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by Christina E. Bannier

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Abstract

This paper studies the effects that heterogeneous multiple bank financing has on a firm’s risk- and information-policy, particularly with respect to credit renegotiation efficiency. We find that a significant, yet limited, degree of relationship lending enables firms with high asset specificity to credibly signal their desire to abstain from strategic default. This allows the firm’s policy to eliminate the risk of inefficient liquidation even in the case of bleak cash-flow expectations. This “hold-up benefit” comes at a cost, though: firms with low asset specificity cannot always eliminate the risk of coordination failure by their banks. (JEL: D82, G21, L14)

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Multiple bank financing is a widespread phenomenon, both among small and large firms. Most often, firms do not obtain equal financing shares from the banks but tend to borrow more from one relationship lender and smaller amounts from multiple arm’s-length lenders (Ongena et al. [2000, 2008]). Among the benefits of relationship lending, more efficient credit decisions for borrowers facing financial distress are typically emphasized (Sharpe [1990]). Yet, the hold-up costs associated with a relationship bank’s informational advantage and the ensuing rent extraction may be sufficiently severe to make additional borrowing from several, more distant “transactional” lenders attractive (Detragiache et al. [2000]). By complicating the refinancing process, multiple arm’s-length lending moreover hardens the firm’s budget constraints and reduces entrepreneurial incentives to default strategically (Dewatripont and Maskin [1995]). At the same time, however, it suffers from the problem that financially distressed but fundamentally solvent firms may be forced into inefficient default due to a coordination failure among arm’s-length banks (Morris and Shin [2004]).

Recent work on the subject of heterogeneous multiple bank financing has focussed mainly on the firm’s choice of lending structure and on the effects of lenders’ refinancing decisions on efficiency (e.g., Elsas et al. [2004]; Schuele and Stadler [2005]). However, these papers mostly do not analyze the debtor firm’s investment strategy and treat several parameters in the bank-firm relationships as exogenous. In particular, the firm’s risk-taking and choice of transparency vis-à-vis its lenders have hardly been taken into account explicitly. The current paper tries to fill this gap and emphasizes exactly these strategic policy decisions by the debtor firm. The paper hence contributes to the literature by (i) deriving a firm’s optimal choice of transparency and business risk within a setting of asymmetric bank financing and by (ii) studying how the optimal policy and financing regime interact.

In our model, the firm’s policy choice balances the relationship bank’s potential to coordinate arm’s-length banks’ refinancing decisions against the hold-up costs that arise
from relationship lending, in order to reduce the incidence of inefficient firm liquidation. The optimal policy parameters crucially depend on the firm’s liquidation value and its cash-flow expectations, as these variables determine the strategic incentives of banks and firm: the higher the liquidation value - for instance due to low asset specificity - the more attractive it is for banks not to roll over credit, and the lower the expected cash-flows, the higher is the firm’s incentive to default strategically.

As one of the key insights, the model shows that heterogeneous multiple bank financing enables firms to choose a multi-faceted policy mix. While the degree of heterogeneity does not impact the firm’s optimal policy in the most favorable and the most unfavorable circumstances for efficient project continuation, it is decisive for the intermediate cases. Particularly firms with *highly-specific assets* may fully eliminate the risk of inefficient liquidation, provided that the degree of relationship lending is limited. If this is the case, the hold-up concern turns into a benefit, as it allows the firm to credibly signal to abstain from strategic default even for low cash-flow expectations: refinancing relationship credit cannot become too costly, while the risk of coordination failure among arm’s-length banks is nevertheless bounded. Still, this beneficial effect comes at a cost: firms with *lowly-specific assets* suffer from a higher risk of inefficient liquidation unless expected cash-flows are extremely high.

This result may contribute to an explanation of why particularly small and medium-sized firms, which tend to be specialized in one or only a small number of production lines that typically employ highly specific assets,\(^1\) enjoy the services of several banks of which one - the so-called housebank - most often has a significant but not overwhelming status (GUISO AND MINETTI [2004, 2007]). For these firms, heterogeneous multiple bank financing reduces the incidence of inefficient liquidation and, consequently, increases the access to finance, a result that has been emphasized in several empirical studies (FOGLIA ET AL. [1998]; ELSAS AND KRAHNEN [1998]).

\(^{1}\)DIETSCHE AND PETEY (2004) show, for instance, that French and German small and medium-sized enterprises have a lower asset correlation than large businesses. LEHMANN ET AL. (2004) conclude that smaller companies tend to be involved in innovative and often regional businesses.
Overall, we can show that, except for the most unfavorable circumstances where firms optimally “gamble for resurrection”, a heterogeneous multiple bank financing regime clearly moderates firms’ risk appetite or even eliminates it completely. In this respect, asymmetric bank financing proves to support economic stability by reducing firms’ risk taking. At the same time, it induces firms to disclose very precise information to their main lender, which contributes to overall transparency as well.

The main contribution of this paper lies in uncovering the “flip-side” of the hold-up problem that arises once informed relationship bank financing coexists with less-informed arm’s-length bank financing. It delivers an instrument for the firm to commit to efficient continuation of the business project and to abstain from strategic default even for bleak business prospects. This is in contrast to Bolton and Scharfstein [1996], where a multiplicity of homogeneous lenders reduces the firm’s incentive to default strategically, while in our model the hold-up “benefit” necessarily requires a significant, albeit limited degree of relationship banking stemming from the coexistence of relationship and arm’s-length banking.

The remainder of the paper is organized as follows. Section 2 gives a brief overview on related literature. Section 3 delineates the model of heterogeneous multiple bank financing. Section 4 derives the equilibrium structure, while the subsequent section analyzes optimal risk-taking and information disclosure for a firm that aims at reducing inefficient project liquidation. Section 6 studies the interaction of optimal policy choice and financing regime and section 7 concludes.

2 Related literature

Seminal papers on bank financing focussed on the differences between single relationship lending on the one hand and multiple arm’s-length lending on the other. According to Fischer [1990], Boot [2000], and Elsas [2005], relationship banking is characterized by long-term relations between bank and customer, a large proportion of total firm debt held by the relationship bank, and preferred access to firm-specific information through
multiple interactions. As the ongoing sharing of information between borrower and lending bank strengthens the relationship, commitment between the two parties allows for intertemporal transfers: the bank may reasonably expect to earn high rents in the future, so that she may be willing to accept low profits (or even losses by forgiving debt, reducing interest rates etc.) in financial distress situations (Petersen and Rajan [1995]; Allen and Gale [1999]). However, due to the information monopoly of the relationship lender, a hold-up problem may arise: a borrower’s trial to raise debt capital from alternative sources tends to be interpreted as a negative signal about his creditworthiness by these outside lenders and is, hence, accompanied by a high risk premium. Anticipating this effect, the incumbent relationship lender may extract a substantial monopoly rent (Sharpe [1990]; Rajan [1992]). In our model, the relationship bank’s potential to hold-up a borrower becomes manifest in the refinancing costs that jump up once relationship debt is not prolonged.

Multiple arm’s-length lending avoids the risk of capture by a single housebank (Von Thadden [1992]). It also helps to mitigate managerial incentive problems by making refinancing decisions more complicated. In this context, Dewatripont and Maskin [1995] argue that a relationship bank may not be able to credibly commit to stop refinancing unprofitable projects and thus reduces entrepreneurial incentives to prevent default by softening the firm’s budget constraints. Bolton and Scharfstein [1996], in contrast, show how complicated credit renegotiations among several creditors may limit debtor firms’ incentives to default strategically. While, according to Detra-

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2 As an alternative in order to reduce the hold-up problem from relationship bank financing, rather than changing the financing system, Mahrt-Smith (2006) also suggests to let the bank hold a small equity stake in the firm. This reduces the bank’s ability to extract rents in multiple rounds of financing and has been common in the German banking system, particularly in the 1970s and 1980s.

3 For an overview on the theory of soft budget constraints, see Kornai et al. [2003].

4 Strategic default refers to a debtor’s incentive not to repay the full amount of credit in order to force lenders to forgive (part of the) debt (Bolton and Scharfstein [1990]; Mella-Barral [1999]). In contrast to Bolton and Scharfstein [1996], Bergloef et al. [2008] prove that imperfect renegotiation may also lead to increasing incentives to default strategically along with a larger number of creditors.
GIACHE ET AL. [2000] multiple arm’s-length bank financing may moreover help to overcome liquidity shortages of a single lender and hence reduces the probability of an early liquidation of the debtor firm, coordination failures among several arm’s-length banks may lead to the opposite effect (MORRIS AND SHIN [2004]). In this respect, fear of premature credit foreclosure by other lenders may lead to pre-emptive actions by individual lenders that undermine the sustainability of the debtor firm. Similarly to the coordination problem among bank depositors in DIAMOND AND DYBVIG [1983] and GOLDSTEIN AND PAUZNER [2005], MORRIS AND SHIN [2004] show that this pre-emptive action may force inherently solvent but illiquid companies into an inefficient default. Our paper stresses both coordination effects and the mitigation of adverse managerial incentives as the key features of multiple arm’s-length banking.

