

Trenczek, Jan; Wacker, Konstantin M.

Working Paper

Human Capital Misallocation and Output per Worker Differences: Beyond Cobb-Douglas

GLO Discussion Paper, No. 1331

Provided in Cooperation with:

Global Labor Organization (GLO)

Suggested Citation: Trenczek, Jan; Wacker, Konstantin M. (2023) : Human Capital Misallocation and Output per Worker Differences: Beyond Cobb-Douglas, GLO Discussion Paper, No. 1331, Global Labor Organization (GLO), Essen

This Version is available at:

<https://hdl.handle.net/10419/278063>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Human Capital Misallocation and Output per Worker Differences: Beyond Cobb-Douglas

Jan Trenczek^a, Konstantin M. Wacker^b

^a *University of Mainz; jan.trenczek@gmx.de*

^b *University of Groningen & GLO Fellow; k.m.wacker@rug.nl*

This version: September 28, 2023

Abstract

Misallocation of human capital across sectors can have substantial negative implications for aggregate output. So far, the literature examining this type of labor misallocation has assumed a Cobb-Douglas production function. Our paper departs from this assumption and instead considers more flexible CES production functions with different labor skill types as individual inputs. Our estimates from sectoral data of 39 countries suggest that physical and human capital are less substitutable than Cobb-Douglas assumes. Our counterfactual results indicate that human capital misallocation can explain approximately 15% of output per worker variation across countries, which is substantially less than under a Cobb-Douglas specification (21%).

JEL classifications: O41, O47, E24

Keywords: Misallocation, Human Capital, Sector level, Production function, Elasticity of substitution, Total factor productivity

We thank Beata Javorcik, Fransceco Caselli, Chris Parmeter, Daniel Henderson, Rainer Klump, and Jakub Muck for beneficial discussions and scholars from Oxford, Mainz, Heidelberg, Frankfurt and Mannheim, as well as participants at the NAPWX conference in Miami for helpful comments and suggestions. Financial support from the Foundation of German Business (sdw) is gratefully acknowledged. This paper is the third chapter of Jan Trenczek's dissertation "Essays on Structural Transformation".

1 Introduction

Countries do not allocate production factors to their most efficient use and some countries manage this allocation better than others. Such factor misallocation across sectors, firms, or plants lowers aggregate productivity and output. In fact, variation in the degree of factor misallocation across countries offers to be one of the best candidates to answer the question why some countries are so much richer than others (Banerjee and Duflo, 2005; Hsieh and Klenow, 2010; Jones, 2016; Restuccia and Rogerson, 2017).

To quantify the costs of misallocation, researchers need to specify how production units generate output from inputs (see, e.g., Restuccia and Rogerson, 2008; Vollrath, 2009; Hsieh and Klenow, 2009).¹ Empirical work on misallocation generally assumes a Cobb-Douglas production function of the form

$$Y = AH^\alpha K^{1-\alpha} \quad \alpha \in (0, 1) \quad (1)$$

where Y and K are output and physical capital, respectively, and H is human capital, which is skill-augmented labor. The term A represents total factor productivity (TFP) and α defines the factor output shares. Allocative efficiency demands production factors K and H to be allocated to sectors or plants with the highest productivity A .

The assumption of a Cobb-Douglas production function in the misallocation literature is analytically convenient but imposes strong assumptions. Those assumptions can be particularly problematic when quantitatively assessing the degree of factor reallocation that is necessary to achieve allocative efficiency. In the neoclassical framework, the latter requires equalization of marginal products across sectors (or firms). How strongly marginal productivity responds to such reallocation depends on the production function and, particularly, the elasticity of factor substitution, which is equal to one in the Cobb-Douglas case.

By now, there is vast empirical evidence that production factors are more complementary in aggregate production than implied by Cobb-Douglas (e.g., León-Ledesma et al., 2015; Knoblach et al., 2019; Gechert et al., 2022). Quantifying what this means for the misallocation literature is plagued by several challenges. First, estimation and parameterization of a more flexible production structure is a challenge by itself. Parameters of the most natural alternative production functions, the constant-elasticity-of-substitution (CES) function, are not ‘deep’. Instead, they are a function of the elasticity of substitution itself. This feature significantly undermines their use for comparative-static exercises (Klump and de La Grandville, 2000; Klump et al., 2007; Klump and Saam, 2008; Klump et al., 2012). Second, output elasticities are not constant in a CES framework. I.e., once a unit of a production factor

¹In this paper, output refers to value added. *Gross* output comprises valued added and intermediate inputs.

is reallocated to another sector, the production function takes another form. Finally, lack of sector-specific data on internationally comparable prices and inputs constitute another challenge in the literature.

The main contribution of our paper is to tackle those challenges of moving beyond Cobb-Douglas. In line with Vollrath (2014), Herrendorf and Schoellman (2018), and motivated by the neoclassical growth model, where labor is the constrained production factor, we focus on quantifying the role of human capital misallocation for explaining cross-country output per worker differences but consider a more flexible aggregate production structure. We therefore combine data from the World-Input-Output Database (WIOD) for 39 countries² and relative industry-level prices from Inklaar and Timmer (2014) and rely on the class of CES production functions. Those CES functions relax the assumption of a unitary substitution elasticity of inputs while still imposing a structure that can be used to calculate counterfactuals that are necessary to gauge misallocation costs. More precisely, we build on theoretical and empirical advances for so-called ‘normalized’ CES production functions. As introduced by Klump and de La Grandville (2000) in the context of growth models, normalizing creates “families” of CES functions that are only distinguished by their constant elasticity of substitution. To address the issue of varying output elasticities, we rely on iterative solution techniques. This allows us to conduct suitable counterfactuals in the context of a standard misallocation framework.

A second contribution of our paper is to relax the assumption of perfect substitutability of workers with different educational achievements (henceforth: skill levels). Human capital H in equation (1) is usually constructed as unskilled worker equivalents, which imposes perfect substitutability (Jones, 2014; Caselli and Ciccone, 2019). Again, this assumption influences the output and marginal product levels for a given allocation of factors and hence the aggregate cost of factor misallocation. Since WIOD’s Socio-Economic Accounts (SEA) include information on hours worked and labor compensation for three different skill types, we can consider more flexible forms of the human capital aggregate than commonly done in the literature.

Our empirical strategy first estimates sector-specific values for the elasticity of substitution between inputs, given the lack of reliable estimates on the sector level in the literature. The second empirical task is to examine the role of human capital misallocation across sectors for cross-country output per worker differences under different production function specifications. We build on the standard misallocation framework, where market frictions cause gaps

²Despite only ranging until 2007, this data set has several attractive features. First, it covers the total economy for a sample of 40 countries, accounting for 85% of world output. Second, it differentiates between several service sectors. Third, it covers a time horizon of more than a decade from 1995 onward, allowing to track changes of the role of misallocation over time. Moreover, it contains detailed information for different skill levels (see below).

in the marginal product of human capital across sectors within countries. We then first estimate by how much aggregate output in each country would hypothetically grow, assuming that human capital misallocation is eliminated so that the marginal products across sectors are equalized. Given each country’s hypothetical output per worker, we obtain two ratios and compare each across different production function specifications. The first ratio, $R_{c,USA}$, tells us how much of the output per worker gap between each country c and the U.S. is left to explain *after* we have eliminated differences in the degree of misallocation. The second ratio, R_{World} , extends this exercise to the overall output per worker variation in the sample.

Our results indicate that cross-country differences in human capital misallocation can account for 15% of the output per worker variation in the sample in 2007 if we assume that sectors produce with a CES technology; considerably less than the 21% that a Cobb-Douglas production function implies for our sample. The reason for this difference are the elasticities of substitution between physical and human capital, which we estimate to range between 0.3 and 0.7. As both production factors are found to be gross complements in all sectors, the marginal products respond more strongly to human capital reallocation across sectors. This implies that less human capital needs to be reallocated for marginal products to equate across sectors, leading to lower hypothetical output gains of reallocation. Importantly, however, the extent to which the gains decrease in the CES case differs across countries and is (at least weakly) correlated with the output per worker level. This results in a smaller role of human capital misallocation for explaining the cross-country variation in output per worker. Moreover, we find that deviating from the assumption of perfect substitutability of skill types affects the estimated hypothetical reallocation only marginally in most countries.

As a sensitivity check, we consider different parameter values and sample compositions. The 10th-to-90th percentiles intervals from different simulations suggest that human capital misallocation across sectors can account for 12-21% of the variation in output per worker in our sample. Overall, we hence conclude that imposing a Cobb-Douglas production function specification can have sizable implications for the estimated gains from reallocation.

This paper links to the growing research on the role of factor misallocation for aggregate productivity and output. Specifically, it relates to previous efforts that follow an *indirect approach* (cf. Restuccia and Rogerson, 2013, 2017). This approach aims to quantify the cost of factor misallocation that results from many possible frictions without identifying the underlying source of misallocation.³ In their seminal contribution to this literature, Hsieh and Klenow (2009) propose a methodology to quantify the cost of factor misallocation that involves fitting a production function to firm level data to infer distributions of marginal

³Contrary to this, the *direct approach* aims to assess the consequences of a specific source of misallocation. See e.g., Buera et al. (2011) and Midrigan and Xu (2014) for the role of financial frictions, Caselli and Gennaioli (2013) for the role of failures of meritocracy due to dynastic management, or Garcia-Santana and Pijoan-Mas (2014) for the role of size-dependent policies and regulations in India.

products within manufacturing industries in the U.S., India, and China. Numerous papers have followed this approach.⁴ What unifies all these studies, but also the misallocation analyses in Vollrath (2009), Caselli and Feyrer (2007), Restuccia and Rogerson (2008), Alfaro et al. (2008), and Bartelsman et al. (2013), is that they rely on a Cobb-Douglas function as in equation (1).

We are not the first to note that the assumed functional form of the production function is central to measures of allocative efficiency (see, e.g., Restuccia and Rogerson, 2017). However, previous work focused on relaxing the assumption that producers use the same Cobb–Douglas technology by allowing for heterogeneous capital intensities in production (Hsieh and Klenow, 2009; Oberfield, 2013; Schelkle, 2016). The two assumptions on the substitutability of inputs implied by equation (1) remain unaddressed. Our paper highlights the need to take this aspect of the production function more seriously in future research on misallocation – irrespective of whether one works with sectoral macro data, as we do in our paper, or with micro data sets and micro-founded structural models, which are increasingly popular in this literature.⁵

The remainder of this paper is structured as follows. Using a development accounting framework, the next section outlines how human capital misallocation translates into aggregate output per worker differences within and across countries. Section 3 presents the theoretical framework and optimization approach applied to calculate efficiency losses from labor misallocation. Section 4 discusses the data and model calibration. Particular focus is placed on “normalization” as a means to conduct suitable counterfactuals and facilitate the estimation of the elasticity of substitution. Section 5 presents the results, while section 6 contains robustness tests and extensions. Finally, we conclude.

⁴Among others, Busso et al. (2013); Brandt et al. (2013); Oberfield (2013); Vries (2014); Chen and Irarrazabal (2015); Dabla-Norris et al. (2015); Dias et al. (2016); Ryzhenkov (2016); Bento and Restuccia (2017). See also Restuccia and Rogerson (2013) and Restuccia and Rogerson (2017) for surveys of the misallocation literature.

⁵Microeconomic estimates of firm- or plant-level substitution elasticities are rare but suggest an elasticity around 0.4 for the U.S. case (Chirinko et al., 2011; Raval, 2019), well within the range of our sector-level estimates.

2 Human capital misallocation and output per worker differences

Before conducting the empirical exercise, it will be useful to recap how human capital misallocation across sectors relates to cross-country differences in output per worker. The standard approach in the development accounting literature is to attribute differences in output per worker in country c and a benchmark country to the sum of differences in factor endowments (e.g., physical and human capital), sector productivity (e.g., technology), and factor market efficiency.⁶ Formally:

$$y_{Benchmark}^0 - y_c^0 = \underbrace{(y_{Benchmark}^0 - y_c^1)}_{\text{Factor Endowments}} + \underbrace{(y_{Benchmark}^1 - y_c^2)}_{\text{Sector Productivity}} + \underbrace{(y_c^2 - y_c^0)}_{\text{Factor Market Efficiency}} \quad (2)$$

Although the U.S. are generally chosen as a benchmark country, note that for each of the three channels we can also choose the benchmark setting to be a hypothetical version of country c . In that case, we can write the latter term as $(y_c^2 - y_c^0)$. It measures the difference within country c between its maximized output per worker given its endowments as well as sector productivities and its actual output per worker. If factor markets are operating efficiently in country c , then this difference should be at or close to zero. Note that if we assume that all countries operate at y_c^2 , we effectively eliminate all cross-country differences in the degree of misallocation, while keeping the assumption that countries operate with different sector productivities and are endowed with different amounts of inputs per worker. In other words, we isolate the contribution of factor market efficiency in explaining output per worker differences across countries.

Importantly, we can also look at just a portion of $(y_c^2 - y_c^0)$ that can be attributed to the allocative efficiency (AE) of specific factors. Equation (3) illustrates this extension for the case of two inputs, physical and human capital.

$$y_{Benchmark}^0 - y_c^0 = \underbrace{(y_{Benchmark}^0 - y_c^1)}_{\text{Factor Endowments}} + \underbrace{(y_{Benchmark}^1 - y_c^2)}_{\text{Sector Productivity}} + \underbrace{(y_c^2 - y_c^3)}_{\text{Physical capital AE}} + \underbrace{(y_c^3 - y_c^0)}_{\text{Human capital AE}} \quad (3)$$

⁶For long, empirical work focused on the first two channels. The third channel, different degrees of inefficiency in the allocation of factors, was rather neglected. The reason for this was the use of an aggregate production function as the workhorse model in both growth and development accounting exercises. In order for the aggregate production function to be a valid representation of what happens in an economy, however, a key assumption is that prices reflect marginal product values and that inputs and outputs are perfectly mobile. In that case, the market allocates the available supply of inputs to maximize total output, or put differently, all factor markets operate efficiently (Banerjee and Dufo, 2005).

This paper focuses on y_c^3 , which represents the output per worker level in country c if human capital is allocated across sectors so that it maximizes aggregate output. It should be stressed that y_c^3 is a *hypothetical* output per worker level. To the extent that some amount of misallocation can appropriately be considered the result of measurement error or factors omitted from the model (e.g., adjustment costs), the estimated gains from reallocating human capital, captured by $y_c^3 - y_c^0$, are likely overstated. This paper therefore focuses on the role of misspecification for examining *differences* in misallocation across economies, rather than the levels per se.

Using y_c^3 , the next step is to estimate the variation in aggregate output across countries that can be attributed to differences in the misallocation of human capital across sectors. More precisely, we are interested in how the current variance in logarithmized output per worker across countries, y_c^0 , compares to the counterfactual variance given each country reaches y_c^3 . Formally, following the approach of Vollrath (2009), we compute the ratio of variances as

$$R_{World} = \frac{\text{var}(\ln y_c^3)}{\text{var}(\ln y_c^0)} \quad (4)$$

The ratio R_{World} tells us how much of the total variation in output per worker across countries is left to explain after we have eliminated differences due to the misallocation of human capital across sectors.⁷ In the same spirit, we calculate for each country:

$$R_{c,USA} = \frac{\frac{y_c^0}{y_{US}^0}}{\frac{y_c^3}{y_{US}^3}}, \quad (5)$$

which states how much of the output per worker gap between country c and the U.S remains when both countries reach their hypothetical output per worker level y_c^3 .

⁷The ratio R is analogous to the ratio V in Vollrath (2009), when only frictions in the labor market are eliminated.

3 Calculating efficiency losses in a CES framework

3.1 Model specifications

To calculate R_{World} and $R_{c,US}$, we need to calculate the hypothetical output per worker, y_c^3 , which differs from the actual output per worker, y_c^0 , only in that human capital is reallocated across sectors so that gaps in the marginal product values are eliminated.⁸ In each country, aggregate output per worker, y_c , is defined as

$$y_c = \frac{Y_c}{N_c} \quad (6)$$

where N is the total number of workers.⁹ We assume that there is a single final good Y produced by a representative firm in a perfectly competitive and undistorted final output market. This firm combines the value added Y_i of $i = 1, \dots, J$ sectors using a Cobb-Douglas production technology¹⁰

$$Y_c = \prod_{i=1}^J Y_i^{v_i}, \quad \text{where } \sum_{i=1}^J v_i = 1 \quad (7)$$

In each sector, a representative firm produces value added using a constant-returns-to-scale CES production function of the form (the country index is omitted for clarity)

$$Y_i = C_i [\beta_i H_i^{\rho_i} + (1 - \beta_i) K_i^{\rho_i}]^{\frac{1}{\rho_i}} \quad \beta_i \in (0, 1) \quad (8)$$

⁸To be precise, we do not compute y_c^3 directly, but calculate Y_c^3 first. That is, the aggregate output level of country c given an efficient allocation of labor across sectors. In a second step, we then calculate y_c^3 by dividing Y_c^3 by the total workforce N .

⁹Note that we work with each country's output *per worker* level and not their output *per capita* level. This choice is motivated by the main data set that we work with, the Socio-Economic Accounts in the World-Input-Output Database (WIOD-SEA), which contains employment data for the aggregate economy. The correlation of the aggregate employment data in WIOD-SEA and the employment data in the PWT 8.1 (Feenstra et al., 2015) is 0.9997 with equal means and medians across the countries of my sample. Using population data from PWT 8.1 does not affect the results in any significant way.

