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Fighting cartels: some economics of council regulation (EC) 1/2003

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Abstract

This paper investigates the effectiveness of the new Council Regulation (EC) 1/2003 which replaces the mandatory notification and authorization system by a legal exception system. Effectiveness is operationalized via the two subcriteria compliance to Art. 81 EC Treaty and the probabilities of type I and type II errors committed by the European Commission. We identify four different types of Perfect Bayesian Nash Equilibria: full-compliance, zero-compliance, positive-compliance and full-deterrence. We show that the Commission can, in principle, hit the full-compliance equilibrium, where the cartelizing firms fully obey the requirements of Art 81(3) EC Treaty and both error probabilities are zero.

Keywords: competition law, cartel law enforcement, legal exception, imperfect decision making, type I error, type II error

JEL Classification: K21, K42, L40

1 Introduction

The European ban on cartels follows from article 81(1) of the EC Treaty. It states that “all agreements between undertakings, decisions by associations of undertakings and concerted practices which may affect trade between Member States

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and which have as their object or effect the prevention, restriction or distortion of competition within the common market” are “prohibited as incompatible with the common market”.

However, exemption from this general ban can be granted if the requirements stated in article 81(3) EC Treaty are fulfilled. To be exempted, an agreement has to “contribute[s] to improving the production or distribution of goods or to promoting technical or economic progress” (economic benefits), the consumers are to be given “a fair share of the resulting benefit” (fair share for the consumers), and the agreement must not “impose on the undertakings concerned restrictions which are not indispensable to the attainment of these objectives” (indispensability) nor “afford such undertakings the possibility of eliminating competition in respect of a substantial part of the products in question” (no elimination of competition).

The enforcement of article 81 EC Treaty was first laid down in Council Regulation (EEC) No 17/62. But on May 1st 2004 a new regulation on the enforcement of EC competition rules (Council Regulation (EC) No 1/2003) came into force. This new regulation has been the result of the so called modernization package invoked by the European Commission in 1999. A main feature of the new regulation is the replacement of the mandatory notification system by a system of legal exception: while under Reg. 17/62 firms had to notify their agreements in order to be allowed to sign them, which constitutes some sort of ex ante control, Reg. 1/2003 imposes a regime of ex post control on the firms (abuse control).

The aim of the reform as stated by the Commission in its White Paper is to refocus its resources on prosecuting the most serious restrictions on competition. This had become necessary because theadministrational overload caused by an increasing number of notifications every year started to endanger the effec-

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1 Article 81(1) EC Treaty. Article 81(2) specifies the legal consequences: such “agreements or decisions” are “automatically void”.

2 In this paper we will use the term agreement to cover any behavior prohibited according to article 81(1).

3 Article 81(3) EC Treaty.


5 Council Regulation (EC) No 1/2003 of 16 December 2002 on the implementation of the rules on competition laid down in Articles 81 and 82 of the Treaty, OJ L1, 04.01.2003, p. 1-25, hereafter Reg. 1/2003. The regulation also covers the abuse of a dominant market position as prohibited in article 82 EC Treaty. The focus of this article is firm behavior that falls under article 81 EC treaty. Nevertheless, similar results could be derived for the case of abuse of dominant positions.


7 Two further changes were made: decentralization of the application of articles 81 and 82 of the Treaty to member states and strengthening of private enforcement.
tiveness of cartel law enforcement by the Commission.\textsuperscript{8} However, the proposed reform and the new regulation gave rise to a comprehensive discussion of the reform in general and the introduction of a legal exception system in particular. Especially German lawyers appeared quite reluctant towards the replacement of the notification system by a legal exception system. The main allegation can be summarized as the fear that the enforcement of the cartel ban might become ineffective if the notification system is dropped, driving Europe towards a “cartel paradise”.\textsuperscript{9}

However, for two reasons the legal reasoning against the legal exception system can be widely considered flawed: First, despite using economic terminology, most allegations are pure assertions and lack a sound economic underpinning. Second, the reasoning frequently suffers from the so called “nirvana fallacy”.\textsuperscript{10} Instead of comparing the legal exception system with its real alternative, the notification system, only the legal exception’s disadvantages in relation to an ideal enforcement system are listed.

The aim of this paper is to investigate the effectiveness of the new Council Regulation. For this purpose it is not necessary to conduct a fully fledged institutional comparison of Reg. 17/62 and Reg. 1/2003.\textsuperscript{11} Rather, it is sufficient to build a model that represents the relevant aspects of the interaction between the European Commission as cartel authority and potentially cartelizing firms under Reg. 1/2003. Analyzing this interaction, i.e. deriving the equilibrium strategies of the players, one can predict the behavior of the involved parties. In a second step, this behavior can be evaluated in accordance with preassigned criteria operationalizing effectiveness. We have chosen to measure effectiveness via the firms’ compliance to article 81 EC Treaty on the one hand and the probability of decision-making errors —may they be false positives or false negatives— committed by the European Commission on the other.\textsuperscript{12}

\textsuperscript{8}See Commission (1999, para 41-45).

\textsuperscript{9}The term “cartel paradise” is borrowed from Pirrung (2004). For a comprehensive survey on the (German) legal objections against legal exception see e.g. Krumstroh (2004); for the reactions on the White Paper by the member states see Commission (2000), for those by European legal experts see Ehlermann/Atanasiu (2001).

\textsuperscript{10}This term common to economists to describe improper comparisons was first used by Harold Demsetz to characterize the comparison between real markets and idealized state institutions. As a result, state intervention is often seen as the one and only means to cure market failure while neglecting the fact that state intervention may lead to regulatory failure as well. See Demsetz (1969). As for regulatory failure in the field of competition policy see e.g. Schinkel/Tuinstra (2006).

\textsuperscript{11}However, such a fully fledged institutional comparison is possible, see Will (2008). Still, this would go far beyond the scope of this paper.

\textsuperscript{12}The relevance of both error types becomes evident when considering their repercussions on the firms’ behavior. A high probability of type I errors, i.e. good cartels are objected to and fined, may induce firms to sign bad agreements precautionary. In addition, a high probability of type II errors, i.e. bad cartels are exempted, may also stipulate the signing of bad agreements, as this type of error reduces expected fines. For similar reasoning see e.g. Schinkel/Tuinstra
rather than efficiency in terms of welfare effects, the criticism of the reform is more aptly addressed.

In contrast to the literature on regulation, only few economic papers have so far focused on optimal competition policy design and enforcement problems. To our knowledge, there exist only seven: three of them—Di Federico/Manzini (2004), Pirrung (2004) and Wils (2000)—deal with the topic in a merely verbal manner, while Barros (2003) and Neven (2002) address the problem using a decision theoretic framework. Only Hahn (2002) and Loss et al. (2008) use game theoretic models to gain insights in the interdependent actions of cartel authorities on the one and cartelizing firms on the other hand.

In Hahn (2002) both enforcement regimes, the legal exception as well as the notification system, are interpreted as means to deter agreements that fall under article 81(1) EC Treaty. This is surprising as such an approach negates the regulating nature of the notification system and puts it on a par with the legal exception system. Clearly, notification and authorization as under Reg. 17/62 is fundamentally different in its effects on the firms from the abuse control constituted by Reg. 1/2003. Further, Hahn (2002) assumes not only benevolent behavior by the cartel authority but also its perfectness in decision making. As do Barros (2003) and Neven (2002).

The hitherto most sophisticated paper on the topic is Loss et al. (2008). As we do in this paper, the authors assume an imperfectly deciding cartel authority committing both types of error. But despite some commonalities as for the methods utilized, our paper is based on a significantly different approach. First, we allow for different error probabilities. This is a more general way of modelling the possibility of erroneous cartel authority decisions. Second, we model the firms’ decision to sign an agreement and then to shape the agreement according to article 81 EC Treaty. In other words, in our model the type of the agreement, i.e. whether the agreement fulfills the requirements of article 81(3) EC Treaty (good) or not (bad), is endogenous while it is exogenously given in Loss et al. (2008). From this it follows directly that Loss et al. (2008) can only predict the probability of signing a predefined type of agreement while we can predict not only whether an agreement is deterred or not but also—in case it is not deterred—the probability of the signed agreement being good or bad. Third, we define behavioral strategies not only for the firms’ actions but also for the

\footnote{Arrangement (2006).}

\footnote{As do Barros (2003) and Neven (2002).}
Commission’s. This allows for a wider scope of actions for the latter. Finally, Loss et al. (2008) also follow the somewhat simplifying approach to assume benevolent behavior on the cartel authority’s side. Instead, we have chosen a different way by assuming not benevolent but self-interested behavior of the Commission: our Commission seeks to fell decisions which are not overruled by the European Court of Justice, since being overruled leads to reputational losses.

The remainder of the paper is organized as follows: In section 2 the game theoretic model is specified while it is solved in section 3. Section 4 briefly depicts the main findings. In section 5 the conclusions are drawn.

2 The Model

We build a game theoretic model that represents the main features of the legal exception system. For this purpose we consider two risk neutral players: the European Commission, denoted as $C$, and a group of firms deliberating to form a cartel, denoted as $F$.

