'(x) > 0, \ell''(x) > 0 \) for all \( x > 0 \). Thus the principal’s benefit reaches a unique maximum for \( d = d_P \), and the agent’s benefit is maximized for \( d = d_A \). The principal and the agent have conflicting interests over the selection of a project because\(^5\)

\[
0 \leq d_A < d_P \leq 1.
\]

The ‘loss’ function \( \ell(\cdot) \) represents each party’s utility loss as an increasing function of the distance between his ideal and the actual project. The weights \( k_P \) and \( k_A \) describe how much the principal and the agent care about the selection of a project. These weights will turn out to be important for the optimal allocation of decision rights.

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\(^4\) A similar preference structure is used in e.g. Crawford and Sobel (1982) and Dessein (2002).

\(^5\) The assumption that \( d_A < d_P \) is not significant. What is important is that the principal and the agent have different ideal projects.
Success and failure of the project are publicly verifiable. If the project succeeds, the principal pays the agent the wage $w_S$; in the case of failure the agent receives the wage $w_F$. Let $w = (w_S, w_F)$. Then the expected payoffs of the principal and the agent are

$$U_P(d, e, w|k_P) \equiv p(e)[u_P(d|k_P) - w_S] - (1 - p(e))w_F, \quad \quad (3)$$
$$U_A(d, e, w|k_A) \equiv p(e)[u_A(d|k_A) + w_S] + (1 - p(e))w_F - c(e).$$

As the agent’s outside option payoff is $U_A = 0$, the principal has to design a contract so that

$$U_A(d, e, w|k_A) \geq 0 \quad \quad (4)$$
guarantees the agent’s participation. We assume that neither the agent nor the principal can credibly threaten to quit after a project has been selected.

In addition to the agent’s participation constraint, the principal faces the usual incentive constraint because the agent’s effort is not observable. The agent selects the effort

$$e = \tilde{e}(d, w) \equiv \arg\max_{e \in \{e_L, e_H\}} U_A(d, e, w|k_A), \quad \quad (5)$$
where as a tie-breaking rule we assume $\tilde{e}(d, w) = e_H$ if the agent is indifferent between high and low effort. Note that the agent’s effort incentives depend not only on the wage schedule $w$ but also on the project $d$. The higher his private benefit $u_A(d|k_A)$, the more inclined is the agent to exert high effort.

Since $d$ is not contractible, the principal offers the agent a contract which in addition to the wages $w$ specifies which party gets the authority to select the project. We describe the allocation of authority by $h \in \{P, A\}$. Thus, if $h = P$ the principal retains the right to select $d \in D$; if $h = A$ he delegates the selection of a project to the agent. If party $h \in \{P, A\}$ has the authority over the project decision, it will select $d$ to maximize its own expected payoff ex post after wages have been set at the contracting stage. Therefore, $d$ will satisfy

$$d = \tilde{d}(h, w) \equiv \arg\max_{d \in D} U_h(d, \tilde{e}(d, w), w|k_h) \quad \quad (6)$$
Note that according to (5) the agent’s effort depends on $d$. Therefore, if $h = P$, the principal’s decision $\tilde{d}(P, w)$ takes this incentive effect into account. In contrast, if $h = A$, the agent’s decision $\tilde{d}(A, w)$ is simply $d_A$, independently of $w$. 


The time structure of the model is summarized in Figure 1. First the principal and the agent sign a contract that specifies the wage schedule $w$ and the party $h$ who has the authority to select a project $d$ at the subsequent stage. After a project has been determined, the agent chooses his effort $e$. This choice affects the probability of success and failure in the final stage.

In the following we study the optimal contract in two settings: We first consider the case without restrictions on the wage schedule $w$. In this case, the principal’s problem is to choose $(h, d, e, w)$ so that his expected payoff $U_P(d, e, w|k_P)$ is maximized subject to the constraints (4)–(6). Then we consider the case where limited liability or wealth restrictions prevent payments from the agent to the principal. In this case, the principal faces the additional constraint $w \geq 0$. In both cases, we illustrate our analytical results by a numerical example with a quadratic loss function $\ell(x) = x^2$. Further we set $d_A = 0$, $d_P = 1$, $r_A = r_P = 1$, $p_H = 8/10$, $p_L = 4/10$ and $k_A = 1/2$. This allows us to describe how the optimal contract depends on the agent’s effort cost $c$ and the principal’s preference intensity $k_P$.

### 3 Authority and Incentives

In this section we study the optimal allocation of authority in the absence of non-negativity restrictions on the wage schedule $w$. Thus the agent is not protected by limited liability and he may face a penalty $w_F < 0$ if the project fails. Obviously, in this situation the agent’s participation constraint (4) is always binding for a solution of the principal’s problem. This means that the principal can appropriate the entire expected surplus

$$p(e)\left[u_P(d|k_P) + u_A(d|k_A)\right] - c(e).$$

Effectively, without limited liability restrictions the principal’s problem is equivalent to maximizing the expected surplus in (7) subject to (5) and (6).
The principal’s problem would be trivial if the decision $d$ was contractible, i.e., in the absence of restriction (6). In this case, the principal could achieve the first-best by contractually committing to the surplus maximizing decision

$$d^*(k_P, k_A) \equiv \arg \max_{d \in D} \left[u_P(d|k_P) + u_A(d|k_A)\right].$$

and to a wage schedule that induces the agent to exert effort whenever this is optimal. Note that the specification of preferences in (1) implies that $d_A < d^* < d_P$. It is also useful to note that, due to the symmetry of $\ell$, the joint surplus in (7) is the larger the closer is the decision $d$ to the surplus maximizing decision $d^*$.

When only decision rights are contractible, the principal faces a fundamental commitment problem when he keeps the decision right: From an ex ante point of view, he would like to commit to the first-best project $d^*$, which maximizes the joint surplus. However, ex post, after the agent has accepted the contract, he selects the project which maximizes his expected private benefits net of expected wage payments. Thus, the principal’s ex ante and ex post interests diverge. This is a basic consequence of the non-contractibility of $d$.

To gain further insights into the principal’s commitment problem, it is useful to consider the benchmark case in which the probability of success, $p$, is fixed, and there is no effort choice for the agent. Suppose that the principal keeps authority. Then ex post, at the project selection stage, he maximizes $p[u_P(d) - w_S] - (1 - p)w_F$. With a given success probability, he thus selects $d_P$, and hence, under $P$-authority he realizes the ex ante surplus $p[u_P(d_P) + u_A(d_P)]$.

When he delegates the decision instead, the agent obviously chooses his ideal project $d_A$. Hence, under $A$-authority, the principal realizes the ex ante surplus $p[u_P(d_A) + u_A(d_A)]$. It is easy to see that delegation is better than $P$-authority when the agent cares more about the decision than the principal, i.e., $k_A > k_P$. This is so since in this case the agent’s decision $d_A$ is closer to the first-best decision $d^*$ than is the principal’s decision $d_P$. In other words, when $k_A > k_P$, the principal can mitigate his commitment problem by delegating the decision to the agent.

We now turn to the problem when incentive considerations matter. We first study the optimal contract under $A$-authority, where the principal delegates the decision right to the agent by setting $h = A$. In this case, constraint (6) immediately implies that the

\[6^*\text{This observation is identical to Proposition 4 in Bester (2005), who studies the optimal allocation of decision rights in the absence of incentive effects.}\]
agent always selects his ideal project \( d_A \). Thus, the principal can only decide whether he wants to implement high effort by a steep wage schedule or low effort by a flat schedule. The next Proposition states this formally.

**Proposition 1** There is a \( \tilde{c}_I(k_P, k_A) > 0 \) such that the optimal project-effort combination under \( A \)-authority has the following properties:

(i) If \( c \leq \tilde{c}_I \), then \( d_A \) and \( e_H \) are implemented.

(ii) If \( c > \tilde{c}_I \), then \( d_A \) and \( e_L \) are implemented.

