

Jeanjean, François

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Fair cost sharing in telecommunication industry, a virtuous circle

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Fair cost sharing in telecommunication industry, a virtuous circle.*

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Working paper

Abstract

This article studies the impact of the sharing of traffic costs between an Internet access provider and a content provider, both of which have a monopoly on their market. It shows that when the content provider charges consumers for content, cost sharing triggers a virtuous circle that incentivizes the content provider to reduce its traffic, which lowers prices for the end consumer and thus increases, not only the consumers surplus but also the profits of the ISP as well as to some extent, those of the content provider. When the content provider chooses an ad-business model, if it charges at ad-level, the cost sharing also favors consumers surplus and in a wide range of cases, the total surplus. If it charges at content level, the result is always favorable to consumers provided, however, that content provider is able to sufficiently monetize ads. The results are robust to different billing modes for traffic, pay-per-use or flat rate.

Key Words: *Telecommunication, fair share, cost sharing*

JEL Classification: D61, L11, L86

*Any opinions expressed here are those of the authors and not those of Orange. All errors are our own.

1 Introduction

Exploding Internet data generated by contents (video, series, etc.) entails increasing costs and requires regular and massive investments for Internet access providers while their revenues tend to stagnate. At the same time revenues of content providers increase exponentially much faster than their costs or investments. Currently, the cost of data routing is mainly borne by access providers who cannot, or very little, pass it on to content providers. In this context, the question of the sharing of these costs arises with increasing insistence.

This paper models the relationship between on the one hand, Content Providers (CP) and Internet Service Providers (ISP) and on the other hand between content providers and advertisers. It shows that, in most cases, sharing the cost between content and access providers increases demand for content, consumer surplus, welfare and even, in many cases content provider profits. This is at odd to the claim of some big tech companies that it is harmful to end users and the internet ecosystem.

At first glance, one could consider, like Big Tech companies, that content and traffic being complementary services, cost sharing has no effect since it only shifts the costs from one service to another such that the total bill for the end customer is not affected. However, this vision is incomplete because it does not take into account the incentives for content providers to invest in traffic reduction. Indeed, cost sharing gives content providers an incentive to invest to reduce the traffic generated by their content (data compression software, optimization of congestion management protocols, etc.) Cost sharing creates a virtuous circle. Less data-intensive contents reduces cost both for content and access providers. This leads to lower overall prices for consumers and to an increase in demand for content. This benefits both producers and consumers and increases welfare. This also reduces CO2 emissions due to redundant traffic.

The impact of cost sharing depends on the business model chosen by content provider that can choose to charge consumers or opt for a free service. When the free option is chosen, the content provider is remunerated through advertising. In that case, content and access are no more complementary and thereby, cost sharing has a positive impact even without relying on incentives to invest in reducing traffic. Consumers incur a disutility because of advertising which

is not necessarily correlated to content providers revenues. The document suggests two ways to charge advertisers. Ad-level billing or content-level billing. Content-level billing restores the complementarity between content and traffic because the advertising disutility incurred by consumers, as well as the revenues of content providers, is directly dependent on the demand for content. In the content-level billing model, revenues of content providers also depends on their ability to monetize advertising, consumer data usage for targeting for instance. In that case, the higher the ability to monetize advertising, the greater the impact of cost sharing.

The paper shows that cost sharing also improves the incentives to invest of the access providers because the higher demand for content increases the need for access provider to reduce traffic costs. Finally the paper shows that the virtuous circle holds for different traffic billing options, usage or flat rate.

The rest of the article is as follows: section 2 is a literature review, section 3 is the model, section 4 is an extension for Internet Service Providers investments, section 5 is an extension with flat rate models and section 6 concludes.

2 Literature review

Academic literature on the relationship between internet service providers and content providers initially focused on net neutrality. More specifically, on the possibility for internet service provider to discriminate between different content, allowing different priorities according to the sensitivity to time of the contents or by charging differently those contents, like Economides & Hermalin (2012), Bourreau *et al.* (2015) or Reggiani & Valletti (2016), and more recently Baranes & Vuong (2022) who study also the impacts on investment incentives, to give (non exhaustive) examples.

Only a few articles specifically study the issue of cost sharing between ISPs and content providers. Kamiyama (2014) or Im *et al.* (2016) find that a charge of content traffic by an ISP in competition with another ISP, can have a positive impact on CPs in some cases, however they did not study consumer surplus or welfare. Peitz & Schuett (2016) underline the positive impact of cost sharing on the allocation of traffic and contributes to reduce congestion. More recently,

Jullien & Bouvard (2022) developed a model to study the impact of cost sharing between ISP and CP. Using a flat-rate business model for ISPs, they showed that cost sharing increases consumer surplus provided the content provider is able to sufficiently monetize advertisements. They assumed relying on Anderson & Coate (2005) that content providers per consumer ad revenues are proportional (or at least positively related) to ad-induced consumer disutility. This paper builds on these insights but consider the advertising market explicitly and consider two ways for ad-billing: ad-level billing or content level billing. Content-level billing joins the work of Jullien & Bouvard (2022).

3 The model

I consider an Internet Service Provider, called "ISP" that provides internet traffic to end-users. The ISP incurs a cost c per unit of traffic. Traffic can be sold per unit (per megabyte) at price p or through a flat-rate subscription at price P or even by a combination of the two (two-part tariff).

A content provider, called "CP" delivers content to end-users via the ISP's network. I assume that each content generates β units of traffic. The PC can choose between two business models. A paid model where the CP sells each content at price p_c or an ad-based model. I assume that ISP sells traffic to CP at price a to share the cost of traffic ($0 \leq a \leq c$). I assume that consumer utility for content follows a quadratic utility such that $U_0 = \alpha q - \frac{q^2}{2}$ where q is the quantity of content consumed by end-users and α a constant term. I assume α is high enough to assure a positive equilibrium. In particular, I assume $\alpha > \beta c$

3.1 Paid model

I consider first the paid model case where ISP sells traffic per unit, without flat rate subscription.