In a first step, the firms have to decide whether to form a cartel. The decision to do so is denoted as $f = \text{in}$, while the decision to abstain from any agreement is denoted as $f = \text{out}$. If the firms decide not to sign an agreement, the status quo persists and both players receive a payoff normalized to zero. In a second step, if the firms have chosen $f = \text{in}$, they can decide whether to form a good or a bad agreement, denoted as $a \in \{g, b\}$ where $g$ stands for a good agreement and $b$ for a bad one. Note that under Reg. 1/2003 a notification of the agreement is no more necessary.

A good agreement fulfills article 81(3) EC Treaty while a bad one does not. It is convenient to define the behavioral strategy $\gamma = Pr(a = g) \in [0; 1]$. A good agreement leads to a gain $G > 0$ for the firms, while a bad agreement yields an additional profit $A > 0$, so that the firms are tempted to sign a bad agreement. The assumption that the profit of a bad agreement ($G + A$) exceeds the profit of a good agreement ($G$) follows directly from the economic interpretation of article 81(3) EC Treaty: only if the firms abstain from some part of their gain realized through the agreement and pass this on to the customers may an agreement be exempted from the general cartel ban.

A signed agreement is controlled by the Commission with probability $\xi \in (0; 1)$. The Commission is assumed to be an imperfect decision maker. Imperfection means that the Commission is able to distinguish better than by pure chance between good and bad agreements but that it makes mistakes in doing

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14 Both players are unitary actors in the sense that we ignore intra-group decision making problems.


16 Thus, the decision whether to control a certain agreement is not an endogenous variable in the game. Instead, it is assumed that the Commission can credibly commit to a control probability $\xi$.  

5
Two types of mistakes are possible: First, a good agreement can be falsely
deemed bad and therefore prohibited and fined which constitutes an error called
type I error (false positive). Second, a bad agreement can be falsely deemed
good and therefore not prohibited; this is called a type II error (false negative).
When the firms have decided upon the type of the agreement by choosing a value
for $\gamma$, nature $N$ produces an informative signal, denoted as $i \in \{g, b\}$, where
$i = g$ stands for a signal indicating a good agreement and $i = b$ indicating a
bad one. The probability of receiving a particular signal realization is contingent
on the firms’ choice: $\rho = Pr(i = g|a = g)$, $\varphi = Pr(i = g|a = b)$. The
introduction of nature producing a signal is a technical method to describe the
following underlying idea: although the Commission does not know the actual
type of the agreement because it is not able to observe the firms’ choice, it is
not blind. It is able to gather information on the case by looking at market data
and interrogating the firms and/or other market participants. The parameters
$\rho$ and $\varphi$ provide a measure for the Commission’s skill to assess the type of the
agreement based on this information. With $\rho = 1$ and $\varphi = 0$ it perfectly assesses
the agreement, i.e. without making any mistakes; with $\rho = \varphi$ its assessment skills
are zero, i.e. it is not able to distinguish good from bad agreements better than
by pure chance. In our model we assume the intermediate case $1 > \rho > \varphi > 0$;
this reflects imperfect, but positive assessment skills. The Commission updates
its beliefs using Bayes’ rule. The Commission’s ex post beliefs are denoted as
$\mu = Pr(a = g|i = g)$, $\nu = Pr(a = g|i = b)$. The Commission then has to decide
between prohibiting and fining the agreement or not to object to it, denoted as$c \in \{p, \neg p\}$ where $p$ stands for prohibiting and fining an agreement and $\neg p$ for
not doing so.\textsuperscript{17} It is again convenient to define behavioral strategies for these
actions: $\alpha = Pr(c = \neg p|i = g) \in [0; 1]$ and $\beta = Pr(c = \neg p|i = b) \in [0; 1]$.\textsuperscript{18} The
(monetary) fine is denoted as $M > 0$.

The Commission’s decisions are subject to external control by the Court of
First Instance of the European Communities (CFI).\textsuperscript{19} An appeal before the CFI
takes place with probability $\chi \in (0; 1)$.\textsuperscript{20} The court is assumed to be a perfect
decision maker in the sense that its decisions are error-free in a legal meaning.\textsuperscript{21}
In its judgement the court states whether the Commission’s legal subsumption of
the agreement under scrutiny has been correct. Whatever conclusion the court

\textsuperscript{17}These decision options are defined in articles 7, 10, and 23 Reg. 1/2003.
\textsuperscript{18}Note that our model therefore allows for the possibility that the Commission could prohibit
an agreement although it believes the agreement to be good as the Commission’s decision is not
bound by the received signal. Instead, the Commission’s choice depends solely on its expected
payoffs.
\textsuperscript{19}Article 31 Reg. 1/2003.
\textsuperscript{20}This is a simplifying assumption in two respects: First, potential plaintiffs are not modeled
as players but are only represented by the exogenous appeals probability $\chi$. Second, it neglects
the fact that the cartelizing firms or third parties may have different incentives to take action.
\textsuperscript{21}To keep the analysis manageable, the court is not modeled as a player but is only represented
by the exogenous probability of whether a decision is repealed or not.
comes to is final.\textsuperscript{22} We assume the Commission to be averse to a repeal of its
decisions by the CFI. If the decision of the Commission is correct, i.e. whenever
the Commission has not objected to a good agreement \((c = \neg p|a = g)\) or pro-
hibited a bad one \((c = p|a = b)\), the appeal before the CFI is not successful and
the Commission receives a payoff normalized to zero. If the decision is wrong,
i.e. the commission has committed either a type I \((c = p|a = g)\) or a type II
error \((c = \neg p|a = b)\), the appeal is successful and the Commission receives a
negative payoff of 1. This negative payoff can be interpreted as the reputation
the Commission loses when it is overruled.\textsuperscript{23}

Figure 1 shows the corresponding game tree consisting of the interaction of
the two players \(F\) and \(C\) and the signals generated in the Legal Exception Game.
The payoff parameters \((G, A, M)\), the control and appeals probabilities \((\xi, \chi)\)
as well as the assessment skill parameters \((\rho, \phi)\) are exogenously given and are
common knowledge. Thus, the endogenous variables are \(f, \alpha, \beta, \gamma, \mu\) and \(\nu\).

An equilibrium in the Legal Exception Game consists of the firms’ equilib-
rium strategy \((f^*, \gamma^*)\), the Commission’s equilibrium strategy \((\alpha^*, \beta^*)\) and the
corresponding ex post beliefs \((\mu^*, \nu^*)\). To evaluate the equilibria of the Legal
Exception Game in order to assess the effectiveness of European cartel law en-
forcement it is necessary to operationalize the criterion effectiveness. We do this
in defining two subcriteria: compliance, measured as the equilibrium probability
of signing a good agreement \((\gamma^*)\), and the equilibrium error probabilities \(p_I\) and
\(p_{II}\) which are derived as follows:

\[
p_I = \begin{cases} 
0, & f^* = \text{out}; \\
\xi \gamma^*(1 - \chi)[1 - \beta^* - \rho(\alpha^* - \beta^*)], & f^* = \text{in}; 
\end{cases} 
\]

\[
p_{II} = \begin{cases} 
0, & f^* = \text{out}; \\
(1 - \gamma^*)[1 - \xi [1 - (1 - \chi)(\beta^* + \varphi(\alpha^* - \beta^*))]], & f^* = \text{in}. 
\end{cases} 
\]

Note that both error probabilities are zero whenever the firms decide not to sign
an agreement, i.e. choose \(f^* = \text{out}\), as a state identical with the status quo per
definition cannot be erroneous.

\textsuperscript{22}Another justification for assuming a perfect court is the fact that this assumption is wide-
spread in economic literature, see for similar assumptions e.g. Shavell (1995) or Shavell (2006).
To slightly weaken the assumption one could also think of the court monitoring the methods
applied by the Commission. Such a kind of ex post monitoring can also set suffi-
cient incentives for better ex ante decisions by the Commission.

\textsuperscript{23}Possible reputation losses from not engaging in cartel control (this is the case with probabili-
ty \(1 - \xi\)) are not considered in the model. However, introduction of such additional reputation
effects would not change the derived results, see Will (2008, p. 202).
3 Equilibrium Analysis

The Legal Exception game has a proper subgame starting in decision node $F_1$. We solve this subgame using perfect Bayesian Nash equilibria $(\gamma^*; \alpha^*, \beta^*; \mu^*, \nu^*)$. Having replaced the subgame with its subgame value, the entire game can be solved by applying simple backward induction. The resulting equilibria are denoted as $(f^*; \gamma^*; \alpha^*, \beta^*; \mu^*, \nu^*)$.