¹⁰Assuming that the final good is aggregated using a Cobb-Douglas production technology is analogous to assuming that aggregate preferences are Cobb-Douglas. The aggregator function thus explicitly states the underlying assumption how relative sector output prices respond to the change in sector quantities caused by the reallocation of labor types. More specifically, using a Cobb-Douglas function implies the assumption that the elasticity of substitution is the same for any pair of sector value added and takes a common value of 1. As a consequence, the share of the total budget spent on each good is assumed to be unaffected by the presence or absence of factor misallocation. This benchmark scenario is also used in the Hsieh and Klenow (2009) framework to aggregate output from different manufacturing industries. Similarly, Hallak and Schott (2011) assume that 1-digit SITC manufacturing industries, or selected 2-digit SITC manufacturing industries sectors are aggregated via Cobb-Douglas preferences. Recently, Redding and Weinstein (2018) estimated a substitution elasticity of 1.36 across 4-level NAICS sectors in the U.S.

where K is physical capital, C is a (Hicks-neutral) efficiency parameter, β is the distribution parameter and the elasticity of substitution between physical and human capital is defined as $\sigma = 1/(1-\rho) > 0$. We define H_i in each sector as a human capital input aggregator

$$H_i = F(LH_i, LM_i, LL_i, \gamma_i) \quad (9)$$

that is defined by the hours worked of low-skilled (LL), medium-skilled (LM), and high-skilled (LH) workers and the elasticity of substitution between the skill types defined as $\eta = 1/(1-\gamma) > 0$. Building on (9), we modify (8) to a two-level nested CES production function of the form

$$Y_i = C_i \left[\beta_i \left[\alpha_i^{LH} (LH_i)^{\gamma_i} + \alpha_i^{LM} (LM_i)^{\gamma_i} + (1 - \alpha_i^{LM} - \alpha_i^{LH}) (LL_i)^{\gamma_i} \right]^{\frac{\rho_i}{\gamma_i}} + (1 - \beta_i) (K_i)^{\rho_i} \right]^{\frac{1}{\rho_i}} \quad (10)$$

Here, α^{LM} , α^{LH} and β are the distribution parameters, while ρ and γ govern the substitutability between inputs. Each function is assumed to be “well-behaved” such that it satisfies standard regularity conditions like continuity, differentiability and is strictly increasing and concave in all factors. Note that in the traditional approach where $\gamma \rightarrow 1$ (i.e., perfect substitution), the human capital input is calculated as *low-skilled labor equivalents* by weighting the hours worked by each skill type with its relative efficiency, captured by the distribution parameters α^{LM} and α^{LH} . Importantly, this means that the relative marginal productivity of each skill type is independent from the skill composition of the labor force. A main contribution of this paper is to relax this assumption and work with the generalized function (10) to allow $1 < \eta \leq \infty$ (i.e., imperfect substitutability).

3.2 Optimization problem

In each country c , the optimal allocation of each skill type $L_k^* \in \{LL, LM, LH\}$ across sectors, ceteris paribus, can be determined by solving the problem of a social planner who maximizes aggregate value added.¹¹

$$L_{ki}^* = \arg \max_{L_{ki}} Y_c \quad (11)$$

¹¹Ceteris paribus implies that the optimization is subject to the constraint that technology and the physical capital stock in each sector stay constant and that the overall supply of each labor skill type in each country is fixed.

The first order condition of this maximization problem is that the marginal product value (MPV) of each skill type is equated across sectors. In reality, mobility barriers distort the allocation of workers across sectors. The sources of these barriers are numerous, including sector-specific labor regulations, skill requirements, or direct mobility costs (non-monetary and monetary). These distortions cause a wedge between the current MPV in each sector and the hypothetical sector-spanning MPV obtained at an efficient allocation, so that

$$MPV_i^{L_k} = (1 + \lambda_i^{L_k}) MPV_{hyp}^{L_k} \quad \forall i \quad (12)$$

where $\lambda_i^{L_k}$ displays the wedge. The set up presented in equation (12) is the starting point to find y_c^3 . Following Aoki (2012), we use equation (12) and $L_{kc} = \sum_{i=1}^J L_{kci}$, to write the current sector allocation of human capital within each country (omitting the country index c for clarity) as

$$L_{ki} = \frac{\frac{(1 + \lambda_i^{L_k}) MPV_{hyp}^{L_k} L_{ki}}{(1 + \lambda_i^{L_k}) MPV_{hyp}^{L_k}}}{\sum_{j=1}^J \left(\frac{(1 + \lambda_j^{L_k}) MPV_{hyp}^{L_k} L_{kj}}{(1 + \lambda_j^{L_k}) MPV_{hyp}^{L_k}} \right)} L_k, \quad (13)$$

which can be written as

$$L_{ki} = \frac{MPV_i^{L_k} L_{ki}}{\sum_{j=1}^J (MPV_j^{L_k} L_{kj})} \tilde{\lambda}_i^{L_k} L_k, \quad (14)$$

where $\tilde{\lambda}_i^{L_k}$ does not display the absolute, but relative wedge for each sector. The optimal quantity of each skill type in each sector can be obtained by setting $\tilde{\lambda}_i^{L_k}$ equal to unity. That is, we assume that distortions do not bias the allocation of skill types across sectors.

Some additional remarks are in order. A major assumption underlying the reallocation exercise is that human capital (or labor skill type respectively) is perfectly substitutable across sectors within each country. Clearly, this assumption is still strong. Note, however, that if workers in each sector are seen as idiosyncratic, this would render an investigation of human capital misallocation obsolete. Further note that equating the MPV of human capital instead of raw hours worked or number of workers allows for the possibility that wages per hour or worker across sectors may vary because of cross-sector differences in actual human capital. In the case of skill types being imperfect substitutes, optimization within skill types allows for wage gaps between different skills types after reallocation.

The optimal sector allocation of each factor is essentially determined by the sector's relative value added and factor output elasticity. If one assumes that both consumer preferences and production in each sector are best represented by a Cobb-Douglas function, the optimal

sector allocation can be readily obtained from equation (14). However, in general, the output elasticities of inputs are not constant, but depend on the form of the production function and the current allocation of factors. While this emphasizes the restrictiveness of using Cobb-Douglas functions, it also highlights that the flexibility of the CES framework demands some adjustments.

3.3 Normalization

To advance the reallocation framework outlined above into the CES environment, we need to address two key challenges. First, parameters of the non-normalized CES functions are not *deep* in the sense that they depend on the elasticity of substitution (León-Ledesma et al., 2010). As such, non-normalized CES functions with different elasticities of substitution are unsuitable for comparative-static analysis. We address this issue by normalizing the production functions, which has the important advantage that it allows to fix the distribution parameters so that they are no longer a function of the elasticity of substitution. The second challenge is that even with fixed distribution parameters, output elasticities are not constant in a CES framework. The optimal worker allocation is therefore determined numerically based on equation (14).

To be more specific, we replace the sector production functions in (8) and (10) by the normalized functions

$$Y_i = Y_{0i} \left[\phi_{0i} \left(\frac{H_i}{H_{0i}} \right)^{\rho_i} + (1 - \phi_{0i}) \left(\frac{K_i}{K_{0i}} \right)^{\rho_i} \right]^{\frac{1}{\rho_i}} \quad (15)$$

and

$$Y_i = Y_{0i} \left[\phi_{0i} \Omega_i^{\frac{\rho_i}{\gamma_i}} + (1 - \phi_{0i}) \left(\frac{K_i}{K_{0i}} \right)^{\rho_i} \right]^{\frac{1}{\rho_i}} \quad (16)$$

where $\Omega_i = \left[\theta_{0i}^{LH} \left(\frac{LH_i}{LH_{0i}} \right)^{\gamma_i} + \theta_{0i}^{LM} \left(\frac{LM_i}{LM_{0i}} \right)^{\gamma_i} + (1 - \theta_{0i}^{LH} - \theta_{0i}^{LM}) \left(\frac{LL_i}{LL_{0i}} \right)^{\gamma_i} \right]$. The subscript 0 stands for the point of normalization.

Given the production function (15), the optimal allocation of human capital input in each sector is then determined by (see appendix 7 for derivation):

$$H_i^* = \frac{\phi_{0i} Y_i \left(\frac{H_i}{H_{0i}} \right)^{\rho_i} \left[\phi_{0i} \left(\frac{H_i}{H_{0i}} \right)^{\rho_i} + (1 - \phi_{0i}) \left(\frac{K_i}{K_{0i}} \right)^{\rho_i} \right]^{-1}}{\sum_{j=1}^J \left(\phi_{0j} Y_j \left(\frac{H_j}{H_{0j}} \right)^{\rho_j} \left[\phi_{0j} \left(\frac{H_j}{H_{0j}} \right)^{\rho_j} + (1 - \phi_{0j}) \left(\frac{K_j}{K_{0j}} \right)^{\rho_j} \right]^{-1} \right)} H \quad (17)$$

and in the case of the sector production function with the generalized human capital aggregate the optimal allocation of each skill type is found by

$$L_{ki}^* = \frac{\phi_{0i}\theta_{0ki}Y_i\left(\frac{L_{ki}}{L_{0ki}}\right)^{\gamma_i}\Omega_i^{-1}\Gamma_i^{\frac{\rho_i-\gamma_i}{\gamma_i}}}{\sum_{j=1}^J\left(\phi_{0j}\theta_{0kj}Y_j\left(\frac{L_{kj}}{L_{0kj}}\right)^{\gamma_j}\Omega_j^{-1}\Gamma_j^{\frac{\rho_j-\gamma_j}{\gamma_j}}\right)}L_k \quad (18)$$

where $\Omega_i = \left[\phi_{0i}\Gamma_i^{\frac{\rho_i}{\gamma_i}} + (1 - \phi_{0i})K_i^\rho\right]$

and $\Gamma_i = \left[\theta_{0i}^{LH}\left(\frac{LH_i}{LH_{0i}}\right)^{\gamma_i} + \theta_{0i}^{LM}\left(\frac{LM_i}{LM_{0i}}\right)^{\gamma_i} + (1 - \theta_{0i}^{LH} - \theta_{0i}^{LM})\left(\frac{LL_i}{LL_{0i}}\right)^{\gamma_i}\right]$

3.4 Choosing a normalization point

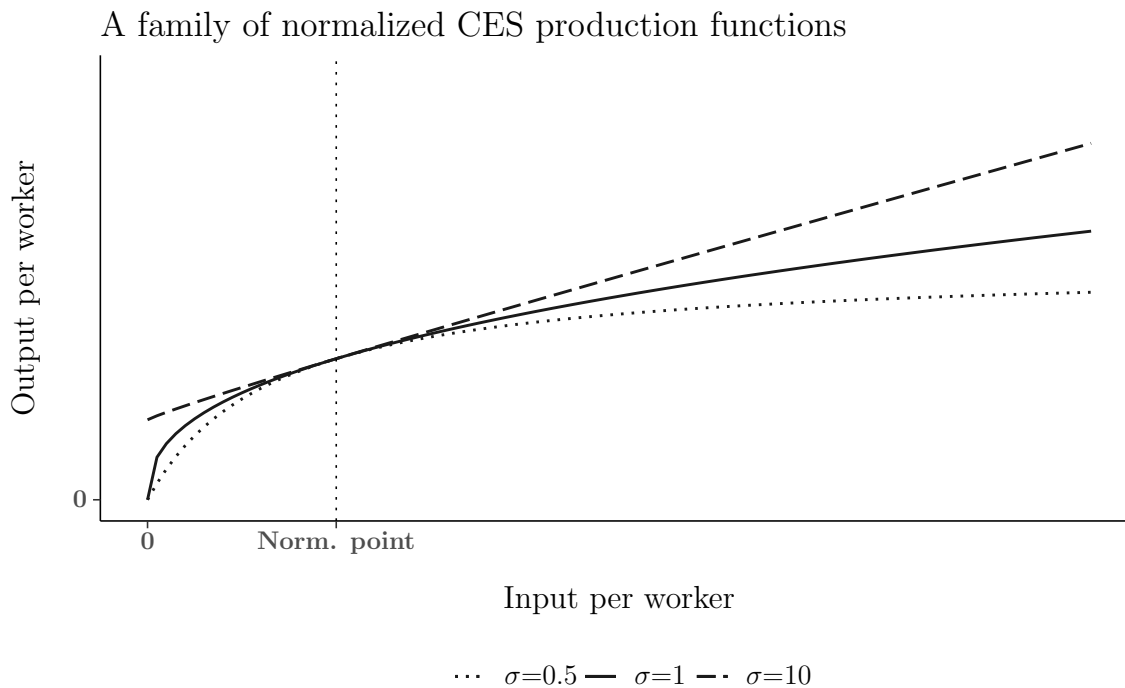
The choice of the normalization point warrants some discussion. Essentially, normalization reshapes the surface of the production function without shifting it so that CES functions characterized by the same normalization points and distribution parameters but different elasticities of substitutions are tangents. Figure 1 illustrates a family of normalized CES functions. Importantly, the point of tangency is not random, but the chosen normalization point. Setting the normalization point therefore influences how the production surface is reshaped by changing the elasticity of substitution. This means that the counterfactuals are not invariant to the choice of the normalization point (Klump and Saam, 2008; Temple, 2012).¹²

This begs the question what normalization point to choose. From an empirical standpoint, the choice of the normalization point should be motivated by defining appropriate values of the fundamental production technology (Klump et al., 2007).¹³ An attractive feature of normalization is that the distribution parameters can be linked to the factor shares of production at the normalization point, given the maintained assumption of perfect competition and firm optimization. It is important to point out that all CES production functions, if not explicitly, are at least implicitly normalized in the point where the input values equal one. In this implicit case, the distribution parameter ϕ_0 can be interpreted as the labor revenue share that will arise if all input ratios are unity (León-Ledesma et al., 2010; Klump et al., 2012;

¹²More precisely, *ceteris paribus*, the increase in output from an increase of an input factor is positively related to the elasticity of substitution, except at the point of normalization. The farther away the normalization point is from a given level, the stronger is the effect of the substitution elasticity on the shape of the production surface.

¹³As criticized by Temple (2012), formally, the choice of the normalization points is arbitrary. Although the choice of the normalization point has quantitative implications, there is “no inherent reason to prefer one normalization of the CES production function to another.”

Figure 1: A family of CES functions



Temple, 2012). In practice, however, gauging a sensible magnitude for the revenue shares at this points is quite infeasible.¹⁴ Instead, Klump et al. (2007) argue that appropriate values for the distribution parameter are best detected from the data and propose to calculate the baseline values on the basis of sample averages to use as much information as possible, while also netting out short-term fluctuations.

Following this argument, two explicit normalization points offer to be appropriate in this paper: 1) sector-specific sample averages or 2) sector-country-specific averages of Y , K , H , LH , LM , and LL . Recall, though, that the counterfactuals are based on sector-country-specific production functions and countries differ greatly in their absolute input quantities. Hence, in case of a common normalization point, the effect of σ and η on the production surface will differ strongly across countries. This biases the gains from reallocation across countries, and consequently, the ratios $R_{c,USA}$ and R_{World} . Sector-country-specific normalization points are therefore the appropriate choice for the counterfactuals in this paper.

¹⁴As pointed out by León-Ledesma et al. (2010), one consequence of normalizing the CES function by $K_0 = H_0 = Y_0 = 1$ is the counterfactual outcome that the factor prices at the normalization point are equal to the factor revenue shares.

4 Data and calibration

4.1 Output and input data

All output and input data used in this study come from the Socio-Economic Accounts in the World-Input-Output Database (WIOD-SEA).¹⁵ The WIOD-SEA database has been constructed on the basis of national accounts data and harmonization procedures were applied in order to ensure international comparability. For the period 1995-2007, WIOD-SEA contains annual data for 35 industries in 40 countries (27 EU countries and 13 other major countries) on gross output and intermediate inputs at current basic prices, capital stock, as well as hours worked and labor compensation by skill type (low-, medium- and high-skilled) (Timmer et al., 2015). Since information on the latter is not available at the industry level for a large number of countries, We aggregate the 35 industries into 14 sectors. Moreover, we drop the Mining and Quarrying sector as well as Public Utilities sector. The motivation to do so is that both sectors have high labor productivity levels, but are unlikely able to absorb large quantities of workers in reality. This would considerably inflate the estimated (hypothetical) reallocation gains. The final sample therefore consists of the 12 sectors listed in table 1.

Table 1: Sector overview

Abbr	Sector Name	ISIC Rev3.1 description
AtB	Agriculture	Agriculture, hunting and forestry, fishing
D	Manufacturing	Manufacturing
F	Construction	Construction
G	Trade Service	Wholesale and retail trade, repair of motor vehicles, motorcycles and personal and household goods
H	Hotels & Restaurants	Hotels and restaurants
I	Transport & Storage	Transport, storage and communications
J	Finance Services	Financial intermediation
K	Business Services	Real estate, renting and business activities
L	Public & Social Services	Public administration and defense
M	Education	Education
N	Health & Social Work	Health and social work
OtP	Other Services	Other community, social and personal service activities, Private households with employed persons

A central issue for cross-country comparison of output and productivity levels is that output is nominated in international purchasing power parities (PPP). To account for the substantial variation in relative prices across industries, we use multilateral industry-level

¹⁵This paper uses the 2014 release of WIOD-SEA. Later global input-output tables, such as WIOD 2016 released in February 2018, do not include hours worked by skill types.

PPPs from Inklaar and Timmer (2014) (see table 6 in the appendix). As no information on relative prices are available for Taiwan, the sample reduces to 39 countries. Also, relative price data is only available for the year 2005. We therefore proceed as follows. We deflate both gross output and inputs using the industry-level price deflators integrated in WIOD-SEA and obtain growth rates of both series. An advantage of the double-deflated value-added method, oppose to a single-deflated value-added measure, is that it can account for changes in the relative prices of intermediate inputs to gross outputs. The real growth rates are then used to extrapolate the gross output and intermediate input series, which are converted into PPPs for the available year 2005.¹⁶ In a last step, we construct a value added series in constant 2005 PPPs by subtracting intermediate inputs from gross output, both in constant PPPs. For the physical capital stock, we use growth rates from the real capital stock data nominated in national currencies in WIOD-SEA and the exchange rate from PWT 8.1 (Feenstra et al., 2015) to convert the series into constant 1995 US\$.

The data on hours worked in WIOD-SEA includes both self-employed and employees.¹⁷ Worker skill types are classified on the basis of educational attainment levels as defined in the International Standard Classification of Education (ISCED): low-skilled (ISCED categories 1 and 2), medium-skilled (ISCED 3 and 4) and high-skilled (ISCED 5 and 6). We define the general normalized human capital input in country c in sector i as

$$\left(\frac{H_{ci}}{H_{0ci}}\right)^{\gamma_{ci}} = \left[\theta_{0ci}^{LL} \left(\frac{LL_{ci}}{LL_{0ci}}\right)^{\gamma_{ci}} + \theta_{0ci}^{LM} \left(\frac{LM_{ci}}{LM_{0ci}}\right)^{\gamma_{ci}} + \theta_{0ci}^{LH} \left(\frac{LH_{ci}}{LH_{0ci}}\right)^{\gamma_{ci}}\right]^{\frac{1}{\gamma_{ci}}} \quad (19)$$

where the subscript 0 indicates the normalization point defined as the average over the period 1995-2007. Recall that both the Cobb-Douglas and the standard two-factor CES production function implicitly assume $\gamma = 1$ (i.e., perfect substitutability of skill types), in which case equation (19) can be rewritten as

$$\frac{H_{ci}}{H_{0ci}} = \theta_{0ci}^{LL} \times \left[\frac{LL_{ci}}{LL_{0ci}} + \frac{\theta_{0ci}^{LM}}{\theta_{0ci}^{LL}} \frac{LM_{ci}}{LM_{0ci}} + \frac{\theta_{0ci}^{LH}}{\theta_{0ci}^{LL}} \frac{LH_{ci}}{LH_{0ci}} \right] \quad (20)$$

Further, note that the general normalized form in (20) includes the implicitly normalized human capital input H_{ci} if $H_{0ci} = LL_{0ci} = LM_{0ci} = LH_{0ci} = 1$. In this case, θ_{0ci}^{LL} , θ_{0ci}^{LM} , and θ_{0ci}^{LH} collapse to measures of relative wages.¹⁸ The general normalized human capital input

¹⁶While this approach is not uncommon, it should be pointed out that the underlying assumption is that cross-country industry-level prices deflators are constant over time. Given the time horizon 1995-2007, this assumption seems defensible. Also note that we do not compare output levels of different countries at different time periods (e.g. the U.S. in 2000 to Germany in 2005) as for instance Inklaar and Diewert (2016).

