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Working Paper

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GLO Discussion Paper, No. 1342

Provided in Cooperation with:

Global Labor Organization (GLO)

Suggested Citation: Ponthiere, Gregory (2023) : Social Insurance against a Short Life: Ante-Mortem versus Post-Mortem Policies, GLO Discussion Paper, No. 1342, Global Labor Organization (GLO), Essen

This Version is available at:

<https://hdl.handle.net/10419/279473>

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Social Insurance against a Short Life: Ante-Mortem *versus* Post-Mortem Policies

Gregory Ponthiere*

October 26, 2023

Abstract

Welfare States do not insure citizens against the risk of premature death, i.e., the risk of having a short life. Using a dynamic OLG model with risky lifetime, this paper compares two insurance devices reducing well-being volatility due to the risk of early death: (i) an *ante-mortem* age-based statistical discrimination policy that consists of an allowance given to *all* young adults (including the unidentified adults who will die early); (ii) a *post-mortem* subsidy on accidental bequests due to early death. Each policy is financed by taxing old-age consumption. Whereas each device can yield full insurance, the youth allowance is shown to imply a higher lifetime well-being at the stationary equilibrium. The marginal utility of consumption exceeding the marginal utility of giving when being dead, the youth allowances system is, despite imperfect targeting, a more efficient mechanism of insurance against the risk of early death.

Keywords: premature death, mortality risk, social insurance, inheritance, lifecycle models.

JEL classification codes: J10, J17, I31, E21, H55.

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1 Introduction

Existing Welfare States insure citizens against various risks of life, such as the risk of losing one's job (through the unemployment insurance), the risk of having a disease (*via* the social health insurance) or the risk of old-age poverty (*via* public pension schemes). However, Welfare States do not insure citizens against the risk of premature death, that is, the risk of having a short life.¹

The non-existence of a social insurance against the risk of a short life is paradoxical, because empirical evidence shows that the risk about the duration of life is high and persistent.² Using Shannon's lifetime entropy index as a measure of the mathematical expectation of the quantity of information revealed by the event of death at a particular age, Meyer and Ponthiere (2020) showed that, over the last 250 years, the risk about the duration of life has exhibited an inverted-U pattern with a peak in the early 20th century, but remains high nowadays, at about 5.5 bits. Learning one's age at death is equivalent, in terms of information revelation, to learning the outcome of 5-6 draws of a fair coin.³

The absence of a social insurance against the risk of premature death is even more paradoxical once one acknowledges that an early death deprives the person from a substantial amount of well-being. Losing 40 or 50 years of life causes a well-being loss that is much larger than the ones due to other risks of life, such as unemployment. In monetary terms, the opportunity cost of dying early was shown to be substantial and to exceed the amount of foregone consumption.⁴

Although existing Welfare States do not insure citizens against the risk of a short life, it could be possible, in theory, to construct a new 'pillar' of social protection against that risk. True, this new 'pillar' of social protection would not provide insurance against a short life in the sense of 'insurance' as defined by Arrow (1965, p. 45) as 'the exchange of money now for money payable contingent on the occurrence of certain events'. Given that victims of a premature death are, by definition, no longer alive, an insurance against a short life cannot take the form of a standard system of payment of benefits to the insurees in case of occurrence of their premature death. However, if one defines 'insurance' more broadly, as 'a procedure that reduces uncertainty' (Zweifel and Eisen, 2012 p. 3), it is possible to construct a social insurance against the risk of early death.

Two classes of social insurance systems against premature death were studied in the recent years.⁵ On the one hand, *ante mortem* policies use age-based

¹It is rather the opposite: by insuring citizens against the risk of having a long life, existing public pension systems redistribute resources from *short-lived* persons towards *long-lived* persons, and thus worsen the lot of the short-lived. This makes existing social insurance systems regressive. See Coronado et al. (2011), Fleurbaey et al. (2016) and Ponthiere (2023).

²One possible explanation for this paradox is the social invisibility of prematurely dead persons, as argued by Sen (1998).

³On measures of risk about the duration of life, see also Hill (1993) and Kannisto (2000).

⁴On the measurement of well-being losses caused by premature mortality by means of the equivalent income approach, see Usher (1973, 1980), Murphy and Topel (2003), Becker et al. (2005), Fleurbaey and Gaulier (2009), Ponthiere (2016) and Da Costa (2020, 2023).

⁵By 'premature death', we mean here 'death when being a young adult', that is, death before having reached age 60. Throughout the paper, we abstract from infant mortality, which is a topic that goes beyond the scope of this paper.

statistical discrimination favouring all young individuals in order to decrease the opportunity cost of dying prematurely. The intuition is that, even if one cannot identify persons who will die prematurely when they are still alive, it is possible to make short-lived persons better off by improving the situation of all young adults (who necessarily include the - unidentified - persons who will die early). *Ante mortem* policies include increasing consumption at young ages either by taxing savings (Fleurbaey et al. 2014) or by postponing the (final) retirement age (Fleurbaey et al. 2016), and providing a retirement period to all young adults before their career (Ponthiere 2020, 2023).⁶ On the other hand, *post mortem* policies consist of intervening *after* the early death of the person to increase her well-being *indirectly*, by improving the lives of persons who matter for the deceased. For instance, subsidizing accidental bequests left by a prematurely dead person to her descendants is an indirect way to make the short-lived better off, and, hence, to provide some compensation for the risk of dying early (Fleurbaey et al. 2022, Pestieau and Ponthiere 2023). Indeed, in case of (im)perfect altruism or joy of giving, subsidizing accidental bequests left by prematurely dead persons improves the lot of short-lived persons. This *post mortem* policy can reduce well-being volatility caused by a premature death.

Up to now, *ante mortem* and *post mortem* policies of insurance against the risk of premature death have been studied separately. Comparing these policies reveals that each insurance device exhibits specific strengths/weaknesses. The main strength of *ante mortem* policies is to treat persons who will die prematurely directly, by improving their lives when these persons are still alive. However, *ante mortem* policies exhibit a major weakness: they target *all* young individuals (because persons who will die prematurely are not identifiable before the duration of their life is revealed), and, hence, these treat lots of persons who will not die prematurely. This imperfect targeting dilutes the resources dedicated to compensating the short-lived. On the contrary, *post mortem* policies do not suffer from imperfect targeting. *Ex post*, that is, once individual durations of life are known, short-lived persons can be identified and the treatment can then be targeted towards these persons only. But *post mortem* policies have also a drawback: these can only improve the well-being of prematurely dead persons *indirectly*, by improving the lot of living persons whose life matters for the deceased, either through altruism or joy of giving. Thus *post mortem* policies may have a weaker effect on the well-being of prematurely dead persons.⁷

In the light of these various effects, the comparison of *ante mortem* and *post mortem* policies is challenging. What is the best insurance mechanism: either an *ante mortem* policy with imperfect targeting (and thus dilution of resources) but with direct effects on the lives of short-lived persons, or, alternatively, a *post mortem* policy with perfect targeting but indirect effects on the short-lived?

⁶ Another *ante mortem* policy consists of imposing working time regulations that imply a leisure time profile that is decreasing with age (see Leroux and Ponthiere 2018).

⁷ Another problem is that *post mortem* policies cannot improve the lives of prematurely dead persons who exhibit neither altruism nor joy of giving, because in those cases it is impossible to make a prematurely dead person better off by improving the lives of other persons who are still alive (Ponthiere 2023).

The goal of this paper is to propose a quantitative assessment of these two classes of insurance systems against the risk of premature death. For that purpose, we will develop a dynamic overlapping generations (OLG) model with risky lifetime and physical capital accumulation, and study, within that economy, the effects of introducing either an *ante mortem* policy, which consists of a youth allowances system, or a *post mortem* policy, which consists of a subsidy on accidental bequests left by prematurely dead persons. Each insurance system is financed by taxing old-age consumption. We assume a quasi-linear utility, so as to make the intergenerational process of wealth accumulation Markovian.⁸

Anticipating our results, we first show that, under each device, the sufficient condition for full insurance is satisfied for an interior level of the tax rate on old-age consumption when the marginal utility of consumption when being old exceeds the marginal utility of giving when being dead. Second, assuming that each system is calibrated so as to yield full insurance, we compare the levels of well-being under each system, first at the temporary equilibrium (taking production factor prices as given) and, then, at the stationary equilibrium (allowing for the effects of policies on accumulation and prices). We derive necessary and sufficient conditions for one system to dominate the other system in terms of well-being outcomes. Third, we calibrate our model, and we simulate the stationary equilibria reached under the two insurance systems. Whereas each device can, under our calibrations, yield full insurance, the monetary compensation given to each short-lived person is higher under the subsidy on accidental bequests than under the youth allowance. The reason is that, unlike the bequest subsidy, the youth allowances system exhibits imperfect targeting, and, hence, dilutes resources on a larger number of beneficiaries. However, although it provides lower monetary compensation, the youth allowances system has a larger effect on the well-being of the short-lived. The steady-state level of lifetime well-being under the youth allowances system is higher than the one under the subsidy on accidental bequests. Even if the monetary compensation is lower under the youth allowance, its effects on well-being are larger. *The marginal utility of consumption exceeding the marginal utility of giving when being dead, the youth allowances system is, despite an imperfect targeting, a more efficient way of insuring citizens against the risk of a premature death.*

As such, this paper casts light on the design of a social insurance against the risk of a short life. When facing the choice between *ante mortem* statistical discrimination policies favouring the young and *post mortem* policies targeting short-lived persons through their descendants, it is tempting to believe that the latter policies dominate the former ones, on the ground of a more precise targeting of the persons to be compensated. However, our quantitative comparison of these policies reveals that, even if it suffers from imperfect targeting, the youth allowances system is a more efficient insurance mechanism, because it involves direct - rather than indirect - compensation of short-lived persons.

This paper is related to the literature about the design of a social insurance

⁸By a ‘Markovian’ accumulation process, we mean a process where the wealth inherited by an individual depends only on the wealth and the age at death of her parent, but not on the lot of her grandparents or her great-grandparents (Fleurbaey et al. 2022).

against the risk of premature death, either by means of *ante mortem* policies (Fleurbaey et al. 2014, Fleurbaey et al. 2016, Leroux and Ponthiere 2018, Ponthiere 2020, 2023), or by means of *post mortem* policies (Fleurbaey et al. 2022, Pestieau and Ponthiere 2023). Our contribution is to provide a quantitative comparison of an *ante mortem* policy - a youth allowance - and a *post mortem* policy - a subsidy on accidental bequests -, and to show that the former policy is a more efficient way of insuring citizens against the risk of premature death. Moreover, this paper is also linked to the literature on dynamic OLG models with risky lifetime, such as de la Croix and Licandro (1999, 2013), Boucekine et al. (2002), Chakraborty (2004), Cervelatti and Sunde (2005, 2011) and Dalgaard and Strulik (2014). Our contribution consists here of studying, within an OLG model with risky lifetime and physical capital accumulation, the implications of introducing a social insurance against the risk of premature death. Finally, this paper is also related to the literature measuring well-being losses due to a premature death, such as Usher (1973, 1980), Murphy and Topel (2003), Becker et al. (2005), Hall and Jones (2007), Fleurbaey and Gaulier (2009), Ponthiere (2016) and Da Costa (2020, 2023). Our contribution is here twofold. On the one hand, we study how the structure of the ‘evil of death’ (i.e., the well-being loss due to premature death) affects the design of insurance devices. On the other hand, we examine the implications of developing a new ‘pillar’ of social protection reducing these well-being losses.