3.1 The Commission’s optimal choices

After having received a positive signal $i = g$, the Commission reaches information set $I_1$: it does not know whether it has reached decision node $C_1$ or $C_2$, i.e. it does not know the actual choice of the firms ($a \in \{g, b\}$). In this situation the Commission has to decide whether to prohibit the agreement under scrutiny or not, i.e. it must choose $\alpha$. In doing so the Commission faces the following maximization problem:

$$\max_{\alpha} \quad EP_{I_1}^C = -\mu(1-\alpha)\chi - (1-\mu)\alpha\chi = [(2\mu - 1)\alpha - \mu]\chi,$$  \hspace{1cm} (3)
which yields the following first order condition:

\[
\frac{dEP_C}{d\alpha} = (2\mu - 1)\chi = 0. \tag{4}
\]

Using Bayes’ rule, substituting \( \mu := \frac{\gamma\rho}{\gamma\rho + (1 - \gamma)\varphi} \) yields

\[
\frac{dEP_C}{d\alpha} = \left(2\frac{\gamma\rho}{\gamma\rho + (1 - \gamma)\varphi} - 1\right)\chi. \tag{5}
\]

The right-hand side of equation (5) is positive whenever the following inequality holds:

\[
\gamma > \frac{\varphi}{\rho + \varphi} := \gamma_1. \tag{6}
\]

In this case, the Commission faces a payoff increasing in \( \alpha \) and thus chooses \( \alpha \) as high as possible: \( \alpha^* = \alpha^*(\gamma) = 1 \). If the opposite is true (\( \gamma < \gamma_1 \)) the Commission chooses \( \alpha^* = \alpha^*(\gamma) = 0 \), and the Commission is indifferent (\( \alpha^* = \alpha^*(\gamma) \in [0; 1] \)) whenever \( \gamma = \gamma_1 \). \( ^{24} \) This correlation of the Commission’s behavior \( \alpha \) and the firms’ behavior \( \gamma \) can be summarized in the following best response function\( ^{25} \):

\[
\alpha^*(\gamma) = \begin{cases} 
0 & \forall \gamma < \gamma_1; \\
x \in [0; 1] & \text{for } \gamma = \gamma_1; \\
1 & \forall \gamma > \gamma_1. 
\end{cases} \tag{7}
\]

In the same vein the Commission’s behavioral strategy \( \beta^*(\gamma) \) after having received a negative signal can be derived again using Bayes’ rule which yields the Commission’s ex post belief of facing a bad agreement \( \nu := \frac{\gamma(1 - \rho)}{\gamma(1 - \rho) + (1 - \gamma)(1 - \varphi)} \). The Commission’s maximization problem amounts to

\[
\max_{\beta} EP^C_{I_2} = -\nu(1 - \beta)\chi - (1 - \nu)\beta\chi = [(2\nu - 1)\beta - \nu]\chi. \tag{8}
\]

The first order condition is given by

\[
\frac{dEP^C_{I_2}}{d\beta} = (2\nu - 1)\chi = 0 \tag{9}
\]

and by substituting \( \nu \) the first derivative can be rewritten as

\[
\frac{dEP^C_{I_2}}{d\beta} = \left(2\frac{\gamma(1 - \rho)}{\gamma(1 - \rho) + (1 - \gamma)(1 - \varphi)} - 1\right)\chi. \tag{10}
\]

\( ^{24} \)Note that it is sufficient for maximization purposes to consider the first order condition of \( C \)’s maximization problem only. Since its payoff is linear in \( \alpha \) the second order condition always equals zero.

\( ^{25} \)Which is, to be precise, a relation and not a function in mathematical sense, as for \( \gamma = \gamma_1 \) there exist more than one corresponding values \( \alpha^*(\gamma) \). Nevertheless, economists tend to call it best response function.
The right-hand side of equation (10) is positive whenever the following inequality holds:

\[ \gamma > \frac{1 - \varphi}{1 - \rho + 1 - \varphi} := \gamma_2. \tag{11} \]

In this case, the Commission faces a payoff increasing in \( \beta \) and therefore chooses \( \beta \) as high as possible: \( \beta^* = \beta^*(\gamma) = 1 \). Analyzing the remaining possibilities of the relation between the firms’ behavior \( \gamma \) and the threshold \( \gamma_2 \) yields the Commission’s best response function \( \beta^*(\gamma) \) having received a negative signal in regard to the firms’ behavior \( \gamma \).

\[
\beta^*(\gamma) = \begin{cases} 
0 & \forall \gamma < \gamma_2; \\
x \in [0; 1] & \text{for } \gamma = \gamma_2; \\
1 & \forall \gamma > \gamma_2. 
\end{cases} \tag{12}
\]

The Commission’s best response functions \( \alpha^*(\gamma) \) and \( \beta^*(\gamma) \) are depicted in figure 2. Note that \( \gamma_1 < \gamma_2 \) is guaranteed by assuming \( 1 > \rho > \varphi > 0 \).

![Figure 2: C’s best response functions \( \alpha^*(\gamma) \) and \( \beta^*(\gamma) \)](image)

### 3.2 The firms’ optimal choices

The firms have to make two choices in the Legal Exception Game: they have to chose their optimal compliance \( \gamma^* \) which is their best response to the Commission’s behavioral strategies \( \alpha^*(\gamma) \) and \( \beta^*(\gamma) \) and they have to decide whether to sign an agreement in the first place.

\footnote{Again, the second order condition equals zero for all values of \( \beta \), as \( C \)’s expected payoff is linear in \( \beta \), and can therefore be neglected.}
3.2.1 F’s optimal choice in the subgame

First, let us consider the subgame starting in decision node $F_1$. The firms face the following maximization problem:

$$
\max_{\gamma} \ E P_{\text{sub}}^F = G + (1 - \gamma) A \\
- \gamma [\rho(1 - \alpha) + (1 - \rho)(1 - \beta)] \xi(1 - \chi)M \\
- (1 - \gamma)[\varphi \alpha + (1 - \varphi)\beta] \xi \chi M \\
- (1 - \gamma)[\varphi(1 - \alpha) + (1 - \varphi)(1 - \beta)] \xi M, \tag{13}
$$

which yields the following first order condition:

$$
\frac{dEP_{\text{sub}}^F}{d\gamma} = -A - [\rho(1 - \alpha) + (1 - \rho)(1 - \beta)] \xi(1 - \chi)M \\
+ [\varphi \alpha + (1 - \varphi)\beta] \xi \chi M \\
+ [\varphi(1 - \alpha) + (1 - \varphi)(1 - \beta)] \xi M = 0. \tag{14}
$$

The first derivative can be rewritten as

$$
\frac{dEP_{\text{sub}}^F}{d\gamma} = (\rho - \varphi)(1 - \chi)\xi M (\alpha - \beta) + \chi \xi M - A. \tag{15}
$$

which allows to separate terms depending on $\alpha$ or $\beta$ from others which do not. The right-hand side of equation (15) is positive if the following inequality holds:

$$
\beta < Y + \alpha, \tag{16}
$$

where $Y$ is defined as

$$
Y := \frac{\xi \chi M - A}{(1 - \chi)(\rho - \varphi)\xi M}. \tag{17}
$$

In this case, the firms face a payoff increasing in $\gamma$ and thus choose $\gamma$ as high as possible: $\gamma^* = \gamma^*(\alpha, \beta) = 1$. If the opposite is true ($\beta > Y + \alpha$) the firms choose $\gamma^* = \gamma^*(\alpha, \beta) = 0$, and the firms are indifferent ($\gamma^* = \gamma^*(\alpha, \beta) \in [0; 1]$) whenever $\beta = Y + \alpha$.\(^{27}\) This correlation of the firms’ behavior $\gamma$ and the Commission’s behavior $(\alpha, \beta)$ can be summarized in the following best response function:

$$
\gamma^*(\alpha, \beta) = \begin{cases} 
0 & \forall \ \beta > Y + \alpha; \\
x \in [0; 1] & \text{for } \beta = Y + \alpha; \\
1 & \forall \ \beta < Y + \alpha. 
\end{cases} \tag{18}
$$

Figure 3 shows a possible best response function as the cascaded gray shaded area in the unitary cube. The exact location of the function depends on the

\(^{27}\)Again, it is sufficient to consider the first order condition only. The second order condition equals zero for all values of $\gamma$, as $F$’s expected payoff is linear in $\gamma$. 

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parameters \((\rho, \varphi, \xi, \chi, M, A)\) which determine the axis intercept \(Y\). Due to the parameter assumptions \(Y\) can reach any value in \((-\infty, +\infty)\).

Five relevant parameter constellations (PC) can be identified in each of which the firms’ best response functions have different shapes:

- \(Y > 0\) (PC1),
- \(Y = 0\) (PC2),
- \(-1 < Y < 0\) (PC3),
- \(Y = -1\) (PC4),
- \(Y < -1\) (PC5).