Recent empirical studies show that the combination of single relationship banking with multiple arm’s-length banking is frequently observed in reality, both among small and large companies. According to HOUSTON AND JAMES [1996], more than 60 percent of listed firms in the U.S. have multiple heterogeneous bank relationships. GUISO AND MINETTI [2007] conclude from the 1993 and 1998 U.S. National Survey of Small Business Finance that among firms with two lenders, one bank provides approximately 77 per cent of total credit, for firms with three lenders, the largest financing share is 65 per cent. Concentrated borrowing is also prevalent in other countries with strong firm-bank relationships such as Germany, Japan or Italy.5

Heterogeneous bank financing is also found in a slightly different context in the form of loan syndication, i.e. a loan issued to a firm jointly by at least two financial institutions. In a sample of syndicated loans to U.S. nonfinancial firms between 1992 and

5ONGENA AND SMITH [2000] conclude from a sample of 1129 large firms from 20 European countries that less than 15% of the firms maintain single-bank relationships while more than 20% use eight banks or more. MACHAVER AND WEBER [2001] and ELSAS [2005] show that for German firms the number of bank relationships increases in firm size but decreases with the existence of a housebank. For medium-sized German firms, BRUNNER AND KRAHNEN [2008] find an average of 6 (heterogeneous) bank relationships.
2003, Sufi [2007] finds an average of 8.1 syndicate lenders. Usually, one lead arranger establishes the relationship with the borrower, collects participant lenders to fund part of the loan and monitors the firm over the life of the loan. Comparable to a relationship bank, the lead arranger typically obtains confidential information about the borrowing firm and holds a larger share of the loan than the participant lenders. Interestingly, the particular syndicate structure (i.e. degree of concentration of lenders, amount of loan syndicated etc.) is significantly correlated with the transparency of the borrowing firm (Dennis and Mullineaux [2000]; Lee and Mullineaux [2004]; Sufi [2007]), the firm’s default probability and liquidation value (Jones et al. [2005]). While not catered to this particular instance of heterogeneous multiple bank financing, our results may - at least partly - be applied to this framework as well.

Recent theoretical work on heterogeneous multiple bank financing by Hubert and Schaefer [2002], Elsas et al. [2004], Janda [2006], Egli et al. [2006] and Bannier [2007] has mainly focussed on the optimal structure (or concentration) of bank debt. Hubert and Schaefer [2002] contrast renegotiation efficiency in a single relationship lending regime with the efficiency obtained from multiple arm’s-length banking. They find that the negative effects from coordination failure among transaction banks may exceed the hold-up costs from relationship banking if information asymmetries between debtor firm and banks are not too strong. Elsas et al. [2004] derive the optimal debt structure directly from the trade-off between the bargaining power of the relationship bank and the risk of coordination failure from arm’s-length banks. They find that firms with high expected cash-flows prefer a homogeneous multiple bank financing regime without a relationship bank, while firms with low expected profits or high asset specificity tend to be financed within a heterogeneous multiple banking system. Janda [2006] derives heterogeneous multiple bank financing as the optimal financing regime that leads to renegotiation proofness in a costly state verification framework. Egli et al. [2006] show that the choice of financing regime is dependent on the likelihood of strategic default: in an environment where strategic defaults are likely, firms choose relationship banking over arm’s-length lending. Bannier
[2007] complements the earlier work by Hubert and Schaefer [2002] by comparing the efficiency effects of a heterogeneous multiple banking regime to the isolated regimes of single relationship banking and multiple arm’s-length lending, respectively.

In-depth analyses of refinancing efficiency with respect to agents’ characteristics have recently sprung up from the theory of global games, introduced by Carlsson and van Damme [1993] and generalized by Morris and Shin [2003]. Bannier [2005] analyzes the effects that large and small speculators’ short-selling decisions may have on the likelihood of currency crises. Focussing on credit renegotiation, Takeda [2003] and Schuele and Stadler [2005] examine how the amount of relationship lending and the housebank’s information advantage vis-à-vis the arm’s-length banks may help to reduce the incidence of coordination failure. Schuele [2007] analyzes the relationship bank’s signalling ability and its effects on the debtor firm’s soft budget constraints.

A firm’s choice of risk- and information-policy as the consequence of a particular financing regime has rarely been analyzed before. As one of the first studies, Heinemann and Metz (2002) examined the optimal policy-mix for a firm that aims at minimizing the probability of a liquidity crisis following early withdrawal of credit by a continuum of homogeneous lenders. Bannier and Heinemann [2005] analyze a similar question with respect to a central bank’s policy choice in order to avoid currency crises. Due to the simple frameworks, both papers arrive at clear-cut recommendations regarding the optimal policy choice. The current paper extends this earlier work in two ways. First, it assumes a richer (asymmetric) structure of creditor types, thereby allowing for various strategic incentives to affect the policy choice. Second, whereas the paper by Heinemann and Metz (2002) was limited to firms with high liquidation value, the current study considers both firms with high and low liquidation value. In contrast to the papers by Bester (1985), Besanko and Thakor (1987) or Chen (2006), however, the liquidation value is no strategic choice variable to the firms. Rather, this paper is related to the work by Diamond (1991) where the effect of debt structures on the efficiency of liquidation decisions is analyzed.
3 The model

We consider a simple model where a firm is financed by a relationship bank and a continuum of arm’s-length banks. The firm runs a project with stochastic return \( \theta \) that matures within two time periods. The firm has promised to repay the banks an amount of \( r(>0) \) per unit of credit if the project succeeds in the second period. If it fails, the firm will go into bankruptcy and repayment will be zero. However, lenders may withdraw their loans after the first period, so that the project is threatened by early liquidation. Premature withdrawal yields a liquidation value of \( K(<r) \) per unit of credit. All agents are assumed to be risk-neutral.

The bank financing system is heterogeneous in three respects. First, the relationship bank is a large lender, i.e. she finances a proportion \( \lambda \in [0,1] \) of the total amount of debt, while the arm’s-length banks are small lenders in that their individual investment is negligible. The combined mass of loans provided by the small banks amounts to \( (1-\lambda) \), however. Parameter \( \lambda \) may hence be seen as characterizing the degree of heterogeneity of the financing regime.

Second, it is assumed that the relationship bank has been having long-term relations with the firm, so that she receives more precise information about the project than any of the arm’s-length banks. More specifically, it is assumed that the relationship bank obtains a private signal, \( x_R \), about project quality \( \theta \), with \( x_R|\theta \sim N(\theta, \frac{1}{c}) \). Small banks observe individual private signals \( x_S|\theta \sim N(\theta, \frac{1}{b}) \) with \( b \leq c \). Hence, any arm’s-length banks’ private information is at most as precise as the relationship bank’s signal. Noise in private signals is mutually independent and independent of \( \theta \). The distributions of

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6The assumption of a continuum of arm’s-length banks is made for simplicity. It can be shown that considering a finite number of banks instead does not qualitatively impair the results. See also Morris and Shin [2003].

7For the extreme cases of \( \lambda = 1 \) and \( \lambda = 0 \), the model considers single relationship banking and homogeneous multiple banking, respectively.

8As will be explained below, \( b \) is an exogenous parameter in the model whereas \( c \) is a choice variable for the firm.
private signals are common knowledge.

Third, due to the informational advantage of the relationship bank it is assumed that it is more costly for the firm to refinance credit withdrawn by the relationship bank (at costs of $W_R$ per unit of capital) than to refinance credit withdrawn by any of the arm’s-length banks (at costs of $W_S$), i.e. $W_R > W_S$. This mirrors the typical hold-up problem, indicating that, over the course of the relationship, it gets ever more difficult for the debtor to set up a new financing with an outside lender. However, refinancing informed capital must not become inefficiently costly; therefore, we assume that $W_R \leq 1 + r$. Altogether, it holds that $0 \leq r < W_S < W_R \leq 1 + r$.

The complete sequence of moves in this game is as follows:

1. In $t = 0$, the firm sets up the project. It commits to providing the relationship bank with information of precision $c$ and chooses a level of operating risk that leads to a variance of project cash-flow of $1/a$ in order to maximize its expected payoff. The chosen policy parameters become common knowledge for the banks.

2. In $t = 0$, nature chooses project quality $\theta$ from the commonly known distribution $\theta \sim N(y, 1/a)$. The realized value of $\theta$ is observed by the firm’s managers but remains unobservable to bank lenders. Nature disseminates signals $x_R$ and $x_S$ about project quality to the banks.

3. In $t = 1$, banks decide whether to extend or withdraw their loans. Simultaneously, the firm has to choose whether to commit to additional effort $V$ that is necessary for successful completion of the project in $t = 2$, or to terminate the project altogether. The decision to undertake additional effort is tied to refinancing the withdrawn fraction of debt.

4. In $t = 2$, project cash-flow is realized and equals $\theta$ if the firm did sink effort and refinance. Banks then receive their repayment of $r$. Otherwise, the project fails and credit cannot be repaid. The final liquidation value of assets is assumed to be zero.
Note that in $t = 1$ banks and firm decide on their actions simultaneously. This implies that the firm does not wait to observe the actual amount of financial disruption, but makes its decision based on the anticipated amount of capital withdrawn. Such preemptive behavior may be reasonable for firms whose main business is at stake and whose managers fear a depreciation of human capital should they be forced into default after all. It may also be argued that closing down a firm when its capital is still positive is less costly than closing down when it is bankrupt (Gilson et al. [1990]; Egli et al. [2006]).\(^9\) Section 4 will show that this assumption allows a straightforward derivation of equilibrium. Fig. 1 portrays the sequence of moves in the model.