This paper is organized as follows. Section 2 presents the model. Section 3 introduces the youth allowances system, and examines the conditions under which this *ante mortem* policy can yield full insurance against the risk of premature death. Section 4 carries out a similar task for a *post mortem* subsidy on accidental bequests left in case of premature death. Section 5 compares well-being outcomes under these two policies at the temporary equilibrium. Section 6 makes a similar comparison at the stationary equilibrium. Numerical explorations are made in Section 7. Section 8 concludes.

2 The model

Let us consider a dynamic OLG economy with risky lifetime. Time is discrete and goes from $t = 0$ to $t = +\infty$. The first period of life is the young age, during which each individual has n children, consumes c_t , saves an amount s_t for her old days, and supplies one unit of labour inelastically.⁹ The second period is the old age, during which persons consume d_{t+1} and do not work. The old age is reached with a probability $\pi \in]0, 1[$, which is assumed to be constant. In case of a premature death, the person transmits the proceeds of her savings to her descendants. The transmitted amount b_{t+1} is called ‘accidental bequest’.¹⁰

⁹As usual in the OLG literature, we keep the period of infancy implicit, and focus only on adulthood.

¹⁰This bequest is ‘accidental’ in the sense that this transmission of wealth arises because of the occurrence of the event of premature death, and would not have occurred otherwise (i.e., in case of a long life). However, as we shall see, individuals, when making savings decisions,

Production Production involves capital and labour, and is represented by a Cobb-Douglas production function:

$$Y_t = AK_t^\alpha L_t^{1-\alpha} \quad (1)$$

where $A > 0$, K_t is the capital stock and L_t is the labour supplied by young persons, while $\alpha \in]0, 1[$ is the elasticity of output with respect to capital.

Given that each young adult has $n > 0$ children, the labour supply follows the dynamics:

$$L_{t+1} = nL_t \quad (2)$$

In intensive terms, output per worker y_t is:

$$y_t = Ak_t^\alpha \quad (3)$$

where k_t is the capital to labour ratio.

Capital is assumed to fully depreciate after one period of use.

Markets being perfectly competitive, production factors are paid at their marginal productivity:

$$w_t = (1 - \alpha)Ak_t^\alpha \text{ and } R_t = \alpha Ak_t^{\alpha-1} \quad (4)$$

where w_t is the wage rate and R_t is the interest factor.

Preferences In the presence of accidental bequests, there is, under general preferences, a growing heterogeneity of received endowments within each successive cohort, because children of long-lived persons receive no accidental bequests, whereas children of short-lived persons receive some accidental bequest, which can then be potentially transmitted to future descendants. As shown in Fleurbaey et al (2022), one solution to keep the model analytically tractable is to assume that the process of wealth accumulation is Markovian, that is, a process where the amount of wealth inherited by a person depends only on the wealth and on the age at death of her own parent (and not on the wealth or age at death of her grand-parents or great-grand parents).¹¹

In order to make the dynamics of wealth Markovian, we assume that preferences are quasi-linear, and take a Von Neumann Morgenstern form:

$$EU_t = c_t + \pi\beta \log(d_{t+1}) + (1 - \pi)[\Omega + \gamma \log(b_{t+1})] \quad (5)$$

where the preference parameter β captures the pure discount factor, with $0 \leq \beta \leq 1$. The preference parameter γ captures the person's 'joy of giving' in case of a premature death ($0 \leq \gamma \leq 1$) (Andreoni 1989).

anticipate that, in case of premature death, their savings are not lost but transmitted to their descendants.

¹¹This paper does not consider the possibility of non-accidental bequests, i.e., amounts of wealth transmitted independently from the duration of life. These non-accidental bequests being unrelated to the timing of death, they cannot play a significant role in the construction of a social insurance against a short life.

The preference parameter Ω captures the ‘pure’ disutility of an early death, that is, the welfare loss that arises independently from the opportunity cost of an early death. When the only welfare loss due to a premature death consists of the opportunity cost of dying early (lost consumption), we have $\Omega = 0$. This parametrization prevails under what philosophers call the Deprivation Account: death is bad because it deprives us from good things that would have been enjoyed provided death did not take place (Nagel 1979, Broome 2004). However, following Kamm (1993), there are reasons to believe that the welfare loss due to a premature death cannot be reduced to the opportunity cost of death. Kamm provides two additional accounts of the badness of death: the Insult factor (a premature death causes a welfare loss merely by the disutility of its occurrence, independently from deprivations caused by death) and the Irreversibility factor (death is bad by causing the irreversible disappearance of the person, independently from deprivations caused by death). Under each of these two accounts of the badness of death, the evil of death cannot be reduced to the opportunity cost of death, leading to $\Omega < 0$. For the sake of analytical tractability, we assume that the pure evil of death is finite: $\Omega > -\infty$.

Budget constraints Given the presence of accidental bequests, the endowments of young individuals depend on whether their parent is long-lived or short-lived. Let us denote that received endowment by B_t . We have $B_t = 0$ when the parent is long-lived, and $B_t > 0$ when the parent is short-lived.¹²

In the first period, the individual earns a wage w_t and a bequest B_t (which is either zero or positive, depending on the longevity of her parent) and must decide how to allocate these resources between consumption c_t and savings s_t . The budget constraint is:

$$w_t + B_t = c_t + s_t \quad (6)$$

In the absence of annuity markets, the second-period budget constraint is:

$$R_{t+1}s_t = \begin{cases} d_{t+1} & \text{in case of a long life} \\ b_{t+1} & \text{in case of a short life} \end{cases} \quad (7)$$

If the person dies prematurely, and leaves an accidental bequest b_{t+1} , each of his n children will receive a bequest:

$$B_{t+1} = \frac{b_{t+1}}{n} = \frac{R_{t+1}s_t}{n} \quad (8)$$

that is, each of the n children receives an equal share of the consumption that his parent would have enjoyed in case of survival to the old age, but which was not enjoyed, because of the occurrence of a premature death.

¹²Note that, given quasi-linear preferences, the level of B_t will not affect the savings of the individual. This yields the Markovian process for wealth accumulation (see *infra*).

Optimal saving behavior Substituting for budget constraints, the problem of the individual can be written as:

$$\max_{s_t} EU_t = w_t + B_t - s_t + \pi\beta \log(R_{t+1}s_t) + (1 - \pi) [\Omega + \gamma \log(R_{t+1}s_t)]$$

Using the first-order condition (FOC) for optimal interior saving, we obtain:

$$s_t = \pi\beta + (1 - \pi)\gamma \quad (9)$$

The quasi-linearity of the utility function makes savings independent from whether the young person received an accidental bequest or not, that is, independent from the level of B_t . This means that, in case of a short-lived parent, the child consumes all the accidental bequest received when being young, and, thus, consumes more, at the young age, than the child of a long-lived parent. However, the chosen savings is exactly the same across all young adults, whatever their parent was long-lived or short-lived. This feature makes the process of wealth accumulation Markovian: the inherited wealth depends only on the previous generation, but not on anterior ones.

Note also that the optimal saving does not depend on the preference parameter Ω capturing the pure evil of death, independently from the opportunity cost of death. Regarding the influence of other preference parameters, savings is increasing in the joy of giving parameter γ and in the weight given to the future in case of survival to the old age, β .

3 A youth allowance

As studied in Fleurbaey et al. (2014), one possibility to insure all citizens against the risk of a short life consists of implementing age-based statistical discrimination policies favouring all young individuals. By improving the situation of all young adults when these are still alive, one can also improve the lives of the unidentified persons who will turn out to die prematurely in the future. This rationale motivates the construction of *ante mortem* policies aimed at insuring all individuals against the risk of a short life.

Following that intuition, let us now consider that, in order to insure persons against the risk of premature death, the Welfare State adopts age-based statistical discrimination favouring all young individuals, and gives an allowance P_t to each young individual.¹³ This youth allowance would be financed by taxing old-age consumption at a rate τ_t . Assuming that the budget is balanced, we have thus:¹⁴

$$L_t P_t = \tau_t \pi L_{t-1} R_t s_{t-1} \quad (10)$$

The left-hand-side (LHS) of the equation is the total expenditure in terms of providing an allowance to each young adult, whereas the right-hand-side (RHS)

¹³ P_t , the youth allowance, is a kind of "basic income for the young".

¹⁴ We define the tax rate in a general, time-varying, manner. The reason is that the design of a youth allowances system that yields full insurance will require to adjust the tax rate to economic circumstances (see *infra*).

consist of the revenues obtained by taxing the consumption of all elderly persons (who thus belong to the previous cohort) at a rate τ_t .

Simplifications yield:

$$P_t = \frac{\tau_t \pi R_t s_{t-1}}{n} \quad (11)$$

The youth allowance P_t is increasing in the survival probability to the old age π , in the interest factor R_t and in the savings of the previous cohort s_{t-1} . However, the youth allowance is decreasing in the fertility rate n . Indeed, in case of fertility above the replacement rate ($n > 1$), the size of cohorts grows over time, making it more difficult to finance a youth allowance by taxing the consumption of a relatively smaller number of (surviving) old persons. On the contrary, when fertility is below the replacement rate ($n < 1$), the size of cohorts decreases over time, making it easier to finance a youth allowance by taxing old-age consumption of the previous (thus larger) cohort. Note that another key demographic parameter is π . When the proportion of survivors to the old age is low, it is difficult to finance a high youth allocation, whereas a higher π makes the youth allowances system more generous to the young.

Savings being endogenously determined in our economy, let us rewrite the savings problem under the youth allowances system. The problem becomes:

$$\max_{s_t} EU_t = w_t + B_t + P_t - s_t + \pi \beta \log(R_{t+1} s_t (1 - \tau_{t+1})) + (1 - \pi) [\Omega + \gamma \log(R_{t+1} s_t)]$$

Using the FOC for optimal interior saving, we obtain, here again:

$$s_t = \pi \beta + (1 - \pi) \gamma \quad (12)$$

Due to quasi-linearity of preferences, saving does not depend on the youth allowance P_t . A more generous allowance will only imply a higher consumption at the young age, without affecting savings and old-age consumption.

Full insurance against the risk of a short life is obtained when there is equalization of lifetime well-being under a long life, denoted by U_t^{LL} , with lifetime well-being under a short life, denoted by U_t^{SL} , for persons in the same cohort:

$$U_t^{SL} = w_t + P_t + B_t - s_t + \Omega + \gamma \log(R_{t+1} s_t) \quad (13)$$

$$U_t^{LL} = w_t + P_t + B_t - s_t + \beta \log(R_{t+1} s_t (1 - \tau_{t+1})) \quad (14)$$

Is it possible to find values of policy parameters (P_t, τ_{t+1}) such that full insurance prevails? Proposition 1 summarizes our results.