The shapes of \(F\)’s best response functions differ with respect to the side or edge of the unitary cube which is intersected by the vertical part of the function. Figure 4 shows examples for the five possible shapes of \(\gamma^*(\alpha, \beta)\).\(^{28}\)

Setting the axis intercept \(Y\) equal to zero or minus one yields the following thresholds:

\[
Y = 0 \iff A_1 := \xi \chi M , \quad (19)
\]

\[
Y = -1 \iff A_2 := [\chi + (1 - \chi)(\rho - \varphi)]\xi M . \quad (20)
\]

It is convenient to define the following intersections of the firms’ best response function \(\gamma^*(\alpha, \beta)\) with the unitary cube’s sides:

\[
\beta_1 := Y + 1 = 1 + \frac{\xi \chi M - A}{(1 - \chi)(\rho - \varphi)\xi M} , \quad (21)
\]

\[
\alpha_1 := -Y = \frac{A - \xi \chi M}{(1 - \chi)(\rho - \varphi)\xi M} . \quad (22)
\]

\(^{28}\)Note that PC1 comprises three subcases, \(Y > 1\), \(Y = 1\) and \(0 < Y < 1\), all of which lead to the same subgame equilibrium.
Figure 4: Five classes of $F$'s best response function $\gamma^*(\alpha, \beta)$
At this stage, the perfect Bayesian Nash equilibria of the subgame starting in decision node $F_1$ can be derived. A perfect Bayesian Nash equilibrium lies in the intersection of the three best response functions $\alpha^*(\gamma)$, $\beta^*(\gamma)$ and $\gamma^*(\alpha, \beta)$. Figure 5 shows the resulting equilibria for the five identified parameter constellations. An unambiguous equilibrium is depicted as a black circular area while equilibrium sets where an infinite number of equilibria is possible are depicted as a black rectangle on the corresponding segment.

In the first row of figure 5, the equilibria for the parameter constellations PC1 and PC2 are depicted. Equilibrium 1 is realized if the condition $A < A_1$, i.e. $Y > 0$ (PC1), holds:

$$EQ1 = (1; 1, 1; 1, 1).$$

(23)

In this case the firms fully comply to cartel law by choosing a compliance level $\gamma^* = 1$, i.e. signing a good agreement with certainty, and the Commission never prohibits a controlled agreement regardless the received signal ($\alpha^* = \beta^* = 1$). Consistently, the posterior beliefs $\mu^*$ and $\nu^*$ also amount to 1, i.e. the Commission knows for certain that it has reached the decision nodes $C_1$ and $C_3$ respectively, which is only possible if the firms sign a good agreement.\(^{29}\)

The equilibrium set EQS2 is reached if the condition $A = A_1$, i.e. $Y = 0$, holds. In this case (PC2) an infinite number of equilibria exists:

$$EQS2 = \{(x; 1, 1; m, n) | x \in [\gamma_2; 1]; m \in [\mu_1; 1]; n \in [\frac{1}{2}; 1]\}.$$ 

(24)

In every equilibrium out of this set the firms choose a good agreement with probability $x \in [\gamma_2; 1]$, while the Commission again never prohibits an agreement regardless the received signal ($\alpha^* = \beta^* = 1$). The corresponding posterior beliefs depend on the firms’ choice and range from $\mu_1 := \frac{(1-\varphi)^\rho}{(1-\varphi)^\rho+(1-\gamma)^\rho}$(1-\rho)\varphi}$ to 1 for $\mu^*$ and from $\frac{1}{2}$ to 1 for $\nu^*$.\(^{30}\) In addition to this, a second set of equilibria exists in parameter constellation PC2:

$$EQS6 = \{(x; 0, 0; m, n) | x \in [0; \gamma_1]; m \in [0; \frac{1}{2}]; n \in [0; \nu_1]\}.$$ 

(25)

Here, the firms choose a compliance level $x \in [0; \gamma_1]$ and the Commission prohibits every agreement with certainty regardless the received signal ($\alpha^* = \beta^* = 0$). The corresponding posterior beliefs again depend on the firms’ choice and range from 0 to 1 for $\mu^*$ and from 0 to $\nu_1 := \frac{(1-\rho)^\varphi}{(1-\rho)^\varphi+(1-\varphi)^\rho}$ for $\nu^*$.\(^{32}\)

\(^{29}\)The posterior beliefs in equilibrium $\mu^*$ and $\nu^*$ are calculated by replacing $\gamma$ with its equilibrium value $\gamma^*$ in the terms $\mu := \frac{\gamma^*(\gamma)^\rho}{\gamma^*(\gamma)^\rho+(1-\gamma)^\rho}$ and $\nu := \frac{\gamma^*(1-\rho)^\varphi}{\gamma^*(1-\rho)^\varphi+(1-\rho)^\varphi}$.

\(^{30}\)Note that $\mu_1$ is always greater than $\frac{1}{2}$ as long as the condition $\rho > \varphi$ is not violated.

\(^{31}\)The numbering of the equilibria/equilibrium sets is arranged in accordance to the inherent compliance level $\gamma^*$. EQ1 is the equilibrium with the highest compliance level ($\gamma^* = 1$), EQS2 the equilibrium set with the second highest compliance levels ($\gamma^* = x \in [\gamma_2; 1]$) and so on, down to EQ7 with the lowest compliance level ($\gamma^* = 0$).

\(^{32}\)Note that $\nu_1$ is always smaller than $\frac{1}{2}$ as long as the condition $\rho > \varphi$ is not violated.
Figure 5: Perfect Bayesian Nash equilibria \((\gamma^*; \alpha^*; \beta^*; \mu^*; \nu^*)\) in the subgame
In the second row of figure 5, the equilibria for the parameter constellations PC3 and PC4 are depicted. If the condition \( A_1 < A < A_2 \), i.e. \(-1 < Y < 0\) (PC3), holds three different equilibria can be realized:

\[
\begin{align*}
\text{EQ3} &= (\gamma_2; 1; \beta_1; \mu_1; \frac{1}{2}), \\
\text{EQ5} &= (\gamma_1; \alpha_1; 0; \frac{1}{2}; \nu_1) \\
\text{EQ7} &= (0; 0; 0; 0, 0).
\end{align*}
\]  

(26) (27) (28)

In equilibrium EQ3 the firms sign a good agreement with probability \( \gamma^* = \gamma_2 \). This agreement is exempted from the cartel ban with certainty if the signal is good (\( \alpha^* = 1 \)) and with probability \( \beta_1 \) if the signal is bad. The corresponding posterior beliefs \( (\mu^*, \nu^*) \) amount to \( (\mu_1, \frac{1}{2}) \). In equilibrium EQ5 the firms’ compliance level is smaller: \( \gamma^* = \gamma_1 \). The Commission reacts to this behavior by prohibiting the agreement with certainty whenever the signal is bad (\( \beta^* = 0 \)) and by not objecting to it only with probability \( \alpha_1 \) whenever the signal is good. This decision is based on the equilibrium posterior beliefs \( (\mu^*, \nu^*) = (\frac{1}{2}, \nu_1) \). Finally, equilibrium EQ7 is characterized by zero compliance of the firms \( \gamma^* = 0 \) and an unconditional ban decision by the Commission \( (\alpha^* = \beta^* = 0) \). Consistently, the Commission’s posterior beliefs that it has reached decision node \( C_1 \) or \( C_3 \), which is the case when the firms have signed a good agreement, also equal zero \( (\mu^* = \nu^* = 0) \).

If the condition \( A = A_2 \), i.e. \( Y = -1 \) (PC4) is fulfilled, there again exist multiple equilibria, namely the equilibrium set EQS4 and the equilibrium EQ7 already known from parameter constellation PC3. EQS4 is defined as follows:

\[
\text{EQS4} = \{(x; 1, 0; m, n)| x \in [\gamma_1; \gamma_2]; m \in [\frac{1}{2}; \mu_1]; n \in [\nu_1; \frac{1}{2}]\}.
\]  

(29)

Each of these equilibria implies the firms choosing a good agreement with probability \( x \in [\gamma_1; \gamma_2] \) and the Commission prohibiting the agreement whenever the signal is bad \( (\beta^* = 0) \) and not objecting to it whenever the signal is good \( (\alpha^* = 1) \). The corresponding posterior beliefs range from \( \frac{1}{2} \) to \( \mu_1 \) for \( \mu^* \) and from \( \nu_1 \) to \( \frac{1}{2} \) for \( \nu^* \).

Finally, in the third row of figure 5, the equilibrium of parameter constellation PC5 is depicted. If the condition \( A > A_2 \), i.e. \( Y < -1 \), holds, equilibrium EQ7—already known from parameter constellations PC3 and PC4—is realized. Table 1 gives an overview of all derived subgame equilibria.

\[3.2.2 \text{ Equilibrium selection}\]

In three of the five parameter constellations—PC2, PC3 and PC4—we face an equilibrium selection problem. In the following, we will briefly introduce a reasonable mechanism to choose one subgame equilibrium amongst the others in each parameter constellation.