3.1.1 basic paid model

In the basic paid model, there is no investment stage. The net utility of consumers is:

$$U = \alpha q - \frac{q^2}{2} - p_c q - p \beta q \quad (1)$$

Profit of ISP is:

$$\pi_{ISP} = (p - c + a) \beta q \quad (2)$$

Profit of CP writes:

$$\pi_{CP} = (p_c - \beta a) q \quad (3)$$

We can notice between equations 2 and 3, the transfer of $\beta a q$ from the CP to the ISP which constitutes the cost sharing of traffic.

equation 1 leads to the following demand function:

$$q = \alpha - p_c - p \beta \quad (4)$$

First order conditions for equations (2) and (3) provides prices p and p_c at equilibrium.

$$p = \frac{\alpha - \beta(3a - 2c)}{3\beta} \quad (5)$$

$$p_c = \frac{\alpha + \beta(3a - c)}{3} \quad (6)$$

Equations (5 and 6 combined with equation (4) lead to the quantity of content at equilibrium:

$$q = \frac{\alpha - \beta c}{3} \quad (7)$$

We can notice that the cost sharing a , as expected, increases the price of content and decreases the cost of traffic. However, the quantity of content q is not affected by the cost sharing because the decrease in traffic price compensates for the increase in content price such that the total price paid by end consumers, $p + p_c = \beta c/3$ is not affected by the cost sharing a . This

is the result of the complementary between content and traffic. Consumer surplus, profits and total welfare are directly related to q , as a result, they are not impacted by the cost sharing. $CS = \frac{q^2}{2}$; $\pi_{ISP} = \pi_{CP} = q^2$ and $W = \frac{5q^2}{2}$ (see proof in appendices)

This result does not longer hold if one consider the incentives of the content provider to decrease the traffic of the contents it provides.

3.1.2 paid model with incentives to decrease traffic

In the previous model, the price of content increases with cost sharing a while the amount of content paid by end users remains stable. As a result, as the equation (??) shows, the content provider's profit decreases with cost sharing, and the higher the traffic generated by the contents, the lower the profit. The content provider has therefore incentives to reduce the traffic β generated by its contents.

Let us add an investment term at equation (3) in content provider profit:

$$\pi_{CP} = (p_c - \beta a)q - k \frac{x^2}{2} \quad (8)$$

The term $k \frac{x^2}{2}$ represents the investment of the content provider in the reduction of traffic (compression algorithms or new data format that consumes less bit rate,...) where k is a positive constant. I assume that this investment reduces the traffic of the content β by x $\beta = \beta_0 - x$ with $x < \beta_0$

With the addition of the investment term, contrarily to the previous model, the quantity of content at equilibrium increases with the cost sharing a . The addition of the investment term requires an additional first order condition for the choice of investment x by the content provider. $\frac{\partial \pi_{PC}}{\partial x} = aq - kx$ which leads to:

$$x = \frac{aq}{k} \quad (9)$$

Replacing $\beta = \beta_0 - \frac{aq}{k}$ in equation (7) provides:

$$q = \frac{(\alpha - \beta_0 c)k}{3k - ac} \quad (10)$$

Equation (10) shows that an increase in cost sharing a increases the consumption of content.

The condition $x < \beta_0$ implies that $3k - ac > 0$.

Indeed, using equations (9) and (10), $3k - ac > (\alpha - \beta_0 c)a/\beta_0 > 0$.

Notice that $\lim_{k \rightarrow +\infty} x = 0$ and $\lim_{k \rightarrow +\infty} q = \frac{\alpha - \beta c}{3}$, we find again the result of the paid model without investment.

Investment in traffic reduction impacts consumer surplus, profits and total welfare. Their expressions are related to q following:

$$CS = \frac{q^2}{2}; \pi_{ISP} = q^2; \pi_{CP} = q^2 \left(1 - \frac{a^2}{2k}\right) \text{ and } W = \frac{5k - a^2}{2k} q^2 \text{ (see proof in appendices)}$$

Proposition 1. *i In paid model, cost sharing a increases the demand for content, the profit of the ISP, consumer surplus and Welfare.*

ii Cost sharing increases CP profit if $a \leq 2x/3$ which means CP profit reaches a maximum for $a = 2c/3$ see proof in appendices

Proposition 1 shows that the cost sharing provides incentives to invest in traffic reduction to the content provider. With investment, the cost sharing increases both consumer surplus, ISP profit and welfare. CP profit reaches a maximum for $a = 2c/3$.

3.2 ad-based model

I consider first ad-based model where ISP sells traffic per unit, without flat rate subscription and without investment stage.

3.2.1 ad-based model with ad-level billing.

The content provider can choose an ad-based model where the content provider sells advertising medium to advertisers. The results depends on how the content provider chooses to charge advertisers.

First I consider the case where content provider chooses to charge ads. In this case, content and ad markets are completely separate. Contents are free for end-users, however, I assume that advertising impacts negatively utility of consumer that lose λ per ad. In that case, net utility of consumer writes:

$$U = \alpha q - \frac{q^2}{2} - p\beta q - \lambda q_a \quad (11)$$

where q_a is the quantity of ads.

I assume the content provider sells advertising media to advertisers and earns p_a per ad.

ISP profit is akin to the paid model described in equation (2). Profit of content provider is slightly different:

$$\pi_{CP} = p_a q_a - \beta a q \quad (12)$$

Furthermore, I assume that the net utility of advertisers depends on the number of ads included in the contents following:

$$U_a = \delta q_a - \frac{q_a^2}{2} - p_a q_a \quad (13)$$

The maximisation of the utility of advertisers leads to: $q_a = \delta - p_a$. The maximisation of its profit leads the content provider to choose: $p_a = q_a$ and therefore, $q_a = p_a = \delta/2$. The maximisation of consumer utility leads consumers to consume $q = \alpha - p\beta$. The maximisation of ISP profit leads, as in paid model, to $p\beta = q - (a - c)\beta$ and finally, at equilibrium (see proof in the appendices):

$$q = \frac{\alpha + (a - c)\beta}{2} \quad (14)$$

$$p = \frac{\alpha - (a - c)\beta}{2\beta} \quad (15)$$

We can notice that the cost sharing a increases consumption of content and decreases the price of traffic.