Proposition 1 *Assume that: $(\beta - \gamma) \log(R_{t+1} (\pi \beta + (1 - \pi) \gamma)) > \Omega$.*

There exists a unique tax rate $\tau_{t+1}^ \in]0, 1[$ that yields full insurance against the risk of a short life. It is equal to:*

$$\tau_{t+1}^* = 1 - \left[\frac{\exp(\Omega)}{(R_{t+1} (\pi \beta + (1 - \pi) \gamma))^{\beta - \gamma}} \right]^{\frac{1}{\beta}}$$

The youth allowance implying full insurance is:

$$P_t^* = \left(1 - \left[\frac{\exp(\Omega)}{(R_{t+1}(\pi\beta + (1-\pi)\gamma))^{\beta-\gamma}} \right]^{\frac{1}{\beta}} \right) \frac{\pi}{n} R_t (\pi\beta + (1-\pi)\gamma)$$

Proof. See the Appendix. ■

The condition stated in Proposition 1 is sufficient to guarantee the existence of an interior $\tau_{t+1}^* \in]0, 1[$ that implies full insurance. Note that, if the evil of a premature death is reduced to the opportunity cost of an early death, that is, if $\Omega = 0$, and if the joy of giving matters as much as the joy of living the old age, that is, $\gamma = \beta$, then full insurance is obtained for $\tau_{t+1}^* = 0$ and $P_t^* = 0$. Indeed, in that particular case, lifetime well-being is equalized across the two possible scenarios of life (long life and short life) at the laissez-faire. Therefore no youth allowance is needed in that special case. But that case is highly specific. If, as in Hurd (1989), we assume, more generally, that the marginal utility of consuming exceeds the marginal utility of giving (i.e. $\beta > \gamma$), then, under $\Omega \leq 0$, the condition of Proposition 1 is necessarily satisfied, and a strictly positive tax rate is needed, implying a positive allowance P_t^* given to each young individual. In that case, the smaller Ω is, the higher τ_{t+1}^* is.

Figure 1 illustrates the level of τ_{t+1}^* that yields full insurance as the one that equalizes, on the one hand, second-period utility of a prematurely dead person, i.e. $\Omega + \gamma \log(R_{t+1}s_t)$, and, on the other hand, second-period utility of a long-lived person, i.e., $\beta \log(R_{t+1}s_t(1 - \tau_{t+1}))$, which is decreasing in τ_{t+1} . When the pure evil of death increases, that is, when the parameter $\Omega \leq 0$ goes down, this reduces the second-period utility of short-lived persons for a given τ_{t+1} , which pushes the horizontal line down. As a consequence, the horizontal line crosses the curve at a higher level of τ_{t+1} . Thus, when the pure disutility of dying early increases (for a given opportunity cost of death), full insurance against a short life can only be achieved by increasing the tax rate on old-age consumption and by increasing the youth allowance accordingly. Therefore, whether or not the utility loss due to a premature death can be reduced to the opportunity cost of death (foregone consumption) is a key determinant of the youth allowances system that yields full insurance against a short life.

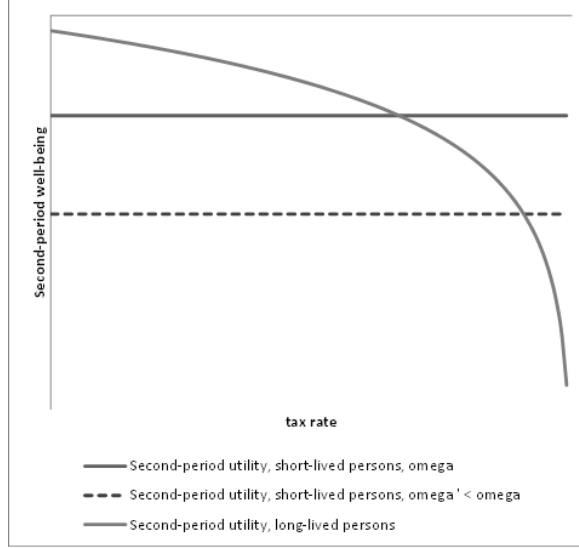


Figure 1. Impact of Ω on the tax rate that yields full insurance against a short life.

Another key determinant of τ_{t+1}^* is the interest factor on savings R_{t+1} . *Ceteris paribus*, the higher the interest factor is, and the higher the tax rate τ_{t+1}^* yielding full insurance is. The underlying intuition is the following. When the interest rate is higher, persons surviving to the old age can, *ceteris paribus*, enjoy a higher consumption. This increases the opportunity cost of dying prematurely. Hence, in order to have full insurance against the risk of a short life, it is necessary, under a higher interest factor, to tax old-age consumption at a higher rate. This explains why τ_{t+1}^* is increasing in R_{t+1} .

Finally, it should be stressed that the level of the youth allowance that yields full insurance is, *ceteris paribus*, decreasing with the fertility rate n . The intuition is that the tax rate on old-age consumption, which determines the revenues, does not depend on fertility. Hence, given that fiscal revenues per each old adult are redistributed towards young adults of the next cohort, it follows that the higher the fertility rate n is, the lower the youth allowance is.

4 A subsidy on accidental bequests

Consider now a *post-mortem* insurance device against the risk of a short life. *Post-mortem*, that is, once the identity of short-lived persons is known, there is no identification problem, and it becomes possible for the Welfare State to improve the situation of short-lived persons without having to do statistical discrimination targeting all young adults. As an example of *post-mortem* insurance device against the risk of a short life, let us assume that the Welfare State sub-

sidizes the accidental bequests left by prematurely dead persons.¹⁵ The subsidy is equal to $S_{t+1} \geq 0$. This policy is financed by taxing old-age consumption at a rate $\sigma_{t+1} \geq 0$, so that:

$$(1 - \pi)L_t S_{t+1} = \pi L_t R_{t+1} s_t \sigma_{t+1} \quad (15)$$

Simplifications yield:

$$S_{t+1} = \frac{\pi}{1 - \pi} R_{t+1} s_t \sigma_{t+1} \quad (16)$$

This budget constraint makes appear a major difference with respect to the system of youth allowances studied in Section 3. Unlike in that system, the fertility rate n plays here no role, because the redistribution occurs *within* a given cohort, from the long-lived individuals of a cohort to the short-lived individuals of the same cohort. On the contrary, the system of youth allowances involved a redistribution from surviving old persons of a cohort born at $t - 1$ towards all young adults of a cohort born at t , implying that the fertility rate - determining the sizes of successive cohorts - was playing a key role.

Let us rewrite the savings problem under the subsidy on accidental bequests:

$$\max_{s_t} EU_t = w_t + B_t - s_t + \pi \beta \log(R_{t+1} s_t (1 - \sigma_{t+1})) + (1 - \pi) [\Omega + \gamma \log(R_{t+1} s_t + S_{t+1})]$$

The FOC for optimal interior saving is:

$$1 = \frac{\pi \beta}{s_t} + \frac{(1 - \pi) \gamma R_{t+1}}{R_{t+1} s_t + S_{t+1}} \quad (17)$$

Hence, substituting for $S_{t+1} = \frac{\pi}{1 - \pi} R_{t+1} s_t \sigma_{t+1}$, saving is:

$$s_t = \frac{\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1 - \pi}\right) + (1 - \pi) \gamma}{\left(1 + \frac{\pi \sigma_{t+1}}{1 - \pi}\right)} \quad (18)$$

Comparing this level of savings with the one derived in the previous section yields the following result.

Lemma 1 *Ceteris paribus, saving is lower under the subsidy on accidental bequests than under the youth allowances system.*

Proof. Optimal saving under the bequest subsidy can be rewritten as:

$$\pi \beta + \frac{(1 - \pi) \gamma}{\left(1 + \frac{\pi \sigma_{t+1}}{1 - \pi}\right)} < \pi \beta + (1 - \pi) \gamma$$

¹⁵This paper considers accidental bequests as a tool for compensating prematurely dead persons, in line with Fleurbaey et al. (2022). However, there exist other reasons to tax/subsidize accidental bequests. On the optimal taxation/subsidy of accidental bequests, see Blumkin and Sadka (2004) and Cremer et al. (2012).

The RHS being the saving under the system of youth allowances, we have that saving is, *ceteris paribus*, smaller under the bequests subsidy than under the system of youth allowances. ■

The intuition behind that result is the following. Introducing a youth allowances system affects neither the marginal utility of young-age and old-age consumptions, nor the marginal utility of giving in case of premature death, and, hence, leaves the saving decision unaffected. On the contrary, a subsidy on accidental bequests reduces the marginal utility of giving in case of premature death, and, hence, discourages savings.

Full insurance is obtained when there is perfect equalization of lifetime well-being under a long life, denoted by U_t^{LL} , with lifetime well-being under a short life, denoted by U_t^{SL} . We have:

$$U_t^{SL} = w_t + B_t - s_t + \Omega + \gamma \log(R_{t+1}s_t + S_{t+1}) \quad (19)$$

$$U_t^{LL} = w_t + B_t - s_t + \beta \log(R_{t+1}s_t(1 - \sigma_{t+1})) \quad (20)$$

The following proposition summarizes our results.

Proposition 2 Assume that: $(\beta - \gamma) \log(R_{t+1}(\pi\beta + (1 - \pi)\gamma)) > \Omega$.

There exists a unique tax rate $\sigma_{t+1}^* \in]0, 1[$ that yields full insurance against the risk of a short life. This is defined implicitly by:

$$\Omega - (\beta - \gamma) \log\left(R_{t+1}\left(\pi\beta\left(1 + \frac{\pi\sigma_{t+1}^*}{1 - \pi}\right) + (1 - \pi)\gamma\right)\right) = \beta \log\left(\frac{1 - \sigma_{t+1}^*}{1 + \frac{\pi\sigma_{t+1}^*}{1 - \pi}}\right)$$

The subsidy on accidental bequests implying full insurance is:

$$S_{t+1}^* = \frac{\pi}{1 - \pi} \sigma_{t+1}^* R_{t+1} \left(\pi\beta\left(1 + \frac{\pi\sigma_{t+1}^*}{1 - \pi}\right) + (1 - \pi)\gamma\right)$$

Proof. See the Appendix. ■

If the evil of death is reduced to the opportunity cost of dying early (i.e. $\Omega = 0$), and if the joy of giving is as strong as the joy of living the old age ($\beta = \gamma$), full insurance is achieved at the laissez-faire, that is, when $\sigma_{t+1}^* = 0$. Indeed, in that particular case, the realized duration of life does not affect the realized lifetime well-being. However, if the marginal utility of the joy of giving is lower than the marginal utility of consuming, that is, if $\gamma < \beta$, then, under $\Omega \leq 0$, full insurance requires a positive tax σ_{t+1}^* .

Figure 2 illustrates the level of the tax rate σ_{t+1} that yields full insurance as the one that equalizes, on the one hand, second-period utility in case of a short life, that is, $\Omega + \gamma \log(R_{t+1}s_t + S_{t+1})$, and, on the other hand, second-period utility in case of a long life, that is, $\beta \log(R_{t+1}s_t(1 - \sigma_{t+1}))$. As illustrated on Figure 2, a higher pure disutility of a premature death (that is, a lower Ω) pushes the curve corresponding to second-period utility in case of a short life down, which implies that full insurance requires a higher tax on old-age consumption, and, hence, a higher subsidy on accidental bequests left by prematurely dead

persons. In the light of this, it appears that whether or not the utility loss due to a premature death can be reduced to the opportunity cost of dying early is a key determinant of the subsidy on accidental bequests that yields full insurance against the risk of dying early.