4 Equilibrium in the refinancing stage

Before deriving the optimal firm policy in $t = 0$, we have to analyze the equilibrium behavior of firm and banks in $t = 1$, that will - in turn - be influenced by the firm’s policy choice. In order to do so, we follow the solution method by Morris and Shin [2003, 2004] and Bannier [2005] for global games.\(^10\) As has been shown in these earlier studies, a unique (perfect Bayesian) equilibrium in trigger strategies may be derived, provided that private information about the uncertain payoff (parameter $\theta$ in our case) is sufficiently precise. The equilibrium is characterized by the triple $(x_R^*, x_S^*, \theta^*)$. The relationship bank will roll over credit whenever her private signal about project quality, $x_R$, is greater than the switching point $x_R^*$. Likewise, any small bank will prolong credit if it observes a private signal $x_S$ larger than $x_S^*$. Correspondingly, the firm will decide to continue the project if the realized project quality, $\theta$, is greater than $\theta^*$ and will default early otherwise. In the following, we derive the conditions that jointly determine the

\(^9\)According to Gilson et al. [1990] only few companies in financial distress wait for a default to happen due to illiquidity. Egli et al. [2006] relate this pre-emptive behavior to accounting standards, the quality of law enforcement and legal protections for creditors.

\(^10\)In a global game, each player noisily observes the game’s payoff structure, i.e. cash-flow parameter $\theta$, which itself is determined by a random draw from a given class of games, in our case from a given distribution $N(y, 1/a)$. 

INVESTMENT AND POLICY CHOICE:

t = 0

firm chooses

- transparency vis-à-vis relationship bank
- operating risk

INFORMATION STAGE:

t = 0+

nature realizes project quality \( \theta \)

and disseminates private signals to banks

REFINANCING STAGE:

t = 1

- banks choose: prolong / withdraw
- firm chooses: strategic default / continue

REALIZATION STAGE:

t = 2

- cash-flow \( \theta \) and repayment \( r \) if project continued
- 0 otherwise

Figure 1: Sequence of actions

switching points \( x^*_R, x^*_S \) and \( \theta^* \).

Having received private information \( x_R \), the relationship bank updates her prior beliefs. Given the bivariate normal distribution of \( \theta \) and \( x_R \), her posterior belief about project quality is characterized by an expected value\(^{11} \) of \( E(\theta|x_R) = a/(a+c)y + c/(a+c)x_R \), which is the precision-weighted average of the face value of public information \( y \) and her private signal \( x_R \), and a variance of \( \text{Var}(\theta|x_R) = 1/(a+c) \). Her expected payoff

\(^{11}\text{When } \theta \text{ and } x \text{ are bivariate normal, the conditional expectation is obtained as } E(\theta|x) = E(\theta) + \frac{\text{Cov}(\theta,x)}{\text{Var}(x)}(x - E(x)).\)
from extending the loan is therefore given by

\[
\pi_R(\text{extend}|x_R) = r \, \text{prob}(\theta > \theta^*|x_R) = r \left(1 - \Phi\left(\sqrt{a + c}(\theta^* - \frac{a}{a + c} y - \frac{c}{a + c} x_R)\right)\right),
\]

where \(\Phi(\cdot)\) denotes the cumulative standard normal distribution. Consequently, the critical signal \(x_R^*\) is implicitly defined by the large bank’s cutoff condition

\[
\pi_R(\text{withdraw}|x_R) = \pi_R(\text{extend}|x_R)
K = r \left(1 - \Phi\left(\sqrt{a + c}(\theta^* - \frac{a}{a + c} y - \frac{c}{a + c} x_R)\right)\right),
\]

i.e. by indifference between extending and foreclosing the loan.

Likewise, indifference for any small bank is given at\(^{12}\)

\[
\pi_S(\text{withdraw}|x_S) = \pi_S(\text{extend}|x_S)
K = r \, \text{prob}(\theta \geq \theta^*|x_S)
K' = r \left(1 - \Phi\left(\sqrt{a + b}(\theta^* - \frac{a}{a + b} y - \frac{b}{a + b} x_S)\right)\right),
\]

so that \(x_S^*\) may implicitly be derived.

Having observed the realized project quality \(\theta\), the cutoff condition for the firm - deciding between continuing the project and terminating it early - is given by\(^{13}\)

\[
\pi_F(\text{effort and refinance}|\theta) = \pi_F(\text{terminate}|\theta)

\theta - V - \lambda r \, \text{prob}(x_R \geq x_R^*|\theta) - (1 - \lambda) r \, \text{prob}(x_S \geq x_S^*|\theta)
- \lambda W_R \, \text{prob}(x_R < x_R^*|\theta) - (1 - \lambda) W_S \, \text{prob}(x_S < x_S^*|\theta) = 0 .
\]

I.e., if the firm decides to continue the project, it will receive the project’s cash-flow net of effort costs, credit repayments and refinancing costs.

The following Lemma solves for the equilibrium switching values.

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\(^{12}\)Updating of beliefs follows the same routine as for the large bank and delivers the following conditional distribution of project quality: \(\theta|x_S \sim N(a/(a + b)y + b/(a + b)x_S, 1/(a + b))\).

\(^{13}\)Note that due to the assumed independence of signals, the proportion of small banks withdrawing their money is equivalent to the probability with which any single small bank observes a private signal lower than \(x_S^*\).
Lemma 1 The equilibrium in the refinancing stage is given by the triple \((x_R^*, x_S^*, \theta^*)\) with

\[ x_R^* = \frac{a + c}{c} \theta^* - \frac{a}{c} y + \frac{\sqrt{a + c}}{c} \Phi^{-1} \left( \frac{K}{r} \right), \]

\[ x_S^* = \frac{a + b}{b} \theta^* - \frac{a}{b} y + \frac{\sqrt{a + b}}{b} \Phi^{-1} \left( \frac{K}{r} \right), \]

\[ \theta^* = V + r + \lambda(W_R - r) \Phi \left( \frac{a}{\sqrt{c}} (\theta^* - y) + \sqrt{\frac{a + c}{c}} \Phi^{-1} \left( \frac{K}{r} \right) \right) \]

\[ + (1 - \lambda)(W_S - r) \Phi \left( \frac{a}{\sqrt{b}} (\theta^* - y) + \sqrt{\frac{a + b}{b}} \Phi^{-1} \left( \frac{K}{r} \right) \right). \]

The equilibrium is unique provided that private information is sufficiently precise relative to public information about \(\theta\).

Appendix A will outline the proof.

Note that with common knowledge (i.e. fully precise public information) about \(\theta\), multiple equilibria would be obtained for \(V + r < \theta < V + \lambda W_R + (1 - \lambda) W_S\). Why? Assume that project cash-flows were extremely low: \(\theta < V + r\). In this case, the firm’s expected payoff from continuing the project would be negative even in the most favorable case where all banks decided to prolong credit and no refinancing were necessary, so that the costs from continuing the project were as low as \(V + r\). Therefore, the firm would certainly terminate the project, irrespective of the banks’ actions. For very high project cash-flows, \(\theta > V + \lambda W_R + (1 - \lambda) W_S\), in contrast, the expected payoff from continuing would be positive, even in the most unfavorable case where all banks decided to withdraw early, resulting in extremely high continuation costs of \(V + \lambda W_R + (1 - \lambda) W_S\). Hence, the firm would certainly continue, again irrespective of the banks’ actions. In-between these two threshold values, however, self-fulfilling expectations among banks may lead to both continuation and termination being optimal.\(^\text{14}\)

\(^\text{14}\)Strategic complementarities in this set-up make one action the more attractive the higher the proportion of other banks that choose the same action.
withdraw their money it is very likely that the firm - anticipating high refinancing costs - will terminate the project, which makes it optimal not to prolong credit in the first place, thereby vindicating the initial belief.

If project cash-flow $\theta$ is not common knowledge, but instead banks observe private signals about $\theta$, a “grain of doubt” is put into this deliberation that eliminates the self-fulfilling feature of agents’ beliefs. $^{15}$ Hence, a unique equilibrium may be obtained that assigns a uniquely optimal action for the banks to any signal value $x$ and for the firm to any project quality $\theta$. This, in turn, allows to study in which way the firm’s policy will influence behavior and equilibrium outcome by conducting comparative static analyses.

Throughout the rest of the paper, we will always assume that the banks’ private information is sufficiently precise so that a unique equilibrium exists. Furthermore, we will focus on the case of intermediate project qualities, i.e. we assume that $\theta \in [V + r, V + \lambda W_R + (1 - \lambda) W_S]$, as this is the interesting interval to study: if the project quality were below (above) this interval, the project would never (always) be continued, irrespective of the firm’s policy. Note that within this interval, equilibrium may still be inefficient: the firm terminates the project whenever a project quality lower than $\theta^*$ is realized; however, for all $\theta \in [V + r, \theta^*]$, terminating the project is inefficient since the firm would be able to continue if only the expected proportion of debt withdrawn prematurely were lower, a problem denoted as “coordination failure”. As a consequence, the firm’s objective of maximizing its expected profit by choosing appropriate policy parameters coincides with minimizing the probability of inefficient project liquidation: a positive net-payoff can only be achieved by the firm if the project is continued, which becomes more likely, the lower threshold $\theta^*$ can be pushed.

