

Curello, Gregorio

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Incentives for Collective Innovation*

Gregorio Curello[†]

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Abstract

Identical agents exert hidden effort to produce randomly-sized improvements in a shared technology. Payoffs grow as the technology develops, but so does the opportunity cost of effort, due to an ‘exploration-exploitation’ trade-off. There exists a *unique* strongly symmetric equilibrium. Moreover, small innovations may hurt all agents in this equilibrium, as they severely reduce effort. Allowing agents to discard innovations increases equilibrium effort and welfare. If agents can conceal innovations, improvements are refined in secret until they are sufficiently large in equilibrium, and progress stops after a single disclosure. Under natural conditions, this equilibrium improves on all equilibria with forced disclosure.

Keywords: dynamic games, imperfect monitoring, public goods, private information.

1 Introduction

Innovation often has the features of a collective-action problem. Innovators gradually improve a ‘technology’ that they share, while simultaneously using it. For example, firms collaborate to refine their products,¹ nonprofit organisations draw

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[†]Institute for Microeconomics, University of Bonn. E-mail: gcurello@uni-bonn.de.

1. Knowledge exchange occurs within R&D partnerships but also among firms in the same ‘network’ (Pénin (2007)). The latter can generate a ‘collective’ kind of innovation, in which no single firm is the main driver of progress or its major beneficiary (Powell and Giannella (2010)). This occurred for biotechnologies (Powell, Koput, and Smith-Doerr (1996)), semiconductors (Chesbrough (2003) and Lim (2009)), and in the steel industry (von Hippel (1987)). See Bessen and Nuvolari (2016) for historical examples. ‘User’ innovation is collective in nature as well (Harhoff and Lakhani (2016)).

from a pool of shared knowledge to improve their programmes,² and workers in a team learn from each other how to perform their tasks better.³

However, innovation differs from traditional games of public-good provision in at least two respects. First, research effort is often unobserved and its returns are uncertain.⁴ As a consequence, it is difficult to monitor the effort exerted by one's partners. Second, innovators face a resource trade-off between using and improving the shared technology. For example, firm that collaborate towards a technological improvement allocate funds between its achievement and private activities that use the technology being developed.⁵ Nonprofits split resources between developing new ideas and managing existing programmes,⁶ and employees allocate their time and effort between 'being creative' and routine behaviour.⁷ Thus, as the shared 'technology' develops and using it becomes more profitable, the opportunity cost of improving it rises.⁸

In this paper, I analyse a parsimonious game of public-good provision with these two key features. Long-lived identical agents exert hidden effort to induce randomly-sized increments in the stock of a good (their frequency being increasing with effort). Agents' flow payoffs are a general function of their effort and of the current stock, and the marginal cost of effort increases (weakly) with the stock. The 'stock' is the quality of the technology that the agents share, and 'effort' measures the quantity of resources that each agent invests in the advancement of the technology, the rest being devoted to its use.

I characterise the social-welfare benchmark (Proposition 1) and show that the game admits a *unique* strongly symmetric equilibrium (Theorem 1). As monitoring

2. Performance monitoring, which facilitates the identification and imitation of effective programmes, has recently become prominent in the social sector (Kroeger and Weber (2014)). Moreover, innovation is known to play a critical role for nonprofits (Dover and Lawrence (2012)).

3. Research on workplace innovation has grown substantially in the recent years (Anderson, Potočník, and Zhou (2014)), and knowledge exchange within teams is a known predictor (van Knippenberg (2017)). Knowledge transfer among different *units* within a firm is also a key driver of innovation (Van Wijk, Jansen, and Lyles (2008)).

4. Geroski (1995) notes that 'R&D ventures may be unable to overcome moral hazard problems that lead participants to invest less than promised in the joint venture, divert the energies of people nominally assigned to [it, and] assign less talented researchers to [it...]'

5. The trade-off is due to financial constraints, a known barrier to innovation for firms (Hottenrott and Peters (2012)).

6. Balancing this 'exploration-exploitation' trade-off is a major challenge for nonprofits. See Dover and Lawrence (2012) and references therein.

7. This is Ford (1996)'s influential model of employee innovation. Unsworth and Clegg (2010) find confirming evidence for this model; they show that employees choose 'creative action' based on whether they judge it 'worthwhile', and argue that time pressure is pivotal to the decision.

8. In the example involving firms, 'using' the technology may also include running 'private' R&D which does not benefit the other firms. Meyer (2003) highlights this incentive effect: 'With the establishment of a profitable industry, technological uncertainty is reduced and the collective invention process evaporates. Surviving firms run private R&D.' Similarly, Powell and Giannella (2010) note that 'as technological uncertainty recedes, firms develop private R&D and focus on their own specific applications. Reliance on collective invention accordingly wanes.'

is imperfect, equilibrium effort is pinned down by the current stock; that is to say, no form of punishment is sustainable. Moreover, effort in *any* equilibrium ceases whenever it would cease in the single-agent setting (Proposition 2), and the Folk Theorem need not hold.

Continuation payoffs in the strongly symmetric equilibrium may fall after a small innovation (i.e. an increment in the stock), as all agents severely reduce their effort, delaying further increments. Thus mediocre advances in a technology can hinder its *overall* progress, and decrease its expected profitability.⁹ This phenomenon hinges on agents facing a trade-off between using and improving the technology, *and* on innovations (being ‘lumpy’ and) having random size (Proposition 3). I obtain a necessary and sufficient condition for the risk of such ‘detrimental’ innovations to persist as long as effort is exerted in equilibrium (Proposition 4), and characterise its occurrence when payoffs are linear (Corollary 2).