Figure 2 illustrates the optimal implementation of effort under \( A \)-authority. For parameter values of \( k_P \) and \( c \) that lie in region \( I \), the effort cost \( c \) is sufficiently small so that the principal optimally induces the agent to exert high effort. The borderline between regions \( I \) and \( II \) is defined by \( c = \tilde{c}_I(k_P, 1/2) \). Above this line, in region \( II \), the effort cost is too large and so the principal optimally implements low effort under \( A \)-authority.

Next, we study the optimal contract under \( P \)-authority, where the principal maintains the decision right by setting \( h = P \). When the principal selects the project ex post, he takes into account the agent’s effort incentives as described in (5). This is the critical difference to the previously described benchmark case in which the success likelihood is fixed. In fact, to induce the agent to select high effort, it may now be optimal for the
principal not to select his ideal project $d_P$ but some $d < d_P$. Of course, this can happen only if the bonus $w_S - w_F$ by itself is not sufficient to provide effort incentives.

To understand the interaction between the principal’s project choice and the bonus, consider the extreme cases in which the bonus $w_S - w_F$ is either very large or very small relative to the effort cost $c$. Then the agent either works hard anyway or shirks anyway, irrespective of project choice, and in either case the principal will ex post choose his ideal project. Yet, if the bonus is in a moderate range relative to $c$, the principal’s decision makes a difference to the agent’s effort choice. We call a bonus in this range critical. That is, a bonus $w_S - w_F$ is critical if

$$\bar{c}(d_A, w) = e_H \text{ and } \bar{c}(d_P, w) = e_L.$$  

(9)

A critical bonus determines a largest project $d^c \in (d_A, d_P)$ that is still compatible with high effort by the agent. We call $d^c$ the critical project:

$$d^c = \max \{ d \in (d_A, d_P) | \bar{c}(d, w) = e_H \}.$$  

(10)

Note that the critical project becomes larger as the critical bonus increases. Indeed, if monetary incentives become stronger the agent is more inclined to exert effort also on projects that yield lower private benefits for him.

If the contract specifies a critical bonus, then ex post the principal selects either the critical project, thereby inducing high effort, or his ideal project, thereby forgoing effort but saving in expected wage payments. Thus, a critical bonus generates a commitment effect if the principal selects $d^c$ rather than $d_P$. By (6) this happens if

$$p_H[u_H(d^c|k_p) - w_S] - (1 - p_H)w_F \geq p_L[u_L(d_p|k_p) - w_S] - (1 - p_L)w_F.$$  

(11)

Clearly, the principal can exploit this commitment effect only if ex post he wants the agent to select high effort. If this is not the case, (11) implies that he will choose $d_P$.

Ideally the principal would use a critical bonus that forces him to select the first-best project $d^*$. This, however, is possible only if the agent’s effort cost is sufficiently low. If this cost rises, then also the bonus must rise. This increases expected wages and raises the principal’s incentive to choose $d_P$ rather than $d^*$ ex post. Accordingly, for some intermediate level of the effort cost $c$, the commitment effect becomes weaker and under the optimal contract the critical project $d^c$ moves away from the first-best project. Of course, if $c$ becomes too large then it is no longer optimal to implement high effort and so, without the commitment effect, the principal selects $d_P$. Proposition 2 describes the optimal contract under $P$-authority.
Figure 3: Project–effort combinations under $P$–authority

**Proposition 2** There is a $\tilde{c}_{II}(k_P, k_A)$ and a $\tilde{c}_{III}(k_P, k_A)$ with $0 < \tilde{c}_{II} < \tilde{c}_{III}$ such that the optimal project–effort combination under $P$–authority has the following properties:

(i) If $c \leq \tilde{c}_{II}$, then $d^*$ and $e_H$ are implemented.

(ii) If $c \in (\tilde{c}_{II}, \tilde{c}_{III}]$, then some $d \in (d^*, d_P)$ and $e_H$ are implemented.

(iii) If $c > \tilde{c}_{III}$, then $d_P$ and $e_L$ are implemented.

Figure 3 illustrates Proposition 2 for our numerical example by showing how the optimal project–effort combination under $P$–authority depends on the parameters $k_P$ and $c$. The borderline between regions $I$ and $II$ is defined by $c = \tilde{c}_{II}(k_P, 1/2)$. Thus in region $I$, where the agent’s effort cost is rather low, the principal optimally selects the first–best decision $d^*$, which in combination with the wage schedule induces the agent to select high effort. High effort is also induced for intermediate effort costs in region $II$; but there the principal selects a decision $d \in (d^*, d_P)$. Finally, in region $III$, which lies above the $c = \tilde{c}_{III}(k_P, 1/2)$ schedule, implementing high effort is too costly so that the principal chooses his ideal project $d_P$ and provides no effort incentives by a flat wage schedule with $w_S = w_F$.

By comparing the expected surplus in (7) from the optimal project–effort combinations in Propositions 1 and 2, we can now determine whether maintaining the decision right or
delegating authority to the agent is optimal for the principal. We begin with a technical point which identifies the intersection of the curves $\bar{c}_I$ and $\bar{c}_{III}$.

**Lemma 1** There is a critical $\bar{k}_P(k_A) \in (0, k_A)$ such that $\bar{c}_I(k_P, k_A) > \bar{c}_{III}(k_P, k_A)$ if and only if $0 < k_P < \bar{k}_P(k_A)$.

Together with Propositions 1 and 2, the lemma implies that for $k_P \geq \bar{k}_P$, $P$–authority always (for all $c$) implements at least the same effort as $A$–authority. The next proposition characterizes the optimal allocation of authority.

**Proposition 3** In the absence of limited liability restrictions, the optimal allocation of authority has the following properties:

(i) If $k_P > k_A$, then $P$–authority is uniquely optimal.

(ii) If $k_P \in (\bar{k}_P, k_A)$, then $P$–authority is uniquely optimal for $c < \bar{c}_{III}$ and $A$–authority is uniquely optimal for $c > \bar{c}_{III}$.

(iii) If $k_P \in (0, \bar{k}_P)$, then there is a $\bar{c}_{IV}(k_P, k_A) \in (\bar{c}_{II}, \bar{c}_{III}]$ such that $P$–authority is uniquely optimal for $c < \bar{c}_{IV}$ and $A$–authority is uniquely optimal for $c > \bar{c}_{IV}$.
Figure 4 summarizes Proposition 3 for our example. In regions I and II the optimal contract entails $P$-authority; high effort is implemented in region I and low effort in region II. Delegating authority to the agent is optimal in regions III and IV; in region III high effort and in region IV low effort is implemented.

Proposition 3 captures the main insight of our paper: when effort considerations matter, there is less delegation relative to the benchmark case with given success probability. Indeed, in region I, the principal maintains authority even if he cares less about the decision than the agent, i.e. if $k_P < k_A$. The reason is the commitment effect, which by Proposition 2 induces the principal to implement high effort and to select a critical project $d^c$ rather than his ideal project $d_P$. Under delegation, in contrast, high effort is implemented together with the agent’s preferred choice $d_A$. Since $d^c$ is closer to $d^s$ than $d_A$, the provision of effort incentives favours $P$- over $A$-authority. Perhaps surprisingly, this happens even when the principal becomes less and less concerned about the choice of project as $k_P$ tends to zero. The explanation is that both $d^c$ and $d^s$ converge to $d_A$ in the limit $k_P \to 0$.

Implementing high effort under $A$-authority is only optimal for values of $k_P$ and $c$ in region III. Here, the commitment effect is relatively weak such that the critical project $d^c$ implemented under $P$-authority is relatively close to $d_P$. Thus, since in this region the principal cares relatively little about the decision, $d^c$ is less close to $d^s$ than $d_A$, and this favours $A$- over $P$-authority.

Finally, consider regions II and IV where low effort is implemented. In these regions, the commitment effect disappears, and whichever party has authority chooses its ideal project. Hence, under these parameter constellations, the logic is the same as in the benchmark case without effort considerations: Authority is optimally assigned to the party who cares more about the decision.