$^{15}$Uncertainty about the realized cash-flow value $\theta$ allows banks to assign probabilities (different from zero or one) to the proportion of banks who choose one action over the other and hence to calculate the expected profit from this action as a smooth function. If banks’ private signals are not too imprecise, this process enables them to determine the exact point of indifference at which they should optimally switch from one action to the other.
5 Optimal information disclosure and risk-taking

The firm’s optimal policy aims at maximizing its expected payoff by reducing the probability of inefficient project liquidation. The optimization problem may therefore be stated as follows

\[
\min_{a,c} \{ \text{prob}(\theta \leq \theta^*) = \Phi(\sqrt{a}(\theta^* - y)) \} \quad \text{s.t. equilibrium uniqueness},
\]

with \( \theta^* \) given by (3). Note that the firm’s choice of parameters is restricted to ensure uniqueness of equilibrium, i.e. there is a lower bound on the precision of information to be disclosed to the relationship bank: \( c \geq c_{\min}. \)\(^{16} \)

While the optimal policy should induce banks to choose the efficient action and prolong credit, the banks succumb to various types of incentives to deviate from efficiency. We will briefly delineate these strategic deliberations in the following before deriving the optimal policy parameters. First, note that it is the more attractive for banks to withdraw their money early, the higher the liquidation value \( K \) is. Second, the incentive not to prolong credit is also influenced by the banks’ anticipation of the firm’s behavior. If the firm decides on a strategic default and terminates the project prematurely, each bank would rather withdraw credit, which delivers a payment of \( K \) as compared to the case of a zero payoff if credit were prolonged. In contrast, if the firm decides on project continuation, each bank would like to extend credit in order to obtain the repayment of \( r (> K) \) after successful completion of the project rather than foreclose early.

Yet, the firm’s behavior cannot be observed at the time banks have to make their decisions. Still, they know that the firm is more likely to terminate the project prema-

\(^{16}\)Based on the derivation in appendix A, the minimal precision values are given as \( \sqrt{c_{\min}} = \frac{\lambda(W_R - r)a}{\sqrt{2\pi(1 - \lambda)(W_S - r)}} \) and \( \sqrt{b_{\min}} = \frac{(1 - \lambda)(W_S - r)a}{\sqrt{2\pi(1 - \lambda)(W_R - r)}} \). The mutual dependence of the two minimum precision values mirrors the fact that each bank type’s information precision may make up for a lack in precision of the other type’s information in order to insure uniqueness of equilibrium. Without restricting our results, we may assume in the following that arm’s-length banks’ private information takes on a fixed precision given by: \( \sqrt{b} = \frac{(1 - \lambda)(W_S - r)a}{\sqrt{2\pi(1 - \lambda)(W_R - r)}} \). It then follows that \( c_{\min} = \frac{a^2}{2\pi} \).
turely the higher the costs from refinancing the withdrawn parts of credit are relative to the realizable project cash-flow in $t = 2$. Unfortunately, both refinancing costs and realized project quality $\theta$ are not known by the banks and give rise to two different types of uncertainty. “Fundamental uncertainty” arises because banks cannot observe the realized project quality. As, furthermore, banks cannot observe each others actions, there is also “strategic uncertainty” about the resulting refinancing costs for the firm. The firm’s policy now determines both types of uncertainty. The higher the chosen business risk, the larger is the project quality’s variance and the more difficult is it for banks to assess the unknown value of $\theta$. Via the precision of private information disclosed to the relationship bank, the firm influences strategic uncertainty among lenders. The more precise the relationship bank’s private information becomes, the less weight will the large bank attach to the common prior $y$ in calculating the posterior expected value of the project. This makes her action less predictable for the small banks and increases strategic uncertainty unless business risk were so small that the realized project quality $\theta$ were very close to its ex-ante expected value $y$. In this case, the relationship bank’s private signal will also be very narrowly distributed around $y$, which reduces strategic uncertainty.

Note that the relationship bank plays a prominent role in arm’s-length banks’ deliberations about their optimal behavior. First, since it is more expensive for the firm to refinance credit withdrawn by the relationship bank, this bank has a much stronger influence on the firm’s decision of whether or not to terminate the project early, i.e. default strategically. Because, second, the relationship bank moves a significant proportion of total firm debt by her decisions, it is very important for arm’s-length banks to anticipate her behavior as precisely as possible. On the other hand, the relationship bank’s decision can be directly influenced by the firm via the disclosure of information. Arm’s-length banks behavior, in contrast, cannot be directly influenced. This latter effect is strengthened by the fact that arm’s-length banks’ strategic choices succumb to aggregate uncertainty, while the relationship bank moves a fixed part of total firm debt by her decision.
In the following, we will derive the optimal precision of information to be disclosed to the relationship bank first. Then, based on the results for optimal information policy, we will in a second step establish the optimal business risk to be chosen.

5.1 Optimal information disclosure

The directional impact of information precision \( c \) on the probability of inefficient project liquidation depends solely on its effect on \( \theta^* \), since

\[
\frac{\partial \text{prob}(\theta \leq \theta^*)}{\partial c} = \varphi(\sqrt{a(\theta^* - y)}) \sqrt{a} \frac{\partial \theta^*}{\partial c},
\]

where \( \varphi(\cdot) \) denotes the standard normal distribution function.

\[
\frac{\partial \theta^*}{\partial c} > 0 \text{ whenever}^{17}
\]

\[
\theta^* < y - \frac{1}{\sqrt{a + c}} \Phi^{-1}\left(\frac{K}{r}\right) \equiv M(\cdot).
\]

For \( \theta^* \) larger than threshold function \( M(\cdot) \), in contrast, trigger value \( \theta^* \) decreases in \( c \). For infinitely precise information, i.e. \( c \to \infty \), threshold function \( M(\cdot) \) converges to \( y \). Convergence is from above for \( K < 1/2r \) and from below for \( K > 1/2r \). Equilibrium value \( \theta^* \), in contrast, converges to

\[
\theta^*(c \to \infty) = V + r + \lambda(W_R - r)\frac{K}{r} + (1 - \lambda)(W_S - r)\Phi\left(\frac{a}{\sqrt{b}(\theta^* - y) + \sqrt{a + b}} \Phi^{-1}\left(\frac{K}{r}\right)\right) \equiv \theta^c.
\]

In finding the optimal policy parameter \( c^* \), we therefore have to differentiate between the cases of a low early liquidation value, \( K < 1/2r \), and a high early liquidation value, \( K > 1/2r \), with the two subcases of low expected cash-flow, \( y < \theta^c \), and high expected cash-flow, \( y > \theta^c \). A detailed derivation of results will be given in appendix C.

The following proposition sums up the results with regard to optimal information disclosure to the relationship bank:

**Proposition 1** For given riskiness \( 1/a \), optimal information disclosure requires providing the relationship bank with information of precision as given in Tab. 1. Here, \( c^{\text{min}} = \frac{a^2}{2\pi} \).

\(^{17}\)Appendix B will derive this result.
Table 1: Optimal information precision, $c^*$

<table>
<thead>
<tr>
<th></th>
<th>$K &gt; 1/2r$</th>
<th>$K &lt; 1/2r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low expected</td>
<td>$c^* \rightarrow \infty$</td>
<td>$c^* = c_{\text{min}}$ for $a \leq \bar{a}$</td>
</tr>
<tr>
<td>cash-flow $y$</td>
<td></td>
<td>$c^* = \infty$ for $a &gt; \bar{a}$</td>
</tr>
<tr>
<td>high expected</td>
<td>$c^* = c_{\text{min}}$ for $\hat{c} &lt; c_{\text{min}}$</td>
<td>$c^* = c_{\text{min}}$</td>
</tr>
<tr>
<td>cash-flow $y$</td>
<td>$c^* = \hat{c}$ for $\hat{c} \geq c_{\text{min}}$</td>
<td></td>
</tr>
</tbody>
</table>

$\hat{c}$ is implicitly defined by $\theta^*(\hat{c}) = y - 1/\sqrt{a + \hat{c}} \Phi^{-1}(K/r)$ and $\bar{a}$ by equality of $\theta^*(c_{\text{min}})$ and $\theta^e$.

For projects with high expected cash-flows, the firm optimally provides the relationship bank with information of relatively low precision. This reduces the weight that the relationship bank attaches to her private signal in her posterior belief about project quality $\theta$. Hence, the bank will rely more on publicly available information, i.e. the ex-ante expected cash-flow $y$, so that her posterior expectation about $\theta$ remains high. At the same time, strategic uncertainty’ among banks is reduced because each arm’s-length bank knows that the relationship bank will place more weight on the common part $y$ in calculating posterior beliefs. For low expected cash-flow, in contrast, the firm tends to disclose more precise information, so that the relationship bank is induced to neglect the bad ex-ante expected value $y$. However, if business risk $(1/a)$ is relatively high, fundamental uncertainty about project quality is already sufficiently high, so that the need to further distract the bank from a low value of $y$ is not that urgent and disclosing information of minimal precision is adequate to this end.

Note that in the upper left cell in Tab. 1 (high $K$ and low $y$), banks experience the highest incentive to foreclose their loans early. This is due to the fact that they expect a relatively high payment of $K$ (while still lower than $r$) in this case and anticipate that the firm will tolerate only relatively low refinancing costs before opting for a strategic default. In the lower right cell (low $K$ and high $y$) the opposite holds. Here, it is most attractive for banks to prolong credit. Clearly, the latter case is the most favorable.
for the desired continuation of the project for the firm. Interestingly, both cases are characterized by clear-cut results concerning the choice of optimal information precision. In the two remaining cases, in contrast, optimal information disclosure is dependent on the chosen business risk. The next section will therefore examine the optimal degree of riskiness $1/a$ that the firm should choose for its project, given that it has already decided on the optimal precision of information to be disclosed to the relationship bank.

