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## The state-dependent impact of changes in bank capital requirements

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## Working Paper Series

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The state-dependent impact of  
changes in bank capital requirements

No 2828

## Abstract

Based on a non-linear equilibrium model of the banking sector with an occasionally-binding equity issuance constraint, we show that the economic impact of changes in bank capital requirements depends on the state of the macro-financial environment. In "normal" states where banks do not face problems to retain enough profits to satisfy higher capital requirements, the impact on bank loan supply works through a "pricing channel" which is small: around 0.1% less loans for a 1pp increase in capital requirements. In "bad" states where banks are not able to come up with sufficient equity to satisfy capital requirements, the impact on loan supply works through a "quantity channel", which acts like a financial accelerator and can be very large: up to 10% more loans for a capital requirement release of 1pp. Compared to existing DSGE models with a banking sector, which usually feature a constant lending response of around 1%, our state-dependent impact is an order of magnitude lower in "normal" states and an order of magnitude higher in "bad" states. Our results provide a theoretical justification for building up a positive countercyclical capital buffer in "normal" macro-financial environments.

**Keywords:** Bank capital requirements, loan supply, dynamic stochastic equilibrium model, financial accelerator, global solution methods

**JEL classification:** D21, E44, E51, G21, G28

## Non-technical summary

Various empirical papers have shown that the lending impact of changes in bank capital requirements varies considerably depending on bank conditions and the state of the macro-financial environment. However, this feature of state-dependence is missing from standard macro models with a banking sector.

To study the state-dependent impact of changes in bank capital requirements within a structural setting, we develop a stylised model of the banking sector that features two occasionally binding constraints. The first one is a (time-varying) regulatory capital requirement. The second one is an equity issuance constraint, or equivalently a non-negativity constraint on dividends, which implies that banks can only accumulate equity through retained earnings. We show that the interaction of these two occasionally binding constraints induces strong state-dependence in the impact of changes in bank capital requirements on loan supply, in line with the empirical findings.

In "normal" states of the world where banks do not face problems to retain enough profits to satisfy higher capital requirements, the impact on bank loan supply works through a "pricing channel" which is quantitatively small: loans change by around -0.1% (or even less) for a 1pp increase in capital requirements. In "bad" states of the world where banks are not able to come up with sufficient equity to satisfy capital requirements, the impact on loan supply works through a "quantity channel", which acts like a financial accelerator and can be very large: up to 10% more loans for a capital requirement release of 1pp. These state-dependent magnitudes are consistent with the magnitudes found in empirical studies.

Compared to existing models in this literature, which usually feature a constant lending response of around -1% for a 1pp capital requirement increase, our state-dependent impact is an order of magnitude lower in "normal" states and an order of magnitude higher in "bad" states. Our results on state-dependence can also help to explain why empirical studies often find different magnitudes for the impact of changes in bank capital requirements on lending: in our structural model a 1pp increase in capital requirements can lead to a change in equilibrium loans of anything between 0% and -10%, depending on how much each of the transmission channels becomes "activated" by the policy change.

Our findings regarding the state-dependent impact of changes in bank capital requirements have a number of policy implications.

First, transition costs to higher bank capital requirements in terms of reduced loan supply (and therefore economic activity) can be kept low if higher capital requirements are introduced gradually over time in "normal" states of the world, where banks make positive profits so that capital ratios can be increased through retained earnings. The level of banking system profitability, in combination with available capital headroom, should be a sufficient statistic to determine the pace of capital requirement increases that can be met by banks without constraining loan supply much.

Second, moderate increases in bank capital requirements during boom phases are unlikely to have a big dampening effect on credit growth. This is because during boom phases banks tend to be profitable and have voluntary capital buffers available to absorb increases in capital requirements without becoming capital constrained. Hence, only the "pricing channel" is likely to be active with its limited impact on equilibrium loan growth. In other words, moderate capital requirement increases are unlikely to have a big dampening effect on potentially "excessive" credit growth during the upswing phase of the financial cycle. The only way that a big dampening effect on credit growth during boom phases could be achieved is by raising capital requirements significantly and abruptly, so that binding capital constraints for the banking system are induced.

Third, the release of capital requirements in "bad" states, where banks make substantial losses and become capital constrained can have a big supporting effect on bank loan supply and is therefore important to mitigate bank deleveraging in crisis times. With constant capital requirements, the voluntary capital buffers held by banks due to self-insurance motives do not prevent the banking system from occasionally becoming capital constrained and being forced to deleverage in "bad" states of the world. If the objective of policy makers is to avoid large volatility in loan supply, time-varying capital requirements will be important.

Finally, our results provide a theoretical justification for building up a positive countercyclical capital buffer (CCyB) in "normal" macro-financial environments, i.e. before clear signs of excessive credit growth emerge. In particular, such a policy strategy will impose limited economic costs during the buffer build-up phase, while minimizing the probability that a bad shock hits the economy before a sufficient level of capital buffers has been built up that can be released to support bank loan supply. In other words, such a policy strategy can create insurance at low economic costs against systemic risks that are inherently difficult to identify and measure.

# 1 Introduction

How large is the impact of changes in bank capital requirements on lending? Various empirical papers have shown that this impact varies considerably depending on bank conditions and the state of the macro-financial environment: When banks are well capitalised, are profitable, or the economy is doing well, an increase in capital requirements tends to have little to no impact on lending. When banks have limited capital headroom, make losses, or the economy is in recession, a 1 percentage point (pp) increase in capital requirements tends to reduce lending by several pp.<sup>1</sup> The most relevant empirical study from a macro stabilisation perspective is by Jiménez et al. (2017), who show that the introduction of higher bank capital buffers during "good times" does not have a significant effect on credit to firms, while availability of a 1pp higher capital buffer during "bad times", which can be used by banks, increases firm credit by 9pp. This empirical feature of state-dependence is missing from standard macro models with a banking sector, which commonly feature a constant lending response of around -1% for a 1pp increase in bank capital requirements.<sup>2</sup>

To study the state-dependent impact of changes in bank capital requirements within a structural setting, we develop a non-linear equilibrium model of the banking sector that features monopolistic competition and two occasionally binding constraints. The first one is a (time-varying) regulatory capital requirement. The second one is an equity issuance constraint, or equivalently a non-negativity constraint on dividends, which implies that banks can only accumulate equity through retained earnings.<sup>3</sup> The key difference to many existing macro models with a banking sector is that in equilibrium the equity issuance constraint will be binding only in certain states of the world, instead of all the time. The interaction of this occasionally-binding equity issuance constraint with the occasionally-binding capital requirement constraint is the crucial model feature that induces state-dependence in the impact of changes in bank capital requirements on loan supply, as it implies that a financial accelerator channel is only present in some states of the world. In line with empirical findings, our model also features a substantial bank equity premium over deposit funding costs due to deposit insurance, which will tend to reduce bank loan supply as capital requirements go up.<sup>4</sup>

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<sup>1</sup>See for example the findings in Mora and Logan (2012), Carlson et al. (2013), Bridges et al. (2014), Arbatli-Saxegaard and Juelsrud (2020), De Jonghe et al. (2020), and Sivec and Volk (2021).

<sup>2</sup>See for example Gertler and Kiyotaki (2010), Gerali et al. (2010), Darracq Pariès et al. (2011), Clerc et al. (2015), Mendicino et al. (2018), Cozzi et al. (2020).

<sup>3</sup>This assumption is standard in macro models with a banking sector, see e.g. Gertler et al. (2020) or Mendicino et al. (2020).

<sup>4</sup>There is ample empirical evidence that the cost of bank equity is considerably higher than the

Within this framework, we derive various analytical results and conduct policy experiments based on a calibrated model for the euro area that is solved with a global solution method. The main insight of our paper is that there is indeed strong state-dependence in the impact of changes in capital requirements on bank loan supply. In "normal" states of the world where banks hold voluntary capital buffers and do not face problems to retain enough profits to satisfy higher capital requirements, the impact on bank loan supply works through a "pricing channel" which is quantitatively small: loans change by around -0.1% (or even less) for a 1pp increase in capital requirements. In "bad" states of the world where banks are not able to come up with sufficient equity to satisfy capital requirements, the impact on loan supply works through a "quantity channel", which acts like a financial accelerator and can be very large: up to 10% more loans for a capital requirement release of 1pp. This "quantity channel" will tend to be present in states of the world where banks have limited capital headroom above requirements and make losses. Our quantitative results regarding state-dependence are consistent with the magnitudes found in empirical studies (see references above).

The intuition behind our results is simple. If banks have no problem to come up with additional equity through retained earnings to satisfy higher capital requirements, the only effect on loan quantities is through a "pricing channel": as equity funding is more costly than debt funding, the marginal cost of loans increases, which is passed on to customers via higher interest rates and this reduces equilibrium loan quantities (Figure 1 panel a). However, this "pricing channel" is extremely small, which can be illustrated with a back-of-the-envelope calculation. Under full pass-through, a 8% cost of equity, a 2% cost of debt, and a 50% bank risk-weight, funding costs and therefore bank lending rates will increase by merely 3 basis points (bps) in response to a 1pp higher capital requirement.<sup>5</sup> The response of loan quantities to this increase in interest rates then depends on the interest rate semi-elasticity of loan demand, which is usually estimated to be around 3.<sup>6</sup> Hence, the "pricing channel" should only reduce equilibrium loan quantities by around 0.1% for a 1pp

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cost of bank debt, see e.g. [Altavilla et al. \(2021\)](#). Our model assumption of a zero Modigliani-Miller off-set between equity and debt funding costs is conservative and will tend to make higher bank capital requirements costly for banks. Our impact estimates on bank loan supply should therefore be seen as upper bounds.

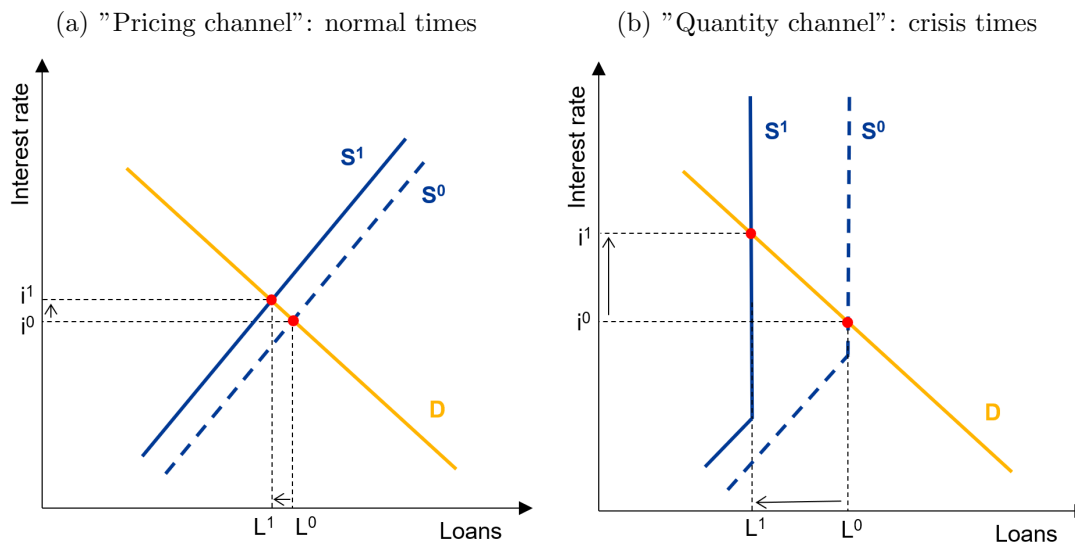
<sup>5</sup>Note that 0.5% more of each loan now needs to be funded by equity, which is 6pp more expensive than debt, leading to a 3 bps increase in the funding cost of a loan. In our model this increase in funding cost will be passed on slightly more than 1-for-1 to interest rates due to a mark-up resulting from monopolistic competition. However, the overall intuition underlying the "pricing channel" still holds in our structural model set-up.

<sup>6</sup>See e.g. [DeFusco and Paciorek \(2017\)](#), [Davis et al. \(2020\)](#), [Best et al. \(2020\)](#), or [Bhutta and Ringo \(2021\)](#) for evidence on the interest elasticity of mortgage credit.



increase in bank capital requirements.

Figure 1: Stylised state-dependent transmission channels of capital requirements



Notes: Stylised exposition. Bank equity is assumed to be more costly than bank debt. An increase in bank capital requirements therefore increases the marginal cost of loans and shifts the bank loan supply curve leftwards from  $S^0$  to  $S^1$  ("pricing channel"). Under the assumption that the bank cannot issue equity and the initial capital requirement already induces a hard limit on loan supply, the "quantity channel" of changing bank capital requirements comes into play.

However, if banks cannot come up with sufficient equity to satisfy higher capital requirements a second, potentially very large, "quantity channel" comes into play, which acts like a financial accelerator: the only way for banks to meet higher capital requirements is through capping the quantity of loans (Figure 1 panel b). Because banks are highly leveraged, adjustments via the "quantity channel" will be large, which can again be illustrated with a back-of-the-envelope calculation. Consider a bank with a 10% capital requirement, no voluntary capital buffers, and current period profits of zero. If the capital requirement is increased by 1pp to 11%, the only way for the bank to meet this higher requirement is through reducing loans by 9.1%, as this will "free-up" just enough equity to meet the higher requirement.<sup>7</sup> More generally, in situations where the capital requirement is binding and banks cannot build equity through retained earnings, the policy-induced percentage change in loans needs to equal the inverse of the percentage change in the capital requirement.<sup>8</sup> Capital requirement releases can therefore provide strong support to bank lending

<sup>7</sup>As the old and new capital requirement hold with equality ( $E_j/(L_j\omega) = R_j$  for  $j = old, new$ ) and equity is fixed ( $E_{new} = E_{old}$ ) we know that  $L_{new}/L_{old} = R_{old}/R_{new} = 0.1/0.11 = 0.909$ , which implies a 9.1% reduction in loans.

<sup>8</sup>This can be seen from taking logs of the equation in the previous footnote:  $\log(L_{new}) - \log(L_{old}) = \log(R_{old}) - \log(R_{new}) = -[\log(R_{new}) - \log(R_{old})]$ .



in states of the world where the "quantity channel" is active.

Whether the low impact "pricing channel" or the high impact "quantity channel" applies when changing bank capital requirements depends crucially on existing voluntary capital buffers, current bank profitability, and the size of the requirement change. We use a calibrated model version for the euro area to provide further insights into this. The quantitative model reveals that in states of the world where voluntary capital buffers and bank profitability are at the average level, an unanticipated 1pp increase in capital requirements has almost no effect on lending, i.e. only the "pricing channel" will be active.<sup>9</sup> More generally, the model simulations reveal that the impact on lending of a 1pp increase in capital requirements is around -0.1% or lower whenever banks hold voluntary capital buffers before and after the requirement change, i.e. when existing capital headroom and profitability are sufficient to easily absorb the requirement increase. These results suggest that the gradual build-up of regulatory capital buffers in "normal" states of the world where banks hold voluntary capital buffers and make positive profits should only have a minimal impact on bank lending, and therefore have low economic costs.

We also use the quantitative model to study an unanticipated 1pp "release" of capital requirements in "bad" states where banks are hit by a relatively large negative shock so that they make considerable losses.<sup>10</sup> Compared to an economy with a constant capital requirement, equilibrium loans drop by 9pp less, as the 1pp requirement release leaves banks additional capital headroom to absorb their losses with the equity that is freed-up. Hence, the "quantity channel" is muted and banks deleverage less compared to an economy without the release. Our analysis shows that capital requirement releases are most effective in supporting lending in states of the world where bank losses materialise and both the equity issuance constraint and the capital requirement constraint are binding. The release is ineffective in states of the world where banks face adverse shocks but still make positive profits.

We go on to show analytically and through model simulations that the "quantity channel" can be shut-off by a policy rule which ensures that capital requirements are always lower or equal to the current capital ratio plus the return on risk-weighted assets, while never turning negative. Such a policy rule implies that gradual increases in capital requirements should start as soon as the economy enters "normal"

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<sup>9</sup>In the quantitative model banks hold voluntary capital buffers above capital requirements. Average profitability and the voluntary capital buffers are high enough to "absorb" a 1pp increase in capital requirements without deleveraging.

<sup>10</sup>We consider a shock that corresponds to the 5th percentile of the stationary equilibrium distribution of banks' return on assets.

states of the world, where banks make positive profits. This is because many periods of gradual requirement increases are needed to ensure that requirement releases in "bad" states can be sufficiently large to allow banks to fully absorb losses without deleveraging and still be able to meet a positive capital requirement. Such a "gradual but early" build-up rule will impose limited economic costs during the build-up phase, while minimizing the probability that a bad shock hits the economy before a sufficient level of releasable capital requirements has been built up. Our model simulations show that such a state-dependent capital requirement rule is able to substantially reduce tail risk to loan supply, while on average loan supply is not reduced much compared to an economy with a constant (and lower) capital requirement.<sup>11</sup> Overall, our results provide a theoretical justification for building up a positive countercyclical capital buffer in "normal" macro-financial environments, as such a policy strategy creates insurance at low economic costs against tail risks that are inherently difficult to identify and measure ex-ante.

Our paper is related to three literature strands that study banks and capital requirements in structural settings: non-linear microeconomic models, linearized dynamic stochastic general equilibrium (DSGE) models, and non-linear macroeconomic models. To our knowledge we are the first to study in a structural model the state-dependent impact of changes in capital requirements on bank loan supply, and in particular that there are states of the world where the impact is low and others where the impact is high.

First, compared to microeconomic models of banks with an equity issuance constraint or costly equity issuance, such as [Van den Heuvel \(2006\)](#), [De Nicolò et al. \(2014\)](#), [Behn et al. \(2019\)](#) or [Mankart et al. \(2020\)](#), we solve a banking sector equilibrium model where interest rates are determined endogenously instead of given exogenously. This feature of endogenous interest rates is essential to study the aggregate loan supply implications of changes in capital requirements for the entire banking sector. Without such an endogenous response of interest rates (or the revenue function of the bank), the impact of changes in aggregate capital requirements on bank loan supply will be vastly overestimated. This is because in equilibrium marginal loan revenue needs to equal marginal cost. If the aggregate interest rate is

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<sup>11</sup>We use our calibrated model to study the transition from an economy with a constant 10% capital requirement to an economy with a state-dependent capital requirement of 10% to 15% that is consistent with the policy principles described in this paragraph. The rule is implemented under full commitment and is known to the banks, so banks take the rule into account when making their optimal decisions. The 2nd and 10th percentiles of the simulated loan distribution compared to the economy with a constant 10% capital requirement improve by 11.5pp and 3pp respectively, while the median goes down by less than 1pp.

given exogenously, the only way to equate marginal revenue to an increased marginal cost due to higher capital requirements is through moving up the *individual* loan demand curve. As banks tend to have limited market power, i.e. the *individual* loan demand curve is rather elastic, large loan quantity adjustments will be needed to increase marginal revenue even a bit. Hence, the "pricing channel" will be vastly overestimated in absence of an endogenous interest rate.