¹⁷To be precise, the latter is estimated based on the assumption that average hours worked by employees and self-employed are equal.

¹⁸Following neoclassical theory, the assumption of $\gamma = 1$ implies that relative wages are constant. This

thus incorporates the standard human capital input measured as *unskilled worker equivalents* with different worker types being weighted by their relative wage. Importantly, however, $\frac{H_{ci}}{H_{0ci}}$ also contains θ_{0ci}^{LL} , which measures the contribution of low-skilled workers to human capital services in sector i in country c .¹⁹ In growth or development accounting exercises, θ_{0ci}^{LL} is often (implicitly) assumed to be fix across time or equal across countries and can therefore be ignored (see, e.g., Jones, 2014).²⁰ Conversely, by working with the nested CES function in equation (18) directly, we implicitly assume that θ_{0ci}^{LL} differs across sectors and countries. Considering θ_{0ci}^{LL} in equation (20) is thus important to ensure consistency, and therefore, allow a direct comparison between the obtained cost of misallocation based on the different specifications.

4.2 Distribution parameters

Aside from output and input data, we need values for the distribution parameters. An important benefit of normalizing the production functions is that it allows to follow the common approach to calibrate the distribution parameters based on the factor shares at the point of normalization, given the maintained assumption of perfect competition and firm optimization. A unique feature of WIOD-SEA is that it contains information on labor compensation on the sector level. We thus calculate ϕ_{0ci} as the geometric average labor share in value added (measured based on current prices in national currency) in sector i in country c over all time periods T . Notably, WIOD-SEA also provides information on the compensation shares for each skill type. This allows us to calculate θ_{0ci}^{LM} , and θ_{0ci}^{LH} based on the relative skill-shares in total labor compensation of each skill type working in sector i in country c at the normalization point. Specifically, we calculate

$$\theta_{0ci}^{LH} = \left[\frac{\bar{w}_{ci}^{LH} L\bar{H}_{ci}}{\bar{w}_{ci}^{LL} L\bar{L}_{ci} + \bar{w}_{ci}^{LM} L\bar{M}_{ci} + \bar{w}_{ci}^{LH} L\bar{H}_{ci}} \right] \quad (21)$$

$$\theta_{0ci}^{LM} = \left[\frac{\bar{w}_{ci}^{LM} L\bar{M}_{ci}}{\bar{w}_{ci}^{LL} L\bar{L}_{ci} + \bar{w}_{ci}^{LM} L\bar{M}_{ci} + \bar{w}_{ci}^{LH} L\bar{H}_{ci}} \right] \quad (22)$$

$$\theta_{0ci}^{LL} = 1 - \theta_{0ci}^{LM} - \theta_{0ci}^{LH} \quad (23)$$

where the bars indicate geometric averages over time.

Despite the effort put into the construction of WIOD-SEA, important limitations exist and warrant discussion. It is a well know problem that hours worked and labor income of

allows to construct θ_{0ci}^{LL} , θ_{0ci}^{LM} , and θ_{0ci}^{LH} based on average country-sector wages.

¹⁹More precisely, θ_{0ci}^{LL} governs the contribution of low skilled workers to the total (i.e., collective) human capital services in sector i in country c .

²⁰On reason for this assumption is that θ_{0ci}^{LL} is often unobserved (whether directly or indirectly) in the data.

self-employed is not registered in the National Accounts in many countries. As discussed in detail in appendix B, WIOD-SEA addresses this issue through imputation. A drawback of this imputation is that it can result in economically unreasonable labor revenue shares. Indeed, in 16 out of the 486 country-sector observations, ϕ_{0ci} exceeds the value of 1. We deal with these questionable parameter values in the baseline counterfactual by setting the maximum value of ϕ_{0ci} to 0.9. Admittedly, this ad-hoc adjustment is somewhat arbitrary. We therefore run robustness tests, where we treat ϕ_{0ci} as a structural parameter that is common across countries.²¹ It is worth noting that the relative efficiency parameters of skill types, θ_{0ci}^{LM} and θ_{0ci}^{LH} , are unaffected by a potential bias in ϕ_{0ci} as long as there is no systematic relation between compensation shares and self-employment.²²

4.3 Elasticity of substitution parameters

Lastly, we require values for the unobserved substitution parameters, σ and η . While there is now a large set of estimates for the aggregate elasticity of substitution between physical and human capital, evidence for the sector level is scarce. Also, the few available estimates do not directly map to the sectors considered in this paper. A secondary contribution of this paper is thus to estimate elasticities of substitution on the sector level that can then be used in the reallocation exercise. More precisely, we identify the parameters based on a normalized supply-side system of equations (Klump et al., 2007; Herrendorf et al., 2015; McAdam and Willman, 2018). As shown by León-Ledesma et al. (2010), this approach is superior to single-equation estimation or factor-demand-equation systems, as it not only increases the degrees of freedom but also considers both technology (via the underlying production function) and optimization behavior (via FOC equations).²³

Despite the advantages of the normalized system approach, identifying two elasticities simultaneously in a highly nonlinear function remains challenging.²⁴ The identification is further constrained by the fact that the harmonized sector data in WIOD-SEA only cover a rather short time horizon. Against this background, we focus on the estimation of σ , which we assume to differ across sectors, but not across countries and time. To consider the impact

²¹Specifically, we consider two different values for ϕ_{0i} . First, We set ϕ_{0ci} to the geometric sample-average labor share over time. Second, we set ϕ_{0ci} to the geometric average of the U.S. labor share over time. This approach is also applied by Hsieh and Klenow (2009); Vollrath (2009); Busso et al. (2013); Oberfield (2013); Brandt et al. (2013). Note that the assumption of homogeneous industry production functions can be in line with varying aggregate labor shares across countries due to differences in the sector composition.

²²Unfortunately, we do not have country-sector-specific information on this. We therefore continue by relying on the assumption that no systematic relation exist.

²³In addition, the normalized system of equations performs well even for relatively small samples and is found not to be sensitive with respect to simultaneity biases or estimation technique (León-Ledesma et al., 2010).

²⁴This challenge is also noted by Papageorgiou et al. (2017). The authors therefore proceed by assuming a CES-in-CD production function.

of η on σ , we follow a grid search approach as proposed by Henningsen and Henningsen (2012). That is, we impose plausible estimates of η that are motivated by the literature and obtain corresponding estimates for σ .²⁵ While clearly restrictive, note that these assumptions implicitly also work in all misallocation studies that use a Cobb-Douglas production function, as this function implies $\sigma = 1$ and $\eta \rightarrow \infty$ (see, e.g., Vollrath, 2009; Hsieh and Klenow, 2009; Inklaar et al., 2017). Nonetheless, as a sensitivity test, we follow a more agnostic approach with regard to the true values of σ_{ci} and η_{ci} and compute bootstrap confidence intervals for $R_{c,USA}$ and R_{World} .²⁶

For matter of space, the estimation specification of the normalized supply-side system and additional results are placed in the appendix. Table 2 presents sector-specific estimates of σ_i and corresponding 90%-confidence intervals (CI) based on bootstrap standard errors. Three findings stand out. First, in all sectors, and regardless of the imposed value of η_i , σ_i is estimated to be statistically significant below unity. Second, σ_i is lower for lower values of η_i and approximates the estimated value of the 1-level CES specification for $\eta \rightarrow \infty$. Third, σ_i is estimated quite precisely.

There is some evidence to which we can compare these estimates, but mostly restricted to the U.S. For instance, Young (2013) finds that the elasticity of substitution between physical and human capital is less than unity for a large majority of U.S. 2-digit SIC level industries. Aggregating industries into sectors based on value-added-share-weights, Young (2013) obtains an elasticity of 0.57 in the manufacturing sector and of 0.66 in services. Using capital price series and applying a low-pass filter to better capture the long-run substitution elasticity, Chirinko and Mallick (2017) find industry-level estimates for U.S. to be below unity with most estimates being in a range of 0.2-0.7. Similar estimates are also found by Disou et al. (2015) in ten Canadian manufacturing industries for the period 1962-1997 and by Oberfield and Raval (2014), who recover elasticities for two-digit SIC U.S. industries from plant-based estimates. The latter estimate a weighted average plant-level elasticity of 0.52 in manufacturing. Taking substitution across plants and industries into account, they recover

²⁵Specifically, we impose $\eta \in [1.6, 2, 4]$. These values are motivated by estimated aggregate elasticities of substitution between skill types. For instance, Ciccone and Peri (2005) use variation across U.S. cities in the relative skill supply instrumented with compulsory-schooling laws over the period 1950-1990. They obtain an elasticity of substitution of 1.5 between unskilled and unskilled worker, where high-school completion functions as the threshold. Autor et al. (2008) estimate an estimated aggregate elasticity of substitution of 1.62 between college equivalents and high-school equivalents for the time period 1963–2005 in the U.S. Mollick (2011) obtains values for a global elasticity of substitution between “unskilled” and “skilled” workers varying from 2.00 to 3.21 for the “college-completed” definition, of 3.53 to 4.07 for the “secondary- completed” definition and of 6.25 to 12.66 for the “primary- completed” definition. Imposing values for η is a common strategy in the development accounting literature (see e.g. Caselli and Coleman, 2006; Jones, 2014; Caselli, 2016). In addition, we also ran regressions to estimate η_i by imposing the estimated $\hat{\sigma}_i$ and starting the optimization algorithm from an initial value of $\eta_i = 2$. The results suggest a high substitution elasticity of $\eta_i > 15$ in multiple sectors. However, for some sectors, η_i is estimated to be negative.

²⁶See section 6.1.1 for details.

Table 2: Sector elasticities of substitution with 90%-CI in parentheses

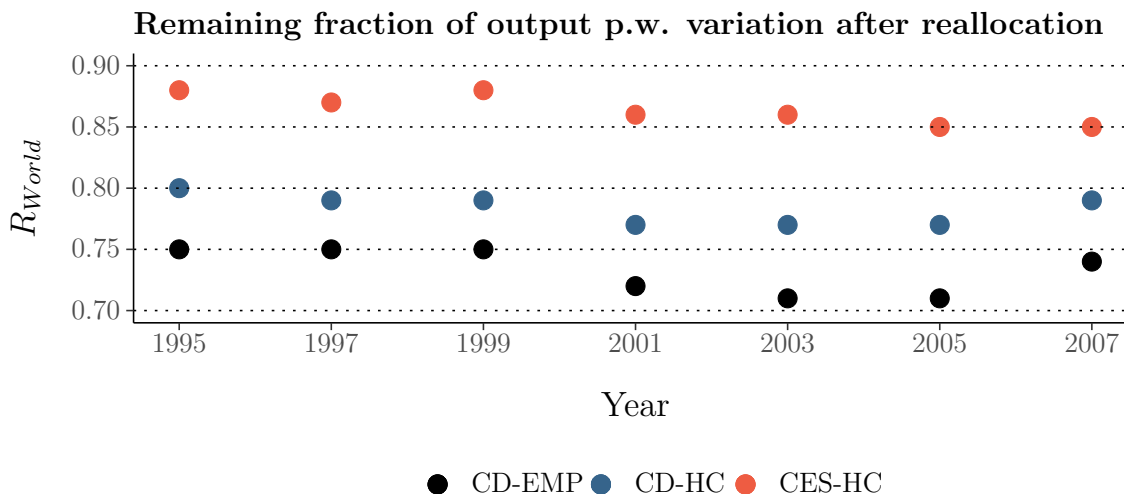
Sector	2-Level CES				1-Level CES
	$\eta = 1.6$	$\eta = 2$	$\eta = 4$	$\eta = 100$	(implicit $\eta \rightarrow \infty$)
Agriculture	0.49 (0.44-0.55)	0.51 (0.46-0.57)	0.55 (0.50-0.61)	0.60 (0.54-0.67)	0.62 (0.54-0.73)
Manufacturing	0.46 (0.43-0.49)	0.47 (0.44-0.50)	0.49 (0.46-0.52)	0.52 (0.49-0.55)	0.53 (0.50-0.57)
Construction	0.41 (0.34-0.51)	0.42 (0.35-0.52)	0.45 (0.38-0.55)	0.47 (0.41-0.57)	0.46 (0.38-0.61)
Trade	0.51 (0.43-0.63)	0.52 (0.44-0.63)	0.54 (0.47-0.65)	0.58 (0.51-0.67)	0.63 (0.56-0.72)
Hotels&Restaurants	0.43 (0.38-0.49)	0.44 (0.39-0.50)	0.46 (0.41-0.51)	0.48 (0.44-0.54)	0.49 (0.44-0.55)
Transport&Storage	0.46 (0.41-0.52)	0.46 (0.41-0.53)	0.48 (0.43-0.54)	0.50 (0.46-0.55)	0.52 (0.47-0.58)
Financial Intermediation	0.50 (0.47-0.54)	0.51 (0.48-0.54)	0.52 (0.49-0.56)	0.54 (0.50-0.58)	0.67 (0.60-0.75)
Business Services	0.44 (0.38-0.52)	0.44 (0.38-0.52)	0.44 (0.38-0.53)	0.45 (0.39-0.54)	0.54 (0.46-0.66)
Public&Social Services	0.41 (0.37-0.45)	0.41 (0.37-0.45)	0.41 (0.38-0.46)	0.43 (0.39-0.47)	0.45 (0.40-0.51)
Education	0.36 (0.34-0.39)	0.36 (0.34-0.39)	0.37 (0.35-0.39)	0.38 (0.36-0.41)	0.40 (0.37-0.44)
Health&Social Work	0.38 (0.35-0.41)	0.38 (0.35-0.41)	0.39 (0.36-0.42)	0.40 (0.37-0.43)	0.44 (0.40-0.48)
Other Services	0.42 (0.37-0.49)	0.43 (0.38-0.50)	0.45 (0.40-0.52)	0.47 (0.42-0.54)	0.49 (0.44-0.57)

Note: Estimates of σ (i.e., the elasticity of substitution between physical capital and the labor input aggregate with three skill types) based on different imposed values of η . Estimation applies a two-step FGNLSUR on a normalized supply side system. 90%-CI in parentheses. Standard errors are bootstrapped based on 500 iterations.

an aggregate elasticity of 0.75 for the manufacturing sector in 2007. For broad sectors, Herrendorf et al. (2015) find that capital and labor are gross complements in U.S. manufacturing and services. Conversely, they estimate an elasticity of above one in agriculture. This stands in contrast to Mundlak’s (2005) examination of the U.S. agriculture sector, advocating an elasticity of equal or below one.²⁷ Overall, the sector estimates for the elasticity of substitution between physical capital and the human capital aggregate obtained in this paper seem to be fairly consistent with previous findings. Moreover, the estimates strongly suggest substitution elasticities that are inconsistent with Cobb-Douglas and in favor of a more flexible CES specification.

²⁷It is worth noting that the ratio R_{World} is not sensitive to this value.

Figure 2: Baseline results for R_{World}



Note: Own calculation based on WIOD-SEA. R_{World} states the remaining fraction of output p.w. variation after misallocation across sectors is eliminated. *Specifications:* CD: Cobb-Douglas production function. CES: Constant elasticity substitution function. EMP: Labor input measured in raw hours worked. HC: Labor input measured as quality adjusted hours worked. All calculations based on value added nominated in constant PPPs for 12 sectors.

5 Counterfactual results

Equipped with the parameter values, we can now calculate the hypothetical output per worker level for each country, y_c^3 , that would be implied by an optimal allocation of human capital across sectors. Based on y_c^3 , we can then obtain the ratio R_{World} . A central result of this paper is depicted in figure 2, which shows values of R_{World} for three different specifications. For now, we will focus on specifications that assume perfect substitutability between different skill types of workers (HC) and the final year 2007.

The value of 0.79 obtained under the Cobb-Douglas set up with human capital as labor input indicates that 21 percent of the cross-country output per worker variation can be explained by differences in the efficiency of human capital allocations across sectors within countries. This share reduces by 6 percentage points to 15 percent if we assume sectors to be characterized by a CES function and impose the estimated sectoral elasticities of substitution. To put this into perspective, the quantitative implications of the functional form are similar to the change due to human capital replacing raw hours worked (0.74; EMP).

To better understand what drives these results, we look at the individual country results. Table 3 presents summary statistics for the year 2007. The first row presents results for a simple labor input measured in raw hour worked. On average, output per worker increases by about 12.8 % if countries allocate labor efficiently across sectors. The large spreads between the respective percentiles and the large gap between the mean and median indicate, however, that countries record sizable differences in their current misallocation costs. We replace the

Table 3: Gains from reallocation (2007)

Percentile	Min	10th	25th	Median	Mean	75th	90th	Max
CD-EMP $\eta \rightarrow \infty$	2.4	3.5	4.3	6.0	12.8	9.9	22.5	141.2
CD-HC $\eta \rightarrow \infty$	1.2	3	3.5	5.6	10.6	9.8	17.9	105.3
CES-HC $\eta \rightarrow \infty$	0.8	2.2	2.8	4.2	7.6	8.5	16.4	65.9

Note: Hypothetical output per worker gains in percentage points. Calculation based on different specifications of normalized sector production functions. *Specifications:* EMP: Labor input measured in raw hours worked. HC: Labor input measured as quality adjusted hours worked. CD: Cobb-Douglas production function. CES: Constant elasticity substitution function. All calculations based on value added nominated in PPPs for 12 sectors.

simple labor input with a measure of human capital in row (2). In line with previous findings, the hypothetical gains decrease (to 10.6 %). This indicates that some of the misallocation of labor across sectors can be explained by cross-sector differences in human capital (Gollin et al., 2014; Vollrath, 2014; Herrendorf and Schoellman, 2018).

Most important for this paper is the change in the distribution of the reallocation gains when we depart from the assumption of an unitary elasticity of substitution between physical and human capital. Row (3) shows that the average gains decrease to 7.6 % and are smaller than in the Cobb-Douglas case at each percentile. At the mean (median), human capital reallocation gains are 28 (25) % lower under the more plausible CES production structure.

