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## Complementarity Effect of Corporate Advertising in a Multimedia World: A Comparison of Online Advertising and Mass Media Advertising

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# **Complementarity Effect of Corporate Advertising in a Multimedia World: A Comparison of Online Advertising and Mass Media Advertising**

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## **Abstract**

This study analyzes the type of advertisements firms pursue when they engage in Cournot competition, especially when goods are complementary in a multimedia environment. Advertisements are classified into search-linked advertisements for online advertisements and TV commercials for mass media advertisements. Which one should the firm choose? This study also analyzes how corporate advertising strategies affect social welfare and provides insight into the role of advertising in multimedia. A firm's advertisement selection depends on the degree of complementarity and differentiation between advertisements. This analysis also focuses on the advertising strategies of duopoly firms in an extended model, such as hardware firms that are complementary to software products. In that case, both choose mass media advertising when the differentiation is moderate, and this choice raises both aggregate surplus and producer surplus. When advertising complementarity is high for both mass media and online advertising, firms choose different types of advertisements, but social welfare is low. This result is one of the considerations of a firm's sales strategy, how on using advertisements to increase demand and maximize profits.

**Keywords:** Online media advertising, Mass media advertising, Complementary relationship, Advertising Strategy, Duopoly model

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## **Introduction**

This study analyzes how firms choose to advertise when goods are complementary. The growth of online advertising is not limited to niche market advertising, but is embedded in the fact that firms focus on the targeting effect that allows them to provide advertising to specific consumers at a low costs. Recent research has focused on the relationship between traditional and online advertising, factors contributing to the growth of online advertising, and the high targeting effectiveness of online advertising. Aislabie and Tisdell (1988) showed that a firm's sales strategy rotates demand through advertising and product concepts. Johnson and Myatt (2006) presented a new taxonomy of advertising distinguishing between mass media advertising that targets "mass markets" and online advertising that focuses on specific needs. Tremblay et al. (2013) analyze the advertisements of competing Bertrand–Cournot firms based on the taxonomy of advertisements developed by Aislabie and Tisdell's (1988) and Johnson and Myatt's (2006). They found that in a Bertrand–Cournot competition, firms have different selling strategies: firms competing by selling quantities prefer mass advertisements, in contrast, firms competing on price select online advertisements suitable for niche markets. Park, et al. (2016) observe that firms engage in excessive advertising in Cournot competition.

For example, that even if coffee and milk do not have a strong complementary relationship, consumers purchasing behavior is influenced if advertising emphasizes that the complementary use of these two goods increases satisfaction. If there is a complementary revenue relationship, such as the hardware-software relationship in the home video game console market or the relationship between apps and smartphones, then which advertising is more profitable for hardware vendors, online or mass media advertising? This paper includes targeting effects in Johnson and Myatt's (2006) classification and analyze firms' choices in terms of mass media advertising, which has low targeting effects but high product awareness, while online advertising

has high targeting effects but generates low product awareness.

There are several studies on complementary goods; however, we did not find any work examining the impact of web and mass media advertising on firms' choices, as they have distinct, complementary outcomes. Therefore, this study analyzes how firms choose advertisements when goods are complementary. The method of analysis is a two-stage game of the behavior of complementary, bilateral monopolistic firms in the basic model. In the extended model, we use a duopoly model to analyze advertising choices in a three-stage game where two firms with complementary goods, such as hard and soft good, differentiate themselves through advertising to maximize their profits.

The analysis shows that the advertising choices of firms selling complementary goods depend on the strategic influence of advertising, complementarity. However, in the extended model that analyzes duopoly market, the advertising strategies of hardware firms that are complementary to soft goods depend on the degree of differentiation between online and mass media advertising; when the degree of differentiation is moderate, both firms choose mass media advertising. In this case, both the total surplus and producer surplus are high. For example, in the game device market, Sony and Nintendo mainly use mass media advertising, and increase demand. In addition, to choosing both mass media and online advertising, firms choose different types of promotion when complementarity is high. This firm choice reduces social welfare gains due to excessive advertising.

The remainder of this paper is organized as follows. Section 2 reviews the relevant previous studies. Section 3 describes the assumptions and the model setup. Sections 4 and 5 examine three advertising strategies to determine the optimal advertising strategy and search for the optimal advertising strategies to determine the optimal strategy and explore an optimal advertising choice for both firms in equilibrium. Section 6 discusses the appropriate advertising strategy for firms

under the extended model, including social welfare. Finally, Section 7 concludes the paper.

## **Literature Review**

Prior studies on advertising relevant to this research are as follows. Aislabie and Tisdell (1988) showed the relationship between advertising and demand changes, wherein firms' sales strategies use advertising to capture new demand and cause demand turnover. Katz and Shapiro (1994) focused on network externalities, analyzing hardware, software, other communication network goods, corporate behavior, and public and private institutions in markets where network effects are profitable. The development of communication network technology has given rise to online advertising, highlighting the differences from traditional advertising effects. Johnson and Myatt (2006) discussed the differences between traditional and online advertising, distinguishing between mass media advertising for "mass markets" and online advertising for "specific needs." Tremblay et al. (2013) proposed a classification of advertising based on Aislabie and Tisdell's (1988) and Johnson and Myatt's (2006) taxonomy, wherein they analyzed a sales strategy and advertising choices of Bertrand–Cournot competitors and found that different sales strategies lead to distinct advertising choices.

As this study analyzes the advertising strategies of firms with complementary goods, it is similar to the vertical relationship between producers and retailers, but differs from previous studies in that these firms are producers who need each other's goods. However, it is important to analyze advertising expenditures between producers and retailers in a mutually complementary or vertical relationship rather than a direct complementary relationship between goods, and to consider the external effects of advertising and the problems caused by these outcomes. For studies on complementarities, Lee et al. (2010) theoretically develop and empirically validated the concept that dynamic complementarities between software product markets can increase

industry hyper-competition. Ennen and Richter (2010) provide a comprehensive review of empirical studies published in leading journals in management, economics, and related fields from 1988 to 2008 on the subject of complementarities. In addition, if retailers in vertical relationships invest in advertising to lower their cost of sales, retailers lower their marginal cost of sales because consumers have access to information about goods. However, if producers are unable to accurately determine the cost of retailers' advertising investments, they will have problems collecting franchise and other fees; however, Zhang et al. (2020) showed that manufacturers and retailers should invest in information advertising between supply chains; Aust and Buscher (2014) found that vertical and cooperative advertising taken by supply chains in horizontal relationships. Karray et al. (2022) analyzed the impact of cooperative advertising between firms in monopolistic relationships and found that if retailer advertising has a positive impact on long-term sales, manufacturers should not only advertise on their own but also provide coordinated advertising support to retailers. We showed that manufacturers and retailers should engage in cooperative advertising, and advertising has a complementary effect when manufacturers and retailers engage in cooperative advertising, and if retailers' sales increase due to advertising, manufacturers' sales will also increase.

