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## Conservation by Lending <br> Bård Harstad, Kjetil Storesletten

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# Conservation by Lending 


#### Abstract

This project analyzes how a principal can motivate an agent to conserve rather than exploit a depletable resource. This dynamic problem is relevant for tropical deforestation as well as for other environmental problems. It is shown that the smaller is the agent's discount factor (e.g., because of political instability), the more the principal benefits from debt-for-nature contracts compared to flow payments (in return for lower deforestation). The debt-for-nature contract combines a loan to the agent with repayments that are contingent on the forest cover.


Keywords: environmental conservation, sovereign debt, sustainability-linked bonds, default, hyperbolic discounting, time inconsistency.

Bård Harstad<br>Stanford Graduate School of Business<br>Standford / CA / USA<br>harstad@stanford.edu

Kjetil Storesletten
University of Minnesota
Minneapolis / MN / USA
kstoresl@umn.edu

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## Introduction

This project analyzes and derives the principal's optimal contract in a dynamic game in which the agent extracts a resource. The resource in question can be a tropical forest, and the principal can be the UN, the World Bank, or an individual donor country, such as Norway. (Norway has spent billions on tropical forest conservation through the UN program REDD+).

Traditionally, the REDD+ contracts take the following form: If the deforestation level $\left(x_{t}\right)$ is below some threshold, then the principal, P , will pay the government in the South, S. The amount of payment is linear in the distance between $x_{t}$ and the threshold. The purpose of this analysis is to show when P can do better with more clever contracts. In particular, Proposition 3 shows that the more myopic S is (e.g., because the government may, with some probability, loose power), the more P benefits from offering a debt contract, instead. The optimal debt contract is an up-front loan, combined with a repayment that will be requested if and only if the forest cover falls below a certain level. By accepting the debt contract, the current government in $S$ "ties the hands" of any future government. The current government values this commitment, and thus accept more conservation at a lower price.

If $S$ did not have access to a foreign credit market, then REDD+ is shown to implement the optimal contract. The existence of a foreign credit market leads to more deforestation in the absence of a principal (/donor), but, if a donor exists, it can take advantage of the credit market (and the possibility to default) in order to obtain more conservaiton at a lower cost.

We draw on the seminal work of Bulow and Rogoff (1989) when we model sovereign debt and the incentives to default. The time inconconsistency of Laibson (1997) follows naturally when we add political rotation, as in Amador (2003), Chatterjee and Eyigungor (2016), and Harstad (2020). This leads to strong incentives to extract natural resources and benefits from tying the hands of future governments (Harstad, 2023). Our "conservation by lending" approach is clearly different from the traditional green bonds, and the associated literature (Mok et al., 2020).

## Assumptions

The planner or principal (P), and private lenders, apply a discount factor $\bar{\beta} \in$ $(0,1)$, so that the required rate of return is $r=R-1$, where $R \equiv 1 / \bar{\beta}$. The sovereign/south (S) applies the discount factor $\beta \leq \bar{\beta}$. The possibility $\beta<\bar{\beta}$ is reasonable when the government in the south can be replaced with some probability. The larger is the replacement probabiliy, the smaller is $\beta$. (See, for instance, the microfoundation in Harstad, 2020.)

S enters period $t$ with forest stock $F_{t}$, agricultural stock $A_{t}=F_{0}-F_{t}$, and extracts or $\operatorname{logs} x_{t} \in\left[0, q F_{t}\right]$ to obtain $F_{t+1}=F_{t}-x_{t}$ and income:

$$
w_{t}=w_{0}+a A_{t}+(a+l) x_{t}
$$

where $a$ is the value of agriculture (e.g., beef) and $l$ of the lumber (or the extracted units), while $q$ is an exogenously given upper boundary. In addition, S enjoys the environmental benefit $\left(F_{t}-x_{t}\right) e$.

If S can borrow privately, S enters period $t$ with debt $b_{t}$ and consumes $c_{t}=w_{t}-R b_{t}+b_{t+1}$.

If S defaults, S can no longer borrow and faces the cost $\phi=k w_{t}$ in every future period. I.e., S thereafter produces $(1-k) w_{t}$.