However, the number of ads, q_a , and the negative effect of ads on consumer utility, λ , have no impact on the results. Indeed, in this case, the advertising market and the content market are completely separate, and an increase in the number of ads does not depend on the number

of content, so it does not change either the consumption of content or the price of traffic.

In this case, consumer surplus and ISP profit increase with cost sharing a . The profit of content provider decreases with cost sharing a . Advertiser surplus S_a does not depend on cost sharing a and welfare is maximum if the cost sharing $a = a^*$ where $a^* = \frac{\alpha - c\beta}{\beta} \geq 0$.

$CS = \frac{q^2 - \lambda\delta}{2}$; $\pi_{ISP} = q^2$; $\pi_{CP} = \frac{\delta^2}{4} - \beta a q$; $S_a = \frac{\delta^2}{8}$ and $W = \frac{12q^2 + 3\delta^2 - 4\delta\lambda - 8\beta a q}{8}$
(see proof in the appendices).

3.2.2 Ad-based model with content-level billing.

Now, I assume that the content provider does not charge for each ad, but rather for each content. In that case, the link between consumer bill (or more specifically here, disutility which acts like a bill) is restored. I assume that the negative effect of advertising on content consumers, λ , is proportional to the price paid by advertisers for each content, p_a . Let us denote r , the ratio of proportionality between p_a and λ , such that $p_a = r\lambda$.

in this case, net utility of end users becomes:

$$U = \alpha q - \frac{q^2}{2} - p\beta q - \lambda q \quad (16)$$

the profit of content provider:

$$\pi_{CP} = (p_a - \beta a)q \quad (17)$$

and utility of advertisers:

$$U_a = \delta q_a - \frac{q_a^2}{2} - p_a q \quad (18)$$

The profit of ISP is the same as the previous case.

replacing $p_a = r\lambda$ in equation (17) provides $\pi_{CP} = (r\lambda - \beta a)q$

The resolution of that case is very similar to the paid model and provides:

$$q = \frac{\alpha + ((\frac{r-1}{r})a - c)\beta}{3} \quad (19)$$

$$p = \frac{\alpha - ((\frac{2r+1}{r})a - 2c)\beta}{3\beta} \quad (20)$$

and

$$p_a = \frac{r\alpha + ((r+2)a - rc)\beta}{3} \quad (21)$$

We can notice that if $r = 1$ we find again the results of paid model. We can also notice that if $r > 1$, consumption of content increases and price of traffic decreases. As a consequence, consumer surplus, profits and welfare increase. On the contrary, if $r < 1$, without investment in traffic reduction, consumer surplus, profit and welfare decrease. These results are similar to Jullien & Bouvard (2022).

$$CS = \frac{q^2}{2}; \pi_{ISP} = q^2 \quad \pi_{CP} = rq^2; S_a = \frac{\delta^2}{2} - rq^2 - a\beta q \text{ and } W = \frac{3q^2 + \delta^2 - 2a\beta q}{2}$$

(see proof in the appendices).

Ad based model with $r = 1$ is very similar to paid model except for welfare where advertisers surplus is added. In this case, price p_a which is equal to λ acts like a hedonic price and replace the real price p_c . In the model where content and ad market are separate, an increase in the number of ads does not decrease content consumption. Content provider can increase the number of ads without fearing the negative consequences which are borne only by the end customers. As a result, content provider can internalize a part of advertisers utility (δ appears in the expression of its profit). In the model where content and ad markets are linked, an increase in ad number increases the volume of content and, therefore, content provider can no longer internalize advertisers utility. However, the profit of content provider depends on the parameter r that represents its ability to monetize ads.

The difference between the separate model and the linked model is how the content provider charges for advertisements. The relative values of r and δ in this regard are important. If r is relatively high compared to δ , the content provider earns more with the linked model, otherwise it is better off with the separate model.

3.3 Ad-based model with traffic reducing investment

As for the paid model, equation (8), we add an investment term to the expression of the content provider's profit. I develop the incentives to invest in the case of ad-level billing and in the case of content-level billing.

In both cases $\beta = \beta_0 - x$. The first order condition for the investment stage leads to the same result as paid model, i.e. equation (9), $x = aq/k$.

3.3.1 ad-level billing

In this case the profit of content provider becomes:

$$\pi_{CP} = p_a q_a - \beta a q - k \frac{x^2}{2} \quad (22)$$

This result combined with the results of previous subsection for ad-level billing leads to the expression of the quantity of content at equilibrium:

$$q_s = \frac{(\alpha + (a - c)\beta_0)k}{2k + a^2 - ac} \quad (23)$$

It is assumed that $k > (a^2 - ac)/2$ to assure a positive value to q_s . In this case, $CS_s = \frac{q_s^2 - \lambda\delta}{2}$, $\pi_{ISP_s} = q_s^2$, $S_a = \frac{\delta^2}{8}$, $\pi_{CP_s} = \frac{\delta^2}{4} - \beta_0 a q_s + \frac{a^2 q_s^2}{2k}$ and $W_s = \frac{(12k + 4a^2)q_s^2 + (3\delta^2 - 4\delta\lambda - 8\beta_0 a q_s)k}{8k}$

Proposition 2. *i. In ad-based model with ad-level billing, the possibility for content provider to invest in traffic reduction increases its profit.*

ii. Under investment, the cost sharing a increases demand for content, consumer surplus, ISP profit and decreases CP profit. Total welfare is maximum for a value of $a > 0$. (see proof in the appendices.)