5.2 Optimal risk-taking

In contrast to information precision $c$, risk parameter $a$ influences the probability of inefficient project liquidation in two ways, as can be seen from the term in brackets in the following derivative

$$
\frac{\partial \text{prob}(\theta \leq \theta^*)}{\partial a} = \varphi(\sqrt{a}(\theta^* - y)) \left[ \frac{1}{2\sqrt{a}}(\theta^* - y) + \sqrt{a} \frac{\partial \theta^*}{\partial a} \right].
$$

In order to minimize the probability of inefficient project liquidation, the firm not only has to be concerned with the impact of $a$ on $\theta^*$, but also with the difference between $\theta^*$ and the expected cash-flow $y$.\footnote{In the following, the “double star” ($a^{**}$) indicates that this value of $a$ takes account of both aspects and hence minimizes the overall probability of inefficient project liquidation.} As has already been stated above, the derivation of the firm’s optimal risk policy is based on the optimal information disclosure results. We therefore again have to consider different cases regarding the value of $K$ and the expected cash-flow $y$. A detailed derivation of results is given in appendix D.

The following proposition combines the results with respect to optimal risk-taking and information disclosure.

**Proposition 2** Optimal risk-taking and information disclosure depend on the ratio of the project’s liquidation value $K$ to repayment $r$ and on the expected cash-flow $y$. The optimal policy-mix is contingent on the degree of heterogeneity in bank financing, whenever exogenous circumstances are neither too favorable nor too unfavorable for successful completion of the firm’s project. A detailed account of results is presented in Tab. 2.
Table 2: Results regarding optimal information precision, $c^*$, and business risk, $1/a^{**}$

<table>
<thead>
<tr>
<th>Low expected cash-flow, $g &lt; \theta^c$</th>
<th>$K &gt; 1/2r$</th>
<th>$K &lt; 1/2r$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c^* \to \infty$, $a^{**} = 0$</td>
<td>$\lambda &gt; \bar{\lambda}$: $c^* = c^{\text{min}}$, $a^{**} = 0$ $\Rightarrow \text{prob}(\theta \leq \theta^*) = 1/2$</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow \text{prob}(\theta \leq \theta^*) = 1/2$</td>
<td>$\lambda \leq \bar{\lambda}$: $c^* = c^{\text{min}}$, $a^{**} \to \infty$ $\Rightarrow \text{prob}(\theta \leq \theta^*) = 0$</td>
</tr>
</tbody>
</table>

| High expected cash-flow, $g > \theta^c$ | $\lambda > \bar{\lambda}$: $c^* = c^{\text{min}}$, $a^{**} \to \infty$ $\Rightarrow \text{prob}(\theta \leq \theta^*) = 0$ |
|                                       | $\lambda \leq \bar{\lambda}$ and $g < \theta^*(c^{\text{min}}, a \to \infty)$: $c^* = \hat{c}$, $a^{**} = \hat{a}$ $\Rightarrow 0 < \text{prob}(\theta \leq \theta^*) \leq 1/2$ |
|                                       | $\lambda \leq \bar{\lambda}$ and $g > \theta^*(c^{\text{min}}, a \to \infty)$: $c^* = c^{\text{min}}$, $a^{**} \to \infty$ $\Rightarrow \text{prob}(\theta \leq \theta^*) = 0$ |

5.3 Interpretation of results

Starting with the most unfavorable case for efficient project continuation in the upper left cell (high $K$ and low $g$) in Tab. 2, we find that optimal firm policy maximizes both fundamental and strategic uncertainty by choosing maximum business risk and disclosing maximally precise information to the relationship bank. By doing so, the firm “gambles for resurrection” and is able to reduce the probability of inefficient project.
liquidation to a level of 1/2, which is the best that can be achieved in this case. Since banks have a high incentive to foreclose their credit early due to the high liquidation value \( K \) and to the low anticipated tolerance with regard to the firm’s refinancing costs, the best the firm can do is to create a maximum of uncertainty both with respect to the underlying project quality \( \theta \) and with respect to aggregate financial disruption. Note that optimal firm policy is completely independent of the structure of bank debt, i.e. of heterogeneity parameter \( \lambda \).

In the opposite case (lower right cell), where a firm with highly-specific assets (low liquidation value \( K \)) runs a project with high expected cash-flow, it faces only a low risk of coordination failure among banks. Banks have only a low ex-ante incentive to withdraw their money early. Additionally, they anticipate only a low propensity of the firm towards a strategic default. Optimal firm policy is then given by minimum business risk and a disclosure of minimally precise information to the relationship bank. The subsequent minimization of both fundamental and strategic uncertainty fully eliminates the remaining risk of inefficient credit withdrawal among banks and hence of inefficient project liquidation. Again, the degree of heterogeneity in the bank financing system is not decisive for optimal firm policy.

In the remaining two cases, in contrast, the degree of heterogeneity in bank debt plays a crucial role for optimal firm policy. If a firm with highly-specific assets conducts a project with low expected cash-flow (low \( K \) and \( y \), upper right cell), we find that it can reduce the probability of inefficient project liquidation to a level of only .5, if the degree of relationship banking is relatively high. In order to do so, the firm has to resort to a maximum of business risk and to disclosing minimally precise information to the relationship bank. Note that in contrast to the situation in the upper left cell, however, the banks’ ex-ante incentive to decide on premature credit withdrawal is less urgent, as the early liquidation payment \( K \) is much lower. Still, due to the low expected cash-flow \( y \), banks anticipate only a low threshold of refinancing costs to be tolerated by the firm before resorting to a strategic default.

If, however, relationship lending in this situation is bounded above by the amount
of arm’s-length financing, i.e. if $\lambda \leq \bar{\lambda}$, the optimal firm policy is able to fully eliminate the remaining coordination risk among banks and hence to reduce the probability of inefficient project liquidation to a value of zero. Why is this possible despite the low expected project cash-flow and why is it important that the degree of relationship banking is bounded above? Remember that it is more costly (per unit of capital) for the firm to refinance relationship credit than refinancing arm’s-length credit, i.e. $W_R > W_S$. Hence, whenever the degree of relationship bank financing is limited, the maximum level of refinancing costs is limited as well. It may therefore be worthwhile for the firm to continue the project despite the low expected project cash-flow. In any way, the firm can use the limited degree of relationship banking as a credible signal that it will abstain from a strategic default, despite the low value of $y$. This reduces the ex-ante risk of coordination failure among banks and allows the firm’s policy to eliminate it completely and, thus, to reduce the probability of inefficient project liquidation to a value of zero. All the firm has to do is to choose minimum business risk and to disclose maximally precise information to the relationship bank,\footnote{Note that information precision of $c_{\text{min}}$ combined with a risk parameter of $a \to \infty$ implies infinitely high precision: $c_{\text{min}} \to \infty$.} thereby minimizing fundamental and strategic uncertainty.

In the remaining case (lower left cell), the firm faces a relatively high ex-ante incentive of early foreclosure by her banks due to the high liquidation value $K$. With sound cash-flow expectations (high $y$), however, banks anticipate a high tolerance of the firm with regard to refinancing costs. This reduces their incentive to withdraw credit early. If in this instance, the firm makes use of a high degree of relationship banking ($\lambda > \bar{\lambda}$), this leaves a large amount of credit that can directly be influenced by the firm’s information policy and only a small amount of credit that succumbs to an uncontrolled risk of coordination failure (among arm’s-length banks). A high degree of relationship financing therefore allows to completely eliminate the remaining probability of inefficient project liquidation by choosing zero business risk and disclosing fully precise information to the relationship bank. By doing so, both fundamental and
strategic uncertainty are minimized.

Yet, if the degree of relationship bank financing is relatively low, a high remaining fraction of total firm debt succumbs to aggregate uncertainty among arm’s-length banks and hence to the risk of coordination failure that is less easily resolved by the firm’s optimal policy. In this case, the firm is able to fully eliminate the probability of inefficient project liquidation only if the expected project cash-flow is extremely high. If this condition is satisfied, arm’s-length banks will have a high incentive to prolong credit as they anticipate that the firm will tolerate almost any level of refinancing costs without resorting to a strategic default. Despite the high offered payment of $K$ in case of early withdrawal, banks will then rather target the full repayment of credit $r$ at the end of the project’s maturity and prolong credit. However, if expected cash-flow $y$ takes on only moderate values (without being classified as “low”, which would lead to the upper left cell), optimal firm policy is not able to fully eliminate the risk of coordination failure. By choosing intermediate values of business risk and information precision the firm can reduce the probability of inefficient project liquidation only to a level between 0 and .5.

6 Interaction of firm policy and financing regime

Heterogeneous multiple bank financing hence enables firms to choose a multi-faceted policy mix. While the degree of heterogeneity does not influence the firm’s optimal policy in the most favorable and the most unfavorable circumstances, it is decisive for the intermediate cases. In particular for firms with highly-specific assets, the bank financing system may help to fully eliminate the probability of inefficient project liquidation in a case where a low expected project cash-flow exposes the firm to a significant risk of coordination failure. However, this beneficial effect necessitates the degree of relationship bank financing to be limited above, which requires the existence of arm’s-length banks to fill this financing gap. In this case, the hold-up concern, that arises due to the firm’s inability to “replace” the relationship lender in a cost-efficient way, turns into
a hold-up benefit. It takes effect, as the fact that the relationship bank’s proportion of total firm debt does not succumb to aggregate uncertainty and can, moreover, be directly influenced by the firm’s policy, outweighs the higher refinancing costs from a foreclosure of credit by this lender.