Because innovations have adverse effects on incentives, agents who obtain them may wish to delay their disclosure and adoption. To explore this idea, I first enrich the model by allowing agents to *discard* innovations, after observing their size. While this is a strong assumption, it leads to sharp results.¹⁰ Namely, the ensuing game admits a unique strongly symmetric equilibrium in which, after any history, both effort and continuation payoffs are higher than in the equilibrium of the baseline game (Theorem 2). Moreover, if innovations are beneficial with certainty when disclosure is compulsory, disposal does not occur even if allowed, and the equilibria coincide. Otherwise, the ex-ante payoffs in the equilibrium with disposal are strictly higher. Thus, in spite of the fact that discarding innovations is clearly inefficient, allowing the agents to do so enhances equilibrium welfare.

In reality, an agent who does not wish to disclose an innovation is more likely to conceal it than to discard it, and may disclose a refined version at a more profitable time. To check the robustness of the social benefits of selective disclosure, I enrich the baseline game by allowing agents who obtain innovations to conceal them from their partners, forgoing the larger payoff flow. At any later time, possibly after having obtained and concealed further innovations, each agent may disclose any portion of the *total* increment, increasing her payoffs and the public stock. Moreover, undisclosed increments obtained by different agents are ‘perfect substitutes’, so that concealment leads to redundant innovations.

If payoffs are linear, no strongly symmetric equilibrium with full disclosure exists in this environment (Lemma 1). Instead, I construct a symmetric perfect

9. Dover and Lawrence (2012) note that, among nonprofits, ‘successful past innovation can act as brakes on new ideas. Past success [...] has been shown to lead to complacency [...], structural barriers to innovation [...], and a lack of questioning the status quo [...]’.

10. Nothing would change if agents could instead conceal innovations, provided they cannot covertly refine the improvements that they hide. See the discussion at the end of Section 7.2.

Bayesian equilibrium in which a single innovation is disclosed (the first to exceed a common, time-varying cutoff) and, after this occurs, no effort is exerted (Theorem 3). If the number of agents is sufficiently large and the size of innovations sufficiently heavy-tailed,¹¹ payoffs exceed those of any equilibrium with forced disclosure (Proposition 5). Thus, even though concealing innovations is clearly inefficient, doing so is incentive-compatible and can improve equilibrium welfare.

The rest of the paper is organised as follows. I summarise the relevant literature in Section 2. I describe the model in Section 3, present the social-welfare benchmark in Section 4, and analyse the equilibria in Section 5. In Section 6, I examine when innovations are detrimental in the symmetric equilibrium. I analyse the game with disposal in Section 7, and the game with concealment in Section 8.

2 Literature review

This paper contributes to understanding the extent to which free-riding can be overcome in partnership games with frequent actions. Strongly Symmetric Equilibria (SSE) are known to sustain higher payoffs than symmetric Markov equilibria in dynamic games with perfect monitoring of the aggregate contribution, sometimes achieving efficiency.¹² In repeated games with imperfect (public) monitoring, only ‘bad news’ Poisson signals are helpful in SSE: there exist efficient SSE if perfectly-revealing bad news are available, but no SSE improves on the stage-game equilibrium payoffs if bad news are completely absent.¹³ As noted by Georgiadis (2015), the impossibility of cooperation under Brownian noise extends to dynamic (non-repeated) games: no SSE induces higher payoffs than the symmetric Markov equilibrium.¹⁴ Moreover, the Folk Theorem typically holds in partnership games with perfect monitoring, and a Folk Theorem for public perfect equilibria (PPE) applies in stochastic games with imperfect monitoring that satisfy irreducibility and identifiability conditions.¹⁵ However, Guéron (2015) shows that, in a partner-

11. See Azevedo et al. (2020) and the references therein for the importance of large and rare innovations.

12. Marx and Matthews (2000) analyse a dynamic game of private provision of a public good, and show that efficient SSE exist if payoffs jump upon reaching an exogenous goal, or if payoffs are ‘kinked’ at the goal and agents are sufficiently patient. Hörner, Klein, and Rady (2022) compute the payoffs attainable in SSE of games of experimentation, and show that efficiency is reached if payoffs have a diffusion component. See also Lockwood and Thomas (2002).

13. See Proposition 5 of Abreu, Milgrom, and Pearce (1991) for the former result and Sannikov and Skrzypacz (2010) for the latter. Other major contributions to the literature on repeated games with imperfect monitoring include Green and Porter (1984), Fudenberg, Levine, and Maskin (1994), Abreu, Pearce, and Stacchetti (1986), and Radner, Myerson, and Maskin (1986).

14. Cetemen, Hwang, and Kaya (2020) and Cetemen (2021) feature Brownian noise as well.

15. The Folk Theorem holds in the games of Marx and Matthews (2000), Lockwood and Thomas (2002), and Hörner, Klein, and Rady (2022), which feature perfect monitoring. Dutta (1995) derived a Folk Theorem for general stochastic games with perfect monitoring. Analogues

ship game with irreversible investments and monitoring subject to ‘smooth’ noise, there exists no PPE sustaining investments beyond the individually-rational level. In the model I analyse, free-riding cannot be overcome in SSE: the unique SSE is a symmetric Markov equilibrium. As I note in Section 5, the result is not driven by the lack of ‘bad news’ signals, or the ‘smoothness’ of noise. Moreover, the Folk Theorem need not hold (the aforementioned irreducibility condition fails).

The baseline model is closely related to the dynamic contribution games of Fershtman and Nitzan (1991), Marx and Matthews (2000), Lockwood and Thomas (2002), and Battaglini, Nunnari, and Palfrey (2014), as agents gradually add to the stock of a public good, and the incentives to produce drop as the stock grows. I contribute to this strand of the literature in two ways. First, I allow the production cost to depend on the current stock and show that, although a higher initial stock is beneficial absent its incentive effects, it may lead to lower equilibrium payoffs. Second, by allowing the stock to make discrete, randomly-sized jumps, I show that agents may have an incentive to discard or conceal increments in the stock, and I analyse the impact of allowing the agents to do so.