3.3.2 content-level billing

In the case of linked markets it is:

$$\pi_{CP} = (p_a - \beta a)q - k \frac{x^2}{2} \quad (24)$$

Combined with the results of the ad-based model with content-level billing, the volume of content at equilibrium is written:

$$q_l = \frac{(\alpha + ((\frac{r-1}{r})a - c)\beta_0)k}{3k + (\frac{r-1}{r})a^2 - ac} \quad (25)$$

It is assumed that $k > (r(a^2 - ac) - a^2)/3r$ to assure a positive value to q_l

In this case, $CS_l = \frac{q_l^2}{2}$; $\pi_{ISPl} = q_l^2$; $\pi_{CPl} = \left(r - \frac{a^2}{2k}\right) q_l^2$; $S_a = \frac{\delta^2}{2} - (r - \frac{a^2}{k})q_l^2 - \beta_0 a q_l$
and $W_l = \frac{3q_l^2 + \delta^2 - 2\beta_0 a q_l}{2}$

Proposition 3. *i. In ad-based model with content-level billing, under investment, if $r > 1 - \frac{(\alpha - \beta_0 c)c}{3\beta_0 k}$, q_l , consumer surplus, ISP profit and total welfare are maximum for $a = c$. Otherwise, if $r \leq 1 - \frac{(\alpha - \beta_0 c)c}{3\beta_0 k}$, q_l , consumer surplus, ISP profit and total welfare are maximum for $a = 0$.*

ii. if $r > \frac{3\beta_0 k}{3\beta_0 k + (\alpha - \beta_0 c)c}$, there is a positive value of cost sharing $a > 0$ for which the content provider maximizes its profit.

(see proof in the appendices.)

4 Model with ISP cost reducing investment

Internet service provider may be encouraged to reduce costs. Does cost sharing a increases or not its investment? I assume that the cost of reducing marginal cost by x' is $k' \frac{x'^2}{2}$. Marginal cost $c = c_0 - x'$ where c_0 is the marginal cost without investment.

Profit of ISP writes:

$$\pi_{ISP} = (p - c_0 + x' + a)\beta q - k' \frac{x'^2}{2} \quad (26)$$

In that case, the result depends on the business model of the content provider.

4.1 Paid model with ISP cost reducing investment

Equations (1) and (3) remains valid.

I assume the timing is as follows: First investment stage, ISP chooses its investment x' , then ISP and CP choose their prices p and p_c or p_a simultaneously.

The first stage provides the value of investment $x' = \frac{\beta q}{k'}$. The second stage provides the demand of content:

$$q = \frac{(\alpha - c_0\beta)k'}{3k' - \beta^2} \quad (27)$$

(see proof in the appendices). In the paid model, without investment of content provider for traffic reduction, cost sharing a has no impact on ISP investment in cost reduction. In this case, like in the first section, complementarity between ISP and CP prevents the growth of q .

4.2 Ad-models with ISP cost reducing investment

4.2.1 ad-level billing

Equations (11), (12) and (13) remain available.

The resolution of the two stages provides:

$$q = \frac{(\alpha + (a - c_0)\beta)k'}{2k' - \beta^2} \quad (28)$$

(see proof in the appendices). In that case, an increase in cost sharing a increases the demand of content, ISP profit and consumer surplus.

4.2.2 content-level billing

Equations (16), (17) and (18) remain available.

The resolution of the two stages provides:

$$q = \frac{(\alpha + ((\frac{r-1}{r})a - c_0)\beta)k'}{3k' - \beta^2} \quad (29)$$

(see proof in the appendices). In that case, an increase in cost sharing a increases the demand of content, profits, consumer surplus and welfare when $r > 1$ and decreases it when $r < 1$.

Investment of ISP in cost reduction increases demand for content. It amplifies the impact of cost sharing a .

5 Models with flat rate subscriptions for traffic

In the previous sections traffic was billed on a pay-per-use basis. What does it change if the traffic is billed on a flat rate basis. In this case, consumers have a utility in subscribing to the ISP which does not depend only on the content they consume, but also on the fact of being a subscriber and being able to use the services of the ISP (mail, personal content, etc.)

This affects the complementarity between ISP and content provider. Cost sharing no longer impacts content price and ISP equally and oppositely. As a result, cost sharing has a positive impact on consumers even in the paid model.

5.1 Paid model with flat rate

5.1.1 Without traffic reducing investment

The timing of the game is as follows. First ISP sets the price of the flat rate subscription P , then the content provider sets the price of the content p_c .

I assume that the N subscribers are uniformly distributed according to their utility for ISP subscription and the variable l measures the decline in utility. In that case, the net utility of consumers is:

$$U = u + \alpha q - \frac{q^2}{2} - p_c q - P - l \quad (30)$$

Profit of ISP is:

$$\pi_{ISP} = (P - (c - a)\beta q)N \quad (31)$$

and CP profit:

$$\pi_{CP} = (p_c - a\beta)qN \quad (32)$$

The quantity of content chosen by consumers is given by equation (30) and the first order condition:

$$\frac{\partial U}{\partial q} = \alpha - q - p_c - \frac{\partial P}{\partial q} = 0 \text{ thus } \frac{\partial P}{\partial q} = \alpha - q - p_c$$

The number of subscribers, N is given by equation (30). N is the value of l for which $U = 0$. This corresponds to the subscriber who is indifferent between subscribing or not subscribing.

$$N = u + \alpha q - \frac{q^2}{2} - p_c q - P$$

Price P chosen by ISP is given by equation (31) and the first order condition:

$$\frac{\partial \pi_{ISP}}{\partial P} = N + (P - (c - a)\beta q) \frac{\partial N}{\partial P} = 0.$$

Price can be written: $P = N + (c - a)\beta q$ and $\frac{\partial P}{\partial q} = (c - a)\beta$

The choice of p_c by the content provider is given by equation (32) and the first order condition:

$$\frac{\partial \pi_{CP}}{\partial p_c} = qN - (p_c - a\beta)N = 0 \text{ which leads to } q = p_c - a\beta$$

These results lead to: $q = \frac{\alpha - c\beta}{2}$ and $p_c = \frac{\alpha + (2a - c)\beta}{2}$

In that case, as in the previous section, when the dynamic of investment is not considered, the cost sharing a increases the price of content but does not impact the number of content per user. The increase in the price of the content being compensated by the reduction in the price of the subscription.

$$P = \frac{8u + (\alpha - c\beta)(\alpha - (8a - 7c)\beta)}{16} \text{ and } N = \frac{8u + (\alpha - c\beta)^2}{16}$$

Notice that, as the number of content per subscriber, the number of subscribers is not impacted by cost sharing a . Therefore, the total number of content consumed $Q = Nq$ is not impacted by cost sharing.