Let us first consider parameter constellation PC2. Here we face a twofold equilibrium selection problem: (1) There are two sets of equilibria EQS2 and
Table 1: Subgame equilibria

<table>
<thead>
<tr>
<th>PC</th>
<th>equilibrium cond.</th>
<th>equilibrium ((\gamma^<em>, \alpha^</em>, \beta^<em>; \mu^</em>, \nu^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1</td>
<td>(A &lt; A_1)</td>
<td>EQ1: ((1; 1, 1; 1, 1))</td>
</tr>
<tr>
<td>PC2</td>
<td>(A = A_1)</td>
<td>EQ2: ({(x; 1, 1; m, n)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EQ6: ({(x; 0, 0; m, n)</td>
</tr>
<tr>
<td>PC3</td>
<td>(A_1 &lt; A &lt; A_2)</td>
<td>EQ3: ((\gamma_2; 1, \beta_1; \mu_1, \frac{1}{2}))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EQ5: ((\gamma_1; \alpha_1, 0, \frac{1}{2}, \nu_1))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EQ7: ((0; 0, 0; 0, 0))</td>
</tr>
<tr>
<td>PC4</td>
<td>(A = A_2)</td>
<td>EQ4: ({(x; 1, 0; m, n)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EQ7: ((0; 0, 0; 0, 0))</td>
</tr>
<tr>
<td>PC5</td>
<td>(A &gt; A_2)</td>
<td>EQ7: ((0; 0, 0; 0, 0))</td>
</tr>
</tbody>
</table>

with \(A_1 := \xi \chi M\), \(A_2 := [\chi + (1 - \chi)(\rho - \varphi)]\xi M\);
\[\gamma_1 := \frac{\varphi}{\rho + \varphi}, \quad \gamma_2 := \frac{1 - \varphi}{1 - \rho + 1 - \varphi},\]
\[\mu_1 := \frac{\rho(1 - \varphi)}{\rho(1 - \varphi) + (1 - \rho)\varphi}, \quad \nu_1 := \frac{(1 - \rho)\varphi}{\rho(1 - \varphi) + (1 - \rho)\varphi},\]
\[\alpha_1 := -Y = -\frac{\xi \chi M - A}{(1 - \chi)(\rho - \varphi)\xi M} \quad \text{and} \quad \beta_1 := Y + 1 = \frac{\xi \chi M - A}{(1 - \chi)(\rho - \varphi)\xi M} + 1.\]

EQS6. (2) Within these sets of equilibria there exists an infinite number of equilibria which differ only with respect to the firms’ compliance level while the Commission’s action remains unchanged. It is worthwhile noting that the multiple equilibria within one set have an interesting feature: they all lead to the same payoff for the firms irrespective the concrete value taken by F’s behavioral strategy \(\gamma^* = x\). In the equilibrium set EQS2 this payoff amounts to \(G\), in EQS6 to \(G - (1 - \chi)\xi M\). Thus, the firms are indifferent with respect to their actual compliance level within the given range. In other words, the firms do not prefer a specific equilibrium within EQS2 and EQS6. It is therefore possible to make the following assumption: Whenever the firms are indifferent regarding their compliance level, they chose the highest possible compliance level as they do not face any opportunity cost in doing so. Consequently, the equilibrium sets EQS2 and EQS6 can be reduced to two single unambiguous equilibria: In EQS2 the firms
choose $\gamma^* = 1$ so that EQ1 is reached given the Commission’s equilibrium strategy $\alpha^* = \beta^* = 1$ and its equilibrium posterior beliefs $\mu^* = \nu^* = 1$; in EQS6 the firms choose $\gamma^* = \gamma_1$, while the Commission chooses ($\alpha^*, \beta^*$) = (0, 0) based on its posterior beliefs $(\mu^*, \nu^*) = (\frac{1}{2}, \nu_1)$. This latter equilibrium $(\gamma_1, 0, 0, \frac{1}{2}, \nu_1)$ is a special case of equilibrium EQ5 = $(\gamma_1; \alpha_1, 0, \frac{1}{2}, \nu_1)$ in parameter constellation PC3 ($A_1 < A < A_2$): it is the limit of EQ5 for $A$ to $A_1$ (yielding $\lim_{A \to A_1} \alpha_1 = 0$) while $A = A_1$ is the defining criterion for parameter constellation PC2.

Let us now turn to the second problem in this parameter constellation. Reducing the equilibrium sets EQS2 and EQS6 to one equilibrium each does not fully solve the equilibrium selection problem; there still exist two equilibria in parameter constellation PC2, namely EQ1 and EQ5. Which of those will be realized? Both equilibria are consistent combinations of equilibrium strategies and corresponding beliefs and therefore are both equally feasible. Still, EQ1 features a focal point quality which EQ5 does not. The firms’ payoff is higher in EQ1 than in EQ5.\(^{34}\) Although the game is simultaneous rather than sequential because of the imperfectly received signal the firms can be considered pseudo-first-mover as their “real life” move lies before the Commission’s choice.\(^{35}\) As a pseudo-first-mover the firms will try to reach an equilibrium that maximizes their expected payoff while the Commission as a pseudo-second-mover anticipates this behavior. Thus, the players act as if the moves were observable. This concept is known as virtual observability. This kind of equilibrium selection is not only intuitively reasonable but numerous experiments have also provided evidence for it.\(^{36}\) Hence, the multiple equilibria in parameter constellation PC2 can be reduced to one single equilibrium, namely EQ1.

In parameter constellation PC3 three different subgame equilibria exist, namely EQ3, EQ5 and EQ7. The concept of virtual observability can again be applied as the firms’ payoff in EQ3 is higher than the ones realized in EQ5 and EQ7.\(^{37}\) Thus, equilibrium EQ3 dominates the other two equilibria.

In parameter constellation PC4 the equilibrium set EQS4 and the equilibrium

\(^{34}\) The firms’ expected payoff in EQ1 amounts to $G$, in EQ5 to $G - (1 - \chi)\xi M$ (see table 2), the latter being necessarily smaller than the former given $\chi < 1$ and $\xi, M > 0$.

\(^{35}\) The game theoretic standard assumption of simultaneous moves whenever moves are not observable neglects the fact that unobservability and simultaneousness in the sense of contemporaneity are not congruent concepts. Simultaneity implies unobservability but not vice versa, see e.g. Weber/Camerer/Knez (2004, p. 26-27).

\(^{36}\) For the concept of virtual observability see Weber/Camerer/Knez (2004, p. 25-31). Experiments on the subject can be found e.g. in Abele/Ehrhart (2005), Rapoport (1997) and also Weber/Camerer/Knez (2004).

\(^{37}\) The firms’ expected payoff in EQ3 amounts to $G - \frac{1 - \delta}{\rho - \phi}(A - \xi \chi M)$, in EQ5 to $G + A - \xi M + \frac{\delta}{\rho - \phi}(A - \xi \chi M)$ and in EQ7 to $G + A - \xi M$, see table 2. Obviously, the firms’ payoff in EQ5 is greater than the one in EQ7, as $\frac{\delta}{\rho - \phi} > 0$ and because of $A > A_1 := \xi \chi M$ in PC3 also $(A - \xi \chi M) > 0$. More sophisticated algebraic reformulation shows that the firms’ expected payoff in EQ3 is greater than the one in EQ5 as long as $A < A_2$ which is fulfilled by definition of PC3.
EQ7 exist. Within Eqs4 the firms’ expected payoff amounts to a single value regardless the compliance level $\gamma^* = x \in [\gamma_1; \gamma_2]$ chosen, namely $G - (1 - \rho)(1 - \chi)\xi M$. Hence, following the same reasoning as above, it can be assumed that the firms choose the highest compliance level feasible, i.e. $\gamma^* = \gamma_2$. This equilibrium $(\gamma_2; 1, 0; \mu_1, \frac{1}{2})$ is a special case of equilibrium $EQ3 = (\gamma_2, \beta_1; \mu_1, \frac{1}{2})$ in parameter constellation $PC3$ ($A_1 < A < A_2$): it is the limit of $EQ3$ for $A$ to $A_2$ (yielding $\lim_{A \to A_2} \beta_1 = 0$) while $A = A_2$ is the constituting condition for parameter constellation $PC4$. The firms’ expected payoff in equilibrium $EQ3$ is greater than the one in $EQ7$. Thus, applying the concept of virtual observability, equilibrium $EQ7$ is dominated by equilibrium $EQ3$.

### 3.2.3 F’s optimal choice in the Legal Exception Game

Having solved the subgame starting in decision node $F_1$, the subgame can be substituted by its subgame value which equals the vector of the expected subgame payoffs of both players $[EP_{sub}^F(\gamma^*; \alpha^*; \beta^*; \mu^*; \nu^*)]; EP_{sub}^C(\gamma^*; \alpha^*; \beta^*; \mu^*; \nu^*)]$. The firms’ expected subgame payoff for each parameter constellation is given in table 2. The firms’ equilibrium payoffs can be derived by replacing in $EP_{sub}^F$ as given in equation (13) the behavioral strategies $\alpha$, $\beta$ and $\gamma$ with their equilibrium values $\alpha^*$, $\beta^*$ and $\gamma^*$.

Thus, the firms face the following decision problem in node $F_0$: whenever the firms choose $f^* = out$ status quo persists and the firms earn a (joint) profit of zero. If they choose $f^* = in$ they receive their expected subgame payoff $EP_{sub}^F(\gamma^*; \alpha^*; \beta^*; \mu^*; \nu^*)$ which may be positive or negative depending on the relative heights of the agreement gains ($G, A$) to the fine ($M$) and also depending on the Commission’s assessment skills ($\rho, \varphi$) and the control and the appeals probability ($\xi, \chi$). The firms will sign an agreement, i.e. choose $f^* = in$, whenever their expected payoff ($EP_{sub}^F(\gamma^*; \alpha^*; \beta^*; \mu^*; \nu^*)$) is non-negative. Setting the firms’ expected payoff equal to zero, the fine $M^*$ can be derived where the firms are indifferent whether to sign the agreement or not. Table 3 lists these fines $M^*$ for every parameter constellation.

