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The genesis of the Black-Scholes option pricing formula

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No. 98

**The genesis of the Black-Scholes
option pricing formula**

by Prof. Dr. Thomas Heimer, Sebastian Arend

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Abstract

Innovations in the finance industry are an important tool to enhance profitability and to increase a nation's wealth. It, therefore, is not astonishing that there is much empirical work on innovations in finance. Most of the work however is concerned with the design of innovative products. The question on how innovations are established and pushed through in financial markets is mostly neglected. Hardly any asks: How do we develop new ways of pricing derivatives, how do we enhance risk control, how do we generate new processes that may enhance the profitability of finance business?

The second sector innovation theory in the last decades has taken a different approach. To understand innovation better researchers have focused on the question on how innovations have been emerging. Studies on the history of innovations opened a promising line of research that helps to understand innovation processes much better (see Hughes 1983 und Callon 1986).

A similar approach has yet not been adapted to innovation theories in financial markets.¹ Accordingly it is the articles objective to evaluate the outcome of a transfer of innovation theories from the second into the third sector. The transfer is conducted on the example of the Black-Scholes option pricing formula, an innovation with a strong influence on the efficiency of decisions in the option market. The article shows how the innovation emerged and what factors influenced the diffusion process.

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¹ It is D. McKenzie 2002 and 2003 who tried first to transfer those theories to explain the emergence of innovations in financial markets

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1 The blue print: Innovation theory for the second sector and its transferability

There is a long tradition in innovation theory in the second sector: for many decades the emergence of innovations was discussed under the scheme of technology push. In that line of theory the main hypothesis was based on the idea of an independent scientific world creating inventions that would change the world. The main concern in that theory was how to provide a sufficient flow of knowledge from science to research and industry. The emergence of innovations itself was not in the focus of the approach.

This did not change with demand-pull theories either. Schmookler (1966), in a study tried to explain that the innovation process was driven by the demand of people. Using the example of the railway industry Schmookler showed empirical proof for his hypothesis. However, even Schmookler's theory was not concerned with the generation of innovations itself. Rather, it gave an explanation on how resources devoted to innovative activities were distributed (see Mowery, David, Nathan Rosenberg, 1979).

The generation of innovations as a topic of theory building entered the scene when several scholars noticed that there is a social influence on the emergence of innovations (see Mowery, David, Nathan Rosenberg, 1989; Nelson, R.R., 1987; Nelson, R.R., S.G. Winter, H.L. Schuette, 1976; Pinch, Trevor J., Bijker, Wiebe E., 1984; Rammert, W., 1988; Rosenberg, Nathan, 1982). It became obvious that the idea of a technological determinism – as assumed in technology-push as well as demand-pull theories – is no more convincing as an explanation of innovation processes. The innovation process is far more complex and evolutionary than assumed in the theory of technological determinism. Even a solely economic approach is not sufficient. It was shown in the literature that innovations are shaped by a large number of scientific, technical, economic and social factors (see Hughes 1983, MacKenzie/Wajcman 1985). The conjunction of these factors could only be explained by an evolutionary approach.

It was Giovanni Dosi (1983, 1984) who developed - in reference to Polany - the concept of a technological paradigm as an explanation for the complex emergence of innovations. In a technological paradigm the definition of problems is provided and promising paths to solve problems are developed. Accordingly a technological paradigm can be understood as a bunch of "puzzle-solving" activities used by people involved in generating innovations. In several empirical studies the concept had been used to explain the emergence of single innovations successfully (Orsenigo 1989; Dosi 1984; Heimer 1993).

The discoveries made by evolutionary studies have a strong impact on the way industrial innovation processes are designed today. As has been shown by Phillip Vergragt (1988) the emergence of innovations can be explained by three factors: Problem definition, agreement on a problem solving approach and critical events that prevent the problem solution from being successful.

Based on these three factors the emergence of innovations follows the described path. The theory starts from the assumption that the innovation process is socially shaped. Accordingly individuals play a major role in innovations. In his rejection of technological determinism he assumes that all technological problems in principle are solvable by human actors.

Actors involved create a problem to be solved. In the process of problem creation a definition of various perspectives of what a problem is come together. As will be shown below the definition of a problem is going along with a completely new perspective on the construction of the corridor of opportunities. The Black Scholes formula, for instance, is no answer to problems linked to a world of equilibrium. With the new definition of a world of arbitrage, however, the problem solved by Black and Scholes become a dominant perspective.

If there is an agreement among the actors involved on the dominant problem definition, all actors involved attempt to get the problem solved. During that process minor adjustments of the problem definition might take place. The main activity is, however, is focused on using existing paradigms to solve the problem defined.

These activities continue until a critical event occurs. Critical events are either a successful completion of the problem solving activity or a new major occurs that can not be solved by the actors involved. Second type critical events might be solved by a major adjustment of the problem definition or by the integration of new actors into the problem solving activity.

