

Bofinger, Peter; Haas, Thomas

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## R-star: A new approach to estimate the polar star of monetary policy

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# R-Star: A new approach to estimate the polar star of monetary policy

Peter Bofinger and Thomas Haas\*

\* University of Würzburg

September 2023



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Peter Bofinger

Department of Economics, University of Würzburg  
97070 Würzburg, Germany  
[peter.bofinger@uni-wuerzburg.de](mailto:peter.bofinger@uni-wuerzburg.de)

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Julius-Maximilians-Universität Würzburg  
Volkswirtschaftliches Institut  
Sanderring 2  
97070 Würzburg

<https://www.wiwi.uni-wuerzburg.de/fakultaet/institute-und-lehrstuehle/vwl/>

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Am Hubland                                      <https://opus.bibliothek.uni-wuerzburg.de>  
97074 Würzburg  
Tel.: +49 931 - 31-85906

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# R-Star: A new approach to estimate the polar star of monetary policy

Peter Bofinger and Thomas Haas \*

September 2023

## Abstract

*The necessary adjustments to prominent measures of the neutral rate of interest following the COVID pandemic sparked a wide-ranging debate on the measurement and usefulness of r-star. Due to high uncertainty about relevant determinants, trend patterns and the correct estimation method, we propose in this paper a simple alternative approach derived from a standard macro model. Starting from a loss function, neutral periods can be determined in which a neutral real interest rate is observable. Using these values, a medium-term trend for a neutral interest rate can be determined. An application to the USA shows that our simple calculation of a neutral interest rate delivers comparable results to existing studies. A Taylor rule based on our neutral interest rate also does a fairly good job of explaining US monetary policy over the past 60 years.*

**Keywords:** Neutral rate of interest; equilibrium real interest rate; monetary policy rules.

**JEL codes:** E3; E4; E5.

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\*University of Würzburg

Corresponding author: [peter.bofinger@uni-wuerzburg.de](mailto:peter.bofinger@uni-wuerzburg.de). We are grateful to Pascal Frank and the participants of the Luxembourg Central Bank research workshop on 6 June 2023 for helpful comments, suggestions, and discussions.

# 1 Introduction

The necessary adjustments to prominent measures of the neutral rate of interest following the COVID pandemic ([Holston, Laubach, & Williams, 2023](#)) sparked a wide-ranging debate on the measurement and usefulness of  $r^*$ . At a time of elevated uncertainty, the publication of one of the most important anchors for monetary policy was interrupted for two years. Moreover, the adjustments to the estimation of the neutral interest rate altered not only the most recent estimates of  $r^*$ , but also the full time series going back to the 1960s. This paper discusses the concept of the natural rate of interest, reviews some of the criticisms of the measure and presents a simple but theoretically sound derived approach to determining the level of the neutral rate of interest.

The neutral rate of interest - or  $r^*$  - is an important concept in economics, used mainly to describe the stance of monetary policy in an economy, but also to discuss economic conditions and long-term developments in economies (for example, the secular stagnation debate ([Summers, 2016](#))). A recent comment by Philip ([Lane, 2023](#)) - Member of the Executive Board of the ECB - underlines the importance of  $r^*$  for monetary policy:

*"Conceptually, the neutral interest rate is the hypothetical level of the interest rate that, when all temporary shocks have faded out, can set the economy on a sustainable path of balanced growth with inflation durably at target. As actual rates move beyond that level, policy becomes restrictive – as is necessary when inflation is otherwise set to remain above the central bank's target for an extended period."*

The quote describes the basic idea behind  $r^*$ : In a period with no exogenous shock, the central bank should set its policy rate at the level of the natural rate of interest in order to achieve a favourable economic development with sustainable growth and price stability at the same time. If the economy is hit by a shock, the central bank can respond by raising or lowering its policy rate relative to the neutral rate. A situation where the policy rate is below the neutral rate is considered to be a loose monetary policy stance, i.e. an expansionary monetary policy, and a policy rate above the neutral rate is considered to be a restrictive monetary policy stance.

While this description sounds intuitive and straightforward, the r-star concept has an important shortcoming: As a purely theoretical concept, it cannot be observed, but must be estimated based on economic theory. Jerome [Powell \(2018\)](#) - chairman of the Federal Reserve - has highlighted the problems this poses, particularly for active policymaking:

*"Navigating by the stars can sound straightforward. Guiding policy by the stars in practice, however, has been quite challenging of late because our best assessments of the location of the stars have been changing significantly."*

This has already been pointed out by [J. H. Williams \(1931, p. 578\)](#):

*"The natural rate is an abstraction; like faith, it is seen by its works. One can only say that if the bank policy succeeds in stabilizing prices, the bank rate must have been brought in line with the natural rate, but if it does not, it must not have been."*

As we show in our paper, the most prominent current approaches to determining r-star suffer from serious challenges in terms of their determination, the high uncertainty of their estimates and their informational value for policymakers. We therefore propose a new and simplified approach to determining a neutral level of interest rates to guide monetary policy decisions. An estimation of a simple Taylor rule for the United States illustrates the merits of our approach.

The remainder of this paper is structured as follows: Section [2](#) provides a comprehensive definition of the neutral interest rate and an overview of some widely used measures of r-star. We also discuss some of the shortcomings of these approaches. Section [3](#) presents a simple IS-MP-PC macroeconomic model, which we use as the theoretical workhorse for our determination of r-star. In section [4](#) we provide our own measure of the neutral interest rate and apply it to an empirical test for the United States. Section [5](#) concludes the paper.

## 2 Existing approaches towards r-star

### 2.1 Definition

The concept of the neutral rate of interest was first introduced by the Swedish economist Knut Wicksell (1898, p. 102):

*"There is a certain rate of interest on loans which is **neutral in respect to commodity prices**, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be **determined by supply and demand** if no use were made of money and all lending were effected in the form of real capital goods. It comes to much the same thing to describe it as the current value of the **natural rate of interest on capital**."* (emphasis by authors)

Thus, Wicksell provided three different definitions of r-star that should lead to the same rate: First, it is the rate of interest consistent with **price stability**. Second, it is the rate of interest determined by supply and demand in an economy without money (*real analysis economy* (Schumpeter, 1954, p. 264)). This is equal to the real interest rate that **equates saving to investment** in a simple loanable funds model (Amato, 2005). Finally, it is also equal to the natural rate of return on capital, i.e. the **net marginal product of capital**.

### 2.2 Approaches to measure r-star

From these definitions, two theoretical approaches to measuring r-star can be derived. First, if r-star is equal to the net marginal product of capital, it can be represented in a simple neoclassical growth model:

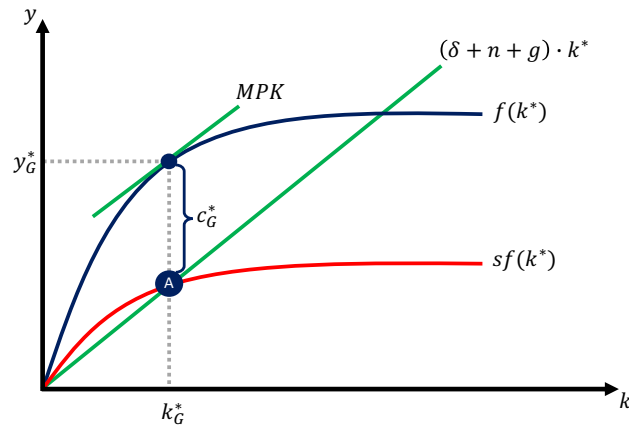


Figure 1: Golden rule standard growth model

In the *Golden Rule* state (Phelps, 1961) of a standard growth model, such as the Solow growth model (Solow, 1956) shown in figure 1, the equilibrium interest rate  $r^*$  is equal to the net marginal product of capital, which is equal to population growth plus technological progress, which is equal to the trend growth rate. Thus, in this equilibrium state,  $r^*$  is equal to the trend growth rate of the economy.