## Benchmark A (Autarky)

Suppose, first, that S cannot borrow.

Lemma 1. In autarky, iff $e \leq e^{A}, x_{t}=q F_{t}$ and $S$ obtains the continuation value $V^{A}\left(F_{t}\right)$, where:

$$
\begin{align*}
e^{A} & \equiv a+(1-\beta) l  \tag{1}\\
V^{A}\left(F_{t}\right) & =\frac{w_{0}+a A_{t}}{1-\beta}+\frac{q(l+a /(1-\beta))+e(1-q)}{1-(1-q) \beta} F_{t} . \tag{2}
\end{align*}
$$

Proof. The continuation value is:

$$
V^{A}\left(F_{t}\right)=\max _{x_{t} \in\left[0, q F_{t}\right]}\left\{\left(w_{0}+a\left(A_{t}+x_{t}\right)+x_{t} l\right)+\left(F_{t}-x_{t}\right) e+\beta V^{A}\left(F_{t}-x_{t}\right)\right\} .
$$

So, $V^{A}\left(F_{t}\right)$ is linear, takes the form $V^{A}\left(F_{t}\right)=\mu+\nu F_{t}$, and $x_{t} \in\left\{0, q F_{t}\right\}$. Suppose, first, $x_{t}=q F_{t}$. Then, from the Envelope theorem:

$$
\begin{aligned}
\nu & =(-a(1-q)+q l)+e(1-q)+\beta(1-q) \nu, \text { while } \\
\mu & =\left(w_{0}+a F_{0}\right)+\beta \mu
\end{aligned}
$$

which gives (2). It is indeed optimal for S with $x_{t}>0$ (i.e., $x_{t}=q F_{t}$ ) iff

$$
\begin{aligned}
a+l-e & >\beta \nu=\beta \frac{(q l-a(1-q))+e(1-q)}{1-(1-q) \beta} \Rightarrow \\
e & <[1-(1-q) \beta](a+l)-\beta(q l-a(1-q))
\end{aligned}
$$

which gives (1).

If S never extracts (e.g., $e>e^{A}$ ), $V^{A}\left(F_{t}\right)$ is:

$$
V_{0}^{A}\left(F_{t}\right)=\frac{w_{0}+a A_{t}+F_{t} e}{1-\beta}
$$

## Benchmark B (Borrowing possible)

Timing: At the beginning of $t$, the state is $\left(b_{t}, F_{t}\right)$. S decides whether to repay $R b_{t}$ and decides on $x_{t} \in\left[0, q F_{t}\right]$. If S repays, S can borrow $b_{t+1}$ and consumes $c_{t}=w_{t}-R b_{t}+b_{t+1}$. If S defaulted, S enjoys $V^{D}\left(F_{t}\right)$, which equals $V^{A}\left(F_{t}\right)$ where all income parameters (i.e., not $e$ ) are multiplied by $1-k$ :

$$
V^{D}\left(F_{t}\right)=(1-k) \frac{w_{0}+a A_{t}}{1-\beta}+\frac{(1-k) q(l+a /(1-\beta))+e(1-q)}{1-(1-q) \beta} F_{t}
$$

Lemma 2. (i) With private borrowing, $S$ is more inclined to log and logs if $e \leq e^{B}=e^{A}+a(1-R \beta) k / r$. (ii) In equilibrium, $S$ borrows more if $a, l, q$ and $F_{t}$ are large (given $A_{t}$ ).

Proof. Because $\beta \leq 1 / R$, S prefers to borrow and repay later. S borrows as much as possible, i.e., up to the point where S will be indifferent to default.
(i) First, consider the situation in which S would never want to log. Then, $V^{D}\left(F_{t}\right)$ is:

$$
V_{0}^{D}\left(F_{t}\right)=\frac{(1-k)\left[w_{0}+a A_{t}\right]+F_{t} e}{1-\beta}
$$