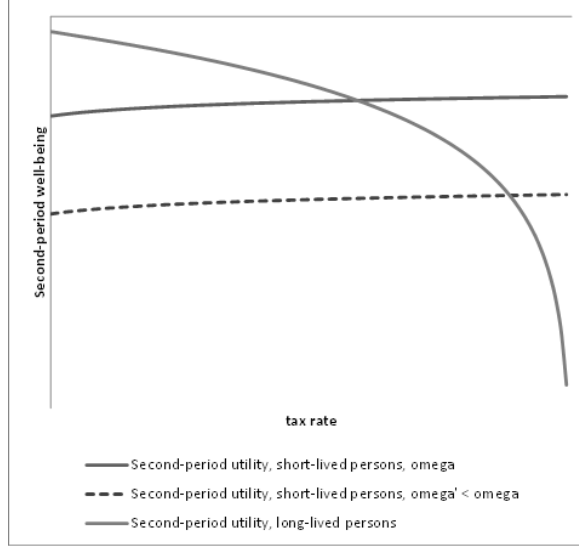


Figure 2. Impact of Ω on the tax rate that yields full insurance against a short life.

Another determinant of σ_{t+1}^* is the interest factor R_{t+1} . The higher R_{t+1} is, the higher the tax rate yielding full insurance is. The intuition is that, as under the youth allocations policy, a higher interest factor increases old-age consumption, and, hence, the extent to which a premature death deprives the person. As a consequence, in order to have full insurance against the risk of a short life, it is necessary to tax old-age consumption to a larger extent.

An interesting difference with respect to the youth allowances system is that, whereas the youth allowance implying full insurance P_t^* is decreasing in the fertility rate n , the subsidy on accidental bequest S_{t+1}^* does not depend on fertility. The intuition is that the latter insurance device works through transfers from long-lived persons to short-lived persons who all belong to the *same* cohort, unlike the former system, which works through transfers from long-lived persons of a cohort to all young adults of the next cohort.

5 Policy comparison: the short run

To compare the *ante mortem* policy (youth allowance) with the *post mortem* policy (subsidy on accidental bequests), this section will examine the consequences of these policies on well-being at the temporary equilibrium, under the

assumption that full insurance is provided.¹⁶ For pedagogical purposes, we will here assume that production factor prices w_t , R_t and R_{t+1} are given and equal across the two insurance systems. By taking production factor prices as given, this section provides a comparison of the two insurance devices in the *short run*. As a complement, Section 6 will compare the two insurance devices in the long run, by studying the stationary equilibrium, and, hence, by comparing the effects of insurance devices on capital accumulation and prices.

For the sake of simplicity, we will denote variables under the youth allowances system with the superscript (a) and variables under the bequest subsidy with the superscript (b). Lifetime well-being levels are, under the two systems:

$$U_t^{(a)} = w_t - (\pi\beta + (1-\pi)\gamma) + P_t^* + \Omega + \gamma \log(R_{t+1}(\pi\beta + (1-\pi)\gamma))$$

$$U_t^{(b)} = w_t - \left(\pi\beta + \frac{(1-\pi)\gamma}{1 + \frac{\pi\sigma_{t+1}^*}{1-\pi}} \right) + \Omega + \gamma \log \left[R_{t+1} \left(\pi\beta + \frac{(1-\pi)\gamma}{1 + \frac{\pi\sigma_{t+1}^*}{1-\pi}} \right) + S_{t+1}^* \right]$$

where policy parameters P_t^* , σ_{t+1}^* and S_{t+1}^* take levels guaranteeing full insurance against a short life.

Proposition 3 *Compare the youth allowance policy (τ_t^*, P_t^*) and the accidental bequest subsidy policy $(\sigma_{t+1}^*, S_{t+1}^*)$ under full insurance. Under fixed production factor prices R_t and R_{t+1} , we have:*

$$U_t^{(a)} \geq U_t^{(b)}$$

if and only if

$$\left(1 - \left[\frac{\exp(\Omega)}{(R_{t+1}(\pi\beta + (1-\pi)\gamma))^{\beta-\gamma}} \right]^{\frac{1}{\beta}} \right) \frac{\pi}{n} R_t (\pi\beta + (1-\pi)\gamma)$$

$$\geq \gamma \log \left(\frac{\left(\pi\beta \left[1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right] + (1-\pi)\gamma \right)}{(\pi\beta + (1-\pi)\gamma)} \right) + \gamma \frac{\pi\sigma_{t+1}^*}{\left(1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right)}$$

where σ_{t+1}^* is defined implicitly by the equality:

$$\Omega - (\beta - \gamma) \log \left(R_{t+1} \left(\pi\beta \left(1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right) + (1-\pi)\gamma \right) \right) = \beta \log \left(\frac{1 - \sigma_{t+1}^*}{1 + \frac{\pi\sigma_{t+1}^*}{1-\pi}} \right)$$

Proof. See the Appendix. ■

Proposition 3 provides a necessary and sufficient condition so that lifetime well-being is, in the short run, higher under the youth allowances system than

¹⁶ Given that we will focus on situations where full insurance prevails, there is, by construction, equality of achieved lifetime well-being levels across long-lived and short-lived persons who received the same endowment from their own parents. For simplicity, this section will show well-being levels for individuals who have long-lived parents (and, thus, did not receive accidental bequests).

under the subsidy on accidental bequests. That condition is hard to interpret, because it depends on a large number of determinants: present and future interest factors (R_t and R_{t+1}), preference parameters (β , γ and Ω) and demographic parameters (π and n). However, some observations can be made.

First of all, a higher current interest factor R_t , by raising the LHS of the condition of Proposition 3 - whereas the RHS is left unchanged - makes the system of youth allowances more likely to yield a higher level of lifetime well-being than the subsidy on accidental bequests. The intuition is that a higher R_t raises, *ceteris paribus*, the revenues available for financing the youth allowance, and, hence, leads to a higher lifetime well-being under the youth allowances system. However, the effect of the future interest factor R_{t+1} is unknown.

Second, regarding preference parameters, Proposition 3 highlights the key role played by the joy of giving parameter γ . When the joy of giving is low, that is, when γ tends to zero, the condition of Proposition 3 collapses to:

$$\frac{\left(1 - \left[\frac{\exp(\Omega)}{(R_{t+1}(\pi\beta))^\beta}\right]^{\frac{1}{\beta}}\right) \pi R_t (\pi\beta)}{n} > 0$$

Thus, when the joy of giving is low, well-being is unambiguously higher under the youth allowances system than under the subsidy on accidental bequest. The intuition is that, in that case, the link between the well-being of the short-lived person and the accidental bequest left to her descendants is too weak, so that an insurance policy that taxes old-age consumption to subsidize accidental bequests, although it brings full insurance, leads nonetheless to a lower level of realized lifetime well-being in comparison to what a youth allowance can achieve. However, the roles of other preference parameters β and Ω are ambiguous.

Another important corollary of Proposition 3 concerns the role of the fertility rate n . Interestingly, n only affects the LHS of the condition of Proposition 3, but not the RHS of that condition (since σ_{t+1}^* does not depend on n). Hence, the higher the fertility rate n is, and the lower is the LHS of the condition, whereas the RHS is left unchanged. As a consequence, it follows that, *ceteris paribus*, the well-being of persons living in societies with high fertility is more likely to be higher under the subsidy on accidental bequests than under the youth allowances system. The intuition is that, in an economy with a high fertility, a full insurance device based on transfers from the old to the young can only provide a low youth allowance to each young adult. This reduces the lifetime well-being of adults who will turn out to die prematurely. Thus, in economies with a high fertility, the *post-mortem* policy (subsidizing accidental bequests) dominates the *ante-mortem* policy (youth allowance), because the targeting of the latter is so large under a high fertility that it leads to a too low improvement of lives for the prematurely dead persons. On the contrary, in societies with a low fertility, the opposite takes place, and, *ceteris paribus*, the youth allowances system is more likely to dominate the subsidy on accidental bequests.

Whereas these corollaries are worth being highlighted, it should be stressed that Proposition 3, which takes production factor prices as given, provides only a comparison of the two insurance policies in the short run. However, each

insurance system has its own effects on capital accumulation, and, hence, on wages and interest rates. The next section analyzes those long-run effects.

6 Policy comparison: the long run

Let us now compare the two insurance devices from a long-run perspective. For that purpose, we will consider first the economy with a youth allowances system, and, then, the economy with a subsidy on accidental bequests.

6.1 The youth allowances system

Given the full depreciation of capital after one period of use, the dynamic equation for capital accumulation is, in intensive terms:

$$k_{t+1}^{(a)} = \frac{s_t^{(a)}}{n} = \frac{\pi\beta + (1-\pi)\gamma}{n} \quad (21)$$

Hence the steady-state capital to labor ratio is:

$$k^{(a)} = \frac{\pi\beta + (1-\pi)\gamma}{n} \quad (22)$$

Steady-state wage and interest factor are:

$$w^{(a)} = (1-\alpha)A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha \quad \text{and} \quad R^{(a)} = \alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^{\alpha-1} \quad (23)$$

Using Proposition 1, full insurance at the steady-state is obtained when:

$$\tau = 1 - \left[\frac{\exp(\Omega)}{(\alpha A n^{1-\alpha} (\pi\beta + (1-\pi)\gamma)^\alpha)^{\beta-\gamma}} \right]^{\frac{1}{\beta}} \quad (24)$$

At the stationary equilibrium under full insurance, the level of the youth allocation is:

$$P = \left[1 - \left[\frac{\exp(\Omega)}{(\alpha A n^{1-\alpha} (\pi\beta + (1-\pi)\gamma)^\alpha)^{\beta-\gamma}} \right]^{\frac{1}{\beta}} \right] \frac{\pi\alpha A (\pi\beta + (1-\pi)\gamma)^\alpha}{n^\alpha} \quad (25)$$

6.2 The subsidy on accidental bequests

The dynamic equation for capital accumulation is, in intensive terms:

$$k_{t+1}^{(b)} = \frac{s_t^{(b)}}{n} = \frac{\pi\beta \left(1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right)} \quad (26)$$

where $\sigma_{t+1}^* \in]0, 1[$ is the tax rate that yields full insurance for the risk of dying prematurely.

As stressed in Proposition 2, σ_{t+1}^* is a function of R_{t+1} , and, hence, of k_{t+1} . We have:

$$\Omega - (\beta - \gamma) \log \left(R_{t+1}^{(b)} \left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}^*}{1 - \pi} \right) + (1 - \pi) \gamma \right) \right) = \beta \log \left(\frac{1 - \sigma_{t+1}^*}{1 + \frac{\pi \sigma_{t+1}^*}{1 - \pi}} \right)$$

After some manipulations, this equality can be rewritten as:

$$R_{t+1}^{(b)} = \exp \left(\frac{\Omega}{(\beta - \gamma)} - \log \left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}^*}{1 - \pi} \right) + (1 - \pi) \gamma \right) - \frac{\beta}{(\beta - \gamma)} \log \left(\frac{1 - \sigma_{t+1}^*}{1 + \frac{\pi \sigma_{t+1}^*}{1 - \pi}} \right) \right)$$

Given that $R_{t+1}^{(b)} = \alpha A k_{t+1}^{(b)\alpha-1}$, we have:

$$k_{t+1}^{(b)} = \left[\frac{\alpha A}{\exp \left(\frac{\Omega}{(\beta - \gamma)} - \log \left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}^*}{1 - \pi} \right) + (1 - \pi) \gamma \right) - \frac{\beta}{(\beta - \gamma)} \log \left(\frac{1 - \sigma_{t+1}^*}{1 + \frac{\pi \sigma_{t+1}^*}{1 - \pi}} \right) \right)} \right]^{\frac{1}{1-\alpha}} = \Gamma(\sigma_{t+1}^*) \quad (27)$$

Hence, $\sigma_{t+1}^* = \Gamma^{-1}(k_{t+1}^{(b)})$.