To understand the impact of online advertising, we referred to the following studies. Landry (2022), who studied the economic impact of search-linked advertising, a representative of web advertising, shows that relentless advertising by monopolistic firms using online advertising lowers prices. Edelman, et al. (2007) analyzed the pricing mechanism of search-linked advertising; Chen and He (2011) analyzed the impact of paid search engine listings on the output and profits of advertiser firms; de Corniere (2016) analyzed the extent to which search-linked advertising improves the matching of consumers with goods. Bergemann and Bonatti (2011) analyzed the relationship between the increase in online media dissemination and advertising prices.

### **3. Model**

We believe that advertising influences consumers buying behavior, and if firms provide information on the differences and complementarities of goods in their advertising, consumers will base their purchase decisions not only on price but also on the characteristics and utility of the goods. Therefore, we posit that goods with low advertising effectiveness have low advertising costs and demand effectiveness. Goods with high advertising effectiveness are assumed to have high advertising costs and demand effectiveness. The proposed basic model first analyzes advertising decisions in monopolistic competition by exploiting the complementarity of goods. The extended model analyses the advertising differentiation behavior of firms with complementary goods in a duopolistic market. A large body of research on whether firms engage in Bertrand or Cournot competition. Firms prefer to engage in Cournot competition because they have a ceiling on their production capacity, as we witnessed in the recent shortages related to the coronavirus pandemic, such as masks, disinfectants, and semiconductors. Advertising expenditure is also a constraint on firms, therefore, we believe that Cournot is more appropriate than Bertrand competition.

Here, using Johnson and Myatt's (2006) taxonomy of advertising, it is assumed that mass media advertising is widely available to an unspecified number of general consumers, advertising costs are incurred in lump sums rather than units. Conversely, the current mainstream of online advertising is search-related, and the cost of this advertising is linked to the queries (or keywords on the business side) that consumers explore on search engines (e.g., Google and Yahoo) and the unit cost per click is determined by bidding. This search-linked advertising is shown to users whose needs are known from their search behavior and can therefore be considered closer to purchase. Therefore, cost per click is considered as purchase cost, and online advertising

expenditure is based on cost per click. The basic model uses the following two-stage game as the method of analysis.

Stage 1: Firm  $i$  ( $i = 1, 2$ ) provides consumers with information about the complementarity of goods; to increase demand, the firm determines advertising costs and chooses between mass media and online advertising.

Stage 2: Both firms engage in Cournot competition, given the other firm's advertising choice.

This two-stage game is solved using backward induction to obtain each firm's optimal advertisements choice.

In the extended model, a three-stage game is used.

### **3.1 Consumer demand**

Consumers have homogeneous utility  $U(q_i, q_j)$  and are uniformly distributed in the market. Firms base their advertising decisions on the difference between these types of advertising and their costs. Advertising costs are assumed to be proportional to advertising-size effect and are  $e^2/2$  in the case of mass media advertising. Online advertising costs are assumed to be lower than mass media advertising costs, with a unit cost of  $e^2/3$ . The model assumes that the degree of complementarity varies with the type of advertising and denotes  $s$  as the degree of complementarity when mass media advertising is used and  $k$  as the degree of complementarity when online advertising is used.  $s, k \in (0, 1)$ . The closer  $s$  or  $k$  is to 1, the greater the degree of complementarity between the two goods, and the closer  $s$  or  $k$  is to 0, the smaller the degree of complementarity. In the extended model, complementarity was set to 1 and  $b$  and  $g$  are assumed to represent the degree of differentiation. Through advertising, in addition to  $(i = 1, 2)$  and initial



willingness to pay  $a (> 0)$  for good  $i$ , consumers receive a valuation of the additional good from the advertising information:  $e_i$  in the case of online advertising and  $e_j (> 0, i \neq j)$  in the case of mass media advertising. Simultaneously, the complementarity of the goods affects their additional valuation, and the satisfaction obtained by the consumer is  $ke_i$  or  $se_j$ . Thus, consumer's willingness to pay in the presence of advertising is expressed as  $a + ke$  or  $a + se_j$ .

Our demand structure is derived from the model proposed by Matumura and Ogawa (2012) and Li and Ji (2010). Let  $q_i (i = 1, 2, i \neq j)$  be the quantity of goods produced by firm  $i (i = 1, 2)$ . When two firms use mass media advertising, consumers' valuation of goods is affected by the degree of complementarity. Consumers determine each quantity demanded from the maximization condition of  $CS^m$  to maximize consumer surplus  $CS^m = U^m(q_i, q_j) - p_i q_i - p_j q_j$ , taking the prices of goods  $i$  and  $j$  as given and viewing both firms' advertising.<sup>3</sup> Thus, the inverse demand function for good  $i$  (denoted by subscript  $i$ ) in mass media advertising is given as follows:

$$p_i^m(q_i, q_j) = a + se_j - q_i + sq_j \quad (1)$$

Superscript  $m$  indicates that both firms choose mass media advertising.

If both firms use online advertising, the consumer utility of goods  $i$  and  $j$  takes the same form as in Equation (1), with  $g$  replaced by  $b$  and  $U^o(q_i, q_j)$ . From the maximization condition for consumer surplus  $CS^o = U^o(q_i, q_j) - p_i q_i - p_j q_j$ , the inverse demand function for good  $i$  is

$$p_i^o(q_i, q_j) = a + ke_i - q_i + kq_j \quad (2)$$

Superscript  $o$  indicates the case where both firms choose online advertising.

If firm  $i$  uses online advertising and firm  $j$  uses mass media advertising, the inverse demand functions for goods  $i$  and  $j$  derived from the consumer surplus maximization condition  $CS^a = U^a(q_i, q_j) - p_i q_i - p_j q_j$  are as follows:<sup>4</sup>

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<sup>3</sup> See Equation A1 in Appendix A.

<sup>4</sup> See Equation A2 in Appendix A.

$$p_i^{ao}(q_i, q_j) = a + ke_i - q_i + q_j(k + s)/2 \quad (3)$$

$$p_j^{am}(q_i, q_j) = a + se_j + q_i(k + s)/2 - q_j \quad (4)$$

Superscript *ao* indicates that firm *i* opts for online advertising; superscript *am* indicates that firm *j* opts for mass media advertising.

#### 4. Firms' Advertising Strategies

This section analyzes the advertising strategies of bilateral monopolies in which goods *i* and *j* are complementary.

Firm *i* chooses advertising based on the complementarity of goods and the cost of advertising based on advertising in mass media and online media, given the advertising decision of firm *j*.

The combined strategies of both firms are as follows: 1) both choose mass media advertising; 2) both choose online advertising; 3) one chooses mass media advertising and the other chooses online advertising.