In this case, $S$ defaults now, instead of being tempted to default in the next period, unless:

$$
\begin{gathered}
-R b_{t}+b_{t+1}+w_{t}+e F_{t}+\beta V_{0}^{D}\left(F_{t}\right) \geq V_{0}^{D}\left(F_{t}\right) \Rightarrow \\
b_{t}+\frac{1}{r}\left(b_{t}-b_{t+1}\right) \leq b^{0}(F) \equiv \frac{1}{r}\left(w_{t}+e F_{t}-(1-\beta) V_{0}^{D}\left(F_{t}\right)\right)=\frac{k w_{t}}{r}
\end{gathered}
$$

If S extracts a (marginal) unit, S produces $a+l$ more, borrows $k a / r$ more, suffers $e$, and, in the next period, S's payoff is reduced by $\partial V_{0}^{D}\left(F_{t}\right) / \partial F_{t}$. The total marginal surplus is (if $x_{t+1}=0$ ):

$$
\begin{equation*}
\left(l+a+\frac{k a}{r}-e\right)+\beta \frac{(1-k) a-e}{1-\beta}=l+\frac{a(1+(1-R \beta) k / r)-e}{1-\beta}=\frac{e^{B}-e}{1-\beta}, \tag{3}
\end{equation*}
$$

which is negative iff $e>e^{B}$.
(ii) Now, suppose $e<e^{B}$ and that S always $\operatorname{logs} x_{t}=q F_{t} .{ }^{1}$ Then,

$$
\begin{aligned}
w_{t} & =w_{0}+F_{0}\left(q+(1-q) q+(1-q)^{2} q \ldots(1-q)^{t-1} q\right) a+F_{0}(1-q)^{t-1} q l \\
& =w_{0}+q F_{0}\left[\frac{1-(1-q)^{t}}{1-(1-q)} a+(1-q)^{t-1} l\right]=w_{0}+F_{0}\left(\left[1-(1-q)^{t}\right] a+q(1-q)^{t-1} l\right) .
\end{aligned}
$$

S defaults now, instead of being tempted to default in the next period, unless:

$$
\begin{array}{r}
-R b_{t}+b_{t+1}+w_{t}+e(1-q) F_{t}+\beta V^{D}\left((1-q) F_{t}\right) \geq V^{D}\left(F_{t}\right) \Rightarrow \\
-R b_{t}+b_{t+1}+w_{0}+a\left(F_{0}-(1-q) F_{t}\right)+l q F_{t}+e(1-q) F_{t} \\
(1-k)\left[\frac{(1-\beta)\left(w_{0}+a F_{0}\right)}{1-\beta}+\frac{q l-a(1-q)+\frac{e}{1-k}(1-q)}{1-(1-q) \beta}[1-\beta(1-q)] F_{t}\right] \\
-R b_{t}+b_{t+1} \geq-k\left(w_{0}+a F_{0}+[q l-a(1-q)] F_{t}\right) \Rightarrow \\
b_{t}+\frac{1}{r}\left(b_{t}-b_{t+1}\right) \leq \frac{k}{r}\left[w_{0}+a A_{t}+q(l+a) F_{t}\right] \tag{5}
\end{array}
$$

S will borrow so much that this first-order difference equation always binds. We can thus solve for $b(F)$. Here, it suffices to note that more debt can be served for a large r.h.s. of (5), which increases in $q, a$, and in $F_{t}$ (given $A_{t}$ ). Over time, when $F_{t}$ declines and $A_{t}$ grows, the debt grows if $a(1-q)>q l$ and declines

[^0]otherwise. Because S borrows as much as possible, (4) binds and S's payoff is $V^{D}\left(F_{t}\right)$ in equilibrium.

## Planner Allocation

Assume that the planner seeks to conserve at least cost. Let $C_{t}=\sum_{\tau=t}^{\infty} R^{-(\tau-t)} c_{\tau}$ be the present-discounted costs of S's consumption flow. It seems natural to require that the resulting value to $\mathrm{S}, V_{t}^{C}=\sum_{\tau=t}^{\infty} \beta^{\tau-t}\left(c_{\tau}+e F_{t}\right)$, must be larger than $V^{A}\left(F_{t}\right)$ :

$$
\begin{equation*}
V_{t}^{C} \geq V^{A}\left(F_{t}\right) \tag{A}
\end{equation*}
$$