Consumer surplus, profits and total welfare are also unaffected by cost sharing.

$$CS = \int_0^N U(l)dl = \int_0^N (N - l)dl = \frac{N^2}{2} \quad (33)$$

Profit of ISP and profit of CP at equilibrium, from equations (31) and (32) and the previous results are respectively:

$$\pi_{ISP} = N^2 \text{ and } \pi_{CP} = Nq^2$$

$$\text{Welfare at equilibrium is thus: } W = \frac{3N^2}{2} + Nq^2$$

5.1.2 With traffic reducing investment

As in the previous section, the traffic reducing investment gives a signal to content provider which can internalize the negative externality generated by its traffic.

In that case, the profit of the content provider becomes:

$$\pi_{CP} = (p_c - a(\beta_0 - x))qN - k\frac{x^2}{2} \quad (34)$$

Equations (30) and (31) remain valid, therefore, as $\beta = \beta_0 - x$, we can write:

$$q = \frac{\alpha - c(\beta_0 - x)}{2}$$

From equation (34), the first order condition for investment x becomes:

$$\frac{\partial \pi_{CP}}{\partial x} = aqN - kx = 0 \text{ which leads to: } x = \frac{aqN}{k}$$

This result with the previous one lead to:

$$q = \frac{(\alpha - c\beta_0)k}{(2k - aNc)} \quad (35)$$

The consumption of content is maximum for a positive value of cost sharing $a > 0$. Derivative of q with respect to a provides

$$\frac{\partial q}{\partial a} = \frac{Nqc}{2k - (3N - u)ac} \text{ and this expression is positive for } a = 0$$

5.2 ad-model with flat rate and ad-level billing

We first study the case where content and ad markets are separate, i.e. content provider sells availability for each ad rather than availability for a density of ad per content.

The timing of the game is as follows: First ISP sets the price of the flat rate' subscription P , then the content provider sets the price of the ad, p_a . In this case, net utility of consumers writes:

$$U = u + \alpha q - \frac{q^2}{2} - P - l - \lambda q_a \quad (36)$$

Profit of the ISP:

$$\pi_{ISP} = (P + (a - c)\beta q)N \quad (37)$$

Profit of the CP:

$$\pi_{CP} = p_a q_a - a\beta qN \quad (38)$$

and the utility of advertisers is as defined previously in equation (13).

the model is solved like in the previous subsection and leads to the following results: The contents consumed per subscriber: $q = \alpha + (a - c)\beta$,

The number of ads: $q_a = p_a = \frac{\delta}{2}$

The number of subscribers: $N = \frac{2u - \lambda\delta + (\alpha + (a - c)\beta)^2}{4}$

The consumption of content: $Q = Nq$

The price of subscription: $P = \frac{2u - \lambda\delta + \alpha^2 - 2(a - c)\beta\alpha - 3(a - c)^2\beta^2}{4}$

The profit of ISP: $\pi_{ISP} = N^2$

The profit of CP: $\pi_{CP} = \frac{\delta^2 - 4a\beta qN}{4}$

Consumer surplus: $CS = \frac{N^2}{2}$

Advertiser Surplus: $S_a = \frac{\delta^2}{8}$

Welfare: $\frac{3\delta^2 + 3(2u - \lambda\delta)^2 + (6\alpha + (4a - 6c)\beta)q + (3\alpha + (a - 3c)\beta)q^3}{8}$

Cost sharing a increases the subscriptions N , the consumption of contents Q , consumer surplus and welfare.

5.3 Ad-model with flat rate and content-level billing

Then we study the case where content and ad market are linked, i.e. content provider sells availability for a density of ad per content rather than availability for each ad.

In this case, net utility of consumers is:

$$U = u + \alpha q - \frac{q^2}{2} - P - l - \lambda q \quad (39)$$

Profit of the ISP:

$$\pi_{ISP} = (P + (a - c)\beta q)N \quad (40)$$

Profit of the CP:

$$\pi_{CP} = (p_a - a\beta)qN \quad (41)$$

and the utility of advertisers is as defined previously in equation (18).

The model is solved like in the previous subsections and leads to the following results:

The contents consumed per subscriber: $q = \frac{\alpha + ((\frac{r-1}{r})a - c)\beta}{2},$

The price of ads: $p_a = \frac{\alpha r + ((r+1)a - rc)\beta}{2}$

The number of subscribers: $N = \frac{2u + q^2}{4}$

The consumption of content: $Q = Nq$

The price of subscription: $P = \frac{8u + (\alpha - ((\frac{7r+1}{r})a - 7c)\beta)(\alpha + ((\frac{r-1}{r})a - c)\beta)}{16}$

The profit of ISP: $\pi_{ISP} = N^2$

The profit of CP: $\pi_{CP} = rq^2N$

Consumer surplus: $CS = \frac{N^2}{2}$

Advertiser Surplus: $S_a = \frac{\delta^2}{2} - p_a q$

Welfare: $\frac{3N^2 + \delta^2}{2} + rq^2N - p_a q$

In this case, provided $r > 1$, cost sharing a increases the consumption of content per user, the number of subscriptions and consumer surplus but decreases advertiser surplus. Its impact on welfare is ambiguous and depends on the value of the parameters. In particular, u , the content-independent utility of subscribing, increases the probability that cost-sharing a increases welfare.

6 Conclusion

The content provider can choose between a paid or an ad business model. Concerning the paid business model, the sharing of the costs incurred by service providers in carrying traffic to end consumers, between content providers and Internet service providers, has a positive impact on consumer surplus because it creates an incentive for the content provider to invest in the optimization of the traffic of its contents. The fact that the content provider shares traffic costs encourages it to invest in limiting the traffic of its content. As content and traffic are complementary. Cost sharing reduces the price of traffic but increases the price of content in return, so that the resulting effect would be nil for the end consumer if the content provider were not encouraged to optimize traffic. In the paid model, cost sharing increases content demand, consumer surplus, welfare, ISP profit, and even, up to a certain level, content provider profit.