Second, compared to linearized DSGE models with a banking sector, such as [Gertler and Kiyotaki \(2010\)](#), [Gerali et al. \(2010\)](#), [Darracq Pariès et al. \(2011\)](#), [Clerc et al. \(2015\)](#) or [Mendicino et al. \(2018\)](#), we study the impact of changes in bank capital requirements in a non-linear model. By definition, linearized DSGE models cannot generate state-dependence: most papers find that a 1pp increase in capital requirements leads to a drop in loans of 1% to 3.5% (see Table 2 in [Cozzi et al. \(2020\)](#)).<sup>12</sup> Compared to our state-dependent model results, this is an order of magnitude larger than the impact in "good" states, and up to an order of magnitude lower than the impact in "bad" states. When doing cost-benefit assessments of time-varying capital requirements, neglecting this state-dependence can provide misleading results.

Third, compared to existing non-linear macroeconomic models with a banking sector, we are the first to focus explicitly on the state-dependent impact of changes in bank capital requirements. The banking sector frameworks used by [Gertler and Karadi \(2011\)](#), [Gertler and Kiyotaki \(2015\)](#), [Gertler et al. \(2020\)](#), [Mendicino et al. \(2020\)](#), and [Elenev et al. \(2021\)](#) either assume dividends are only paid upon exogenous bank exit or dividends are assumed to be a constant fraction of net worth.<sup>13</sup> Hence, within such set-ups banks are always capital constrained and some form of the "quantity channel" is always present, whereas in our model the "quantity channel" is only present occasionally when both the equity issuance constraint and the capital requirement constraint become binding.<sup>14</sup> While our banking set-up with

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<sup>12</sup>In these models banks are assumed to engage in strong dividend smoothing: every period banks aim to pay out a constant fraction of net worth as dividends. Potential deviations from the dividend target are penalised by a cost function. The impact of changes in capital requirements on loans depends crucially on the calibration of the cost parameter (dividend smoothing motive).

<sup>13</sup>In contrast, our model does not impose a dividend smoothing motive, which is in line with the empirical evidence for euro area banks as shown in section 2 of the paper. [Elenev et al. \(2021\)](#) allow banks to deviate from the dividend target but the cost of doing so is calibrated to be relatively high. Another common assumption in the literature is that banks belong to risk averse bankers that actually consume the dividends stemming from the banking operations, see e.g. [Iacoviello \(2015\)](#).

<sup>14</sup>[Mendicino et al. \(2020\)](#) solve the perfect foresight transition path for a gradual increase of the regulatory capital requirement from 8% to 10.5%. The focus of [Gertler et al. \(2020\)](#) is on the ability of macroprudential policy in form of regulatory capital requirements to prevent bank

an occasionally-binding equity issuance constraint is closely related to the modelling of financial intermediaries in Brunnermeier and Sannikov (2014), Van der Ghote (2018), Holden et al. (2020), and Schroth (2021), we are the first to focus on the state-dependent impact of changes in bank capital requirements.<sup>15</sup> Finally, Corbae and D’Erasmus (2021) and Jamilov and Monacelli (2020) study non-linear macroeconomic models with heterogeneous banks, but also do not focus on the state-dependent impact of changes in bank capital requirements.

The remainder of our paper is structured as follows. Section 2 lays out our structural model set-up. Section 3 presents analytical results that prove some of our key insights. Section 4 uses a calibrated model for the euro area to provide quantitative insights about the impact of time-varying capital requirements. Section 5 discusses qualitatively how our findings would be affected by relaxing some of the model assumptions. Section 6 concludes.

## 2 The model

We study a discrete time infinite horizon non-linear banking sector equilibrium model with monopolistic competition. To keep the framework as transparent as possible, we take the *loan demand* side as given and do not derive it explicitly from a firm or household optimisation problem. This allows us to focus attention on the core issue of the state-dependent impact of changes in bank capital requirements on *loan supply*. However, it would be straightforward to also derive the loan demand side from optimising household or firm behaviour and make it time-varying.

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runs. Elenev et al. (2021) study welfare consequences of changes in average regulatory capital requirements. In one exercise, they consider a simple state-dependent capital regulation rule that varies with the aggregate uncertainty shock. Our paper, in contrast, provides analytical results on how a regulator can mitigate non-linearities arising in the intermediation sector by insuring banks via a state-dependent capital requirement rule.

<sup>15</sup>Brunnermeier and Sannikov (2014) and Schroth (2021) study the non-linear dynamics of an economy where banks face an occasionally-binding equity issuance constraint and a (market based) no default constraint. Van der Ghote (2018) studies the optimal interaction between macroprudential and monetary policy in a similar framework as Brunnermeier and Sannikov (2014) with nominal rigidities. Holden et al. (2020) consider an intermediation sector with an occasionally binding capital constraint and costly equity issuance with a focus on quantitative assessment of the dynamics of macro-variables during financial recessions.

## 2.1 Loan demand

Aggregate loan demand ( $L_t^A$ ) is assumed to be downward sloping with a constant interest rate semi-elasticity ( $\epsilon$ ):

$$\log(L_t^A) = \lambda - \epsilon \cdot i_t^A \quad (1)$$

where  $i_t^A$  is the aggregate interest rate and  $\lambda$  is a demand shifter. Similar to [Jamilov and Monacelli \(2020\)](#) we assume that there is a continuum of banks that operate in segmented loan markets, which are indexed by  $i$ . Each loan market segment enters aggregate loan demand through a constant elasticity of substitution (CES) aggregator with elasticity  $\mu$ :

$$L_t^A = \left( \int_0^1 L_{i,t}^{\frac{\mu-1}{\mu}} di \right)^{\frac{\mu}{\mu-1}} \quad (2)$$

As borrowers aim to minimize interest expenditure for a given level of aggregate loans, demand for loans of bank  $i$  is given by:

$$L_{i,t} = \left( \frac{i_{i,t}}{i_t^A} \right)^{-\mu} L_t^A \quad (3)$$

This set-up of bank loan demand is analogous to the monopolistic competition set-up employed for firms in standard New Keynesian models. If  $\mu$  is very high, market power of banks is low because loans across banks are almost perfect substitutes, and banks will make little "excess profit" over marginal cost. If  $\mu$  is very low, market power of banks is high because loans across banks cannot be substituted easily, and banks will make considerable "excess profits" over marginal cost. Inverting the loan demand curve yields the following expression for the interest rate that bank  $i$  will be able to charge:

$$i_{i,t} = \left( \frac{L_{i,t}}{L_t^A} \right)^{-\frac{1}{\mu}} i_t^A \quad (4)$$

## 2.2 Banks

There is a unit mass of identical risk neutral banks that maximise the present discounted value of expected dividend streams paid to shareholders. Let  $i$  denote a given bank and let  $t$  denote a given time period. The balance sheet of banks consists

of loans ( $L$ ) on the asset side and deposits ( $D$ ) and equity ( $E$ ) on the liability side:

$$L_{i,t} = D_{i,t} + E_{i,t} \quad (5)$$

Each period, a time-varying fraction  $\theta_{i,t}$  of loans default and are not paid back. Hence, the period impairment cost (cost of risk) is given by:

$$COR(\theta_{i,t}, L_{i,t}) = \theta_{i,t} L_{i,t} \quad (6)$$

In addition to impairment costs, banks face operating costs for maintaining the current loan book, which can be thought of as expenses for staff, rent, IT systems, and other current expenses:

$$OC(L_{i,t}) = \kappa L_{i,t} \quad (7)$$

All loans mature at the end of each period. Banks then choose how many new loans to originate. These new loans start yielding interest income in the subsequent period. This assumption can be justified by the fact that it takes time to originate new loans and to conclude the associated contractual arrangements. The interest rate banks receive on their loan book is given by the inverted loan demand equation (4), and depends on aggregate and bank-specific loans at time  $t$ , which are pre-determined by past loan origination decisions:

$$i_{i,t} = \left( \frac{L_{i,t}}{L_t^A} \right)^{-\frac{1}{\mu}} \frac{\lambda - \log(L_t^A)}{\epsilon} \quad (8)$$

Deposits, which can be thought of within the model as comprising all non-equity financing (e.g. customer deposits, interbank borrowing, bank bond issuance), can be freely adjusted by banks in each period. We assume that deposits are fully covered by deposit insurance, so that the interest rate that banks need to pay for deposit funding ( $i^D$ ) does not depend on bank leverage and is constant. In other words, we assume that the Modigliani-Miller theorem does not hold. We make this assumption for two reasons. First, in the real world deposit insurance covers large parts of bank liabilities and there is ample evidence that bank equity financing is always more costly than deposit funding.<sup>16</sup> Second, it eases the exposition of our analysis and helps to focus on the key mechanism driving state-dependence in the

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<sup>16</sup>See e.g. [Altavilla et al. \(2021\)](#).

impact of changes in bank capital requirements.<sup>17</sup>

Given the specifications for loan and deposit interest rates, and using the balance sheet identity in equation (5), banks' net interest income can be defined as:<sup>18</sup>

$$NII(L_{i,t}, E_{i,t}, L_t^A) = \left[ \left( \frac{L_{i,t}}{L_t^A} \right)^{-\frac{1}{\mu}} \frac{\lambda - \log(L_t^A)}{\epsilon} - i^D \right] L_{i,t} + i^D E_{i,t} \quad (9)$$

Net profits of banks for the period are given by the difference between net interest income, loan impairments (cost of risk) and operating costs:

$$\pi^N(\theta_{i,t}, L_{i,t}, E_{i,t}, L_t^A) = NII(L_{i,t}, E_{i,t}, L_t^A) - COR(\theta_{i,t}, L_{i,t}) - OC(L_{i,t}) \quad (10)$$

In contrast to deposits, bank equity at time  $t$  is fixed and can only be adjusted for the next period through retaining net profits ( $\pi^N$ ) after taking into account any dividend payouts ( $d$ ):

$$E_{i,t+1} = E_{i,t} + \pi_{i,t}^N - d_{i,t} \quad (11)$$

It is assumed that banks cannot issue equity, i.e. that  $d \geq 0$ . This simplifying assumption can be justified on the grounds that equity issuance by banks occurs infrequently (see Figure 2 panel a) and is either not possible or very costly in crisis times. In addition, a large fraction of euro area banks are unlisted, making new equity issuance difficult.<sup>19</sup> Finally, the assumption of an equity issuance constraint is standard in many macro models with a banking sector.<sup>20</sup> The key difference to existing macro models with a banking sector is that in equilibrium the equity issuance constraint - or equivalently the non-negativity constraint on bank dividends - will be binding only in certain states of the world, instead of all the time. This property is again in line with the data (see Figure 2 panel b).

<sup>17</sup>With the assumption of constant deposit funding costs, the impact of increasing bank capital requirements on loan supply that we derive below will be an upper bound. If deposit funding costs were assumed to go down as bank leverage decreases, this would reduce the cost of increasing bank capital requirements further.

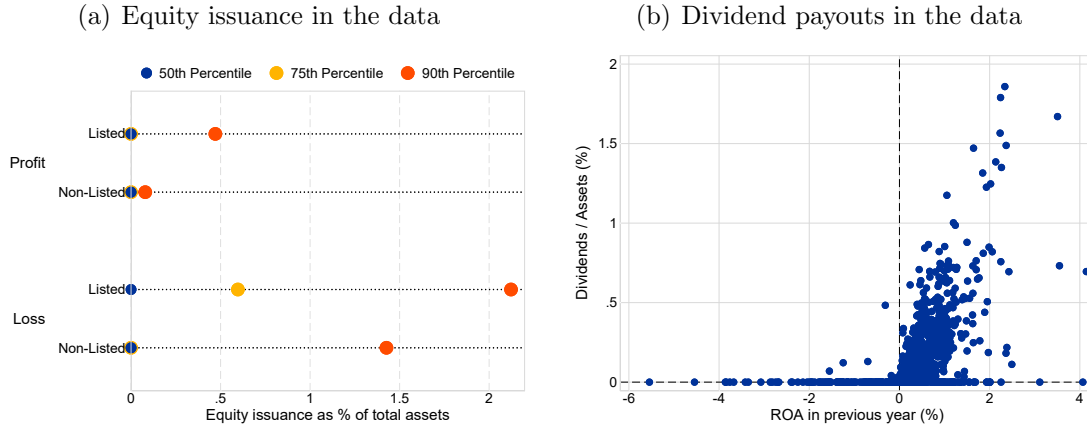
<sup>18</sup>This can be derived from the identity  $NII_{i,t} = i_{i,t}L_{i,t} - i^D D_{i,t}$ .

<sup>19</sup>As shown in Figure 2 panel a, unlisted banks do not issue equity in more than 75% of the situations where they incur losses.

<sup>20</sup>See e.g. Gertler et al. (2020) or Mendicino et al. (2020).



Figure 2: Empirical evidence on equity issuance and dividend payouts



Sources: SNL Financial. Authors' calculations.

Notes: Based on a panel of around 320 euro area banks since 2005 at highest level of consolidation. Data on dividend payouts is available for around one third of these banks. Dividends tend to be paid out of previous year profits. As equity issuance is not directly reported in balance sheet data, it is approximated as the difference between the change in common equity and net profits, whenever this difference is positive and growth in the number of issued shares is greater than 1%.

## 2.3 Policy rule(s) for capital requirements

The regulatory capital ratio of banks is defined in terms of risk-weighted assets, just as in the real world. For analytical tractability, the loan risk-weight  $\omega$  of banks is assumed to be constant over time. Hence, the risk-weighted capital ratio is given by:

$$CR(L_{i,t}, E_{i,t}) = \frac{E_{i,t}}{\omega L_{i,t}} \quad (12)$$

Banks face a potentially time-varying capital requirement  $R_t$  that is announced by the supervisor one period in advance. Without loss of generality, we assume that next period's capital requirement can be expressed as a function of the current aggregate state vector. Banks are not allowed by the supervisor to breach the capital requirement. Hence, at all times the capital ratio must satisfy  $CR_{i,t} \geq R_t$  or equivalently  $E_{i,t} \geq R_t \omega L_{i,t}$ . As equity and loans at the beginning of period  $t$  are both predetermined by past decisions, the choices for loans and equity by banks always need to be done such that the capital requirement is met in the following period.

To keep the model as tractable as possible, we abstain from modelling a specific economic mechanism that justifies having a positive capital requirement. We simply rely on findings from the literature that in the presence of limited commitment or

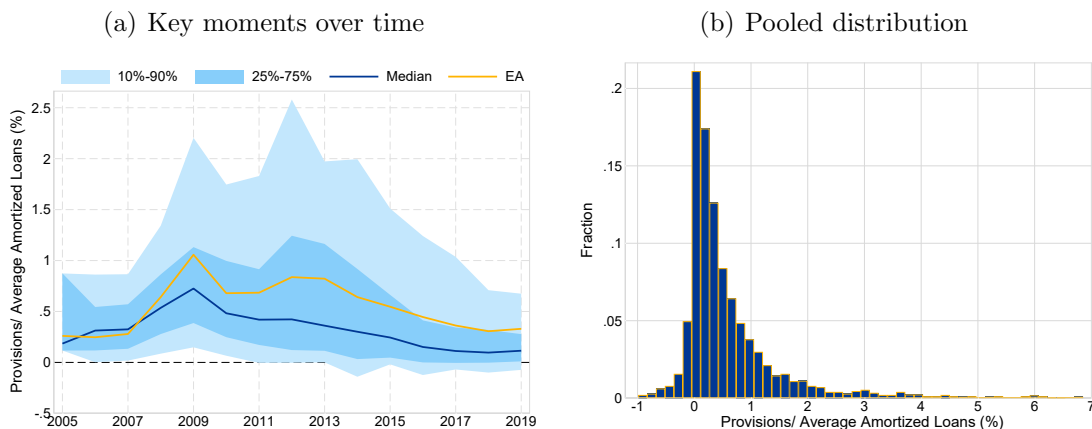
costly bank default and deposit insurance, a positive regulatory capital requirement can be justified.<sup>21</sup> To close our model, we simply assume that the regulatory capital requirement is always high enough to prevent that there are states where aggregate equity of the banking sector would turn negative, i.e. states where the banking sector would need to default in presence of the equity issuance constraint.<sup>22</sup> This model set-up allows us to focus on our key object of study: how *changes* in capital requirements affect bank loan supply in different states of the world.

## 2.4 Stochastic processes

As we abstract from bank heterogeneity, there is just a single stochastic process for loan impairments that drives the dynamics of the model. The dynamic evolution of credit risk is specified as a log AR(1) process in order to get a fatter right tail of credit impairments, in line with the empirical distribution of provisioning across euro area banks (see Figure 3):

$$\ln(\theta_{i,t}) = \alpha_0 + \alpha_1 \ln(\theta_{i,t-1}) + \alpha_2 \varepsilon_{i,t} \quad (13)$$

Figure 3: Properties of bank-level provisioning rates



Sources: SNL Financial.

Notes: Based on a panel of around 320 euro area banks since 2005 at highest level of consolidation.

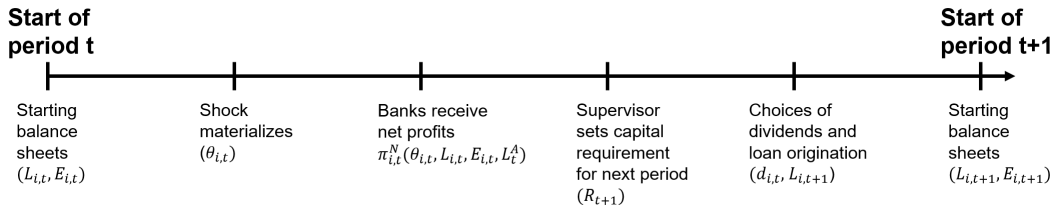
<sup>21</sup>See e.g. [Gertler and Kiyotaki \(2010\)](#) or [Clerc et al. \(2015\)](#).

<sup>22</sup>The empirical model calibration to euro area data in section 4 has the property that aggregate bank equity never turns negative, i.e. that bank default does not occur in equilibrium.

## 2.5 Timing of events

Banks start the period with pre-determined loans and equity from prior decisions. At the beginning of the period, the credit risk shock materialises. Based on the realised shock and starting loans and equity, banks receive net profits for the period. The supervisor observes the state of credit risk and bank profitability and then sets capital requirements for the next period. Banks then decide how much dividends to pay to shareholders and how many loans to originate before the start of the next period. The timing of events is illustrated in Figure 4.

Figure 4: Overview of the timing of events



## 2.6 Decision problem and value function of banks

Banks' objective is to maximise the present discounted value of expected dividend payments. We assume risk neutrality of banks, i.e. no curvature in the utility function, as in the real world banks often pay out zero dividends (see Figure 2 panel b) and dividend smoothing is limited.<sup>23</sup> Expectations are rational, i.e. banks know the laws of motion for all stochastic variables and equilibrium objects. The discount factor  $\beta$  is determined by the required return on bank equity from the market  $\rho$ :

$$\beta = \frac{1}{1 + \rho} \quad (14)$$

For ease of exposition, we assume that the required return on equity is constant over time. Moreover, we assume that the required return on bank equity is greater than the cost of deposit funding, i.e. that  $\rho > i^D$ . These assumptions are motivated by the fact that there is ample empirical evidence that the cost of bank equity is between 8% and 12%<sup>24</sup>, i.e. much higher than deposit interest rates, and does not

<sup>23</sup>In euro area data dividend payouts are zero around 30% of the time. Moreover, in a tobit model of the dividend payout ratio (dividends / assets) for euro area banks, the coefficient on the lagged dividend payout ratio is just 0.12 when controlling for bank profitability.