#### Stage 2: The firms' profit maximization problem

##### 4.1. Both firms choose mass media advertising

If firms *i* and *j* both choose mass media advertising, they incur advertising costs of  $e^2/2$ . The advertising costs are treated as fixed costs and the firms' decision problem for forecasting consumer demand is as follows:

$$\max_{q_i} \pi_i^m = q_i \cdot p_i^m - e_j^2/2 = q_i (a + se_j - q_i + sq_j) - e_j^2/2 \quad (5)$$

Solving the profit maximization problem for firms *i* and *j* leads to the following demand, price and profit in equilibrium as follows:

$$q_i^{m*} = (a + se_j)/(2 - s). \quad (6)$$

$$p_i^{m*} = (a + se_j)/(2 - s). \quad (7)$$

$$\pi_i^{m*} = (2a^2 + 4ase_j - e_j^2(4 - s(4 + s)))/2(2 - s)^2 \quad (8)$$

The  $\bar{e}$  that guarantee  $\pi_i^m \geq 0$ , optimal  $e_i (> 0)$  are obtained as follows:

$$e_j^* = 2as/(4 - 4s - s^2) \quad (9)$$

$$\bar{e} = \sqrt{2}\sqrt{a^2(2 - s)^2 + 2as} / (4 - 4s - s^2) \quad (10)$$

From Equation (10), the possible range of  $s$  is  $0 < s < 0.75$ . The superscript \* indicates the equilibrium value.<sup>5</sup>

#### 4.2. in the case of the use of online advertising by both firms

If both firms use online advertising, they incur an advertising cost of  $e_i^2/3$  per good. Under this cost, forecasting consumer demand, the decision problem for the firms is as follows.

$$\max_{q_i} \pi_i^o = q_i(p_i^o - e_i^2/3) = q_i(a + ke_i - q_i + kq_j - e_i^2/3) \quad (9)$$

The equilibrium demand, price and optimal  $e_b$ , is derived by solving the firm's profit maximization problem as follows:

$$q_i^{o*} = (3a + 3ke_i - e_i^2)/(6 - 3k). \quad (11)$$

$$p_i^{o*} = (3a + e_i^2 + 3ke_i - ke_i^2)/(6 - 3k) \quad (12)$$

$$\pi_i^{o*} = (3a + e(3k - e_i))^2/(9(2 - k)^2) \quad (13)$$

$$e_i^* = 3k/2 \quad (14)$$

#### 4.3. If firm $i$ uses online advertising and firm $j$ uses mass media advertising

The profit maximization problem for firms forecasting consumer demand is as follows.

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<sup>5</sup> The condition of  $e_j \leq \bar{e}$  guarantees  $\pi_i^{m*} \geq 0$

$$\max_{q_i} \pi_i^{ao} = q_i(p_i^{ao} - e_i^2/3) = q_i(a + ke_i - q_i + q_j(k+s)/2 - e_i^2/3) \quad (15)$$

$$\max_{q_j} \pi_j^{am} = q_j(p_j^{am} - e_j^2/2) = q_j(a + ke_j + q_j(k+s)/2 - q_i) - e_j^2/2 \quad (16)$$

Solving the above maximization problem, demand, price,  $e_{ao}$  and  $e_{am}$  in equilibrium are derived as follows:

$$q_i^{ao*} = 2(3aD_2 + 8e_i(3k - e_i) + 6ke_j s + 6e_j s^2)/(3D_1 D_2) \quad (17)$$

$$q_j^{am*} = 2(3aD_2 + b(3k - e_i)e_i + (3ke_i - e_i^2 + 12e_j)s)/(3D_1 D_2). \quad (18)$$

$$p_i^{ao*} = (6aD_2 + e_i(24k + (8 - k^2)e_i) + 2k(e_i^2 - 3e_j)s + (e_i^2 - 6e_j)s^2)/(3D_1 D_2). \quad (19)$$

$$p_j^{am*} = 2(3aD_1 + 2(k(3k - e_i)e_i + (3ke_i - e_i^2 + 12e_j)s)/(3D_1 D_2) \quad (20)$$

$$\pi_i^{ao*} = (4(4e_i(3k - e_i) + 3ke_j s + 3e_j s^2 + 3a D_2^2)/(9D_1^2 D_2^2) \quad (21)$$

$$\pi_j^{am*} = (4(-3k^2 e_i + ke_i(e_i - 3s) + (e_i^2 - 12e_j)s - 3aD_2^2)/(9D_1^2 D_2^2) - e_j^2/2 \quad (22)$$

$$e_{ao} = 3k/2 \quad (23)$$

$$e_{am} = (8s(3k^2(k+s) + 4aD_2))/(D_3^2 - 4kD_3s - 2(80 - 3k^2)s^2 + 4ks^3 + s^4) \quad (24)$$

$$D_1 \equiv (4 - k - s), \quad D_2 \equiv (4 + k + s), \quad D_3 \equiv (16 - k^2)$$

## 5 Advertising Choice and Social Welfare

### 5.1. Advertising Choice

If complementarity,  $s = k = 0.30$ , then given firm  $j$ 's choice of online advertising, firm  $i$  chooses online advertising because  $\pi_i^{am}(k, s) = 0.369 < \pi_i^o(k) = 0.394$ . Given firm  $j$ 's choice of mass media advertising, firm  $i$  chooses online advertising because  $\pi_i^{ao}(s, k) = 0.393 > \pi_i^m(s) = 0.369$ . Due to symmetry, both firms choose online advertising. In this case,  $e_{am} = 0.191$  and  $e_j^* = 0.45$ . If  $s = 0.6$  and  $k = 0.3$ , then given firm  $j$ 's online ad, firm  $i$ 's ad choice is  $\pi_i^{am}(s, k) = 0.369 < \pi_i^o(k) = 0.394$ , so firm  $i$  chooses mass media advertising. Given firm  $j$ 's mass media advertising, firm  $i$ 's advertising choice is  $\pi_i^{ao}(s, k) = 0.514 < \pi_i^m(s) = 0.806$ , so firm  $i$  chooses

mass media advertising. Due to symmetry, both firms choose mass media advertising.<sup>6</sup> In this case,  $e_{am} = 0.515$  and  $e_{ao} = 0.45$ . If  $s = k = 0.6$ , given the mass media ad of firm  $j$ , firm  $i$  chooses online advertising because  $\pi_i^m(s) = 0.806 < \pi_i^{ao}(s, k) = 0.857$ . Given the online ad of firm  $j$ , firm  $i$  chooses the online ad because  $\pi_i^{am}(s) = 0.736 < \pi_i^o(s, k) = 0.823$ . Due to symmetry, both firms choose online advertising.<sup>7</sup> In this case,  $e_j^* = 0.968$  and  $e_i^* = 0.9$ . We now set A and B as follows to obtain the following proposition.

$$A \equiv \text{def}\{s, k \mid \pi_j^{am}(s, k) > \pi_j^o(k), \pi_j^{ao}(s, k) < \pi_j^m(s)\}$$

$$B \equiv \text{def}\{s, k \mid \pi_j^{ao}(s, k) > \pi_j^m(s), \pi_j^{am}(s, k) < \pi_j^o(k)\}$$

### Proposition 1

If  $(s, k) \in A$ , both firms choose mass media advertising.

If  $(s, k) \in B$ , both firms choose online advertising.