Still, the hold-up benefit comes at a cost. Whenever the bank financing system is sufficiently heterogeneous, i.e. if the degree of relationship banking is limited, firms with lowly-specific assets and moderate expected project cash-flows face a relatively high probability of inefficient project liquidation unless the expected cash-flow becomes extremely high. For them, the limited degree of relationship lending does not allow the optimal firm policy to fully eliminate the remaining risk of a coordination failure. Comparing these results to the ones obtained in a model of homogeneous multiple bank financing (Heinemann and Metz, 2002) shows that firms with lowly-specific assets hardly benefit from using both relationship and arm’s-length lending. Rather, a homogeneous bank financing regime allows to fully eliminate the risk of inefficient liquidation for projects with high expected cash-flows. Firms with highly-specific assets, however, may be put at an advantage by using a heterogeneous bank financing system.

One further aspect may be raised with regard to the employment of heterogeneous multiple bank financing. It concerns the question who benefits from such a bank financing system. From the analysis above, we know that firms with highly-specific assets profit most, since for them the hold-up benefit is most relevant, while firms with low asset specificity may suffer from the remaining risk of coordination failure that cannot be fully eliminated by the optimal firm policy. Apart from the firms themselves, the involved banks profit as well due to an increased probability of credit repayment following the hold-up benefit. Taking an even broader perspective, we find that, eventually, the hold-up benefit allows firms to choose low business risk despite low expected project cash-flows, which would otherwise incite firms to maximize fundamental uncertainty by choosing high business risk. In this way, a heterogeneous multiple bank financing system supports economic stability by reducing the firms’ incentives to conduct highly risky projects.
Our study underlines the subtle effects that bank financing may have on a firm’s optimal risk- and information-policy and, subsequently, on the policy’s efficiency. We find that heterogeneous multiple bank financing allows multi-faceted firm decisions. Firms adjust their optimal risk-taking and information disclosure to the degree of lending heterogeneity whenever circumstances are not so favorable that the firm will almost always be viable nor so unfavorable that the firm will almost always have to default. In particular, firms with highly-specific assets may benefit, as their optimal policy leads to a full elimination of ex-ante default probability, provided that the degree of relationship lending is limited by arm’s-length lending.

One of the main contributions of this paper focuses on the hitherto undetected flip-side of the hold-up problem that arises once relationship lending is combined with arm’s-length lending so that the degree of relationship bank financing is limited. The coexistence of both types of banks allows the firm to make use of the tradeoff between the relationship bank’s bargaining power due to her informational advantage on the one hand and the risk of coordination failure among arm’s-length banks on the other hand. The degree of bank financing heterogeneity hence delivers an instrument for the firm to credibly signal its commitment to the business project and its willingness to abstain from strategic default. This allows the optimal firm policy to fully eliminate the remaining risk of inefficient credit withdrawal and hence the probability of firm default - an effect denoted as “hold-up benefit”, as it relies on the characteristics of the hold-up problem, albeit weakened by the existence of arm’s-length bank financing.

Certainly, the hold-up benefit requires the existence of sufficiently heterogeneous types of banks, respectively the willingness and ability of banks to act either as a relationship bank or as an arm’s-length lender to a borrower. The paper may therefore also contribute to an explanation of the wide variety of - particularly European - commercial banks that often specialize on lending to particular borrowers with regard to, e.g., industry, size or location.
Appendix

Appendix A - Derivation of the uniqueness condition

For the equilibrium derived in section 3 to be unique, the indifference curves of banks and firm have to intersect exactly once (necessary and sufficient condition). For this to be the case, the following conditions have to be satisfied

\[
\frac{\partial x^*_R}{\partial \theta^*} < \frac{\partial x^*_S}{\partial \theta^*} \quad \text{and} \quad \frac{\partial x^*_S}{\partial \theta^*} < \frac{\partial x^*_R}{\partial \theta^*},
\]

where \( \frac{\partial x^*_R}{\partial \theta^*} \) denotes the derivative of the firm’s indifference condition (i.e. equilibrium equation (3)), solved for \( x^* \), i.e.,

\[
x^* = \theta^* + \frac{1}{\sqrt{c}} \Phi^{-1} \left[ \frac{\theta^* - V - r - (1 - \lambda)(W_S - r)\Phi(\sqrt{b} (x^*_S - \theta^*))}{\lambda(W_R - r)} \right].
\]

The respective partial derivatives are given by

\[
\frac{\partial x^*_R}{\partial \theta^*} = \frac{a + c}{c}, \quad \frac{\partial x^*_S}{\partial \theta^*} = \frac{a + b}{b}, \quad \frac{\partial x^*}{\partial \theta^*} = 1 + \frac{1}{\sqrt{2\pi} \varphi \left( \frac{\theta^* - V - r - (1 - \lambda)(W_S - r)\Phi(\sqrt{b} (x^*_S - \theta^*))}{\lambda(W_R - r)} \right)} \cdot \frac{1 - (1 - \lambda)(W_S - r)\varphi(\sqrt{b} (x^*_S - \theta^*))}{\lambda(W_R - r)} \cdot \frac{\frac{a}{\sqrt{2\pi} b}}{\lambda(W_R - r)}.
\]

Since the partial derivatives of the banks’ indifference curves are constant, the uniqueness conditions are satisfied if they hold for the minimum of the firm’s indifference curve’s derivative. In order to calculate this minimum, it has to be taken into account that both standard normal distributions that enter the partial derivative have to reach their maximum, which is given by \( \frac{1}{\sqrt{2\pi}} \). This delivers

\[
\min \left( \frac{\partial x^*}{\partial \theta^*} \right) = 1 + \sqrt{\frac{2\pi}{c}} \cdot \frac{1 - (1 - \lambda)(W_S - r)\frac{a}{\sqrt{2\pi} b}}{\lambda(W_R - r)}.
\]

For the uniqueness condition to hold, it therefore has to be the case that

\[
\lambda(W_R - r)\frac{a}{\sqrt{c}} + (1 - \lambda)(W_S - r)\frac{a}{\sqrt{b}} < \sqrt{2\pi}.
\]

This inequality is satisfied provided that the banks’ private information is sufficiently precise relative to public information, i.e. \( b \) and \( c \) are sufficiently large relative to \( a \).
Appendix B - Partial derivative of $\theta^*$

$$\frac{\partial \theta^*}{\partial c} = -\frac{\lambda(W_R - r)\varphi_c(\cdot)\left[\frac{1}{2}\sqrt{\frac{a}{a+c}}\Phi^{-1}(\frac{K}{r}) + \frac{a}{2\sqrt{a+c}}(\theta^* - y)\right]}{1 - \lambda(W_R - r)\varphi_c(\cdot)\frac{a}{\sqrt{c}} - (1 - \lambda)(W_S - r)\varphi_b(\cdot)\frac{a}{\sqrt{b}}}$$

Here, $\varphi_c(\cdot) = \varphi\left(\frac{a}{\sqrt{c}}(\theta^* - y) + \sqrt{\frac{a+c}{c}}\Phi^{-1}\left(\frac{K}{r}\right)\right)$ and $\varphi_b(\cdot) = \varphi\left(\frac{a}{\sqrt{b}}(\theta^* - y) + \sqrt{\frac{a+b}{b}}\Phi^{-1}\left(\frac{K}{r}\right)\right)$. Since under the uniqueness condition the denominator is positive, the partial derivative is positive whenever $\theta^* < y - 1/\sqrt{a+c}\Phi^{-1}(K/r)$ and negative for $\theta^* > y - 1/\sqrt{a+c}\Phi^{-1}(K/r)$.

Appendix C - Optimal information disclosure

Case 1: high early liquidation value, $K > 1/2 r$

When the early liquidation value $K$ is sufficiently high, threshold function $M(c \to \infty)$ converges to $y$ from below. Let us first analyze the case of low expected cash-flow, i.e. $y < \theta^c$. Since $\theta^*$ is decreasing along with $c$ whenever $\theta^* > y - 1/\sqrt{a+c}\Phi^{-1}(K/r)$, we find that $\theta^*$ decreases in $c$ for the whole range of parameter values. Hence, the firm can minimize the probability of inefficient project liquidation by providing the relationship bank with completely precise information, i.e. $c^* \to \infty$.

If, in contrast, expected cash-flow is high, so that $y > \theta^c$, the following situation is obtained (see Fig. 2). For low precision values $c$, equilibrium value $\theta^*$ will be higher than threshold function $M(\cdot)$, so that $\theta^*$ is decreasing in $c$. Once $\theta^*$ equals the threshold, a minimum is reached and $\theta^*$ starts increasing along with $c$ for higher precision values. The minimum value of $\theta^*$ is obtained for a precision denoted $\tilde{c}$, where the two curves cross. However, in order to ensure uniqueness of equilibrium, we require $c$ to be at least as high as $c^{\min}$. The optimal precision value $c^*$ in this case is therefore given as

$$c^* = \begin{cases} c^{\min} & \text{if } c^{\min} > \tilde{c} \\ \tilde{c} & \text{if } c^{\min} \leq \tilde{c} \end{cases}$$

where $\tilde{c}$ is implicitly defined by $\theta^*(c) = y - 1/\sqrt{a+c}\Phi^{-1}(K/r)$. 