It may be useful to compare our results to the standard principal–agent model in which project decisions are given and only effort incentives play a role. In this context it is well-known that the first-best effort can be implemented if the the agent is risk neutral and not protected by limited liability (see e.g. Sappington (1983)). To induce the agent to select the appropriate effort, the principal simply sets the bonus $w_S - w_F$ such that the agent’s private return equals the social return to effort. As the following result shows, a similar efficiency property holds in our framework:

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$^7$ For the parameter values in our example we have $\tilde{c}_{IV}(k_P, k_A) = \tilde{c}_{III}(k_P, k_A)$. 

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Proposition 4  In the absence of limited liability restrictions, if the optimal contract implements the project–effort combination \((d, e)\) then
\[
p(e)[u_P(d|k_P) + u_A(d|k_A)] - c(e) \geq p(e')[u_P(d|k_P) + u_A(d|k_A)] - c(e').
\]
for all \(e' \in \{e_L, e_H\}\). Thus, the agent’s effort maximizes the overall expected surplus from project \(d\).

This result is not a direct implication of the standard principal–agent model because the implementation of high effort generates a beneficial commitment effect under \(P\)-authority. This effect could make implementing high effort attractive beyond pure efficiency considerations. Nonetheless, the proof of Proposition 4 shows that high effort is not excessive when the principal selects the critical project defined in (10).

By Proposition 4, inefficiencies relative to the first–best occur only because for some parameter constellations the optimal contract fails to implement the surplus maximizing project \(d^*\). Since the bonus \(w_S - w_F\) is the only instrument to provide incentives for both project and effort choice, the overall first–best is not always achievable. Indeed, under \(A\)-authority, our model resembles a multi–task principal–agent environment (see Holmstrom and Milgrom (1991)) since the agent selects both \(d\) and \(e\). Similarly, a two–sided moral hazard problem (see e.g. Cooper and Ross (1985), Dybvig and Lutz (1993), and Bhattacharyya and Lafontaine (1995)) occurs under \(P\)-authority because the principal selects \(d\) and the agent \(e\). As is well–known from the literature, such extensions of the standard principal agent problem create additional sources of inefficiencies.

4 Limited Liability

We now turn to the case in which the agent is protected by limited liability. Thus, the principal’s objective is to find a contract which maximizes his expected payoff \(U_P(d, e, w|k_P)\), as defined in (3), subject to the constraints (4)–(6), and the additional non-negativity constraints on transfers
\[
w_S \geq 0, w_F \geq 0.
\]
Since \(u_A(d|k_A) > 0\) and \(c(e_L) = 0\), the limited liability constraint (12) and the moral hazard constraint (5) imply that \(U_A(d, e, w|k_A) \geq U_A(d, e_L, w|k_A) > 0\). Therefore, the agent’s participation constraint (4) is never binding and the principal will optimally set \(w_F = 0\). Effectively, under limited liability the principal’s constraints reduce to (5), (6),
$w_S \geq 0$, and $w_F = 0$. The next proposition describes the allocation of authority under the optimal contract.

**Proposition 5** Under limited liability, $P$–authority is always uniquely optimal.

The intuition is that under limited liability the ex ante and ex post interests of the principal coincide. Since he cannot extract the agent’s surplus, the principal does not seek to maximize total surplus ex ante. Ex ante as well as ex post his objective is to select the project which maximizes his expected private benefits net of expected wage payments. Thus, if $A$–authority is optimal ex ante, then implementing the agent’s ideal project $d_A$ must be optimal also ex post. Therefore, if the principal instead of the agent had the decision right, the principal would also choose $d_A$ ex post. In other words, if a contract with $h = A$ is optimal, it can be replicated by a contract with $h = P$. The proof of Proposition 5 actually shows that the principal can always do better than implementing $d_A$, which proves the stronger claim that $P$–authority is uniquely optimal.

Proposition 5 focuses on the optimal allocation of authority and does not provide information on the project and effort levels. The next proposition describes the project–effort combinations under the optimal contract.

**Proposition 6** There is a $\bar{c}_V(k_P, k_A)$ and a $\bar{c}_{VI}(k_P, k_A)$ with $0 < \bar{c}_V < \bar{c}_{VI}$ such that with limited liability the optimal project–effort combination under $P$–authority has the following properties:

(i) If $c \leq \bar{c}_V$, then $d_P$ and $e_H$ are implemented.

(ii) If $c \in (\bar{c}_V, \bar{c}_{VI}]$, then some $d \in [d^*, d_P)$ and $e_H$ are implemented.

(iii) If $c > \bar{c}_{VI}$, then $d_P$ and $e_L$ are implemented.

Since the principal’s ex ante and ex post interests are aligned, he simply trades off higher effort against higher wage payments. If the effort cost is very small (part (i)), the agent will provide high effort even if the bonus is small, and so the principal maximizes his payoff by choosing his ideal project. In Figure 5 this is the case for parameter constellations in region $I$. The principal chooses his ideal project also in region $III$, which corresponds to part (iii) of the proposition. Here inducing high effort is not attractive for the principal since this would require a rather high bonus $w_S$. If costs are moderate (part (ii)), high effort is desirable. In principle, the principal could induce high effort along
with his ideal project by paying a large bonus. Yet, in region \( \Pi \) of Figure 5 it turns out that it is more profitable to provide effort incentives by paying a moderate bonus and selecting a project which is closer to the agent's ideal project.

We conclude this section by considering how effort incentives are affected by limited liability. As our analysis in Section 3 shows, \( P \)-authority and high effort are optimal in the absence of limited liability as long as \( c \leq \bar{c}_{III} \). By Proposition 8 in the presence of limited liability the optimal contract leads to high effort if and only if \( c \leq \bar{c}_{VI} \).

**Proposition 7** For all \((k_P, k_A)\), it is the case that \( \bar{c}_{VI}(k_P, k_A) < \bar{c}_{III}(k_P, k_A) \). Therefore, the range of parameter constellations under which high effort is implemented is strictly larger without than with limited liability.

Limited liability makes the provision of effort incentives less profitable for the principal. The reason is the same as in the standard principal agent problem: In the absence of limited liability the principal can extract the whole surplus, whereas with limited liability he faces a trade-off between effort incentives and the surplus share that he can extract. This leads him sometimes to implement low effort even though total surplus would be higher with high effort.
5 Conclusion

Organizational decisions affect the various members of the organization in different ways. Thus, when decisions are non-contractible, the allocation of decision rights becomes a central issue for optimal organizational design. In this paper, we investigate how the allocation of the right to control organizational projects affects the incentives of an agent who has to work on these projects. If the agent is not protected by limited liability and in the absence of incentive concerns, the principal faces a fundamental commitment problem when he keeps the decision right. In this case, delegation mitigates the commitment problem and is the best authority regime when the agent is strongly affected by project choice.

In contrast, we show that when incentives matter, the principal can better solve the commitment problem by keeping authority even if the agent is strongly affected by project choice. In fact, if effort costs are low, the principal can implement the first-best by keeping authority and offering an appropriate bonus payment in case of project success. The reason is that when the principal selects the project, he takes the effect on the agent’s effort into account, leading him to choose less opportunistically.

In light of previous work which has emphasized the beneficial incentive effects of delegation on pre-decisional investments such as information acquisition, our findings indicate that the optimal allocation of authority depends critically on the nature and sequencing of the various decisions involved in completing organizational projects. Our analysis suggests that transferring decision rights to the agent might be a suboptimal way to induce post-decisional incentives.

On a related note, previous research has argued that delegation creates information revelation incentives when the agent possesses decision relevant private information. This is so because delegation protects the agent from the principal’s opportunism once the information is revealed. The commitment effect discussed in this paper suggests that when the principal needs the agent to exert effort ex post, the principal can credibly promise not to abuse the information revealed even if he keeps authority. A full analysis of this issue is the object of future research.
6 Appendix

Throughout the appendix, we use the notation

\[ S(d|k_P, k_A) \equiv u_P(d|k_P) + u_A(d|k_A). \tag{13} \]

We also sometimes suppress the dependency of \( S, u_P, u_A \) on \( k_P \) and \( k_A \).