The second theoretical approach is based on a simple Keynesian macro model. Here, the neutral interest rate is the interest rate at which aggregate demand equals potential output. A graphical representation is given in figure 2:

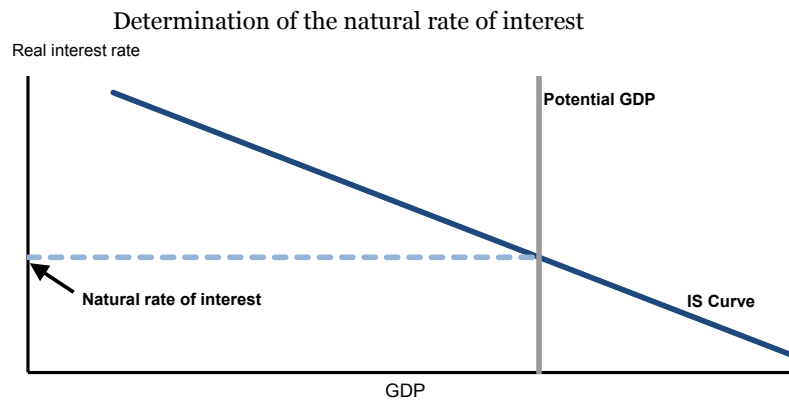


Figure 2: Determining  $r^*$  (Source: Laubach and Williams (2016, p. 2).

The figure shows a downward sloping IS curve, representing equilibria in the aggregate goods

market as a function of the real interest rate. Potential output is represented by a vertical line, which indicates a certain level of output consistent with the economy's potential. At the intersection of the IS curve and the vertical potential output line, the economy's actual output is equal to its potential output level. The interest rate at this point of intersection is therefore the interest rate that keeps economic output at its target level, i.e. r-star. [Laubach and Williams \(2001, p. 1\)](#), who provide the most widely used measure of r-star, define r-star in accordance with figure 2 as follows

*"[The equilibrium rate of interest is] the real short-term interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation"*

While the theoretical explanation is based on a Keynesian short-term macro model, Laubach-Williams use the theoretical concept based on the equivalence of r-star with the net marginal product of capital, i.e. the trend growth rate of the economy:<sup>1</sup>

$$r_t^* = c \cdot g_t + z_t, \quad (1)$$

With  $c \approx 1$ ,  $g_t$  equals the trend growth of real GDP, and  $z_t$  captures "other determinants", such as households' rate of time preference. And indeed, [Laubach and Williams \(2001\)](#) state: "Our natural rate estimates vary about one-for-one with changes in the trend growth rate." This can also be observed in the presentation in figure 3:

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<sup>1</sup>This implies a conflation of different models. The Keynesian model is a short-term business cycle model and the neoclassical growth model is a long-term growth model. These models are based on different assumptions, some of which are diametrically opposed ([Bofinger, 2020](#)).

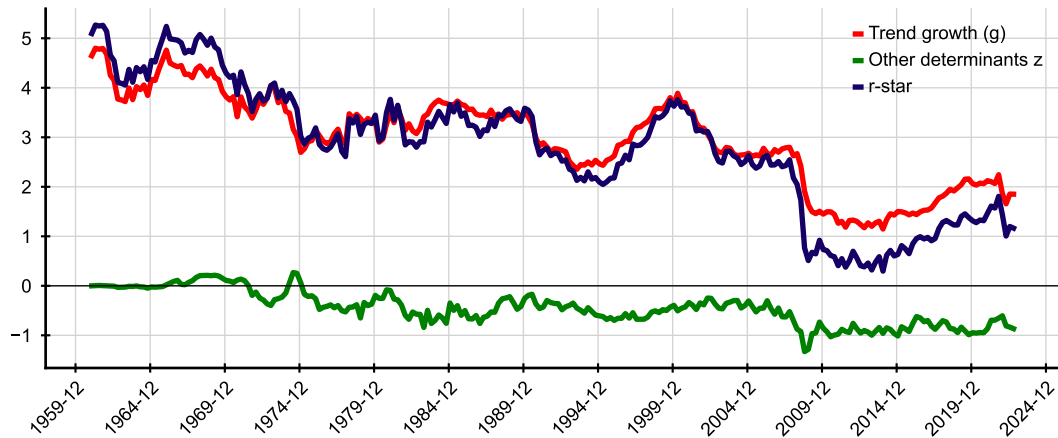


Figure 3: R-star and its components based on the one-sided estimates by [Laubach and Williams \(2003\)](#) (Source: NY Fed).

While a clear similar movement can be observed from the 1960s to the early 2000s, there is a visible and persistent gap between r-star and the trend growth rate since the financial crisis. A similar abrupt downward movement can be seen in the "other factors  $z$ " variable at this time. Overall, the figure 3 shows a persistent downward trend in r-star over the last 6 decades.

### 2.3 Problems of existing approaches in measuring r-star

There are several problems with Laubach and Williams' approach to estimating r-star: First, the strong and persistent downward trend in their r-star estimate is mainly driven by the *other factors*  $z_t$  variable, which is an autoregressive random walk term. For the related r-star estimation by [Holston, Laubach, and Williams \(2017\)](#), [Buncic \(2021\)](#) shows that the implementation of *other factors*  $z_t$  suffers from serious econometric weaknesses and that accounting for these problems leads to "estimates of 'other factor  $z_t$  [...] markedly lower, very close to zero, and highly statistically insignificant" ([Buncic, 2021](#), p. 2). The high sensitivity of the estimation procedure to small changes in some of the estimation variables in [Laubach and Williams \(2003\)](#) has also been shown by [Beyer and Wieland \(2015\)](#). The Laubach-Williams approach, like all r-star estimates, also suffers from a high degree of uncertainty, with confidence intervals of up to 3 percentage points above and below the point estimate. Furthermore, [Buncic \(2021, p. 2\)](#) shows that the estimates of

the natural rate and its components are extremely sensitive to the starting date of the sample used to estimate the model. A simple practical example shows that the relatively high values of  $r$ -star at the beginning of the estimated series are at least questionable: In the early 1960s, the real federal funds rate in the US (the federal funds rate minus the inflation rate) was between 1% and 2%, while inflation was around 2%. For this period, the neutral interest rate estimated by Laubach and Williams is around 4.5% to 5%. Thus, a neutral monetary policy stance at that time would have required an increase of more than 3 percentage points. The high values in the 1960s, which explain the secular decline in the neutral interest rate, are also an exception among the various estimates of the neutral interest rate, as Figure 14 in section 4.2 shows.

As we have outlined above, the Laubach-Williams approach is based on the conceptual idea that  $r$ -star corresponds to the net marginal product of capital in a neoclassical growth model. This is also reiterated by Holston et al. (2017, p. 60), who state that: "A useful starting point for modeling the natural rate of interest is the neoclassical growth model." However, an empirical examination of the net returns on the net capital stock does not indicate a decline, as Figure 4 shows:

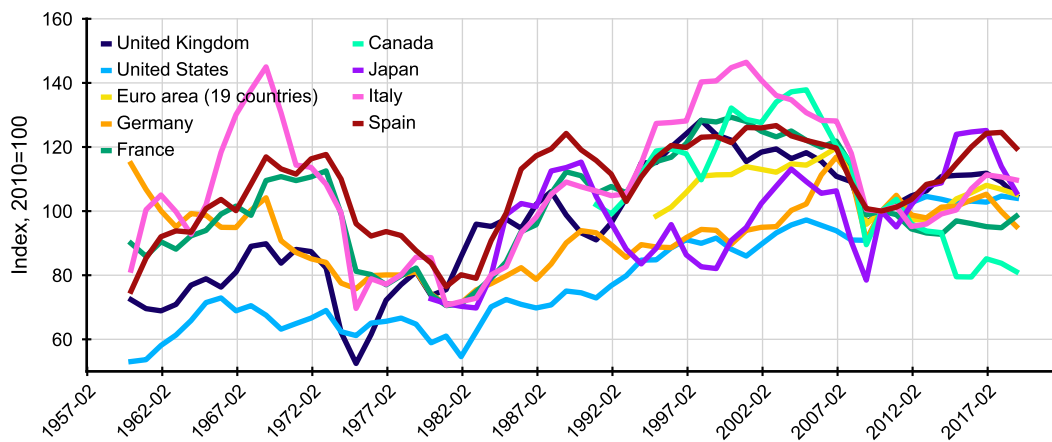


Figure 4: Net returns on net capital stock (Source: Ameco).