If complementarity  $k$  of online advertising is small, the complementarity effect ( $s$ ) of mass media advertising increases the demand; therefore, both firms will likely choose mass media advertising. If the complementarity between the two advertisements is equally small,

$\partial \pi_i^{ao}(s, k)/\partial k > \partial \pi_i^m(s)/\partial s > 0$  and  $\partial \pi_i^o(k)/\partial k > \partial \pi_i^{am}(s, k)/\partial k > 0$ , the price effect of online advertising is larger, and both firms will choose online advertising.<sup>8</sup>

## 5.2. Social welfare

Let  $CS^o$  denote consumer surplus,  $PS^o$  denote producer surplus, and  $PS^o$  denote total

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<sup>6</sup> See Figure 1, 3 in Appendix B.

<sup>7</sup> See Figure 2, 4 Appendix B.

<sup>8</sup>  $\partial \pi_m(s)/\partial s > \partial \pi_{ao}(s, k)/\partial s > 0$ ,  $\partial \pi_{am}(s, k)/\partial k > \partial \pi_o(k)/\partial k > 0$

surplus when both firms choose online advertising. When firms choose different advertisements, we denote consumer surplus as  $TS^a$ , producer surplus as  $PS^a$ , and total surplus as  $TS^a$ . If both firms choose mass media advertising, we denote consumer surplus as  $CS^m$ , producer surplus as  $PS^m$ , and total surplus as  $TS^m$ . When  $s = k = 0.3$ , both firms choose online advertising. Total surplus is  $TS^o > TS^m > TS^a$ , consumer surplus is  $CS^o > CS^m > CS^a$ , and producer surplus is  $PS^o > PS^m > PS^a$ . When  $s = 0.6, k = 0.3$ , both firms choose mass media advertising: consumer surplus is  $CS^m > CS^o > CS^a$ , and producer surplus is  $PS^m > PS^o > PS^a$ . However, total surplus is in the order of  $TS^o > TS^m > TS^a$ . When  $s = k = 0.6$ , both firms choose online advertising; total surplus is  $TS^m > TS^o > TS^a$ , consumer surplus is  $CS^m > CS^o > CS^a$ , and producer surplus is  $PS^m > PS^o > PS^a$ . Firms' choices are determined by the magnitudes of  $s$  and  $k$ , and the magnitudes of total surplus and consumer surplus also depend on  $k$  and  $s$ . As  $s$  increases, firms' choices and each surplus become different. Therefore, we set  $\hat{s}(k)$  as the boundary between the profit from mass media advertising and online advertising as follows: when  $k = 0.3$ , online advertising increases consumer surplus and total surplus, when  $s$  is lower than  $\hat{s}(k)(=0.332)$ . When  $s$  is higher than  $\hat{s}(k)(=0.332)$ , mass media advertising effects increase consumer surplus and total surplus.<sup>9</sup>

$$\hat{s}(k) \equiv \text{def} \{ s \in \hat{s}(k) \mid TS^o(k) = TS^m(s) > TS^a(k, s) \}$$

### **Proposition 2**

When  $s < \hat{s}(k)$ ,  $TS^o(k) > TS^m(s) > TS^a(k, s)$

When  $s > \hat{s}(k)$ ,  $TS^m(k) > TS^o(s) > TS^a(k, s)$

## **6. Advertising Strategies of Hardware Firms in Complementary Relationships With Software Firms**

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<sup>9</sup> See Figure 5 ~10 in Appendix B.

This section analyzes the behavior of firms that sell hard goods, which are complementary to the goods offered by software firms, and maximize their profits by differentiating themselves through advertising.

## 6.1 Expansion Model

Consider a duopolistic market of hardware firms with complementary goods. Hardware firms can use royalty income from complementary software, such as applications, to keep the price of hardware products low and increase the demand for hardware. A software firm produces a soft good with complementarity 1 for hardware firms  $i$  and  $j$  and pays royalties  $t_i$  and  $t_j$  per unit of soft goods  $i$  and  $j$ . For example, demand for soft goods in a complementary relationship, such as game software or applications, and hardware devices, such as game consoles and smartphones, will increase if the demand for hard good increases. However, this will decrease if the demand for other firms' hardware goods increases. Advertising costs are assumed to be the same as those in the basic model. At this point, if advertising affects the demand for hard and soft goods, which type of advertising hardware do firms choose, online or mass media advertising?

The combination of the strategies of two hard firms is the same as in Chapter 4: 1) both choose mass media advertising; 2) both choose online advertising; and 3) one chooses mass media advertising and the other chooses online advertising. The timing of the play is as follows:

First stage: Hardware firm  $i$  determine on advertising costs, given the choice of rival firm  $j$ , and chooses between mass media or online advertising, considering advertising effectiveness.

Second stage: Hardware firm  $i$  decides on royalty  $t_i$  and engages in Cournot competition.

Third stage: Software firms sell soft goods  $i$  and  $j$ , given the demand for hard goods,

royalties, and advertising effects.

Superscript  $z$  denotes the price of the software products. The inverse demand function for software  $i$  (denoted by the subscript  $i$ ) for firm  $i$  is obtained from the consumer surplus maximization condition  $CS^c$  as follows:<sup>10</sup>

$$p_i^z(z_i, z_j) = a + q_i - z_i \quad (25)$$

The inverse demand function for hardware firm  $i$ , adding the complementary demand for soft goods  $z_i$ , is:

$$p_i^{hm}(q_i, q_j) = a + ge_i - q_i - gq_j + z_j \quad (26)$$

Superscript  $hm$  indicates that hardware firms choose mass media advertising,  $ho$  indicates online advertising, and  $hao$  and  $ham$  different choices.  $g$  indicate

### Stage 3: Profit Maximization Problem for Software Firms

Given the demand for hardware  $i$  and  $j$  and royalties  $t_i$  and  $t_j$ , the decision problem for a firm selling software with development cost  $F$  is as follows:

$$\begin{aligned} \max_z \pi_i^z &= z_i \cdot p_i^z + z_j \cdot p_j^z - t_i z_i - t_j z_j - F \\ \text{s.t. } F &\leq \{ z_i \cdot p_i^z + z_j \cdot p_j^z - t_i z_i - t_j z_j \} \end{aligned} \quad (27)$$

To solve the software firm's maximization problem, the equilibrium demand  $z_i$  and price  $p_i^z$  for the software is:

$$z_i^* = (a - t_i + q_i)/2 \quad (28)$$

$$p_i^{z*} = (a + t_i + q_i)/2 \quad (29)$$

$z_i^*$  and  $p_i^{z*}$  will be the same values if the firm chooses a different type of advertisement.

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<sup>10</sup> See Equations (A3) ~ (A5) in Appendix A.