**Proof of Proposition 1**: By (6), the agent chooses his ideal decision, i.e. \( \tilde{d}(A, w) = d_A \). Thus, by (7), when the principal implements high effort by a steep wage schedule, he obtains the surplus \( p_H S(d_A|k_P, k_A) - c \). When he implements low effort by a flat wage schedule, he obtains the surplus \( p_L S(d_A|k_P, k_A) \). Thus, implementing high effort is optimal if and only if \( c \) is lower than the critical value

\[ \bar{c}_I(k_P, k_A) = (p_H - p_L) S(d_A|k_P, k_A), \tag{14} \]

and this proves the claim. Q.E.D.

**Proof of Proposition 2**: To establish the properties stated in the proposition, we characterize the solution to the principal’s problem. To do so, we proceed in two steps. First, we characterize the project-effort combinations that are contractually implementable under \( P \)-authority. Second, we determine the optimal contract by identifying the project-effort combinations that maximize the principal’s surplus among all those that are contractually implementable.

**Step 1**: We say that \((d, e)\) is implementable if there is a wage \( w \) such that \((d, e, w)\) satisfies the constraints (5) and (6) for \( h = P \). The next two claims characterize which \((d, e)\) are implementable:

(a) \((d, e_L)\) can be implemented if and only if \( d = d_P \).

(b) Define

\[ \varphi(d|k_P, k_A) \equiv p_H S(d|k_P, k_A) - p_L [u_P(d_P|k_P) + u_A(d|k_A)]. \tag{15} \]

Then \((d, e_H)\) can be implemented if and only if either \( d = d_P \) or \( d \in [d_A, d_P) \) and \( \varphi(d|k_P, k_A) - c \geq 0 \).

As for (a). “\( \Rightarrow \)”: Let \((d, e_L)\) be implementable. Then there is a \( w \) such that \((d, e_L, w)\) satisfies (5) and (6). By (5), \( \tilde{e}(d, w) = e_L \). Thus, (6) implies

\[ U_P(d, e_L, w) \geq U_P(d', \tilde{e}(d', w), w) \quad \text{for all} \quad d'. \tag{16} \]
Since \( d = d_P \) uniquely maximizes \( U_P(\cdot, e, w) \), (16) implies \( d = d_P \), which is what we sought to prove.

“\( \Leftarrow \)”: We have to show that there is a \( w \) such that \((d_P, e_L, w)\) satisfies (5) and (6). Let \( w \) be such that \( w_S - w_F = -u_A(d_A) \). Then it is easy to see that the agent chooses \( e = e_L \) for all \( d \), i.e. \( \tilde{e}(d, w) = e_L \) for all \( d \), implying (5). Thus, since \( U_P(\cdot, e_L, w) \) attains a unique maximum at \( d = d_P \), the principal optimally selects \( d = d_P \), implying (6). This completes the proof of (a).

As for (b). “\( \Rightarrow \)”: Let \((d, e_H)\) be implementable. Then there is a \( w \) such that \((d, e_H, w)\) satisfies (5) and (6). We have to show that either either \( d = d_P \) or \( d \in [d_A, d_P) \) and \( \varphi(d) - c \geq 0 \). To see this, we distinguish two cases.

First, let \( \tilde{e}(d_P, w) = e_H \). Then since \( U_P(\cdot, \cdot, w) \) attains its unique maximum at \((d_P, e_H)\), (6) implies that \( d = d_P \).

Second, let \( \tilde{e}(d_P, w) = e_L \). We show that \( d \in [d_A, d_P) \) and \( \varphi(d) \geq c \). Indeed, since \((d, e_H)\) is implementable, (5) implies that \( \tilde{e}(d, w) = e_H \), i.e.,

\[
p_H[u_A(d) + w_S] + (1 - p_H)w_F - c \geq p_L[u_A(d) + w_S] + (1 - p_L)w_F. \tag{17}\]

Note that (17) must hold with equality. Indeed, by (6),

\[
U_P(d, e_H, w) \geq U_P(d', \tilde{e}(d', w), w) \quad \text{for all } d' \neq d. \tag{18}\]

Hence, if (17) holds with strict inequality, then there is an \( \epsilon > 0 \) such that \( \tilde{e}(d + \epsilon) = e_H \). Since \( U_P(\cdot, e_H, w) \) is increasing, \( d + \epsilon \) is a better decision for the principal than \( d \), a contradiction to (18). Moreover, (18) holds in particular for \( d' = d_P \). Thus, since \( \tilde{e}(d_P, w) = e_L \) by assumption, we obtain \( U_P(d, e_H, w) \geq U_P(d_P, e_L, w) \), which can be written as

\[
p_H[u_P(d) - (w_S - w_F)] - w_F \geq p_L[u_P(d_P) - (w_S - w_F)] - w_F. \tag{19}\]

From the equality (17), we compute

\[
w_S - w_F = \frac{c}{p_H - p_L} - u_A(d). \tag{20}\]

Using this in (19) and re-arranging gives the desired condition \( \varphi(d) \geq c \).

Finally, note that two cases are mutually exclusive. This implies the “either-or” condition in the statement of claim (b) and establishes the “\( \Rightarrow \)”-part of claim (b).
“⇐”: We first show that \((d_P, e_H)\) is implementable by setting \(w\) such that
\[
p_H[u_A(d_P) + ws] + (1 - p_H)wF - c > p_L[u_A(d_P) + ws] + (1 - p_L)wF.
\] (21)
Indeed, in this case, the agent selects \(e = e_H\) for all \(d\), i.e. \(\hat{e}(d, w) = e_H\) for all \(d\). Thus, since \(U_P(\cdot, e_H, w)\) is uniquely maximized by \(d = d_P\), the principal optimally selects \(d = d_P\).

This proves that \((d_P, e_H, w)\) satisfies (5) and (6).

Next, consider \(d \in [d_A, d_P)\) with \(\varphi(d) \geq c\). Then by setting \(w\) such that (17) holds with equality, the agent chooses \(e = e_I\) if and only if the principal selects a project \(d' > d\). The same arguments as in the second part of the proof of the “⇒”-part now imply that the principal optimally selects \(d\). Thus, \((d, e_H, w)\) satisfies (5) and (6), and this completes the proof of part (b).

Step 2: We now use claims (a) and (b) to determine the project-effort combinations that maximize the expected surplus in (7). The following two projects, \(d^0\) and \(\hat{d}\), defined via the function \(\varphi\) will be central. Define
\[
d^0(k_P, k_A) \equiv \arg\max_{d \in D} \varphi(d | k_P, k_A), \quad \hat{d}(k_P, k_A) \equiv \min\{d \in D | \varphi(d | k_P, k_A) \geq c\}.
\] (22)
Note that \(d^0\) is uniquely defined because \(\varphi(\cdot | k_P, k_A)\) is strictly concave. From the first-order condition \(\partial \varphi(d^0 | k_P, k_A) / \partial d = 0\) it follows immediately that \(d^\ast < d^0 < d_P\). Also note that \(\varphi(d^0 | k_P, k_A) > 0\) because \(\varphi(d^0 | k_P, k_A) > \varphi(d_P | k_P, k_A) = (p_H - p_L)S(d_P) > 0\).

Therefore, \(\hat{d}\) is well-defined as long as \(\varphi(d^0 | k_P, k_A) \geq c\).

The optimal project-effort combinations are then given as follows.

(A) If \(\varphi(d^\ast) \geq c\), then \(d = d^\ast\) and \(e = e_H\) are implemented.

(B) If \(\varphi(d^\ast) < c \leq \varphi(d^0)\), then \(\hat{d}\) and \(e = e_H\) are implemented.

(C) If \(\varphi(d^0) < c\), then \(d = d_P\) and \(e = e_L\) are implemented.

To prove (A)–(C), we first have to show that the stated projects are implementable. But this is immediate from (a) and (b) of step 1. Second, we have to show that the stated project-effort combination gives the principal a higher surplus \(p(e)S(d) - c(e)\) than any other implementable project-effort combination.