Concerning the ad business model, this paper proposes two alternatives. The content provider may charge at ad level or at content level. In both cases, cost sharing increases demand for content, consumer surplus and ISP profit. The ad model vanishes the complementarity between content and traffic. Cost sharing decreases the price of traffic but this decrease is not compensated by an increase in content price. If content provider charges at ad level, cost sharing decreases content provider profit and if it charges at content level, cost sharing increases its profit. Advertising-level billing creates a separation between content and ad markets. More content does not necessarily mean more ads and more revenues for the content provider. Content-level billing re-establishes the link between the two markets. The lack of link between the markets prevents content provider profit to compensate for the cost sharing. In both cases welfare is maximized for a positive level of cost sharing. Under content-level billing, results can be reversed if the content provider is unable to monetize advertising at an equal or higher level

of the paid model. In both cases, the results are robust to incentives to invest.

All the results are robust to different billing models, usage billing or flat rate. Furthermore, cost sharing gives also incentives to the internet service provider to invest to reduce marginal costs which amplifies the results, therefore, regulation authorities should implement cost sharing between content and access providers.

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7 Appendices

7.1 Paid model without investment

$$CS = (\alpha - p_c - p\beta)q - \frac{q^2}{2} = q^2 - \frac{q^2}{2} = \frac{q^2}{2}$$

from equation (4) $\alpha - p_c - p\beta = q$, therefore, $CS = \frac{q^2}{2}$

$\pi_{ISP} = (p - c + a)\beta q$ using equations (5) and (7), $p - c + a = q/\beta$, therefore $\pi_{ISP} = q^2$

$\pi_{CP} = (p_c - a\beta)q$. Using equations (6) and (7), $p_c - \beta a = q$, therefore $\pi_{CP} = q^2$

$$W = CS + \pi_{CP} + \pi_{ISP} = \frac{5q^2}{2}$$

7.2 Paid model with investment in traffic reduction

The calculations of CS and π_{ISP} are the same as the Paid model without investment, however, the value of q is higher.

From equation (8), $\pi_{CP} = (p_c - \beta_0 a + xa)q - k\frac{x^2}{2}$

The choice of p_c by the content provider is given by the first order condition:

$$\frac{\partial \pi_{CP}}{\partial p_c} = q - p_c - \beta_0 a + xa = 0$$

Using equations (9) and (10), $\pi_{CP} = q^2 \left(1 - \frac{a^2}{2k}\right)$

$$W = CS + \pi_{ISP} + \pi_{CP} = q^2 \left(\frac{5k - a^2}{2k}\right)$$

7.3 Proof of Proposition 1

i: $q^2 = \frac{(\alpha - \beta_0 c)^2 k^2}{(3k - ac)^2}$ and $\frac{\partial q^2}{\partial a} = \frac{2c}{(3k - ac)} q^2$

Therefore, $\frac{\partial CS}{\partial a} \geq 0$ and $\frac{\partial \pi_{ISP}}{\partial a} \geq 0$

$$\frac{\partial W}{\partial a} = \left(\frac{5c - 3a}{3k - ac}\right) q^2$$

we know that $c \geq a$, therefore $5c \geq 3a$ and $\frac{\partial W}{\partial a} \geq 0$

ii: For π_{CP} We have: $\frac{\partial \pi_{CP}}{\partial a} = \left(\frac{2c - 3a}{3k - ac}\right) q^2$, therefore, $\frac{\partial \pi_{CP}}{\partial a} \geq 0$ if $a \leq 2c/3$

7.4 ad-model with ad-level billing

Maximisation of consumer net utility yields:

$$\frac{\partial U}{\partial q} = \alpha - q - p\beta = 0 \text{ which leads to}$$

$$q = \alpha - p\beta$$

Maximisation of ISP profit is the same as the paid model and leads to $p\beta = q - (a - c)\beta$ and

finally

$$q = \frac{\alpha + (a - c)\beta}{2}$$

Maximisation of advertiser utility yields:

$$\frac{\partial U_a}{\partial q_a} = \delta - q_a - p_a = 0 \text{ which leads to: } q_a = \delta - p_a$$

Maximisation of CP profit yields: $\frac{\partial \pi_{CP}}{\partial p_a} = q_a - p_a = 0$ and leads to $p_a = q_a = \delta/2$. There is

no link between q and q_a .

$$\text{Consumer surplus writes: } CS = (\alpha - p\beta)q - \frac{q^2}{2} - \lambda q_a = \frac{q^2 - \lambda\delta}{2}$$

$$\text{ISP profit: } \pi_{ISP} = (\alpha - c + a)\beta q = q^2$$

$$\text{CP profit: } \pi_{CP} = \frac{\delta^2}{4} - \beta a q$$

$$\text{Advertiser surplus: } S_a = \delta^2/2 - \delta^2/8 - \delta^2/4 = \delta^2/8$$

$$\text{Welfare: } W = CS + \pi_{ISP} + \pi_{CP} + S_a = \frac{12q^2 + 3\delta^2 - 4\delta\lambda - 8\beta a q}{8}$$

q is increasing in a , therefore $\pi_{ISP} = q^2$ and CS that depend on q^2 are also increasing. S_a

does not depend on a and π_{CP} is decreasing in a .

The derivative of w according to a is:

$$\frac{\partial W}{\partial a} = \frac{(q - a\beta)\beta}{2}$$

$$W \text{ reach a maximum for } a^* = \frac{\alpha - c\beta}{\beta}$$

$a^* \geq 0$ because, on the one hand, $\alpha + (a - c)\beta \geq 0$ to assure that $q \geq 0$ and on the other hand $\alpha - (a - c)\beta \geq 0$ to assure the price of traffic $p \geq 0$. As a result, the sum of these two expressions yields $\alpha + (a - c)\beta + \alpha - (a - c)\beta \geq 0$ or $\alpha - c\beta \geq 0$

7.5 ad-model with content-level billing

Maximisation of consumer net utility yields:

$$\frac{\partial U}{\partial q} = \alpha - q - p\beta - \lambda = 0 \text{ which leads to}$$

$$q = \alpha - p\beta - \lambda$$

Maximisation of ISP profit is the same as the paid model and leads to $p\beta = q - (a - c)\beta$
 Maximisation of CP profit yields: $\frac{\partial \pi_{CP}}{\partial p_a} = q - \frac{p_a - \beta a}{r} = 0$ and leads to $p_a = qr + \beta a$ or
 $\lambda = q + \frac{\beta a}{r}$.