In terms of the structural change in the  $r$ -star estimates after the financial crises, it is also interesting to compare different estimates of the US output gap. Figure 5 provides a comparison between the Laubach-Williams estimates and the output gap values provided by FRED using

data from the Congressional Budget Office (CBO):

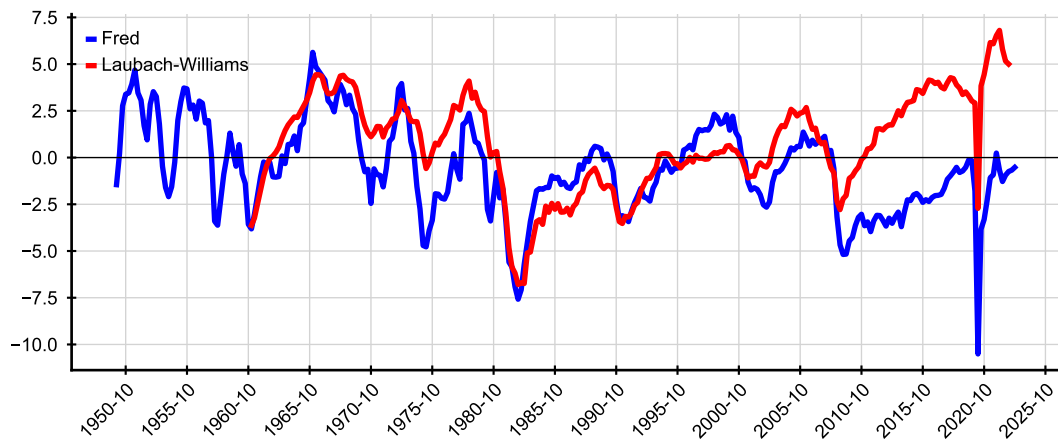


Figure 5: Output Gaps estimated by [Laubach and Williams \(2003\)](#) and Output Gaps as provided FRED, St. Louis FED based on CBO data (Source: CBO, NY Fed).

While there is a clear overlap in the series until the onset of the financial crisis, a large and persistent gap has emerged since then. These differences are not only significant, but also imply different states of the economy. While FRED consistently reports negative output gaps, suggesting that the economy is operating below potential, Laubach-Williams consistently report positive output gaps, suggesting that the economy is overheating, as it is operating above potential.

Another structural shock was the COVID-19 pandemic starting in 2020. The shock led to an abrupt change in the r-star estimate from levels around 1% to around -2.5% ([Holston et al., 2023](#)). This led to the decision to stop publishing r-star estimates:

*"After the onset of the pandemic, we suspended publication of r-star estimates due to this extreme economic volatility and the elevated uncertainty about how the pandemic would evolve."* ([J. C. Williams, 2023](#))

To correct for these highly volatile estimates, [Holston et al. \(2023\)](#) added a persistent supply shock to their model and a statistical procedure to account for outliers in their estimation. However, this led to some significant changes in r-star and its components, not only for the affected periods but for the whole estimation period, as shown in figure 6. The values for 2020Q2 are the last

published real time values before the COVID hiatus and the model adjustments.

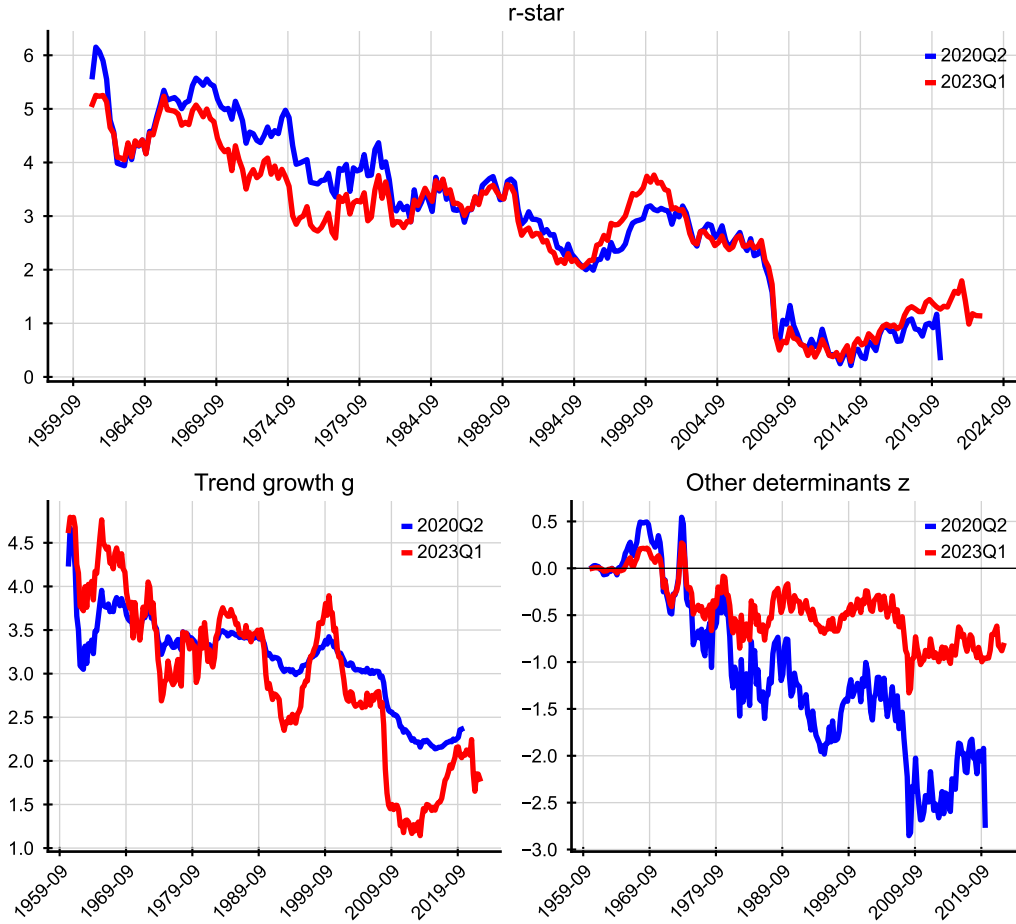


Figure 6: Changes in r-star and components pre and post Covid (Source: [Laubach and Williams \(2003\)](#), real time estimates, NY FED).