Therefore, if one denotes $\Theta(k_{t+1}^{(b)}) \equiv \Gamma^{-1}(k_{t+1}^{(b)})$, the dynamics of the economy can be described by a two-dimensional system:

$$\begin{cases} k_{t+1}^{(b)} = \frac{\pi \beta \left(1 + \frac{\pi \sigma_{t+1}^*}{1 - \pi} \right) + (1 - \pi) \gamma}{n \left(1 + \frac{\pi \sigma_{t+1}^*}{1 - \pi} \right)} \\ \sigma_{t+1}^* = \Theta(k_{t+1}^{(b)}) \end{cases}$$

A stationary equilibrium is a pair $(k^{(b)}, \sigma^*)$ such that:

$$k^{(b)} = \frac{\pi \beta \left(1 + \frac{\pi \sigma^*}{1 - \pi} \right) + (1 - \pi) \gamma}{n \left(1 + \frac{\pi \sigma^*}{1 - \pi} \right)} \quad (28)$$

$$\sigma^* = \Theta(k^{(b)}) \quad (29)$$

Substituting the second equation in the first one, we obtain that the steady-state capital to labour ratio satisfies:

$$k^{(b)} = \frac{\pi \beta \left(1 + \frac{\pi \Theta(k^{(b)})}{1 - \pi} \right) + (1 - \pi) \gamma}{n \left(1 + \frac{\pi \Theta(k^{(b)})}{1 - \pi} \right)} \quad (30)$$

Hence, at the stationary equilibrium, production factors prices are:

$$w^{(b)} = (1 - \alpha)A \left(\frac{\pi\beta \left(1 + \frac{\pi\Theta(k^{(b)})}{1-\pi} \right) + (1 - \pi)\gamma}{n \left(1 + \frac{\pi\Theta(k^{(b)})}{1-\pi} \right)} \right)^\alpha \quad (31)$$

$$R^{(b)} = \alpha A \left(\frac{\pi\beta \left(1 + \frac{\pi\Theta(k^{(b)})}{1-\pi} \right) + (1 - \pi)\gamma}{n \left(1 + \frac{\pi\Theta(k^{(b)})}{1-\pi} \right)} \right)^{\alpha-1} \quad (32)$$

The level of σ^* that yields full insurance against a short life is the solution of the equation:

$$\sigma^* = \Theta \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi} \right) + (1 - \pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi} \right)} \right) \quad (33)$$

6.3 Comparing the two economies

In order to compare the long-run dynamics of the economy under either the youth allowance or the subsidy on accidental bequests, it is crucial to remind Lemma 1, which states that the saving per worker under the youth allowances system exceeds the saving per worker under the subsidy on accidental bequests:

$$s_t^{(a)} = \pi\beta + (1 - \pi)\gamma > s_t^{(b)} = \pi\beta + \frac{(1 - \pi)\gamma}{\left(1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right)}$$

where $\sigma_{t+1}^* \in]0, 1[$.

Given Lemma 1, it follows that, for a given fertility rate n , the strength of the accumulation process is higher under the youth allowances system than under the subsidy on accidental bequests. The two economies under comparison do not have the same capacity of accumulating physical capital. The economy with a youth allowances system exhibits, *ceteris paribus*, a stronger accumulation power. The intuition is that the subsidy of accidental bequests discourages individuals to save, so that they save necessarily less than in the absence of such a subsidy, and, also, less than under the youth allowances system.

Given the accumulation law $k_{t+1} = \frac{s_t}{n}$, the inequality $s_t^{(a)} > s_t^{(b)}$, which prevails at any given level of k_t , allows us to deduce that the steady-state capital to labour ratio is higher under the youth allowances system than under the subsidy on accidental bequests:

$$k^{(a)} > k^{(b)}$$

Based on that corollary of Lemma 1, Proposition 4 compares the major economic variables prevailing at the stationary equilibrium under the two economies.

Proposition 4 *When comparing the two economies, with youth allowance (a) and accidental bequest subsidy (b), we obtain:*

$$\begin{aligned} k^{(a)} &> k^{(b)} \text{ and } y^{(a)} > y^{(b)} \\ w^{(a)} &> w^{(b)} \text{ and } R^{(a)} < R^{(b)} \end{aligned}$$

We have also:

$$\begin{aligned} c^{(a)} &\geq c^{(b)} \\ \Leftrightarrow A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha &\left[1 - \frac{\pi\alpha [\exp(\Omega)]^{\frac{1}{\beta}}}{(\alpha A n^{1-\alpha} (\pi\beta + (1-\pi)\gamma)^\alpha)^{\frac{\beta-\gamma}{\beta}}} \right] - (\pi\beta + (1-\pi)\gamma) \\ &\geq (1-\alpha)A \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha - \frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{\left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \end{aligned}$$

and

$$\begin{aligned} d^{(a)} &\geq d^{(b)} \\ \Leftrightarrow \frac{\left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha [\exp(\Omega)]^{\frac{1}{\beta}}}{\left(\alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^{\alpha-1} (\pi\beta + (1-\pi)\gamma) \right)^{\frac{\beta-\gamma}{\beta}}} &\geq \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha (1 - \sigma^*) \end{aligned}$$

and

$$\begin{aligned} b^{(a)} &\geq b^{(b)} \\ \Leftrightarrow \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha &\geq \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha \left[1 + \frac{\pi\sigma^* (1-\pi + \pi\sigma^*)}{(1-\pi)^2} \right] \end{aligned}$$

Proof. See the Appendix. ■

The comparison of the two economies in the long run is more complicated than in the short run. The reason is that each system of insurance leads to its own dynamics of capital accumulation, with various effects on production factor prices (unlike in Proposition 3, where prices were supposed to be unaffected from a short run perspective). As a consequence, even if we know that the economy with a youth allowance is characterized by a higher saving, this does not allow us to conclude concerning the ranking of well-being, because that economy is characterized also by a lower interest factor in comparison with the economy with a subsidy on accidental bequests. A lower interest factor matters for the issue at stake here, because each policy is financed by taxing old-age consumption, which depends crucially on the interest rate.

In the light of the conditions of Proposition 4, it is unlikely to expect some analytical results concerning well-being comparisons in the long-run. Some numerical simulations are needed here, to be able to compare the two insurance systems from a long-run perspective. This is the task of the next section.

7 Numerical illustrations

The goal of this section is to provide a quantitative comparison, from a long-run perspective, of the two systems of social insurance against the risk of premature death studied in this paper. For that purpose, this section will assign some values to the structural parameters of the economy and will compare the implications of the two insurance systems in terms of (realized) lifetime well-being under full insurance against a short life.

Calibration From a quantitative perspective, a key issue is to see how the comparison of the two systems of insurance varies when demographic parameters - survival conditions and fertility patterns - vary. Thus, we will consider several demographic scenarios (Table 1). The medium value for fertility corresponds to a replacement fertility rate ($n = 1$), that is, a TFR equal to 2 children per women. The low fertility value ($n = 0.5$) amounts to one child per women, whereas the high fertility value ($n = 1.5$) corresponds to 3 children per women. Regarding survival conditions, the medium value for π is 0.90, that is, 90 % of a cohort reaches the old age (i.e. survives beyond age 60). This proportion is close to the one prevailing in advanced economies. For the sake of examining the robustness of our results, we consider also $\pi = 0.85$ and $\pi = 0.95$.

	low	medium	high
fertility	$n = 0.50$	$n = 1.00$	$n = 1.50$
survival probability	$\pi = 0.85$	$\pi = 0.90$	$\pi = 0.95$

Table 1: Demographic scenarios.

Each period of time lasting 30 years, applying a quarterly discount factor of 0.99 leads to the time preference parameter $\beta = [0.99]^{120} = 0.30$.

Taking the baseline values for demographic parameters ($n = 1$ and $\pi = 0.90$), we can calibrate all other parameters to be in line with stylized facts. First, fixing α to 1/3 is in line with a labor share in production of 2/3 (de la Croix and Michel 2002, p. 340). Moreover, fixing $\gamma = 0.15$ and $A = 6$ allows us to obtain, under $n = 1$ and $\pi = 0.90$, a steady-state annual capital to output ratio equal to about 2.15 (de la Croix and Michel, 2002, p. 340). Having $\gamma = 0.15 < \beta = 0.30$ is in line with Hurd (1989), who emphasized that the marginal utility of giving is lower than the marginal utility of consuming directly.

Finally, the parameter Ω capturing the pure evil of a premature death - independently from the opportunity cost of an early death - can be calibrated by using empirical estimates of the value of a statistical life. The value of a statistical life is the marginal rate of substitution between the risk of premature death and current income:

$$VSL = -\frac{\frac{\partial EU_t}{\partial(1-\pi)}}{\frac{\partial EU_t}{\partial w_t}} = -\frac{-\beta \log(d) + \Omega + \gamma \log(b)}{1}$$

Assuming that the VSL is, on average, about 120 times annual GDP per capita (Miller 2000), we have that the VSL in our model can be rewritten as:

$$\frac{120(6)K^{1/3}L^{2/3}}{30(3L)3} = (0.30)\log(d) - \Omega - (0.15)\log(b)$$

$$\iff \Omega = -(2.66)k^{1/3} + (0.30)\log(d) - (0.15)\log(b)$$

This yields $\Omega = -1.74$. This negative value is in line with Kamm (1993), who argued that the welfare loss due to premature death is not reducible to the welfare loss due to foregone consumption, but includes also other welfare losses due to the insult factor of death and the irreversibility factor of death. Table 2 summarizes the calibration of structural parameters.

parameter	value	source
A	6.00	an annual capital to output ratio of 2.15
α	0.33	a labor share in production of 2/3
β	0.30	a quarterly discount factor of 0.99
γ	0.15	an annual capital to output ratio of 2.15
Ω	-1.74	a VSL of 120 times annual GDP per capita

Table 2: Calibration of structural parameters.

Results Let us now compare the two systems of insurance against a short life, i.e., the youth allowance and the subsidy on accidental bequests, by studying their consequences in terms of well-being. For that purpose, we assume that the two devices are calibrated so as to yield full insurance. Given the low value of the parameter Ω capturing the pure evil of a premature death (beyond the opportunity cost of death), full insurance can only be reached for high levels of tax rates τ and σ , which lie, in our numerical simulations, in the interval $[0.99, 1]$. Bringing a full compensation for a premature death is feasible in our model, but requires substantial redistribution, under each insurance device.