Games of strategic experimentation model social learning about the value of a *given* technology or project.¹⁶ Flow payoffs and incentives to produce (information) move jointly in these games: ‘good news’ simultaneously makes agents better off and experimentation more attractive. This implies that good news always increases continuation payoffs in equilibrium, and agents have no incentive to conceal it.¹⁷

In dynamic contribution games with a fixed goal, the incentive to contribute *rises* as the aggregate contribution grows.¹⁸ While this is not the case in the baseline model that I analyse, a similar structure arises *in equilibrium* if agents are allowed to conceal the innovations that they obtain (Section 8). However, in contrast to the literature, increments obtained by different agents are perfect substitutes.¹⁹

for games with imperfect monitoring were obtained by Fudenberg and Yamamoto (2011), Hörner et al. (2011), and Pęski and Wiseman (2015).

16. Important contributions to this literature include Bolton and Harris (1999), Keller, Rady, and Cripps (2005), Keller and Rady (2010), Klein and Rady (2011), Bonatti and Hörner (2011), Heidhues, Rady, and Strack (2015), and Keller and Rady (2015, 2020).

17. In the model I analyse, innovations *decrease* the incentive to exert effort. In a richer variant where agents learn about the productivity of their effort, the effect of innovations on incentives would be ambiguous, as they would raise the opportunity cost of effort but alter its (conjectured) productivity. However, it is reasonable to assume that sufficiently small innovations would decrease productivity, thus exacerbating their effect on incentives. Therefore some innovations are likely to remain harmful in equilibrium.

18. See e.g. Admati and Perry (1991), Strausz (1999), and Georgiadis (2017).

19. In Campbell, Ederer, and Spinnewijn (2014), agents obtain ‘successes’ at random times, and may conceal them in order to preserve their partners’ incentive to exert effort. Concealing innovations in my model is beneficial for the same reason. However, successes obtained by different agents in their model are perfect *complements*, whether or not they are concealed. If

This paper is also related to the large theoretical literature on innovation.²⁰ Reinganum (1983) argued that industry leaders are likely to be overtaken by new entrants in the innovation race. Even though agents do not compete in my model, overtaking occurs as well. However, it is more severe in that the ‘leader’ may be ex-ante worse off than the ‘follower’.²¹ Cetemen, Urgan, and Yariv (Forthcoming) analyse a model of collective search where discoveries accumulate over time, building on past ones. At any point in time, agents may quit unilaterally and irreversibly, and exploit the best discovery to date. Just as in the model I analyse, (search) effort yields stochastic rewards and its marginal cost rises as progress is made. Yet continuation payoffs are guaranteed to increase with progress in equilibrium, as discoveries are arbitrarily small.²² In the endogenous-growth models of Lucas and Moll (2014) and Perla and Tonetti (2014), agents face a resource trade-off between using an existing technology and improving it, exactly as in my model. However, innovations are always beneficial, as aggregate technological progress is deterministic.

3 Model

In this section, I describe the baseline model and discuss its main assumptions.

Time is continuous, indexed by $t \in \mathbb{R}_+$, and discounted at rate $r > 0$. There are $n \geq 2$ identical agents, indexed by $i \in \{1, \dots, n\}$, and a public good. Its time- t stock is denoted x_t . At any time $t \geq 0$, agent i exerts effort $a_t^i \in [0, 1]$ and receives a flow payoff $ru(a_t^i, x_t)$, where $u : [0, 1] \times \mathbb{R}_+ \rightarrow \mathbb{R}$. Agents’ effort is hidden.

The stock x_t takes some initial value $x_0 \geq 0$ and, for $t > 0$, is determined as follows. Agent i produces an increment in x_t at rate λa_t^i , where $\lambda > 0$. Each increment has (possibly) random size z , drawn from a CDF F with mean $\mu < \infty$ and such that $F(0) = 0$. The production and the size of increments are independent

innovations have a fixed value in my model (as successes do in theirs), agents do not conceal them in equilibrium, as concealed increments are perfect substitutes. See Online Appendix L.1.

20. Important contributions include Brander and Spencer (1983), Spence (1984), Katz (1986), d’Aspremont and Jacquemin (1988), Kamien, Muller, and Zang (1992), Suzumura (1992), and Leahy and Neary (1997).

21. In detail, consider two groups of agents (e.g. two distinct R&D partnerships) playing the equilibrium of the baseline game (Theorem 1). If the first innovation obtained across groups makes the group who obtained it (the ‘leaders’) worse off, the group of ‘followers’ (which therefore has a lower stock at the time of the innovation) is likely to have a higher stock in the near future. This is because the continuation payoffs of the followers are higher, even though their *flow* payoffs are lower as long as their stock lies below that of the leaders.

22. In detail, agents in Cetemen, Urgan, and Yariv (Forthcoming) exert costly effort to increase the variance of a Brownian motion, and may quit irreversibly at any time, to enjoy its highest realised value. Given a realised path $(X_t)_{t \geq 0}$ and times $s < t$, continuation payoffs (net of past exerted effort) are higher at time t than at time s if either (i) $\max_{[0,s]} X = \max_{[0,t]} X$ and $X_s \leq X_t$, or (ii) $\max_{[0,s]} X \leq \max_{[0,t]} X = X_t$.

across agents, time, and from each other. The arrival of increments, their size, as well as the identity of the agents inducing them, are public. Suppose without loss of generality that $r = 1$ (this is equivalent to rescaling λ and payoffs by $1/r$).