### 3 A simple macroeconomic model

#### 3.1 The static version

Given these limitations, we propose a new and simpler approach to determine a neutral interest rate based on a consistent theoretical framework. Similar to [Holston et al. \(2017\)](#), we start from a

simple macroeconomic model consisting of an IS curve, a Phillips curve and a monetary policy (MP) curve. This model was originally proposed by [Bofinger, Mayer, and Wollmershäuser \(2006\)](#). The model equations are given by

$$\text{IS: } y = a - b \cdot r + \varepsilon_1 \quad (2)$$

$$\text{PC: } \pi = \pi^e + d \cdot y + \varepsilon_2 \quad (3)$$

$$\text{MP: } r = \bar{r}(\varepsilon_1, \varepsilon_2), \quad (4)$$

where monetary policy is conducted according to a loss function  $L$ :

$$\text{Loss function: } L = (\pi - \pi^*)^2 + \lambda \cdot y^2 \quad (5)$$

Figure 7 shows a graphical representation of our 3-equation model. The *IS* curve (equation 2) represents all goods market equilibria. It is downward sloping in the  $y;r$  space, where  $y$  is the output gap and  $r$  is the real policy rate, i.e. the nominal policy rate  $i$  minus the inflation rate  $\pi$ . The downward slope indicates a higher level of output at lower interest rates. The Phillips curve (PC, equation 3) is upward sloping in the  $\pi;y$  space, indicating a positive relationship between the inflation rate  $\pi$  and the output gap  $y$ . By adjusting the slope of the Phillips curve, different degrees of correlation between the two variables can be taken into account. While the original Phillips curve ([Phillips, 1958](#)) shows a correlation between unemployment and the inflation rate, based on Okun's law ([Okun, 1962](#)), which suggests a negative relationship between unemployment and GDP, the unemployment rate can be replaced by the output gap. The *MP* curve (equation 4) represents the monetary policy of the central bank. The horizontal slope indicates that the central bank perfectly controls the real interest rate  $r$ , hence the  $\bar{r}$  in equation 4.<sup>2</sup> The loss function in equation 5 explains how the central bank conducts its monetary policy ([Svensson, 1999](#)). In equilibrium, the inflation rate is at its target value  $\pi^*$  and the level of output is at its potential, i.e. the output gap is closed ( $y = 0$ ). Demand shocks ( $\varepsilon_1$ ) and supply shocks

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<sup>2</sup>In reality, the central bank controls the nominal interest rate  $i$ . We use the real interest rate  $r$  for consistency between the top and bottom graphs and for ease of interpretation. Based on the Fisher equation  $i = r + \pi$ , for a given inflation rate  $\pi$ , the central bank can actually control the real interest rate by adjusting the nominal interest rate:  $r = i - \pi$ .

( $\varepsilon_2$ ) can lead to deviations from the target values. The central bank responds to these shocks according to the loss function, hence the  $r = \bar{r}(\varepsilon_1, \varepsilon_2)$  in equation 4. The terms in the loss function are squared so that both positive and negative deviations are considered losses. The squaring also ensures that larger deviations lead to a disproportionate increase in the total loss. The  $\lambda$  parameter is a preference parameter that takes into account the different importance of the two objectives. For example, a central bank that only targets the inflation rate would set  $\lambda = 0$ .

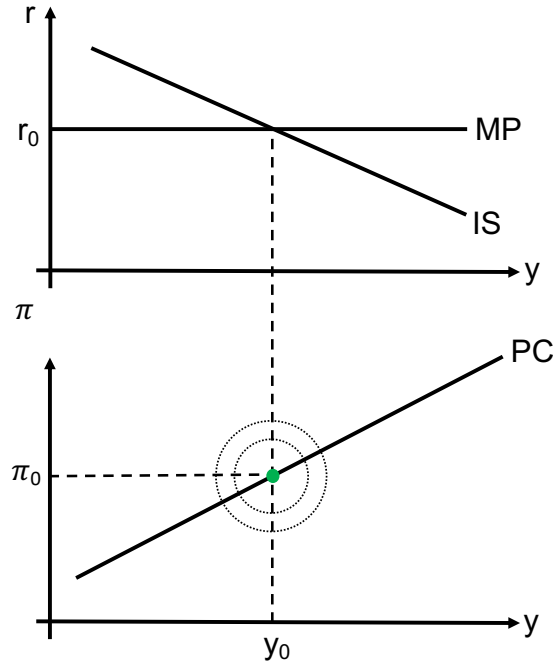


Figure 7: IS/MP/PC model

The simple and static structure of the model allows us to solve for an optimal monetary policy path. Assuming that the expected inflation rate  $\pi^e$  is equal to the inflation target  $\pi^*$ , we can derive the optimal policy rate in response to the exogenous shocks  $\varepsilon_1$  and  $\varepsilon_2$ , given that the central bank tries to minimise the loss function. To do this, we first plug the Phillips curve into the loss function

$$L = ((\pi^* + d \cdot y + \varepsilon_2) - \pi^*)^2 + \lambda \cdot y^2 \quad (6)$$

$$\Leftrightarrow L = (d \cdot y + \varepsilon_2)^2 + \lambda \cdot y^2 \quad (7)$$

Next, we derive an optimum value for the output gap  $y^{opt}$  from the loss function in equation 7:

$$\frac{\partial L(y)}{\partial y} = 2 \cdot (d \cdot y + \varepsilon_2) \cdot d + 2 \cdot \lambda y \stackrel{!}{=} 0 \iff d^2 \cdot y + d \cdot \varepsilon_2 + \lambda \cdot y = 0 \quad (8)$$

$$\iff y^{opt} = -\frac{d}{(d^2 + \lambda)} \cdot \varepsilon_2 \quad (9)$$

By inserting  $y^{opt}$  into the Phillips curve equation, we can derive an optimal value for the inflation rate  $\pi^{opt}$ :

$$\pi^{opt} = \pi^* + d \cdot y^{opt} + \varepsilon_2 \quad (10)$$

$$\iff \pi^{opt} = \pi^* + d \cdot \left(-\frac{d}{(d^2 + \lambda)} \cdot \varepsilon_2\right) + \varepsilon_2 \quad (11)$$

$$\iff \pi^{opt} = \pi^* + \frac{\lambda}{(d^2 + \lambda)} \cdot \varepsilon_2 \quad (12)$$

Finally, we can derive  $r^{opt}$  by inserting  $y^{opt}$  into the IS equation:

$$-\frac{d}{(d^2 + \lambda)} \cdot \varepsilon_2 = a - b \cdot r + \varepsilon_1 \quad (13)$$

$$\iff r^{opt} = \frac{a}{b} + \frac{1}{b} \cdot \varepsilon_1 + \frac{d}{b \cdot (d^2 + \lambda)} \cdot \varepsilon_2 \quad (14)$$

Equation 14 shows the central bank's optimal monetary policy response. The policy rate is a function of the two shock terms  $\varepsilon_1$  and  $\varepsilon_2$ . It is possible to include a third error term  $\varepsilon_3$  which accounts for policy shocks, i.e. when the central bank itself acts as a shock and sets the interest rate to a non-optimal value:

$$r^{opt} = \frac{a}{b} + \frac{1}{b} \cdot \varepsilon_1 + \frac{d}{b \cdot (d^2 + \lambda)} \cdot \varepsilon_2 + \varepsilon_3 \quad (15)$$

The optimal solution requires knowledge of the shock terms. Since these shock terms are not observable in reality, central banks often use heuristics to conduct monetary policy. The best known heuristic is the Taylor rule (Taylor, 1993):

$$r^{Taylor} = r^* + e \cdot (\pi - \pi^*) + f \cdot y \quad (16)$$

Similar to the loss function, the Taylor rule takes the inflation gap and the output gap and adds preference parameters  $e$  and  $f$  for the two gaps. The main difference is the addition of the neutral interest rate  $r^*$  as a baseline for the actual interest rate  $r$ . Unlike the loss function, the Taylor rule provides a direct measure of the policy rate. The main difference between the optimal and the Taylor rate is that the former is a function of the shocks ( $\varepsilon_1$  and  $\varepsilon_2$ ) and the latter is a function of the target variables ( $\pi^*$  and  $y = 0$ ). Of course, one can also include a monetary policy shock in the Taylor rule, indicating a policy rate that deviates from the actual Taylor rate:

$$r^{Taylor} = r^* + e \cdot (\pi - \pi^*) + f \cdot y + \varepsilon_3 \quad (17)$$