## Stage 2: Profit Maximization Problem for Hardware Firms

### 6.2. Both hardware firms choose mass media advertising

The decision problem for the hardware firms is as follows:

$$\max_{q_i} \pi_i^{hm} = q_i \cdot p_i^{hm} - e_j^2/2 + t_i \cdot z_i = q_i (a + ge_j - q_i - gq_j + z_i) - e_j^2/2 + t_i \cdot z_i \quad (28)$$

The superscript *hm* indicates that the hardware firms choose mass media advertising, *g* represents the degree of products differentiation, and *t<sub>i</sub>z<sub>i</sub>* is the royalty income from the software firms. The demand  $q_i^{hm*}$ , price  $p_i^{hm*}$ , royalty  $t_i^{hm*}$  and  $e_j^{hm*}$  in equilibrium with the hardware firm's maximization problem solved are obtained as follows:

$$q_i^{hm*} = (3a + 2eg)/(2 + 2g) \quad (29)$$

$$p_i^{hm*} = (3a + 2eg - 2(1 + g)t_j)/(4(1 + g)) \quad (30)$$

$$t_i^{hm*} = t_j^{hm*} = a/2 \quad (31)$$

$$e_j^{hm*} = 3ag/(2(1 + 2g)) \quad (32)$$

### 6.3. If both hardware firms use online advertising

The decision problem for the hardware firms is as follows:

$$\max_{q_i} \pi_i^{ho} = q_i(p_i^{ho} - e_i^2/3) + t_i \cdot z_i = q_i (a + be_i - q_i - bq_j + z_i - e_i^2/3) + t_i \cdot z_i \quad (33)$$

The superscript *ho* indicates that hardware firms choose online advertising, and *b* indicates the degree of differentiation of hardware goods that use online advertising.

Solving the maximization problem equation (33) for hardware firms *i* and *j*, demand and price and royalties  $t_i^{ho*}$  and  $e_i^{ho*}$  in equilibrium are obtained as follows:<sup>11</sup>

$$q_i^{ho*} = (9a + 6be - 2e^2)/(6 + 6b) \quad (34)$$

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<sup>11</sup> Please see Equation (A3) in Appendix A.

$$p_i^{hao*} = (9a + 2e(e + b(3 + 2e)) - 6(1 + b)t_i)/(12(1 + b)) \quad (35)$$

$$t_i^{hao*} = a/2 \quad (36)$$

$$e_i^{hao*} = 3b/2 \quad (37)$$

### 6.3. When firm i uses mass media advertising and firm j uses online advertising

The decision problem for firms  $i$  and  $j$  forecasting consumer demand is as follows:

$$\max_{q_i} \pi_i^{hao} = q_i(p_i^{hao} - e_i^2/3) + t_i z_i = q_i(a + be_i - q_i - q_j(b + g)/2 + z_i - e_i^2/3) + t_i z_i \quad (38)$$

$$\max_{q_j} \pi_j^{ham} = q_j(p_j^{ham} - e_j^2/2) + t_j z_j = q_j(a + ge_j - q_j(b + g)/2 + z_j - q_j) - e_j^2/2 + t_j z_j \quad (39)$$

When using different ads from each other, we put the online ad effect as  $e_i$  and the mass media ad effect as  $e_j$ , since different ad types have different ad effects.

Solving the decision problem for both firms with the rivals' behavior as given, the price and demand in equilibrium, royalty  $t_i^{hao*}$ ,  $t_j^{ham*}$  and advertising effectiveness  $e_i^{hao*}$  and  $e_j^{ham*}$  are obtained as follows:<sup>12</sup>

$$q_i^{hao*} = (9aD_3 + 4e_i(3b - e_i) - 6be_jg - 6e_jg^2)/(3D_3D_4) \quad (38)$$

$$q_j^{ham*} = (9aD_3 - 6b^2e_i + 2be_i(e_i - 3g) + 2(e_i^2 + 6e_j)g)/(3D_3D_4) \quad (39)$$

$$p_i^{hao*} = (9aD_3 + 2e_i^2(2 - b^2 - 2bg - g^2) - 3e_j(2g - 3)g + 3t_i(b^2 - 4 + 2bg + g))/(6D_3D_4) \quad (40)$$

$$p_j^{ham*} = (9aD_3 - 6b^2e_i + 2be_i(e_i - 3g) + 2(e_i^2 + 6e_j)g - 3D_3D_4 t_j)/(6D_3D_4) \quad (41)$$

$$t_j^{ham*} = t_i^{hao*} a/2 \quad (42)$$

$$e_i^{hao*} = 3b/2 \quad (43)$$

$$e_j^{ham*} = (6g(2aD_3 - b^2(b + g)))/((4(1 - g) - (b + g)^2)(4(1 + g) - (b + g)^2)) \quad (44)$$

$$D_3 \equiv (2 - b - g), \quad D_4 \equiv (2 + b + g)$$

### 6.4. Stage 1: Advertising choice of the hardware firm

<sup>12</sup> Please see Equation (A6) ~ (A12) in Appendix A. From the condition of  $e_j^{ham*} > 0$ , we set  $b$  and  $g \in (0, 0.6)$ .

Given firm  $i$ 's choice of mass media advertising, if the degree of differentiation is  $g = b = 0.30$ , then firm  $j$ 's profit is  $\pi_j^{hao^*}(g, b) = 0.829 > \pi_j^{hm}(g, b) = 0.828$ , and firm  $j$  chooses online advertising. In this case,  $e_i^{hao^*} = 0.45$  and  $e_j^{hm^*} = 0.24$ . Given firm  $i$ 's choice of online advertising, firm  $j$  chooses online advertising because  $\pi_i^{ham^*}(g, b) = 0.843 < \pi_i^{ho^*}(g, b) = 0.852$ . In this case,  $e_i^{ho^*} = 0.45$  and  $e_j^{ham^*} = 0.418$ . In terms of symmetry, both firms choose online advertising.  $g = 0.50$ ,  $b = 0.30$ , given firm  $j$ 's choice of online advertising, firm  $i$  compares its profit,  $\pi_i^{ham^*}(g, b) = 0.961 > \pi_i^{ho^*}(b) = 0.852$ , so firm  $i$  chooses mass media advertising. Given the mass media advertising choice of firm  $i$ , when firm  $j$  compares the profits from both advertisements, it chooses mass media advertising because  $\pi_j^{hao^*}(g, b) = 0.551 < \pi_j^{hm^*}(g) = 0.688$ . Based on symmetry, both firms choose mass media advertising.  $g = b = 0.6$ , given firm  $i$ 's choice of mass media advertising, profit is  $\pi_j^{hm^*}(g) = 0.636 < \pi_j^{hao^*}(b, g) = 1.439$ , so firm  $j$  chooses online advertising. In this case,  $e_i^{hao^*} = 0.90$  and  $e_j^{hm^*} = 0.66$ . In addition, given firm  $i$ 's choice of online advertising, firm  $j$ 's profit is  $\pi_j^{ham^*}(g) = 2.059 > \pi_j^{ho}(b, g) = 0.737$ , thus, firm  $j$  chooses mass media advertising. From symmetry, firms choose different type of advertising.<sup>13</sup>

We now place the range of  $b$  and  $g$  as follows to obtain the following propositions.

$$C(b, g) \equiv_{\text{def}} \{ b, g \mid \pi_j^{hao}(b, g) > \pi_j^{hm}(g), \pi_j^{ham}(b, g) < \pi_j^{ho}(b) \}$$

$$D(b, g) \equiv_{\text{def}} \{ b, g \mid \pi_j^{hao}(b, g) < \pi_j^{hm}(g), \pi_j^{ahm}(b, g) > \pi_j^{ho}(g) \}$$

$$E(b, g) \equiv_{\text{def}} \{ b, g \mid \pi_j^{hao}(b, g) > \pi_j^{hm}(g), \pi_j^{ham}(b, g) > \pi_j^{ho}(b) \}$$

### Proposition 3

If  $C(b, g)$ , both firms choose online advertising.