Figure 3 shows the levels of the youth allowance P and the subsidy on accidental bequests S that yield full insurance, under different survival conditions (parameter π). Each simulation assumes the baseline fertility ($n = 1$).

Under all scenarios, the subsidy on accidental bequests is much larger than the youth allowance. The monetary compensation given to each short-lived person is much higher under the subsidy on accidental bequests than under the youth allowances system. The reason lies in the unequal degree of *targeting* of the two systems. The resources obtained by taxing old-age consumption are transferred precisely towards the children of prematurely dead persons under the bequest subsidy system, which thus concentrates resources on short-lived persons *only*. However, under the youth allowances system, the resources available for the compensation of the short-lived are distributed towards all members of the young cohort, which implies a much larger number of beneficiaries (exceeding the number of short-lived persons). *Due to imperfect targeting, the youth allowances system dilutes resources among a large number of beneficiaries. This leads to a lower level of monetary compensation for each short-lived person.*

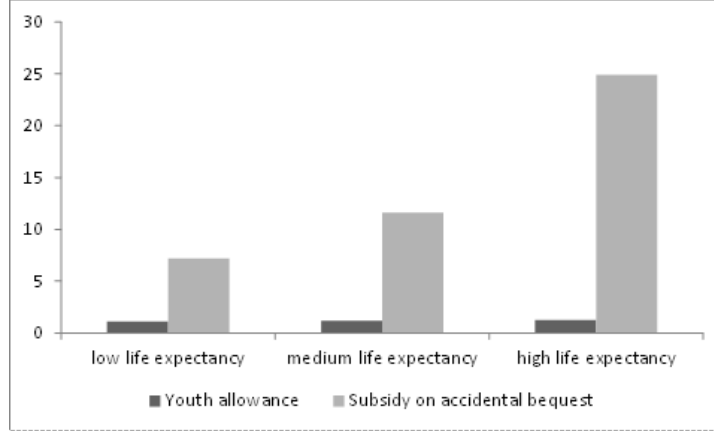


Figure 3. Youth allowance and subsidy on accidental bequests that yield full insurance, under different levels of life expectancy.

Another observation concerns the impact of life expectancy. Under both devices, a rise in life expectancy, by increasing savings, raises capital intensity, which affects the revenues available for compensating the short-lived in an ambiguous way (because of the associated fall in the interest factor). Figure 3 shows that the effect is positive for both devices, but varies in intensity across insurance systems. The slight rise of the youth allowance comes from the rise in the number of contributors to the system thanks to a higher proportion of persons reaching the old age. However, the rise of π has a much larger effect on the subsidy on accidental bequests. The reason is that, under that system, the rise in the number of contributors is combined with a second effect, which is the fall in the number of beneficiaries. This double effect - instead of a single one - explains why the subsidy on accidental bequests increases more with life expectancy. Figure 3 confirms our analytical intuitions: the *post-mortem* insurance system benefits from perfect targeting of the short-lived persons, and, as such, avoids a dilution of resources on a too large number of beneficiaries (unlike the *ante-mortem* youth allowances system).

But is this perfect targeting sufficient to make the *post-mortem* bequest subsidy a better insurance mechanism? Figure 4, which compares the levels of realized lifetime well-being under the two insurance systems, again for low, medium and high levels of life expectancy, suggests that the answer is: No.¹⁷

¹⁷ Given that we will compare economic outcomes under full insurance, focusing on the realized lifetime well-being of short-lived or long-lived persons does not make any difference, since these levels are equalized by the insurance device. Therefore, the following figures will present levels of well-being under each system of social insurance against the risk of premature death, without specifying the well-being of which group of persons. In each graph, we will focus on the well-being of individuals whose parents were long-lived (the large majority of persons), and leave aside the well-being of persons whose parents were short-lived.

Actually, under all scenarios, the youth allowances system dominates, in terms of well-being outcomes, the subsidy on accidental bequests. The intuition behind this result is the following. While both insurance systems are financed by taxing old-age consumption, the youth allowances system has the advantage of transferring these resources *directly* to young persons, for whom the marginal utility of consumption is larger, in comparison to the transfers made under the bequest subsidy, which only affect the well-being of short-lived persons through a joy of giving function. The marginal utility of consuming when being young exceeding the marginal utility of giving when being dead, the youth allowances system leads to a higher level of well-being for the prematurely dead (and for everyone in the society) in comparison to the bequest subsidy system.

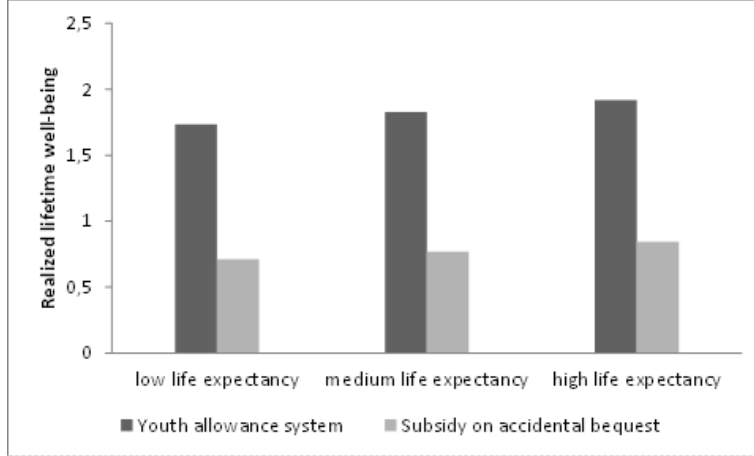


Figure 4: Comparison of well-being under the youth allowance and the accidental bequest subsidy, under various survival conditions.

Another observation concerning Figure 4 is that, for each insurance system, an improvement of survival conditions allows to reach higher levels of well-being. The reason is that better survival conditions favor savings and capital accumulation, which leads to a rise in the wage levels, leading to a rise in well-being. True, a higher steady-state capital intensity leads also to lower interest rates, but this effect is here dominated by the previous one.

Let us now compare the two insurance devices under various fertility levels. For that purpose, Figure 5 shows the levels of the youth allowance P and the subsidy on accidental bequests S that yield full insurance, for n equal to 0.5, n equal to 1 and n equal to 1.5, while survival conditions take the intermediate levels ($\pi = 0.90$). Figure 5 shows that a higher level of fertility has opposite effects on P and S . A higher level of fertility decreases the youth allowance, whereas it increases the subsidy on accidental bequests. The intuition goes as follows. Note first that the generosity of the two systems depends on the level

of old-age consumption, which is increasing in the interest factor, and, hence, decreasing in steady-state capital per worker. Since a higher fertility leads to more capital dilution, this implies a rise in the interest factor, and, hence, of resources available for either P or S . But this effect is, in the case of the youth allowances system, outweighed by the fact that a higher fertility increases the proportion of beneficiaries of the allowance with respect to the number of contributors. This second, negative effect, explains why the youth allowance is decreasing in the level of fertility, despite the associated rise in the interest rate. However, under the subsidy on accidental bequests, this effect is absent. This explains the different patterns for P and S .

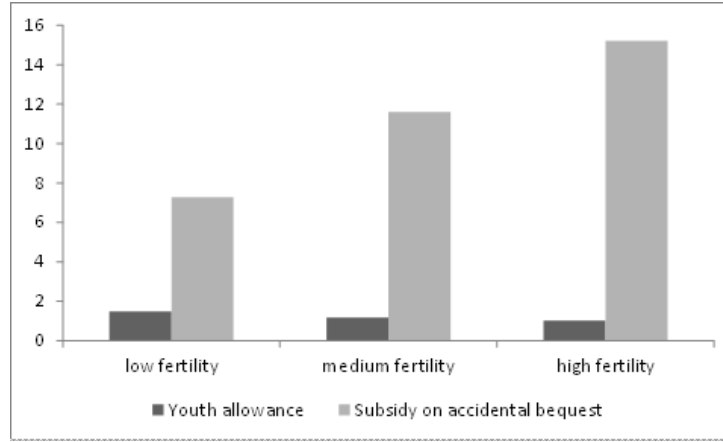


Figure 5. Youth allowance and subsidy on accidental bequests that yield full insurance, under different levels of fertility.

Although the subsidy on accidental bequests is a perfectly targeted policy, which provides a larger monetary compensation per short-lived person, this is an *indirect* compensation, through the joy of giving of the deceased to his children. This indirect compensation has a lower effect on well-being than the one under the (direct) youth allowances system. Figure 6 shows that the steady-state levels of realized well-being under the youth allowances system are unambiguously higher than under the subsidy on accidental bequests. The reason is that the marginal utility of consuming when being young is much larger than the marginal utility of giving when being dead. Hence, even though more resources are transferred towards the short-lived under the bequest subsidy system, the effect, in terms of well-being, is smaller than under the youth allowances system.

Figure 6 shows also that the higher fertility is, the lower well-being is, under all insurance systems under comparison. This effect is due to the fact that a higher fertility leads to more capital dilution, that is, to a lower steady-state capital intensity. This reduces wages and increases interest rates. But the former effect dominates the latter one, leading to a decrease of well-being.

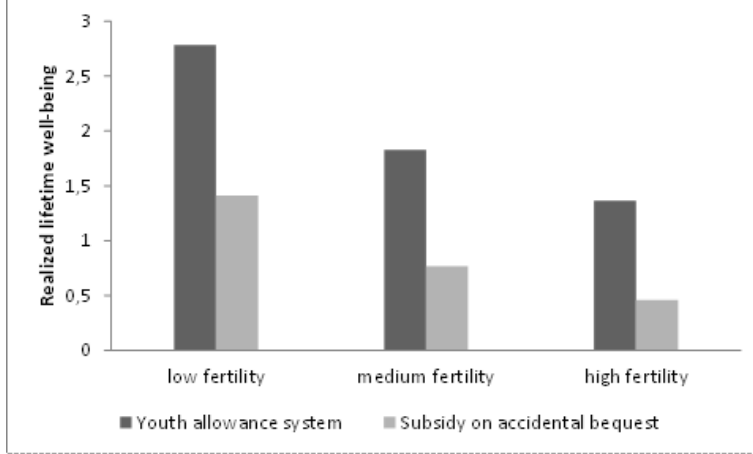


Figure 6: Comparison of well-being under the youth allowance and the accidental bequest subsidy, under various fertility levels.

All in all, the findings of this section provide unambiguous support for the youth allowances system: in all scenarios for survival conditions and fertility, the youth allowance leads to higher steady-state well-being levels than the subsidy on accidental bequests. This result arises despite the fact that the youth allowances system exhibits imperfect targeting, which dilutes resources available for the compensation of short-lived persons on a much larger number of beneficiaries, unlike the subsidy on accidental bequests (which exhibits perfect targeting). But despite imperfect targeting, the youth allowances system dominates, in welfare terms, the subsidy on accidental bequests, because the marginal utility of consuming at the young age is higher than the marginal utility of giving once dead. Hence, the youth allowances system is a more efficient way of insuring persons against the risk of early death.