Combining these results yields:

$$q = \alpha - q + (a - c)\beta - q - \frac{\beta a}{r} \text{ and finally } q = \frac{\alpha + ((\frac{r-1}{r})a - c)\beta}{3} \text{ which is equation (19).}$$

Replacing equation (19) in $p\beta = q - (a - c)\beta$ provides equation (20) and replacing equation (19) in $p_a = qr + \beta a$ provides equation (21)

$$\text{Consumer surplus writes: } CS = (\alpha - p\beta - \lambda)q - \frac{q^2}{2} = q^2 - \frac{q^2}{2} = \frac{q^2}{2}$$

$$\text{ISP profit: } \pi_{ISP} = (p - c + a)\beta q = q^2$$

$$\text{CP profit: } \pi_{CP} = (p_a - \beta a)q = rq^2$$

$$\text{Maximisation of advertisers utility writes: } \frac{\partial U_a}{\partial q_a} = \delta - q_a = 0 \text{ which yields } q_a = \delta$$

$$\text{Advertisers surplus: } S_a = \frac{\delta^2}{2} - p_a q$$

$$S_a = \frac{\delta^2}{2} - rq^2 - a\beta q$$

$$\text{As a result, welfare is written: } W = CS + \pi_{ISP} + \pi_{CP} + S_a$$

$$W = \frac{3q^2 + \delta^2 - 2a\beta q}{2}.$$

The derivative of q is $\frac{\partial q}{\partial a} = \frac{(r-1)\beta}{3r}$. This means q is strictly increasing in a if $r > 1$ and strictly decreasing in a if $r < 1$.

Consumer surplus and profits depend on q^2 .

$$\text{We can write: } \frac{\partial CS}{\partial a} = q \frac{\partial q}{\partial a}; \frac{\partial \pi_{ISP}}{\partial a} = 2q \frac{\partial q}{\partial a}; \frac{\partial \pi_{CP}}{\partial a} = 2rq \frac{\partial q}{\partial a}$$

The sign of these expressions depend on the sign of $\frac{\partial q}{\partial a}$. As a result, consumer surplus, the profits and welfare are all strictly increasing if $r > 1$ and decreasing if $r < 1$.

$$\text{Advertiser surplus decreases in a: } \frac{\partial S_a}{\partial a} = -(2r + a\beta) \frac{\partial q}{\partial a},$$

Welfare is maximum for $a = r(\alpha - \beta c)$ if $r(\alpha - \beta c) \leq c$ and for $a = c$ otherwise.

$$\frac{\partial W}{\partial a} = (3q - a\beta) \frac{\partial q}{\partial a} \text{ and } \frac{\partial W}{\partial a} = 0 \text{ if } a = r(\alpha - \beta c)$$

7.6 Ad-model with ad-level billing under investment

In this case, the expression of Consumer surplus, ISP profit and advertisers surplus are derived from the expression without investment changing q by q_s .

$$CS_s = \frac{q_s^2 - \lambda\delta}{2}$$

$$\pi_{ISP_s} = q_s^2$$

$$S_a = \frac{\delta^2}{8}$$

The expression of CP profit takes the term $-kx^2/2$ into account and provides:

$$\pi_{CP_s} = \frac{\delta^2}{4} - \beta_0 a q_s + \frac{a^2 q_s^2}{2k}$$

The welfare is thus:

$$W_s = \frac{(12k + 4a^2)q_s^2 + (3\delta^2 - 4\delta\lambda - 8\beta_0 a q_s)k}{8k}$$

7.7 proof of proposition 2

i. The growth of the profit of the content provider due to investment is:

$$\frac{(\alpha + (a - c)\beta_0)k}{2k + a^2 - ac} - \frac{(\alpha + (a - c)\beta_0)}{2} = \frac{(c - a)a(\alpha + (a - c)\beta_0)}{2(2k + a^2 - ac)} = \frac{(c - a)a}{2k} q_s$$

We know that $c \geq a$, therefore, this expression is positive. Investment in traffic reduction increases the volume of content.

ii. The derivative of q_s according to cost sharing a is:

$$\frac{\partial q_s}{\partial a} = \frac{\beta_0 k(2k + a^2 - ac) - (\alpha + (a - c)\beta_0)k(2a - c)}{(2k + a^2 - ac)^2}$$

which can be rewritten:

$$\frac{\partial q_s}{\partial a} = \frac{\beta_0 k - (2a - c)q_s}{(2k + a^2 - ac)}$$

We know that $x \leq \beta_0$, because β must remain positive, therefore, using equation (9), $\beta_0 k - (2a - c)q_s \geq (c - a)q_s \geq 0$ because $c \geq a$.

As a result, $\frac{\partial q_s}{\partial a} \geq 0$.

$\frac{\partial \pi_{CP}}{\partial a} = \left(\frac{a q_s}{k} - \beta_0\right) \left(q + a \frac{\partial q}{\partial a}\right)$. This expression is negative because $\beta_0 \geq \frac{a q_s}{k}$. As a result, a increases demand for content, profit of ISP, consumer surplus and decreases profit of CP.

$$\frac{\partial W}{\partial a} = \left(\frac{(3k + a^2)}{k} q_s - a\beta_0\right) \frac{\partial q_s}{\partial a} + \left(\frac{a q_s}{k} - \beta_0\right) q_s.$$

$$\frac{\partial W}{\partial a}(0) = \frac{\beta_0 k + 3c q_s}{2k} q_s \geq 0.$$

Welfare is increasing for $a = 0$ which means that welfare is maximum for $a > 0$.