Assume that $u(a, x)$ is twice continuously differentiable, and (weakly) increasing and (weakly) concave in x . That is, keeping the effort fixed, the payoff increases as the stock grows, but at a decreasing rate. Note that

$$u(a, x) = b(x) - c(a, x)$$

where $b(x) := u(0, x)$ are gross benefits, and $c(a, x) := u(0, x) - u(a, x)$ is the opportunity cost of effort. Suppose that $c(a, x)$ is increasing and convex in a , and *strictly* increasing in a if $x > 0$. Suppose also that the first and second partial derivatives of c with respect to a (denoted c_1 and c_{11} , respectively) are increasing in x ; that is, the cost of effort becomes steeper and more convex as the stock grows. Assume also that

$$\lim_{x \rightarrow \infty} \lambda \mu n b'(x) - c_1(a, x) < 0 \quad \text{for all } 0 < a \leq 1. \quad (1)$$

This means that any fixed amount of positive effort is inefficiently large if the stock is large enough. (Note that $\lambda \mu n b'(x)$ approximates the marginal social benefit of effort when the stock x is large, whereas $c_1(a, x)$ is its marginal cost.)

For some of the results,²³ we shall further assume that payoffs take the following *linear multiplicative* form:

$$b(x) = x \quad \& \quad c(a, x) = ax, \quad (2)$$

and that

$$F(z) > 0 \quad \text{for all } z > 0. \quad (3)$$

This means that the size of increments is random, and that arbitrarily small (but strictly positive) sizes are possible. (To analyse the game with concealment in Section 8, we shall instead assume that F is Lipschitz.)

In the main interpretation of the model, the stock x_t denotes the (quality of a shared) *technology*, and increments in x_t are *innovations*. These terms are used throughout the discussion. Given this interpretation, linear multiplicative payoffs may be understood as follows. In each period, (risk-neutral) agents face a binary decision between using and improving the technology. Improving the technology

23. In detail, I impose (2) to derive closed-form expressions for the equilibrium I study (Corollary 1), as well as in the game with concealment (Section 8); I assume (3) for some results regarding detrimental innovations (in Section 6) and to characterise the incentive to conceal progress in equilibrium (Lemma 1).

($a_t^i = 1$) yields no payoff, and using it ($a_t^i = 0$) yields a payoff equal to its current quality, x_t . (We may interpret $0 < a_t^i < 1$ as improving the technology with probability a_t^i and using it with probability $1 - a_t^i$.)

We shall illustrate the results using linear multiplicative payoffs and the distribution

$$F(z) = \rho \mathbf{1}_{z \geq \zeta} + (1 - \rho)(1 - e^{-\frac{z}{\epsilon}}), \quad (4)$$

where $\rho \in [0, 1)$ and $\zeta > \epsilon > 0$. Thus, with probability ρ , the innovation is ‘substantial’ and it has size ζ ; otherwise, it is a small improvement, with size drawn from the exponential distribution with mean ϵ .

Below is a brief description of histories, strategies and continuation payoffs. Formal definitions are in Online Appendix H.1. Note that (almost surely) only finitely many innovations are produced within any bounded period of time. Since effort is hidden, a (public) *history* is a finite sequence

$$h_m := (x_0, (t_1, z_1, i_1), \dots, (t_m, z_m, i_m)) \quad (5)$$

such that agent i_1 obtains an innovation of size z_1 at time t_1 , agent i_2 one of size z_2 at time $t_2 > t_1$, and so on. In particular, the stock after the m th innovation is

$$X(h_m) := x_0 + \sum_{l=1}^m z_l.$$

Agents simultaneously reach a new history whenever an innovation is produced. Since past exerted effort has no direct payoff relevance, we may without loss of generality restrict attention to public strategies (i.e., strategies that can be expressed as functions of public histories). Moreover, for any mixed perfect Bayesian equilibrium (in public strategies), there exists a public perfect equilibrium in pure strategies inducing the same distribution over terminal public histories, and the same ex-ante payoffs (Lemma 8 in Online Appendix H.1). Thus, we may without loss restrict attention to pure strategies.

A (public, pure) *strategy* σ^i specifies, for each history h_m (including $h_0 := x_0$), an effort schedule $\sigma^i(h_m) : (t_m, \infty) \rightarrow [0, 1]$ (where $t_0 := 0$). Agent i exerts effort $[\sigma^i(h_m)](t)$ at any time $t > t_m$ such that no innovation was produced within the time interval $[t_m, t)$. If agents play a strategy profile $\sigma := (\sigma^i)_{i=1}^n$, agent i 's continuation payoff at a history h_m (is well-defined and) may be expressed as

$$v_\sigma^i(h_m) := \mathbb{E} \left(\sum_{l=m}^{\tilde{m}} \int_{\tilde{t}_l}^{\tilde{t}_{l+1}} e^{t_m - t} [b(X(\tilde{h}_l)) - c([\sigma^i(\tilde{h}_l)](t), X(\tilde{h}_l))] dt \right) \quad (6)$$

where $\tilde{m} \in \{m, m + 1, \dots, \infty\}$ is the total number of innovations produced, \tilde{h}_l is

the history reached upon the l^{th} innovation (at time \tilde{t}_l), $\tilde{h}_m := h_m$, $\tilde{t}_m := t_m$ and, if $\tilde{m} < \infty$, $\tilde{t}_{\tilde{m}+1} := \infty$.

Discussion of the assumptions. The map from effort to increments in the stock is stationary and ensures that (i) the stock is increasing and (ii) its trajectory does not perfectly reveal the trajectory of aggregate effort. If effort were binary, these properties would uniquely pin down the map. Alternatively, effort could influence the distribution F ; this would complicate the analysis but is likely to yield similar insights, provided higher effort not only increases the size of increments on average, but also its dispersion.

In the literature on dynamic public-good games, payoffs are typically assumed to be separable (formally, c is a function of a alone). The assumptions I impose on c are weaker and ensure that the efficient level of effort decreases as the stock grows, a central feature in the contribution games without a fixed goal. Moreover, (1) weakens the standard assumption of vanishing returns to production (i.e., $\lim_{x \rightarrow \infty} b'(x) = 0$).

4 Social-welfare benchmark

In this section, I describe how non-strategic agents should behave in order to maximise aggregate payoffs. The main features of this benchmark are common in dynamic public-good games. In particular, any innovation is beneficial.