### 3.2 The dynamic version

Since the static version of the model has no time structure, monetary policy actions have no time lags, and demand shocks, which unlike supply shocks can be offset without trade-offs, would never actually materialise because the central bank would always react and offset the shock perfectly. In actual monetary policy making, however, these time lags can be large and long (e.g. [Friedman \(1960\)](#); [Friedman and Schwartz \(1963, 1982\)](#)). We therefore briefly present a dynamic version of the above model that takes account of these shortcomings:

$$\textbf{IS: } y_t = a + \gamma \cdot y_{t-1} - b \cdot r_{t-1} + \varepsilon_{1,t} \quad (18)$$

$$\textbf{PC: } \pi_t = \pi_{t-1} + d \cdot y_{t-1} + \varepsilon_{2,t} \quad (19)$$

$$\textbf{Taylor rule: } r_t = r_t^* + e \cdot (\pi_t - \pi^*) + f \cdot y_t + \varepsilon_{3,t}, \quad (20)$$

The dynamic equations correspond to the static versions, but an autoregressive term is added. The transmission channel of monetary policy starts from a change in the policy rate in period  $t$ , which affects output in period  $t + 1$  and inflation via the change in output in period  $t + 2$ . This is also shown in the simulations in [figure 8](#):

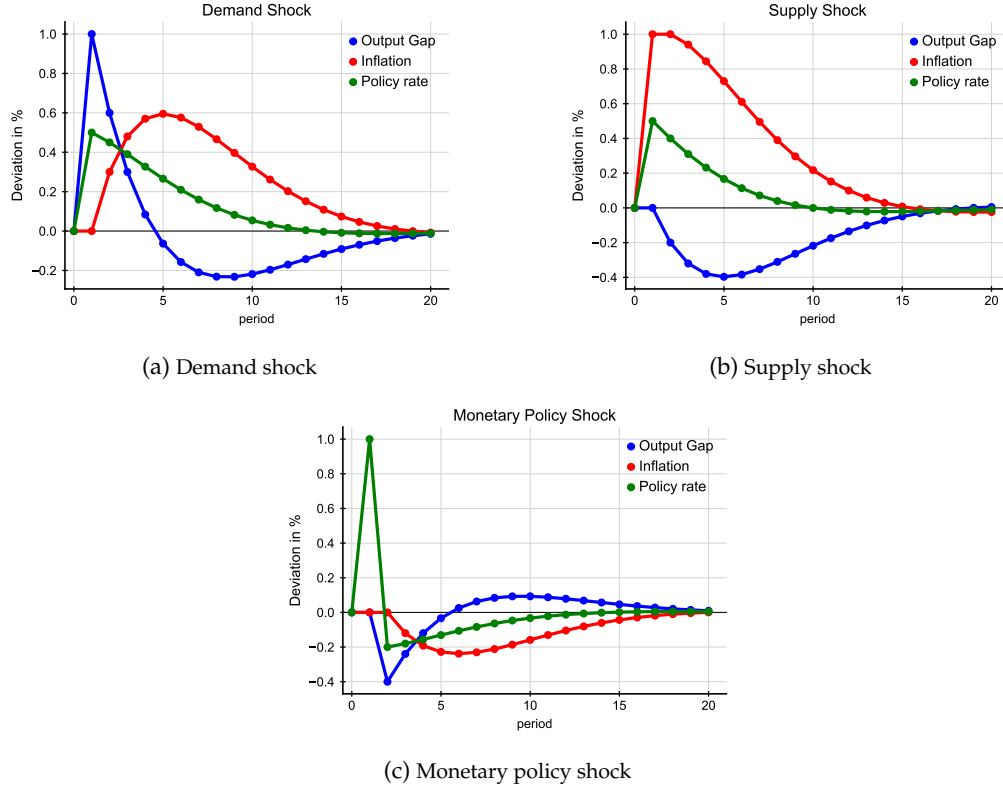


Figure 8: Transmission structure of shocks

## 4 A new approach towards r-star

### 4.1 Theoretical considerations

In this section we develop our strategy for deriving a measure of the neutral rate. To do this, we use the concept of the loss function and identify neutral periods, i.e. periods in which the loss function is close to zero.<sup>3</sup> Thus, in these periods,  $\pi \approx \pi^*$  and  $y \approx 0$ . We assume that a neutral period indicates the absence of shocks in the economy, i.e.  $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0$ . In this situation, the Taylor rule would simply say

$$r_t = r_t^* \quad (21)$$

<sup>3</sup>A loss function of exactly zero is an unrealistic goal in reality, so we set a low threshold for a loss and consider values below this threshold as negligible deviations from the target variables.

Thus, the Taylor rule would suggest setting the policy rate at the neutral rate. It could therefore be argued that the fact that a period is a neutral period with inflation and output at target is an indication that the actual real policy rate in that period is close to the neutral rate. Any shock would lead to a deviation from target and, given the time lags in monetary policy, the response to a shock would take time. This means that it is unlikely that a neutral period is actually a period in which the economy is hit by a demand shock, but the central bank has responded immediately and there are no time lags in the transmission of the monetary policy response.

From these observed manifestations of the neutral rate, we compute a moving average to smooth the adjustments between different manifestations. In non-neutral periods, we cannot observe new manifestations of the neutral rate and therefore insert the latest value of our averaged time series.

## 4.2 Empirical assessment

Figure 9 shows a loss function for the United States. We use the following loss function, and follow Taylor (1999) by setting the weight of the output gap to 1 and the inflation gap to 0.5 (neutral rate estimates for different weights are provided in the Appendix). The inflation target  $\pi^*$  is set at 2%:

$$L = y_t^2 + 0.5(\pi_t - \pi^*)^2 \quad (22)$$

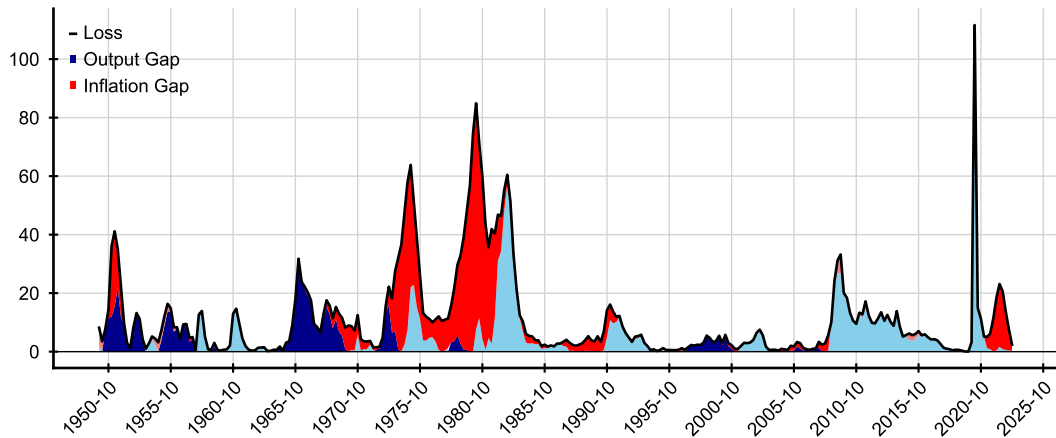


Figure 9: Loss function  $L = y_t^2 + 0.5(\pi_t - \pi^*)^2$  for the United States, lighter coloring indicates negative gaps, darker coloring indicates positive gaps (Source: FRED).

The loss function does a good job of capturing the evolution of the US economy over the past 70 years. The 1960s were fairly stable with low inflation. This is followed by a period of high inflation in the 1970s and early 1980s, the reaction of the Volcker Fed which brought inflation down but also a recession, the period of great moderation with low losses, the financial crises with rather persistent losses, and the COVID recession followed by high inflation rates.