As for (A). Since \(d^\ast\) maximizes \(p(e)S(d) - c(e)\) for \(e = e_H\), \((d^\ast, e_H)\) dominates all other project-effort combinations that have \(e = e_H\). It remains to show that it also dominates \((d_P, e_L)\) (which is the only implementable project-effort combination with \(e = e_L\)). To see this, note that \(\varphi(d^\ast) \geq c\) implies that
\[
p_H S(d^\ast) - c \geq p_L[u_P(d_P) + u_A(d^\ast)].
\] (23)
Observe that the right hand side of (23) is larger than \( p_L S(d_P) = p_L[u_P(d_P) + u_A(d_P)] \) since \( u_A(\cdot) \) is decreasing in \( d \). Thus, \((d_P, e_L)\) yields a lower surplus than \((\hat{d}^*, e_H)\), and this proves (A).

As for (B). Since \( \varphi(d^*) < c \leq \varphi(d^0) \), claim (b) of step 1 implies that \( e = e_H \) can only be implemented in combination with projects \( d \geq \hat{d} \). Note that since \( S(\cdot) \) is single-peaked with a maximum in \( d^* \), the project \( \hat{d} \) maximizes \( p(e)S(d) - c(e) \) for \( e = e_H \) and \( d \geq \hat{d} \). Thus, \((\hat{d}, e_H)\) dominates all other implementable project-effort combinations with \( e = e_H \). It remains to show that it also dominates \((d_P, e_L)\). But this follows with analogous steps as in (A), and this establishes (B).

As for (C). By (a) and (b) from step 1, the only implementable project-effort combinations, when \( \varphi(d^0) < c \) are \((d_P, e_L)\) and \((d_P, e_H)\). Thus, we have to show that \( p_L S(d_P) \geq p_H S(d_P) - c \). To see this, note that \( \varphi(d^0) > \varphi(d_P) \). So \( \varphi(d^0) < c \) implies \( \varphi(d_P) < c \) which can be written as
\[
p_H S(d_P) - c \leq p_L[u_P(d_P) + u_A(d_P)].
\] (24)
Since the right hand side equals \( p_L S(d_P) \), this establishes (C).

To complete the proof, we have to establish the threshold values \( \tilde{c}_{II} \) and \( \tilde{c}_{III} \). Define
\[
\tilde{c}_{II}(k_P, k_A) = \varphi(d^*|k_P, k_A), \quad \tilde{c}_{III}(k_P, k_A) = \varphi(d^0(k_P, k_A)|k_P, k_A).
\] (25)
Then, since \( \varphi(d^+) > 0 \) and \( d^0 \) maximizes \( \varphi \), it follows that \( 0 < \tilde{c}_{II} < \tilde{c}_{III} \). Moreover, by definition, the three ranges of \( c \) defined by \( \tilde{c}_{II} \) and \( \tilde{c}_{III} \) correspond to the regions defined by (i) to (iii) in the statement of the proposition. This completes the proof. Q.E.D.

**Proof of Lemma 1**: Recall from (14) and (25) that \( \tilde{c}_{III}(k_P, k_A) = \varphi(d^0(k_P, k_A)|k_P, k_A) \) and \( \tilde{c}_{II}(k_P, k_A) = (p_H - p_L)S(d_A|k_P, k_A) \).

For given \( k_A \) denote by \( g(k_P) \) the difference \( \tilde{c}_{III}(k_P, k_A) - \tilde{c}_{II}(k_P, k_A) \). We show that \( g \) satisfies the following properties:

(a) \( \lim_{k_P \to 0} g(k_P) = 0 \); (b) \( \lim_{k_P \to 0} g'(k_P) < 0 \); (c) \( g \) is convex; (d) \( g(k_A) > 0 \).

With this, the claim follows by noting that since \( g \) is continuous, (a)–(d) imply that there is a unique \( \bar{k}_P \in (0, k_A) \) such that \( g(\bar{k}_P) = 0 \).

To see (a)–(d), note first that a little bit of algebra yields
\[
g(k_P) = (p_H - p_L)[k_P\ell(d_P - d_A) - k_P\ell(d_P - d^0) - k_A\ell(d^0 - d_A)]
- p_L k_P\ell(d_P - d^0).
\] (26)
Moreover, recall that \( d^0(k_P) \) maximizes \( \varphi(d|k_P, k_A) \), and thus satisfies the first-order condition

\[
(p_H - p_L)[k_P \ell'(d_P - d^0) - k_A \ell'(d^0 - d_A)] + p_L k_P \ell'(d_P - d^0) = 0.
\]  

(27)

Hence, \( \lim_{k_P \to 0} d^0(k_P) = d_A \).

As for (a). Using \( \lim_{k_P \to 0} d^0(k_P) = d_A \) in (26) yields the claim.

As for (b). Using (27) we obtain

\[
g'(k_P) = (p_H - p_L)[\ell'(d_P - d_A) - \ell(d_P - d^0)] - p_L \ell(d_P - d^0).
\]

(28)

Since \( \lim_{k_P \to 0} d^0(k_P) = d_A \), this expression converges to \(-p_L \ell(d_P - d_A)\), which is negative, implying (b).

As for (c). By (28)

\[
g''(k_P) = \left( (p_H - p_L)\ell'(d_P - d^0) + p_L \ell(d_P - d^0) \right) \frac{\partial d^0}{\partial k_P}.
\]

(29)

Since the term in the square brackets is positive, it remains to show that \( \partial d^0/\partial k_P \) is positive. Indeed, differentiating (27) with respect to \( k_P \) delivers

\[
\frac{\partial d^0}{\partial k_P} = -\frac{(p_H - p_L)\ell'(d_P - d^0) + p_L \ell(d_P - d^0)}{(p_H - p_L)[-k_P \ell'(d_P - d^0) - k_A \ell'(d^0 - d_A)] - p_H k_P \ell'(d_P - d^0)}.
\]

(30)

Our assumptions on \( \ell \) imply that the denominator is negative and the numerator is positive. This completes (c).

As for (d). Since \( \varphi(\cdot|k_P, k_A) \) attains its maximum at \( d^0 \), it follows that \( g(k_P) \) is larger than \( \varphi(d|k_P, k_A) - (p_H - p_L)S(d_A|k_P, k_A) \) evaluated at \( d = d_P \), i.e.

\[
g(k_P) > (p_H - p_L)[k_P \ell(d_P - d_A) - k_A \ell(d_P - d_A)].
\]

(31)

Since the right hand side is 0 at \( k_P = k_A \), it follows that \( g(k_A) > 0 \), which is (d). This completes the proof. Q.E.D.

**Proof of Proposition 3.** We will use that \( S(d_P) \geq S(d_A) \) if and only if \( k_P \geq k_A \).

As for (i). We consider first the case \( c \leq \bar{c}_i \). Then Proposition 1 implies that \( (d_A, e_H) \) is optimally implemented under \( A \)-authority, resulting in the surplus \( p_H S(d_A) - c \) for the principal. We now show that the principal can guarantee himself a higher surplus under \( P \)-authority. Indeed, by step 1(b) in the proof of Proposition 2, \( (d_P, e_H) \) is implementable under \( P \)-authority. This guarantees the principal the surplus \( p_H S(d_P) - c \), which is higher
than the surplus from $A$–authority since $S(d_P) > S(d_A)$. This establishes the claim for the first case. Now consider the case $c > \bar{c}_I$. Then, similarly, by Proposition 1 the principal obtains the surplus $p_L S(d_A)$ under $A$–authority, while, by step 1(a) in the proof of Proposition 2, he can guarantee himself the larger surplus $p_L S(d_P)$ under $P$–authority. This completes the proof of (i).