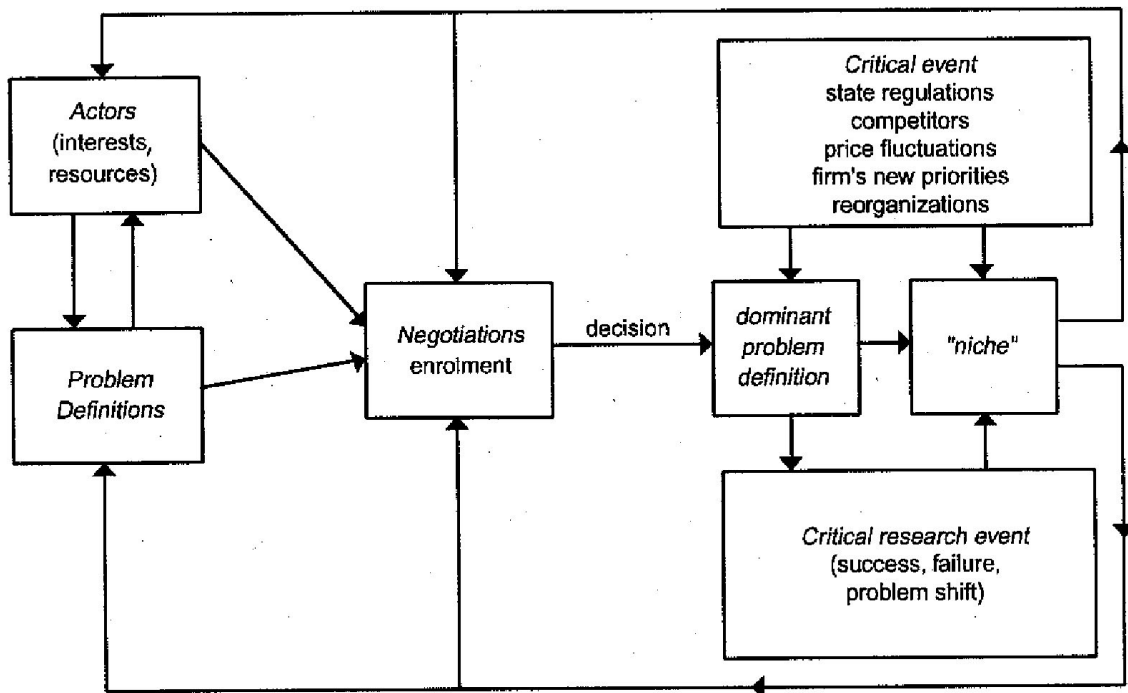


Figure 1: Industrial Innovation Model

In Vergragt's case study on the emergence of innovations in the chemical industry in the Netherlands he could proof his hypothesis. The transfer of evolutionary innovation theories from the use in the second to the third sector means to evaluate the fruitfulness of explaining and understanding the emergence of financial innovations under the frame of the above mentioned three factors. What is the problem definition of the actors involved in the emergence of the financial innovation? How did they agree to solve the problem? Have there been any critical events that threatened the success of the problem solving strategy?

2 The innovation of the Black-Scholes option pricing formula

In the following the innovative emergence of the Black-Scholes option pricing formula will be discussed in terms of innovation theory. In doing this, we transfer the innovation theory of the second sector to the third sector. First, the paradigm change exemplified by the formula will be described. In a second step, we show how this paradigm change was achieved: we describe the process of how the formula was generated and apply an evolutionary model to clarify how the actors rejected the existing paradigm and introduced a new paradigm in finance.

2.1 Paradigm Change

The Black-Scholes option pricing formula is a pricing algorithm for European call options on stocks without dividends.² The formula was first published in 1973 by Fisher Black and Myron Scholes; however, in the same year, Robert C. Merton provided extensions and an alternative derivation of the formula.³ From the viewpoint of innovation theory, the Black-Scholes option pricing formula is of interest because it constitutes a paradigm change. Before the publication of the formula, finance was dominated by the paradigm of risk-based pricing models rooted in an equilibrium (neoclassic) thinking with the Capital Asset Pricing Model (CAPM) being the most prominent example.