If  $D(b, g)$ , both firms choose mass media advertising.

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<sup>13</sup> Please see Figure 13 ~ 16 in Appendix B.

If  $E(b, g)$ , firms choose different type of advertisements.

Two firms that have complementary goods and earn royalties will both choose low-cost online advertising because competition between the two goods is low when advertising has a large differentiating impact on the goods. When the differentiation effects of  $g$  and  $b$  are low, competition is high, therefore, they choose mass media advertising, which has an average differentiation effect. However, when the impacts of both differentiation  $g$  and  $b$  are low, competition is high and prices are low; thus if firm  $j$  chooses mass media advertising, firm  $i$  chooses online advertising with a low price decline. Thus, the choice of hardware firms is determined by the impacts of demand and price, including advertising differentiation of goods and royalty income.

## 6.5 The social welfare

Let  $CS^{oh}$  denote consumer surplus when both firms choose online advertising,  $PS^{oh}$  denotes producer surplus, and  $TS^{oh}$  denotes total surplus. Let  $CS^{ah}$  denotes consumer surplus,  $PS^{ah}$  denote producer surplus, and  $TS^{ah}$  denote total surplus when firms choose different type of advertising. Let  $CS^{hm}$  denote consumer surplus,  $PS^{hm}$  denote producer surplus, and  $TS^{hm}$  denote total surplus when both firms choose mass media advertising.<sup>14</sup> When  $b = g = 0.30$  and both firms choose online advertising, then total surplus is  $TS^{hm} > TS^{ho} > TS^{ha}$ , the consumer surplus is  $CS^{hm} > CS^{ho} > CS^{ha}$ , and the producer surplus is  $PS^{ha} > PS^{ho} > PS^{hm}$ . When  $g = 0.5$  and  $b = 0.3$  and both firms choose mass media advertising, the total surplus is  $TS^{ho} > TS^{hm} > TS^{ha}$ , consumer surplus is  $CS^{ho} > CS^{hm} > CS^{ha}$ , and producer surplus is  $PS^{ho} > PS^{ha} > PS^{hm}$ .  $g = b = 0.6$ , when one firm chooses online advertising and the other firm opts for mass media advertising, then total surplus is  $TS^{ha}$

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<sup>14</sup>  $PS^{hm} = \pi_j^{hm}(g) + \pi_i^{hm}(g) + \pi^{zm}$ ,  $PS^{ha} = \pi_j^{hao}(b, g) + \pi_i^{ham}(b, g) + \pi^{xa}$   $PS^{ho} = \pi_j^{ho}(b) + \pi_i^{ho}(b) + \pi^{zo}(b)$

$> TS^{ho} > TS^{hm}$ ,  $CS^{hm} > CS^{ho} > CS^{ha}$  for consumer surplus, and  $PS^{ha} > PS^{ho} > PS^{hm}$  for producer surplus. We now place  $(b, g)$  to obtain the following proposition.<sup>15</sup>

$$g(b) \equiv \text{def} \{ b, g \mid TS^{ho}(b) = TS^{hm}(g) > TS^{ha}(b, g) \text{ and } CS^{ho}(b) > CS^{hm}(b, g) > CS^{ha}(g) \}$$

#### Proposition 4

$$g > g(b) \text{ and } g \in C(b, g) \text{ or } g \in D(b, g) \text{ or } g \in E(b, g) : TS^{ho} > TS^{hm} > TS^{ha}, CS^{ho} > CS^{hm} > CS^{ha}$$

$$g < g(b) \text{ and } g \in D(b, g) \text{ or } g \in C(b, g) : TS^{hm} > TS^{ho} > TS^{ha}, CS^{hm} > CS^{ho} > CS^{ha}$$

Each advertising strategy using hardware firm differentiation has a different magnitude of surplus based on the type of advertising. For  $g > g(b)$ , differentiation using mass media advertising increases advertising costs but has a low demand effect. For  $g < g(b)$ , low differentiation using mass media advertising has a high demand effect but low advertising cost. In contrast, as online advertising has a low demand effect, total surplus, consumer surplus, and producer surplus are higher with mass media advertising.  $g > g(b)$  and  $g \in D(b, g)$ , hardware firms choose mass media advertising, but total surplus, consumer surplus, and producer surplus are higher for online advertising. Conversely, if  $g < g(b)$  and  $g \in C(b, g)$ , hardware firms choose online advertising, but total surplus, consumer surplus, and producer surplus are higher with mass media advertising.

## 7. Conclusion.

Monopolistic firms' advertising strategies with mutually complementary goods are determined by the magnitude of their complementarity. In this case, the total and consumer surpluses increase according to the firm's choice. However, in a differentiation strategy using

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<sup>15</sup> Please see Figure 17, 18 in Appendix B.

advertising in a duopoly market, when a company obtains royalty income from complementary goods, the hardware firm's choice depends on the amount of software produced and the degree of hardware differentiation. If differentiation is low, the firm's choice of advertising may not be consistent with the magnitude of social welfare. First, royalty contracts raise the price of software and lower consumer surplus. Second, the strategy of using royalties to lower hardware prices and increase demand does not unilaterally increase the consumer surplus. However, corporate advertising choices and social welfare coincide when mass-media advertising with moderate differentiation is preferred. Nintendo and Sony, the manufacturers of home gaming machines, use TV commercials. Smartphone companies such as Apple also use TV commercials (this study did not consider corporate websites as online advertisements). This indicates that hardware manufacturers, such as game-console manufacturers, believe that the advertising effect of mass media, that is, stimulating consumer demand in the mass market, will increase the demand for hardware and software. Although this depends on the judgment of hardware differentiation, if there is no significant difference between companies' gaming machines, companies' decisions of mass media advertisements increase social welfare. Furthermore, for these hardware manufacturers, it is strategically important not only to provide the appeal of their software products along with their own products and services through advertising but also to contribute to the expansion of complementary product markets, leading to greater demand for their products. As part of this, Sony's support for software makers in developing their own software contributed to the expansion of the PlayStation market. From these examples, royalty payments to hardware manufacturers may play an important role in advertising expenses and investments in market expansion. Future research should clarify the relationship between this and social welfare.

## Appendix A

### Basic model where both software firm and hardware firm.

The consumer's utility from goods  $i$  and  $j$  under mass media advertising is:

$$U^m(q_i, q_j) = (a + se_j)(q_i + q_j) - (q_i^2 - 2sq_iq_j + q_j^2)/2 \quad (\text{A1})$$

where  $a$ ,  $k$ , and  $s$  are the parameters. Superscript  $m$  denotes mass media advertising, and  $U^m$  denotes the utility under it.