As for (ii). Let $k_P \in (\bar{k}_P, k_A)$, and consider first the case $c > \bar{c}_{III}$. We have to show that $A$–authority is optimal. Indeed, since $k_P \in (\bar{k}_P, k_A)$, Lemma 1 together with Propositions 1 and 2 imply that low effort is implemented under both $P$– and $A$–authority. Thus, the principal’s surplus is $p_L S(d_P)$ under $P$– and $p_L S(d_A)$ under $A$–authority. Since $k_P < k_A$ by assumption, we have that $S(d_A) > S(d_P)$. Thus, $A$–authority is uniquely optimal, and this shows the claim for $c > \bar{c}_{III}$. Consider next the case $c < \bar{c}_{III}$. We have to show that $P$–authority is optimal. Proposition 2 and its proof yield that under $P$–authority high effort is implemented together with the decision

$$d^{**} = \max \{\bar{d}, d^*\}. \tag{32}$$

Hence, the principal’s surplus under $P$–authority is $p_H S(d^{**}) - c$. Moreover, the principal’s surplus under $A$–authority is $\max \{p_L S(d_A), p_H S(d_A) - c\}$. Thus, we have to show that $p_H S(d^{**}) - c > \max \{p_L S(d_A), p_H S(d_A) - c\}$.

We show first that $p_H S(d^{**}) - c > p_H S(d_A) - c$ by showing that $S(d^{**}) > S(d_A)$. Indeed, since $k_P \in (\bar{k}_P, k_A)$, Lemma 1 implies that $\bar{c}_{III} - \bar{c}_I > 0$ which, by definitions (14) and (25) can be written as

$$(p_H - p_L) S(d^0) - (p_H - p_L) S(d_A) + p_L [u_P(d^0) - u_P(d_P)] > 0. \tag{33}$$

Since the term in square brackets is negative, it follows that $S(d^0) > S(d_A)$. Moreover, since $d^{**} \in [d^*, d^0]$ and $S(\cdot)$ is single-peaked, we find that $S(d^{**}) \geq S(d^0)$ and thus, $S(d^{**}) > S(d_A)$.

Next, we show that $p_H S(d^{**}) - c > p_L S(d_A)$. Indeed, since $c < \bar{c}_{III}$, the definition of $\bar{c}_{III}$ in (25) implies that

$$p_H S(d^0) - c > p_L [u_A(d^0) + u_P(d_P)]. \tag{34}$$

Note that the right hand side is larger than $p_L S(d^0)$. Hence, we have that $p_H S(d^0) - c > p_L S(d^0)$. Now, since $p_H > p_L$ and $S(d^{**}) \geq S(d^0)$, we can deduce that $p_H S(d^{**}) - c > p_L S(d^{**}) > p_L S(d_A)$, where the last inequality follows from the above observation $S(d^{**}) > S(d_A)$. This establishes (ii).
As for (iii). Let $k_P \in (0, \tilde{k}_P)$. We distinguish two cases. Let first $c > \bar{c}_{III}$. We have to show that $A$–authority is optimal. By Proposition 2, the principal’s surplus under $P$–authority is $p_L S(d_P)$. Under $A$–authority, the principal can guarantee himself a surplus of $p_L S(d_A)$. But since $k_P < k_A$ by assumption, we have that $S(d_A) > S(d_P)$, and this shows that $A$–authority is uniquely optimal.

Next, let $c < \bar{c}_{III}$. By Proposition 2, the principal’s surplus under $P$–authority is $p_H S(d^{**}) - c$. Moreover, since $k_P < \tilde{k}_P$ by assumption, Lemma 1 implies that $c < \bar{c}_I$, and thus, by Proposition 1, the principal’ surplus under $A$–authority is $p_H S(d_A) - c$. Hence, $P$–authority is optimal if and only if $S(d^{**}) > S(d_A)$. The following claim (proven below) shows that there is a unique cost level $\bar{c}_{IV}$ at which $S(d^{**}) = S(d_A)$. Recall that $\hat{d} = \min \{d \in D | \varphi(d) \geq c\}$ and thus $d^{**}$ depend on $c$. Recall also that $d^{**} = \hat{d}$ if and only if $c > \bar{c}_{II}$.

Claim A Let $c \in [\bar{c}_{II}, \bar{c}_{III}]$. For all $k_A, k_P$ there is at most one \( c \in [\bar{c}_{II}, \bar{c}_{III}] \) such that

\[
S(\hat{d}(c)) - S(d_A) = 0. \tag{35}
\]

Moreover, if there is a solution \( \hat{c} \), then $S(d^{**}(c)) - S(d_A) > 0$ if and only if $c < \hat{c}$. If there is no solution, then $S(d^{**}(c)) - S(d_A) > 0$ for all $c \leq \bar{c}_{III}$.

By Claim A, the following boundary is well-defined:

\[
\bar{c}_{IV}(k_P, k_A) = \begin{cases} 
\hat{c} & \text{if there is a unique solution to (35)} \\
\bar{c}_{III}(k_P, k_A) & \text{otherwise}.
\end{cases}
\tag{36}
\]

Moreover, it follows by construction that $P$–authority is optimal if and only if $c < \bar{c}_{IV}$. Finally, note that $\bar{c}_{IV} \in (\bar{c}_{II}, \bar{c}_{III}]$.

To complete the proof, it remains to prove Claim A. Define the function

\[
r(c) = S(\hat{d}(c)) - S(d_A). \tag{37}
\]

We show: (a) If $c = \bar{c}_{II}$, then $r(c) > 0$; (b) $r'(c) < 0$ for all $c \in (\bar{c}_{II}, \bar{c}_{III}]$.

With this, the claim follows, because (b) implies that there is at most one $\hat{c}$ such that $S(\hat{d}(\hat{c})) - S(d_A) = 0$, and (a) implies that if there is such a $\hat{c}$, then $S(d^{**}(c)) > S(d_A)$ for all $c < \hat{c}$. Otherwise, if there is no such $\hat{c}$, then (a) and (b) imply that $S(d^{**}(c)) > S(d_A)$ for all $c < \bar{c}_{III}$.

As for (a). If $c = \bar{c}_{II}$, then $\hat{d}(c) = d^*$, and so $r(c) > 0$ follows from the fact that $S(\cdot)$ is maximal at $d = d^*$.
As for (b). We have that \( r'(c) = S'(\hat{d})(\partial \hat{d}/\partial c) \). Observe first that, in the range of \( c \) considered, \( \hat{d} > d^* \). Therefore, since \( S(\cdot) \) is single-peaked, \( S'(\hat{d}) < 0 \). Second, since \( \hat{d} \) solves \( \varphi(d) = c \), we obtain that \( (\partial \hat{d}/\partial c) = 1/\varphi'(d) \). Now recall that \( \varphi(\cdot) \) is single-peaked and attains its maximum at \( d = \hat{d}^0 \). Thus, since \( \hat{d} < \hat{d}^0 \) it follows that \( \varphi'(d) > 0 \). The two observations yield that \( r'(c) < 0 \), which is what we sought to prove. Q.E.D.

Proof of Proposition 4: Let \((d, e)\) be the project–effort combination implemented under the optimal contract. We have to show that \( p(e)S(d) - c(e) \geq p(e')S(d) - c(e') \) for \( e' \neq e \).

If \( A\)-authority is optimal, the claim follows immediately from Proposition 1. Let now \( P\)-authority be optimal. Recall from the proof of Proposition 2 that implementation of high effort is optimal if \( \varphi(d^0) - c > 0 \) and that the implemented project \( d \) satisfies \( \varphi(d) - c \geq 0 \). Since, by definition, \( \varphi(d) = p_H S(d) - p_L [u_F(d_P) + u_A(d)] \leq p_H S(d) - p_L S(d) \), it follows that \( p_H S(d) - c \geq p_L S(d) \). Likewise, implementation of low effort is optimal if \( \varphi(d^0) - c < 0 \), and the implemented project is \( d_P \). Since \( \varphi(d^0) > \varphi(d_P) \), we also have that \( \varphi(d_P) - c < 0 \). But this is equivalent to \( p_L S(d_P) > p_H S(d_P) - c \), and this completes the proof.

Q.E.D.

Proof of Proposition 5: To prove the result, we distinguish three cases. Since the three cases exhaust all possible cases, they imply Proposition 3. Recall that under limited liability, the optimal contract has \( u_F = 0 \) and the principal’s payoff is \( p(e)[u_P(d) - w_S] \).