<sup>24</sup>See Altavilla et al. (2021) for comprehensive empirical evidence for euro area banks.

fluctuate too much over time. By assuming that the cost of bank equity is always greater than the cost of deposit funding, and that the difference between the two does not react to bank leverage, we are making conservative assumptions that will tend to make it costly to increase bank capital requirements. The results derived in the rest of the paper should therefore be seen as conservative upper bound estimates regarding the impact of changes in capital requirements on bank loan supply. The decision problem of banks is given by the following dynamic maximization problem:

$$\begin{aligned}
& \max_{\{d_{i,t+j}, L_{i,t+1+j}\}} \mathbb{E}_t \left[ \sum_{j=0}^{\infty} \beta^j d_{i,t+j} \right] \\
& \quad s.t. \\
& \quad E_{i,t+j+1} = E_{i,t+j} + \pi^N(\theta_{i,t+j}, L_{i,t+j}, E_{i,t+j}, L_{t+j}^A) - d_{i,t+j} \\
& \quad E_{i,t+j+1} \geq R_{t+j+1} \omega L_{i,t+j+1} \\
& \quad d_{i,t+j} \geq 0
\end{aligned} \tag{15}$$

The decision problem of banks can also be represented in recursive notation by a value function with four state variables: the credit risk shock ( $\theta$ ), current loans of the bank ( $L$ ), current equity of the bank ( $E$ ), and current aggregate loans ( $L^A$ ). As the capital requirement is assumed to be a function of the current state vector, it does not enter as a separate state variable. In addition, there are two choice variables: next period loans ( $L'$ ) and next period equity ( $E'$ ). The value function is given by the following expression:<sup>25</sup>

$$\begin{aligned}
V(\theta, L, E, L^A) = & \max_{L', E'} d(\theta, L, E, L^A, E') + \beta \mathbb{E} \left[ V(\theta', L', E', L^A) \right] \\
& + \chi^1 \left[ E' - R' \omega L' \right] + \chi^2 \left[ d(\theta, L, E, L^A, E') \right]
\end{aligned} \tag{16}$$

The variables  $\chi^1$  and  $\chi^2$  are the lagrange multipliers on the occasionally binding capital requirement and equity issuance constraints. Equilibrium dynamics of the model will be governed by the policy functions that emerge from the solution to this non-linear dynamic optimization problem, together with the condition that in equilibrium aggregate and idiosyncratic bank variables will be identical, due to the representative bank assumption. The state vector characterising the model equilibrium has therefore three dimensions:  $(\theta, L, E)$ .

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<sup>25</sup>Dividends are given by  $d(\theta, L, E, L^A, E') = \pi^N(\theta, L, E, L^A) + E - E'$ . Net profits are given by  $\pi^N(\theta, L, E, L^A) = NII(L, E, L^A) - COR(\theta, L) - OC(L)$ .

### 3 Analytical results

In this section we derive analytical results that characterise the optimal bank choices for loans and the capital ratio in equilibrium, and how changes in bank capital requirements affect them.

#### 3.1 First-order conditions

As a starting point for the analysis it is useful to derive the system of first-order conditions for  $E'$  and  $L'$  that characterise the equilibrium of the full model.<sup>26</sup>

$$\underbrace{1 + \chi^2}_{\text{Marginal cost of more } E'} = \underbrace{\chi^1 + \frac{1 + i^D}{1 + \rho} \mathbb{E}[1 + \chi^{2'}]}_{\text{Marginal benefit of more } E'} \quad (17)$$

$$\underbrace{\chi^1 R' \omega}_{\text{Marginal cost today of more } L'} = \mathbb{E} \left[ \frac{1 + \chi^{2'}}{1 + \rho} \left( \underbrace{\frac{\mu - 1}{\mu} \frac{\lambda - \log(L')}{\epsilon}}_{\text{Marginal benefit tomorrow of more } L'} \underbrace{- i^D - \theta' - \kappa}_{\text{Marginal cost tomorrow of more } L'} \right) \right] \quad (18)$$

In addition to these first-order conditions, the complementary slackness conditions, the non-negativity conditions for the lagrange multipliers, and the original inequality constraints must be satisfied in equilibrium. The interpretation of the first-order conditions is straightforward.

Let's start with the equity choice in equation (17). Holding one more unit of equity has a direct unit cost as these own funds are not paid out as dividends to shareholders. In states of the world where the equity issuance constraint is binding, i.e. where  $\chi^2$  is positive, the cost of holding one more unit of equity increases by the shadow value of the constraint. Optimal bank behaviour equates these marginal costs to the marginal benefits: on the one hand more equity relaxes the potentially binding capital requirement constraint, which is captured by  $\chi^1$ ; on the other hand the additional unit of equity is available tomorrow and reduces the need for deposit funding, which is captured by  $1 + i^D$ , discounted to the present value with  $1 + \rho$ . In states of the world where the equity issuance constraint next period is binding, i.e.

<sup>26</sup>See Appendix B1 for the derivation. In particular, as all banks are identical, in equilibrium it has to be the case that  $L_t^A = L_{i,t}$  for all  $t$ .

where  $\chi^{2'}$  is positive, the marginal benefit of the additional equity is enhanced.

Equation (17) further shows that the shadow value of the capital requirement constraint  $\chi^1$  is increasing with the shadow value of the equity issuance constraint  $\chi^2$ . Hence, the marginal value to banks of relaxing the capital requirement constraint will be larger in states where the equity issuance constraint is binding, than in states where this constraint is slack. This feature shows that the two occasionally-binding constraints interact and that this interaction will yield state-dependence in the impact of changes in bank capital requirements.

Let's now turn to the optimal loan choice in equation (18). As additional loans need to be partly financed by equity, this creates a direct cost of  $\chi^1 R' \omega$  whenever the capital requirement constraint is binding. These marginal costs incurred today need to equate the expected discounted net marginal benefit of loans in the next period. Net marginal benefit in the next period is in turn given by the difference between the marginal interest revenue and the marginal cost<sup>27</sup> of providing loans. In states of the world where the equity issuance constraint next period is binding, i.e. where  $\chi^{2'}$  is positive, the marginal net benefit of having more profits is enhanced. Next period marginal net benefits are discounted using the required return on bank equity  $\rho$ .

### 3.2 Equilibrium without an equity issuance constraint

The model with a capital requirement, but without the equity issuance constraint is a useful benchmark, as it allows for the derivation of a closed-form solution.

**Proposition 1 (Equilibrium without an equity issuance constraint).** *In an economy without a constraint on bank equity issuance:*

- *The capital ratio is always equal to the capital requirement.*
- *The marginal value of changing the capital requirement is:*

$$\chi^1 = 1 - \frac{1 + i^D}{1 + \rho} > 0 \quad (19)$$

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<sup>27</sup>Which is given by the sum of marginal interest expenses for funding the loan, impairment costs, and operating costs

- *Equilibrium loans are given by:*

$$\log(L') = \lambda - \epsilon \underbrace{\left( \frac{\mu}{\mu - 1} \right) \left[ \mathbb{E}(\theta') + \kappa + i^D + (\rho - i^D)\omega R' \right]}_{\substack{\text{Interest rate charged by banks} \\ = \text{Mark-up over marginal cost}}} \quad (20)$$

*Proof.* See Appendix B2. ■

The results of Proposition 1 are intuitive. In the absence of an equity issuance constraints, there are no reasons for banks to hold voluntary capital buffers. Banks can always go to the market and raise new equity from shareholders in case of need, e.g. when losses are incurred. As bank equity is assumed to be more costly than deposit funding, banks will minimise the equity they hold. Hence, banks will always hold just enough equity to meet the capital requirement. As the capital requirement constraint is always binding, the shadow value of relaxing the constraint is always positive: its value is given by the difference between the cost of a unit of equity, i.e. foregone dividends of 1, and the benefit it brings next period  $(1 + i^D)$  discounted with the required return on equity. Finally, as banks can always obtain as much equity as needed, equilibrium loans do not depend on banks' current equity. Due to monopolistic competition, banks charge a mark-up  $\mu/(\mu - 1)$  over the marginal cost of loans. The interest rate semi-elasticity of loan demand  $\epsilon$  then determines how the charged interest rate translates into equilibrium loan quantities.

**Proposition 2 ("Pricing channel" of changing capital requirements).** *In the absence of an equity issuance constraint, equilibrium loans respond to changes in bank capital requirements through a "pricing channel", which is given by:*

$$\Delta \log(L') = -\epsilon \left( \frac{\mu}{\mu - 1} \right) \underbrace{\left[ (\rho - i^D)\omega \Delta R' \right]}_{\substack{\text{Policy-induced change} \\ \text{in loan funding cost}}} \quad (21)$$

*Proof.* See Appendix B3. ■

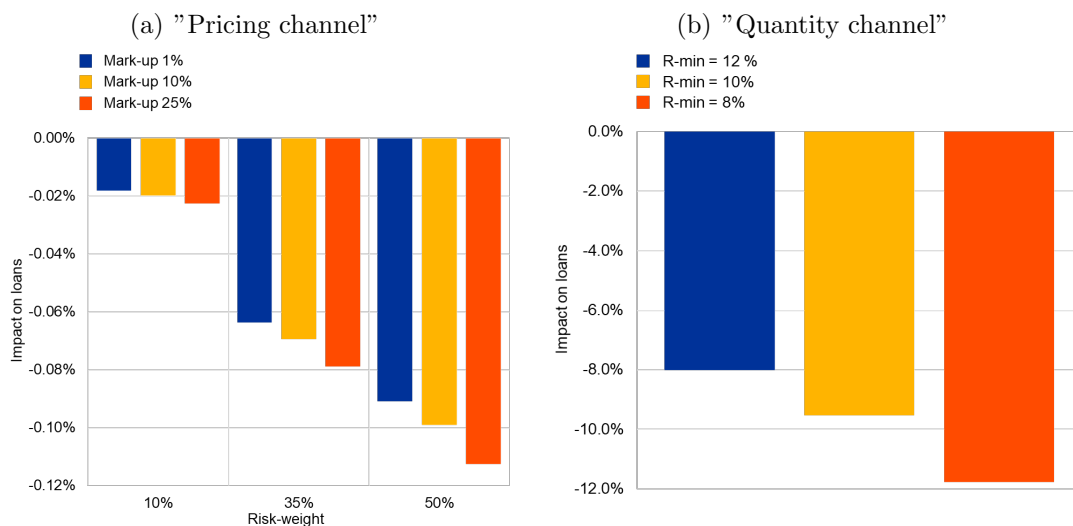
In the absence of an equity issuance constraint, banks will never face situations with less equity than desired as they can always go to the market to raise new equity. Hence, the only channel through which an increase in bank capital requirements affects loan supply is through a "pricing channel" (see Proposition 2): as equity is more costly than deposit funding, an increase in capital requirements raises the weighted average funding cost of loans by  $(\rho - i^D)\omega \Delta R'$ . This increase in funding



cost, multiplied by the mark-up due to monopolistic competition, is passed on to borrowers. The response of equilibrium loans to this leftward shift in the loan supply curve is then determined by the interest rate semi-elasticity of loan demand  $\epsilon$ .

The quantitative relevance of this "pricing channel" will depend on model parameters. However, for empirically plausible parameter values the impact of the "pricing channel" will be quantitatively very small. This can be easily seen from the following realistic example. Assume a constant cost of equity of 8%, a deposit funding cost of 2%, an average risk-weight of 50%, and a 1pp increase in capital requirements. This will push up the funding cost of loans by merely 3 basis points, i.e. just one eighth of a standard monetary policy rate hike increment. As market power of banks is rather low<sup>28</sup>, the mark-up charged will be low, and  $\mu/(\mu - 1) \approx 1$ . Finally, empirical estimates of the interest rate semi-elasticity of loan demand are on average around 3.<sup>29</sup> Hence, equilibrium loan quantities should merely drop by around -0.1% in response to a 1pp increase in capital requirements, if only the "pricing channel" is at play (see Figure 5 panel a). This impact through the "pricing channel" should even be an upper bound, as the conservative assumption is made that deposit funding costs do not go down in response to higher bank capitalisation.

Figure 5: Examples for the magnitudes of the "pricing" and "quantity" channels



Notes: In the examples an increase in capital requirements by +1pp is assumed. For the illustration of the "pricing channel" a loan demand interest rate semi-elasticity of 3, a cost of equity of 8%, and a cost of bank debt of 2% are assumed. The mark-ups of 1%, 10%, and 25% correspond to parameter values for  $\mu$  of 100, 11, and 5 respectively.

<sup>28</sup>Since the global financial crisis, profitability of euro area banks has been low and price-to-book ratios have often been below 1. Both observations indicate low market power.

<sup>29</sup>See e.g. [DeFusco and Paciorek \(2017\)](#), [Davis et al. \(2020\)](#), [Best et al. \(2020\)](#), or [Bhutta and Ringo \(2021\)](#) for evidence on the interest elasticity of mortgage credit.

Another important insight from Proposition 2 is that the impact on equilibrium loan quantities through the "pricing channel" depends on the risk-weight of loans: the higher the risk-weight, the larger will be the impact on loan funding costs and therefore loan quantities in response to a change in capital requirements. For example, risk-weights for retail mortgage loans under the internal ratings-based approach (IRB) can be as low as 10% in some European countries, which implies a "pricing channel" impact of just -0.02% (Figure 5 panel a). The risk-weight for retail mortgage loans under the standardised approach (SA) is 35%, implying a "pricing channel" impact in the range of -0.06% to -0.08%, depending on the market power of banks. For corporate loans under the IRB approach, which carry a risk weight of around 50%, the "pricing channel" impact should be in the range of -0.09% to -0.11% (Figure 5 panel a).<sup>30</sup> These are of course just indicative numbers, that would change if the equity premium or the loan demand semi-elasticity were different. However, they give a good sense of the orders of magnitude one should expect for the impact of changes in bank capital requirements through the "pricing channel". Moreover, these insights can help to explain why empirical studies have generally found that a given change in bank capital requirements tends to have a bigger impact on corporate lending than on household lending.

### 3.3 Equilibrium with an equity issuance constraint

Now that we have established benchmark results for the "pricing channel" of changes in bank capital requirements, it is instructive to see how imposing an equity issuance constraint changes the picture.

**Proposition 3 (Voluntary capital buffers).** *In an economy with an equity issuance constraint banks may hold voluntary capital buffers to partially insure against the risk that this constraint becomes binding in the future. In particular, banks will hold voluntary capital buffers above capital requirements in states of the world where the probability is large enough that the equity issuance constraint in the next period becomes binding and its shadow value is sufficiently high, i.e. whenever:*

$$\mathbb{E}[\chi^2] = \frac{1 + \rho}{1 + i^D}(1 + \chi^2) - 1 > 0 \quad (22)$$

*Proof.* See Appendix B4. ■

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<sup>30</sup>For details on risk-weights of euro area banks, see pp. 58 - 60 of the [ECB supervisory banking statistics for 2022 Q3](#).

The first insight is that imposing an equity issuance constraint can induce banks to hold voluntary capital buffers in some states of the world (see Proposition 3). The reason is that the equity issuance constraint may become binding in the next period. In these situations banks will be forced to hold fewer loans and/or a lower capital ratio than desired, because they cannot come up with sufficient equity. If the probability is large enough that the equity issuance constraint could become binding in the future, it can pay off for banks to hold voluntary capital buffers above the capital requirement to partially insure against this risk.

**Proposition 4 (“Pricing channel” with voluntary capital buffers).** *In states of the world where banks hold voluntary capital buffers before and after a change in bank capital requirements, equilibrium loans respond to changes in bank capital requirements via a “pricing channel”, which is given by:*

- *Case 1 when banks pay dividends:*

$$\Delta \log(L') = -\epsilon \left( \frac{\mu}{\mu - 1} \right) \left( \frac{1 + i^D}{1 + \rho} \right) \underbrace{\Delta \text{Cov}(\chi^{2'}, \theta')}_{\text{Policy-induced change in covariance term}} \quad (23)$$

- *Case 2 when banks do not pay dividends:*

$$\Delta \log(L') = -\epsilon \left( \frac{\mu}{\mu - 1} \right) \left( \frac{1 + i^D}{1 + \rho} \right) \underbrace{\Delta \frac{\text{Cov}(\chi^{2'}, \theta')}{1 + \chi^2}}_{\text{Policy-induced change in covariance term}} \quad (24)$$

*Proof.* See Appendix B5. ■

The second insight is that in states of the world where banks hold voluntary capital buffers<sup>31</sup>, changes in bank capital requirements affect loan supply through a “pricing channel”, which is similar to the “pricing channel” in the absence of an equity issuance constraint (see Proposition 4). However, compared to the model without an equity issuance constraint it is no longer possible to derive an exact closed-form solution for the magnitude of the pricing channel: the magnitude will depend on how the covariance between the credit risk shock and the shadow value of the equity issuance constraint in the next period is affected by the change in capital requirements.<sup>32</sup> Moreover, the magnitude of the “pricing channel” with

<sup>31</sup>I.e. banks hold voluntary capital buffers under the old and the new capital requirement.

<sup>32</sup>The shadow value of the equity issuance constraint is an equilibrium object and its functional form is unknown. Hence, it is not possible to derive an expression for the covariance term.

voluntary capital buffers will differ depending on whether banks are able to pay dividends before and after the capital requirement change or not, as in the latter case also the current shadow value of the equity issuance constraint is affected by the requirement change. As we will show in section 4 for the euro area model calibration, the "pricing channel" is close to zero when banks are able to pay dividends and hold their desired level of voluntary capital buffer, and it is on average around -0.1% for a 1pp increase in capital requirements when banks do not pay dividends but have sufficient voluntary capital buffers to absorb the increase in capital requirements.<sup>33</sup> Hence, the "pricing channel" with voluntary capital buffers is of similar or lower magnitude as the "pricing channel" in the absence of an equity issuance constraint.

**Proposition 5 ("Quantity channel" of changing capital requirements).** *In states of the world where the equity issuance constraint is more binding today than it is expected to be in the next period, i.e. in states where  $\chi^2 > \mathbb{E}(\chi^2)$ , banks will not hold voluntary capital buffers and equilibrium loans respond to changes in bank capital requirements via a "quantity channel", which is given by:*

$$\Delta \log(L') = -\Delta \log(R') \quad (25)$$

*Proof.* See Appendix B6. ■

The third insight is that in states of the world where the equity issuance constraint and the capital requirement constraint are both binding, i.e. where banks are capital constrained<sup>34</sup>, the maximum loan choice of banks is determined by available equity, net profits for the period, and the capital requirement. Even if banks would like to hold more loans on their balance sheets, they cannot do so because there is no other option to obtain additional equity than through retained earnings. In these states of the world, changes in capital requirements will impact bank loan supply through a "quantity channel", which acts like a financial accelerator (see Proposition 5): a change in the capital requirement will directly change the maximum loan quantity that banks are able to hold on their balance sheets, while fulfilling the requirement.

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<sup>33</sup>The magnitude of the "pricing channel with buffers" when banks do not pay dividends is increasing with the level of credit risk  $\theta$ . However, even for extremely high credit risk (e.g. 350 bps) its magnitude is only around -0.16%. For "normal" credit risk (below 100 bps) its magnitude is around -0.1%. For details see section 4.

<sup>34</sup>We refer to banks being "capital constrained" when the equity issuance constraint and the capital requirement constraint are both binding. When only the equity issuance constraint is binding we refer to banks being "equity issuance constrained".