A strategy is Markov if effort is pinned down by the current stock. Formally, a *Markov strategy* (for agent i) is a Borel measurable map $\alpha : \mathbb{R}_+ \rightarrow [0, 1]$ such that $a_t^i = \alpha(x_t)$ for all $t \geq 0$. If agents play a Markov profile $\alpha := (\alpha^i)_{i=1}^n$, the time- t continuation payoff of each agent i is a function $v_\alpha^i(x_t)$ of the current stock x_t . Moreover, for all $x \geq 0$,

$$v_\alpha^i(x) = b(x) - c(\alpha^i(x), x) + \lambda \sum_{j=1}^n \alpha^j(x) \{ \mathbb{E}_F [v_\alpha^i(x + \tilde{z})] - v_\alpha^i(x) \}. \quad (7)$$

That is to say, agent i 's continuation payoff is the sum of the current payoff flow $b(x) - c(\alpha^i(x), x)$, and the net expected future benefit. This is given by the rate $\lambda \sum_{j=1}^n \alpha^j(x)$ at which innovations occur, multiplied by their expected social value; that is, the difference between the continuation payoff $v_\alpha^i(x+z)$ after an innovation of size z , and the current payoff $v_\alpha^i(x)$, weighted by the distribution F of z .

Welfare is the average of ex-ante payoffs across agents. Since agents are identical and the cost of effort $c(a, x)$ is convex in $a \in [0, 1]$, and the rate of arrival of innovations λa is linear in effort, it is efficient for all agents to exert the same amount of effort. Then, from (7), the Bellman equation for the maximal welfare

achievable in the game is

$$v(x) = \max_{a \in [0,1]} \{b(x) - c(a, x) + a\lambda n \{\mathbb{E}_F[v(x + \tilde{z})] - v(x)\}\}. \quad (8)$$

Standard dynamic-programming arguments imply that, if (8) admits a (well-behaved) solution v_* then, for any initial stock $x_0 \geq 0$, $v_*(x_0)$ is the maximal achievable welfare, across all (Markov and non-Markov) strategy profiles.²⁴ Moreover, any Markov strategy α solving

$$\alpha(x) \in \arg \max_{a \in [0,1]} \{a\lambda n \{\mathbb{E}_F[v_*(x + \tilde{z})] - v_*(x)\} - c(a, x)\} \quad (9)$$

for all $x \geq 0$, induces welfare $v_*(x_0)$.

The following result shows that, in order to maximise welfare, agents should exert less effort as the stock grows. Moreover, any increment in x_t makes agents better off.

Proposition 1. *There exists a Markov strategy α_* that,²⁵ if played by all agents, maximises welfare for all initial stocks $x_0 \geq 0$. Effort $\alpha_*(x_t)$ is decreasing in the stock x_t , and $\alpha_*(\tilde{x}_t) \rightarrow 0$ a.s. as $t \rightarrow \infty$. Maximal welfare $v_*(x_0)$ is increasing in x_0 , and $v_*(x_0) - b(x_0)$ is decreasing and vanishes as $x_0 \rightarrow \infty$.*

I prove Proposition 1 in Online Appendix I. There, I also characterise the long-run value of the stock x_t when α_* is played (Corollary 1).

The fact that effort α_* is decreasing is standard in dynamic public-good games without a fixed goal, and is generally due to concave payoffs. In this model, not only are payoffs concave in x_t , but the cost of effort becomes steeper and more convex as x_t grows (formally, $c_1(a, x)$ and $c_{11}(a, x)$ are increasing in x). The latter force adds to the former and does not alter the qualitative features of the benchmark.

Maximal welfare v_* is increasing in the initial stock x_0 because higher x_0 yields a larger payoff flow ($b(x) - c(a, x)$ is increasing in x) without altering the productivity of effort (the frequency and size of innovations does not depend on x). Moreover, since each agent's time- t continuation payoff when α_* is played equals $v_*(x_t)$, any increment in the stock (that is, any innovation) is beneficial. We shall see that, despite this, welfare need not increase with x_0 in equilibrium, so that innovations may be detrimental.

24. See e.g. Theorem 3.1.2 of Piunovskiy and Zhang (2020). It is sufficient that (8) admits a Borel solution $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ such that $b(x) \leq v(x) \leq b(x) + n\lambda[b(\mu) - b(0)]$ for all $x \geq 0$.

25. Unless c is strictly convex, (9) may admit more than one solution α . α_* is the pointwise smallest.

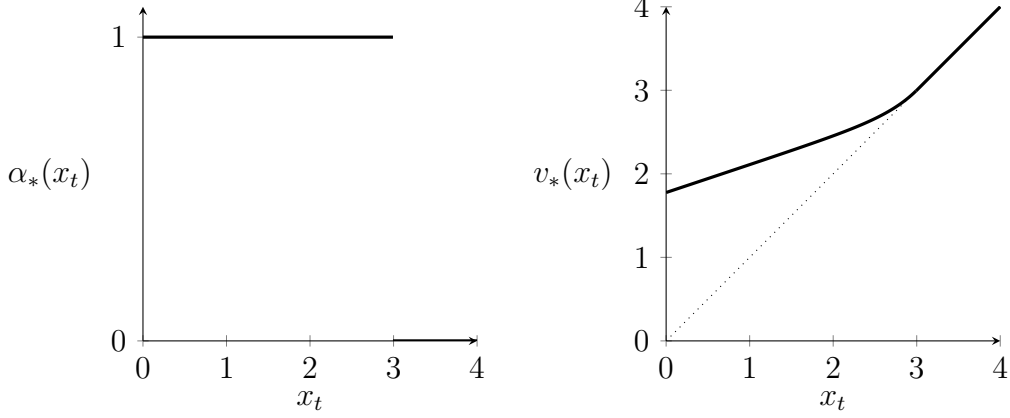


Figure 1: Effort (left) and welfare (right) in the social-welfare benchmark, as functions of the stock x_t . Parameter values are $b(x) = x$, $c(a, x) = ax$, $F(z) = \rho \mathbf{1}_{z \geq \zeta} + (1 - \rho)(1 - e^{-z/\epsilon})$, $\lambda = 10$, $\rho = \epsilon = 0.01$, and $\zeta = n = 5$.