Case 1: We first show that \( P\)-authority is uniquely optimal if the optimal contract implements effort \( e_L \). Indeed, to implement \( e_L \), the principal optimally sets \( w_S = 0 \). Thus, under \( A\)-authority, the agent selects project \( d_A \), and the principal gets \( p_L u_P(d_A) \).

Under \( P\)-authority, the principal selects project \( d_P \) and he gets \( p_L u_P(d_P) \). Thus, since \( u_P(d_P) > u_P(d_A) \), \( A\)-authority is never optimal.

Case 2: We now show that \( P\)-authority is uniquely optimal, if the optimal contract implements effort \( e_H \) and \( (p_H - p_L)u_A(d_A) > c \). First consider the case \( p_H u_A(d_P) - c \geq p_L u_A(d_P) \). In this case the agent selects \( \hat{e}(d, w) = e_H \) for all \( d \in [d_A, d_P] \) and for all wages \( w_S \geq 0 \). Thus, the optimal \( A\)-authority contract has \( w_S = 0 \) and gives the principal the payoff \( p_H u_P(d_A) \). Likewise, the optimal \( P\)-authority contract has \( w_S = 0 \) and gives the principal the payoff \( p_H u_P(d_P) \). Since \( u_P(d_P) > u_P(d_A) \), \( A\)-authority is never optimal in this case.

Now consider the case \( p_H u_A(d_P) - c < p_L u_A(d_P) \). In this case, since \( (p_H - p_L)u_A(d_A) > c \) by assumption, there is a unique \( \hat{d} \in (d_A, d_P) \) such that \( p_H u_A(\hat{d}) - c = p_L u_A(\hat{d}) \). Thus, if \( w_S = 0 \), the agent selects \( \hat{e}(d, 0) = e_H \) for all \( d \in [d_A, \hat{d}] \) and \( \hat{e}(d, 0) = e_L \) for all \( d \in (\hat{d}, d_P] \). In particular, the optimal \( A\)-authority contract has \( w_S = 0 \) and gives the
principal the payoff $p_H u_P(d_A)$. Towards a contradiction, suppose now that this contract is overall optimal. Then one must have
\begin{equation}
    p_H u_P(d_A) \geq p_L u_P(d_P),
\end{equation}
because the principal could always implement $(e_L, d_P)$ with $w_S = 0$ under $P$-authority. Therefore, since $u_P(\hat{d}) > u_P(d_A)$, we have
\begin{equation}
    p_H u_P(\hat{d}) > p_H u_P(d_A) \geq p_L u_P(d_P) = \max_{d > \hat{d}} p_L u_P(d).
\end{equation}
This proves that $\hat{d}$ satisfies the no-commitment constraint \( \square \) with $w_S = 0$ under $P$-authority. Thus, under $P$-authority $(e_H, \hat{d})$ can be implemented with $w_S = 0$ and gives the principal the payoff $p_H u_P(\hat{d})$. Since $p_H u_P(\hat{d}) > p_H u_P(d_A)$, this yield a contradiction to the optimality of $A$-authority.

Case 3: We finally show that $P$-authority is uniquely optimal if the optimal contract implements effort $e_H$ and $(p_H - p_L) u_A(d_A) \leq c$. Suppose the contrary. Then $A$-authority is optimal and $(d_A, e_H)$ is implemented. The corresponding wage $\hat{w}_S > 0$ satisfies the agent’s incentive constraint with equality, i.e.
\begin{equation}
    p_H[u_A(d_A) + \hat{w}_S] - c = p_L[u_A(d_A) + \hat{w}_S].
\end{equation}
We first show that $(d_A, e_H)$ in combination with $\hat{w}_S$ can also be implemented under $P$-authority. Indeed, \( \square \) implies that $\tilde{e}(d_A, \hat{w}_S) = e_H$ and $\tilde{e}(d, \hat{w}_S) = e_L$ for all $d > d_A$. Since implementing $e_H$ is optimal, one must have $p_H[u_P(d_A) - \hat{w}_S] \geq p_L u_P(d_P)$ because the principal could always implement $(e_L, d_P)$ with $w_S = 0$ under $P$-authority. Therefore
\begin{equation}
    p_H[u_P(d_A) - \hat{w}_S] \geq p_L u_P(d_P) \geq p_L[u_P(d_P) - \hat{w}_S] = \max_{d > d_A} p_L[u_P(d) - \hat{w}_S].
\end{equation}
This proves that $d_A$ satisfies the no-commitment constraint \( \square \) under $P$-authority. Thus the principal can get at least the same payoff as under $A$-authority.

To prove that $A$-authority is suboptimal we show that there is a contract with $P$-authority under which the principal gets a higher payoff than $p_H[u_P(d_A) - \hat{w}_S]$. Consider the following maximization problem:
\begin{equation}
    \max_{d, w_S} p_H[u_P(d) - w_S] \quad \text{subject to} \quad p_H[u_A(d) + w_S] - c = p_L[u_A(d) + w_S].
\end{equation}
If $(d, w_S)$ satisfies the constraint in \( \square \) then $\tilde{e}(d, w_S) = e_H$ and $\tilde{e}(d', w_S) = e_L$ for all $d' > d$. Thus $w_S$ implements $e_H$ under $P$-authority, given $d$. Also, by the above argument,
the solution of (42) satisfies the no-commitment constraint (6) under $P$-authority. In summary, if $(d, w_S)$ solves (42), then the principal receives the payoff $p_H[u_P(d) - w_S]$ under $P$-authority.

Substituting $w_S$ from the constraint in (42) into the objective function, simplifies the choice of $d$ to

$$\max_d p_H[u_P(d) + u_A(d)] - \frac{p_H}{p_H - p_L} c. \quad (43)$$

The solution of this problem is $d^*$ rather than $d_A$. This proves that the principal can get a higher payoff than under $A$-authority, a contradiction. Q.E.D.

**Proof of Proposition 6** Define the threshold $\bar{c}_V$ by

$$\bar{c}_V \equiv (p_H - p_L)u_A(d_P). \quad (44)$$

To define $\bar{c}_{VI}$, we distinguish two cases. If $p_H u_P(d^*) < p_L u_P(d_P)$, there is a unique $\bar{d} \in (d^*, d_P)$ such that $p_H u_P(\bar{d}) = p_L u_P(d_P)$. With this we, define

$$\bar{c}_{VI} = \begin{cases} 
(p_H - p_L)u_A(\bar{d}) & \text{if } p_H u_P(d^*) < p_L u_P(d_P) \\
(p_H - p_L)[u_P(d^*) + u_A(d^*) - \frac{p_H}{p_H - p_L} u_P(d_P)] & \text{if } p_H u_P(d^*) \geq p_L u_P(d_P)
\end{cases} \quad (45)$$

A straightforward calculation shows that $\bar{c}_V < \bar{c}_{VI}$. We now derive the optimal ($P$-authority) contract for the cases (i)-(iii).

As for (i). Let $c \leq \bar{c}_V$. Then by agent’s incentive constraint (5), the agent chooses high effort for all $d \in [d_A, d_P]$ and $w_S$. Thus, the optimal contract has $w_S = 0$ and implements $d_P$ with high effort. This proves (i).

As for (ii). Let $\bar{c}_V < c \leq \bar{c}_{VI}$. We first, (a), derive the wage-project combination $(w_S, d)$ that is optimal for implementation of high effort $e_H$. We then, (b), show that this combination $(w_S, d)$ satisfies the principal’s no-commitment constraint and dominates implementation of low effort for the range $\bar{c}_V < c \leq \bar{c}_{VI}$.

As for (a). Note first that $\bar{c}_V < c$ implies that for all $w_S \geq 0$ there is a unique $\hat{d}(w_S)$ such that $(p_H - p_L)u_A(\hat{d}(w_S)) + w_S = c$. Since $u_A(\cdot)$ is strictly decreasing in $d$, we have that $\hat{d}(\cdot)$ is strictly increasing in $w_S$, and it holds that $(p_H - p_L)u_A(\hat{d}(0)) = c$.