In a next step, we set a threshold for the loss function below which we consider a period to be a neutral period. We set this threshold to 5, but also provide the results for different values in the appendix. Figure 10 shows the loss function together with the real federal funds rate. The shaded areas indicate neutral periods. As defined above, we consider the real federal funds rate during a neutral period to be equal to the neutral rate.

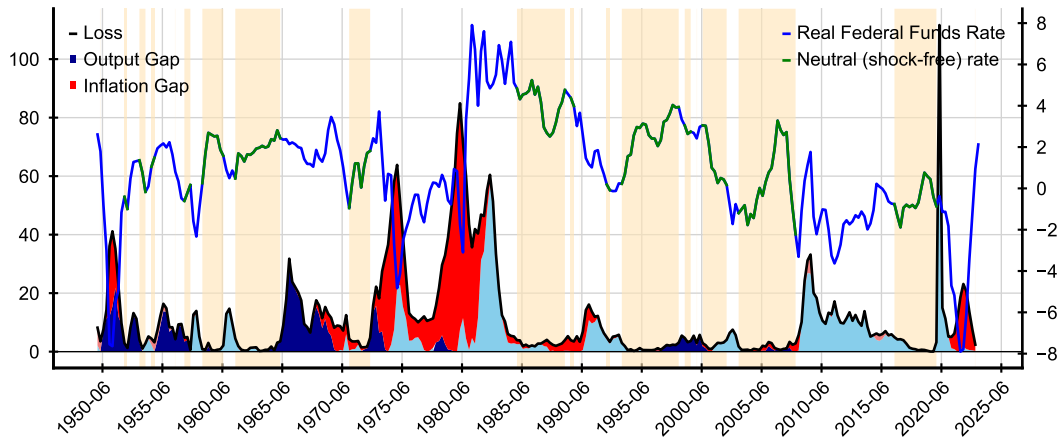


Figure 10: Loss function  $L = y_t^2 + 0.5(\pi_t - \pi^*)^2$ , shaded areas are neutral phases with  $L \leq 5$  (Source: FRED).

We identify several neutral periods in the 1950s and 1960s. The 1970s and early 1980s are largely non-neutral periods due to the high inflation rates and subsequent recession. The Great Moderation is largely a neutral period until the onset of the financial crisis. As our measure of the neutral rate is now 1) highly dependent on the current period and 2) has several gaps in non-neutral periods, making it difficult to compute, for example, a Taylor rate, we now compute a moving average over 12 quarters to get a smoother neutral rate. For non-neutral periods, we take the last available value of the neutral rate and use that value until we are back in a neutral period. This procedure also highlights the importance of using a moving average for our neutral rate. Figure 11 shows the actual and averaged values for the neutral rate:

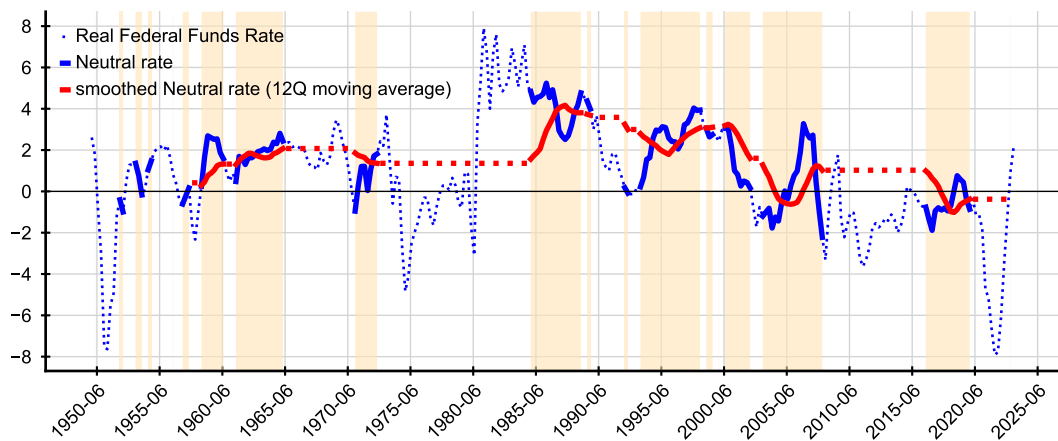


Figure 11: Neutral rate, shaded areas are neutral phases (Source: FRED).

Figure 12 shows a comparison of the neutral rate and r-star as estimated by [Laubach and Williams \(2003\)](#) series. The main observation is that our measure of r-star does not show the widely discussed secular decline in the neutral rate.

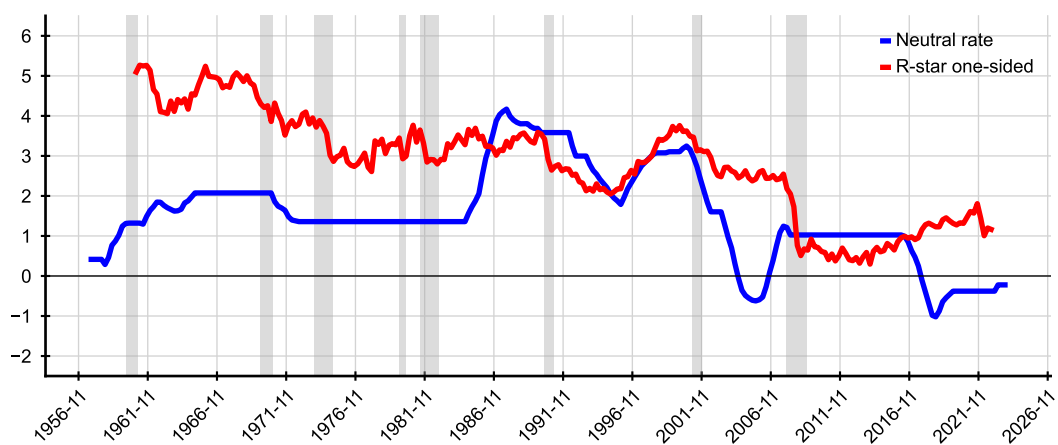


Figure 12: Neutral rate and r-star, shaded areas are recession phases (Source: FRED, NY FED).

The high level of uncertainty around r-star has given rise to a large literature using different methods to measure r-star. Figure 13 compares our measure of the neutral interest rate with some of the most prominent estimates:

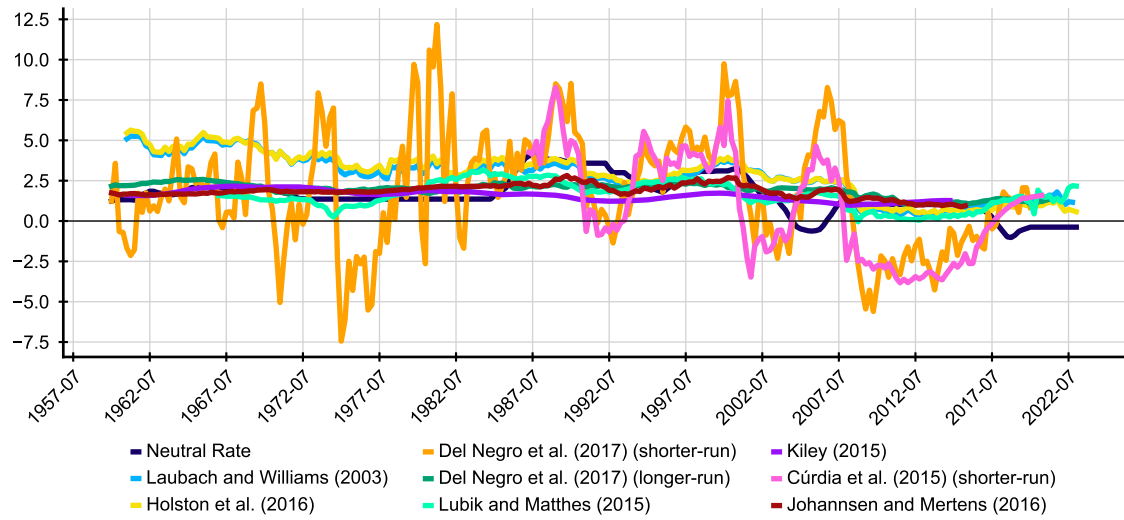


Figure 13: Neutral rate and different measures for r-star (Source: [Laubach and Williams \(2003\)](#), [Holston et al. \(2017\)](#), [Johansen and Mertens \(2016\)](#), [Del Negro et al. \(2017\)](#), shorter and longer rate; shorter rate series cut-off in 2020Q1 due to the strong downward swing to -30%), [Lubik and Matthes \(2015\)](#), [Kiley \(2015, case 3 scenario\)](#), [Cúrdia et al. \(2015\)](#)).