Note that $\left(\mathrm{IC}^{A}\right)$ also serves as a participation constraint.
A weaker constraint might be relevant if the planner can threaten $S$ with default:

$$
\begin{equation*}
V_{t}^{C} \geq V^{D}\left(F_{t}\right) \tag{B}
\end{equation*}
$$

From (5), $b\left(F_{0}\right)$ measures the largest debt S is willing to serve, so suppose $b_{0} \in\left[0, b\left(F_{0}\right)\right]$. If S enters period 0 with $b_{0}=b\left(F_{0}\right)$, S 's equilibrium payoff, if S rejects the planner's plan, is $V^{D}\left(F_{0}\right)$. Again, $\left(\mathrm{IC}^{B}\right)$ also serves as a participation constraint. If $b_{0}<b\left(F_{0}\right), \mathrm{S}$ accepts only if:

$$
\begin{equation*}
V_{0}^{C} \geq V^{D}\left(F_{0}\right)+\left(b\left(F_{0}\right)-b_{0}\right) R \tag{B}
\end{equation*}
$$

## Proposition 1.

(i) Consider the situation where $S$ cannot privately borrow (e.g., $k=0$ ). The planner's solution to $\min C_{0}$ s.t. $\left(I C^{A}\right)$ requires

$$
\begin{equation*}
c_{t}=c^{0} \equiv w_{0}+\frac{e^{A}-e}{1-(1-q) \beta} q F, t \geq 0 \tag{6}
\end{equation*}
$$

(ii) Suppose $S$ has access to private borrowing (i.e., $k>0$ ). The planner solution to $\min C_{0}$ s.t. $\left(I C^{B}\right)-\left(P C^{B}\right)$ requires

$$
\begin{align*}
c_{t} & =c^{k}=w_{0}-k w_{0}\left(1-\frac{F_{0}}{\bar{F}}\right)<c^{0}, \forall t>0, \text { and }  \tag{7}\\
c_{0} & =c^{k}+\left(b\left(F_{0}\right)-b_{0}\right) R .
\end{align*}
$$

Proof.
(i). Because $\beta \leq 1 / R, \min C_{0}$ s.t. $\left(\mathrm{IC}^{A}\right)$ requires that consumption is frontloaded and that the later $\left(\mathrm{IC}^{A}\right) \mathrm{s}$ binds. If $\left(\mathrm{IC}^{A}\right)$ binds at some $t+1>1$, then, for $\left(\mathrm{IC}^{A}\right)$ to bind also at $t, c_{t}$ must equal:
$c^{0}=(1-\beta) V^{A}\left(F_{t}\right)-F_{t} e=w_{0}+q \frac{((1-\beta) l+a)-e}{1-(1-q) \beta} F_{t}=w_{0}+\frac{e^{A}-e}{1-(1-q) \beta} q F$.
(ii) If $\left(\mathrm{IC}^{B}\right)$ binds at some $t+1>1$, then, at $t, c_{t}$ must equal:

$$
\begin{aligned}
c^{k} & =(1-\beta) V^{D}\left(F_{t}\right)-F_{t} e \\
& =(1-k)\left(w_{0}+a A_{t}\right)+(1-\beta) \frac{(1-k) q(l+a /(1-\beta))+e(1-q)}{1-(1-q) \beta} F_{t}-F_{t} e \\
& =(1-k) w_{t}+q \frac{(1-k)((1-\beta) l+a)-e}{1-(1-q) \beta} F_{t}=w_{0}-k w_{0}\left(1-\frac{F_{0}}{\bar{F}}\right) .
\end{aligned}
$$

Note that $c^{k}=c^{0}$ if $k=0$ and $c^{k}<c^{0}$ when $k>0$. When $\left(\mathrm{IC}^{B}\right)$ binds at $t=1$, $\left(\mathrm{PC}^{B}\right)$ implies that, in the first period, $c_{0}=c^{k}+R\left(b\left(F_{0}\right)-b_{0}\right)$.

Corollary 1. Suppose, as is likely in equilibrium, that $b_{0}=b\left(F_{0}\right)$. P's cost $C_{0}$ is smaller when $S$ can borrow privately $\left(c^{k}<c^{0}\right)$.