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Case 2: low early liquidation value, $K < 1/2r$

For $K < 1/2r$, threshold $M(c \to \infty)$ converges to $y$ from above. If expects cash-flow is low, so that $y < \theta^c$, the firm’s optimal information policy is either to disclose completely precise information to the relationship bank or to decrease information precision to its minimally necessary level, as can be seen from Fig. 3.

If $a$ is sufficiently low, so that $c^{\min}$ takes on very low values, it might be the case that $\theta^*(c^{\min}) \leq \theta^c$, so that it is advantageous for the firm to disclose as imprecise information as possible. In any other case, however, the firm can minimize the probability of inefficient project liquidation by granting completely precise information to the relationship bank.

From (3) it follows that

$$\sqrt{2\pi(\theta^* - y)} + \sqrt{\frac{2\pi + a}{a} \Phi^{-1} \left( \frac{K}{r} \right)} = \Phi^{-1} \left( \frac{K}{r} \right)$$

$$a = \frac{2\pi}{\left( \frac{\sqrt{2\pi(y - \theta^*)}}{\Phi^{-1} \left( \frac{K}{r} \right)} \right)^2 - 1} \approx \bar{a}.$$
Hence, for $a \leq \bar{a}$ optimal information precision is given by $c^{\text{min}}$, whereas for $a > \bar{a}$ the firm is best off by providing the relationship bank with completely precise information, i.e. $c^* \to \infty$.

If the market holds very optimistic expectations with regard to cash-flow, i.e. $y > \theta^c$, the optimal information policy is to choose $c^* = c^{\text{min}}$, as the condition for $\theta^*$ to be increasing in $c$ is always satisfied.

Appendix D - Optimal risk-taking

Case 1: high early liquidation value, $K > 1/2r$

If expected cash-flow is low, i.e. $y < \theta^c$, we know that the relationship bank should optimally be provided with completely precise information: $c^* \to \infty$. Examining the extreme values of $a$, i.e. either maximum risk ($a = 0$) or zero risk ($a \to \infty$), while taking into account the optimal information policy, the equilibrium values of $\theta^*$ are
given by\textsuperscript{20}

\begin{equation}
\theta^*(c \to \infty, a = 0) = V + r + K \frac{r}{r} [\lambda(W_R - r) + (1 - \lambda)(W_S - r)]
\end{equation}

and

\begin{equation}
\theta^*(c \to \infty, a \to \infty) = V + r + \lambda(W_R - r) K \frac{r}{r} + (1 - \lambda)(W_S - r).
\end{equation}

The partial derivative of $\theta^*(c \to \infty, a)$ with respect to $a$ delivers

\begin{equation}
\frac{\partial \theta^*(c \to \infty, a)}{\partial a} = \frac{(1 - \lambda)(W_S - r) \varphi(\cdot) \frac{1}{\sqrt{b}}(\theta^* - y) + \frac{1}{2 \sqrt{b(a+b)}} \Phi^{-1}(\frac{K}{r})}{1 - (1 - \lambda)(W_S - r) \varphi(\cdot) \frac{b}{\sqrt{b}}},
\end{equation}

which is positive whenever $\theta^*(c \to \infty, a) > y - 1/(2\sqrt{a+b})\Phi^{-1}(K/r)$. This condition is satisfied, as $\Phi^{-1}(K/r) > 0$ and $y < \theta^c$. Hence, $\theta^*$ is increasing in $a$ and $\theta^* - y > 0$, so that according to (8) the probability of inefficient project termination increases in $a$. The optimal business risk is therefore given by maximum risk, i.e. $a^{**} = 0$. The ex-ante probability of inefficient project liquidation, $\Phi(\sqrt{a(\theta^* - y)})$, is then reduced to a level of $\Phi(0) = 1/2$.

If, in contrast, \textbf{expected cash-flow is high}, i.e. $y > \theta^c$, optimal information precision is given as either $c^* = c^{\text{min}}$ or $c^* = \tilde{c}$. Let us first concentrate on the case of $c^* = c^{\text{min}}$. Here, the equilibrium value $\theta^*$ for $a = 0$ is given by

\begin{equation}
\theta^*(c^{\text{min}}, a = 0) = V + r + K \frac{r}{r} [\lambda(W_R - r) + (1 - \lambda)(W_S - r)] = \theta^*(a = 0).
\end{equation}

We know that $\theta^c = \theta^*(c \to \infty, a) < y$ for all $a$. Hence, it also holds for $a = 0$. $\theta^*(a = 0)$, however, is independent of $c$. Therefore, it must be the case that $\theta^*(c^{\text{min}}, a = 0) < y$ as well.

\textsuperscript{20}Equation (A3) is derived by using the fact that the second term in (3) can also be expressed as $(1 - \lambda)(W_S - r)\Phi(a/\sqrt{b}(\theta^* - y) + \sqrt{a/b + 1} \Phi^{-1}(K/r)) = (1 - \lambda)(W_S - r)\Phi(a[1/\sqrt{b}(\theta^* - y) + \sqrt{1/ab + 1/a^2} \Phi^{-1}(K/r)])$. Since $y < \theta^c$ holds for all values of $a$, it has to hold for $a \to \infty$ as well, so that the latter term converges to $(1 - \lambda)(W_S - r)\Phi(+\infty) = (1 - \lambda)(W_S - r)$.
For the partial derivative of $\theta^*(c_{\min}, a)$ with respect to $a$, we find

$$\frac{\partial \theta^*(c_{\min}, a)}{\partial a} = \frac{1}{1 - \lambda(W_r - r)\varphi_1(\cdot)\sqrt{2\pi} - (1 - \lambda)(W_s - r)\varphi_2(\cdot)\frac{a}{\sqrt{b}}} \left[ -\lambda(W_r - r)\varphi_1(\cdot)\frac{\pi}{a^2} \sqrt{\frac{a}{2\pi + a}} \Phi^{-1}\left(\frac{K}{r}\right) + (1 - \lambda)(W_s - r)\varphi_2(\cdot)\left[ \frac{1}{\sqrt{b}}(\theta^* - y) + \frac{1}{2b} \sqrt{\frac{b}{a + b}} \Phi^{-1}\left(\frac{K}{r}\right) \right] \right],$$

(A6)

where $\varphi_1(\cdot) = \varphi(\sqrt{2\pi}(\theta^* - y) + \Phi^{-1}(K/r))$ and $\varphi_2(\cdot) = \varphi(a/\sqrt{b}(\theta^* - y) + \sqrt{(a + b)/b}\Phi^{-1}(K/r))$. This partial derivative is positive, if $\theta^*$ is higher than

$$\frac{\partial \theta^*(c_{\min}, a)}{\partial a} = \frac{\lambda(W_r - r)\varphi_2(\cdot)\sqrt{a^3(2\pi + a)}}{(1 - \lambda)(W_s - r)\varphi_2(\cdot)\sqrt{a^3(2\pi + a)} - \frac{1}{2\sqrt{a + b}} \Phi^{-1}\left(\frac{K}{r}\right)}.$$

(A7)

What happens to threshold (A7) for $a \to \infty$? As long as

$$\lambda > \frac{(W_s - r)\varphi_2(\cdot)\sqrt{a^3(2\pi + a)}}{(W_r - r)\varphi_1(\cdot)2\pi\sqrt{b(a + b)} + (W_s - r)\varphi_2(\cdot)\sqrt{a^3(2\pi + a)}} \equiv \tilde{\lambda},$$

threshold (A7) converges to $y$ from above, since the term in brackets is then positive and $\Phi^{-1}(K/r) > 0$ in the case considered. For $\lambda > \tilde{\lambda}$, therefore, $\theta^*(c_{\min}, a)$ decreases in $a$ as $\theta^*(c_{\min}, a = 0) < y$. Hence, since $\theta^* - y < 0$, the optimal value of $a$ is given by $a^{**} \to \infty$, so that the probability of inefficient project liquidation is completely eliminated, i.e. $\text{prob}(\theta \leq \theta^*) = \Phi(-\infty) = 0$.

For $\lambda \leq \tilde{\lambda}$, however, threshold (A7) converges to $y$ from below. Here, we have to distinguish two cases: either $\theta^*(c_{\min}, a = 0) < y < \theta^*(c_{\min}, a \to \infty)$ or $\theta^*(c_{\min}, a \to \infty) < \theta^*(c_{\min}, a = 0) < y$. In the first case we find that $a^* = 0$ as given in Fig. 4, since $\partial \theta^*(c_{\min}, a)/\partial a > 0$. However, $\theta^*(c_{\min}, a = 0) < y$, so that the overall optimal business risk $a^{**}$ will take on an intermediate value.

In the second case, i.e. for $\theta^*(c_{\min}, a \to \infty) < \theta^*(c_{\min}, a = 0) < y$, the optimal value of business risk is given by $a^{**} \to \infty$, as can be seen from Fig. 5. Here, $\theta^*$ decreases in $a$ for sufficiently high values of $a$, and $\theta^*(c_{\min}, a \to \infty) < y$, so that $\partial \text{prob}(\theta \leq \theta^*)/\partial a < 0$ and hence projects with zero risk ($a^{**} \to \infty$) will minimize the probability of inefficient liquidation, i.e. $\text{prob}(\theta \leq \theta^*) = \Phi(-\infty) = 0$. 