By the agent’s incentive constraint (5), $\hat{c}(d, w_S) = e_H$ if and only if $d \leq \hat{d}(w_S)$. Since $w_S$ is the lowest wage at which decision $d = \hat{d}(w_S)$ can be implemented with high effort, the optimal combination $(w_S, d, e_H)$ must satisfy $w_S = \hat{d}^{-1}(d) = c/(p_H - p_L) - u_A(d)$ and $d \geq \hat{d}(0)$. Thus, the optimal $d$ maximizes the principal’s payoff

$$p_H[u_P(d) - w_S] = p_H[u_P(d) + u_A(d)] - \frac{p_H}{p_H - p_L} c \quad \text{s.t.} \quad d \geq \hat{d}(0). \quad (46)$$

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Let $d^+$ be the solution to (46). Recall that $d = d^*$ is the unconstrained maximizer of $u_P(\cdot) + u_A(\cdot)$ and that $u_P(\cdot) + u_A(\cdot)$ is single-peaked. Thus, $d^+ = d^*$ if $d^* \geq \hat{d}(0)$, and $d^+ = \hat{d}(0)$ otherwise. To characterize these two conditions in terms of costs, note that $(p_H - p_L)u_A(\hat{d}(0)) = c$. Hence, monotonicity of $u_A(\cdot)$ and $\hat{d}(\cdot)$ implies that $d^* \geq \hat{d}(0)$ if and only if $(p_H - p_L)u_A(d^*) \leq c$. Thus, $d^+ = d^*$ if $(p_H - p_L)u_A(d^*) \leq c$, and $d^+ = \hat{d}(0)$ if $(p_H - p_L)u_A(d^*) > c$. This completes (a).

As for (b). Consider first the case $p_H u_P(d^*) < p_L u_P(d_P)$. Since $c \leq \bar{c}_{VI} = (p_H - p_L)u_A(\bar{d})$, and since $\bar{d} > d^*$, it follows that $(p_H - p_L)u_A(d^*) > c$, and thus, by (a), that $d^{**} = \hat{d}(0)$. Therefore, to prove that the combination $(0, \hat{d}(0), e_H)$ satisfies the principal’s no-commitment constraint (6), we have to show that $p_H u_P(\hat{d}(0)) \geq p_L u_P(d_P)$. To see this, note that since $(p_H - p_L)u_A(\hat{d}(0)) = c \leq \bar{c}_{VI}$, we also have that $\bar{d} \leq \hat{d}(0)$. Hence,

$$p_H u_P(\hat{d}(0)) \geq p_H u_P(\bar{d}) = p_L u_P(d_P).$$ (47)

It remains to show that the principal cannot (ex ante) do better by implementing $e_L$ with $d_P$, i.e. that $p_H u_P(\hat{d}(0)) \geq p_L u_P(d_P)$. But this is the same inequality as the inequality just established. This shows claim (b) for the case $p_H u_P(d^*) < p_L u_P(d_P)$.

Consider now the case $p_H u_P(d^*) \geq p_L u_P(d_P)$. From (a) we have that the optimal $(w_S, d)$ with high effort is either $(0, \hat{d}(0))$ or $(\hat{d}^{-1}(d^*), d^*)$. We first show claim (b) for the combination $(0, \hat{d}(0))$. The principal’s no-commitment constraint (6) is satisfied if $p_H u_P(\hat{d}(0)) \geq p_L u_P(d_P)$. But this follows from the fact that $\hat{d}(0) \geq d^*$ and the assumption $p_H u_P(d^*) \geq p_L u_P(d_P)$.

It remains to show that the principal cannot (ex ante) do better by implementing $e_L$ with $d_P$, i.e. that $p_H u_P(\hat{d}(0)) \geq p_L u_P(d_P)$. But this is the same inequality as the inequality just established. This shows claim (b) for $d^* = \hat{d}(0)$.

We now show claim (b) for the combination $(\hat{d}^{-1}(d^*), d^*)$. The principal’s no-commitment constraint (6) is satisfied if $p_H[u_P(d^*) - \hat{d}^{-1}(d^*)] \geq p_L[u_P(d_P) - d^{-1}(d^*)]$. Thus, it suffices to show that $p_H[u_P(d^*) - \hat{d}^{-1}(d^*)] \geq p_L u_P(d_P)$. To see this, note that

$$p_H[u_P(d^*) - \hat{d}^{-1}(d^*)] = p_H[u_P(d^*) + u_A(d^*)] - \frac{p_H}{p_H - p_L}c$$

$$\geq p_H[u_P(d^*) + u_A(d^*)] - \frac{p_H}{p_H - p_L}\bar{c}_{VI}$$

$$= p_L u_P(d_P).$$ (48)

The first line follows by inserting $\hat{d}^{-1}(d^*)$. The second inequality follows since $c \leq \bar{c}_{VI}$. The final line follows from the definition of $\bar{c}_{VI}$ in (45).
It remains to show that the principal cannot (ex ante) do better by implementing $e_L$ with $d_P$, i.e. that $p_H[u_P(d^*)] - w_S \geq p_L u_P(d_P)$. But this is the same inequality as the inequality just established. This shows claim (b) for $d^+ = d^*$ and completes (ii).

As for (iii). Consider finally the case $c > \varepsilon_{VI}$. We show that it is (ex ante) optimal for the principal to implement low effort. Consider first the case $p_H u_P(d^*) < p_L u_P(d_P)$. From (ii), (a), the best wage project combination with high effort is $(0, d(0))$, yielding a payoff of $p_H u_P(d(0))$. Now, the same argument as in (ii), (a) shows that $c \geq \varepsilon_{VI}$ implies that $d(0) < d$. Thus, $p_H u_P(d(0))$ is smaller than $p_H u_P(d) = p_L u_P(d_P)$ which is the payoff from implementing low effort.

Consider now the case $p_H u_P(d^*) \geq p_L u_P(d_P)$. It is easy to see that $c \geq \varepsilon_{VI}$ implies that $(p_H - p_L) u_A(d^*) \leq c$. Thus it follows from (ii), (a) that the optimal wage project combination with high effort is $(\hat{d}^{-1}(d^*), d^*)$, yielding a payoff of $p_H[u_P(d^*) + u_A(d^*)] - p_H c/(p_H - p_L)$. Since $c > \varepsilon_{VI}$, (45) implies that this is smaller than $p_L u_P(d_P)$, the principal’s payoff from implementing $d_P$ with low effort and $w_S = 0$. This shows (iii) and completes the proof.

**Proof of Proposition 7** We show that $\varepsilon_{VI} \leq \varepsilon_{III}$. Consider first the case $p_H u_P(d^*) < p_L u_P(d_P)$. Recall from (25) that $c_{III} = \varphi(d^0)$ where $d^0$ maximizes $\varphi(\cdot)$. Hence,

$$\varepsilon_{III} = \varphi(d^0) \geq p_H u_P(d^0) - p_L u_P(d_P) + (p_H - p_L) u_A(d^0) \geq p_H u_P(d) - p_L u_P(d_P) + (p_H - p_L) u_A(d) = \varepsilon_{VI}.$$  

The second line is the definition of $\varphi(\cdot)$ in (15), the third line follows since $d^0$ maximizes $\varphi(\cdot)$, and the last line follows by definition of $\varepsilon_{VI}$ and $d$. Consider now the case $p_H u_P(d^*) \geq p_L u_P(d_P)$. By (25) and (45), we have

$$\varepsilon_{III} - \varepsilon_{VI} = \varphi(d^0) - (p_H - p_L) S(d^0) + (p_H - p_L) \frac{p_L}{p_H} u_P(d_P).$$  

With a little bit of algebra, the r.h.s. can be written as

$$\varphi(d^0) - \varphi(d^*) + p_L [u_P(d^*) - \frac{p_L}{p_H} u_P(d_P)].$$  

Since $d^0$ maximizes $\varphi(\cdot)$, $\varphi(d^0) - \varphi(d^*) \geq 0$. Moreover, the term in the square bracket in (51) is positive by the assumption $p_H u_P(d^*) \geq p_L u_P(d_P)$. This completes the proof. Q.E.D.
References