## Implementation with Debt Contracts

Suppose P can lend $s_{t} \geq 0$ to S at every $t$, and that S can require the repayment $\widetilde{r} x_{t}$ if S logs. If S logs, and $\widetilde{r}$ is large, S is likely preferring to default on the debt instead of repaying. If $S$ defaults, $S$ defaults on all debt.

## Proposition 2.

(i) Consider the situation where $S$ cannot privately borrow (e.g., $k=0$ ). $P$ implements (6) by transfering in every period, conditional on $x_{t}=0$,

$$
\begin{equation*}
s^{A} \equiv q \frac{((1-\beta) l+a)-e}{1-(1-q) \beta} F_{0}=\frac{e^{A}-e}{1-(1-q) \beta} q F_{0} \tag{8}
\end{equation*}
$$

(ii) Suppose $S$ has access to private borrowing (i.e., $k>0$ ). Consider $\bar{F}$, where

$$
c^{k}<w_{0} \Leftrightarrow F<\bar{F} \equiv \frac{k w}{q} \frac{1-(1-q) \beta}{(1-k)((1-\beta) l+a)-e} .
$$

(ii-1) Suppose $F_{0}<\bar{F}$. P implements (7) with loan

$$
\begin{align*}
s_{0} & =R b\left(F_{0}\right)-\frac{R}{r} k w_{0}\left(1-\frac{F_{0}}{\bar{F}}\right), \text { and }  \tag{9}\\
\widetilde{r} & \geq \widetilde{r}^{*} \equiv \frac{e^{B}-e}{1-\beta}
\end{align*}
$$

while $s_{t}=0$ for $t>0$. In equilibrium, $S$ 's private debt is $b^{0}>0$, decreasing in $a, l, q$, and $F_{0}$ :

$$
b^{0}=\frac{k w_{0}}{r}\left(1-\frac{F_{0}}{\bar{F}}\right)
$$

A larger $s>0$ increases $S$ 's debt and reduces $s_{0}$, without affecting $C_{0}$.
(ii-2) Suppose $F_{0}>\bar{F}$. P implements (7) with loan

$$
s_{0}=R b\left(F_{0}\right)+k w_{0}\left(\frac{F_{0}}{\bar{F}}-1\right),
$$

with repayment $\widetilde{r} \geq \widetilde{r}^{*}$, and $s=k w_{0}\left(F_{0} / \bar{F}-1\right)$ in every later period.
A larger $s>c^{k}-w_{0}$ increases $S$ 's debt and reduces $s_{0}$, without affecting $C_{0}$.

Proof.
(i) For S to consume $c^{0}, \mathrm{P}$ must transfer $c^{0}-w_{0}$.
(ii-1) Suppose that P , if S conserves, transfers $s_{t}$ at $t \geq 0$ and suppose $s_{t}=s$, $t \geq 1$. At every time, for every $s, \mathrm{~S}$ consumes $w+s-R b_{t}+b_{t+1}$ and S will borrow as much as possible, until S is just willing to default, i.e., ( $\mathrm{IC}^{B}$ ) binds. This implies that S will increase $b_{t}$ until S consumes exactly $c^{k}$, so $b_{t}$ will be:

$$
\begin{aligned}
b^{s} & =\frac{1}{r}\left(w_{0}+s-c^{k}\right)=\frac{1}{r}\left(w_{0}+s-(1-k) w_{0}-q \frac{(1-k)((1-\beta) l+a)-e}{1-(1-q) \beta} F_{0}\right) \\
& =\frac{s}{r}+\frac{k w_{0}}{r}\left(1-\frac{F_{0}}{\bar{F}}\right)
\end{aligned}
$$

assuming $b^{s}>0$. Then, $b^{s}$ is decreasing in $a, l, q$, and $F_{0}$.
At time 0, S's participation constraint is:

$$
\begin{equation*}
-R b_{0}+b^{s}+w_{0}+s_{0}+e F_{0}+\beta V^{D}\left(F_{0}\right) \geq V^{D}\left(F_{0}\right)+\left(b\left(F_{0}\right)-b_{0}\right) R \tag{10}
\end{equation*}
$$