The consumer utility  $U^a(q_i, q_j)$  when firms choose different advertisements is

$$U^a(q_i, q_j) = (a + ke_i)q_i + (a + se_j)q_j - [q_i^2 - (k + s)q_iq_j + q_j^2]/2 \quad (\text{A2})$$

Superscript  $a$  indicates an asymmetric advertising choice situation in which one of firms uses mass media advertising and the other uses online advertising.

### Utility of hardware firms to earn royalties from software firms in a duopoly market.

Let  $z_i$  denote the soft good  $i$  for hardware firm  $i$  and  $z_j$  denote the demand for the soft good for hardware firm  $j$ . Consumer utility is affected by the hard and soft goods, and the consumer determines the quantity of soft goods demand so as to maximize the consumer surplus of the soft good  $CS^s = U^{ho} - p_i z_i - p_j z_j$ . The consumer determines each demand quantity so as to maximize the consumer surplus of the hard good  $C^{hm} = U^{hm} - p_i q_i - p_j q_j$ , where superscript  $hm$  indicates that both firms choose mass media advertising, superscript  $ho$  indicates the utility when both firms choose mass media advertising, and superscript  $ha$  indicates that firm  $i$  chooses mass media advertising and firm  $j$  chooses online advertising.

$$U^{ho} = (a + be_i)(q_i + q_j) - (q_i^2 - 2bq_iq_j + q_j^2)/2 + a(z_i + z_j) - (z_i^2 + z_j^2 - 2(q_i z_i + q_j z_j))/2 \quad (\text{A3})$$

$$U^{hm} = (a + ge_j)(q_i + q_j) - (q_i^2 - 2gq_iq_j + q_j^2)/2 + a(z_i + z_j) - (z_i^2 + z_j^2 - 2(q_i z_i + q_j z_j))/2 \quad (\text{A4})$$

$$U^{ha}(q_i, q_j) = (a + be_i)q_i + (a + ge_j)q_j - [q_i^2 - (b + g)q_iq_j + q_j^2]/2 \\ + (a + be_i)z_i + (a + be_j)z_j - (z_i^2 + z_j^2 - 2(q_i z_i + q_j z_j))/2 \quad (\text{A5})$$

When the hardware firm chooses mass media, the profit margins for the software and hardware companies are as follows:

$$\pi^{zm*} = (2e_j g + a(4 + g))^2 / (8(1 + g)^2) - F \quad (A6)$$

$$\pi_i^{hm*} = (12ae_j g - 4e_j^2 g^2 + a^2(10 + 2g + g^2)) / (8(1 + g)^2) - e_j^2 / 2 \quad (A7)$$

The equilibrium profit for each firm when a hardware firm chooses online advertising is as follows:

$$\pi^{zo*} = (3a(4 + b) + 6be_i - 2e_i^2) / (72(1 + b)^2) - F \quad (A8)$$

$$\pi_i^{ho*} = (9a^2(10 + b(2 + b)) + 36a(3b - e_i)e_i + 4e^2(3b - e_i)^2) / (72(1 + b)^2) \quad (A9)$$

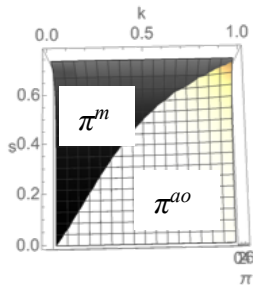
The equilibrium profit of each firm when firm i employs mass media and firm j employs online advertising is:

$$\begin{aligned} \pi^{za*} = & (4be_i(3b - (e_i - 3g) - 4(e_i^2 + 6e_j)g - 3aD_3(8 + b + g))^2 \\ & - (3aD_3(8 + b + g) + 4(2e_i(e_i - 3b) + 3e_j g(b + g))) / (144D_3^2 D_4^2) - F \end{aligned} \quad (A10)$$

$$\begin{aligned} \pi_j^{ham*} = & (144aD_3(3b^2 e_i - be_i(e_i - 3g) - (e_i^2 + 6e_j)g) + 16(-3b^2 e_i + be_i(e_i - 3g) \\ & + (e_i^2 + 6e_j)g)^2 + 9a^2 D_3^2 (40 + b^2 + 2b(2 + g) + g(4 + g))) / (72D_3^2 D_4^2) - e_j^2 / 2 \end{aligned} \quad (A11)$$

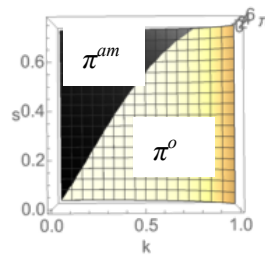
$$\pi_i^{hao*} = (9a^2 / 4 + (4e_i(-3b + e_i) + 6e_j g(b + g) + 9aD_3)^2) / (D_3^2 D_4^2) / 18 \quad (A12)$$

## Appendix B: Basic Model



**Figure 1.**

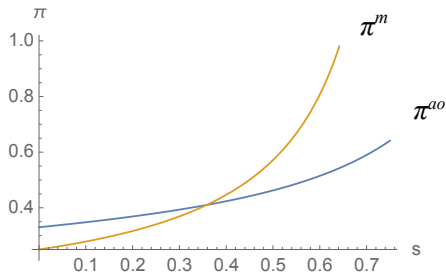
$a=1$ . vertical axis:  $s$ , horizontal axis:  $k$   
Given the mass media advertising of firm  $j$ ,  
 $\pi_{ao}$  or  $\pi_m$  for firm  $i$



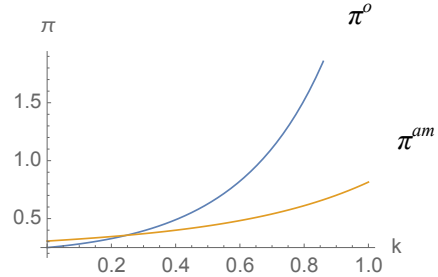
**Figure 2 :**

$a=1$ . vertical axis:  $s$ , horizontal axis:  $k$   
Given the mass media advertising of firm  $j$ ,  
 $\pi_{am}$  or  $\pi_o$  for firm  $i$



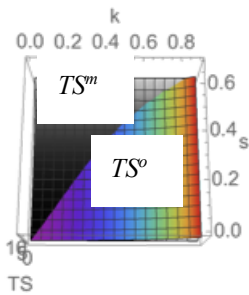


**Figure 3.**  
 $a = 1, k = 0.3$ , vertical axis:  $\pi$ , horizontal axis:  $k$   
 Given the mass media advertising of firm  $j$ ,  
 $\pi^{ao}$  or  $\pi^m$  for firm  $i$



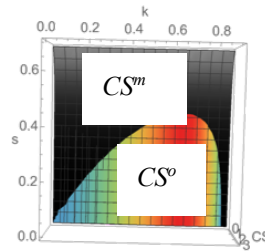
**Figure 4.**  
 $a = 1, s = 0.3$ , vertical axis:  $\pi$ , horizontal axis:  $k$   
 Given the mass media advertising of firm  $j$ ,  
 $\pi^{am}$  or  $\pi^o$  for firm  $i$