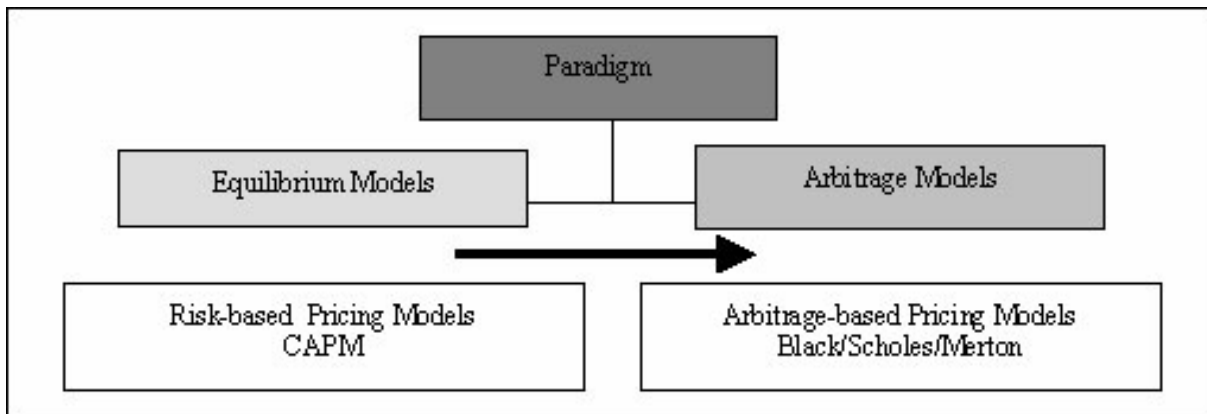


Figure 2: Paradigm Change

The Black-Scholes option pricing formula marks an important step towards contemporary finance with its arbitrage-based⁴ pricing models and the related paradigm of arbitrage theory. The Black-Scholes option pricing formula is the first and - *perhaps* - most important example of the class of arbitrage pricing models. On the one hand, the formula is a scientific innovation; on the other hand, it should be clear that it is also an economic innovation: Using the Black-Scholes option pricing formula it was possible for the first time to compare the price resulting from supply and demand with the analytical value of an option.⁵

² See Black/Scholes, 1973, p. 637-654

³ See Merton, 1973(a), p. 141-183; Black/Scholes, 1973, p. 641, footnote 3

⁴ The Arbitrage-based derivation of the Black-Scholes Option pricing formula goes back to Merton, see Black/Scholes, 1973, p. 641, footnote 3

⁵ The Black-Scholes Option pricing formula has been heavily criticized, among others, because it is based on the unrealistic assumption of a frictionless market (see Shah, 1997, p. 3337-3342). Therefore, it is argued that the formula can never yield the correct price of an option but rather an approximation of the correct market price. The fact that the market price is often identical to the theoretical Black-Scholes price is explained by a 'self-fulfilling prophecy' constituted by the use of the formula (see Shah, 1997, p.3337-3342.) Black admits that: " ... *traders now use the formula and its variations extensively. They use it so much that market prices are usually close to the formula even in situations where there should be a large difference: situations, for example, where a cash takeover is likely to end the life of the option or warrant*" (Black, 1989, p. 8). The analytical correctness of the formula on the basis of its assumptions has been proven numerously.

The creative and invaluable contribution of Black and Scholes to option pricing theory was their "*tinkering*"⁶; starting from the CAPM, their thought process enrolled to recognising that hedging must lead to the risk-free interest rate replacing the stock's expected return. This allowed the Black-Scholes partial differential equation to be derived and subsequently solved for the Black-Scholes option pricing formula. Thereby, they laid the foundation for the first major application of Arbitrage Theory (Discounting). Merton's creative contributions consist of the idea of the arbitrage portfolio and the application of Itô-Calculus, laying the rigorous foundation (Duplication). The term "*risk-neutral valuation*" (See Hull, 2000, pp. 205) has become the generic term for pricing via arbitrage portfolios. The crucial difference between the paradigm of equilibrium models and the paradigm of arbitrage models is, (besides the more technical innovation of using a different math in form of the Itô-Calculus), the fundamentally different assumptions used. In equilibrium models (in general and especially in the CAPM), the user has to make assumptions about risk preferences of individuals whose behavior must be modeled. Therefore, the CAPM always returns the risk-return tradeoff: An investor will only accept a risky security if, and only if, he is adequately compensated by a higher return. By contrast, the risk-return tradeoff is irrelevant for arbitrage models because the valuation is conducted on a *risk-neutral* basis (as the hedging portfolios are risk-free, they must earn the risk-free rate).

2.2 Process of generating the Formula and Application of an Evolutionary Model

Neither of the two approaches of Arbitrage Theory shown in Exhibit 2 was consequently and rigorously employed by Black and Scholes in their 1973 article⁷; the derivation of the Formula was not free from mathematical flaws. Examples of the correct derivation can be found in Baxter/Rennie⁸ (Discounting) and Schindler et al.⁹ (Duplication). Discounting requires the application of the Martingale techniques¹⁰ and solving a stochastic integral, where as duplication requires the Itô-Calculus and solving a stochastic differential equation. Both approaches yield the same results: Using the Black-Scholes differential equation, one obtains the Black-Scholes option pricing formula. Though neither of the approaches was adhered strictly to, Black and Scholes' article introduced both approaches and Arbitrage Theory itself into modern finance.

Therefore, this discussion shall not be prolonged here, but was only mentioned for reasons of completeness. The concept of the 'self-fulfilling prophecy' goes back to Merton's father, Robert King Merton (see Merton, 1948, p. 193-210).