Given that banks are highly leveraged - the capital requirement is usually around 10% - a small change in capital requirements can have a very large impact on bank loan supply when the "quantity channel" is present, i.e. when banks are capital constrained. For example, as can be seen from equation (25) in Proposition 5, when banks are capital constrained an increase in capital requirements from 10% to 11% will induce a drop of almost -10% in loans. If the initial capital requirement is 8% or 12% the "quantity channel" impact of a 1pp increase in the requirement would be -12% and -8% respectively (Figure 5 panel b). This impact on loan supply through the "quantity channel" is almost two orders of magnitude larger than the impact through the "pricing channel"! Hence, whenever the "quantity channel" (financial accelerator) is present, a release of capital requirements can have a large supporting impact on bank loan supply.

Propositions 5 and 4 show that the impact of changes in bank capital requirements on bank loan supply will be strongly state-dependent and determined by whether banks are capital constrained or not. In the former case, the "quantity channel" will be present and even small changes in capital requirements will induce large changes in bank loan supply. In the latter case, the "pricing channel" will be present and moderate changes in capital requirements will induce extremely small changes in bank loan supply. Based on the indicative magnitudes provided above, a 1pp increase in capital requirements can therefore lead to a drop in equilibrium loans of anything between 0% and -10%, depending on how much each of the channels becomes "activated" by the policy change. This state-dependence can be one explanation for why empirical studies often find vastly different magnitudes for the impact of changes in bank capital requirements on lending.

### 3.4 Insights for changes in capital requirements

For an economy with an equity issuance constraint it is instructive to see what kind of time-varying capital requirement rules would ensure that the "quantity channel" is never present at the margin so that capital requirements transmit to the economy only via the "pricing channel". Such rules would ensure that the economic costs of building regulatory capital buffers are low, while the benefits of releasing capital buffers are high, as releases are always large enough to prevent banks from deleveraging to meet capital requirements in situation where they incur losses.

To understand how such rules would have to look like, it is important to note that in the presence of an equity issuance constraint, the maximum capital ratio

that banks are able to fulfill in the next period without deleveraging is determined by the starting period capital ratio, the current return on risk-weighted assets, and the desired loan growth rate.<sup>35</sup> The return on risk-weighted assets is of central importance here. If banks make profits, equity can be increased through retained earnings. If banks make losses, equity will go down. How these increases or decreases in bank equity translate into changes in the capital ratio depends on the amount of risk-weighted assets. However, one needs to keep in mind that if desired loan growth is positive, risk-weighted assets will increase, which need to be partly financed with additional equity due to the capital requirement. Hence, for a given starting capital ratio and return on risk-weighted assets, a higher desired loan growth rate will reduce the maximum capital ratio that banks are able to fulfil in the next period.

**Proposition 6 (Capital requirement rules that avoid the "quantity channel").** *Policy makers can avoid the "quantity channel" with any rule for time-varying capital requirements that satisfies  $R' > 0$  and the following condition in all states of the world:*

$$R' < \left( CR + \frac{\pi^N}{\omega L} \right) \frac{1}{1 + g^*} \quad (26)$$

where  $(1 + g^*)$  is the desired growth of loans, defined as the ratio of the unconstrained optimal loan choice  $L^*$  in equation (20) and current loans  $L$ .

*Proof.* See Appendix B7. ■

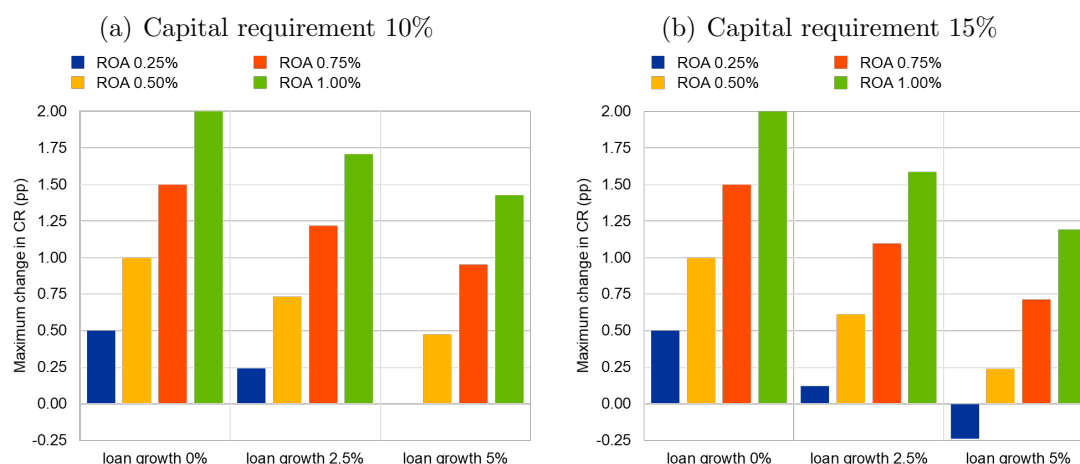
Policy rules which ensure that the time-varying capital requirement is always lower than the maximum attainable capital ratio under the unconstrained loan growth target, will also ensure that banks are never capital constrained and therefore that the "quantity channel" will never be present (Proposition 6). Of course, such policy rules will also need to ensure that the capital requirement never becomes negative. For this to be the case, sufficiently high capital requirements will need to be put in place in normal times when banks make positive profits, so that capital requirement releases in bad states of the world can be sufficiently large to allow banks to fully absorb losses without deleveraging and still be able to meet a positive capital requirement. The following paragraphs provide further intuition on how the build-up and release principles would look like.

The two key variables determining the maximum build-up speed without triggering the "quantity channel" are the return on risk-weighted assets and the desired

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<sup>35</sup>Note that the maximum capital ratio is obtained when dividend payouts are zero.

Figure 6: Examples of maximum changes in capital requirements that can be financed through retained earnings without triggering the "quantity channel"



Notes: A risk-weight of 50% is assumed for the calculations of maximum capital generation capacity.

loan growth rate. The following realistic examples provide insights into possible build-up speeds. Consider a "normal" level of bank profitability with a return on assets of 0.5%, a capital ratio and capital requirement of 10%, a risk-weight of 50%, and a desired loan growth rate of 0%. The maximum capital requirement next period without causing deleveraging would be 11%, i.e. an increase of +1pp would be possible (Figure 6 panel a). If the desired loan growth rate was 5%, the maximum capital requirement next period would be 10.48%, i.e. an increase of almost +0.5pp would be possible. In a "good" profitability state with a return on assets of 0.75%, the maximum increases in capital requirements would be +1.5pp and +0.71pp for target loan growth rates of 0% and 5% respectively and an initial capital requirement of 15% (Figure 6 panel b). In "normal" or "good" states of the world it should therefore be possible to build regulatory capital buffers at the pace of 0.5pp to 1pp per year without imposing big costs on the economy through reduced loan supply.

In "bad" states of the world, where banks make losses, the release of capital requirements would need to be at least as large as the return on risk-weighted assets, to avoid bank deleveraging.<sup>36</sup> Cumulative bank losses amounted to more than 1.6% of total assets during half of the past financial crisis episodes in European countries.<sup>37</sup> In one third of past crises, cumulative losses were larger than 3.4% of total assets. Assuming an average risk-weight of 50%, these numbers imply that capital requirement releases would need to be at least in the range of 3.2% to 6.8%

<sup>36</sup>Assuming that the bank would want to keep the loan book constant in the absence of any binding constraints.

<sup>37</sup>See p.4 in [Lang and Forletta \(2020\)](#).



to allow banks to absorb losses during crisis episodes without triggering deleveraging.

These indicative build-up and release magnitudes provide important lessons for the design of policy rules that manage to avoid the "quantity channel". First, they show that a gradual build-up of regulatory capital buffers is needed during "normal" times to avoid large costs in terms of reduced credit supply. However, the gradual build-up should start as soon as the economy enters "normal" or "good" states of the world, where banks make positive profits. This is because many periods of gradual regulatory buffer increases are needed before a sufficiently high level of capital buffers is built-up to allow for the necessary release magnitude during crisis episodes. Such a "gradual but early" buffer build-up rule will impose limited economic costs during the build-up phase, while minimizing the probability that a bad shock hits the economy before a sufficient level of releasable capital buffers has been built up. Our model therefore provides a theoretical justification for building up a positive countercyclical capital buffer in "normal" macro-financial environments. Quantitative results for different capital requirement rules are discussed in the next section.

## 4 Quantitative Insights

As there is no analytical closed form solution for the model with an equity issuance constraint, we solve it numerically with global solution methods and conduct various policy experiments regarding changes in bank capital requirements.<sup>38</sup>

### 4.1 Calibration

We calibrate our model at annual frequency to match key properties of euro area banking data. As our data source for empirical moments we take a large representative panel dataset of around 320 euro area banks starting in 2005.

The deposit interest rate ( $i^D$ ) is set equal to 0.02, i.e. 2% per annum, based on

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<sup>38</sup>We use a policy function iteration algorithm similar to [Brumm and Grill \(2014\)](#) and [Menno and Oliviero \(2020\)](#) to solve the model. In the absence of loan adjustment costs our model has a "wealth recursive" structure as in [Kubler and Schmedders \(2003\)](#): net worth, defined as beginning of period equity plus realized profits in period  $t$ , is a sufficient statistic for the endogenous state variables. The transformed state space is therefore given by the exogenous shock  $\theta$  (loan impairments) and net worth. We solve the model on an equally spaced grid with 301 grid points for  $\theta$  (using the [Tauchen \(1986\)](#) method) and on 301 equally spaced grid points for net worth. For details on the numerical solution algorithm see Appendix C.

the average cost of liabilities for euro area banks (see Table 1). We calibrate the operating cost parameter ( $\kappa$ ) to match the average cost-to-asset ratio for euro area banks, which is 1.4%.<sup>39</sup> The average risk-weight ( $\omega$ ) is set to 0.48 to match the average value of the risk-weighted assets to total assets ratio for euro area banks.

Table 1: Empirical moments for key bank variables

Variable	mean	sd	p1	p5	p25	p50	p75	p95	p99	N
Average yield on assets	3.4	1.5	0.4	1.2	2.3	3.2	4.3	6.1	7.7	3,527
Average cost of liabilities	2.0	1.3	0.1	0.3	1.0	1.7	2.8	4.5	5.9	3,496
Net interest margin	1.5	0.8	0.0	0.2	0.9	1.5	2.0	2.9	4.1	3,597
Other income to assets	0.8	0.7	-0.5	-0.1	0.4	0.8	1.1	1.8	3.5	3,520
Cost-to-asset ratio	1.4	0.8	0.0	0.1	0.9	1.4	1.9	2.8	4.0	4,046
Provisioning rate	0.6	0.9	-0.5	-0.1	0.1	0.3	0.8	2.4	4.9	3,591
Return on assets	0.5	1.0	-3.5	-1.1	0.2	0.5	0.9	1.7	2.7	4,072
CET1 capital ratio	13.0	5.1	4.3	5.9	9.2	12.5	15.9	22.7	26.8	3,093
Average risk-weight	48.3	20.8	5.0	14.9	32.6	48.0	63.3	82.3	95.6	3,446
Price-to-book ratio	1.2	0.9	0.1	0.2	0.5	0.9	1.7	3.0	4.3	7,311

Source: SNL Financial, Bloomberg

Notes: All variables except the price-to-book ratio are based on an unbalanced annual panel of around 320 euro area banks from 2005 to 2019. All variables are expressed in percent, except the price-to-book ratio. The return on assets is measured before tax. The price-to-book ratio is based on quarterly data from Bloomberg for around 70 listed banks. Additional figures illustrating the properties of the key bank variables can be found in Appendix A.

The capital requirement ( $R$ ) for our benchmark model is set to 10%, based on the aggregate capital requirement for euro area banks (see Table 2).<sup>40</sup> The cost of equity ( $\rho$ ) is set to 0.08, i.e. to 8% per year, which is consistent with the empirical evidence on the cost of equity for euro area banks presented in Altavilla et al. (2021).

The interest rate semi-elasticity of loan demand ( $\epsilon$ ) is set to 3, which is a plausible average estimate based on the empirical findings in DeFusco and Paciorek (2017), Davis et al. (2020), Best et al. (2020), and Bhutta and Ringo (2021) for mortgage credit. The loan demand shifter ( $\lambda$ ) is set to 0.1215, in order to target median steady state loans of 1. However, the specific choice of  $\lambda$  is irrelevant for the properties of the model and only determines the scale of loans. The elasticity of substitution between loans from different banks ( $\mu$ ), which determines banks' market power, is set to 100. This value is chosen to ensure that the average price-to-book ratio in

<sup>39</sup>We use total operating costs to measure costs and we compute the cost-to-asset ratio rather than the cost-to-loan ratio, as loans in the context of our model are equal to total assets.

<sup>40</sup>Capital requirements include Pillar I and Pillar II requirements for CET1, AT1 and AT2 instruments and were on aggregate close to 10% between 2019 and 2021 for banks supervised by the ECB.

Table 2: Calibrated values of model parameters

Parameter	Value	Source
$\rho$	0.08	Based on bank cost of equity estimates in <a href="#">Altavilla et al. (2021)</a>
$i^D$	0.02	Empirical: Average cost of liabilities for euro area banks 2005-2019
$\kappa$	0.014	Empirical: Average cost-to-asset ratio for euro area banks 2005-2019
$\omega$	0.48	Empirical: Average risk-weight for euro area banks 2005-2019
$R$	0.10	Empirical: Aggregate ECB capital requirement 2019 - 2021
$\lambda$	0.1215	Scaling parameter set to target median steady state loans of 1
$\epsilon$	3	Based on average of estimates from various empirical studies
$\mu$	100	Set to target the empirical mean of the price-to-book ratio of 1.2
$\alpha_0$	-2.40	Empirical: Estimated intercept of a log AR(1) process for cost of risk
$\alpha_1$	0.56	Empirical: Estimated persistence of a log AR(1) process for cost of risk
$\alpha_2$	0.67	Empirical: Estimated shock SD of a log AR(1) process for cost of risk

our benchmark model mimics the empirical average price-to-book ratio of 1.2.<sup>41</sup>

Finally, the parameters of the stochastic process for bank provisioning (cost of risk) are obtained from econometric estimates based on panel data.<sup>42</sup> As the distribution of bank provisioning has a fat right tail in the data, the process is specified as a log AR(1) process. The estimation yields coefficients for the intercept term ( $\alpha_0$ ) of -2.40, for the autocorrelation coefficient ( $\alpha_1$ ) of 0.56, and for the shock standard deviation ( $\alpha_2$ ) of 0.67. A summary of all calibrated parameter values can be found in Table 2.

Table 3 summarizes the stochastic steady state values of the benchmark model. In terms of untargeted data moments, the model implies a mean return on assets (ROA) of 0.67 %, compared to 0.5% in the data. The mean interest rate in the model is 4.4%, compared to 4.2% in the data.<sup>43</sup> The model implies that 30.4% of the time banks do not pay dividends, in line with the data (see footnote 23). The benchmark model generates on average a "voluntary" capital buffer above the capital requirement of 51 bps. This means that banks to some extent self insure against downside risk. However, this self insurance is not large enough to avoid occasional severe drops in loan supply: 8% of the years both the equity issuance

<sup>41</sup>Since the global financial crisis, profitability of euro area banks has been low (cost of equity estimates often being above the return on equity) and price-to-book ratios have often been below 1. Both observations indicate low market power of banks.

<sup>42</sup>We aggregate bank-level provisioning at country level and estimate the process based on country panel data with fixed effects using GMM. We exclude Greece and Lithuania from the estimation as these countries represent extreme outliers in terms of provisioning during the global financial crisis and euro area sovereign debt crisis.

<sup>43</sup>The data counterpart for the loan interest rate in the model is the yield on assets (3.4%) plus other income to assets (0.8%), see Table 1.

Table 3: Stochastic steady state of the benchmark economy

Variable	mean	sd	p1	p5	p25	p50	p75	p95	p99
<i>Exogenous shock</i>									
Loan impairments (bps)	58.3	53.2	6.7	11.6	24.9	42.8	72.4	157.9	271.5
<i>Endogenous variables</i>									
Loans	0.99	0.04	0.79	0.95	0.99	<b>1.00</b>	1.00	1.01	1.01
Loan interest rate	4.42	1.82	3.84	3.86	3.91	4.05	4.30	5.69	11.93
Return on assets	0.67	3.88	-1.34	-0.47	0.15	0.37	0.53	1.59	9.32
Capital ratio	10.51	0.61	10.00	10.00	10.00	10.34	10.76	11.66	12.83
Dividends/assets	0.46	0.75	0.00	0.00	0.00	0.32	0.59	1.40	4.08
Price-to-book ratio	<b>1.17</b>	0.20	0.98	1.04	1.10	1.13	1.17	1.47	2.07
$Pr(d = 0)$	30.43								
$Pr(\text{quantity channel})^\dagger$	8.05								

Notes: Calibration targets in bold. All other reported numbers are non-targeted moments of the benchmark model. <sup>†</sup>Fraction of years in which the quantity channel kicks in. This is the case when the equity issuance constraint and the capital requirement constraint are both binding at the same time ( $d_t = 0$  and  $CR_{t+1} = R$ ).

and the capital requirement constraint are binding at the same time and, hence, the "quantity channel" forces banks to substantially reduce loan supply in response to negative shocks. The non-linearity due to the "quantity channel" explains the fat left tail of the loan distribution: 1% of the time loans are 21% below the median value. In 22.38% of the years banks choose zero dividends, all profits are retained to build up voluntary capital buffers, but the capital requirement constraint is not binding.

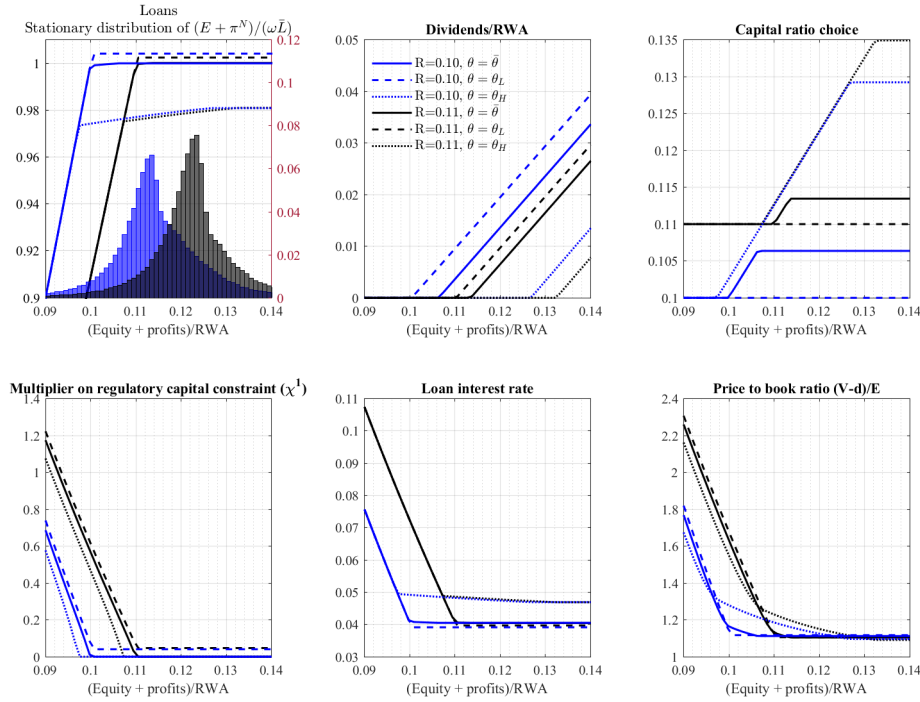
## 4.2 Equilibrium policy functions

Consider first the policy functions for a median credit risk state in our benchmark economy (solid blue lines in Figure 7). The loan and capital ratio policies are flat whenever the endogenous state variable equity plus profits<sup>44</sup> is higher than the "optimal" capital ratio target of 10.65%, i.e. to the right of 0.1065 on the x-axis. In these states, banks are unconstrained in their loan and capital ratio choices and use their remaining available internal funds to pay dividends, which can be seen from the upward sloping solid blue dividend policy to the right of 0.1065 in Figure 7.