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Whenever optimal information precision is given by $\tilde{c}$, we find that for the extreme values of $a$ the equilibrium value $\theta^*$ is given by

\begin{equation}
\theta^*(\tilde{c}, a = 0) = V + r + \frac{K}{r} [\lambda(W_R - r) + (1 - \lambda)(W_S - r)]
\end{equation}

and

\begin{equation}
\theta^*(\tilde{c}, a \to \infty) = y.
\end{equation}

Generally, the partial derivative is given as

\begin{equation}
\frac{\partial \theta^*(\tilde{c}, a)}{\partial a} = \frac{1}{\sqrt{a + \tilde{c}}} \Phi^{-1}\left(\frac{K}{r}\right) > 0.
\end{equation}

Since $\theta^*(\tilde{c}, a) \leq y$ (see figure 1), while the partial derivative $(\partial \theta^*(\tilde{c}, a))/(\partial a)$ is positive, the optimal value of $a$ must be an interior solution. Plugging the partial derivative in (8), the impact of $a$ on the overall probability of inefficient project liquidation is given by

\begin{equation}
\frac{\partial \Phi(\sqrt{a}(\theta^* - y))}{\partial a} = \varphi(\sqrt{a}(\theta^* - y)) \left[ \frac{1}{2\sqrt{a}} (\theta^* - y) + \frac{a}{(a + \tilde{c})^{3/2}} \Phi^{-1}\left(\frac{K}{r}\right) \right].
\end{equation}

The value of $a$ that minimizes this probability is then found as the intermediate value $a^{**} = \tilde{a}$ for which $\theta^*(\tilde{c}, \tilde{a}) = y - 2a/\sqrt{(a + \tilde{c})^{3/2}} \Phi^{-1}(K/r)$. 

Figure 4: $K > 1/2 r$, $y > \theta^*$, $\lambda \leq \bar{\lambda}$ and $\theta^*(c_{\text{min}}, a = 0) < y < \theta^*(c_{\text{min}}, a \to \infty)$.
Figure 5: $K > 1/2 r$, $y > \theta^*$, $\lambda \leq \bar{\lambda}$ and $\theta^*(c^{\min}, a \to \infty) < \theta^*(c^{\min}, a = 0) < y$

Summarizing the different results for the case of high liquidation value and high expected cash-flow, we find the following:

- For $c^* = c^{\min}$:
  
  - with sufficiently high $\lambda$, optimal business risk is characterized by $a^{**} \to \infty$, so that the probability of inefficient project termination amounts to $\Phi(-\infty) = 0$, since $\theta^* < y$.
  
  - with sufficiently low $\lambda$, optimal business risk is either achieved with $a^{**} \to \infty$ and leads to a probability of inefficient project termination of $\Phi(-\infty) = 0$ whenever $y$ is extremely high, i.e. $y > \theta^*(c^{\min}, a \to \infty)$. Otherwise, optimal risk takes on an intermediate value.

- For $c^* = \tilde{c}$, the optimal value of $a$ is given by $a^{**} = \tilde{a}$, so that $0 \leq \Phi(\sqrt{a}(\theta^* - y)) \leq 1/2$.

Hence, for a sufficiently high degree of relationship banking (i.e. for $\lambda > \bar{\lambda}$), optimal firm policy is described by $c^* = c^{\min}$ and $a^{**} \to \infty$, since, due to the high value of $a$, $c^{\min} > \tilde{c}$. For a low degree of relationship banking, in contrast, the firm will either
choose a policy combination of $a^{**} \to \infty$ and $c^* = c^\text{min}$ for projects with extremely high expected cash-flows or select intermediate riskiness and intermediate information precision otherwise.

**Case 2: low early liquidation value, $K < 1/2r$**

For **low expected cash-flow** and $a \leq \bar{a}$, optimal information precision is given by $c^* = c^\text{min}$, whereas with $a > \bar{a}$, optimal precision is given by $c^* \to \infty$.

If we first concentrate on the case of $c^* = c^\text{min}$, we know that due to the assumption of $y < \theta^c$ also $\theta^*(c^\text{min}, a = 0) > y$. Again, it holds that $\theta^*(c^\text{min}, a)$ increases in $a$ whenever $\theta^*$ is higher than threshold (A7). Since in the current case it is assumed that $K < 1/2r$, however, the threshold will converge to $y$ from below for $a \to \infty$ whenever $\lambda > \bar{\lambda}$. It can therefore be shown that for a sufficiently high degree of relationship banking $\theta^*(c^\text{min}, a)$ increases in $a$ and, since $\theta^* > y$, the overall probability of inefficient project termination increases in $a$ as well, so that the optimal risk parameter is given by $a^{**} = 0$.

For $\lambda \leq \bar{\lambda}$, however, threshold (A7) converges to $y$ from above. Again, two different possibilities arise. Either $\theta^*(c^\text{min}, a \to \infty) < y < \theta^*(c^\text{min}, a = 0)$, so that $\theta^*$ decreases in $a$. Since for sufficiently high $a$ equilibrium value $\theta^*$ is lower than $y$, the probability of inefficient project liquidation is minimized by selecting a project risk of $a^{**} \to \infty$. The overall probability of liquidation is then reduced to a level of $\Phi(-\infty) = 0^21$

Alternatively, the case of $y < \theta^*(c^\text{min}, a = 0) < \theta^*(c^\text{min}, a \to \infty)$ could arise as shown in Fig. 6. Since in this case equilibrium value $\theta^*$ is always higher than $y$ and $a^* = \tilde{a}_1$, an intermediate value of $a$ will minimize the overall probability of inefficient project termination but cannot eliminate the risk completely.

For $a > \bar{a}$, in contrast, optimal information precision is given by $c^* \to \infty$. We know that $\theta^*(c \to \infty, a)$ increases in $a$ whenever $\theta^* > y - 1/(2\sqrt{a+b}) \Phi^{-1}(K/r)$ and vice

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21Note that for $c^\text{min}$ to be the optimal precision value, risk parameter $a$ is required to be at most as high as $\bar{a}$, i.e. the value of $a$ that equates $\theta^*(c^\text{min}, a)$ and $\theta^*(c \to \infty, a) = \theta^c$. Since, as it turns out, the optimal risk parameter is given by $a^{**} \to \infty$, $c^\text{min} = a^2/(2\pi) \to \infty$ as well, so that the condition for $c^\text{min}$ being optimal in the first place is indeed satisfied, as $\bar{a} \to \infty$. 

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versa. For $a \to \infty$, this threshold converges to $y$ from above, since $\Phi^{-1}(K/r) < 0$. In the only feasible case, $y < \theta^*(c \to \infty, a = 0) < \theta^*(c_{\text{min}}, a \to \infty)$, so that Fig. 7 is obtained. Again, an intermediate solution for $a$ will be optimal to reduce $\theta^*$. Since equilibrium value $\theta^*$ is always higher than $y$, however, choosing $a^{**} \to 0$ would be the best policy to minimize the overall probability of inefficient project liquidation and reduce it to a level of $1/2$. However, for $c \to \infty$ to be chosen as optimal precision in the first place, $a$ has to be sufficiently high, so that this result has to be ruled out.

Summing up the results for this case of low liquidation value and low expected cash-flow, we find the following: Optimal information precision is only given by $c^* = c_{\text{min}}$:

- For $\lambda > \bar{\lambda}$ optimal riskiness is characterized by $a^{**} = 0$. The prior probability of inefficient project termination is thereby reduced to a value of $1/2$.

- For $\lambda \leq \bar{\lambda}$, the probability of inefficient project termination can be completely eliminated by choosing $a^{**} \to \infty$. Since this is the lowest level that can be achieved, intermediate values of $a$ do not have to be considered as alternative solutions.

Let us finally consider the case where expected cash-flow is high, i.e. $y > \theta^c$. The
Figure 7: $K < 1/2$, $y < \theta^c$, $a > \bar{a}$ and $y < \theta^*(c \to \infty, a = 0) < \theta^*(c \to \infty, a \to \infty)$

optimal information precision is given by $c^* = c^{\min}$. $\theta^*(c^{\min}, a)$ increases in $a$ whenever $\theta^*$ is higher than threshold (A7). For $\lambda > \bar{\lambda}$ and $a \to \infty$, threshold (A7) converges to $y$ from below. In the only feasible case of $\theta^*(c^{\min}, a \to \infty) < \theta^*(c^{\min}, a = 0) < y$, $\theta^*$ is decreasing in $a$ for sufficiently high values of $a$, while at the same time $\theta^* < y$, so that the overall optimal value of $a$ is given by $a^{**} \to \infty$.

For $\lambda \leq \bar{\lambda}$, instead, threshold (A7) converges to $y$ from above. Since $\theta^*(c^{\min}, a = 0) < y$, $\theta^*$ decreases in $a$ and the probability of inefficient project liquidation can be minimized by selecting minimum business risk: $a^{**} \to \infty$.

Hence for both low and high values of $\lambda$, the probability of inefficient project liquidation can be minimized by conducting a policy with parameters $c^* = c^{\min}$ and $a^{**} \to \infty$, so that $\Phi(\sqrt{a^*(\theta^* - y)}) = 0$.\[22\]

\[22\]Choosing maximum risk, i.e. $a^{**} = 0$, for $\lambda > \bar{\lambda}$ would reduce the ex-ante probability of project liquidation to a value of $1/2$. A risk policy of $a^{**} \to \infty$ is therefore more efficient and should be preferred.
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