### 3.3 The equilibria

Having derived the subgame equilibria and the fines $M^*$ determining the firms’ optimal choice in decision node $F_0$, the equilibria of the entire Legal Exception Game, denoted as $(f^*, \gamma^*; \alpha^*; \beta^*; \mu^*; \nu^*)$ can be derived. There exist twelve different equilibria, some of them appearing only in one, some of them appearing in more than one parameter constellation. Table 4 lists the twelve equilibria with

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38 The firms expected payoff in $EQ3$ amounts to $G - (1 - \rho)(1 - \chi)\xi M$, in $EQ7$ to $G - [1 - (\rho - \varphi)](1 - \chi)\xi M$ (see table 2), the latter being necessarily smaller than the former given $\varphi > 0$.

39 Note that the Commission’s expected subgame payoff can be neglected for further analysis as there is only $F$ left to make a move.
the two corresponding equilibrium conditions each. The equilibria are sorted by ascending parameter constellations PC1 to PC5.

Note that according to the equilibrium selection mechanism defined in section 3.2.2, some of the equilibria are dominated. In parameter constellation PC2 the equilibrium sets EQS in-2, EQS in-6 and EQS out-6 are dominated by equilibrium EQ in-1; in parameter constellation PC3 the equilibria EQ in-5 and EQ in-7 are dominated by EQ in-3, while the equilibria EQ out-5 and EQ out-7 are dominated by EQ out-3, and in parameter constellation PC4 the equilibrium set

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<table>
<thead>
<tr>
<th>PC</th>
<th>Payoff $EP_{sub}^F(\gamma^<em>; \alpha^</em>; \beta^<em>; \mu^</em>; \nu^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1: $A &lt; A_1$</td>
<td>$EP^F_{1,1}(1; 1; 1; 1, 1)$ = $G$</td>
</tr>
<tr>
<td>PC2: $A = A_1$</td>
<td>$EP^F_{2,2}(x; 1, 1; m, n)$ = $G$ ( \forall x \in [\gamma_2; 1], m \in [\mu_1; 1], n \in [\frac{1}{2}; 1] )</td>
</tr>
<tr>
<td></td>
<td>$EP^F_{2,6}(x; 0, 0; m, n)$ = $G - (1 - \chi)\xi M$ ( \forall x \in [\gamma_2; 1], m \in [0; \frac{1}{2}], n \in [0; \nu_1] )</td>
</tr>
<tr>
<td>PC3:</td>
<td>$EP^F_{3,3}(\gamma_2; 1, \beta_1; \mu_1, \frac{1}{2})$ = $G - \frac{1-\rho}{\rho-\varphi}(A - \xi M)$</td>
</tr>
<tr>
<td></td>
<td>$EP^F_{3,5}(\gamma_1; \alpha_1, 0; \frac{1}{2}, \nu_1)$ = $G + A - \xi M - \frac{\varphi}{\rho - \varphi}(\xi M - A)$</td>
</tr>
<tr>
<td></td>
<td>$EP^F_{3,7}(0; 0, 0; 0, 0)$ = $G + A - \xi M$</td>
</tr>
<tr>
<td>PC4: $A = A_2$</td>
<td>$EP^F_{4,4}(x; 1, 0; m, n)$ = $G - (1 - \rho)(1 - \chi)\xi M$ ( \forall x \in [\gamma_1; \gamma_2], m \in [\frac{1}{2}; \mu_1], n \in [\nu_1; \frac{1}{2}] )</td>
</tr>
<tr>
<td></td>
<td>$EP^F_{4,7}(0; 0, 0; 0, 0)$ = $G - (1 - \chi)(1 - (\rho - \varphi))\xi M$</td>
</tr>
<tr>
<td>PC5: $A &gt; A_2$</td>
<td>$EP^F_{5,7}(0; 0, 0; 0, 0)$ = $G + A - \xi M$</td>
</tr>
</tbody>
</table>

with $A_1 := \xi M$, $A_2 := [\chi + (1 - \chi)(\rho - \varphi)]\xi M$;

$\gamma_1 := \frac{\varphi}{\rho + \varphi}$, \hspace{1cm} $\gamma_2 := \frac{1 - \varphi}{1 - \rho + 1 - \varphi}$, \hspace{1cm} $\mu_1 := \frac{\rho(1 - \varphi)}{\rho(1 - \varphi) + (1 - \rho)\varphi}$, \hspace{1cm} $\nu_1 := \frac{(1 - \rho)\varphi}{\rho(1 - \varphi) + (1 - \rho)\varphi}$;

$\alpha_1 := -Y = -\frac{\xi M - A}{(1 - \chi)(\rho - \varphi)\xi M}$ and \hspace{1cm} $\beta_1 := Y + 1 = \frac{\xi M - A}{(1 - \chi)(\rho - \varphi)\xi M} + 1$.

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Table 2: $F$’s expected subgame payoff $EP_{sub}^F(\gamma^*; \alpha^*; \beta^*; \mu^*; \nu^*)$
EQS in-4 and the equilibrium EQ in-7 are dominated by the equilibrium EQ in-3, while equilibrium EQ out-3 dominates the equilibrium set EQS out-4 as well as equilibrium EQ out-7.\(^{41}\)

According to the compliance level \(\gamma^*\) the twelve equilibria can be grouped into four different types. Full compliance by the firms, i.e. \(\gamma^* = 1\), is only reached in equilibrium EQ in-1. The equilibrium EQ in-1 is realized if the conditions \(A < A_1\) (PC1) and \(M \in (0; \infty)\) hold and is also part of equilibrium set EQS in-2 which is realized if the conditions \(A = A_1\) (PC2) and \(M \in (0; \infty)\) are fulfilled. Recall that equilibrium EQ in-1 is chosen out of the multiple equilibria in the equilibrium set EQS in-2 if the firms are assumed to chose the highest compliance level feasible when indifferent about moves. We call such equilibria full-compliance equilibria.

In some parameter constellations the firms decide to sign a bad agreement with certainty, i.e. \(\gamma^* = 0\). In such a case equilibrium EQ in-7 is reached. We call this equilibrium zero-compliance equilibrium. This equilibrium is realized if the additional profit \(A\) is greater than the threshold \(A_1\) and if the monetary fine \(M\) is not set sufficiently high to deter the firms from signing an agreement. Recall that the zero-compliance equilibrium is a dominated one as long as \(A \leq A_2\) holds.

In all parameter constellations except PC1 it is possible to deter the firms

<table>
<thead>
<tr>
<th>parameter constellation</th>
<th>fine (M^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1: (A &lt; A_1)</td>
<td>(M^*_{1,1} \in (0; \infty))</td>
</tr>
<tr>
<td>PC2: (A = A_1)</td>
<td>(M^*_{2,2} \in (0; \infty))</td>
</tr>
<tr>
<td></td>
<td>(M^*_{2,6} = \frac{1}{\xi(1-\chi)} G)</td>
</tr>
<tr>
<td>PC3: (A_1 &lt; A &lt; A_2)</td>
<td>(M^*_{3,3} = \frac{1}{\xi\chi} A - \frac{\rho-\varphi}{\xi\chi(1-\rho)} G)</td>
</tr>
<tr>
<td></td>
<td>(M^*_{3,5} = \frac{1}{\xi(\rho-(1-\chi)\varphi)} G + \frac{\rho}{\xi\chi(1-\rho)} A)</td>
</tr>
<tr>
<td></td>
<td>(M^*_{3,7} = \frac{1}{\xi}(G + A))</td>
</tr>
<tr>
<td>PC4: (A = A_2)</td>
<td>(M^*_{4,4} = \frac{1}{\xi(1-\rho)(1-\chi)} G)</td>
</tr>
<tr>
<td></td>
<td>(M^*_{4,7} = \frac{1}{\xi(1-\chi)[1-(\rho-\varphi)]} G)</td>
</tr>
<tr>
<td>PC5: (A &gt; A_2)</td>
<td>(M^*_{5,7} = \frac{1}{\xi}(G + A))</td>
</tr>
</tbody>
</table>