8 Concluding remarks

Existing Welfare States do not insure citizens against the risk of dying prematurely. Insuring all citizens against the risk of a short life requires to build a new ‘pillar’ for social protection: a social insurance against the risk of premature death. The present paper constitutes a first step towards that new ‘pillar’, by comparing, in a dynamic OLG model, two insurance mechanisms: on the one hand, an *ante mortem* age-based statistical discrimination policy that consists of a youth allowances system; on the other hand, a *post mortem* subsidy on accidental bequests left by prematurely dead persons.

Our analyses reveal that the two insurance devices can, under weak conditions, yield full insurance against premature death, that is, they can equalize realized lifetime well-being across short-lived and long-lived persons. However,

these two insurance devices have different implications. While analytical explorations yield ambiguous results, numerical simulations show that, under all demographic scenarios considered, *the system of subsidy on accidental bequests gives to each short-lived person a higher monetary compensation than the youth allowances system*. The reason is that the former system involves perfect targeting of resources towards short-lived persons, unlike the youth allowances system, which exhibits imperfect targeting, and thus dilutes resources on a larger number of beneficiaries. *However, despite an imperfect targeting, the youth allowances system leads, at the stationary equilibrium, to higher well-being levels than the ones under the subsidy on accidental bequests*. The intuition is that the marginal utility of consuming is higher than the marginal utility of giving when being dead. Hence, the youth allowances system is, despite imperfect targeting, a more efficient way of insuring citizens against the risk of early death.

The main lesson to be learnt from this comparison of two insurance mechanisms against the risk of a short life is the following. One may be tempted, at first glance, to prefer a *post-mortem* insurance policy on the ground that, once individual durations of life are known, the Welfare State can identify short-lived persons and target resources towards these persons only. However, our analysis infirms that view: to insure citizens against a short life, a more efficient strategy is to do age-based statistical discrimination favouring the young. The reason is that, even if statistical discrimination leads to an imperfect targeting - and, hence, yields a lower monetary compensation -, this (smaller) monetary compensation is better at increasing the well-being of prematurely dead persons. Thus an *ante-mortem* policy with imperfect targeting of the short-lived does a better job than a *post-mortem* policy with perfect targeting.

To conclude, it should be stressed that other systems of insurance against a short life would also be worth studying, beyond the policies compared in this paper. One of these alternative insurance devices is the reverse retirement system (Ponthiere 2020, 2023). That system would, like the youth allowances system, provide a transfer to each young adult. But this system departs from the youth allowances system by imposing a junior work ban, to make all young adults enjoy as much leisure time as possible. Comparing this system with other insurance systems would require alternative models. Hence much work remains to be done, in the future, to draw the precise contours of a new Welfare State offering a universal protection against the risk of premature death.

9 Ethical statements

The author has no conflict of interests to declare. This paper did not benefit from any funding.

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11 Appendix

11.1 Proof of Proposition 1

Full insurance is obtained when there is equalization of lifetime well-being under a long life, denoted by U_t^{LL} , with lifetime well-being under a short life, denoted by U_t^{SL} :

$$U_t^{SL} = w_t + P_t + B_t - s_t + \Omega + \gamma \log(R_{t+1}s_t)$$

and

$$U_t^{LL} = w_t + P_t + B_t - s_t + \beta \log(R_{t+1}s_t(1 - \tau))$$

Substituting for savings $s_t = \pi\beta + (1 - \pi)\gamma$, we have:

$$U_t^{SL} = U_t^{LL} \iff \Omega + \gamma \log(R_{t+1}(\pi\beta + (1 - \pi)\gamma)) = \beta \log(R_{t+1}(\pi\beta + (1 - \pi)\gamma)(1 - \tau))$$

The LHS is a constant, which does not depend on τ_t . The RHS is decreasing in τ . Let us denote the RHS as:

$$\phi(\tau_t) \equiv \beta \log(R_{t+1}(\pi\beta + (1 - \pi)\gamma)(1 - \tau_t))$$

We have:

$$\begin{aligned} \lim_{\tau_t \rightarrow 0} \phi(\tau_t) &= \beta \log(R_{t+1}(\pi\beta + (1 - \pi)\gamma)) \\ \lim_{\tau_t \rightarrow 1} \phi(\tau_t) &= \beta \log(0) = -\infty \end{aligned}$$

When $\tau_t \rightarrow 1$, we have:

$$\Omega + \gamma \log(R_{t+1}(\pi\beta + (1 - \pi)\gamma)) > \phi(\tau_t) = -\infty$$

Thus for $\tau_t \rightarrow 1$, we have that the LHS of the condition is higher than the RHS.

Hence, a sufficient condition for the existence of a τ_t^* leading to full insurance is that when $\tau_t \rightarrow 0$, the LHS of the condition is smaller than the RHS of the condition, that is:

$$\beta \log(R_{t+1}(\pi\beta + (1 - \pi)\gamma)) > \Omega + \gamma \log(R_{t+1}(\pi\beta + (1 - \pi)\gamma))$$

Due to the monotonicity of the $\phi(\tau_t)$, that condition is also sufficient for the uniqueness of τ_t^* leading to full insurance.

The level of τ_t^* is given by:

$$\begin{aligned} \log\left(\frac{(R_{t+1}(\pi\beta + (1 - \pi)\gamma)(1 - \tau))^{\beta}}{(R_{t+1}(\pi\beta + (1 - \pi)\gamma))^{\gamma}}\right) &= \Omega \\ \iff \tau^* &= 1 - \left[\frac{\exp(\Omega)}{(R_{t+1}(\pi\beta + (1 - \pi)\gamma))^{\beta - \gamma}}\right]^{\frac{1}{\beta}} \end{aligned}$$

11.2 Proof of Proposition 2

Full insurance is obtained when there is equalization of lifetime well-being under a long life, denoted by U_t^{LL} , with lifetime well-being under a short life, denoted by U_t^{SL} . We have:

$$U_t^{SL} = w_t + B_t - s_t + \Omega + \gamma \log(R_{t+1}s_t + S_t)$$

and

$$U_t^{LL} = w_t + B_t - s_t + \beta \log(R_{t+1}s_t(1 - \sigma_{t+1}))$$

Substituting for $S_{t+1} = \frac{\pi}{1-\pi}R_{t+1}s_t\sigma_{t+1}$, this equality holds when:

$$\Omega + \gamma \log\left(R_{t+1}s_t\left(1 + \frac{\pi\sigma_{t+1}}{1 - \pi}\right)\right) = \beta \log(R_{t+1}s_t(1 - \sigma_{t+1}))$$

Substituting for savings, this expression becomes:

$$\begin{aligned} & \Omega + \gamma \log \left(R_{t+1} \frac{\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma}{\left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right)} \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) \right) \\ &= \beta \log \left(R_{t+1} \frac{\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma}{\left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right)} (1 - \sigma_{t+1}) \right) \end{aligned}$$

Hence the condition is

$$\Omega + \gamma \log \left(R_{t+1} \left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma \right) \right) = \beta \log \left(R_{t+1} \frac{1 - \sigma_{t+1}}{1 + \frac{\pi \sigma_{t+1}}{1-\pi}} \left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma \right) \right)$$

or, alternatively:

$$\Omega = \log \left(\left[R_{t+1} \left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma \right) \right]^{\beta-\gamma} \left(\frac{(1 - \sigma_{t+1})}{\left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right)} \right)^{\beta} \right)$$

Hence the condition is:

$$\Omega - (\beta - \gamma) \log \left(R_{t+1} \left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma \right) \right) = \beta \log \left(\frac{1 - \sigma_{t+1}}{1 + \frac{\pi \sigma_{t+1}}{1-\pi}} \right)$$

Let us denote the LHS of the condition as:

$$\begin{aligned} \rho(\sigma) &\equiv \Omega - (\beta - \gamma) \log \left(R_{t+1} \left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma \right) \right) \\ \varsigma(\sigma) &\equiv \beta \log \left(\frac{1 - \sigma_{t+1}}{1 + \frac{\pi \sigma_{t+1}}{1-\pi}} \right) \end{aligned}$$

Assume $\beta > \gamma$. We have: $\rho'(\sigma_{t+1}) < 0$ and $\varsigma'(\sigma_{t+1}) < 0$. We have:

$$\begin{aligned} \lim_{\sigma_{t+1} \rightarrow 0} \rho(\sigma_{t+1}) &= \Omega - (\beta - \gamma) \log (R_{t+1} (\pi \beta + (1-\pi) \gamma)) \\ \lim_{\sigma_{t+1} \rightarrow 1} \rho(\sigma_{t+1}) &= \Omega - (\beta - \gamma) \log \left(R \left(\pi \beta \left(1 + \frac{\pi}{1-\pi} \right) + (1-\pi) \gamma \right) \right) \end{aligned}$$

We have also:

$$\begin{aligned} \lim_{\sigma_{t+1} \rightarrow 0} \varsigma(\sigma_{t+1}) &= \beta \log \left(\frac{1 - \sigma_{t+1}}{1 + \frac{\pi \sigma_{t+1}}{1-\pi}} \right) = \beta \log (1) = 0 \\ \lim_{\sigma_{t+1} \rightarrow 1} \varsigma(\sigma_{t+1}) &= \beta \log \left(\frac{1 - \sigma_{t+1}}{1 + \frac{\pi \sigma_{t+1}}{1-\pi}} \right) = \beta \log \left(\frac{0}{1 + \frac{\pi}{1-\pi}} \right) = -\infty \end{aligned}$$

Note that, since

$$\Omega - (\beta - \gamma) \log \left(R_{t+1} \left(\pi \beta \left(1 + \frac{\pi}{1-\pi} \right) + (1-\pi) \gamma \right) \right) > -\infty$$

the LHS is always higher than the RHS of the condition when $\sigma_{t+1} \rightarrow 1$.

Hence, a sufficient condition for the existence of σ_{t+1}^* guaranteeing full insurance is:

$$\Omega - (\beta - \gamma) \log (R_{t+1} (\pi \beta + (1-\pi) \gamma)) < 0$$

Regarding uniqueness, note that:

$$\begin{aligned} \rho'(\sigma_{t+1}) &\equiv \frac{-(\beta - \gamma) \pi \beta \frac{\pi}{1-\pi}}{\left(\left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma \right) \right)} < 0 \\ \rho''(\sigma_{t+1}) &= \frac{0 - \left[-(\beta - \gamma) R \pi \beta \frac{\pi}{1-\pi} \right] \left(R \left(\pi \beta \frac{\pi}{1-\pi} \right) \right)}{\left(R \left(\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma \right) \right)^2} > 0 \end{aligned}$$

Thus when σ_{t+1} goes up, the slope of $\rho(\sigma_{t+1})$ becomes less negative.