As profits decrease, i.e. going from right to left on the x-axis, banks decrease dividend payouts but keep the voluntary capital buffer at its optimal level. Once banks

<sup>44</sup>Normalised by steady state risk-weighted assets.

Figure 7: Policy functions of the calibrated model



Notes: All policy functions are plotted against the endogenous state variable, which is given by beginning-of-period equity plus realized profits, normalised by median steady state risk weighted assets ( $\omega \cdot 1$ ). Blue lines: benchmark model with capital requirement  $R = 0.1$ . Black lines: counterfactual economy with capital requirement  $R = 0.11$ . The histograms show the stationary distributions of the normalised endogenous state variable for the respective economies (right axis). Solid lines: policy functions for a cost of risk realization equal to its median. Dashed lines: policy functions for a low cost of risk realization equal to the 5th percentile of its stationary distribution. Dotted lines: policy functions for a high cost of risk equal to the 95th percentile of its stationary distribution.

start to incur losses, which will be the case left of 0.1065 on the x-axis for median credit risk, banks stop paying dividends, the equity issuance constraint becomes binding, and banks gradually reduce their voluntary capital buffer. A decreasing capital buffer, in turn, increases the probability of a binding equity issuance constraint next period: banks want to be compensated for this additional risk and therefore the interest rate slightly increases (solid blue line to the left of 0.1065 in the bottom middle panel of Figure 7) while loan supply decreases a little bit. This explains the slightly steeper slope of the loan policy to the left of the point where dividends are zero. The loan policy functions display a pronounced kink when losses start to exceed the available capital buffer, i.e. equity minus losses is below the capital requirement. In these states, both the equity issuance constraint and the capital requirement constraint are binding, and banks are forced to deleverage.<sup>45</sup>

<sup>45</sup>The solid blue loan policy line kinks slightly to the left of 0.1. This is because we normalize the state variable (x-axis) by median steady state RWA and not by actual RWA. Below 0.1065, however, loans are lower than the median level as discussed in the text.

Banks' capital buffer targets increase when banks are hit by high loan impairment shocks (dotted versus solid lines in upper right panel of Figure 7). As credit risk is autocorrelated, a bad shock today implies a higher probability of a bad shock next period, making a binding equity issuance constraint next period more likely, and therefore the expected marginal value of having an additional unit of equity next period increases. When banks are hit by positive shocks with extremely low credit risk, their optimal target capital buffer goes down to zero (dashed lines in upper right panel). In these states, the insurance motive is low relative to the cost of equity because the probability of a binding equity issuance constraint next period is low.

Figure 7 also illustrates the state-dependence of changes in capital requirements. On the one hand, to the right of the x-axis equity plus profits is sufficiently high to absorb an increase in capital requirements from 10% to 11% without the capital requirement becoming binding, and the loan policy functions are very close together (black versus blue lines in upper left panel). This is the "pricing channel" of changing capital requirements. On the other hand, to the left of the x-axis the loan policy functions in Figure 7 are far apart as equity plus realized profits is not enough to absorb the capital requirement increase. In such states, banks' capital ratios will be below requirements after the requirement change: the "quantity channel" kicks in and banks are forced to deleverage. The flip side is that in these states, capital releases have large positive effects on loan supply. As an illustration, consider the economy with capital requirements of 11% (black lines). In this economy, the median capital ratio is 11.32%. Suppose banks experience a bad credit risk shock implying a loss over RWA of -1.32%. The capital ratio net of realized losses would then be 10%, so that banks would need to cut lending by 9.1% to meet capital requirements of 11%. If contemporaneously to the shock capital requirements are lowered to 10%, banks are able to just satisfy the capital requirement and loans drop only by -2.5%.<sup>46</sup>

It is also instructive to use the calibrated model to assess the magnitudes of the "pricing channel" when banks hold voluntary capital buffers (see Proposition 4), as analytical closed-form expressions are not available. As shown in panel (a) of Figure 8, the "pricing channel" impact of a 1pp capital requirement increase is close to zero when banks hold voluntary capital buffers and pay dividends before and after

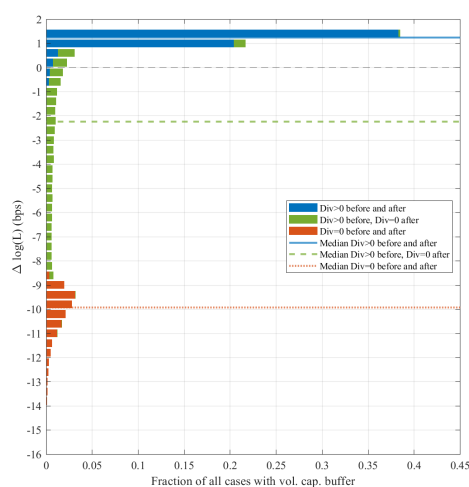
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<sup>46</sup>See solid black line versus dotted blue line in the upper left panel at 0.1 on the x-axis. For the considered shock, the unconstrained loan drop is -1.9%. The actual loan drop of -2.5% is slightly larger. The reason is that banks want to hold voluntary capital buffers. As the equity issuance constraint is binding also after the requirement release, banks have to "finance" the capital buffer by cutting lending relative to the unconstrained loan choice.

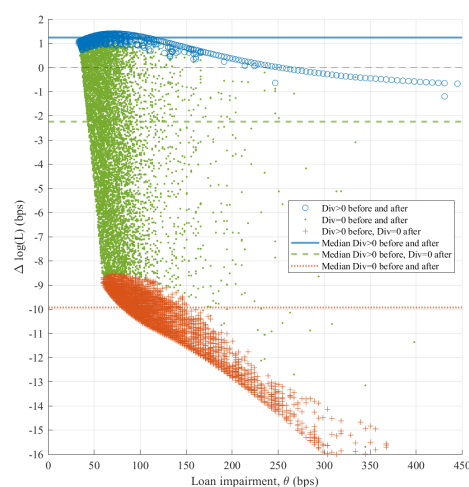
the requirement change. It is even slightly positive at 1 bps for the vast majority of simulated cases. The "pricing channel" impact of a 1pp capital requirement increase is on average -10 bps, and varies mostly between -8.5 and -11.5 bps, when banks hold voluntary capital buffers and do not pay dividends before and after the requirement change.<sup>47</sup> These magnitudes are very similar to the magnitudes of the "pricing channel" in the absence of an equity issuance constraint. Panel (b) of Figure 8 further illustrates that the "pricing channel" impact on lending when banks hold capital buffers is negatively correlated with credit risk, in particular when banks do not pay dividends. However, panel (b) also illustrates that for credit risk lower than its 95th percentile (158bps), the magnitude of the "pricing channel" when banks hold voluntary buffers is less than 12 bps. Hence, as long as banks have sufficient capital headroom and profitability to "absorb" a given increase in capital requirements, the impact on lending should be small as only the "pricing channel" with voluntary buffers will be active.

Figure 8: "Pricing channel" impact on lending when banks hold voluntary capital buffers for a capital requirement increase from 10% to 11%

(a) Simulated distribution of lending impact



(b) Effect of credit risk on lending impact



Notes: We simulate the benchmark economy with the 10% capital requirement for 200,000 periods. We then select all simulated states where banks hold voluntary capital buffers. We then apply the policy functions for the economy with the 11% capital requirement to these states and only keep simulated states where banks again hold voluntary capital buffers. For this set of simulated states we compare the loan policies under the two different capital requirements and produce the charts.

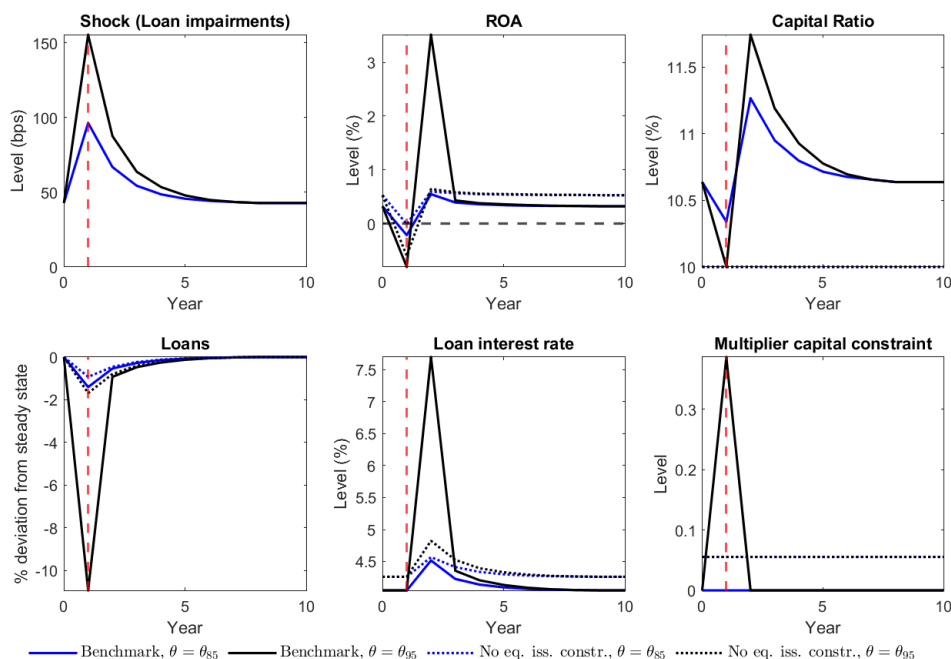
<sup>47</sup>For cases where banks hold voluntary capital buffers but switch from paying dividends before the change to not paying dividends after the change, the impact on lending lies between 1 bps and -10 bps.



### 4.3 Impulse responses to a negative shock

The impulse responses to a negative shock in Figure 9 further illustrate the non-linearity of the model. Consider first an increase in loan impairments to around 85 bps (blue solid lines). Even though banks make losses in response to this shock (ROA is slightly negative, solid blue line, top middle panel) and the equity issuance constraint is binding as dividends are zero in response to this shock (not shown), voluntary buffers are sufficient to absorb these moderate losses. However, after the shock, the capital buffer is below the desired level, implying a higher risk of a binding equity issuance constraint next period. Therefore, the decrease in the capital ratio is accompanied by a decrease in loans. Overall, loans drop by 1.29pp, 0.44pp higher compared to the model without the equity issuance constraint (solid blue vs. dashed blue line in bottom left panel).<sup>48</sup>

Figure 9: Impulse responses to a negative shock



Notes: Impulse responses to an increase in loan impairments for the benchmark economy, i.e. a "bad" shock. Blue lines: impulse responses to an increase in loan impairments to a level equal to the 85th percentile of its stationary distribution. Black lines: impulse responses to an increase in loan impairments to a level equal to the 95th percentile of its stationary distribution. Solid lines: benchmark economy with  $R = 0.1$ . Dotted lines: model without equity-issuance constraint.

Things look quite different for a large increase in loan impairments to 156bps (black solid lines). Banks realize large losses in response to this shock: in year 1 the

<sup>48</sup>The higher impact in the benchmark economy than compared to the economy without equity issuance constraint comes from the effect on the covariance between marginal return on equity and impairment level next period, that is absent in the model without the equity issuance constraint.

ROA decreases to -0.8%. This is equivalent to a return on risk weighted assets of -1.67pp, so the risk weighted loss exceeds the voluntary capital buffers of 0.65pp by 1.02pp. Without adjusting loans, banks would end up with a capital ratio of 8.98%. In order to meet the capital requirement of 10%, banks are forced to deleverage and reduce loans by 10.76% (solid black line, bottom left panel). Hence, loans drop by 9.25pp more compared to the model without the equity issuance constraint (dashed black line, bottom left panel). As a result of reduced loan supply, the equilibrium interest rate shoots up to 7.7%.

The increased interest rate due to lower loan supply leads to profits in year 2, explaining the increase in ROA in year 2. Banks use part of these profits to increase voluntary capital buffers from year 2 onwards. When experiencing a negative shock, banks' self insurance motive increases because of the persistent and fat-tailed loan impairment process. When a negative shock hits, further negative shocks become more likely. This makes a binding equity issuance constraint more likely next period and increases the self-insurance motive to hold additional capital buffers in bad states.

#### 4.4 Unanticipated capital requirement changes

Consider first the transition dynamics for an unanticipated increase in capital requirements (Figure 10), starting from the median of the stochastic steady state of the benchmark economy and keeping the loan impairment shock at the median throughout. In year 0, prior to the unexpected increase in capital requirements, the capital ratio is 10.65%, i.e. banks hold a voluntary capital buffer of 0.65pp, and the return on risk weighted assets is 0.67pp, implying "capital headroom" of 1.32pp above requirements.<sup>49</sup> This headroom is the maximum by how much the regulator can increase capital requirements without pushing banks into the quantity channel, provided the loan impairment shock stays at the median.

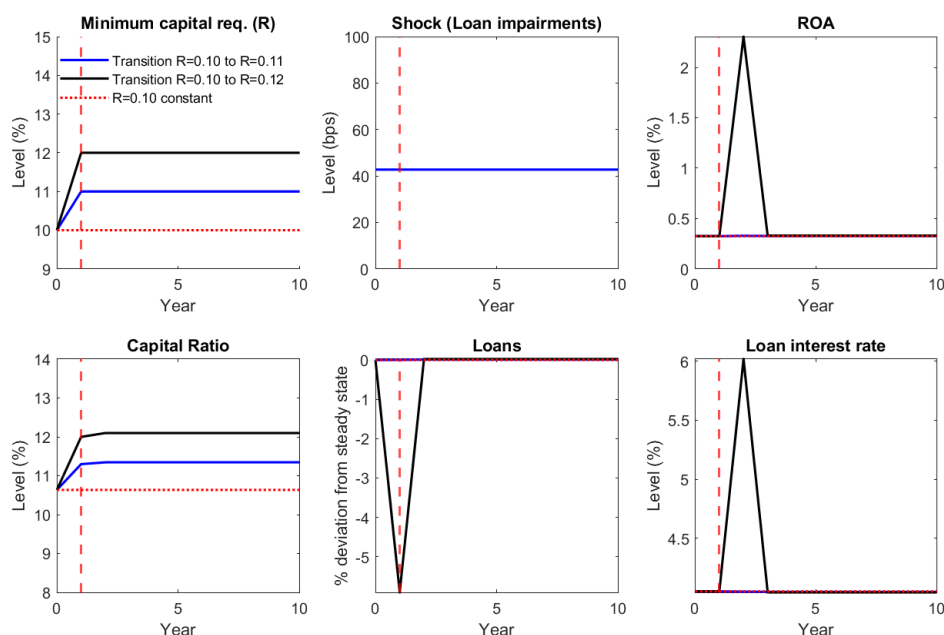
In response to the 1pp unanticipated increase in capital requirements in year 1 (blue lines), banks use part of their profits and existing voluntary buffers to meet the new capital requirements. The rest of their "capital headroom" of 0.32pp is used by banks to hold voluntary capital buffers above the new 11 % capital requirement (blue line bottom left panel). This is slightly below the target capital buffer and banks pay zero dividends. As shown by the blue line in the lower middle panel, bank

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<sup>49</sup>The return on risk weighted assets in year 0 is the ROA of 0.32% (upper right panel) divided by the risk weight of 0.48.

loans remain virtually unchanged in response to the capital requirement increase. The reason is that banks hold voluntary capital buffers before and after the capital requirement increase and also pay dividends before. Hence, the "pricing channel" with voluntary buffers and a dividend payment switch is at work, with its limited impact on loan supply.

Figure 10: Unanticipated increase in capital requirements in normal times

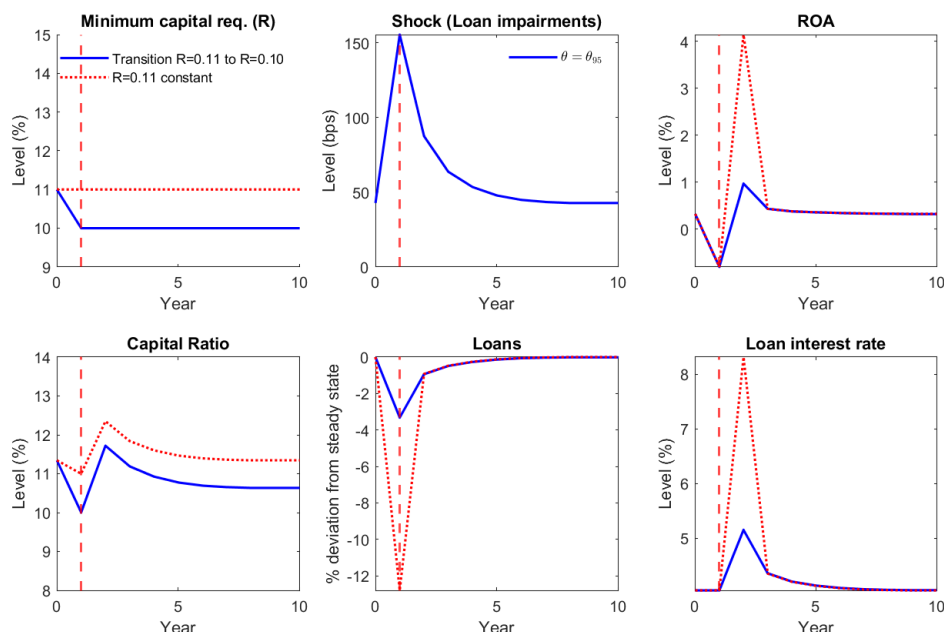


Notes: Blue lines show a 1pp unanticipated increase of the capital requirement in period  $t = 1$ . Black lines: a 2pp unanticipated increase of the capital requirement in period  $t = 1$ . Red dotted line: benchmark model without changes in the capital requirement. The loan impairment shock plotted in the upper middle panel is the same for all shown economies; the shock is kept at the median of its stationary distribution.

In contrast, the unanticipated 2pp capital requirement increase in year 1 exceeds banks' "capital headroom", leaving banks with an effective capital ratio gap of 0.68pp (2pp-1.32pp). As banks cannot issue new equity, the 2pp increase in capital requirements pushes banks into the quantity channel and they are forced to cut lending by 5.8% in year 1 to meet the higher capital requirements (black line bottom middle panel). The drop in loan supply leads to an increase in the equilibrium interest rate in year 2, pushing up net interest income and consequently also the ROA. Because of the increased profits, banks are unconstrained in their loan choice from year 2 onwards, and are able to hold the target voluntary capital buffer and pay dividends.