⁶ which will be described below in further detail

⁷ Although they "proved" the formula via the CAPM and via arbitrage portfolios

⁸ Baxter/Rennie, 2002

⁹ Schindler et al., 1994, p. 1-76

¹⁰ see Musiela/Rutkowski, 1998

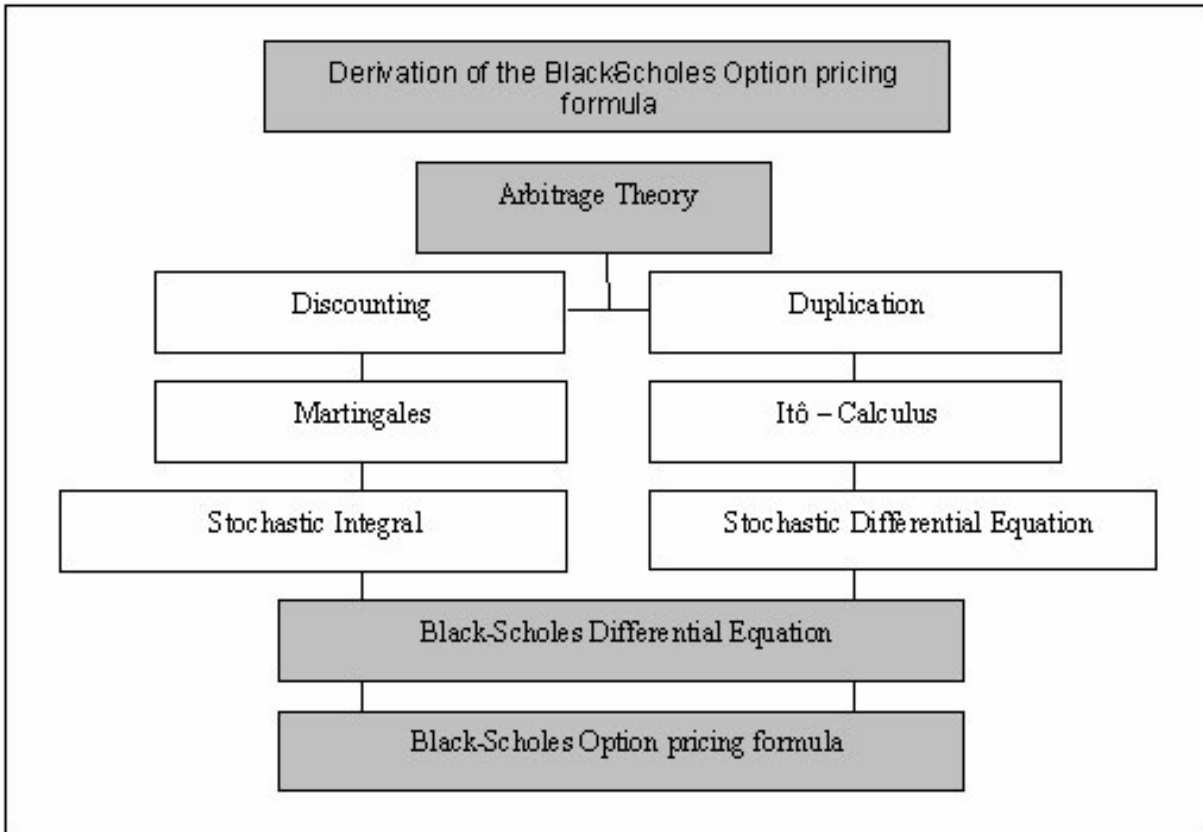


Figure 3: Derivation of the Black-Scholes option pricing formula

The starting point was the search for a warrant pricing formula initiated by Samuelson¹¹, and then taken forward by Black, followed by Black and Scholes and later Merton. Ultimately, the path led to the solution of another problem, i.e. option pricing that will be described and analyzed below.

First, we will describe the dominant problem definition of the actors and give an overview of the process of finding the option pricing formula. Second, the Research Line of the actors along three critical research events will be analyzed. Third, we will show how the fourth critical research event, a second order critical research event as described above, caused re-negotiations among the actors and the shift of the dominant problem definition. This problem shift allowed to overcome the existing paradigm of equilibrium models and to accelerate the establishment of the paradigm of arbitrage models.

2.3 The dominant problem definition and overview of the process of finding the option pricing formula

Fisher Black¹² portrayed the process of the genesis of the Black-Scholes option pricing formula, at least in its beginning, as *"tinkering"*. It can however not be concluded that this tinkering was conducted randomly and that the discovery of the formula was therefore a "random success". On the contrary, it has to be acknowledged that Black, before he started his research, had clearly defined his problem, i.e. finding a warrant pricing formula, as well as a possible road map to achieve it: the CAPM. He convinced Merton and Scholes to join his problem definition. Hence, a dominant problem definition was in place as well as a possible solution within the given economic paradigm of equilibrium models.

¹¹ Samuelson, 1965

¹² Black, 1989, p. 4

Black and Scholes as well as Merton were influenced by the CAPM and its related paradigm. In 1965, Black came in touch with the CAPM for the first time: *"I began to spend more and more time studying the capital asset pricing model and other theories in finance. The notion of equilibrium in the market for risky assets had great beauty for me"*¹³. Mehrling¹⁴ goes as far as to interpret the CAPM and its variations and developments as the center of Black's work as a whole. Scholes received his PhD from the Graduate School of Business in Chicago where he was thoroughly introduced to this paradigm. Together with Fisher Black and Michael Jensen, Scholes tested the model empirically.¹⁵ Merton studied Portfolio Theory and the CAPM when he worked with Samuelson on the pricing of warrants.¹⁶ Furthermore, he developed an intertemporal version of the CAPM.¹⁷ Black's and later Black's and Scholes' first attempts on their way to the option pricing formula were based on the CAPM.¹⁸ Black and Scholes discussed their work with Merton before the publication of their first article in 1973. It was him who pointed out that it was unnecessary to fall back on the CAPM to derive the Black-Scholes option pricing formula.¹⁹