Table 3: Optimal fines \(M^*\) for each parameter constellation

\(^{41}\)Recall from section 3.2.2 that the subgame equilibrium EQ3 = \((\gamma_2; 1, \beta_1; \mu_1, \xi)\) is part of the subgame equilibrium set EQS4 = \(\{(x; 1, 0; m, n)| x \in [\gamma_2; \gamma_2]; m \in [\frac{1}{2}; \mu_1]; n \in [\nu_1; \frac{1}{2}]\}\) (whenever \(x \in [\gamma_1; \gamma_2]\) takes the value \(\gamma_2\)) and the limit of EQ3 for \(A \rightarrow A_2\), see section 3.2.2. For the same reason, EQ in-3 and EQ out-3 are part of the equilibrium sets EQS in-4 and EQS out-4 and the limits of EQ in-3 and EQ out-3 for \(A \rightarrow A_2\), respectively.
<table>
<thead>
<tr>
<th>1&lt;sup&gt;st&lt;/sup&gt; cond.</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; cond.</th>
<th>equilibrium ((f^<em>, \gamma^</em>; \alpha^<em>, \beta^</em>; \mu^<em>, \nu^</em>))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A &lt; A_1) (PC1)</td>
<td>(M \in (0; \infty))</td>
<td>EQ in-1: ((in, 1; 1, 1; 1, 1))</td>
</tr>
<tr>
<td>(A = A_1) (PC2)</td>
<td>(M \in (0; \infty))</td>
<td>EQS in-2: ({(in, x; 1, 1; m, n)\</td>
</tr>
<tr>
<td>&amp; \quad x \in [\gamma_2; 1]; m \in [\mu_1; 1]; n \in [\frac{1}{2}; 1]})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M \leq M^*_{2,6})</td>
<td>EQS in-6: ({(in, x; 0, 0; m, n)\</td>
<td></td>
</tr>
<tr>
<td>&amp; \quad x \in [0; \gamma_1]; m \in [0; \frac{1}{2}]; n \in [0; \nu_1]})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M &gt; M^*_{2,6})</td>
<td>EQS out-6: ({(out, x; 0, 0; m, n)\</td>
<td></td>
</tr>
<tr>
<td>&amp; \quad x \in [0; \gamma_1]; m \in [0; \frac{1}{2}]; n \in [0; \nu_1]})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A_1 &lt; A &lt; A_2) (PC3)</td>
<td>(M \geq M^*_{3,3})</td>
<td>EQ in-3: ((in, \gamma_2; 1, \beta_1; \mu_1, \frac{1}{2}))</td>
</tr>
<tr>
<td>(M \leq M^*_{3,5})</td>
<td>EQ in-5: ((in, \gamma_1; \alpha_1, 0; \frac{1}{2}, \nu_1))</td>
<td></td>
</tr>
<tr>
<td>(M \leq M^*_{3,7})</td>
<td>EQ in-7: ((in, 0; 0, 0; 0, 0))</td>
<td></td>
</tr>
<tr>
<td>(M &lt; M^*_{3,3})</td>
<td>EQ out-3: ((out, \gamma_2; 1, \beta_1; \mu_1, \frac{1}{2}))</td>
<td></td>
</tr>
<tr>
<td>(M &gt; M^*_{3,5})</td>
<td>EQ out-5: ((out, \gamma_1; \alpha_1, 0; \frac{1}{2}, \nu_1))</td>
<td></td>
</tr>
<tr>
<td>(M &gt; M^*_{3,7})</td>
<td>EQ out-7: ((out, 0; 0, 0; 0, 0))</td>
<td></td>
</tr>
<tr>
<td>(A = A_2) (PC4)</td>
<td>(M \leq M^*_{4,4})</td>
<td>EQS in-4: ({(in, x; 1, 0; m, n)\</td>
</tr>
<tr>
<td>&amp; \quad x \in [\gamma_1; \gamma_2]; m \in [\frac{1}{2}; \mu_1]; n \in [\nu_1; \frac{1}{2}]})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M \leq M^*_{4,7})</td>
<td>EQ in-7: ((in, 0; 0, 0; 0, 0))</td>
<td></td>
</tr>
<tr>
<td>(M &gt; M^*_{4,4})</td>
<td>EQS out-4: ({(out, x; 1, 0; m, n)\</td>
<td></td>
</tr>
<tr>
<td>&amp; \quad x \in [\gamma_1; \gamma_2]; m \in [\frac{1}{2}; \mu_1]; n \in [\nu_1; \frac{1}{2}]})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M &gt; M^*_{4,7})</td>
<td>EQ out-7: ((out, 0; 0, 0; 0, 0))</td>
<td></td>
</tr>
<tr>
<td>(A &gt; A_2) (PC5)</td>
<td>(M \leq M^*_{5,7})</td>
<td>EQ in-7: ((in, 0; 0, 0; 0, 0))</td>
</tr>
<tr>
<td>(M &gt; M^*_{5,7})</td>
<td>EQ out-7: ((out, 0; 0, 0; 0, 0))</td>
<td></td>
</tr>
</tbody>
</table>

\[\begin{align*}
\gamma_1 & := \frac{\varphi}{\rho + \varphi}, \quad \gamma_2 := \frac{1 - \varphi}{1 - \rho + 1 - \varphi}, \quad \mu_1 := \frac{\rho(1 - \varphi)}{\rho(1 - \varphi) + (1 - \rho)\varphi}, \quad \nu_1 := \frac{(1 - \rho)\varphi}{\rho(1 - \varphi) + (1 - \rho)\varphi}; \\
\alpha_1 := -Y & = -\frac{\xi_1M - A}{(1 - \chi)(\rho - \varphi)\xi M} \quad \text{and} \quad \beta_1 := Y + 1 = \frac{\xi_1M - A}{(1 - \chi)(\rho - \varphi)\xi M} + 1.
\end{align*}\]

Table 4: Equilibria in the Legal Exception Game
from signing an agreement —may it be good or bad— by setting the monetary fine \( M \) sufficiently high. In such a case the firms choose \textit{out}. The resulting equilibria and/or equilibrium sets are EQS out-6 in parameter constellation PC2, EQ out-3, EQ out-5 and EQ out-7 in parameter constellation PC3, EQS out-4 and EQ out-7 in parameter constellation PC4 as well as EQ out-7 in parameter constellation PC5. We call those equilibria \textit{full-deterrence equilibria}. Recall that only EQ out-3 in PC3 and PC4 and EQ out-7 in PC5 are dominating equilibria.

All remaining equilibria and equilibrium sets (EQS in-6, EQ in-3, EQ in-5, EQS in-4) can be summarized as \textit{positive-compliance equilibria}. Recall that all of them are dominated except EQ in-3. In such a situation the firms choose a compliance level \( \gamma^* \in (0; 1) \) which means that they choose a good agreement only with a certain probability.

\[ A_0 = 1, \quad A_1 = 1.8, \quad A_2 = 2.64, \quad \gamma^* = 0.04, \quad \gamma_1 = 0.3, \quad G = 1, \quad M = 10. \]

Figure 6 shows the firms’ equilibrium compliance level \( \gamma^* \) for different values of the additional profit \( A \).\footnote{The following specific parameter values were chosen for the figure: \( \rho = 0.6, \varphi = 0.4, \xi = 0.6, \chi = 0.3, G = 1 \) and \( M = 10 \).}

Compliance levels resulting from dominated equilibria/equilibrium sets are printed as dashed lines, those resulting from the dominating equilibria EQ in-1 (in PC 1 and PC2), EQ in-3 (in PC3 and PC4) and EQ in-7 (in PC5) are printed as solid bold lines. The firms’ equilibrium compliance level is decreasing for increasing \( A \) which is not surprising: the higher the additional gain that can be received by signing a bad agreement instead of a good one, the higher the probability to do so. Furthermore, as long as

\[ A \leq A_1 := \xi \chi M \quad (30) \]

holds —which is equivalent to setting the monetary fine \( M \) sufficiently high—
the compliance level equals one:

\[ M \geq \frac{1}{\xi \chi} A \iff \gamma^* = 1. \tag{31} \]

The four types of equilibria in the Legal Exception Game not only imply different compliance levels of the firms but also different probabilities of the Commission to commit a type I or type II error. The equilibrium error probabilities can be derived by replacing in equations (1) and (2) the behavioral strategies \( \alpha \), \( \beta \) and \( \gamma \) with their equilibrium values \( \alpha^* \), \( \beta^* \) and \( \gamma^* \). Table 5 shows the resulting error probabilities for every equilibrium.

<table>
<thead>
<tr>
<th>equilibrium</th>
<th>error probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p_I(f^<em>, \gamma^</em>; \alpha^<em>, \beta^</em>; \mu^<em>, \nu^</em>) )</td>
</tr>
<tr>
<td>EQ in-1</td>
<td>0</td>
</tr>
<tr>
<td>EQS in-2</td>
<td>0</td>
</tr>
<tr>
<td>EQ in-3</td>
<td>( \gamma_2 \xi (1 - \chi)(1 - \rho)(1 - \beta_1) )</td>
</tr>
<tr>
<td>EQS in-4</td>
<td>( x \xi (1 - \chi)(1 - \rho) )</td>
</tr>
<tr>
<td>EQ in-5</td>
<td>( \gamma_1 \xi (1 - \chi)(1 - \rho \alpha_1) )</td>
</tr>
<tr>
<td>EQS in-6</td>
<td>( x \xi (1 - \chi) )</td>
</tr>
<tr>
<td>EQ in-7</td>
<td>0</td>
</tr>
<tr>
<td>EQ out-3 to</td>
<td>0</td>
</tr>
<tr>
<td>EQ out-7</td>
<td>0</td>
</tr>
</tbody>
</table>

with \( \gamma_1 := \frac{\varphi}{\rho + \varphi} \), \( \gamma_2 := \frac{1 - \varphi}{1 - \rho + 1 - \varphi} \), \( \alpha_1 := -Y = -\frac{\xi M - A}{(1 - \chi)(\rho - \varphi) \xi M} \) and \( \beta_1 := Y + 1 = \frac{\xi M - A}{(1 - \chi)(\rho - \varphi) \xi M} + 1 \).