What drives the reduction of reallocation gains in the CES case? Recall that our estimates impose substitution elasticities of below unity, which implies that the physical and human capital are gross complements in all sectors. As a consequence, marginal products respond more strongly to changes in the factor allocation across sectors. A laborer moving from a low-productivity sector A to a high-productivity sector B initially experiences a strong increase in marginal productivity: since human capital is scarce in sector B and physical capital K is relatively abundant, the laborer is using this capital in sector B very productively when H and K are complements. Since we only focus on the reallocation of human capital and consider physical capital fixed in each sector, this effect quickly decays: there is not enough physical capital K available in sector B that a large influx of laborers could work with.²⁸ This results in faster convergence of the marginal products across sectors, leading to less

²⁸Remember that this is less of a problem in the Cobb-Douglas case since H and K are more substitutable: labor needs less capital to productively work with.

reallocation, and lower output growth overall.^{29,30}

To what extent these differences affect the ratio R_{World} depends on whether the country gains respond differently to the production function specification and whether this variation is correlated with output per worker. Figure 3 sheds light on this issue. The figure shows each country's value of R_{USA} for different specifications in relation to their output per worker. Recall that $R_{c,USA}$ states the remaining output per worker gap of a country relative to the U.S. after differences in labor allocation efficiency across sectors have been removed, effectively normalizing the hypothetical gains across countries. While many countries seem to not differ too much in their degree of misallocation, those countries that do have large efficiency losses tend to be at the poorer end of the output per worker distribution. In fact, two of the three poorest countries, India and China, have, by far, the largest reduction in the output gap to the U.S. It is further noticeable that while not all countries are estimated to have a lower efficiency gap relative to the U.S. in a CES set up, this does seem to be the case for all countries in the lower half of the distribution. For instance, eliminating differences in human capital misallocation across sectors reduces the output gap to the U.S. in the year 2007 for India (China) by 48% (33%) in the case of Cobb-Douglas, but only by 34% (17%) under the CES specification.

5.1 Imperfect skill substitution

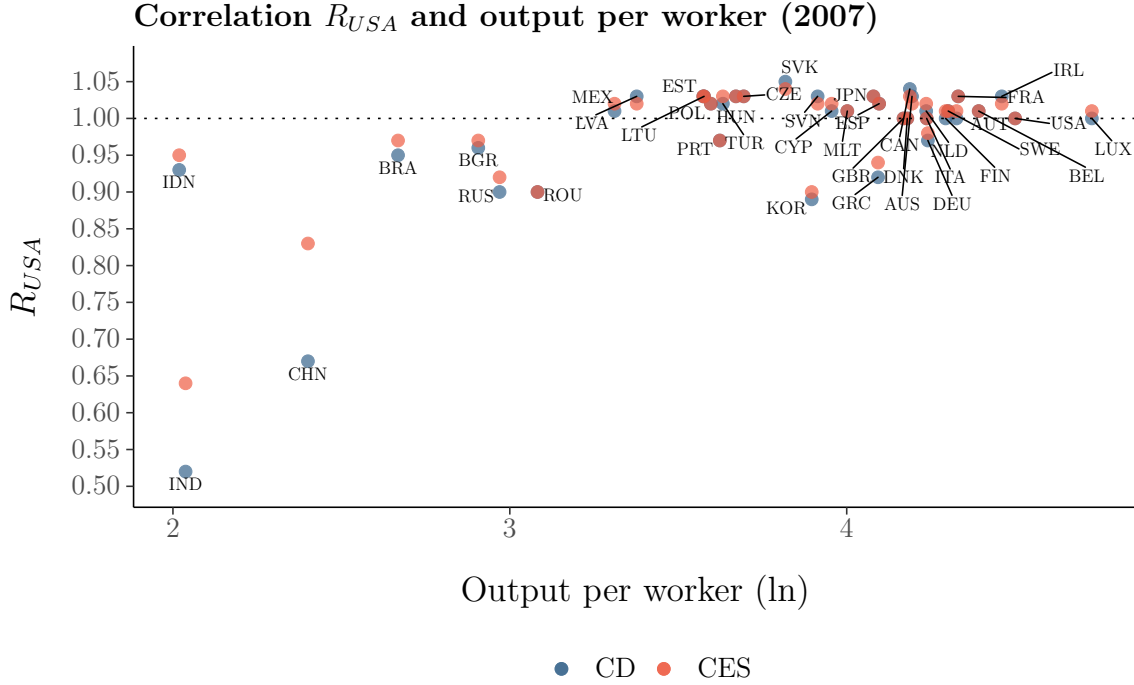
We now relax the implicit assumption that labor skill types are perfect substitutes. Table 4 summarizes the hypothetical gains for the nested CES specifications. Rows (1)-(3) still impose σ_i to be unity, while rows (4)-(6) consider different values of η_i and a simultaneous change in the corresponding σ_i . Some changes in the distributions are visible. However, if we fix σ_i , the reallocation gains change only marginally for different values of η_i , with notable exceptions in the case of India and Romania (see appendix figure 5). Overall, the results suggest that the substitutability of physical capital and the human capital aggregate has larger quantitative implications than relaxing the assumption of perfect substitution across skill types.

There is limited evidence to which we can compare our estimates, as studies differ in

²⁹On average, countries reallocate around 13% and 10% of their aggregate human capital under the Cobb-Douglas and CES specification, respectively. The largest extent of reallocation is found in India (40% in CD and 34% in CES). In many countries, however, less than 15% (CD) or 10% (CES) of the aggregate human capital is reallocated. The difference between the two specifications is on average 3 percentage points. Overall, the extent of human capital reallocation decreases, on average, by around 20% (not percentage points) in the CES set up compared to the Cobb-Douglas set up. The difference is as small as 4% and as large as 28% across countries.

³⁰Another important channel for lower overall output growth in the CES case operates through *average* productivities of sectors: because H and K are gross complements (but are not required in fixed proportions, as in the Leontief case), inflowing laborers in the higher-productivity sector B 'take away' some capital from incumbent laborers in that sector, lowering average productivity.

Figure 3: Baseline results for R_{USA}



Note: Own calculation based on WIOD-SEA. CD: Cobb-Douglas production function. CES: Constant elasticity substitution function with sector-specific σ_i . All calculations based on value added nominated in constant PPPs for 12 sectors.

Table 4: Gains from reallocation (other specifications, 2007)

	Percentile	Min	10th	25th	Median	Mean	75th	90th	Max
(1)	$\sigma = 1$ $\eta = 4$	2.0	3.2	3.7	6.0	10.5	9.5	18.6	92.6
(2)	$\sigma = 1$ $\eta = 2$	1.9	3.1	3.6	5.6	10.0	9.0	17.5	85.0
(3)	$\sigma = 1$ $\eta = 1.6$	1.8	3.1	3.5	5.6	9.8	8.9	17.3	82.0
(4)	$\sigma < 1$ $\eta = 4$	1.4	2.4	3.1	4.2	7.3	7.9	14.3	55.6
(5)	$\sigma < 1$ $\eta = 2$	1.3	2.3	2.8	3.9	6.9	7.4	13.2	51.8
(6)	$\sigma < 1$ $\eta = 1.6$	1.3	2.3	2.8	3.8	6.8	7.3	13.0	50.5

Note: Hypothetical output per worker gains in percentage points. Calculation based on different specifications of normalized sector production functions. Note that η is assumed homogeneous across sectors and determines σ in the 2-level nested CES function in rows (4)-(6). All calculations based on value added nominated in PPPs for 12 sectors.

their focus regarding input factors (physical and/or human capital), aggregation level (sector, industry, or firm), or outcome measure (total factor productivity or output). For instance, Inklaar et al. (2017) estimate that the average observed manufacturing productivity level in their sample of 52 medium- and low-income countries would rise from 23% to 37% of the U.S. level if cross-country differences in both capital and labor misallocation would be eliminated. In contrast, besides differences in the sample composition, this paper examines the role of human capital misallocation across sectors in explaining cross-country differences in *aggregate output* per worker levels. Similar, the seminal study by Hsieh and Klenow (2009), calculates that improving factor allocation across firms to the extent observed in the United States could rise manufacturing productivity by 30%–50% in China and 40%–60% in India. However, as pointed out by Vollrath, this increase in manufacturing productivity would “only” translate into a 10% increase in aggregate output in India, and a 15% increase in China (Vollrath, 2015).

Closer to this study, Vollrath (2009) estimates that variation across countries in the degree of human capital misallocation between the agriculture and non-agriculture sector could account for 30% of the variation in income per capita in his sample of 42 countries. Considering that Vollrath only looks at efficiency losses that occur between agriculture and the remainder of the economy, this estimate appears to be large, but it does not necessarily contract mine. For one, Vollrath’s sample consists of several economies in which the agriculture sector employs a large share of total human capital at low productivity levels, thereby capturing a substantial portion of the inter-sector misallocation in poor countries. Besides this, Vollrath is forced to work with an arguably cruder sectoral measure of human capital. These differences considered, Vollrath’s empirical estimates seem to be consistent with our findings.

6 Robustness and extensions

6.1 Robustness

6.1.1 Parameter assumptions

Until now, we have not only assumed that σ_i and η are homogeneous across countries, but we have also imposed a common η for all sectors. To test how sensitive our results are to these assumptions, we make use of a key advantage of normalization and artificially introduce variation in the substitution parameters.³¹ Specifically, we perform 1000 calculations of

³¹Recall that the normalization procedure allows to fix the distribution parameters while varying the substitution parameters.

R_{World} and $R_{c,USA}$ based on simulated samples. For each sample, we draw random values for η_{ci} from the possibility set [1.6, 2, 4, ∞]. For a given η_{ci} , we then obtain the corresponding σ_{ci} from the 90%-CI around $\hat{\sigma}_i(\eta_i)$.³² The motivation for this simulation is as follows. Provided that the actual degree of cross-country parameter heterogeneity is moderate, in the sense that the 90%-CI envelops most of the true parameter values in each country and sector, increasing the set of possible production structures also raise the probability of approximating the true production side.

Appendix figure 4 shows the 10th-to-90th percentile from the resulting distribution of R_{World} . Two findings stand out. First, R_{World} reacts moderately sensitive to variation in η_{ci} and σ_{ci} (about 3-4 percentage points). Second, departing from the Cobb-Douglas assumption reduces the fraction of output per worker variation explained by misallocation by at least 6 percentage points. Appendix figures 5 and 6 show the 10th-to-90th percentile interval of each country's hypothetical gains and remaining gap to the U.S., respectively. Countries are ordered by their actual output per worker. The figures indicate that for many countries the estimated reallocation gains are robust to the artificial variation in η_{ci} and σ_{ci} . Nevertheless, there are several countries, especially in the lower half of the output per worker distribution in the sample, for which the estimated absolute (and relative) gains from reallocation are sensitive to the imposed parameter values.

So far, the results are based on production functions with a distribution parameter ϕ_{0ci} that is specific to each country and sector. As a robustness test, we replace ϕ_{0ci} by a structural parameter that is common across countries. Appendix figure 7 shows the results for R_{World} . Noticeable, the cross-country differences in the estimated efficiency losses increase when we treat the distribution parameter ϕ_{0i} as structural and are largest if we impose distribution parameter obtained for the U.S., ϕ_{0USAi} , as the true technology parameter for all countries. In this case, the Cobb-Douglas results suggest that differences in human capital misallocation explain as much as 28% of the output per worker variation in 2007.

The explanation for this is visible in appendix figure 8. With imposed ϕ_{0USAi} in all countries, countries in the lower half of the income distribution obtain larger hypothetical gains from reallocation relative to the U.S. Put differently, countries are estimated to be less inefficient in allocating their human capital across sectors if technology parameters are assumed to be country-specific.³³ This seems economically plausible and is supported by the concept that countries opt for technologies that are appropriate for their factor endowments (see, e.g., Caselli, 2005).

In terms of sensitivity regarding η_{ci} and σ_{ci} , the 10th-to-90th inter-percentile range spans

³²In practice, the 90%-interval does not introduce huge variation as the sector substitution elasticities are estimated quite precisely.

³³Note that the reduction in the relative efficiency gaps mirrors less absolute efficiency losses because the gains from reallocation for the U.S. are constant.

from 0.79 to 0.83. This implies that human capital misallocation explains between 17%-21% of the output per worker variation, given that ϕ_{0USAi} resembles the true technology parameter in all countries. Overall, the robustness exercises regarding the assumed parameter values support the previous finding that imposing a Cobb-Douglas production function specification *can* have sizable implications for the estimated gains from reallocation. It does, however, not necessarily have to. The appropriateness and quantitative implications of the production function specification should therefore be evaluated on a case-by-case bases.

6.1.2 Sample composition

We have found that countries not only differ in their degree of human capital misallocation, but also by how much their estimated hypothetical gains differ across production function specifications. This raises the question to what extent the change in R_{World} across specifications is robust to the sample composition. To examine this, we create 1000 simulated samples consisting of 100 countries drawn with replacement from the original set of 39 countries in our sample. For each sample, We calculate R_{World} based on the Cobb-Douglas and CES specification. Appendix figure 9 plots the difference in R_{World} between the two specifications. The 10th and 90th percentile lines mark a difference of 4 and 8 percentage points, respectively. If we set this into relation to the 6 percent difference from the baseline, this indicates that the previous result is moderately sensitive to the countries in the sample.

Evidence on how sensitive the absolute value of R_{World} is to the sample composition is shown in appendix figure 10, which depicts the respective distributions and means of R_{World} for different specifications. Two things are noticeable. On the one hand, there is a considerable degree of variation in R_{World} for each specification, and as a consequence, overlap across the different specifications. On the other hand, there is a noticeable differences in the distributions. This indicates that the previous finding that the Cobb-Douglas specification overstates the role of misallocation in explaining output per worker variation is not just an artifact of the sample composition.

6.1.3 Non-market services

A caveat regarding the data is that output in non-market service sectors (public administration, education, and health care) is potentially poorly measured. While this provides a serious problem for, e.g., sector level comparison across countries, we consider it less severe for the main objective of this paper for a set of reasons. First, the measurement error only biases the estimated hypothetical reallocation gains given that measurement errors in sector outputs correlate systematically with productivity and employment levels. Second, to bias the ratio R_{World} , the bias in the country gains need to systematically correlate with the coun-

try output per worker levels. Third, even if some bias due to measurement error is present, it needs to have a systematically different and sizable effect across the different production function specifications.

Nonetheless, as a check, we run the reallocation exercise on a sample in which we drop the non-market service sectors. While the overall importance of human capital misallocation for explaining cross-country variation in output per worker is reduced in both the Cobb-Douglas and CES specification, the previous qualitative finding still holds. Allowing physical and human capital to be gross complements reduces cross-country differences in efficiency losses due to human capital misallocation. With this being said, the robustness exercises also indicate that the quantitative implications of the imposed production function parameter values depend on the countries and sectors examined.

6.2 Country dynamics in misallocation over time

One interesting finding depicted in figure 2 is the stability in R_{World} over the years. Appendix figure 11 indicates that this hides country dynamics in misallocation over time. The figure plots the hypothetical gains from reallocation for six countries. What we see is a mixed picture. Two countries, namely the Republic of Korea and Czech Republic, improved the allocation of human capital across sectors over the 12 year period. On the contrary, in countries like Belgium, Greece, China, and India, the aggregate output costs of human capital misallocation in 2007 exceed the 1995 level. In light of the widespread structural change that occurred in China and India, this finding seems puzzling. However, underlying these developments are different records of human capital reallocation across sectors and productivity growth within sectors. While a thorough investigation is beyond the scope of this paper, it is worth noting that an increase in the aggregate costs of misallocation does not imply that there has been no growth-enhancing reallocation of workers across sectors over the period. In fact, in both China and India the reallocation of human capital across sectors contributed positively to growth, albeit just marginally (see appendix table 7).³⁴ The dominant contribution came from within sector productivity growth. In fact, in both countries, marginal productivity values of human capital in agriculture remained low, despite substantial reallocation. Conversely, marginal productivity values increased further in other sectors. Consequently, gaps in marginal products across sectors increased. Rapid productivity growth

³⁴We apply a shift-share-approach to decompose human capital productivity growth into three different components. The first component captures productivity gains within sectors. The second (static) component captures gains that result from human capital shifting into sectors with above-average productivity levels. The third (dynamic) component, captured gains that result if human capital shifts, overall, to sectors with above-average productivity growth.

within highly productive sectors can thus increase allocative inefficiency.³⁵ At the same time, however, it raises the potential for growth-enhancing human capital reallocation.

7 Conclusion

In this paper, we have examined whether the strong functional form assumptions imposed by Cobb-Douglas functions have quantitatively important implications for measuring aggregate costs of human capital misallocation across sectors for a sample of 39 countries. We extend the misallocation framework to the CES and nested CES specification by drawing on a normalization approach. This allows suitable counterfactuals and improves the identification of sector-level substitution elasticities. Physical capital and the human capital input are estimated to be gross complements in all sectors. This suggests that counterfactuals based on Cobb-Douglas sector production functions are not appropriate.

We find that relaxing assumptions on the substitutability of input factors affects the hypothetical gains from reallocation considerably in some countries, but not in all. As a consequence, the degree to which human capital misallocation can explain aggregate output per worker variation in the sample drops from 21% (Cobb-Douglas) to 14% (CES). Allowing for imperfect substitution of skill types has by itself only small effects on the estimated reallocation gains. It does, however, imply some changes in the estimated elasticity of substitution between physical capital and the human capital aggregate, which in turn translate into moderate changes in the estimated reallocation gains. Finally, we find that the fraction of output per worker variation in the sample that can be explained by cross-country differences in the degree of human capital misallocation across sectors has been quite stable over the period 1995-2007. This stability, however, hides the fact that countries experienced different developments regarding the extent of cross-sector human capital misallocation.

Our results raise the question whether previous work has overestimated the role of factor misallocation by working with a Cobb-Douglas production function. This conclusion appears premature. First, we have focused on the allocation of one production factor only, keeping the other production factor fixed. In the long run, physical capital may reallocate as well, leading to different quantitative implications.³⁶ Second, and relatedly, the rate at which marginal products of one production factor are equated across sectors or firms depends on how abundant or scarce the mobile factor is in the economy. Additionally, we want to emphasize that output gains from efficiently allocating human capital across sectors, which

³⁵Ceteris paribus, a shift of human capital out of (into) a sector implies an increase (decrease) in the marginal products of the remaining workforce due to an increase (decrease) in the capital intensity. The increase in the marginal product gaps, despite substantial reallocation, therefore indicates changes in the sectoral capital stock and/or technology.

³⁶See the recent contribution by Mallick and Maqsood (2023).

we estimate to be 7.6 % on average, are still substantial.

In any case, the results stress that the appropriateness and quantitative implications of the production function specification should be evaluated on a case-by-case basis. Hence, a test for model specification should be a prerequisite for parametric counterfactuals that gauge the aggregate cost of factor misallocation, irrespective of whether they focus on the macro or micro level. For cases in which the CES specification is found more appropriate, this paper presents an approach to extend the misallocation framework beyond Cobb-Douglas.