The Black-Scholes differential equation which can be solved for the Black-Scholes option pricing formula was discovered by Black in 1969.²⁰ By 1965, Black worked for the consulting firm Arthur D. Little, Inc. where Jack Treynor was one of his colleagues. Treynor had derived the CAPM independently²¹ from Sharpe, Lintner and Mossin and introduced Black to the model.²² In the following years, Black not only applied the model to stocks and bonds but also tried to find a warrant pricing formula²³ based on the CAPM. In 1969, this research led to the partial differential equation which later became known as the Black-Scholes differential equation.²⁴

As Black based his reasoning on the CAPM, it was necessary to calculate the expected value of the stock at the maturity date of the warrant. Depending on this, the expected value of the warrant could be estimated and discounting this expected value yielded the present value of the warrant. However, to apply this method, the expected return of the stock was regarded exogenously as was the appropriate discount rate to discount the expected payoff of the warrant. The warrant's risk depends on both the stock price and on time. Black experimented with the CAPM to mirror these dependencies:

*"I applied the capital asset pricing model to every moment in a warrant's life, for every possible stock price and warrant value. To put it another way, I used the capital asset pricing model to write down how the discount rate for a warrant varies with time and the stock price. This gave me the differential equation."*²⁵

Black was unable to solve the equation, but made the important discovery that some variables which he had initially used were not part of the differential equation:

¹³ Black, 1989, p. 5

¹⁴ Mehrling, 2000, p. 1-39

¹⁵ See Scholes, 1998, p. 353, see Black/Jensen/Scholes, 1972, pp. 79-121

¹⁶ See Bernstein, 1992, p. 214-215

¹⁷ See Merton, 1973(b), p. 867-887

¹⁸ See Black, 1989, p. 6, Dunbar, 2001, p. 32-34

¹⁹ See Black/Scholes, 1973, p. 641, Footnote 3, Dunbar, 2001, p. 35

²⁰ See Black, 1989, p. 5. Bachelier (1900) & Samuelson (1965) found it before.

²¹ - but obviously never bothered publishing it -

²² See Black, 1989, p. 5

²³ In the 1970s warrants were in the center of interest instead of options because warrants were publicly traded. The OTC market for options was regarded as being *"imperfect"* (Black, 1989, S. 5) because of doubtful business practices (see Dunbar, 2001, p. 31-32; Bernstein, 1992, p. 209).

²⁴ See Black, 1989, p. 5

²⁵ Black, 1989, p. 5

" *The warrant value did not seem to depend on how the risk of the stock was divided between risk that could be diversified away and risk that could not be diversified away. It depended only on the total risk of the stock (as measured, for example, by the standard deviation of the return on the stock). The warrant value did not depend on the stock's expected return, or on any other asset's expected return. That fascinated me. But I was still unable to come up with the formula. So I put the problem aside and worked on other things.* " ²⁶

In 1969, when Black and Scholes jointly worked on empirical tests of the CAPM, they discussed the warrant pricing problem.²⁷ Black and Scholes continued where Black had failed: Black had already discovered that the warrant's price was independent of the stock's return itself, but depended on the volatility or standard deviation of the stock's return. " *We decided to try to assuming that the stock's expected return was equal to the [risk-free] interest rate. In other words, we assumed that the stock's beta was zero; all of its risk could be diversified away.* " ²⁸ Going forward, Black and Scholes assumed a constant risk-free interest rate and a constant volatility of the stock's return. Since Samuelson's article on warrant pricing in 1965²⁹ it was known that stock prices are lognormally distributed and follow a geometric Brownian motion. Sprenkle³⁰ had made the same assumptions for the stock price; on this basis Black and Scholes assumed the risk-free rate at the stock's return and used Sprenkle's pricing formula: " *By putting the [risk-free] interest rate for the expected stock return into his formula, we got the expected terminal value of the option under our assumptions.* " ³¹

However, Black and Scholes were looking for a formula giving the present value of an option, not its value at maturity. After having made an assumption about the stock's return, they therefore had to make an additional assumption regarding the appropriate discount rate: " *Rather suddenly, it came to us. We were looking for a formula relating the option value to the stock price. If the stock price had an expected return equal to the [risk-free] interest rate, so would the option. After all, if all the stock's risk could be diversified away, so could all the option's risk. If the beta of the stock were zero, the beta of the option would have to be zero.* " ³²

Hence, from the assumption of the risk-free interest rate as the stock's return, Black and Scholes concluded that the appropriate discount rate for the warrant had to be the same risk-free interest rate. The present value of the warrant could now be calculated. Even more, the assumption of the risk-free interest rate as the warrant's return allowed Black and Scholes to solve the differential equation discovered Black with the analytic solution being the Black-Scholes option pricing formula.³³

Based on the CAPM, Black and Scholes finally came up with the option pricing formula. When they discussed their result with Merton, he pointed out that the CAPM was not necessary to derive the option pricing formula and that the use of the risk-free interest rate could be justified otherwise, but not within the CAPM.³⁴ After all, Black and Scholes assumption of the risk-free interest rate as the stock's return and the warrant's return was arbitrary. The warrant's price was independent from the stock's return; hence, one can only conclude that *every* assumption for the stock's return is justified, *not*, that the assumption of the risk-free interest rate as the stock's return is the *only* justified assumption.