where, if $b\left(F_{0}\right)>b_{0},\left(b\left(F_{0}\right)-b_{0}\right) R$ is S's additional consumption, beyond the
level making $S$ indifferent to default. With (7), (10) becomes:

$$
\begin{align*}
s_{0} \geq & c^{k}-w_{0}-b^{s}+R b\left(F_{0}\right)  \tag{11}\\
= & (1-k) w_{0}+q \frac{(1-k)((1-\beta) l+a)-e}{1-(1-q) \beta} F_{0}-w_{0} \\
& -\frac{1}{r}\left(w_{0}+s-(1-k) w_{0}-q \frac{(1-k)((1-\beta) l+a)-e}{1-(1-q) \beta} F_{0}\right)+R b\left(F_{0}\right) \\
= & \frac{R}{r}\left[-k w_{0}+q \frac{(1-k)((1-\beta) l+a)-e}{1-(1-q) \beta} F_{0}\right]-\frac{s}{r}+R b\left(F_{0}\right) \\
= & R b\left(F_{0}\right)-\frac{R}{r} k w_{0}\left(1-\frac{F_{0}}{\bar{F}}\right) .
\end{align*}
$$

The present-discounted cost of the side payments is:

$$
s_{0}+\sum_{t=1}^{\infty} R^{-t} s=\frac{R}{r}\left[-k w_{0}+q \frac{(1-k)((1-\beta) l+a)-e}{1-(1-q) \beta} F_{0}\right]+R b\left(F_{0}\right)
$$

independent of $s$. Thus, $s=0$ suffices, and P can simply grant S a loan $s_{0}$ in the very first period.

To ensure that S does not prefer to extract and repay P what is owned (instead of defaulting), P must require, for every extracted unit, at least the following (borrowed from (3)):

$$
r^{*} \equiv(l+a+k a / r)+\frac{\beta(1-k) a}{1-\beta}-\frac{e}{1-\beta}=\frac{e^{B}-e}{1-\beta}
$$

Any required repayment at this or higher level is outcome-equivalent in that it discourages S from extracting and repaying the debt to P .
(ii-2) Above, we assumed $b^{s}>0$. However: $b^{0}<0 \Leftrightarrow F_{0}>\bar{F}$. If $b^{0}<0$, i.e., $F>\bar{F}$, S cannot privately borrow if $s=0$ and to satisfy $\left(\mathrm{IC}^{B}\right), s=s^{*} \equiv$ $c^{k}-w>0$. From (11), $s_{0}=s^{*}+R b\left(F_{0}\right)$. Then, S does not borrow privately. An even larger $s$ allows S to borrow (without violating $\left(\mathrm{IC}^{B}\right)$ ) and reduces the required $s_{0}$, without changing $C_{0}$.

Corollary 2. If $F_{0}<\bar{F}, P$ can issue a loan at $t=0$ and no further transfer is necessary. The equilibrium level of $b_{t}, b^{0}$, ensures that $S$ does not log and default. Thus, $b^{0}$ decreasing in $a, l$, $q$, and $F_{0}$, which reverses Lemma 2(ii).

## Cost-Savings Relative to REDD+

Suppose P can pay $z_{t}$ to S in every period $t$, conditional on conservation in that period. As with REDD+, there is no required repayment, and there is no linkage to the international credit market: Logging does not lead to default and defaults do not lead to logging.

## Proposition 3.

(i) Suppose $S$ cannot borrow privately. Then, $z_{t}=s^{A}$, given by (8).
(ii) Suppose $S$ has access to private borrowing (i.e., $k>0$ ). Then, every $z_{t}$ is

$$
z^{*}=(1 / \beta-1) b\left(F_{0}\right)-k w_{0}\left(\frac{1 / \beta-1}{R-1}-\frac{F_{0}}{\bar{F}}\right)
$$

and the additional present-discounted cost of paying $z^{*}$ every period, compared to (7), is:

$$
\begin{equation*}
\left(\frac{\bar{\beta}-\beta}{1-\bar{\beta}}\right) \frac{R}{\beta}\left[b\left(F_{0}\right)-\frac{k w_{0}}{r}\right] . \tag{12}
\end{equation*}
$$