References

- Alfaro, L., Charlton, A., and Kanczuk, F. (2008). Plant Size Distribution and Cross-Country Income Differences. *NBER International Seminar on Macroeconomics*, 5(1):243–272.
- Antras, P. (2004). Is the US aggregate production function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *Contributions to Macroeconomics*, 4(1):1161.
- Aoki, S. (2012). A simple accounting framework for the effect of resource misallocation on aggregate productivity. *Journal of the Japanese and International Economies*, 26(4):473–494.
- Autor, D. H., Katz, L. F., and Kearney, M. S. (2008). Trends in US wage inequality: Revising the revisionists. *The Review of Economics and Statistics*, 90(2):300–323.
- Baccianti, C. (2013). Estimation of sectoral elasticities of substitution along the international technology frontier. *ZEW Discussion Papers*, 13(092).
- Banerjee, A. V. and Duflo, E. (2005). Growth Theory through the Lens of Development Economics. *Handbook of Economic Growth*, 1:473–552.
- Bartelsman, E., Haltiwanger, J., and Scarpetta, S. (2013). Cross-Country Differences in Productivity: The Role of Allocation and Selection. *The American Economic Review*, 103(1):305–334.
- Bento, P. and Restuccia, D. (2017). Misallocation, Establishment Size, and Productivity. *American Economic Journal: Macroeconomics*, 9(3):267–303.
- Bentolila, S. and Saint-Paul, G. (2003). Explaining Movements in the Labor Share. *Contributions in Macroeconomics*, 3(1).
- Brandt, L., Tombe, T., and Zhu, X. (2013). Factor market distortions across time, space and sectors in China. *Review of Economic Dynamics*, 16(1):39–58.
- Buera, F. J., Kaboski, J. P., and Shin, Y. (2011). Finance and Development: A Tale of Two Sectors. *American Economic Review*, 101(5):1964–2002.
- Busso, M., Madrigal, L., et al. (2013). Productivity and Resource Misallocation in Latin America. *The BE Journal of Macroeconomics*, 13(1):903–932.
- Cai, W. and Pandey, M. (2015). The Agricultural Productivity Gap in Europe. *Economic Inquiry*, 53(4):1807–1817.

- Caselli, F. (2005). Accounting for Cross-Country Income Differences. In Aghion, P. and Durlauf, S., editors, *Handbook of Economic Growth*, volume 1, chapter 9, pages 679–741. Elsevier.
- Caselli, F. (2016). *Technology Differences Over Space and Time*. Princeton University Press.
- Caselli, F. and Ciccone, A. (2019). The Human Capital Stock: A Generalized Approach Comment. *American Economic Review*, 109(3):1155–74.
- Caselli, F. and Coleman, W. J. (2006). The World Technology Frontier. *The American Economic Review*, 96(3):499–522.
- Caselli, F. and Feyrer, J. (2007). The Marginal Product of Capital. *The Quarterly Journal of Economics*, 122(2):535–568.
- Caselli, F. and Gennaioli, N. (2013). Dynastic Management. *Economic Inquiry*, 51(1):971–996.
- Chen, K. and Irarrazabal, A. (2015). The role of allocative efficiency in a decade of recovery. *Review of Economic Dynamics*, 18(3):523–550.
- Chirinko, R. S., Fazzari, S. M., and Meyer, A. P. (2011). A New Approach to Estimating Production Function Parameters: The Elusive Capital? Labor Substitution Elasticity. *Journal of Business & Economic Statistics*, 29(4):587–594.
- Chirinko, R. S. and Mallick, D. (2017). The Substitution Elasticity, Factor Shares, and the Low-Frequency Panel Model. *American Economic Journal: Macroeconomics*, 9(4):225–53.
- Ciccone, A. and Peri, G. (2005). Long-run substitutability between more and less educated workers: Evidence from US States, 1950–1990. *Review of Economics and Statistics*, 87(4):652–663.
- Dabla-Norris, M. E., Guo, M. S., Haksar, M. V., Kim, M., Kochhar, M. K., Wiseman, K., and Zdzienicka, A. (2015). The New Normal: A Sector-Level Perspective on Productivity Trends in Advanced Economies. Staff Discussion Notes No. 15/3, International Monetary Fund.
- Dias, D. A., Marques, C. R., and Richmond, C. (2016). Misallocation and productivity in the lead up to the Eurozone crisis. *Journal of Macroeconomics*, 49:46–70.
- Dissou, Y., Karnizova, L., and Sun, Q. (2015). Industry-level econometric estimates of energy-capital-labor substitution with a nested CES production function. *Atlantic Economic Journal*, 43(1):107–121.

- Feenstra, R. C., Inklaar, R., and Timmer, M. P. (2015). The Next Generation of the Penn World Table. *American Economic Review*, 105(10):3150–82.
- Garcia-Santana, M. and Pijoan-Mas, J. (2014). The reservation laws in India and the misallocation of production factors. *Journal of Monetary Economics*, 66:193–209.
- Gechert, S., Havranek, T., Irsova, Z., and Kolcunova, D. (2022). Measuring capital-labor substitution: The importance of method choices and publication bias. *Review of Economic Dynamics*, 45:55–82.
- Gollin, D., Lagakos, D., and Waugh, M. E. (2014). The Agricultural Productivity Gap. *The Quarterly Journal of Economics*, 129(2):939–993.
- Hallak, J. C. and Schott, P. K. (2011). Estimating Cross-Country Differences in Product Quality. *The Quarterly Journal of Economics*, 126(1):417–474.
- Henningsen, A. and Henningsen, G. (2012). On estimation of the CES production function Revisited. *Economics Letters*, 115(1):67–69.
- Herrendorf, B., Herrington, C., and Valentinyi, A. (2015). Sectoral Technology and Structural Transformation. *American Economic Journal: Macroeconomics*, 7(4):104–133.
- Herrendorf, B. and Schoellman, T. (2015). Why is Measured Productivity so Low in Agriculture? *Review of Economic Dynamics*, 18(4):1003–1022.
- Herrendorf, B. and Schoellman, T. (2018). Wages, Human Capital, and Barriers to Structural Transformation. *American Economic Journal: Macroeconomics*, 10(2):1–23.
- Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and Manufacturing TFP in China and India. *The Quarterly Journal of Economics*, 124(4):1403–1448.
- Hsieh, C.-T. and Klenow, P. J. (2010). Development Accounting. *American Economic Journal: Macroeconomics*, 2(1):207–223.
- Inklaar, R. and Diewert, W. E. (2016). Measuring industry productivity and cross-country convergence. *Journal of Econometrics*, 191(2):426–433.
- Inklaar, R., Lashitew, A. A., and Timmer, M. P. (2017). The Role of Resource Misallocation in Cross-country Differences in Manufacturing Productivity. *Macroeconomic Dynamics*, 21(3):733–756.
- Inklaar, R. and Timmer, M. P. (2014). The Relative Price of Services. *Review of Income and Wealth*, 60(4):727–746.

- Jones, B. F. (2014). The Human Capital Stock: A Generalized Approach. *American Economic Review*, 104(11):3752–77.
- Jones, C. I. (2016). The Facts of Economic Growth. *Handbook of Macroeconomics*, 2:3–69.
- Klump, R. and de La Grandville, O. (2000). Economic Growth and the Elasticity of Substitution: Two Theorems and Some Suggestions. *American Economic Review*, 90(1):282–291.
- Klump, R., McAdam, P., and Willman, A. (2007). Factor substitution and factor-augmenting technical progress in the United States: a normalized supply-side system approach. *The Review of Economics and Statistics*, 89(1):183–192.
- Klump, R., McAdam, P., and Willman, A. (2012). The normalized CES production function: theory and empirics. *Journal of Economic Surveys*, 26(5):769–799.
- Klump, R. and Saam, M. (2008). Calibration of normalised CES production functions in dynamic models. *Economics Letters*, 99(2):256–259.
- Knoblach, M., Roessler, M., and Zwerschke, P. (2019). The Elasticity of Substitution Between Capital and Labour in the US Economy: A Meta-Regression Analysis. *Oxford Bulletin of Economics and Statistics*.
- Kukk, M., Paulus, A., Staehr, K., et al. (2018). Income underreporting by the self-employed in Europe: a cross-country comparative study. Working Papers 04/2018, Bank of Estonia.
- León-Ledesma, M. A., McAdam, P., and Willman, A. (2010). Identifying the elasticity of substitution with biased technical change. *The American Economic Review*, 100(4):1330–1357.
- León-Ledesma, M. A., McAdam, P., and Willman, A. (2015). Production technology estimates and balanced growth. *Oxford Bulletin of Economics and Statistics*, 77(1):40–65.
- Mallick, D. and Maqsood, N. (2023). Capital-labor substitution and misallocation. Working Papers 116587, MPRA.
- McAdam, P. and Willman, A. (2018). Unraveling the skill premium. *Macroeconomic Dynamics*, 22(1):33–62.
- Midrigan, V. and Xu, D. Y. (2014). Finance and Misallocation: Evidence from Plant-Level Data. *American Economic Review*, 104(2):422–58.
- Mollick, A. V. (2011). The world elasticity of labor substitution across education levels. *Empirical Economics*, 41(3):769–785.

- Mundlak, Y. (2005). Economic growth: Lessons from two centuries of American agriculture. *Journal of Economic Literature*, 43(4):989–1024.
- Oberfield, E. (2013). Productivity and misallocation during a crisis: Evidence from the Chilean crisis of 1982. *Review of Economic Dynamics*, 16(1):100–119.
- Oberfield, E. and Raval, D. (2014). Micro data and macro technology. NBER Working Paper No. 20452, National Bureau of Economic Research.
- Papageorgiou, C., Saam, M., and Schulte, P. (2017). Substitution between clean and dirty energy inputs: A macroeconomic perspective. *Review of Economics and Statistics*.
- Raval, D. R. (2019). The micro elasticity of substitution and non-neutral technology. *The RAND Journal of Economics*, 50(1):147–167.
- Redding, S. and Weinstein, D. (2018). Accounting for Trade Patterns. mimeograph, Princeton University.
- Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics*, 11(4):707–720.
- Restuccia, D. and Rogerson, R. (2013). Misallocation and productivity. *Review of Economic Dynamics*, 16(1):1–10.
- Restuccia, D. and Rogerson, R. (2017). The Causes and Costs of Misallocation. *Journal of Economic Perspectives*, 31(3):151–174.
- Ryzhenkov, M. (2016). Resource misallocation and manufacturing productivity: The case of Ukraine. *Journal of Comparative Economics*, 44(1):41–55.
- Schelkle, T. (2016). General Methods for Measuring Factor Misallocation. Working Paper Series in Economics 87, University of Cologne, Department of Economics.
- Temple, J. (2012). The calibration of CES production functions. *Journal of Macroeconomics*, 34(2):294–303.
- Timmer, M., Erumban, A. A., Gouma, R., Los, B., Temurshoev, U., de Vries, G. J., Arto, I.-a., Genty, V. A. A., Neuwahl, F., Francois, J., et al. (2012). The World Input-Output Database (WIOD): Contents, Sources and Methods. IIDE Discussion Papers 20120401, Institute for International and Development Economics.
- Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R., and Vries, G. J. (2015). An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production. *Review of International Economics*.

- Vollrath, D. (2009). How important are dual economy effects for aggregate productivity? *Journal of Development Economics*, 88(2):325–334.
- Vollrath, D. (2014). The efficiency of human capital allocations in developing countries. *Journal of Development Economics*, 108:106–118.
- Vollrath, D. (2015). There is more to life than manufacturing. <https://growthecon.com/blog/there-is-more-to-life-than-manufacturing/> (April 2015).
- Vries, G. J. (2014). Productivity in a Distorted Market: The case of Brazil’s retail sector. *Review of Income and Wealth*, 60(3):499–524.
- Young, A. T. (2013). US elasticities of substitution and factor augmentation at the industry level. *Macroeconomic Dynamics*, 17(4):861–897.

Appendix

Appendix A: Tables and figures

Table 5: Reallocation gains for varying η_i and fixed ρ_i

Country	$\eta \rightarrow \infty$	$\eta = 4$	$\eta = 2$	$\eta = 1.6$	Country	$\eta \rightarrow \infty$	$\eta = 4$	$\eta = 2$	$\eta = 1.6$
AUS	3.12	3.23	3.17	3.17	IRL	3.36	3.55	3.52	3.62
AUT	4.61	4.67	4.63	4.64	ITA	4.94	5.85	5.62	5.4
BEL	4.03	4.39	4.13	4.07	JPN	2.05	2.14	2.12	2.11
BGR	8.88	8.21	7.81	7.74	KOR	17.56	18.27	17.98	17.72
BRA	8.48	9.9	9.16	9.06	LTU	2.3	2.38	2.24	2.16
CAN	2.15	2.41	2.41	2.42	LUX	4.16	4.91	4.48	3.98
CHN	27.06	25.98	25.68	25.57	LVA	3.11	3.23	3.12	3.09
CYP	3.81	5.4	5.11	5.04	MEX	3.63	3.7	3.7	3.55
CZE	2.44	3.02	2.73	2.68	MLT	4.73	5.59	4.9	4.71
DEU	7.45	7.85	7.55	7.47	NLD	3.41	3.35	3.18	2.97
DNK	5.55	5.96	5.71	5.63	POL	2.83	3.79	3.73	3.64
ESP	3.09	3.9	3.72	3.72	PRT	8.89	7.98	7.39	7.32
EST	2.62	2.85	2.74	2.72	ROU	16.44	12.29	12.02	11.89
FIN	4.22	4.13	4.06	4.04	RUS	14.41	14.98	14.09	13.47
FRA	2.19	2.57	2.38	2.35	SVK	.84	1.42	1.29	1.26
GBR	4.86	4.81	4.68	4.64	SVN	3.72	3.79	3.66	3.67
GRC	12.28	13.63	13.49	13.47	SWE	4.75	4.55	4.34	4.25
HUN	2.5	3.25	2.94	2.89	TUR	2.72	2.75	2.78	2.76
IDN	11.15	11.93	11.33	11.26	USA	5.37	5.85	5.65	5.55
IND	65.91	59.73	56.52	55.29					

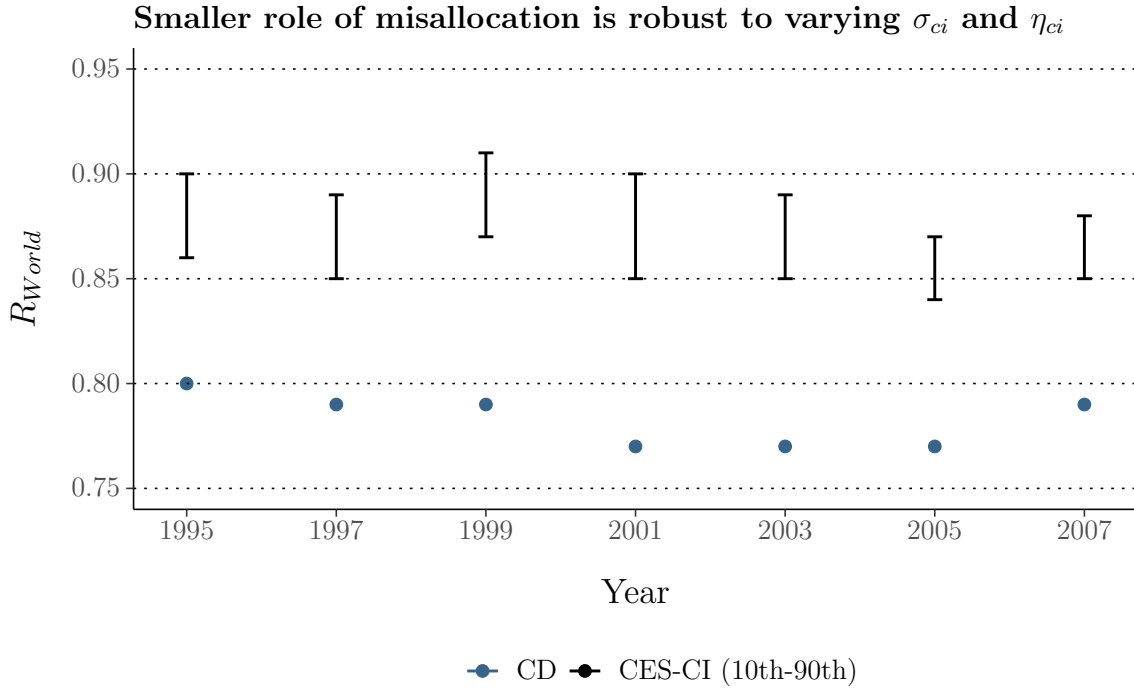
Note: Hypothetical output per worker gains in percentage points. Calculation based on different specifications of normalized sector production functions. η is assumed homogeneous across sectors. σ_i is set to the estimated value of the 1-level CES sector function. All calculations based on value added nominated in PPPs for 12 sectors.