5 Equilibrium

In this section, I show that the game admits a unique strongly symmetric equilibrium (Theorem 1), and derive it in closed-form for linear and multiplicative payoffs (Corollary 1). I also show that effort in any public perfect equilibrium ceases when it would stop in the single agent problem (Proposition 2).

Given a history h featuring $m \in \{0, 1, \dots\}$ innovations, write $h \circ (t, z, i)$ for the history that features $m + 1$ innovations and extends h , and in which the last innovation is produced by agent i at time t , and has size z . Recall that, if agents play a strategy profile $\sigma := (\sigma^i)_{i=1}^n$, $v_\sigma^i(h)$ is the payoff to agent i in the subgame after history h . Given h leading to some time $t_h \geq 0$, for all $t > t_h$, let $v_{\sigma, h}^i(t)$ be agent i 's payoff at time t , given that no innovation was produced within the interval $[t_h, t)$ (and define $v_{\sigma, h}^i(t_h) := v_\sigma^i(h)$). Then, $v_{\sigma, h}^i : [t_h, \infty) \rightarrow \mathbb{R}$ is Lipschitz and, labelling x the stock that h leads to, for almost all $t > t_h$,²⁶

$$v_{\sigma, h}^i(t) = \frac{dv_{\sigma, h}^i}{dt}(t) + b(x) - c([\sigma^i(h)](t), x) + \lambda \sum_{j=1}^n [\sigma^j(h)](t) \{ \mathbb{E}_F[v_\sigma^i(h \circ (t, \tilde{z}, j))] - v_{\sigma, h}^i(t) \}. \quad (10)$$

Compared to (7), effort depends on the current time (as the strategy profile played need not be Markov). As a consequence, agent i 's payoff evolves at rate $dv_{\sigma, h}^i/dt$ even in the absence of innovations.

Let $\hat{v}_\sigma^i(h)$ be the largest payoff that agent i can achieve, across all strategies, in the subgame after history h . Assuming h leads to time $t_h \geq 0$, for all $t > t_h$,

26. See Online Appendix H.1 for details on how to derive (10), (11), (12), (13) and (14).

let $\hat{v}_{\sigma,h}^i(t)$ be the largest payoff achievable by agent i at time t , given that no innovation was produced within the interval $[t_h, t)$ (and define $\hat{v}_{\sigma,h}^i(t_h) := \hat{v}_{\sigma}^i(h)$). Then, the map $\hat{v}_{\sigma,h}^i : [t_h, \infty) \rightarrow \mathbb{R}$ is Lipschitz and, for almost all $t > t_h$,

$$\begin{aligned} \hat{v}_{\sigma,h}^i(t) = & \frac{d\hat{v}_{\sigma,h}^i}{dt}(t) + b(x) + \max_{a \in [0,1]} \{a\lambda \{ \mathbb{E}_F[\hat{v}_{\sigma}^i(h \circ (t, \tilde{z}, i))] - \hat{v}_{\sigma,h}^i(t) \} - c(a, x) \} \\ & + \lambda \sum_{j \neq i}^n [\sigma^j(h)](t) \{ \mathbb{E}_F[\hat{v}_{\sigma}^i(h \circ (t, \tilde{z}, j))] - \hat{v}_{\sigma,h}^i(t) \}. \end{aligned} \quad (11)$$

The strategy σ^i is a best response for agent i at all histories only if, given any history h leading to any time t_h , for almost all $t > t_h$,

$$[\sigma^i(h)](t) \in \arg \max_{a \in [0,1]} \{a\lambda \{ \mathbb{E}_F[\hat{v}_{\sigma}^i(h \circ (t, \tilde{z}, i))] - \hat{v}_{\sigma,h}^i(t) \} - c(a, x) \}. \quad (12)$$

The profile σ is a *public perfect equilibrium* (PPE) if, for each i , σ^i is a best response for agent i at all histories. If moreover, $\sigma^i = \sigma^j$ for all i and j , then σ is a *strongly symmetric equilibrium* (SSE).

Recall the definition of Markov strategies from Section 4. If the opponents of a given agent play a Markov strategy α , the largest continuation payoff that she can achieve, across all strategies, is a (value) function \hat{v}_{α} of the current stock x_t , symmetric across agents. From (7), \hat{v}_{α} is the unique (well-behaved) solution to

$$v(x) = \max_{a \in [0,1]} \{b(x) - c(a, x) + \lambda[a + (n-1)\alpha(x)] \{ \mathbb{E}_F[v(x + \tilde{z})] - v(x) \} \}. \quad (13)$$

Then, α is a best response, after any history, against opponents playing the same strategy if and only if

$$\alpha(x) \in \arg \max_{a \in [0,1]} \{a\lambda \{ \mathbb{E}_F[\hat{v}_{\alpha}(x + \tilde{z})] - \hat{v}_{\alpha}(x) \} - c(a, x) \} \quad (14)$$

for all $x \geq x_0$.

The following result characterises the unique strongly symmetric equilibrium of the game. The proof is in Appendix A.

Theorem 1. *There exists a unique SSE, and it is induced by a Markov strategy α_f . Effort $\alpha_f(x_t)$ is continuous and decreasing in the stock x_t , and lies below the benchmark $\alpha_*(x_t)$. Moreover, $v_f(x_t) - b(x_t)$ is decreasing in x_t , where $v_f(x_t)$ is the equilibrium continuation payoff given x_t .*

Provided (3) holds, the equilibrium α_f is inefficient unless no effort is optimal (Corollary 2 in Online Appendix J). This is caused by ‘intertemporal free riding’ (Marx and Matthews (2000)): since α_f is decreasing, agents are reluctant to exert

effort as this causes their opponents to exert less effort in the future.