Proof.
(i) Follows directly.
(ii) If S can borrow privately, then, when $x_{t}=0, \mathrm{~S}$ defaults unless:

$$
\begin{aligned}
w_{0}-R b_{t}+b_{t+1} & \geq(1-k) w_{0}, \text { so } \\
b_{t}+\frac{1}{r}\left(b_{t}-b_{t+1}\right) & \leq b^{z} \equiv \frac{k w_{0}}{r} .
\end{aligned}
$$

Suppose S, at the end of any given period, ends the agreement with P. Then, S can borrow $b\left(F_{0}\right)$ now, instead of simply $b^{z}$, and S will thereafter receive $V^{D}\left(F_{0}\right) . \mathrm{S}$ still prefers to conserve iff:

$$
\begin{aligned}
& \beta \frac{w_{0}+z+e F_{0}-r b^{z}}{1-\beta} \geq b\left(F_{0}\right)-b^{z}+\beta V^{D}\left(F_{0}\right) \\
= & b\left(F_{0}\right)-\frac{k w_{0}}{r}+\beta \frac{(1-k) w_{0}}{1-\beta}+\beta \frac{(1-k) q(l+a /(1-\beta))+e(1-q)}{1-(1-q) \beta} F_{0} \Rightarrow \\
& \beta \frac{z}{1-\beta} \geq b\left(F_{0}\right)-\frac{k w_{0}}{r}+\beta \frac{(1-k) q(l+a /(1-\beta))+e(1-q)}{1-(1-q) \beta} F_{0}-\beta \frac{e F_{0}}{1-\beta} \Rightarrow \\
& z \geq(1 / \beta-1)\left(b\left(F_{0}\right)-\frac{k w_{0}}{r}\right)+q \frac{(1-k)((1-\beta) l+a)-e}{1-(1-q) \beta} F_{0} \\
= & (1 / \beta-1)\left(b\left(F_{0}\right)-\frac{k w_{0}}{r}\right)+k w_{0} \frac{F_{0}}{\bar{F}}=(1 / \beta-1) b\left(F_{0}\right)-k w_{0}\left(\frac{1 / \beta-1}{R-1}-\frac{F_{0}}{\bar{F}}\right) .
\end{aligned}
$$

The present-discounted cost of paying $z$ forever is $z R / r$. When the inequality binds:

$$
\frac{R}{r} z=\left(\frac{1 / \beta-1}{R-1}\right) R\left[b\left(F_{0}\right)-\frac{k w_{0}}{r}\right]+k w_{0} \frac{R}{r} \frac{F_{0}}{\bar{F}} .
$$

By substracting $s_{0}$, from (9), and with $\bar{\beta}=1 / R$, we get (12). Note that $b\left(F_{0}\right)>\frac{k w_{0}}{r}=b^{z}$, as can be proven from (5).

## Corollary 3.

(i) If $S$ cannot borrow privately, $R E D D+$ implements (6).
(ii) If $S$ can borrow privately, $R E D D+$ is strictly more expensive than (7), and than a debt contract, iff $\beta<\bar{\beta}$.

The explaination for (ii) is that with REDD+, P must satisfy additional incentive constraints: Not only must S be discouraged from logging + defaulting, but also from logging without defaulting, and from defaulting without logging.

If $\beta=\bar{\beta}$, S does not benefit from private borrowing and the possibility to borrow privately is irrelevant.

## Conclusions

This analysis has illustrated that traditional REDD+ flow payments are suboptimal when the receipient government rotates being in office. In that situation, the government's discount factor $\beta$ is smaller and it seeks to tie the hands of future governments. A donor can exploit this time inconsistency and achieve "conservation by lending". The smaller is $\beta$, the larger are the gains from conservation by lending relative to conservation by REDD+ (i.e., a flow of conservation payments).

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[^0]:    ${ }^{1}$ If $e \in\left([1-k][(1-\beta) l+a], e^{B}\right)$, S logs when S borrows but not after defaults. This situation can be considered later.