Table 6: Overview industry prices levels, 2005

Description	Code	mean	s.d.	min	max
Food, Beverages and Tobacco	15t16	0.71	0.22	0.33	1.27
Textiles and Textile Products	17t18	0.53	0.10	0.27	0.93
Leather, Leather and Footwear	19	0.51	0.08	0.27	0.74
Wood and Products of Wood and Cork	20	0.70	0.13	0.40	0.95
Pulp, Paper, Paper, Printing and Publishing	21t22	0.84	0.25	0.27	1.40
Coke, Refined Petroleum and Nuclear Fuel	23	0.88	0.20	0.35	1.21
Chemicals and Chemical Products	24	0.81	0.19	0.21	1.19
Rubber and Plastics	25	0.80	0.24	0.21	1.85
Other Non-Metallic Mineral	26	0.61	0.25	0.34	1.85
Basic Metals and Fabricated Metal	27t28	0.72	0.17	0.33	1.04
Machinery, Nec	29	0.69	0.08	0.51	0.87
Electrical and Optical Equipment	30t33	0.67	0.12	0.37	0.91
Transport Equipment	34t35	0.76	0.12	0.30	0.88
Manufacturing, Nec; Recycling	36t37	0.70	0.12	0.43	0.93
Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel	50	0.81	0.31	0.23	1.63
Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles	51	0.76	0.10	0.49	1.00
Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods	52	0.79	0.24	0.32	1.26
Inland Transport	60	0.84	0.48	0.26	2.02
Water Transport	61	0.78	0.48	0.20	2.48
Air Transport	62	1.16	0.42	0.47	2.59
Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies	63	0.74	0.35	0.27	1.66
Post and Telecommunications	64	0.85	0.24	0.23	1.23
Real Estate Activities	70	0.70	0.38	0.18	1.35
Renting of M&Eq and Other Business Activities	71t74	0.79	0.25	0.34	1.29
Agriculture, Hunting, Forestry and Fishing	AtB	0.61	0.19	0.29	1.33
Mining and Quarrying	C	0.90	0.46	-0.28	2.98
Electricity, Gas and Water Supply	E	0.83	0.36	0.17	1.63
Construction	F	0.82	0.39	0.25	1.74
Hotels and Restaurants	H	0.79	0.29	0.30	1.48
Financial Intermediation	J	0.79	0.25	0.36	1.32
Public Admin and Defence; Compulsory Social Security	L	0.86	0.41	0.10	1.96
Education	M	0.92	0.45	0.11	1.72
Health and Social Work	N	0.78	0.34	0.14	1.39
Other Community, Social and Personal Services	O	0.75	0.36	0.24	1.56
Private Households with Employed Persons	P	1.09	0.64	0.15	2.49

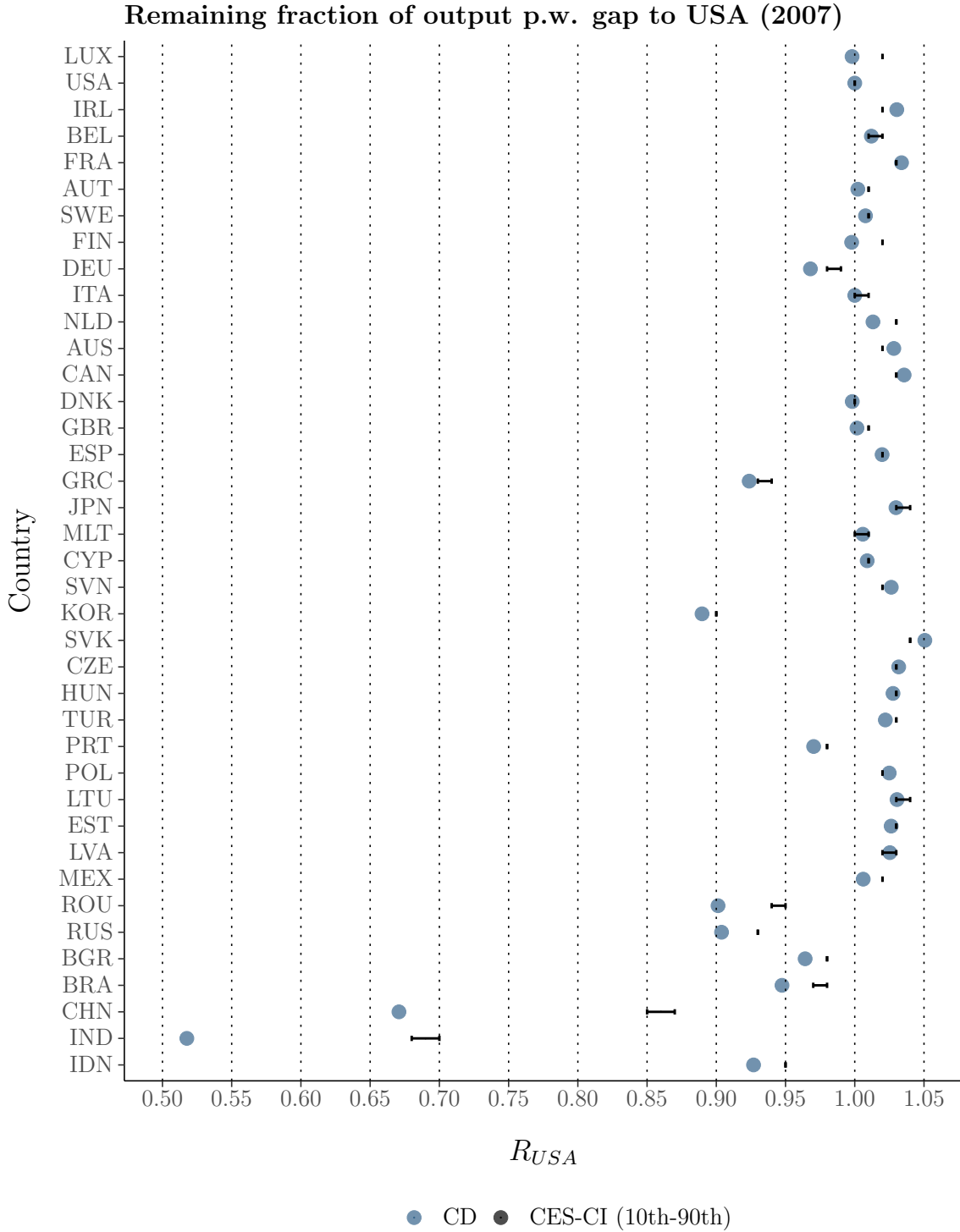
Note: Price levels relative to the aggregate price level of the U.S. in 2005. *Data source:* Inklaar and Timmer (2014).

Figure 4: Robustness test for R_{World} (I)



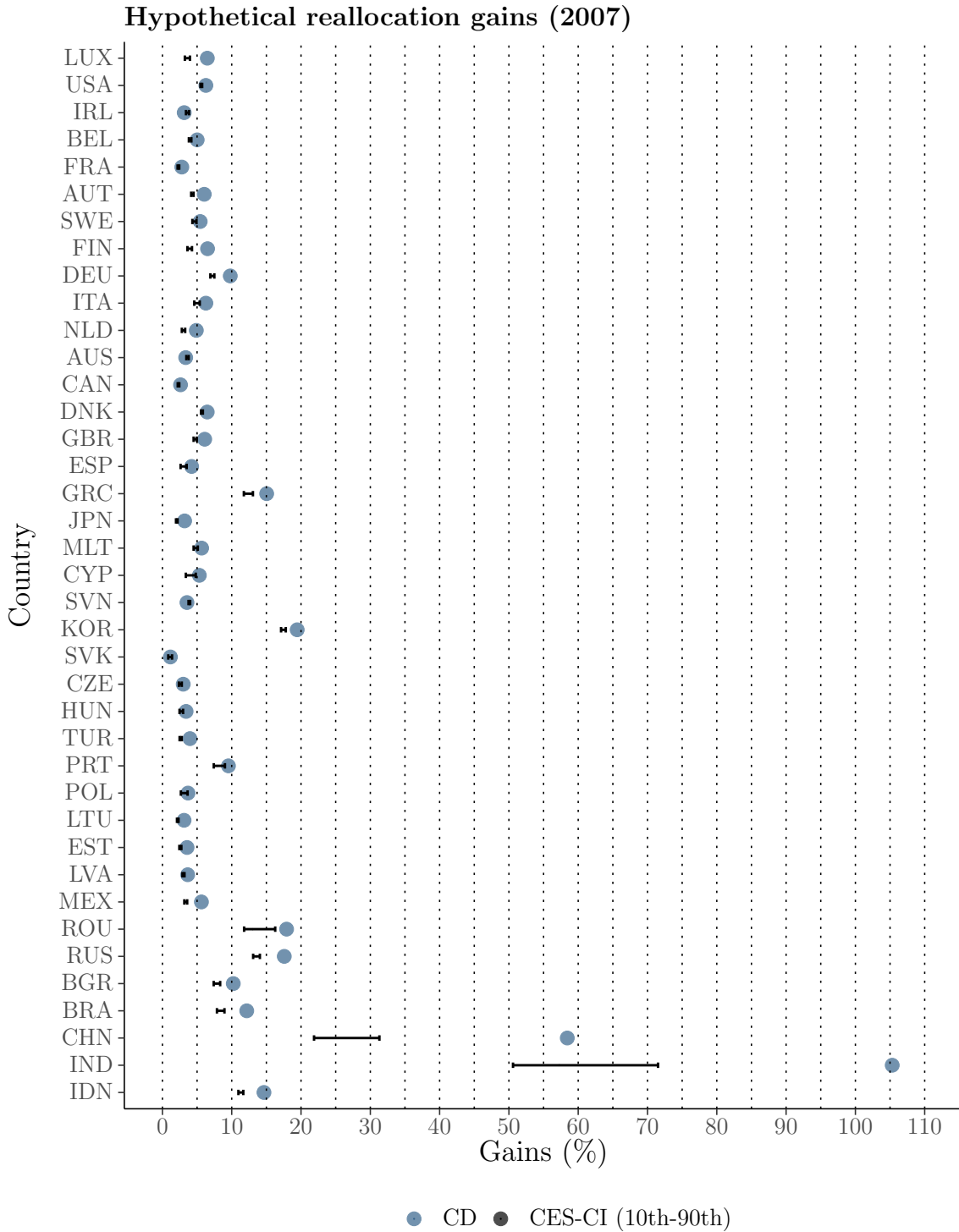
Note: Own calculation based on WIOD-SEA. R_{World} states the remaining fraction of output p.w. variation in 2007 after misallocation across sectors is eliminated. *Specifications:* CD: Cobb-Douglas production function. CES-CI: 10th-to-90th percentile bootstrap interval of R_{World} based on 1000 iterations with random values of σ_{ci} and η_{ci} . For a given η_{ci} , the admissible range of σ_{ci} is the 90%-CI around estimated expected value. For $\eta_{ci} \rightarrow \infty$, R_{World} is calculated based on a 1-level CES function. For $\eta_{ci} = 1.6, 2, 4$, R_{World} is calculated based on a 2-level CES function. All calculations based on value added nominated in constant PPPs for 12 sectors.

Figure 5: Robustness test for R_{USA} (I)



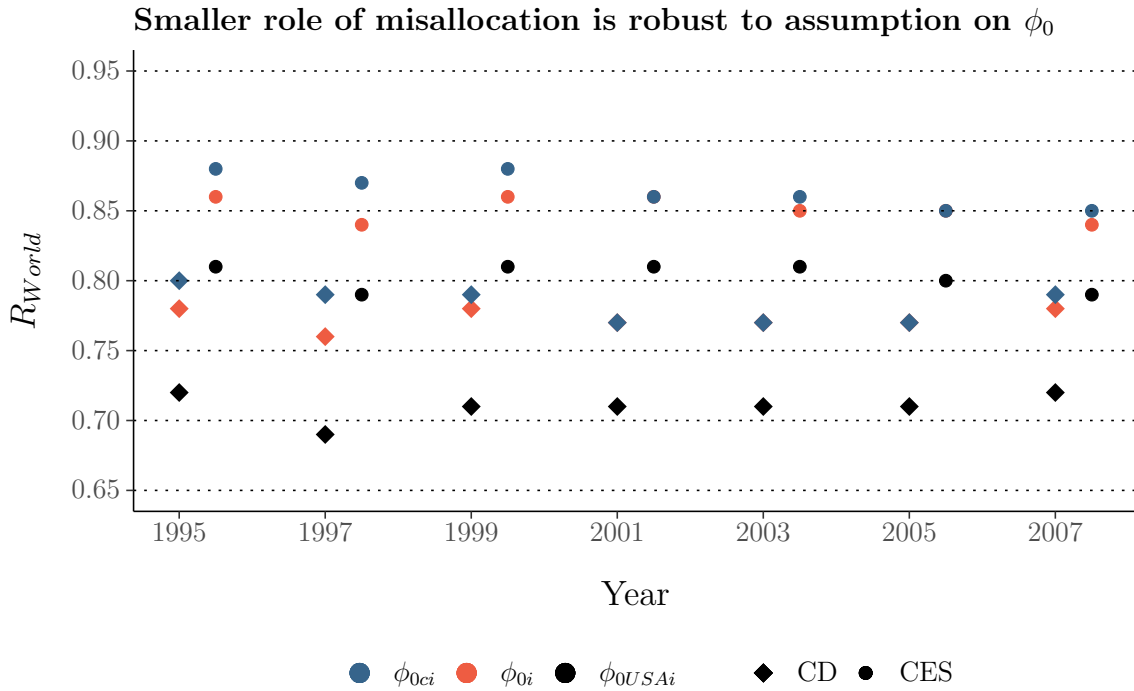
Note: Own calculation based on WIOD-SEA. CD: Cobb-Douglas production function. CES-CI: 10th-90th percentile bootstrap interval based on 500 iterations with varying values of $\rho_{c,i}$ and $\gamma_{c,i}$. For a given η_{ci} , the admissible range of σ_{ci} is the 90%-CI around estimated expected value. For $\eta_{ci} \rightarrow \infty$, $R_{c,USA}$ is calculated based on a 1-level CES function. For $\eta_{ci} = 1.6, 2, 4$, $R_{c,USA}$ is calculated based on a 2-level CES function. All calculations based on value added nominated in constant PPPs for 12 sectors.

Figure 6: Hypothetical reallocation gains



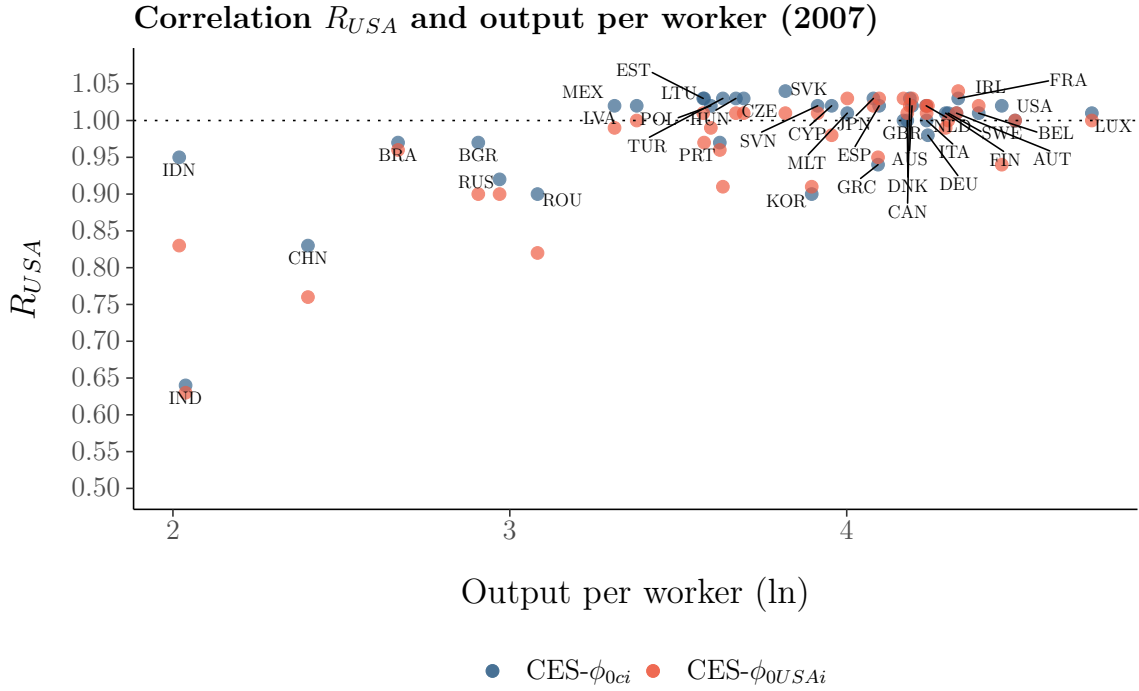
Note: Own calculation based on WIOD-SEA. CD: Cobb-Douglas production function. CES-CI: 10th-90th percentile bootstrap interval based on 500 iterations with varying values of ρ_{ci} and γ_{ci} . For a given η_{ci} , the admissible range of σ_{ci} is the 90%-CI around estimated expected value. For $\eta_{ci} \rightarrow \infty$, gains are calculated based on a 1-level CES function. For $\eta_{ci} = 1.6, 2, 4$, gains are calculated based on a 2-level CES function. All calculations based on value added nominated in constant PPPs for 12 sectors.

Figure 7: Robustness test for R_{World} (II)



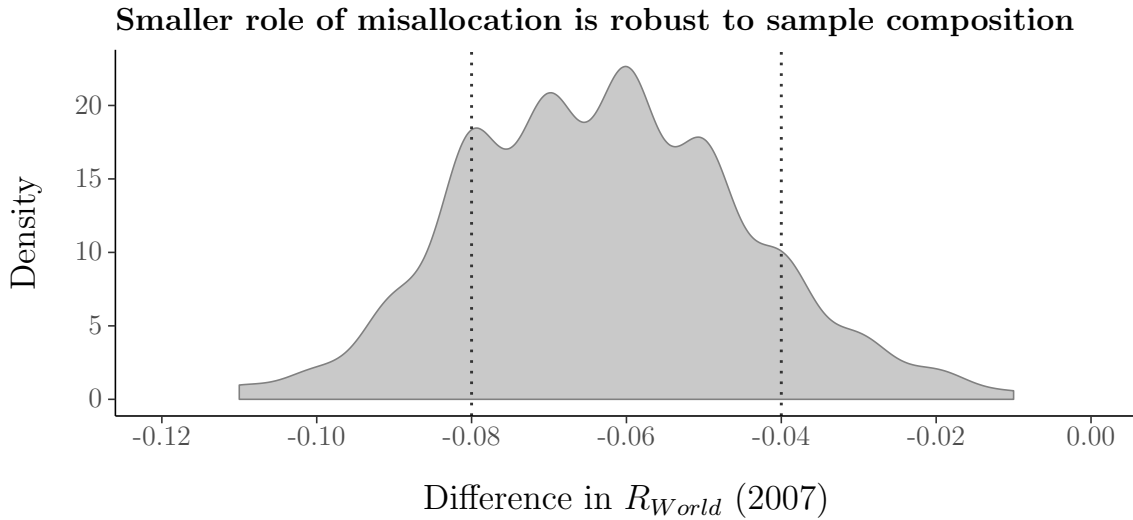
Note: Own calculation based on WIOD-SEA. *Specifications:* CD: Cobb-Douglas production function. CES: Constant elasticity substitution function. $\phi_{0,i}$: sample-average of ϕ_i . $\phi_{0,c}$: country-averages of ϕ_i . $\phi_{0,USA}$: U.S.-average of ϕ_i . All calculations based on value added nominated in constant PPPs for 12 sectors.