The analyses of Marx and Matthews (2000), Lockwood and Thomas (2002), and Hörner, Klein, and Rady (2022) suggests that the uniqueness of the SSE relies on the fact that monitoring is imperfect. If either aggregate effort were observable, or could be perfectly inferred from the trajectory of the stock, there would exist multiple SSE and some would induce higher payoffs than the (unique) symmetric Markov equilibrium. Note however that, although monitoring is imperfect in this model, it may be arbitrarily precise: if the distribution F of the size of innovations equals $z \mapsto F_0(\lambda z)$ for some fixed CDF F_0 , and their arrival rate λ is large, effort essentially induces a deterministic and continuous drift in the stock (with approximate growth rate $\mu_0 \sum_{i=1}^n a_t^i$, where μ_0 is the mean of F_0), so that the past trajectory of the stock is an arbitrarily precise signal of past aggregate effort. Thus, an arbitrarily small amount of noise in the monitoring technology can make the SSE unique. Uniqueness follows from a ‘backward-induction’ logic, as effort can be adjusted flexibly and must vanish in the long run by assumption, and information is revealed only whenever the stock grows. The same logic would apply even if past exerted effort were perfectly revealed after each increment in the stock, so that Theorem 1 would continue to hold.²⁷

This game admits many asymmetric equilibria, some of which yield efficiency gains over the strongly symmetric one, due to the agents’ ability to coordinate. However, as the next result establishes, effort ceases in any equilibrium no later than it would stop in the single-agent problem. In the latter setting, effort can be sustained at any stock x such that

$$\lambda\{\mathbb{E}_F[b(x + \tilde{z})] - b(x)\} \geq c_1(0, x), \quad (15)$$

but not beyond it. (Note that the left-hand side is decreasing in x and the right-hand side is increasing in x).

Proposition 2. *In any PPE, no effort is exerted after any history leading to a stock x such that (15) fails.²⁸*

Proposition 2 is proved in Appendix B. The logic behind it is similar to that explaining the analogous result for the Markov equilibria with ‘finite switching’ of Keller, Rady, and Cripps (2005). Specifically, effort cannot be sustained after

27. It is interesting to see that this logic fails in a repeated game: in a repeated prisoners’ dilemma where actions played are revealed at a fixed rate as long as at least one player cooperates, there exists an SSE that is approximately efficient as the period length vanishes. See Proposition 5 of Abreu, Milgrom, and Pearce (1991).

28. Proposition 2 is similar to the main result of Guéron (2015), but holds for different reasons. In particular, it would continue to hold (with the same proof), if past exerted effort were perfectly revealed after each increment in the stock, even though this monitoring technology would not satisfy the ‘smoothness’ assumption that Guéron’s result hinges on (Assumption 6).

the stock exceeds some cutoff \hat{x} . If the stock is below but close to \hat{x} , with high probability, no effort is exerted after an innovation. Then, the incentive to exert effort is essentially no higher than in the single-agent setting.

Proposition 2 implies that, unless (15) holds for any x , welfare among all equilibria is bounded as the number of agents grows large. However, as we shall see in Section 7, welfare in the unique SSE of the game with disposal may diverge as the number of agents grows. Therefore, the ability to selectively discard innovations yields larger gains (in large populations) than the ability to coordinate.

Proposition 2 also implies that, in the limit as agents become arbitrarily patient, the largest achievable welfare across all equilibria need not approximate that of the welfare benchmark. That is, the ‘Folk Theorem’ need not hold. To see why, recall from Section 3 that payoffs are *linear and multiplicative* if $b(x) = x$ and $c(a, x) = ax$. In this case, (8) and (9) imply that welfare is maximised by the profile $\alpha_*(x) = \mathbb{1}_{x < \lambda\mu}$. Moreover, $\lambda\mu$ is the unique solution to (15) and, using the appropriate analogues of (8) and (9), it is easy to show that among all strategy profiles such that no effort is exerted at stock levels exceeding $\lambda\mu$, welfare is maximised if all agents exert maximal effort until the stock exceeds $\lambda\mu$, and no effort thereafter. Moreover, one can show that the welfare $\bar{v}(x_0)$ induced by this strategy profile satisfies $\limsup_{\lambda \rightarrow \infty} \bar{v}(x_0)/v_*(x_0) < 1$ for all $x_0 \geq 0$.²⁹

In the SSE α_f , effort is exerted at any stock x such that (15) holds strictly, but not beyond it. Thus, if payoffs are linear and multiplicative and $x_0 < \lambda\mu$, effort in the SSE ceases whenever the stock reaches $\lambda\mu$ and the expected number of innovations produced in equilibrium is

$$M(x_0) := \mathbb{E} \left[\min \left\{ m \in \mathbb{N} : x_0 + \sum_{l=1}^m \tilde{z}_l \geq \lambda\mu \right\} \right]$$

where z_1, z_2, \dots are i.i.d. draws from F . Note that there is a unique $y_f \in (0, \lambda\mu)$ such that

$$y_f(n-1) = \int_{y_f}^{\lambda\mu} \frac{M}{\lambda}. \quad (16)$$

The next result describes effort and payoffs in the SSE under the hypothesis that the payoff function is linear and multiplicative. The proof is in Online Appendix J.