Consider now the impact of an unanticipated decrease in capital requirements when banks are hit by a large adverse shock, as shown in Figure 11. We assume that in year 0, the economy has converged to the stochastic steady state with a

Figure 11: Unanticipated decrease in capital requirements during crisis times



Notes: Blue lines show a 1pp unanticipated decrease of the capital requirement in period  $t = 1$ . Black lines: a 2pp unanticipated decrease of the capital requirement in period  $t = 1$ . Red dotted line: benchmark model without changes in the capital requirement. The loan impairment shock plotted in the upper middle panel is the same for all shown economies: the size of the shock is chosen such that in period 1 loan impairments increase to their 98th percentile of distribution conditional on the median impairment level.

capital requirements of 11%. In year 1, loan impairments increase from 43bps to 156bps, implying a decrease in the ROA to -0.8%. Hence, the loss relative to risk-weighted assets exceeds the voluntary capital buffer of 0.32pp by 1.35pp.<sup>50</sup> Without adjustments to their loan portfolio, banks would end up with a capital ratio of just 9.65%. In the absence of policy interventions by the regulator, banks are forced to cut lending by 12.27% in order to meet capital requirements of 11% (red dotted lines).

If the regulator is able to contemporaneously decrease capital requirements by 1pp in year 1, additional headroom is created that can be used by banks to absorb part of the losses. After the 1pp release, banks only have to cover the remaining 0.35pp risk weighted loss through deleveraging, implying a decrease in loans of just -3.2%. This is 9pp less than the drop in loans in the no-release scenario (blue versus red dotted line, bottom middle panel in Figure 11).<sup>51</sup> Accordingly, equilibrium

<sup>50</sup>A ROA of -0.8pp is equivalent to a return of risk weighted assets of -1.67pp.

<sup>51</sup>Note that with the considered size of the shock, banks' capital requirement and equity issuance constraint are both binding after the capital requirements decrease. For smaller negative shocks banks may no longer be capital constrained after the release and may find it optimal to use part of the capital freed up for a buffer: they choose a capital ratio above the post-release capital requirements because of the precautionary motive in bad states to shield against potential additional

lending rates increase much less in year 2 and the ROA overshoot is much less pronounced in the simulation with the capital release. Banks use the realised profits in year 2 to build voluntary capital buffers that are higher than the pre-shock voluntary buffers (the capital ratio increases to 11.72% in year 2) and subsequently decrease self-insurance again as credit risk reverts back to the mean. After year 3, loans and other equilibrium variables (with the exception of the capital ratio) evolve similar to the economy without the release in capital requirements.

## 4.5 State-dependent capital requirements

We now study the equilibrium for an economy where capital requirements follow a state-dependent rule in the spirit of Proposition 6. Specifically, we consider a simple state-dependent capital requirement rule of the form:<sup>52</sup>

$$R_{t+1} = \begin{cases} R^{max} & \text{if } \xi \left( \frac{E_t + \pi_t^N}{\omega \bar{L}} \right) > R^{max} \\ \xi \left( \frac{E_t + \pi_t^N}{\omega \bar{L}} \right) & \text{if } R^{min} \leq \xi \left( \frac{E_t + \pi_t^N}{\omega \bar{L}} \right) \leq R^{max} \\ R^{min} & \text{if } \xi \left( \frac{E_t + \pi_t^N}{\omega \bar{L}} \right) < R^{min} \end{cases}$$

where  $R^{min}$ ,  $R^{max}$ ,  $\xi$  are parameters, and  $\bar{L}$  is the median steady state loan level. Note that without  $R^{max}$  the model would no longer be stationary. As a benchmark, we set  $R^{min} = 0.1$ ,  $R^{max} = 0.15$ , and  $\xi = 0.95$ . We set the lower bound of the capital requirement rule to 10%, i.e. the capital requirements currently observed in the data.<sup>53</sup> The upper bound of the capital requirement rule of 15% implies a maximum releasable capital requirement of 5pp: a full release of that size can almost absorb two consecutive realisations of the 1st percentile of the ROA distribution in the benchmark model. A value of  $\xi = 0.95$  implies that the regulator at any point in time leaves the banks some headroom: increases are gradual and less than the maximum possible amount; for capital releases, the regulator releases always 5% more than equity minus incurred losses would imply, provided  $R^{min}$  is not hit.

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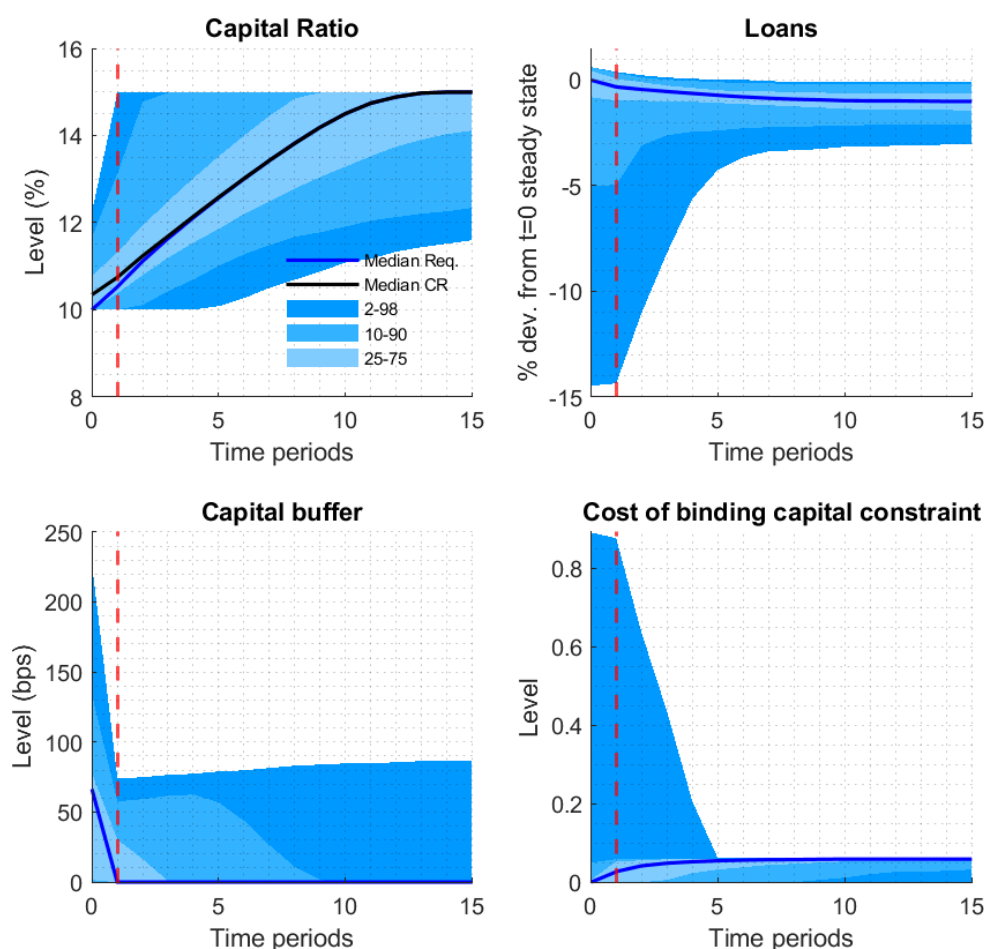
negative shocks in the future. In those states, banks would no longer be quantity constrained after the change. However, as far as the equity issuance constraint continues to bind in order to finance the capital buffer, loans would be still lower than the unconstrained loan choice.

<sup>52</sup>We assume full commitment from the regulators side. In addition, individual banks are atomistic and do not take into account the effect of their choices on the aggregate capital ratio. For details on the model with state dependent rule, see appendix D.

<sup>53</sup>Another reason for not choosing an  $R^{min} < 10\%$  is that in the benchmark model with  $R = 10\%$  there are no states where beginning of period equity plus profits is negative, so that banks always stay solvent in the benchmark economy. For capital requirements below 10% this might no longer be ensured given the calibration of the credit risk shock.

To illustrate the workings of the rule, consider a median return on risk weighted assets of 0.75% and an average capital ratio of 10.65%, a value of  $\xi = 0.95$  implies that the regulator increases capital requirements in normal times according to  $0.95(10.65 + 0.75) = 10.83\%$  and leaves 0.57pp to the banks for dividend pay-out, building buffers, and/or to increase loans. Similarly, when starting from a hypothetical capital ratio of 14%, the rule implies that the regulator increases capital requirements in normal times to  $0.95(14 + 0.75) = 14.0125\%$ . For decreases, given a capital ratio of 15% and a return on risk weighted assets of -1.67%, the regulator would lower the capital requirements to  $0.95(15 - 1.67) = 12.66\%$ , a release of 2.34pp.

Figure 12: Transition from constant to state-dependent capital requirements



Notes: Solid lines show median responses over 100000 simulated economies. The shaded areas show the 2%-98%, 10%-90%, and 25%-75% percentile ranges across the simulated economies.

The gains of the state-dependent rule compared to a constant capital requirement come in form of reduced volatility in bank lending (upper right panel, Figure 12). The gains are significant, as the state-dependent rule lifts in particular the bottom

percentiles of the loan distribution over time. The state-dependent rule works by counteracting the non-linearity induced by the quantity channel: decreasing capital requirements in bad states reduces the marginal cost of a binding capital requirement constraint when the costs are highest (lower left panel, Figure 12). After 5 years, the fat tail of the loan distribution is gone and the loan distribution is similar to an economy without an equity issuance constraint (not shown). The bottom 2%-percentile of the loan distribution increases from -15% in year 0 to -4.2% in year 5. The bottom 10%-percentile increases from -5% to -2.4% after 5 years.

These large gains come at an arguably small cost in the form of reduced median bank loans along the transition path. In year 5, loans are -0.66% lower compared to the benchmark model with a constant 10% capital requirement, and -0.92pp lower compared to the benchmark model after 15 years, when median capital requirements have reached 15%. These are arguably moderate costs, less than one fifth compared to the median increase in capital requirements of 5pp, implying an average loan decrease of 6bps per year over the 15 year horizon.<sup>54</sup>

What happens to voluntary capital buffers? After year 1, median capital requirements increase moderately each period, so that banks can fulfil the additional requirements by retaining profits and still have some headroom for building voluntary capital buffers and/or dividend payouts. The median voluntary capital buffer banks choose decreases with increasing capital requirements, and at a median level of 12% to 13%, banks no longer hold voluntary capital buffers above the capital requirements in median states. The state-dependent rule provides insurance to banks against large negative shocks and therefore the self-insurance motive decreases along the transition.

Regarding the speed of the transition, there are heterogeneous effects, depending on where the economy starts out and what type of shocks hit the economy along the transition, as shown by the confidence bands around median capital requirements.

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<sup>54</sup>The cumulative decrease is larger than the pricing channel derived in the analytical section for a capital requirements change from 10% to 15%. The reason has to do with the buffers and its decrease along the transition. In fact, at capital requirements of 10% in year 0, the median cost of a binding capital requirement constraint is zero because banks hold buffers on average. This means, in year 0 the median interest rate is lower (loans are higher) than in the economy without the equity issuance constraint; in the latter, the capital requirement constraint is always binding and therefore the cost of the binding capital requirement constraint is equal to the equity premium leading to a higher interest rate on average. With the switch to the state dependent rule in year 1 and the associated increase in median capital requirements, there are less and less states where banks hold buffers. Therefore, the median cost of a binding capital requirement constraint increases over time until it converges after 5 years to the maximum possible amount equal to the equity premium. Therefore, until year 5, there is an additional effect on median bank loans, and after year 5, median loans decrease according to the pricing channel.



For economies that have favourable conditions with low impairments along the transition, the increase in capital requirements is steeper than the median. Economies that are hit by unfavourable shocks along the transition experience a slower build up of the capital requirements than the median. In fact, the lower tail of capital requirements distribution stays at 10% for several years hints at the fact that releases are already necessary along the transition for large negative shocks. In addition, the releases are of insufficient size until median capital requirements reach about 12.6% in year 5; only then the lower tail of the capital ratio distribution starts to increase.<sup>55</sup>

To summarize, our numerical exercise shows how the state-dependent capital requirement rule can provide insurance to the banks against bad shocks and prevents the associated detrimental non-linearities. With constant capital requirements, banks do not insure enough against the non-linearities induced by bad shocks. Their voluntary buffers are too small. The regulator sets higher requirements in normal states and then reduces them in bad states. This avoids that banks become occasionally constrained. At the same time this benefit comes at arguably moderate costs.

## 5 Qualitative discussion of model extensions

As our model is stylised in some dimensions, it is useful to discuss qualitatively if and how our main results regarding state-dependence would change with relevant model extensions.

First, what would change if deposit funding costs were to go down in response to higher bank capitalisation, i.e. if at least a partial Modigliani & Miller offset was present? Such a mechanism should mainly lower the lending impact through the "pricing channel" further. This is because the weighted average funding cost of loans would increase less for a given increase in capital requirements. The lending impact through the "quantity channel" should not be affected, as it depends on the presence of a binding equity issuance constraint. However, the probability of the "quantity channel" becoming active could go up, as losses could be amplified due to

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<sup>55</sup>Note that the lower tail of the capital requirements distribution stays at 10% even after year 5 (not shown). This has to do with the parameter value of  $\xi = 0.95$ : the rule releases more than necessary. In fact, the lower tail of the distribution of the capital ratio chosen by banks starts to increase after year 5, so banks choose a capital ratio above capital requirements when large negative shock hit.



increases in deposit funding costs in "bad" states where bank capital ratios decrease. Overall, the state-dependent impact of changes in bank capital requirements should remain.

Second, what would change if some bank equity issuance was possible but costly, e.g. through incurring quadratic issuance costs? Such a model extension would not change the impact through the "pricing channel" in "normal" times, as equity issuance is not needed in these states. However, the impact of the "quantity channel" in "bad" states would likely be reduced or replaced with a second "pricing channel". However, the impact of this second "pricing channel" in "bad" states should be considerably higher than the standard "pricing channel" described in this paper, as the equity issuance cost would need to be passed on to borrowers. Hence, the impact of changes in bank capital requirements should remain state-dependent, but the impact in "bad" states could be lower than derived in this paper, depending on the exact nature and magnitude of the equity issuance costs.

Third, what would change if there were additional mechanisms that induced banks to hold higher voluntary capital buffers, such as loan liquidation (adjustment) costs or a dependence of deposit funding costs on capital headroom? To the extent that voluntary capital buffers would not adjust 1-for-1 upon impact with capital requirement changes, higher voluntary capital buffers would allow banks to absorb larger increases in capital requirements or larger shocks to profitability without becoming capital constrained. Hence, such model extensions should mainly reduce the probability that the "quantity channel" becomes active. However, the state-dependent impact of changes in bank capital requirements should remain.

Fourth, what would change if banks had some preference for dividend smoothing, e.g. through habit persistence or through curvature in the utility function? On the one hand, the impact of the "quantity channel" in "bad" states could be amplified as banks would be reluctant to reduce dividends to zero even when faced with large losses. On the other hand, the self-insurance motive of banks could be larger, inducing them to hold higher voluntary capital buffers, and potentially reducing the probability of the "quantity channel" occurring. However, for empirically plausible (limited) degrees of dividend smoothing the quantitative impact on the results derived in this paper should be small. Overall, the state-dependent impact of changes in bank capital requirements should remain.

Fifth, what would change in general equilibrium if loan demand was stochastic and negatively correlated with credit risk, e.g. because a negative TFP shock pushes

up defaults and reduces loan demand at the same time? Such general equilibrium effects should mainly amplify the negative impact of a given credit risk shock on bank profits, as interest rates would go down in states where credit risk increases. This should mainly impact on the calibration of the model and the probability of the "quantity channel" becoming active. However, the state-dependent impact of changes in bank capital requirements should remain.

Sixth, what would change in general equilibrium if loan demand was also affected by changes in capital requirements (in addition to the impact on loan supply)? For example, lower equilibrium loans due to reduced loan supply could lower production and income, with a negative impact on loan demand. This could potentially amplify the impact of changes in capital requirements on equilibrium loan quantities. However, such amplification is unlikely to be of quantitative importance for the "pricing channel", as the change in equilibrium loans due to reduced loan supply is just around -0.1%. Hence, general equilibrium feedback effects on loan demand should be limited. In cases where the "quantity channel" becomes active, this amplification could be more meaningful. Overall, the state-dependent impact of changes in bank capital requirements should remain with such general equilibrium effects, and if anything should even be more pronounced.

## 6 Conclusion

Various empirical papers have shown that the lending impact of changes in bank capital requirements varies considerably depending on bank conditions and the state of the macro-financial environment. However, this feature of state-dependence is missing from standard macro models with a banking sector.

To study the state-dependent impact of changes in bank capital requirements within a structural setting, we develop a non-linear dynamic stochastic equilibrium model of the banking sector that features monopolistic competition and two occasionally binding constraints. The first one is a (time-varying) regulatory capital requirement. The second one is an equity issuance constraint, or equivalently a non-negativity constraint on dividends, which implies that banks can only accumulate equity through retained earnings. We show that the interaction of these two occasionally binding constraints induces strong state-dependence in the impact of changes in bank capital requirements on loan supply, in line with the empirical findings.

In "normal" states of the world where banks do not face problems to retain enough profits to satisfy higher capital requirements, the impact on bank loan supply works through a "pricing channel" which is quantitatively small: loans change by around -0.1% (or even less) for a 1pp increase in capital requirements. In "bad" states of the world where banks are not able to come up with sufficient equity to satisfy capital requirements, the impact on loan supply works through a "quantity channel", which acts like a financial accelerator and can be very large: up to 10% more loans for a capital requirement release of -1pp. These state-dependent magnitudes are consistent with the magnitudes found in empirical studies.

Compared to existing DSGE models with a banking sector, which usually feature a constant lending response of around 1%, our state-dependent impact is an order of magnitude lower in "normal" states and an order of magnitude higher in "bad" states. Our results on state-dependence can also help to explain why empirical studies often find vastly different magnitudes for the impact of changes in bank capital requirements on lending: in our model a 1pp increase in capital requirements can lead to a change in equilibrium loans of anything between 0% and -10%, depending on how much each of the transmission channels becomes "activated" by the policy change.

Our findings regarding the state-dependent impact of changes in bank capital requirements have a number of important policy implications.

First, transition costs to higher bank capital requirements in terms of reduced loan supply (and therefore economic activity) can be kept low if higher capital requirements are introduced gradually over time in "normal" states of the world, where banks make positive profits so that capital ratios can be increased through retained earnings. The level of banking system profitability, in combination with available capital headroom, should be a sufficient statistic to determine the pace of capital requirement increases that can be met by banks without constraining loan supply much.

Second, moderate increases in bank capital requirements during boom phases are unlikely to have a big dampening effect on credit growth. This is because during boom phases banks tend to be profitable and have voluntary capital buffers available to absorb increases in capital requirements without becoming capital constrained. Hence, only the "pricing channel" is likely to be active with its limited impact on equilibrium loan growth. In other words, moderate capital requirement increases are unlikely to have a big dampening effect on potentially "excessive" credit growth

during the upswing phase of the financial cycle. The only way that a big dampening effect on credit growth during boom phases could be achieved is by raising capital requirements significantly and abruptly, so that binding capital constraints for the banking system are induced.

Third, the release of capital requirements in "bad" states, where banks make substantial losses and become capital constrained can have a big supporting effect on bank loan supply and is therefore important to mitigate bank deleveraging in crisis times. With constant capital requirements, the voluntary capital buffers held by banks due to self-insurance motives do not prevent the banking system from occasionally becoming capital constrained and being forced to deleverage in "bad" states of the world. If the objective of policy makers is to avoid large volatility in loan supply, time-varying capital requirements will be important.