Figure 8: Robustness test for R_{USA} (II)



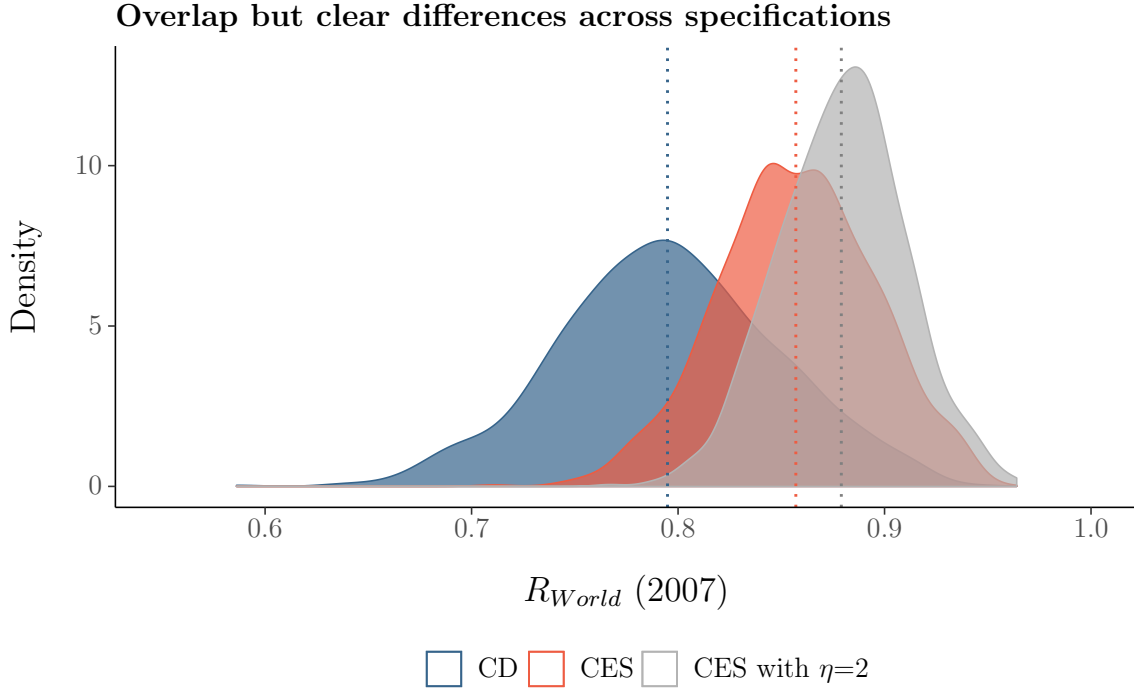
Note: Own calculation based on WIOD-SEA. CES: Constant elasticity substitution function. ϕ_{0ci} : country-averages of ϕ_i . ϕ_{0USAi} : U.S.-average of ϕ_i . All calculations based on value added nominated in constant PPPs for 12 sectors.

Figure 9: Robustness test for R_{World} (III)



Note: Own calculation based on WIOD-SEA. Dependent variable is, for 1000 simulated samples, the difference between R_{World} based on the Cobb-Douglas and CES specification, respectively. The dotted lines indicate the 10th and 90th percentile. All calculations based on value added nominated in constant PPPs for 12 sectors.

Figure 10: Robustness test for R_{World} (IV)



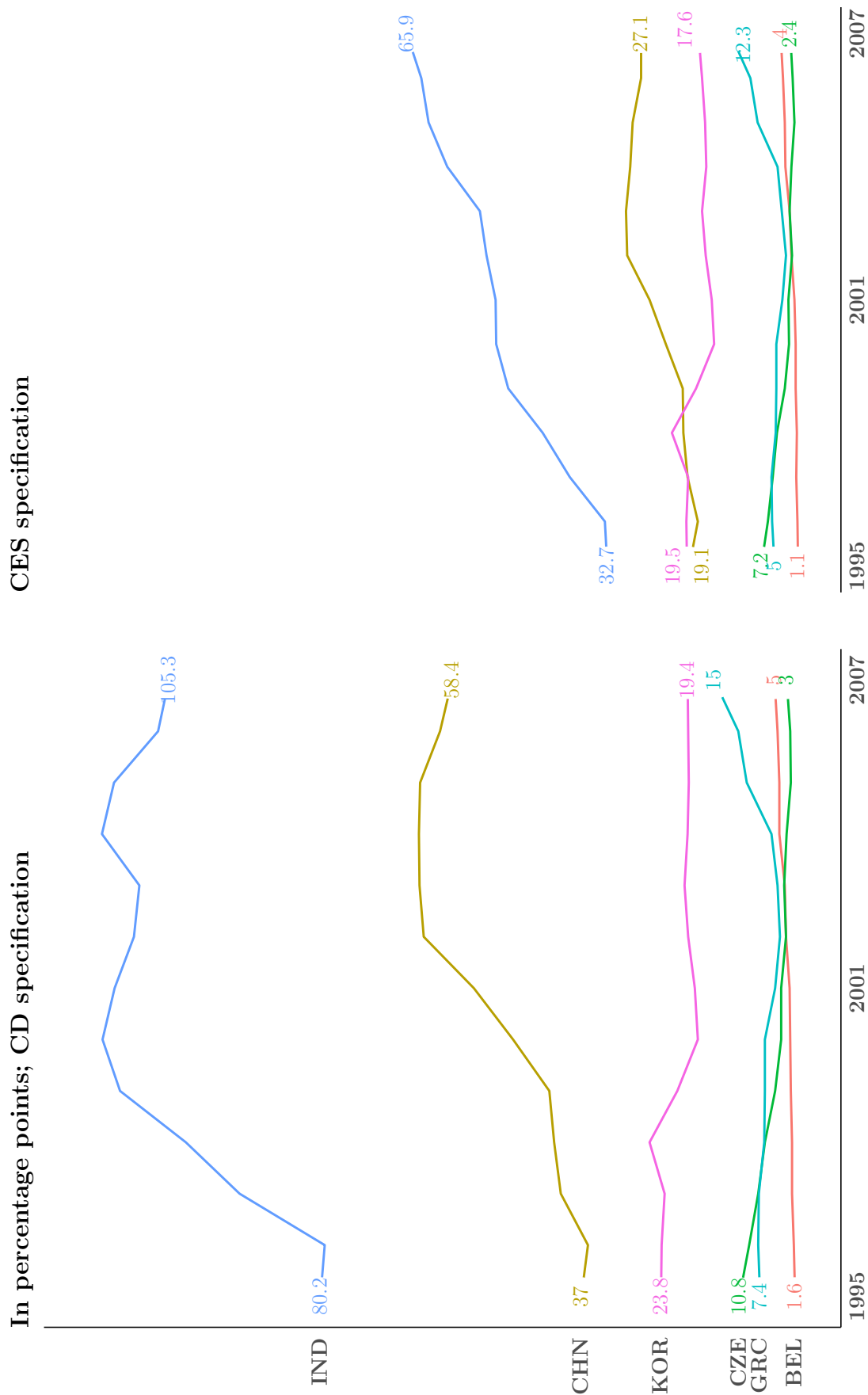
Note: Own calculation based on WIOD-SEA. Dependent variable is R_{World} calculated for a bootstrap sample of 100 countries with replacement based on 1000 iterations. The dotted lines indicate the respective mean value for R_{World} for each specification. All calculations based on value added nominated in constant PPPs for 12 sectors. *Specifications:* CD: Cobb-Douglas production function. CES: Constant elasticity substitution function.

Table 7: Contributions to human capital productivity growth

Country	CAGR	within	static	dynamic
BEL	1.1	1.4	0.4	-0.7
CHN	8.8	7.9	0.4	0.6
CZE	2.4	2.0	0.5	-0.2
GRC	2.7	2.7	1.2	-1.3
IND	5.0	4.2	1.3	-0.5
KOR	3.4	3.9	0.9	-1.4

Note: *CAGR:* Compound annual growth rate of human capital productivity in percentage points over period 1995-2007. *within:* Growth in human capital productivity within sectors. *static:* Growth due to human capital shifting into sectors with above-average productivity levels *dynamic:* Growth due to human capital shifting, overall, to sectors with above-average productivity growth. All calculations based on value added nominated in PPPs for 12 sectors.

Figure 11: Hypothetical output per worker gains over time



Note: Own calculation based on WIOD-SEA. Hypothetical output per worker gains (in percentage points) over time for selected countries. Calculation based on Cobb-Douglas and CES specifications of normalized sector production functions. All calculations based on value added nominated in PPPs for 12 sectors. *BEL*: Belgium; *CHN*: China; *CZE*: Czech Republic; *GRC*: Greece; *IND*: India; *KOR*: Republic of Korea.

Appendix B: Labor value added shares in WIOD-SEA

A key feature of WIOD-SEA is its harmonized information on compensation payments to specific skill-types of workers on the sector level for a panel of countries. Linked to a normalization approach, this data information be used to infer the unobserved distribution parameters. In practice, however, a drawback exist, as there are multiple labor revenue shares which are economically unreasonable. Specifically, in 16 out of the 486 country-sector observations, ϕ_{0ci} exceeds the value of 1.

To explain the large labor shares in value added, it is important to note some assumptions underlying the construction of WIOD-SEA. As is well known, labor income of self-employed is not registered in the National Accounts in many countries. Following standard practice, WIOD-SEA addresses this issue by imputing self-employed income. For advanced countries, it is assumed that the compensation per hour of self-employed is equal to the compensation per hour of employees. Arguing that this assumption is not plausible for emerging countries as a large part of informal workers earns much less than the average wage of low-skilled workers, additional country-specific information is used (Timmer et al., 2012).

The imputation of self-employed income in advanced economies is, however, not without problems. Even if the assumption of equal compensation per hour holds, total labor compensation is mismeasured if average compensation per hour of employees is biased. The latter is calculated by dividing the total compensations to employees by their hours worked, while the total compensations to employees include wages and salaries but also all other costs of employing labor which are borne by the employer (e.g. social security taxes). Especially in cases with high amount of net subsidies and a large share of self-employed, the imputation of self-employed incomes based on average wages per hour of employees can result in total labor compensation being larger than value added. An examples is the agriculture sector in several European countries.

The problem of inflated and implausible labor shares on the sector level is also noted by Bentolila and Saint-Paul (2003) and Inklaar and Diewert (2016), who point to self-employed earning on average a lower wage than employees as a likely source. Both papers deal with the issue in a pragmatic manner by adjusting the assumed average wage ratio of self-employed to employee. Specifically, Bentolila and Saint-Paul (2003) assumes a ratio of two-thirds, while Inklaar and Diewert (2016) assume a ratio of 60 percent and even 30 percent in agriculture to ensure that capital income at the sector level is positive in all countries and years.

Cai and Pandey (2015) propose an adjustment to the imputation approach in WIOD-SEA. Using census data of ten European countries, they impute compensation for self-employed in each sector based on the assumption that a self-employed earns the same hourly wage as a wage worker who has the self-employed's observable characteristics. They find an average ratio of wage of self-employed to that of employee of 1.012 for agriculture and 1.035 in the

remaining non-agriculture economy. Note, though, that based on these estimates the adjusted total labor compensation would in fact even rise.

An alternative source of bias in the labor revenue share is under-reporting of self-employment income (e.g. due to tax evasion) leading to a downward bias in the measurement of value added. For instance, Kukk et al. (2018) reviews the literature on the magnitude of under-reporting of self-employment income and provides own estimates for a set of European countries in the range of 10% to 40%. Although these estimates indeed indicate substantial under-reporting of self-employment income, they measure under-reporting on the aggregate and not the sector level. Indeed, both Herrendorf and Schoellman (2015) and Cai and Pandey (2015) make the case that under-reporting of income is a particular problem in the agriculture sector. Despite this plausible explanation for inflated labor value added shares, we are nevertheless practically constrained, as we do not have estimates of the degree of under-reporting across the sectors and countries in our sample. And, because cross-sector differences are key in this paper, imposing an, arguably arbitrary, aggregate adjustment factor might in fact make matters worse.

In the absence of appropriate adjustment factors, we deal with questionable parameter values in the baseline counterfactual by setting the maximum value of ϕ_{0ci} to 0.9. Additionally, we run robustness tests where we treat ϕ_{0ci} as a structural parameter that is common across countries. Specifically, we consider two different values for ϕ_{0i} . First, we set ϕ_{0ci} to the geometric sample-average labor share. Second, we set ϕ_{0ci} to the geometric average of U.S. labor share. Table 8 states the imposed values in each sector.

Table 8: Labor shares by sector

Abbr	Sector Name	(1)	(2)	(3)	(4)	(5)
		Sample				U.S.
		Mean	Std. Dev.	Min.	Max.	Mean
AtB	Agriculture	0.74	0.19	0.26	0.90	0.47
D	Manufacturing	0.56	0.12	0.32	0.75	0.60
F	Construction	0.66	0.16	0.29	0.90	0.83
G	Trade Service	0.60	0.14	0.23	0.85	0.60
H	Hotels & Restaurants	0.64	0.18	0.28	0.90	0.71
I	Transport & Storage	0.53	0.12	0.29	0.76	0.63
J	Finance Services	0.49	0.13	0.20	0.71	0.50
K	Business Services	0.31	0.11	0.07	0.55	0.42
L	Public & Social Services	0.81	0.10	0.48	0.90	0.85
M	Education	0.88	0.08	0.55	0.90	0.96
N	Health & Social Work	0.80	0.10	0.55	0.90	0.75
OtP	Other Services	0.70	0.13	0.42	0.90	0.68

Note: Column 1 states the sample average labor shares in value added (geometric average over time and countries). Columns 2-4 show summary statistics for country-specific labor shares (geometric average over time). Column 5 shows the labor share in value added in the U.S. (geometric average over time).

Appendix C: Derivation of optimal labor input allocation

Assuming that $H = \sum_{j=1}^J (H_j)$, we can write H_i as:

$$H_i = \frac{H_i}{\sum_{j=1}^J (H_j)} H$$

multiplying the nominator and denominator by $\frac{(1+\lambda_i^H)MPV_{hyp}^H}{(1+\lambda_j^H)MPV_{hyp}^H}$ results in

$$H_i = \frac{\frac{(1+\lambda_i^H)MPV_{hyp}^H H_i}{(1+\lambda_i^H)MPV_{hyp}^H}}{\sum_{j=1}^J \left(\frac{(1+\lambda_j^H)MPV_{hyp}^H H_j}{(1+\lambda_j^H)MPV_{hyp}^H} \right)} H$$

rewrite this as

$$H_i = (1 + \lambda_i^H) MPV_{hyp}^H H_i \frac{\frac{1}{(1+\lambda_i^H)MPV_{hyp}^H}}{\sum_{j=1}^J \left((1 + \lambda_i^H) MPV_{hyp}^H H_i \frac{1}{(1+\lambda_j^H)MPV_{hyp}^H} \right)} H$$

which is

$$H_i = \frac{(1 + \lambda_i^H) MPV_{hyp}^H H_i \frac{1}{(1+\lambda_i^H)}}{\sum_{j=1}^J \left((1 + \lambda_j^H) MPV_{hyp}^H H_j \right) \sum_{j=1}^J \left(\frac{(1+\lambda_i^H)MPV_{hyp}^H H_i}{\sum_{j=1}^J ((1+\lambda_j^H)MPV_{hyp}^H H_j)} \frac{1}{(1+\lambda_j^H)} \right)} H$$

which results in equation (14)

$$H_i = \frac{(1 + \lambda_i^H) MPV_{hyp}^H H_i}{\sum_{j=1}^J \left((1 + \lambda_j^H) MPV_{hyp}^H H_j \right)} \tilde{\lambda}_i^H H$$

where

$$\tilde{\lambda}_i^H = \frac{\frac{1}{(1+\lambda_i^H)}}{\sum_{j=1}^J \left(\frac{(1+\lambda_i^H)MPV_{hyp}^H H_i}{\sum_{j=1}^J ((1+\lambda_j^H)MPV_{hyp}^H H_j)} \frac{1}{(1+\lambda_j^H)} \right)}$$

Note that $(1 + \lambda_i^H) MPV_{hyp}^H = MPV_i^H$ and $MPV_i^H * H_i$ is the sector output multiplied

by the factor output elasticity. Specifically, given equation (15), we have

$$MPV_i^H = \phi_{0i} \frac{Y_i}{H_i} \left(\frac{H_i}{H_{0i}} \right)^{\rho_i} \left[\phi_{0i} \left(\frac{H_i}{H_i} \right)^{\rho_i} + (1 - \phi_{0i}) \left(\frac{K_i}{K_{0i}} \right)^{\rho_i} \right]^{-1}$$

so that

$$H_i = \frac{\phi_{0i} Y_i \left(\frac{H_i}{H_{0i}}\right)^{\rho_i} \left[\phi_{0i} \left(\frac{H_i}{H_i}\right)^{\rho_i} + (1 - \phi_{0i}) \left(\frac{K_i}{K_{0i}}\right)^{\rho_i} \right]^{-1}}{\sum_{j=1}^J \left(\phi_{0j} Y_j \left(\frac{H_j}{H_{0j}}\right)^{\rho_j} \left[\phi_{0j} \left(\frac{H_j}{H_{0j}}\right)^{\rho_j} + (1 - \phi_{0j}) \left(\frac{K_j}{K_{0j}}\right)^{\rho_j} \right]^{-1} \right)} \tilde{\lambda}_i^H H$$

In the case of the sector production function with the generalized human capital aggregate in equation (16), the marginal product with respect to skill type $L_k \in \{LL, LM, LH\}$ is defined as

$$MPV_i^{L_k} = \phi_{0i} \theta_{0ki} \frac{Y_i}{L_{ki}} \left(\frac{L_{ki}}{L_{0ki}}\right)^{\gamma_i} \Omega_i^{-1} \Gamma_i^{\frac{\rho_i - \gamma_i}{\gamma_i}}$$

where $\Omega_i = \left[\phi_{0i} \left[\theta_{0i}^{LH} \left(\frac{LH_i}{LH_{0i}}\right)^{\gamma_i} + \theta_{0i}^{LM} \left(\frac{LM_i}{LM_{0i}}\right)^{\gamma_i} + (1 - \theta_{0i}^{LH} - \theta_{0i}^{LM}) \left(\frac{LL_i}{LL_{0i}}\right)^{\gamma_i} \right]^{\frac{\rho_i}{\gamma_i}} + (1 - \phi_{0i}) K_i^\rho \right]$

and $\Gamma_i = \left[\theta_{0i}^{LH} \left(\frac{LH_i}{LH_{0i}}\right)^{\gamma_i} + \theta_{0i}^{LM} \left(\frac{LM_i}{LM_{0i}}\right)^{\gamma_i} + (1 - \theta_{0i}^{LH} - \theta_{0i}^{LM}) \left(\frac{LL_i}{LL_{0i}}\right)^{\gamma_i} \right]$.