²⁶ Black, 1989, p. 6

²⁷ See Scholes, 1998, p. 354

²⁸ Black, 1989, p. 5

²⁹ See Samuelson, 1965, p. 41-49

³⁰ Sprenkle, 1961, p. 412-474

³¹ Black, 1989, p. 6

³² Black, 1989, p. 6

³³ See Black, 1989, p. 6, Dunbar, 2001, p. 34

³⁴ See Dunbar, 2001, p. 35

Merton proposed to additionally assume that continuous trading in securities was possible. On the basis of this assumption, Merton argued that it should be possible to construct a portfolio of stock and warrant or option that was virtually riskless and therefore yields a return equal to the risk-free interest rate. Simplified, Merton's argument works as follows: As option and stock depend on the same source of risk (volatility), a portfolio with the correct mixture of a long position in stocks and a short position in options or vice versa has to be riskless because the risk is once bought (long) and once sold (short). Further, if short selling was possible without restrictions, it should be possible to keep the portfolio riskless for the lifetime of the option by continuously buying or selling whenever the stock price changes.

By then applying Itô-Calculus³⁵, Merton was able to derive the Black-Scholes option pricing formula based on the concepts of a duplication portfolio and a self-financing, dynamic and stochastic duplication strategy: "*Merton made a number of suggestions that improved our paper. In particular, he pointed out that if you assume continuous trading in the option or the stock, you can maintain a hedged position between them that is literally riskless. In the final version of the paper, we derived the formula that way, because it seemed to be the most general derivation.*"³⁶ Black and Scholes' article was published in the May 1973 edition of the Journal of Political Economy.³⁷

2.4 "Research Line" and first "critical research event"

During the described search process, the "*research line*" can be clearly defined. The first "*critical research event*" on the way to the option pricing formula was Black's meeting with Treynor and his introduction to the CAPM. Thereby, Black's growing interest in finance was nurtured by his mastering the existing economic paradigm. It was Black's task with Arthur D. Little to advise clients in portfolio management.³⁸ Dunbar states: "*Like Treynor, he [Black] was unimpressed by academia*"³⁹ From Black's professional duties as well as from his attitude towards academia, one may conclude that his interest in the CAPM and his attempts to apply the CAPM to many different asset classes was more practically than scientifically motivated (even though he held a PhD in Applied Math⁴⁰). Black was not an academic as Scholes, an assistant professor at the Massachusetts Institute of Technology (MIT) and assistant to Samuelson at MIT. Therefore, the network formed between Black, Scholes and Merton is more reminiscent of industrial R&D networks, as opposed to purely academic networks.

Given the heterogeneity⁴¹ of the actors, it is appropriate to interpret the genesis of the Black-Scholes option pricing formula not only as a scientific invention following solely scientific motivations; but also

³⁵ See Hackenbroch/Thalmeier, 1994, p. 193 ff.

³⁶ Black, 1989, p. 6

³⁷ In 1972, Black & Scholes had a paper in the Journal of Finance testing the formula, p. 399-418

³⁸ See Bernstein, 1992, p. 209

³⁹ Dunbar, 2001, p. 32

⁴⁰ See Black, 1989, p. 5

⁴¹ The fact that Black was not a scientist in the literal sense of the word was an obstacle when he and Scholes tried to publish their work; at least it seemed so to Black: "*I suspect that one reason these journals didn't take the paper seriously was my non-academic return address*" (Black, 1989, S. 7). Scholes had other things in mind: "*I felt that I was an unknown assistant professor and the paper would not be considered broad enough for those academic journals*" (Scholes, 1998, S. 356). It was Merton Miller and Eugene Fama who finally helped Black and Scholes and placed their article in the *Journal of Political Economy* (See Scholes, 1998, p. 356). However, Black and Scholes were not the only ones who, at least for some time, unsuccessfully claimed recognition for their work. They suffered the fate of many "paradigm breakers": Bachelier did not succeed in convincing his professor Poincaré of his dissertation (See Dunbar, 2001, pp. 11) and Markowitz' dissertation on 'Portfolio Selection' was almost rejected in Chicago - Milton Friedman claimed that Markowitz' problem was too special to be considered in economics (See Bernstein, 1992, p. 60). Black and Scholes as well as Markowitz have finally received the Nobel price for their works, while Bachelier remained forgotten during his lifetime (See Dunbar, 2001, pp. 11).

as a commercial innovation because Black's interest in finance was commercially motivated: *"Our first thought was to publish a paper describing the formula. Later, we thought also about trying to use the formula to make money trading in options and warrants."*⁴²

2.5 The second "critical research event"

Black's knowledge of the CAPM and its inability to provide satisfactory answer to the warrant pricing formula required him to move beyond *"normal science"* (i.e. the given equilibrium paradigm) to a *"dominant problem definition"* which finally yielded the Black-Scholes option pricing formula. The second *"critical research event"* in this process is Black's failure to solve the partial differential equation: *"I spent many, many days trying to find the solution to the equation. ... But I was still unable to come up with the formula. So I put the problem aside and worked on other things."*⁴³

One can imagine a scenario in which this failure, creating a discontinuity and being the cause for Black giving up his niche, would have led to the end of the project as a whole. However, the third *"critical research event"* prevented this outcome.