Moreover,

$$\begin{aligned} \varsigma'(\sigma_{t+1}) &= \beta \frac{\left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right)}{(1 - \sigma_{t+1})} \left(\frac{(-1) \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) - (1 - \sigma_{t+1}) \frac{\pi}{1-\pi}}{\left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right)^2} \right) \\ &= \frac{\beta \left(-1 - \frac{\pi}{1-\pi} \right)}{(1 - \sigma_{t+1}) \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right)} < 0 \\ \varsigma''(\sigma_{t+1}) &= \frac{\beta \frac{1}{1-\pi} \left[\frac{-1 + 2\pi(1 - \sigma_{t+1})}{1-\pi} \right]}{\left[(1 - \sigma_{t+1}) \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) \right]^2} \end{aligned}$$

Thus the sign of $\varsigma''(\sigma_{t+1})$ depends on the sign of $-1 + 2\pi(1 - \sigma_{t+1})$. There is an inflexion point, i.e., $\varsigma''(\sigma_{t+1}) = 0$, at $\sigma_{t+1} = 1 - \frac{1}{2\pi}$. When $\sigma_{t+1} < 1 - \frac{1}{2\pi}$, we have $\varsigma''(\sigma_{t+1}) > 0$. When $\sigma_{t+1} > 1 - \frac{1}{2\pi}$, we have $\varsigma''(\sigma_{t+1}) < 0$. Thus, for values of $\sigma_{t+1} < 1 - \frac{1}{2\pi}$, the slope of $\varsigma(\sigma_{t+1})$ becomes less negative when σ_{t+1} goes up. For values of $\sigma_{t+1} > 1 - \frac{1}{2\pi}$, the slope of $\varsigma(\sigma_{t+1})$ becomes more negative when σ_{t+1} goes up.

To prove uniqueness of σ_{t+1}^* , we need to compare the slopes of $\rho(\sigma_{t+1})$ and $\varsigma(\sigma_{t+1})$ at a given σ_{t+1} :

$$\begin{aligned} \rho'(\sigma_{t+1}) &\equiv \frac{-(\beta - \gamma) \pi \beta \frac{\pi}{1-\pi}}{\pi \beta \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right) + (1-\pi) \gamma} \\ \varsigma'(\sigma_{t+1}) &= \frac{\beta \left(-1 - \frac{\pi}{1-\pi} \right)}{(1 - \sigma_{t+1}) \left(1 + \frac{\pi \sigma_{t+1}}{1-\pi} \right)} \end{aligned}$$

We have $\varsigma'(\sigma_{t+1}) < \rho'(\sigma_{t+1})$ iff, for any given σ_{t+1} , we have:

$$\frac{\beta \left(-1 - \frac{\pi}{1-\pi} \right)}{(1-\sigma)(1 + \frac{\pi\sigma_{t+1}}{1-\pi})} < \frac{-(\beta - \gamma)\pi\beta \frac{\pi}{1-\pi}}{\pi\beta(1 + \frac{\pi\sigma_{t+1}}{1-\pi}) + (1-\pi)\gamma}$$

This condition can be rewritten as:

$$\frac{1}{(1-\sigma_{t+1})} \frac{1}{(1 + \frac{\pi\sigma_{t+1}}{1-\pi})} > \frac{(\beta - \gamma)}{\beta} \frac{\pi}{(1 + \frac{\pi\sigma_{t+1}}{1-\pi}) + \frac{(1-\pi)\gamma}{\pi\beta}}$$

Given that $\sigma_{t+1} \in]0, 1[$, the first factor of LHS is higher than 1. The second factor of LHS is higher than the second factor of RHS, which is the same up to a multiplication of the numerator by π and up to the addition of a positive term at the denominator. Moreover, the first factor of RHS is less than 1. Therefore that condition is necessarily satisfied.

Hence, given that the slope of $\varsigma(\sigma_{t+1})$ always exceeds the slope of $\rho(\sigma_{t+1})$, the above conditions suffice so that there exists a unique intersection of the two functions, despite the existence of an inflection point for $\varsigma(\sigma_{t+1})$ at $\sigma_{t+1} = 1 - \frac{1}{2\pi}$.

The level of σ_{t+1}^* is given implicitly by the expression:

$$\Omega - (\beta - \gamma) \log \left(R_{t+1} \left(\pi\beta \left(1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right) + (1-\pi)\gamma \right) \right) = \beta \log \left(\frac{1 - \sigma_{t+1}^*}{1 + \frac{\pi\sigma_{t+1}^*}{1-\pi}} \right)$$

11.3 Proof of Proposition 3

The condition can be written as:

$$\begin{aligned} U_t^{SL(a)} &\geq U_t^{SL(b)} \\ \iff &-(1-\pi)\gamma + P_t^* + \gamma \log(R_{t+1}(\pi\beta + (1-\pi)\gamma)) \\ &\geq -\frac{(1-\pi)\gamma}{(1 + \frac{\pi\sigma_{t+1}^*}{1-\pi})} + \gamma \log \left(R_{t+1} \left(\pi\beta + \frac{(1-\pi)\gamma}{1 + \frac{\pi\sigma_{t+1}^*}{1-\pi}} \right) + S_{t+1}^* \right) \end{aligned}$$

Replacing for P_t^* and S_{t+1}^* yields:

$$\begin{aligned} &\frac{(1-\pi)\gamma}{(1 + \frac{\pi\sigma_{t+1}^*}{1-\pi})} - (1-\pi)\gamma + \frac{\left(1 - \left[\frac{\exp(\Omega)}{(R_{t+1}(\pi\beta + (1-\pi)\gamma))^{\beta-\gamma}} \right]^{\frac{1}{\beta}} \right) \pi R_t(\pi\beta + (1-\pi)\gamma)}{n} \\ &\geq \gamma \log \left(R_{t+1} \left(\pi\beta \left[1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right] + (1-\pi)\gamma \right) \right) - \gamma \log(R_{t+1}(\pi\beta + (1-\pi)\gamma)) \end{aligned}$$

Hence the condition is:

$$\begin{aligned} & \left(1 - \left[\frac{\exp(\Omega)}{(R_{t+1}(\pi\beta + (1-\pi)\gamma))^{\beta-\gamma}} \right]^{\frac{1}{\beta}} \right) \frac{\pi}{n} R_t(\pi\beta + (1-\pi)\gamma) \\ & \geq \gamma \log \left(\frac{\left(\pi\beta \left[1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right] + (1-\pi)\gamma \right)}{\pi\beta + (1-\pi)\gamma} \right) + \gamma \frac{\pi\sigma_{t+1}^*}{1 + \frac{\pi\sigma_{t+1}^*}{1-\pi}} \end{aligned}$$

where $\Omega - (\beta - \gamma) \log \left(R_{t+1} \left(\pi\beta \left(1 + \frac{\pi\sigma_{t+1}^*}{1-\pi} \right) + (1-\pi)\gamma \right) \right) = \beta \log \left(\frac{1 - \sigma_{t+1}^*}{1 + \frac{\pi\sigma_{t+1}^*}{1-\pi}} \right)$.

This is the condition of Proposition 3.

11.4 Proof of Proposition 4

The inequality $k^{(a)} > k^{(b)}$ follows from Lemma 1 and the accumulation law $k_{t+1} = \frac{s_t}{n}$.

Given $y_t = Ak_t^\alpha$, it follows from $k^{(a)} > k^{(b)}$ that $y^{(a)} > y^{(b)}$.

Moreover, using the definitions of the production factor prices, we obtain:

$$w^{(a)} > w^{(b)} \text{ and } R^{(a)} < R^{(b)}$$

Regarding consumption at the young age (and focusing on individuals with long-lived parents), we have:

$$\begin{aligned} c^{(a)} &= w^{(a)} - s^{(a)} + P \\ &= (1-\alpha)A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha - (\pi\beta + (1-\pi)\gamma) \\ &\quad + \frac{\left(1 - \left[\frac{\exp(\Omega)}{(\alpha An^{1-\alpha}(\pi\beta + (1-\pi)\gamma)^\alpha)^{\beta-\gamma}} \right]^{\frac{1}{\beta}} \right) \pi\alpha A (\pi\beta + (1-\pi)\gamma)^\alpha}{n^\alpha} \\ &= A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha \left[1 - \pi\alpha \left[\frac{\exp(\Omega)}{(\alpha An^{1-\alpha}(\pi\beta + (1-\pi)\gamma)^\alpha)^{\beta-\gamma}} \right]^{\frac{1}{\beta}} \right] - (\pi\beta + (1-\pi)\gamma) \end{aligned}$$

We also have:

$$c^{(b)} = w^{(b)} - s^{(b)} = (1-\alpha)A \left(k^{(b)} \right)^\alpha - nk^{(b)}$$

Hence we have:

$$\begin{aligned}
c^{(a)} &\geq c^{(b)} \\
&\Leftrightarrow A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha \left[1 - \pi\alpha \left[\frac{\exp(\Omega)}{(\alpha A n^{1-\alpha} (\pi\beta + (1-\pi)\gamma)^\alpha)^{\beta-\gamma}} \right]^{\frac{1}{\beta}} \right] - (\pi\beta + (1-\pi)\gamma) \\
&\geq (1-\alpha)A \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha - n \frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)}
\end{aligned}$$

Regarding old-age consumption, we have:

$$\begin{aligned}
d^{(a)} &= R^{(a)} s^{(a)} (1 - \tau^*) \\
&= \alpha A \left(k^{(a)} \right)^\alpha n (1 - \tau^*) \\
&= \alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha n (1 - \tau^*) \\
&= \frac{\alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha n [\exp(\Omega)]^{\frac{1}{\beta}}}{\left(\alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^{\alpha-1} (\pi\beta + (1-\pi)\gamma) \right)^{\frac{\beta-\gamma}{\beta}}}
\end{aligned}$$

Moreover, we have also:

$$\begin{aligned}
d^{(b)} &= R^{(b)} s^{(b)} (1 - \sigma^*) \\
&= \alpha A \left(k^{(b)} \right)^{\alpha-1} n k^{(b)} (1 - \sigma^*) \\
&= \alpha A \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha n (1 - \sigma^*)
\end{aligned}$$

Hence we have:

$$\begin{aligned}
d^{(a)} &\geq d^{(b)} \\
&\Leftrightarrow \frac{\alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha n [\exp(\Omega)]^{\frac{1}{\beta}}}{\left(\alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^{\alpha-1} (\pi\beta + (1-\pi)\gamma) \right)^{\frac{\beta-\gamma}{\beta}}} \geq \alpha A \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha n (1 - \sigma^*) \\
&\Leftrightarrow \frac{\left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha [\exp(\Omega)]^{\frac{1}{\beta}}}{\left(\alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^{\alpha-1} (\pi\beta + (1-\pi)\gamma) \right)^{\frac{\beta-\gamma}{\beta}}} \geq \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha (1 - \sigma^*)
\end{aligned}$$

Finally, we have:

$$\begin{aligned}
b^{(a)} &\geq b^{(b)} \\
&\iff R^{(a)} s^{(a)} \geq R^{(b)} s^{(b)} + S^* \\
&\iff \alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha n \geq \alpha A \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha n + S^* \\
&\iff \alpha A \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha n \\
&\geq \alpha A \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha n + \frac{\pi\sigma^* \alpha A \left[n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) \right]^{1-\alpha} \left(\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma \right)^\alpha}{(1-\pi)} \\
&\iff \left(\frac{\pi\beta + (1-\pi)\gamma}{n} \right)^\alpha \\
&\geq \left(\frac{\pi\beta \left(1 + \frac{\pi\sigma^*}{1-\pi}\right) + (1-\pi)\gamma}{n \left(1 + \frac{\pi\sigma^*}{1-\pi}\right)} \right)^\alpha \left[1 + \frac{\pi\sigma^* (1-\pi + \pi\sigma^*)}{(1-\pi)^2} \right]
\end{aligned}$$