Table 5: Equilibrium error probabilities \( p_I \) and \( p_{II} \) in the Legal Exception Game

In the full-compliance equilibrium EQ in-1 both error probabilities amount to zero (\( p_I = p_{II} = 0 \)). This is due to the fact that only good agreements are signed (\( \gamma^* = 1 \)) and that the Commission never prohibits an agreement regardless the signal it has received (\( \alpha^* = \beta^* = 1 \)). The same is true for the equilibrium set EQS in-2 if the firms behave “nicely” and chooses \( \gamma^* = 1 \) instead of any other value \( x \in [\gamma_2; 1] \) since EQS in-2 is reduced to the equilibrium EQ in-1 then.

In the full-deterrence equilibria EQ out-3 to EQ out-7 both error probabilities amount to zero per definition. Whenever the status quo remains because the firms
decide not to sign an agreement \((f^* = \text{out})\), committing an error in the defined sense is not possible.

In the zero-compliance equilibrium EQ in-7 only type II errors occur with probability \(p_{II} = 1 - \xi\). As no good agreements are signed \((\gamma^* = 0)\) a type I error cannot be committed \((p_I = 0)\). Whenever a bad agreement is controlled, it is identified as such and prohibited \((\alpha^* = \beta^* = 0)\). Still, an agreement is only controlled with probability \(\xi\). Thus, whenever the Commission misses to control an agreement the bad agreement persists which is deemed a type II error.

Finally, in the positive-compliance equilibria EQ in-3 to EQS in-6 the error probabilities are positive and differ from each other. If we consider only the dominating equilibria EQ in-3 in parameter constellations PC3 and PC4, the type I error probability \(p_I\) is an increasing function in the additional cartel profit \(A\).

Figures 7 and 8 show all resulting equilibrium error probabilities for different values of the additional agreement profit \(A\). Error probabilities resulting from dominating equilibria are depicted with a solid bold line, those resulting from dominated equilibria with a dashed one.

From the figures follows: as long as \(A\) is smaller than the threshold value \(A_1\) (see inequality (30))—which can be easily achieved by setting the monetary fine sufficiently high— both error probabilities equal zero:

\[
M \geq \frac{1}{\xi \chi} A \iff p_I = p_{II} = 0.
\]  

\(^{43}\)Recall that the thresholds defining the parameter constellations depend on the ratio of the additional profit \(A\) to a weighed value of the monetary fine \(M\): the greater \(A\) in relation to \(M\), the higher the parameter constellation.

\(^{44}\)For these figures, the same specific parameter values were chosen as for figure 6: \(\rho = 0.6, \varphi = 0.4, \xi = 0.6, \chi = 0.3, G = 1\) and \(M = 10\).
4 Main findings

In the previous section, four different types of equilibria in the Legal Exception Game according to their values of the firms’ compliance level and assessment error probabilities were identified:

**Full-compliance equilibrium:** The equilibrium EQ in-1 (appearing in parameter constellations PC1 and PC2) is a full-compliance equilibrium. As the Commission chooses the strategy never to prohibit an agreement that has been controlled regardless of the received signal ($\alpha^* = \beta^* = 1$) and as all agreements signed are good ($\gamma^* = 1$) neither type I nor type II errors exist, i.e. both error probabilities equal zero ($p_I = p_{II} = 0$).

Recall that the equilibrium EQ in-1 is part of the equilibrium set EQS in-2 (appearing in parameter constellation PC2) but that its remaining equilibria are positive-compliance-equilibria (see below) and are dominated by EQ in-1, if the assumption is made that firms behave “nicely”, i.e. choose the highest compliance level feasible, whenever indifferent between different compliance levels.

**Zero-compliance equilibrium:** The equilibrium EQ in-7 (appearing in parameter constellations PC3 to PC5) is a zero-compliance equilibrium. Given that all signed agreements are bad ($\gamma^* = 0$) there is no possibility of committing a type I error ($p_I = 0$). As the Commission then chooses the strategy always to prohibit an agreement regardless the received signal ($\alpha^* = \beta^* = 0$) and as only a fraction of $1 - \xi$ agreements remains uncontrolled, the probability of a type II error amounts to $p_{II} = 1 - \xi$.

Note that following the concept of virtual observability, EQ in-7 is dominated in parameter constellations PC3 and PC4 and exists unchallenged only in PC5.

**Positive-compliance equilibria:** The equilibrium sets EQS in-2 (except for its element EQ in-1, see above) and EQS in-6, the equilibria EQ in-3 and EQ in-5 as well as the equilibrium set EQS in-4 (appearing in parameter constellations PC2 to PC4 in ascending order) are positive-compliance equilibria in the sense that the firms choose neither a good nor a bad agreement with certainty. In such a situation good agreements as well as bad agreements are signed (with
a certain probability $\gamma^*$) and the Commission reacts with an adequate strategy $(\alpha^*, \beta^*)$ corresponding to its posterior beliefs $(\mu^*, \nu^*)$. The result is that both error probabilities are positive ($p_I, p_{II} > 0$).

Recall that all listed equilibria and equilibrium sets of this type are dominated, except the equilibrium EQ in-3, which persists in parameter constellations PC3 (due to virtual observability) and PC4 (due to the assumption of the firms behaving “nicely” when indifferent).\(^{45}\)

**Full-deterrrence equilibria:** The equilibrium set EQS out-6, the equilibria EQ out-3 and EQ out-5, as well as the equilibrium set EQS out-4 (appearing in parameter constellations PC2 to PC4 in ascending order), and the equilibrium out-7 (appearing in parameter constellations PC3 to PC5) are full-deterrrence equilibria, as no agreement is signed ($f^* = \text{out}$). Thus, the status quo persists, compliance is not defined and no legal errors can be made ($p_I = p_{II} = 0$).

Note that following the concept of virtual observability again, only the equilibria EQ out-3 (in parameter constellations PC3 and PC4) and EQ out-7 (in parameter constellation PC5) persist; all other listed equilibria and equilibrium sets of this type are dominated.

## 5 Conclusion

We have defined that an enforcement system is the more effective, the lower the error probabilities and the higher the firms’ compliance level.\(^{46}\) Thus, if the full-compliance equilibrium is reached, the criteria for effectiveness are entirely met. Even if only the full-deterrrence equilibrium is reached one can call the legal exception system effective, as no agreements and therefore no bad agreements are signed.

The interesting question is whether the full-compliance equilibrium is feasible, i.e. whether the equilibrium conditions are achievable in reality. This depends on the exogenous parameters of the model: the gains of an agreement ($G, A$), the fine ($M$), the Commission’s assessment skills ($\rho, \phi$) and the control and appeals probabilities ($\xi, \chi$). For the full-compliance equilibrium to realize only the condition specified in inequality (30) must hold: $A \leq A_1 := \xi \chi M$ which is equivalent to $M \geq \frac{1}{\xi \chi} A$. This condition can always be fulfilled as the fine $M$ can be influenced in principle directly by the Commission; it is part of the Commission’s set of economic policy measures.\(^{47}\) Therefore, the concerns on the legal exception

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\(^{45}\)EQ in-3 is part of the equilibrium set EQS in-4. The equilibrium set EQS in-6 is dominated by the equilibrium EQ in-1 applying the concept of virtual observability again.

\(^{46}\)See section 2.

\(^{47}\)Note that art. 23(2) of Reg. 1/2003 limits fines to 10 % of the involved firms’ annual turnover. Further details are determined in the Commission’s Guidelines on the Method of Setting Fines Imposed Pursuant to Article 23(2)(a) of Regulation No 1/2003, OJ C 210, 01.09.2006, p. 2-5. For some analysis of the guidelines see e.g. Schinkel (2008). However, it must be left to empirical analysis whether this cap of 10 % is set high enough to meet the equilibrium condition
system’s ineffectiveness expressed by its critics are unfounded.

Our model can be seen as a first step to a full institutional comparison of the legal exception system with the notification system. It is essential to understand the working properties established by the new system of legal exceptions before going any further. In a second step, the old system of notifications and authorization should be analyzed using the same techniques and the same level of complexity. This is a precondition to comparing both institutions in a convincing manner.

It would be possible to refine the results presented in this paper by conducting comparative statics. It would be interesting to see in detail how the results are influenced by assuming an improvement of the assessment skills of the Commission, an increase of the monetary fines or an enhancement of the fraction of agreements controlled ex post. In addition, the model could be expanded to include those two main elements of the reform which have not been integrated so far: the decentralized application of the competition rules and the intensified ex post control; the latter being implemented e.g. via private enforcement or leniency programs.

Finally, one might ask how welfare is affected by the legal exception system. We have dealt with this matter implicitly by focusing on the effectiveness of the system. Effectiveness of a legal enforcement scheme is a proxy for efficiency as long as legal norms are considered to be designed to improve welfare. Nevertheless, an explicit welfare assessment of the legal exception regime might be worth pursuing.

References


for full compliance.