This gives

$$L_{ki} = \frac{\phi_{0i} \theta_{0ki} Y_i \left(\frac{L_{ki}}{L_{0ki}}\right)^{\gamma_i} \Omega_i^{-1} \Gamma_i^{\frac{\rho_i - \gamma_i}{\gamma_i}}}{\sum_{j=1}^J \left(\phi_{0j} \theta_{0kj} Y_j \left(\frac{L_{kj}}{L_{0kj}}\right)^{\gamma_j} \Omega_j^{-1} \Gamma_j^{\frac{\rho_j - \gamma_j}{\gamma_j}} \right)} \tilde{\lambda}_i^{L_k} L_k$$

Appendix D: Human capital aggregate

It is important for this paper to ensure that the values of R_{World} and $R_{c,USA}$ can be compared across different specifications. The case of $\gamma \rightarrow 1$ (perfect substitutability) allows a verification. That is, if $\gamma \rightarrow 1$, the hypothetical gains obtained from the 1-level CES function should coincide with the results from the 2-level CES set up. The crucial element here is the consistency of the human capital aggregator. We define the explicitly normalized human capital aggregator as

$$\frac{H_{ci}}{H_{0ci}} = \left[\theta_{0ci}^{LL} \left(\frac{LL_{ci}}{LL_{0ci}} \right)^{\gamma_{ci}} + \theta_{0ci}^{LM} \left(\frac{LM_{ci}}{LM_{0ci}} \right)^{\gamma_{ci}} + \theta_{0ci}^{LH} \left(\frac{LH_{ci}}{LH_{0ci}} \right)^{\gamma_{ci}} \right]^{\frac{1}{\gamma_{ci}}}$$

Assuming that factors get paid their marginal product values the distribution parameters are defined as

$$\theta_{0ci}^{LH} = \frac{w_0^{LH} LH_{0ci}}{w_0^{LH} LH_{0ci} + w_0^{LM} LM_{0ci} + w_0^{LL} LL_{0ci}}$$

$$\theta_{0ci}^{LM} = \frac{w_{0ci}^{LM} LM_{0ci}}{w_{0ci}^{LH} LH_{0ci} + w_{0ci}^{LM} LM_{0ci} + w_{0ci}^{LL} LL_{0ci}}$$

$$\theta_{0ci}^{LL} = 1 - \theta_{0ci}^{LM} - \theta_{0ci}^{LH}$$

where 0 defines the point of normalization. We can rewrite the human capital aggregator as

$$\frac{H_{ci}}{H_{0ci}} = \theta_{0ci}^{LL} \times \left[\frac{LL_{ci}}{LL_{0ci}} + \frac{\theta_{0ci}^{LM}}{\theta_{0ci}^{LL}} \frac{LM_{ci}}{LM_{0ci}} + \frac{\theta_{0ci}^{LH}}{\theta_{0ci}^{LL}} \frac{LH_{ci}}{LH_{0ci}} \right]$$

where

$$\frac{\theta_{0ci}^{LH}}{\theta_{0ci}^{LL}} = \frac{w_{0ci}^{LH}}{w_{0ci}^{LL}} \frac{LH_{ci}^{1-\gamma} LH_{0ci}^{\gamma}}{LL_{ci}^{1-\gamma} LL_{0ci}^{\gamma}}$$

$$\frac{\theta_{0ci}^{LM}}{\theta_{0ci}^{LL}} = \frac{w_{0ci}^{LM}}{w_{0ci}^{LL}} \frac{LM_{ci}^{1-\gamma} LM_{0ci}^{\gamma}}{LL_{ci}^{1-\gamma} LL_{0ci}^{\gamma}}$$

and for $\gamma \rightarrow 1$

$$\frac{\theta_{0ci}^{LH}}{\theta_{0ci}^{LL}} = \frac{w_{0ci}^{LH}}{w_{0ci}^{LL}} \frac{LH_{0ci}^{\gamma}}{LL_{0ci}^{\gamma}}$$

$$\frac{\theta_{0ci}^{LM}}{\theta_{0ci}^{LL}} = \frac{w_{0ci}^{LM}}{w_{0ci}^{LL}} \frac{LM_{0ci}^{\gamma}}{LL_{0ci}^{\gamma}}$$

Note that growth or development accounting exercises usually work with the implicitly normalized human capital input at $H_{0ci} = LL_{0ci} = LM_{0ci} = LH_{0ci} = 1$,

$$H_{ci} = \theta_{0ci}^{LL} \times \left[LL_{ci} + \frac{\theta_{0ci}^{LM}}{\theta_{0ci}^{LL}} LM_{ci} + \frac{\theta_{0ci}^{LH}}{\theta_{0ci}^{LL}} LH_{ci} \right]$$

so that each skill type gets weighted by the relative wage (opposite to the cost share). Further it is often (implicitly) assumed that the quality of low skilled workers in sector i in country c , θ_{0ci}^{LL} , is fix across time or equal across countries and can therefore be ignored (see, e.g., Jones, 2014). In contrast, when we work with the 2-level CES function, we implicitly assume that θ_{0ci}^{LL} differs across sectors and countries. θ_{0ci}^{LL} thus also needs to be considered in the construction of the human capital aggregate that enters the 1-level CES specification.

In theory, as we set H_{0ci} (i.e., country-sector-specific), the relative efficiency levels are equal to

$$\frac{\theta_{0ci}^{LH}}{\theta_{0ci}^{LL}} = \frac{w_{0ci}^{LH} LH_{0ci}}{w_{0ci}^{LL} LL_{0ci}}$$

$$\frac{\theta_{0ci}^{LM}}{\theta_{0ci}^{LL}} = \frac{w_{0ci}^{LM} LM_{0ci}}{w_{0ci}^{LL} LL_{0ci}}$$

while

$$\theta_{0ci}^{LL} = \frac{w_{0ci}^{LL} LL_{0ci}}{w_{0ci}^{LH} LH_{0ci} + w_{0ci}^{LM} LM_{0ci} + w_{0ci}^{LL} LL_{0ci}} = \frac{1}{1 + \frac{w_{0ci}^{LH} LH_{0ci}}{w_{0ci}^{LL} LL_{0ci}} + \frac{w_{0ci}^{LM} LM_{0ci}}{w_{0ci}^{LL} LL_{0ci}}}$$

which for the implicitly normalized case $LL_0 = LM_0 = LH_0 = 1$ becomes

$$\frac{\theta_{0ci}^{LH}}{\theta_{0ci}^{LL}} = \frac{w_{0ci}^{LH}}{w_{0ci}^{LL}}$$

$$\frac{\theta_{0ci}^{LM}}{\theta_{0ci}^{LL}} = \frac{w_{0ci}^{LM}}{w_{0ci}^{LL}}$$

$$\theta_{0ci}^{LL} = \frac{1}{1 + \frac{w_{0ci}^{LH}}{w_{0ci}^{LL}} + \frac{w_{0ci}^{LM}}{w_{0ci}^{LL}}}$$

In practice, the output elasticities with regard to each labor skill type react to changes in the factor allocation across sectors to a degree that prevents convergence in the marginal product values even after numerous iterations. We work around this problem by artificially

reducing the degree of reallocation in each iteration. More precisely, we conduct the following steps:

1. Calculate the optimal sector share of the economy's total human capital H_c based on $\frac{H_{ci}}{H_{0ci}}$.
2. Obtain the ratio between the optimal sector share and the current sector share of H_c . As $\frac{H_{ci}}{H_{0ci}}$ can not be aggregated over sectors, we calculate $H_c = \sum_{i=1}^{12} H_{ci}$ using the relative efficiency levels of low-skilled shown in table 9 to obtain H_{ci} .³⁷
3. Close 70% of the gap between the optimal allocation to the current allocation.
4. Repeat steps (1)-(3) until the marginal product values converged across sectors.

This approach ensures that the hypothetical gains from reallocating the labor input(s) across sectors can be compared between the different specifications.

Table 9: Relative efficiency of low-skilled by sector

Abbr	Sector Name	(1)	(2)	(3)	(4)	(5)
		Sample				U.S.
		Mean	Std. Dev.	Min.	Max.	Mean
AtB	Agriculture	1.03	0.19	0.52	1.43	1.03
D	Manufacturing	0.97	0.08	0.65	1.11	0.89
F	Construction	1.03	0.09	0.72	1.25	1.00
G	Trade Service	0.97	0.09	0.70	1.19	0.98
H	Hotels & Restaurants	1.03	0.11	0.66	1.19	1.02
I	Transport & Storage	1.04	0.11	0.73	1.49	1.05
J	Finance Services	1.03	0.20	0.56	1.47	0.90
K	Business Services	0.91	0.11	0.63	1.40	0.77
L	Public & Social Services	1.03	0.15	0.74	1.38	1.02
M	Education	0.98	0.12	0.72	1.25	1.07
N	Health & Social Work	0.95	0.08	0.80	1.12	0.82
OtP	Other Services	0.97	0.10	0.75	1.20	1.00

Note: Columns 1-4 state sample summary statistics on normalized θ_{0ci} , i.e. the contribution of low skilled workers to the total human capital services in each sector (normalized with respect to each country's median θ_{0i}). Column 5 shows the normalized θ_{0i} for the U.S.

³⁷As Since, for $\gamma = 1$, the relative wages $\frac{w_{0ci}^{LH}}{w_{0ci}^{LL}}$ and $\frac{w_{0ci}^{LM}}{w_{0ci}^{LL}}$ are stable, θ_{0ci}^{LL} is also stable in this case. Hence, for $\gamma = 1$, we can use average values of w_{0ci}^{LL} , w_{0ci}^{LM} , and w_{0ci}^{LH} .

Appendix E: Estimation of sigma across sectors

The estimation of the unobservable elasticities of substitution parameter, ρ_i , in each sector i starts with the sector production functions in (8) and (10):

$$Y_i = C_i [\beta_i H_i^{\rho_i} + (1 - \beta_i) K_i^{\rho_i}]^{\frac{1}{\rho_i}} \quad \beta_i \in (0, 1)$$

and

$$Y_i = C_i \left[\beta_i [\alpha_i^{LH} (LH_i)^{\gamma_i} + \alpha_i^{LM} (LM_i)^{\gamma_i} + (1 - \alpha_i^{LM} - \alpha_i^{LH}) (LL_i)^{\gamma_i}]^{\frac{\rho_i}{\gamma_i}} + (1 - \beta) (K_i)^{\rho_i} \right]^{\frac{1}{\rho_i}}$$

This paper identifies the parameters based on a supply-side system approach using non-linear SUR (Klump et al., 2007; León-Ledesma et al., 2010; McAdam and Willman, 2018). This approach facilitates identification as it increases the degrees of freedom and considers both technology (via the underlying production function) and optimization behavior (via FOC equations). Aside from this, the use of level variables stress the long-run information in the data, which is particularly valuable given the rather short time dimension in the data ($T = 13$). Moreover, as shown by León-Ledesma et al. (2010), the supply-side system approach performs well even for relatively small samples and is not sensitive with respect to simultaneity bias.

In practice, we replace (8) and (10) by the normalized production functions (dropping the sector index for clarity):

$$Y_{ct} = Y_0 A_{0c} \left[\phi_0 \left(\frac{A_H H_{ct}}{H_0} \right)^\rho + (1 - \phi_0) \left(\frac{A_K K_{ct}}{K_0} \right)^\rho \right]^{\frac{1}{\rho}} \quad (24)$$

and

$$Y_{ct} = Y_0 A_{0c} \left[\phi_0 \Omega^{\frac{\rho}{\gamma}} + (1 - \phi_0) \left(\frac{A_K K_{ct}}{K_0} \right)^\rho \right]^{\frac{1}{\rho}} \quad (25)$$

where $\Omega = \left[\theta_0^{LH} \left(\frac{A_{LH} LH_{ct}}{LH_0} \right)^\gamma + \theta_0^{LM} \left(\frac{A_{LM} LM_{ct}}{LM_0} \right)^\gamma + (1 - \theta_0^{LH} - \theta_0^{LM}) \left(\frac{A_{LL} LL_{ct}}{LL_0} \right)^\gamma \right]$.

To improve the estimation, we make three changes compared to the reallocation exercise. First, we define the normalization points to be sector-specific. This choice is motivated by the necessity to estimate the substitution parameter of a global sector production function. Following the argument by Baccianti (2013), intuitively, observations that are represented

by the same production function should have the same normalization point. Consequently, we define $Y_0 = \bar{Y}\xi$, $K_0 = \bar{K}$, $H_0 = \bar{H}$, $LL_0 = \bar{LL}$, $LM_0 = \bar{LM}$, $LH_0 = \bar{LH}$, $\phi_0 = \bar{\phi}$, $\theta_0^{LH} = \bar{\theta}^{LH}$, $\theta_0^{LM} = \bar{\theta}^{LM}$ and $\theta_0^{LH} = (1 - \bar{\theta}^{LH} - \bar{\theta}^{LM})$, where the bar refers to the respective geometric average over countries and time. Note that we introduce two additional parameters ξ and A_{0c} . Analogously to Klump et al. (2007), ξ accounts for the non-linearity of the CES functions, which implies that the sample average of production need not exactly coincide with the level of production implied by the production function with sample averages of the right-hand variables. Unobserved heterogeneity across countries is captured by a set of country dummies, A_{0c} . As ξ and A_{0c} effectively enter the estimation as products, we replace them by the vector ξ_c .

The second change applies to the human capital input. As a consequence of redefining the normalization points, we calculate the human capital input in each sector i in country c as

$$\frac{H_{ci}}{\bar{H}_i} = \theta_0^{LL} \times \left[\frac{LL_{ci}}{\bar{LL}_i} + \frac{\bar{\theta}^{LM}}{\theta_0^{LL}} \frac{LM_{ci}}{\bar{LM}_i} + \frac{\bar{\theta}^{LH}}{\theta_0^{LL}} \frac{LH_{ci}}{\bar{LH}_i} \right] \quad (26)$$

Constructing the human capital input based on (26) also ensures comparability across countries within sectors.³⁸ Given H_{ci} , we calculate the average wage per efficiency unit as total labor compensation, nominated in constant 2005 U.S. dollars, divided by H_{ci} .³⁹

Third, we add a set of parameters, A_K , A_{LH} , A_{LM} and A_{LL} , which capture factor-directed technical change. This choice is motivated by the findings of Antras (2004), who shows that restricting the analysis to Hicks-neutral technological change necessarily biases the estimates of the elasticity of substitution between capital and labor towards one. The importance of factor-directed technical change in the estimation of the substitution parameter is also confirmed by the Monte-Carlo-study of León-Ledesma et al. (2015).

Taking logarithms, assuming that technical change for each factor k takes the form $e^{\nu_k(t-t_0)}$, and adding an error term μ , we estimate the two systems of equations for each sector individually.⁴⁰ First,

$$\log \left(\frac{Y_{ct}}{\bar{Y}} \right) = \log(\xi_c) + \frac{1}{\rho} \log \left[\bar{\phi} \left(\frac{e^{\nu_H(t-t_0)} H_{ct}}{\bar{H}} \right)^\rho + (1 - \bar{\phi}) \left(\frac{e^{\nu_K(t-t_0)} K_{ct}}{\bar{K}} \right)^\rho \right] + \mu_{ct} \quad (27)$$

³⁸This is different from the reallocation exercise, where comparability of the human capital input across sectors within countries is important.

³⁹More precisely, we estimate growth rates of real wages in constant national currency. We use these growth rates to extrapolate the wage in 2005 converted into U.S. dollar using the PWT8.1 exchange rate.

⁴⁰An important feature of the specifying technical change for each factor k as $e^{\nu_k(t-t_0)}$ is that $e^{\nu_k(t-t_0)} = 1$ for $t = t_0$. This ensures that the factor shares are not biased by the growth of factor efficiencies but simply equal to the distribution parameters at the common fixed point.

$$\log(w_{ct}) = \log\left(\frac{\bar{\phi}\bar{Y}}{\bar{H}}\right) + (1 - \rho)\log\left(\frac{Y_{ct}/\bar{Y}}{H_{ct}/\bar{H}}\right) + \rho(\log(\xi_c) + \nu_H(t - t_o)) + \mu_{ct} \quad (28)$$

Second,

$$\log\left(\frac{Y_{ct}}{\bar{Y}}\right) = \log(\xi_c) + \frac{1}{\rho}\log\left[(1 - \bar{\phi})\left(\frac{e^{\nu_K(t-t_o)}K_{ct}}{K_0}\right)^\rho + \bar{\phi}\Omega_{ct}^{\frac{\rho}{\gamma}}\right] + \mu_{ct} \quad (29)$$

$$\begin{aligned} \log(w_{ct}^{LH}) = \log\left(\frac{\bar{\phi}\bar{\theta}^{LH}\bar{Y}}{\bar{LH}}\right) + (1 - \rho)\log\left(\frac{Y_{ct}}{\bar{Y}}\right) + (\gamma - 1)\log\left(\frac{LH}{\bar{LH}}\right) \\ + \frac{\rho - \gamma}{\gamma}\Omega_{ct} + \rho\log(\xi_c) + \gamma(\nu_{LH}(t - t_o)) + \mu_{ct} \end{aligned} \quad (30)$$

$$\begin{aligned} \log(w_{ct}^{LM}) = \log\left(\frac{\bar{\phi}\bar{\theta}^{LM}\bar{Y}}{\bar{LM}}\right) + (1 - \rho)\log\left(\frac{Y_{ct}}{\bar{Y}}\right) + (\gamma - 1)\log\left(\frac{LM_{ct}}{\bar{LM}}\right) \\ + \frac{\rho - \gamma}{\gamma}\Omega_{ct} + \rho\log(\xi_c) + \gamma(\nu_{LM}(t - t_o)) + \mu_{ct} \end{aligned} \quad (31)$$

$$\begin{aligned} \log(w_{ct}^{LL}) = \log\left(\frac{\bar{\phi}\bar{\theta}^{LL}\bar{Y}}{\bar{LL}}\right) + (1 - \rho)\log\left(\frac{Y_{ct}}{\bar{Y}}\right) + (\gamma - 1)\log\left(\frac{LL_{ct}}{\bar{LL}}\right) \\ + \frac{\rho - \gamma}{\gamma}\Omega_{ct} + \rho\log(\xi_c) + \gamma(\nu_{LL}(t - t_o)) + \mu_{ct} \end{aligned} \quad (32)$$

where $\Omega_{ct} = \left[\bar{\theta}^{LH}\left(\frac{e^{\nu_{LH}(t-t_o)}LH_{ct}}{\bar{LH}}\right)^\gamma + \bar{\theta}^{LM}\left(\frac{e^{\nu_{LM}(t-t_o)}LM_{ct}}{\bar{LM}}\right)^\gamma + (1 - \bar{\theta}^{LH} - \bar{\theta}^{LM})\left(\frac{e^{\nu_{LL}(t-t_o)}LL_{ct}}{\bar{LL}}\right)^\gamma\right]$.

While the supply-side system approach does improve the identification of the substitution parameter considerably, estimation problems remain for some sectors when we estimate ρ and γ simultaneously. Against this background, we follow a grid search approach, as proposed by Henningsen and Henningsen (2012). Instead of estimating both parameters simultaneously, we impose plausible estimates of η and obtain corresponding estimates for ρ . Specifically, motivated by Ciccone and Peri (2005) and Mollick (2011), we impose $\eta \in [1.6, 2, 4]$.

As a final note, WIOD-SEA does not contain information on capital income. This impedes the separation of the pure profit (or the markup) component from the rest of non-labor revenue share in WIOD-SEA. As a consequence, we decide to not work with the FOC for the

capital input in the system estimation.