Figure 13 shows that our neutral rate measure is not an outlier among existing approaches. For a better comparison, figure 14 shows a one-to-one comparison of our neutral rate measure and the different r-star estimates:

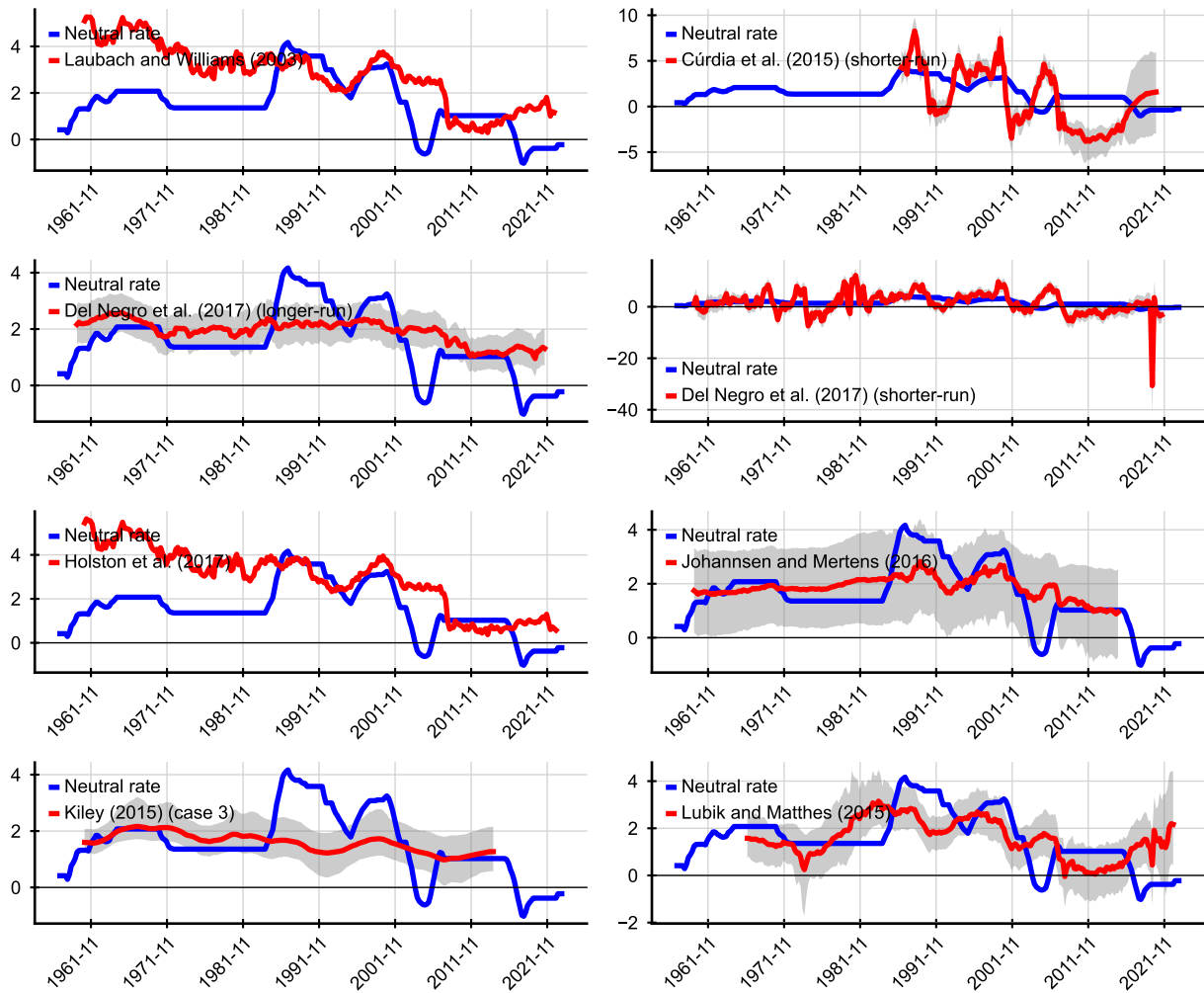


Figure 14: Neutral rate and different measures for  $r^*$ , confidence bands are added when available (Source: [Laubach and Williams \(2003\)](#), [Holston et al. \(2017\)](#), [Johansen and Mertens \(2016\)](#), [Del Negro et al. \(2017\)](#), [Lubik and Matthes \(2015\)](#), [Kiley \(2015, case 3 scenario\)](#), [Cúrdia et al. \(2015\)](#)).

Figure 14 shows that the high estimates of  $r^*$  in the 1960s by [Laubach and Williams \(2003\)](#) and [Holston et al. \(2017\)](#) appear to be an outlier among all estimates, as all other estimates do not show similarly high levels. Second, our simple measure of the neutral rate shows some correlation with most measures. It can also be seen as an intermediate measure between longer-term estimates such as [Laubach and Williams \(2003\)](#) or [Johansen and Mertens \(2016\)](#), which are less sensitive to economic changes, and shorter-term estimates such as [Cúrdia et al. \(2015\)](#) or [Del Ne-](#)

gro et al. (2017), which are much more volatile than our neutral rate. Figure 14 also highlights the high uncertainty of the r-star estimates, with confidence bands of up to 2 percentage points around the mean or median estimate. Our neutral rate measure often lies within the confidence bands of other measures, implying that the estimates are statistically indistinguishable.

Finally, we test the performance of our neutral rate measure for a monetary policy heuristic. We estimate a Taylor rate using our neutral rate and compare it with the actual real federal funds rate (figure 15).

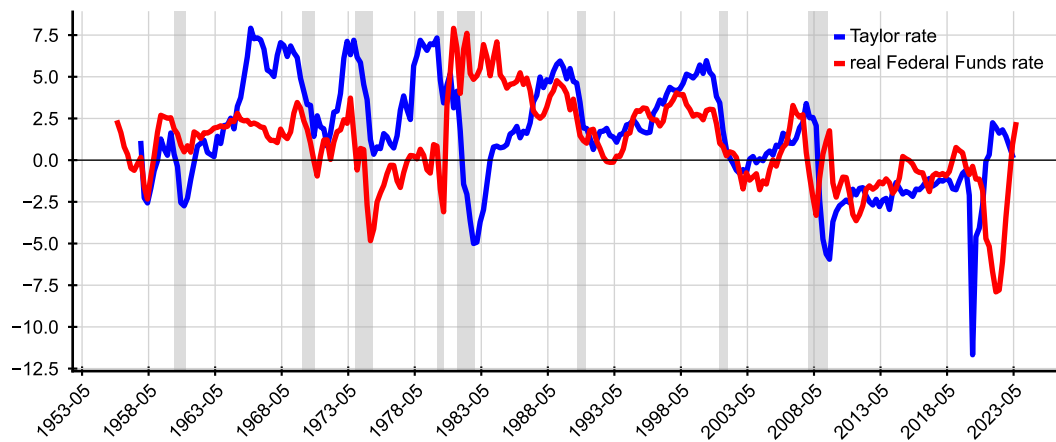


Figure 15: Taylor rate based on neutral rate:  $r^{Taylor} = r^{neutral} + 0.5 \cdot (\pi - \pi^*) + y$ , Shaded areas are recession phases. (Source: Fred).