2.6 The third "critical research event"

*"In 1969, Myron Scholes was at MIT, and I had my office near Boston, where I did both research and consulting. Myron invited me to join him in some research activities at MIT. We started working together on the option problem, and made rapid progress."*⁴⁴ Before Scholes entry, the network consisted of Treynor and Black, two finance professionals, who were now joined by an academic. With respect to the education and experiences of its participants, the network was obviously not homogenous and with respect to its participants the network was not stable. On the contrary, the network carries the characteristics of an open, fragile network as described by Callon and Latour. Black and Scholes' research took place under a *"dominant problem definition"* and within the given equilibrium paradigm; nevertheless, as their research progress, the limitations of this paradigm became obvious. Dosi characterizes research within a given paradigm as a process and an *"... intrinsically uncertain activity of search and problem-solving based upon varying combinations of public and private ... knowledge, general scientific principles and rather idiosyncratic experience, well-articulated procedures and rather tacit competences."*⁴⁵ This description fits Black and Scholes' procedures when they tried to find the warrant pricing formula. It was neither sure that they would come up with a solution at all, nor was it predictable that the solution would be the Black-Scholes option pricing formula. The application of the CAPM clearly proved the limitations of the paradigm; Black and Scholes were only able to circumvent this obstacle by arbitrarily assuming the risk-free rate as both the stock's and the warrant's return, an assumption which could not be justified within the CAPM.

2.7 Fourth "critical research event", "re-negotiation" and shift of "dominant problem definition"

Therefore, the entry of the third decisive actor, Merton, into the network can be interpreted as the result of the partial failure of Black and Scholes, who were able to come up with a formula, but not to rigorously derive it. Merton caused a problem shift by emphasizing the fact that the formula could be derived without actually using the CAPM and thereby independent of the given paradigm. Before his

⁴² Black, 1989, p. 6

⁴³ Black, 1989, p. 5-6

⁴⁴ Black, 1989, p. 6

⁴⁵ Dosi, 1990, p. 233

entry, the "*dominant problem definition*" was finding a warrant pricing formula; with his entry, the "*dominant problem definition*" shifted to finding an alternative derivation of the Black-Scholes option pricing formula. The "*dominant problem definition*" was re-negotiated among the actors and Merton's entry constitutes the fourth "*critical research event*". Furthermore, another discontinuity can now be identified: Black and Scholes were already - at least to some extent - acting outside the given paradigm before Merton entered the network by making their assumptions about the risk-free interest rate not justifiable within the given paradigm. Merton completed this paradigm change by justifying Black and Scholes' assumption, but of course not within the given paradigm. Instead, by arguing with an arbitrage portfolio, he laid the foundations of the new paradigm of Arbitrage Theory. Black was immediately convinced by this argument. Bernstein cites Black: "*A key part of the option paper I wrote with Myron Scholes was the arbitrage argument for deriving the formula. Bob gave us that argument. It should probably be called the Black-Merton-Scholes paper.*"⁴⁶

⁴⁶ Bernstein, 1992, p. 223; In a footnote of their 1973 article, Black and Scholes mention that the arbitrage argument goes back to Merton (See Black/Scholes, 1973, p. 641, footnote 3); in the same year, Merton published an article dealing with option pricing and extending the Black-Scholes Option pricing formula to the case of stock with a constant dividend yield (See Merton, 1973(a), p. 141-183). Merton claims to be the first who labelled Black and Scholes model the Black-Scholes Option pricing formula: "*I am also responsible for naming the model 'the Black-Scholes Option Pricing Model. ... My 1970 working paper was the first to use the 'Black-Scholes' label for their model ...*" (Merton, 1998, p. 326). Though mutual recognition seems to have been strong among the actors, for reasons of completeness a final remark has to be made further illuminating their relationship: "*We were both working on papers about the formula, so it was a mixture of rivalry and cooperation*" (Black, 1989, S. 7).

3 Conclusion

The article's objective was to analyze the emergence of the Black-Scholes option pricing formula in terms of innovation theory and namely by concepts developed for the second sector. In a first step, these concepts have been briefly described. Second, in reference to models on innovation in the second sector, we adopted the evolutionary explanation of innovation and identified the existing paradigm in finance before the invention of the Black-Scholes option pricing formula: the paradigm of equilibrium models with the CAPM being the most well known example. We showed how Black and Scholes, rooted in the paradigm of equilibrium, tried to present an option pricing formula on the basis of this paradigm. We found that they, in creating an option pricing formula, accelerated the advent of a new paradigm in finance: the paradigm of arbitrage models. This is because the correct derivation of the formula was not done in an equilibrium setting but, with the help of Robert Merton, in an arbitrage setting.