Finally, our results provide a theoretical justification for building up a positive countercyclical capital buffer (CCyB) in "normal" macro-financial environments, i.e. before clear signs of excessive credit growth emerge.<sup>56</sup> In particular, such a policy strategy will impose limited economic costs during the buffer build-up phase, while minimizing the probability that a bad shock hits the economy before a sufficient level of capital buffers has been built up that can be released to support bank loan supply. In other words, such a policy strategy can create insurance at low economic costs against systemic risks that are inherently difficult to identify and measure.

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<sup>56</sup>A number of European countries (e.g. UK, LT, IE, EE, NL, SE) have recently moved to CCyB frameworks where buffer build-up starts during "normal" macro-financial conditions, when systemic risks are neither elevated nor subdued. This policy strategy deviates somewhat from the purely risk-based Basel III CCyB build-up rule, where capital buffers should be increased commensurate with the level of cyclical systemic risk (see [Basel Committee on Banking Supervision \(2010\)](#) for details).

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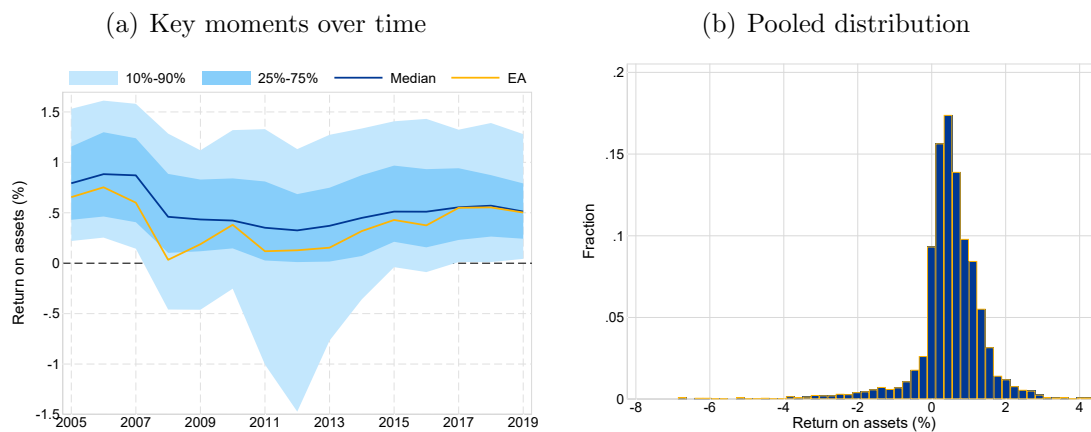
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## Appendix A: Additional figures

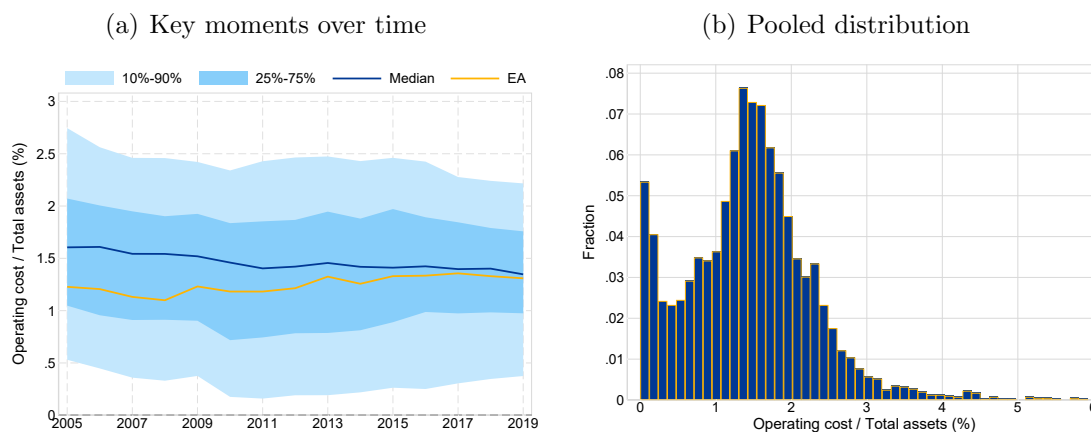
Figure A1: Properties of bank-level pre-tax return on assets



Sources: SNL Financial.

Notes: Based on a panel of around 320 euro area banks since 2005 at highest level of consolidation.

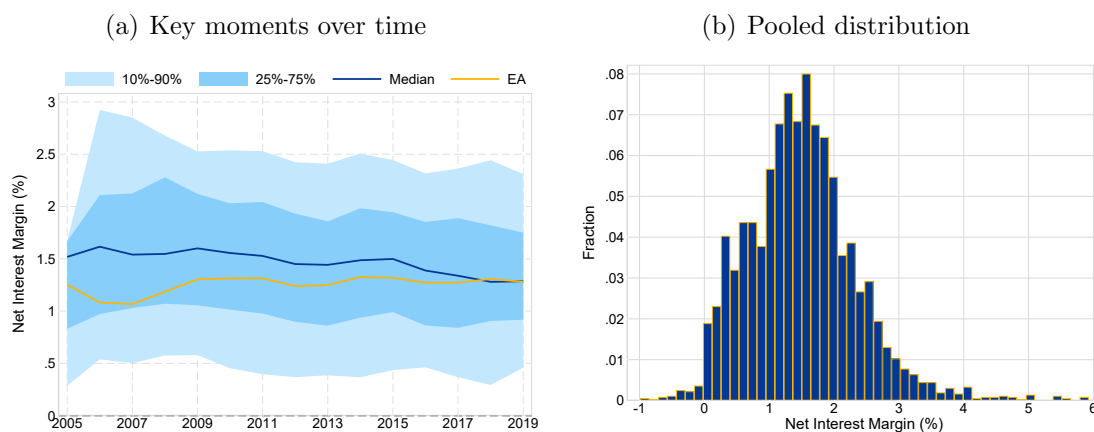
Figure A2: Properties of bank-level cost-to-asset ratios



Sources: SNL Financial.

Notes: Based on a panel of around 320 euro area banks since 2005 at highest level of consolidation.

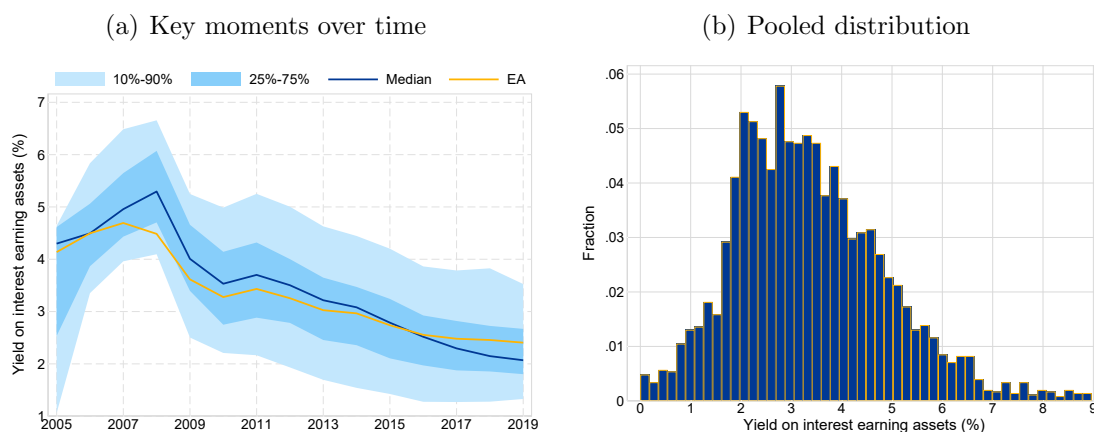
Figure A3: Properties of bank-level net interest margin (NIM)



Sources: SNL Financial.

Notes: Based on a panel of around 320 euro area banks since 2005 at highest level of consolidation.

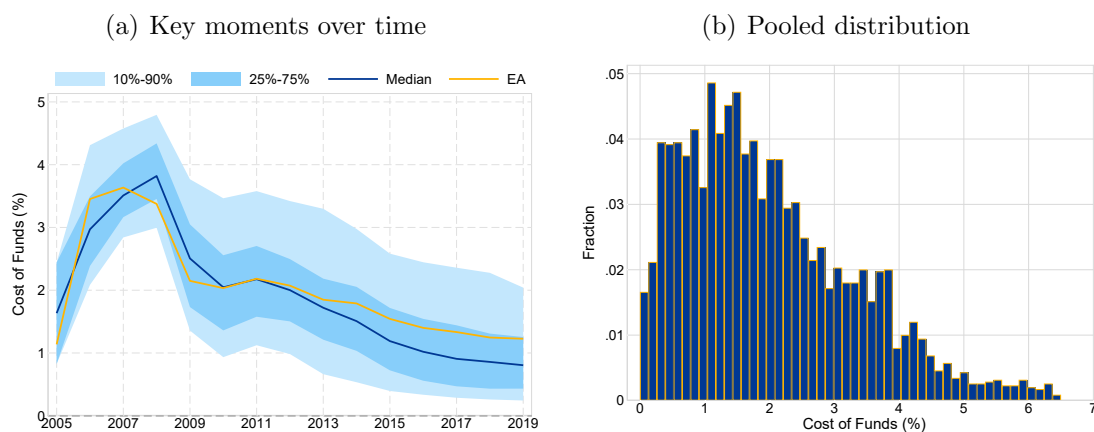
Figure A4: Properties of bank-level yields on assets



Sources: SNL Financial.

Notes: Based on a panel of around 320 euro area banks since 2005 at highest level of consolidation.

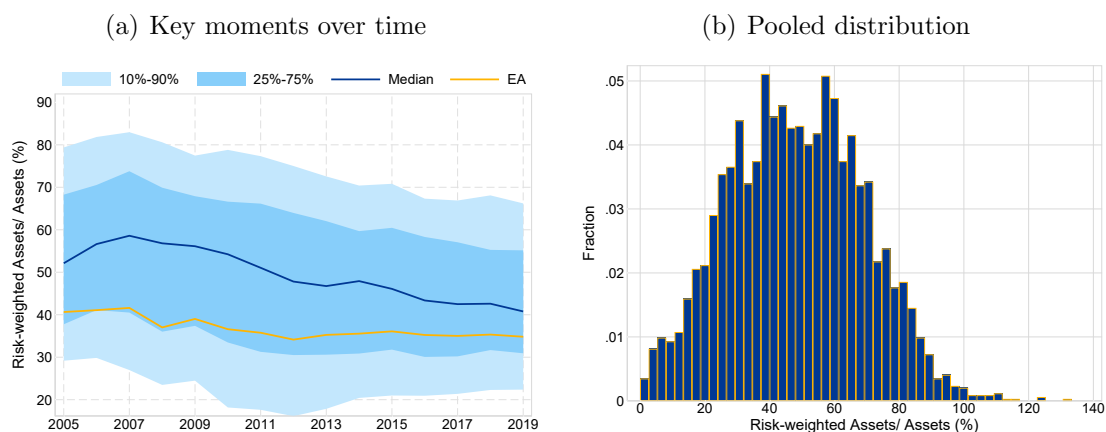
Figure A5: Properties of bank-level cost of funding



Sources: SNL Financial.

Notes: Based on a panel of around 320 euro area banks since 2005 at highest level of consolidation.

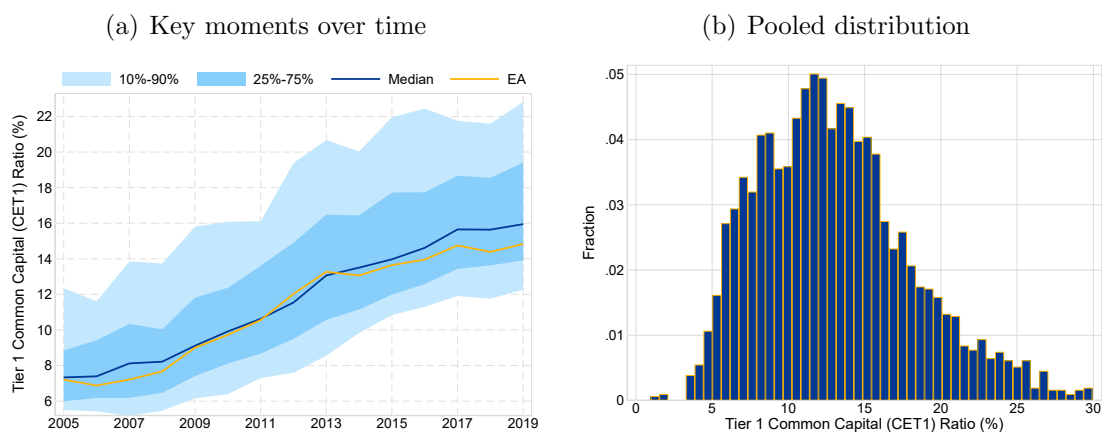
Figure A6: Properties of bank-level average risk-weights



Sources: SNL Financial.

Notes: Based on a panel of around 320 euro area banks since 2005 at highest level of consolidation.

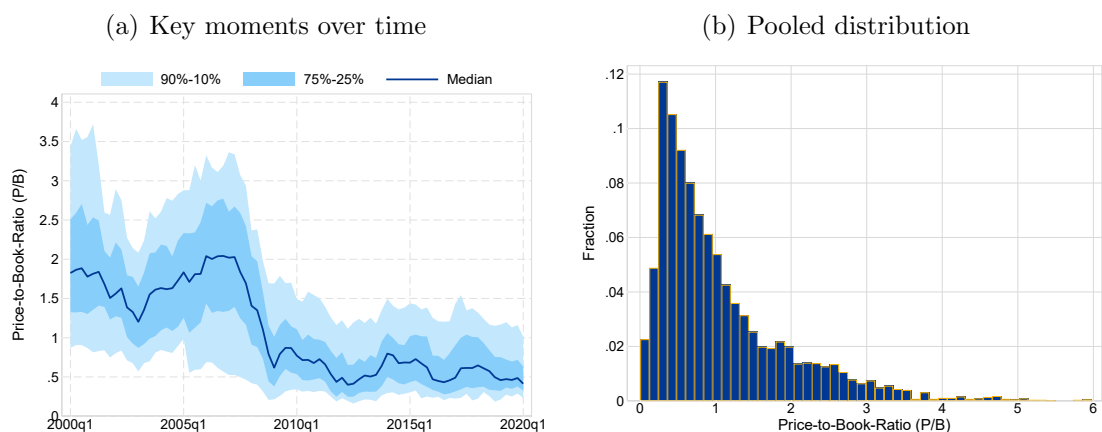
Figure A7: Properties of bank-level CET1 capital ratios



Sources: SNL Financial.

Notes: Based on a panel of around 320 euro area banks since 2005 at highest level of consolidation.

Figure A8: Properties of bank-level Price-to-book ratios



Sources: Bloomberg.

Notes: Based on a quarterly panel of around 70 euro area banks since 2000.

## Appendix B: Derivation of analytical results

### B1 First-order conditions

Taking the derivative of the value function in equation (16) with respect to  $E'$  yields:

$$d_{E'} + \beta \mathbb{E}[V'_{E'}] + \chi^1 + \chi^2 d_{E'} = 0 \quad (27)$$

Making use of the expressions for dividends and the value function next period, we get:

$$d_{E'} = -1 \quad (28)$$

$$V'_{E'} = (1 + \chi^{2'})d'_{E'} \quad (29)$$

$$d'_{E'} = 1 + i^D \quad (30)$$

Using these expressions in equation (27) and rearranging, yields:

$$\chi^1 = 1 + \chi^2 - \frac{1 + i^D}{1 + \rho} \mathbb{E}[1 + \chi^{2'}] \quad (31)$$

Taking the derivative of the value function in equation (16) with respect to  $L'$  yields:

$$\beta \mathbb{E}[V'_{L'}] - \chi^1 R' \omega = 0 \quad (32)$$

Making use of the expressions for dividends and the value function next period, and the fact that in equilibrium  $L' = L^A$ , we get:

$$V'_{L'} = (1 + \chi^{2'})d'_{L'} \quad (33)$$

$$d'_{L'} = \frac{\mu - 1}{\mu} \frac{\lambda - \log(L')}{\epsilon} - i^D - \theta' - \kappa \quad (34)$$

Using these expressions in equation (32) and rearranging, yields:

$$\chi^1 R' \omega = \mathbb{E} \left[ \frac{1 + \chi^{2'}}{1 + \rho} \left( \frac{\mu - 1}{\mu} \frac{\lambda - \log(L')}{\epsilon} - i^D - \theta' - \kappa \right) \right] \quad (35)$$

## B2 Proof of Proposition 1

In the absence of the equity issuance constraint, the first-order condition for  $E'$  reduces to:

$$\chi^1 = 1 - \frac{1 + i^D}{1 + \rho} \quad (36)$$

As  $\rho > i^D$ , this implies that  $\chi^1$  is always positive, i.e. that the capital requirement constraint is always binding, and that  $CR' = R'$ .

Moreover, the first-order condition for  $L'$  reduces to:

$$\chi^1 R' \omega = \mathbb{E} \left[ \frac{1}{1 + \rho} \left( \frac{\mu - 1}{\mu} \frac{\lambda - \log(L')}{\epsilon} - i^D - \theta' - \kappa \right) \right] \quad (37)$$

Using the equilibrium value for  $\chi^1$  from equation (36) in this expression and rearranging yields:

$$\log(L') = \lambda - \epsilon \left( \frac{\mu}{\mu - 1} \right) \left[ i^D + \mathbb{E}(\theta') + \kappa + (\rho - i^D) R' \omega \right] \quad (38)$$

## B3 Proof of Proposition 2

Let  $L'_{New}$  be equilibrium loans under capital requirement  $R'_{New}$  and let  $L'_{Old}$  be equilibrium loans under capital requirement  $R'_{Old}$ . Define  $\Delta \log(L') = \log(L'_{New}) - \log(L'_{Old})$  and  $\Delta R' = R'_{New} - R'_{Old}$ .