Figure 15 shows that our Taylor rate is in fact very close to the actual real federal funds rate. Deviations in the late 1960s, when the actual real federal funds rate was well below the Taylor rate, were followed by high rates of inflation. Similarly, in the early 1980s, the Taylor rate was well below the real federal funds rate, suggesting that monetary policy was too restrictive at the time.

## 5 Conclusion

In this paper we have developed a simple measure of the neutral rate of interest based on the definition that "*[the equilibrium rate of interest is] the real short-term interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation*". (Laubach & Williams, 2001, p.1). We use a standard New Keynesian macroeconomic model with a central bank that follows a simple loss function in the conduct of its monetary policy. Assuming that a low "loss" is an indication of the absence of shocks in the economy, the real policy rate should be close to the unobservable neutral interest rate. By calculating a moving average, our measure of the neutral rate becomes less dependent on the most recent manifestation of the actual policy rate.

A comparison with existing measures of r-star shows that, despite the relative simplicity of the calculation, our measure of the neutral rate is broadly in line with several other measures. Recent revisions by Laubach and Williams (2003) and Holston et al. (2017) have also brought their estimate of r-star closer to our measure.

To check the validity of our measure, we estimate a Taylor rate for the US economy, which explains the FED's monetary policy quite well. Finally, in line with several other approaches estimating r-star, our neutral rate does not show a secular downward trend as estimated by Laubach and Williams (2003) and Holston et al. (2017). Rather, the 1980s were an outlier with rather high real interest rates, but the period before and after had rather similar levels.

Despite its simplicity, our methodology is based on a consistent theoretical derivation from a standard model and largely corresponds to existing approaches to determine r-star. Considering that r-star is a purely theoretical concept and that all approaches to its measurement suffer from very high uncertainty with confidence bands up to 6 percentage points, a simple calculation based on a consistent, purely theoretical model might even be advantageous.

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## A Appendix

As the threshold for neutral periods is somewhat arbitrary, figure 16 shows the neutral rate for different thresholds. The general picture does not change much.

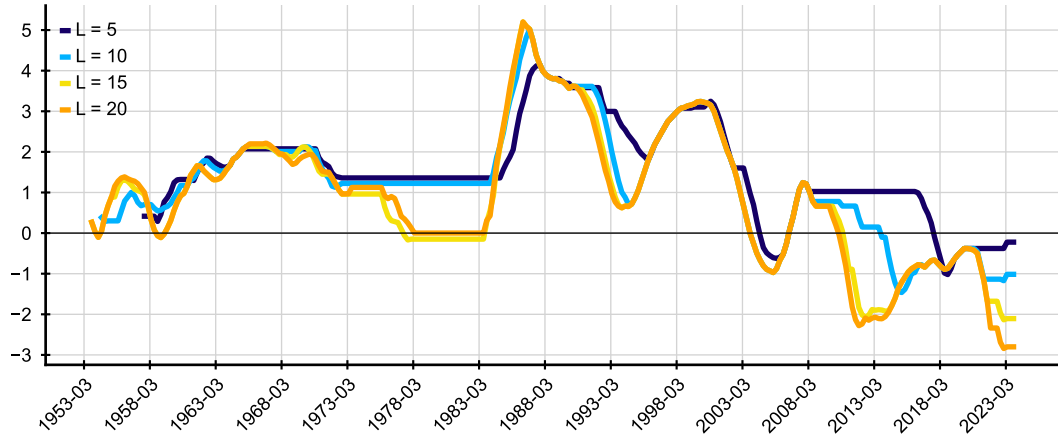


Figure 16: Neutral rate based on Loss function  $L = y_t^2 + 0.5(\pi_t - \pi^*)^2$ , and different thresholds  $L$  for neutral periods.

Figures 17 and 18 show the neutral rate measure calculated for different weights in the loss function. The red line indicates the default value used throughout the paper ( $L = y_t^2 + 0.5(\pi_t - \pi^*)^2$ ).

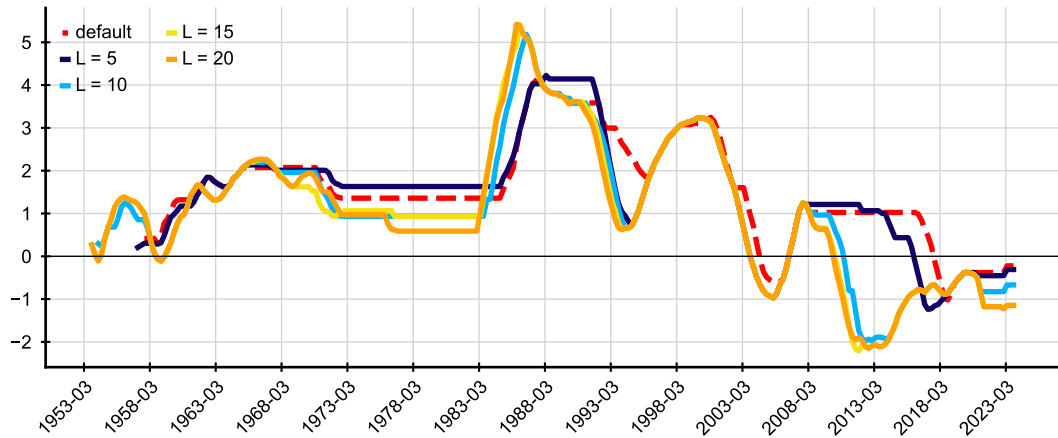


Figure 17: Neutral rate based on Loss function  $L = 0.5 \cdot y_t^2 + (\pi_t - \pi^*)^2$ , and different thresholds  $L$  for neutral periods..

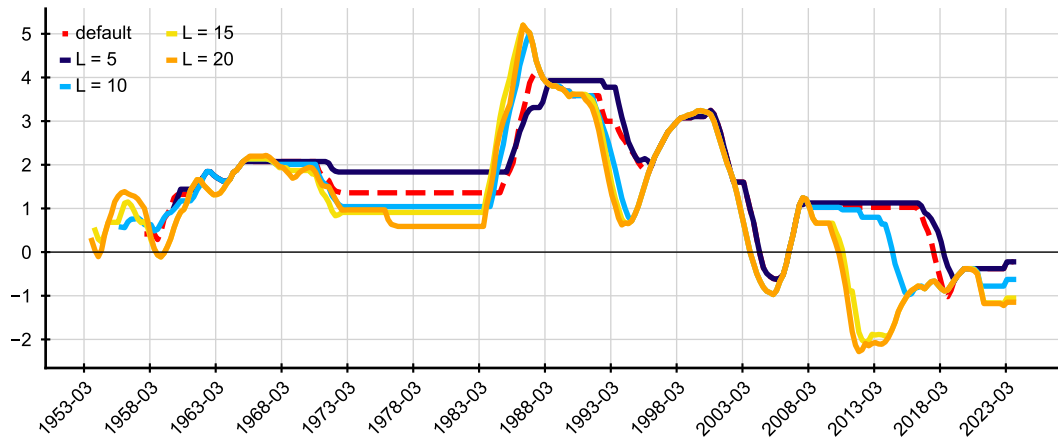


Figure 18: Neutral rate based on Loss function  $L = y_t^2 + (\pi_t - \pi^*)^2$ , and different thresholds  $L$  for neutral periods.