Third, interpreting the search for the Formula as an interaction of actors (most importantly Black, Scholes and Merton) in a network, we analyzed the process of creating the formula and establishing a new paradigm. Analogies can be found between the process in an industrial R&D laboratory identified by Vergragt and the conduct of research that finally culminated in the Black-Scholes option pricing formula, i.e. the dominant problem definition of the actors (*"finding a warrant pricing formula"*)

a research line along three critical research events (*(1) Black meeting Treynor and being introduced to the CAPM, (2) Black's failure in solving the differential equation, (3) Scholes' entry into the network*)

a second order critical research event that finally caused a re-definition of the problem and its solution in form of the Black-Scholes option pricing formula (*Merton's entry and his derivation of the formula without the CAPM*)

We find that the transfers of concepts used for explaining innovations in the second sector are fruitful approaches to explain innovations in the financial sector. We are convinced that the transfer to further innovations in the financial sector will help us to better understand the innovation process in financial markets.

4 References

- Bachelier, L., 1900. *Theory of Speculation*. Gauthier-Villars, Paris.
- Bernstein, P.L., 1992. *Capital Ideas, The Improbable Origins of Modern Wall Street*, New York, London, Toronto, Sydney, Tokyo, Singapore. The Free Press.
- Black, F., 1989. How we came up with the option formula. *Journal of Portfolio Management*, Volume 15, No. 2, pp. 4-8.
- Black, F., Jensen, M.C., Scholes, M., 1972. The Capital Asset Pricing Model: Some Empirical Tests, *Studies in the Theory of Capital Markets*. Michael C. Jensen, ed., Praeger, New York, pp. 79-121.
- Black, F., Scholes, M., 1972. Valuation of Option Contracts and a Test of Market Efficiency. *Journal of Finance*, Volume 27 (May), pp. 399-418.
- Black, F., Scholes, M., 1973. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, May – June, pp. 637-654.
- Baxter, M., Rennie, A., 2002. *Financial Calculus, An introduction to derivative pricing*. University Press, Cambridge.
- Dosi, G., et al., 1990. The nature of the innovative process. *Technical Change and Economic Theory*. Printer Publishers Limited, S. 221-238.
- Dunbar, N., 2001. *Inventing Money. The story of Long-Term Capital Management and the legends behind it*, second ed. John Wiley & Sons, Ltd., Chichester, New York, Weinheim, Brisbane, Singapore, Toronto.
- Hackenbroch, W., Thalmeier, A., 1994. *Stochastische Analysis, Eine Einführung in die Theorie der stetigen Semimartingale*. B.G. Teubner, Stuttgart.
- Hull, J.C., 2000. *Options, Futures and Other Derivatives*, fourth ed. Upper Saddle River, Prentice Hall, New Jersey.
- Merling, P., 2000. *Understanding Fisher Black*, Economics Department, Barnard College. Columbia University, New York, pp. 1-39.
- Merton, R.C., 1973(a). Theory of Rational Option Pricing. *The Bell Journal of Economics and Management Science*, Volume 4, Issue 1, pp. 141 – 183.
- Merton, R.C., 1973(b). An Intertemporal Capital Asset Pricing Model. *Econometrica*, Volume 41, S. 867-887.
- Merton, R.K., 1948. The Self-Fulfilling Prophecy. *Antioch Review*, pp. 193-210.
- Musiela, M., Rutkowski, M., 1998. *Martingale methods in financial modelling: theory and applications*, corrected second printing. Springer, Berlin, Heidelberg, New York, Barcelona, Budapest Hong Kong, Milan, Paris, Santa Clara, Singapore, Tokyo.
- Samuelson, P.A., 1965. Proof That Properly Anticipated Prices Fluctuate Randomly, *Industrial Management Review*, Volume 6, No. 2, S. 41-49
- Schindler, K., et al., 1994. *Finanzinnovationen, Grundlagen und Praxis der Optionspreisbestimmung*. Zentrum für Techno- und Wirtschaftsmathematik: Universität Kaiserslautern.
- Scholes, M.S. 1998. Derivatives in a Dynamic Environment. *American Economic Review*, Volume 88, No. 3, pp. 350-370.

- Shah, A., 1997. Black, Scholes and Merton: Their Work and Their Consequences. *Economic and Political Weekly*, Volume 32, No. 52, pp.. 3337-3342.
- Sprenkle, C., 1961. Warrant Prices as Indicator of Expectations and Preferences, in: Cootner, Paul H. (ed.), *The Random Character of Stock Market Prices*, Cambridge (Mas.), pp. 412-474.
- Vergragt, P.J., 1988. The Social Shaping of Industrial Innovations, *Social Studies of Science*. London, Newbury Hills and New Dehli, Vol. 18, pp. 483-513.

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