Using the definition of equilibrium loans from equation (38) we get:

$$\begin{aligned} \Delta \log(L') &= \lambda - \epsilon \left( \frac{\mu}{\mu - 1} \right) \left[ i^D + \mathbb{E}(\theta') + \kappa + (\rho - i^D) R'_{New} \omega \right] \\ &\quad - \lambda + \epsilon \left( \frac{\mu}{\mu - 1} \right) \left[ i^D + \mathbb{E}(\theta') + \kappa + (\rho - i^D) R'_{Old} \omega \right] \end{aligned} \quad (39)$$

$$\Delta \log(L') = -\epsilon \left( \frac{\mu}{\mu - 1} \right) \left[ (\rho - i^D) R'_{New} \omega - (\rho - i^D) R'_{Old} \omega \right] \quad (40)$$

$$\Delta \log(L') = -\epsilon \left( \frac{\mu}{\mu - 1} \right) \left[ (\rho - i^D) \omega \Delta R' \right] \quad (41)$$

## B4 Proof of Proposition 3

In the presence of an equity issuance constraint, the first-order condition for  $E'$  is:

$$\chi^1 = 1 + \chi^2 - \frac{1 + i^D}{1 + \rho} \mathbb{E}[1 + \chi^{2'}] \quad (42)$$

If banks decide to hold voluntary capital buffers, i.e. if they choose a capital ratio that is higher than the capital requirement, it means that the capital requirement constraint is slack. A necessary condition for the capital requirement constraint to be slack is that the lagrange multiplier  $\chi^1$  must be equal to zero:

$$0 = 1 + \chi^2 - \frac{1 + i^D}{1 + \rho} \mathbb{E}[1 + \chi^{2'}] \quad (43)$$

$$\frac{1 + i^D}{1 + \rho} \mathbb{E}[1 + \chi^{2'}] = 1 + \chi^2 \quad (44)$$

As  $\rho > i^D$ , this condition can only hold if  $\mathbb{E}[1 + \chi^{2'}] > 1$ , i.e. banks expect the equity issuance constraint to be binding in some future states of the world. In particular, the probability and shadow value of the constraint in such states needs to be high enough so that the following condition is met:

$$\mathbb{E}[\chi^{2'}] = \frac{1 + \rho}{1 + i^D} (1 + \chi^2) - 1 > 0 \quad (45)$$

## B5 Proof of Proposition 4

First, note that the first-order condition for loans in equation (35) can be rewritten as:

$$\log(L') = \lambda - \epsilon \left( \frac{\mu}{\mu - 1} \right) \left[ i^D + \kappa + \frac{\mathbb{E}[(1 + \chi^{2'})\theta']}{\mathbb{E}[1 + \chi^{2'}]} + \frac{(1 + \rho)\chi^1 R' \omega}{\mathbb{E}[1 + \chi^{2'}]} \right] \quad (46)$$

We can now use the fact that the covariance between two random variables is defined as:

$$Cov(1 + \chi^{2'}, \theta') = \mathbb{E}[(1 + \chi^{2'})\theta'] - \mathbb{E}[1 + \chi^{2'}]\mathbb{E}[\theta'] \quad (47)$$

Moreover, we can use the fact that:

$$Cov(1 + \chi^{2'}, \theta') = Cov(\chi^{2'}, \theta') \quad (48)$$

Using these definitions in equation (46) we get:

$$\log(L') = \lambda - \epsilon \left( \frac{\mu}{\mu - 1} \right) \left[ i^D + \kappa + \mathbb{E}[\theta'] + \frac{Cov(\chi^{2'}, \theta')}{\mathbb{E}[1 + \chi^{2'}]} + \frac{(1 + \rho)\chi^1 R' \omega}{\mathbb{E}[1 + \chi^{2'}]} \right] \quad (49)$$

Now consider case 1 where banks hold voluntary capital buffers and pay dividends before and after a change in capital requirements, i.e. both constraints are slack. This implies that:

$$\chi^1 = 0 \quad (50)$$

$$\chi^2 = 0 \quad (51)$$

Using these two conditions in the first-order condition for equity in equation (31) gives:

$$\mathbb{E}[1 + \chi^{2'}] = \frac{1 + \rho}{1 + i^D} \quad (52)$$

Using this and the fact that  $\chi^1 = 0$  yields the following equilibrium condition for loans:

$$\log(L') = \lambda - \epsilon \left( \frac{\mu}{\mu - 1} \right) \left[ i^D + \kappa + \mathbb{E}[\theta'] + \frac{1 + i^D}{1 + \rho} Cov(\chi^{2'}, \theta') \right] \quad (53)$$

Let  $L'_{New}$  be equilibrium loans under capital requirement  $R'_{New}$  and let  $L'_{Old}$  be equilibrium loans under capital requirement  $R'_{Old}$ . Define  $\Delta \log(L') = \log(L'_{New}) - \log(L'_{Old})$  and  $\Delta Cov(\chi^{2'}, \theta') = Cov(\chi^{2'}_{New}, \theta') - Cov(\chi^{2'}_{Old}, \theta')$ . Using these definitions yields:

$$\Delta \log(L') = -\epsilon \left( \frac{\mu}{\mu - 1} \right) \left( \frac{1 + i^D}{1 + \rho} \right) \underbrace{\Delta Cov(\chi^{2'}, \theta')}_{\text{Policy-induced change in covariance term}} \quad (54)$$

Now let's turn to case 2 where banks hold voluntary capital buffers and do not



pay dividends before and after a change in capital requirements, i.e. where only the equity issuance constraint is binding. This implies that:

$$\chi^1 = 0 \quad (55)$$

$$\chi^2 \geq 0 \quad (56)$$

Further, from the first-order condition for equity we know that:

$$\mathbb{E}[1 + \chi^{2'}] = \frac{1 + \rho}{1 + i^D} (1 + \chi^2) \quad (57)$$

Using this and the fact that  $\chi^1 = 0$  yields the following equilibrium condition for loans:

$$\log(L') = \lambda - \epsilon \left( \frac{\mu}{\mu - 1} \right) \left[ i^D + \kappa + \mathbb{E}[\theta'] + \frac{1 + i^D}{1 + \rho} \frac{\text{Cov}(\chi^{2'}, \theta')}{1 + \chi^2} \right] \quad (58)$$

Now define:

$$\Delta \frac{\text{Cov}(\chi^{2'}, \theta')}{1 + \chi^2} = \frac{\text{Cov}(\chi_{New}^{2'}, \theta')}{1 + \chi_{New}^2} - \frac{\text{Cov}(\chi_{Old}^{2'}, \theta')}{1 + \chi_{Old}^2} \quad (59)$$

We then get:

$$\Delta \log(L') = -\epsilon \left( \frac{\mu}{\mu - 1} \right) \left( \frac{1 + i^D}{1 + \rho} \right) \underbrace{\Delta \frac{\text{Cov}(\chi^{2'}, \theta')}{1 + \chi^2}}_{\text{Policy-induced change in covariance term}} \quad (60)$$

## B6 Proof of Proposition 5

Assume that the equity issuance constraint in the current period is binding, i.e. that  $\chi^2 > 0$ . Moreover, assume that  $\chi^2 > \mathbb{E}[\chi^{2'}]$ . From equation (42) we know that in such states of the world  $\chi^1 > 0$ , i.e. the capital requirement constraint must be binding. Hence, banks do not hold voluntary capital buffers and  $CR' = R'$ . Moreover, given that the equity issuance constraint is binding, we know that  $d = 0$ .

Using the law of motion for bank equity in equation (11) we can derive:

$$E' = E + \pi^N - d \quad (61)$$

$$CR' L' \omega = E + \pi^N \quad (62)$$

$$R' L' \omega = E + \pi^N \quad (63)$$

$$L' = \frac{E + \pi^N}{R' \omega} \quad (64)$$

$$\log(L') = \log(E + \pi^N) - \log(R') - \log(\omega) \quad (65)$$

Let  $L'_{New}$  be equilibrium loans under capital requirement  $R'_{New}$  and let  $L'_{Old}$  be equilibrium loans under capital requirement  $R'_{Old}$ . Define  $\Delta \log(L') = \log(L'_{New}) - \log(L'_{Old})$  and  $\Delta \log(R') = \log(R'_{New}) - \log(R'_{Old})$ . Using these definitions we get:

$$\begin{aligned} \Delta \log(L') &= \log(E + \pi^N) - \log(R'_{New}) - \log(\omega) \\ &\quad - \log(E + \pi^N) + \log(R'_{Old}) + \log(\omega) \end{aligned} \quad (66)$$

$$\Delta \log(L') = -\log(R'_{New}) + \log(R'_{Old}) \quad (67)$$

$$\Delta \log(L') = -\Delta \log(R') \quad (68)$$

## B7 Proof of Proposition 6

The "quantity channel" will only be present if the equity issuance constraint is binding, i.e. when  $d = 0$  and  $\chi^2 > 0$ . Hence, as long as a rule for time-varying capital requirements ensures that  $d > 0$  under the unconstrained optimal loan choice  $L^*$ , the "quantity channel" will never be present.

If the capital requirement rule ensures that  $d > 0$ , then  $\chi^2 = 0$  will always hold. From equation (36) we know that this implies  $\chi^1 > 0$  in all states of the world, i.e. banks will never hold voluntary capital buffers and therefore  $CR = R$  in all states of the world.

Using these conditions in equation (11) and imposing that  $d > 0$  yields:

$$0 < d \quad (69)$$

$$0 < E + \pi^N - E' \quad (70)$$

$$0 < CR\omega L + \pi^N - CR'\omega L^{*'} \quad (71)$$

$$0 < R\omega L + \pi^N - R'\omega L^{*'} \quad (72)$$

$$R' < R \frac{L}{L^{*'}} + \frac{\pi^N}{\omega L^{*'}} \quad (73)$$

Now define the desired growth rate of loans as the ratio of the unconstrained optimal loan choice  $L^{*'}$  and current loans:

$$(1 + g^*) = \frac{L^{*'}}{L} \quad (74)$$

Using this definition in the above condition that needs to be satisfied by the time-varying capital requirements yields:

$$R' < \left( R + \frac{\pi^N}{\omega L} \right) \frac{1}{1 + g^*} \quad (75)$$

or equivalently:

$$R' < \left( CR + \frac{\pi^N}{\omega L} \right) \frac{1}{1 + g^*} \quad (76)$$

# Appendix C: Numerical Details

## C1 Equilibrium conditions

As all banks are identical, we have in equilibrium  $X_{i,t} = X_t$  for all variables, in particular  $L_{i,t} = L_t$  and  $i_t = i_{i,t}$  for all  $t$ . Also note that the deposit rate is constant and exogenously given  $i_t^D = i^D$  for all  $t$ . The equilibrium conditions in time notation are given by

$$\begin{aligned}\chi_t^1 &= 1 + \chi_t^2 - \beta(1 + i^D)\mathbb{E}_t\{1 + \chi_{t+1}^2\} \\ \chi_t^1 \omega R^{min} &= \beta \mathbb{E}_t \left\{ (1 + \chi_{t+1}^2) \left( \frac{\mu - 1}{\mu} i_{t+1} - \theta_{t+1} - \kappa - i^D \right) \right\} \\ L_{t+1} &= e^{\lambda - \epsilon i_{t+1}}\end{aligned}$$

With the (aggregate) Kuhn-Tucker conditions

$$\begin{aligned}(E_{t+1} - (i_t - \theta_t - i^D) L_t + (1 + i^D) E_t - \bar{d}) \chi_t^2 &= 0 \\ (E_{t+1} - \omega R^{min} L_{t+1}) \chi_t^1 &= 0\end{aligned}$$

where  $\bar{d} = 0$  is the parameter of the dividend constraint bounding the dividends from below (in our simulations at zero, see calibration section). Rewriting the FOCs for loans, we get the following expression for the equilibrium interest rate:

$$i_{t+1} = \frac{\mu}{\mu - 1} \left[ E_t \theta_{t+1} + \frac{Cov_t(1 + \chi_{t+1}^2, \theta_{t+1})}{E_t(1 + \chi_{t+1}^2)} + \kappa + i^D + \beta^{-1} \frac{\chi_t^1}{E_t(1 + \chi_{t+1}^2)} \omega R^{min} \right]$$

The covariance term captures the possibility of a binding equity issuance constraint next period. If the covariance term is positive, it makes the bank effectively risk averse in certain states. Typically, we expect the covariance term to be (weakly) positive and increasing in  $\theta_t$  for autocorrelated shocks, as a higher  $\theta_t$  today implies a positive probability of high  $\theta_t$  next period. The covariance potentially leads to precautionary bank behaviour. Higher effective cost of risk increases the interest rate in certain states and bank loans are ceteris paribus lower.

## C2 Beginning-of-period-equity plus profits (Net Worth) as endogenous state variable

Inspecting the above equilibrium reveals that the equilibrium can be implemented using "beginning-of-period" net worth (Net Worth). Net Worth is defined as equity carried over from last period plus the realized net profits after the shock  $\theta$  has been revealed at the beginning of period  $t$ . Formally, define net worth in period  $t$  as

$$N_t = \underbrace{(i_t - \theta_t - \kappa - i^D) L_t + i^D E_t}_{\text{Net profits } \pi_t} + \underbrace{E_t}_{\text{Equity at beginning of } t}$$

Consequently, net worth in  $t + 1$  is given by

$$N_{t+1} = (i_{t+1} - \theta_{t+1} - \kappa - i^D) L_{t+1} + (1 + i^D) E_{t+1}$$

Note that net worth in  $t + 1$  is a function of  $L_{t+1}$ ,  $i_{t+1}$ , and  $E_{t+1}$  (all determined in period  $t$ ) and importantly also a function of the realized shock in  $t + 1$ , i.e.  $\theta_{t+1}$ .

## C3 Algorithm: "time"-iteration using the equilibrium conditions

1. Define a convergence criterion  $\Delta$
2. Define a discrete grid for net worth  $N = \{N_1, \dots, N_{n_N}\}$ . We use an equally spaced linear grid. Discretize  $\theta$  using the method by [Tauchen \(1986\)](#),  $\Theta = \{\theta_1, \dots, \theta_{n_\theta}\}$ .
3. Define a guess for the endogenous variables, i.e. for  $L_{t+1} = L(\Theta, N)$ ,  $E_{t+1} = E(\Theta, N)$ ,  $i_{t+1} = i(\Theta, N)$ ,  $\chi_t^s = \chi^s(\Theta, N)$  for  $s = 1, 2$  and so on.
4. Given these guesses, can solve above equilibrium conditions and net worth in period  $t + 1$ ,  $N_{t+1} = N'(\theta', \theta, N)$ , using a non-linear solver. To handle the (potentially) occasionally binding constraints and the corresponding Kuhn-Tucker conditions, we use the method proposed by [Zangwill and Garcia \(1981\)](#). Doing so we define  $\chi_t^1$  and  $\chi_t^2$  as variables with support  $(-\infty, +\infty)$ , define auxiliary variables  $\chi^{s+}$  and  $\chi^{s-}$  for  $s = 1, 2$ , and rewrite the equilibrium conditions in

the following way:

$$\begin{aligned}
1 + \chi_t^{2+} &= \beta \mathbb{E}_t \{ (1 + \chi_{t+1}^{2+}) \} (1 + i^D) + \chi_t^{1+} \\
\chi_t^{1+} \omega R^{min} &= \beta \mathbb{E}_t \left\{ (1 + \chi_{t+1}^{2+}) \left( \frac{\mu - 1}{\mu} i_{t+1} - \theta_{t+1} - \kappa - i^D \right) \right\} \\
L_{t+1} &= e^{\lambda - \varepsilon_{t+1}} \\
d_t &= N_t - E_{t+1} \\
0 &= d_t - \bar{d} - \chi_t^{2-} \\
0 &= E_{t+1} - \omega R^{min} L_{t+1} - \chi_t^{1-} \\
N_{t+1} &= (i_{t+1} - \theta_{t+1} - \kappa - i^D) L_{t+1} + (1 + i^D) E_{t+1} \\
\chi_t^{1+} &= \max(0, \chi_t^1)^{\eta_{gz}} \\
\chi_t^{1-} &= \max(0, -\chi_t^1)^{\eta_{gz}} \\
\chi_t^{2+} &= \max(0, \chi_t^2)^{\eta_{gz}} \\
\chi_t^{2-} &= \max(0, -\chi_t^2)^{\eta_{gz}}
\end{aligned}$$

where  $\eta_{gz} > 0$  is an arbitrarily chosen positive integer. As an illustrative example, when the capital requirement constraint is binding, must be the case that  $\chi^{1+} > 0$  and  $\chi^{1-} = 0$ . When the capital requirement constraint is slack, need to have that  $\chi^{1+} = 0$  and  $\chi^{1-} > 0$ . The solver then searches the value of  $\chi^s$   $s = 1, 2$  that satisfies the above conditions along with the other variables.

Note that, when solving the equilibrium, we also impose the solvency constraint  $N_{t+1} \geq 0$ , which has to hold for all possible future states. The solvency constraint can be rewritten in terms of a "market-based" minimum capital requirement constraint:

$$E_{t+1} \geq \max \{ \theta_{t+1} - i_{t+1} + \kappa + i^D \} L_{t+1}$$

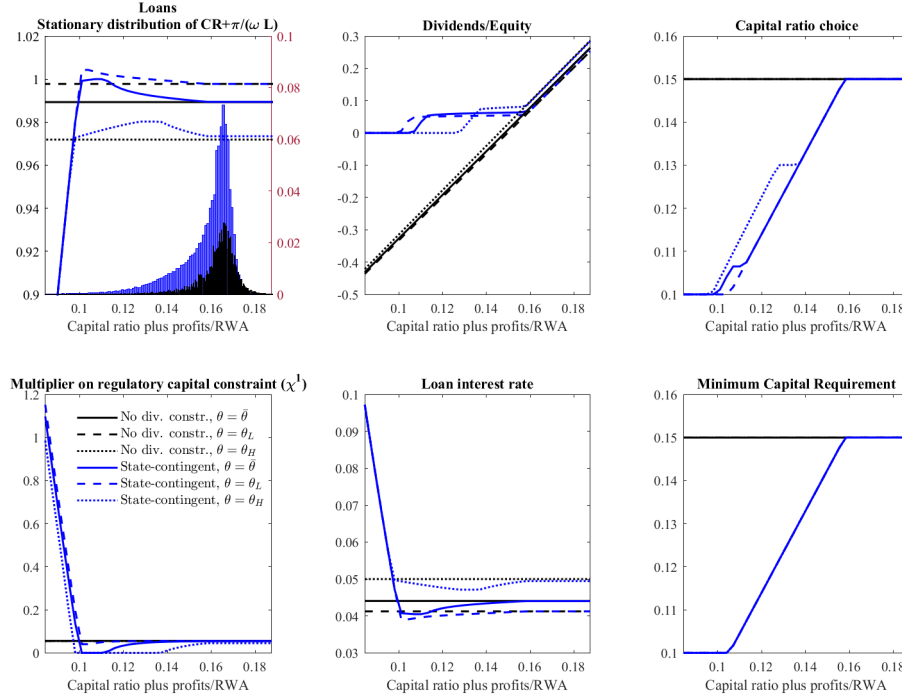
where the max is taken over all possible future values of  $\theta$ . For all states, we check whether this constraint is tighter or the regulatory capital constraint. In all calibrations considered in this paper featuring  $R^{min} \geq 0.1$ , the regulatory constraint is always tighter than the capital requirement implied by the solvency constraint. This means, net worth is always strictly positive, and the solvency constraint is always slack.

5. Given the solution for all variables, update the guesses for all variables.
6. Repeat steps 4 and 5 until convergence criterion is met, i.e. the distance of

the guesses and the implied solution (we use the Euclidean Norm) is smaller than  $\Delta$ .

# Appendix D: Model with state-dependent capital requirements

Figure A9: Policy functions of the model with state-dependent capital requirement



Notes: All policy functions are plotted against the endogenous state variable, which is given by beginning-of-period equity plus realized profits, normalised by median steady state risk weighted assets ( $\omega \cdot 1$ ). Blue lines: model with state-dependent capital requirement  $R^{min} = 0.1$ ,  $R^{max} = 0.15$ , and  $\xi = 0.95$ . Black lines: counterfactual economy without equity issuance constraint and constant capital requirement  $R = 0.15$ . The histograms show the stationary distributions of the normalised endogenous state variable for the respective economies (right axis). Solid lines: policy functions for a cost of risk realization equal to its median. Dashed lines: policy functions for a low cost of risk realization equal to the 5th percentile of its stationary distribution. Dotted lines: policy functions for a high cost of risk equal to the 95th percentile of its stationary distribution.



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