7.8 Ad-model with content-level billing under investment

The expressions of consumer surplus and ISP profit are the same as the case without investment replacing q by q_l .

$$CS_l = \frac{q_l^2}{2}; \pi_{ISPl} = q_l^2;$$

The expression of CP profit and advertiser surplus are changed and become respectively:

$$\pi_{CPl} = (r - \frac{a^2}{2k})q_l^2 \text{ and } S_a = \frac{q_l^2}{2} - (r - \frac{a^2}{k})q_l^2 - a\beta q_l$$

and welfare becomes:

$$W_l = \frac{3q_l^2 + \delta^2 - 2\beta_0 a q_l}{2}$$

7.9 proof of proposition 3

i.

Let us denote $\gamma = \frac{r-1}{r}$.

With this notation $q_l = \frac{(\alpha + (\gamma a - c)\beta_0)k}{3k + (\gamma a - c)a}$

The derivative of q_l is

$$\frac{\partial q_l}{\partial a} = \frac{\gamma\beta_0 k - (2\gamma a - c)q_l}{3k + (\gamma a - c)a}$$

If $r > 1$, $\gamma > 0$

In that case, we know that $\beta_0 > a q_l / k$, thus $\gamma\beta_0 k - (2\gamma a - c)q_l > (c - \gamma a)q_l$.

$(c - \gamma a)q_l \geq 0$ because $c \geq a$ and $\gamma < 1$, therefore $\frac{\partial q_l}{\partial a} \geq 0$ and $q_l(a)$ is increasing and reaches its maximum for $a = c$

If $r < 1$, $\gamma < 0$, and if $\exists a \in [0, c]$ such that $\frac{\partial q_l}{\partial a} = 0$, then there is a minimum $\underline{a} \in [0, c]$ such that $\forall a \in [a, c]$, $q_l(a) \geq q_l(\underline{a})$

The second order condition shows that \underline{a} is a minimum.

$$\frac{\partial^2 q_l}{\partial a^2} = -2 \frac{\gamma q + (2\gamma a - c) \frac{\partial q_l}{\partial a}}{3k + (\gamma a - c)a}$$

If $\frac{\partial q_l}{\partial a} = 0$, then $\frac{\partial^2 q_l}{\partial a^2} = \frac{-2\gamma}{3k + (\gamma a - c)a}$ and if $\gamma < 0$, $\frac{\partial^2 q_l}{\partial a^2} > 0$

There is thus a minimum $\underline{a} \in [0, c]$.

This means that the value that maximizes q_l is $a = c$ or $a = 0$.

$$\text{The difference } q_l(c) - q_l(0) = \frac{(3c\beta_0 k - (\alpha - \beta_0 c)c^2)\gamma + (\alpha - \beta_0 c)c^2}{3(3k + (\gamma - 1)c^2)}$$

q_l reaches its maximum for $a = c$ if $q_l(c) > q_l(0)$, or $0 > \gamma > \frac{-(\alpha - \beta_0 c)c}{3\beta_0 k - (\alpha - \beta_0 c)c}$.

This means $r > 1 - \frac{(\alpha - \beta_0 c)c}{3\beta_0 k}$

and q_l reaches its maximum for $a = 0$ if $0 > \frac{-(\alpha - \beta_0 c)c}{3\beta_0 k - (\alpha - \beta_0 c)c} > \gamma$.

This means $r < 1 - \frac{(\alpha - \beta_0 c)c}{3\beta_0 k}$.

If $r = 1 - \frac{(\alpha - \beta_0 c)c}{3\beta_0 k}$, then $q_l(c) = q_l(a)$, cost sharing a does not increase q_l

The increase in consumer surplus, profits and welfare depends on the increase in q_l .

Remark: This requires $3\beta_0 k - (\alpha - \beta_0 c)c > 0$, however, if $3\beta_0 k - (\alpha - \beta_0 c)c < 0$, then $\gamma > \frac{-(\alpha - \beta_0 c)c}{3\beta_0 k - (\alpha - \beta_0 c)c} > 0$, therefore q_l is maximum for $a = c$

ii. The derivative of π_{CP} is:

$$\frac{\partial \pi_{CP}}{\partial a} = \frac{(2k - (1 - \gamma)a^2)\gamma\beta_0 k q_l - ((3(1 + \gamma)a - 2c)k - \gamma(1 - \gamma)a^3)q_l^2}{(1 - \gamma)(3k + (\gamma a - c)a)k}$$

For $a = 0$, $\frac{\partial \pi_{CP}}{\partial a}(0) = \frac{2\gamma\beta_0 k + 2cq_l(0)}{3(1 - \gamma)k} q_l(0)$. This expression is positive if $\gamma > -\frac{(\alpha - \beta_0 c)c}{3\beta_0 k}$

or $r > \frac{3\beta_0 k}{3\beta_0 k + (\alpha - \beta_0 c)c}$. This means that π_{CP} is increasing for $a = 0$, therefore, the value of a that maximizes π_{CP} is higher than 0.

7.10 models with investment of the ISP in cost reduction

Equation (26) provides the following first order condition for the investment stage:

$$\frac{\partial \pi_{ISP}}{\partial x} = \beta q - k' x' = 0 \text{ which leads to } x' = \frac{\beta q}{k'}$$

replacing x' in equation (26) yields:

$$\pi_{ISP} = (p - c_0 + a)\beta q + \frac{\beta^2 q^2}{2k'}$$

The first order condition for the second stage is:

$$\frac{\partial \pi_{ISP}}{\partial p} = \beta q - (p - c_0 + a)\beta^2 - \frac{\beta^3 q}{k'} = 0 \text{ which leads to } p\beta = \left(\frac{k' - \beta^2}{k'}\right)q - (a - c_0)\beta$$

In the paid model, we have $q = \alpha - p\beta - p_c$ and $p_c = q + a\beta$. combined with the first order conditions, this leads to equation (27).

In the ad-model with ad-level billing, we have $q = \alpha - p\beta$. Combined with the first order conditions this leads to equation (28).

In the ad-model with content-level billing, we have $q = \alpha - p\beta - p_a$ and $p_a = q + \beta a$. Combined with the first order conditions this leads to equation (29).