**Total surplus (TS)**

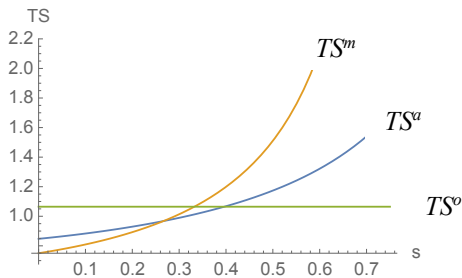


**Figure 5.**  
 $a = 1$ , vertical axis:  $s$ , horizontal axis:  $k$   
 Total surplus under ad selection

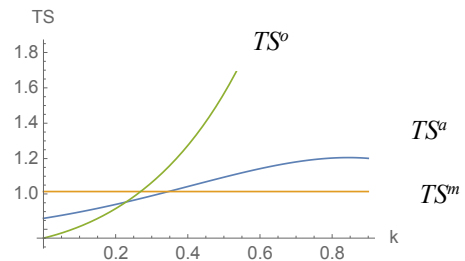
**Consumer surplus (CS)**



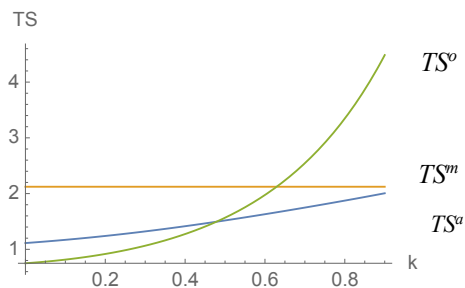
**Figure 6.**  
 $a = 1$ , vertical axis:  $s$ , horizontal axis:  $k$   
 Consumer surplus under ad selection



**Figure 7.**  
 $a = 1$ , vertical axis:  $TS$ , horizontal axis:  $s$   
 Change in total surplus ( $TS$ ) under  $k = 0.3$



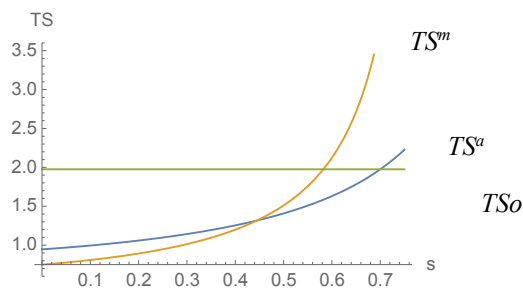
**Figure 8.**  
 $a = 1$ , vertical axis:  $TS$ , horizontal axis:  $k$   
 Change in total surplus ( $TS$ ) under  $s = 0.3$



**Figure 9.**

$a = 1$ , vertical axis:  $TS$ , horizontal axis:  $k$

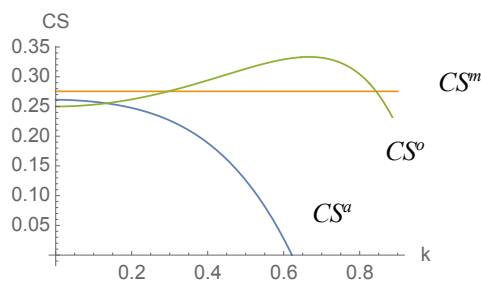
Change in total surplus ( $TS$ ) under  $s = 0.6$



**Figure 10.**

$a = 1$ , vertical axis:  $TS$ , horizontal axis:  $s$

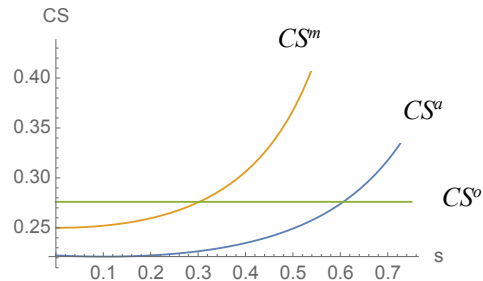
Change in total surplus ( $TS$ ) under  $k = 0.6$



**Figure 11.**

$a = 1$ , vertical axis:  $CS$ , horizontal axis:  $k$

Change in consumer surplus ( $CS$ ) under  $s = 0.3$

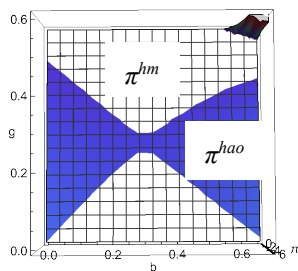


**Figure 12.**

$a = 1$ , vertical axis:  $CS$ , horizontal axis:  $s$

Change in consumer surplus ( $CS$ ) under  $k = 0.3$

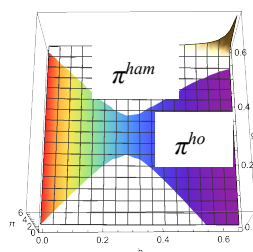
### Duopoly market model: Hardware companies' advertising choices



**Figure 13.**

$a = 1$ , vertical axis:  $g$ , horizontal axis:  $b$

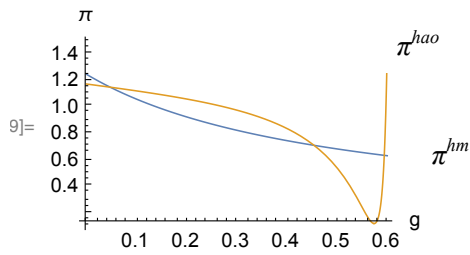
The choice of firm  $i$  given the mass media ads of firm  $j$ .



**Figure 14.**

$a = 1, g = 0.6$ , vertical axis:  $g$ , horizontal axis:  $b$

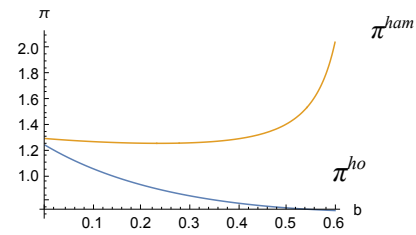
The choice of firm  $i$  given the mass media ads of firm  $j$



**Figure 15.**

$a = 1, b = 0.6$ , vertical axis:  $\pi$ , horizontal axis:  $g$

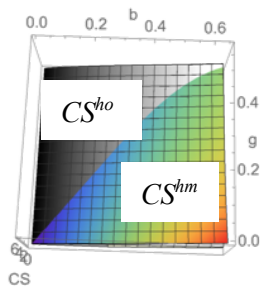
The choice of firm  $i$  given the mass media ads of firm  $j$ .



**Figure 16.**

$a = 1, g = 0.6$ , vertical axis:  $\pi$ , horizontal axis:  $b$

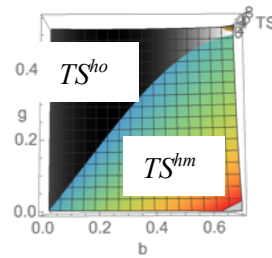
The choice of firm  $i$  given the mass media ads of firm  $j$



**Figure 17.**

$a = 1, F = 1$ , vertical axis:  $g$ , horizontal axis:  $b$

Size of  $CS$  changing with the magnitude of  $g$  and  $b$ ,



**Figure 18.**

$a = 1, F = 1$ , vertical axis:  $g$ , horizontal axis:  $b$

Size of  $TS$  changing with the magnitude of  $g$  and  $b$

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