Corollary 1. *Suppose that payoffs are linear and multiplicative. Then, equilibrium*

²⁹ Recall that, as we normalised the discount rate r to 1, decreasing r to 0 is equivalent to letting λ diverge. For example, if $F(z) = 1 - e^{-\mu z}$ then, for $x_0 \leq \lambda\mu$,

$$v_*(x) = \lambda\mu n e^{\frac{x_0/\mu - \lambda n}{1 + \lambda n}} \quad \text{and} \quad \bar{v}(x) = \frac{\lambda n \mu (1 + \lambda)}{1 + \lambda n} e^{\frac{x_0/\mu - \lambda}{1 + \lambda n}}$$

so that $\bar{v}(x_0)/v_*(x_0) \rightarrow e^{1-1/n}/n < 1$ as $\lambda \rightarrow \infty$, where the inequality holds since $n \geq 2$.

effort α_f is maximal over $[0, y_f]$ and interior over $(y_f, \lambda\mu)$. Moreover, for all $y_f \leq x \leq \lambda\mu$,

$$\alpha_f(x) = \frac{1}{(n-1)x} \int_x^{\lambda\mu} \frac{M}{\lambda} \quad \& \quad v_f(x) = x + \int_x^{\lambda\mu} \frac{M}{\lambda},$$

and v_f is increasing globally if and only if it is increasing over $[y_f, \lambda\mu]$.

Corollary 1 implies that v_f is convex over $[y_f, \lambda\mu]$ and is increasing (is decreasing) over this interval if $M(y_f) \leq \lambda$ (if $\lambda \leq 1$). Otherwise, assuming that (3) holds and that F is atomless, v_f is minimised over $[y_f, \lambda\mu]$ at the unique $x \in (y_f, \lambda\mu)$ such that $M(x) = \lambda$.

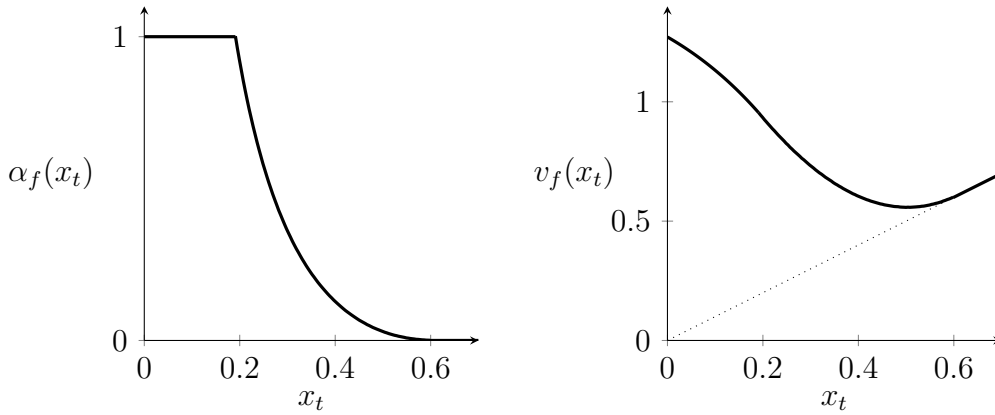


Figure 2: Effort (left) and payoffs (right) in the SSE, as functions of the stock x_t . Parameter values are $b(x) = x$, $c(a, x) = ax$, $F(z) = \rho \mathbb{1}_{z \geq \zeta} + (1 - \rho)(1 - e^{-z/\epsilon})$, $\lambda = 10$, $\rho = \epsilon = 0.01$, and $\zeta = n = 5$.

6 Detrimental innovations

In this section, I argue that detrimental innovations (that is, increments that cause continuation payoffs to drop in the symmetric equilibrium) occur only if agents' marginal cost increases with the stock, and if innovations have random size (Proposition 3). I provide necessary and sufficient conditions for the risk of detrimental innovations to persist as long as effort is exerted (Proposition 4), and characterise its occurrence under linear payoffs (Corollary 1). We will see that the detrimental effects of innovations are driven by the size of the team, and by the dispersion of the distribution of innovation sizes.

Recall the SSE α_f (Theorem 1). Innovations are detrimental whenever continuation payoffs in α_f are not guaranteed to increase after each innovation. Formally:

Definition 1. *Innovations are detrimental* if $\Pr(t \mapsto v_f(\tilde{x}_t) \text{ is increasing}) < 1$, where $(\tilde{x}_t)_{t>0}$ describes the evolution of the stock when α_f is played.

The next result shows that the two distinguishing features of this model relative to the literature on dynamic games of public-good provision are both necessary for innovations to be detrimental. In particular, assuming that increasing the stock has diminishing returns is insufficient.

Proposition 3. *If either innovations have fixed size, or the cost of effort does not increase with the stock, then innovations are not detrimental.*

Proposition 3 is proved in Appendix C, and the logic behind is simple: by (14), if effort is exerted, then innovations must be beneficial when averaging across their size. Thus, they cannot be detrimental if their size is fixed. The fact that innovations are not detrimental if payoffs are separable follows from a fixed-point argument: by Theorem 1, equilibrium effort $\alpha_f(x)$ is decreasing in the stock x . Then, assuming that $\mathbb{E}_F[v_f(x + \tilde{z})]$ is increasing in x , $v_f(x)$ must also be increasing over the interval in which effort is interior, by (14). It is then easy to conclude that v_f is globally increasing, since innovations can be harmful only through their effect on effort provision. In Online Appendix K, I argue that the fact that increments in the stock are ‘lumpy’ is necessary for innovations to be detrimental (I consider an alternative model in which the stock evolves according to a diffusion process, and show that payoffs always grow with the stock).

The next result states a necessary and sufficient condition for the risk of detrimental innovations to persist as long as effort is exerted (provided it is not exerted forever). Recall from Section 5 that effort in α_f ceases eventually if and only if

$$\lambda\{\mathbb{E}_F[b(x + \tilde{z})] - b(x)\} \leq c_1(0, x) \quad (17)$$

admits a solution and, if so, effort ceases as the stock reaches its lowest solution.

Proposition 4. *Suppose that (3) holds and (17) admits a smallest solution x_f . Then, innovations are detrimental for any initial stock $x_0 < x_f$ if (and only if)*

$$b'(x_f)c_{11}(0, x_f) + (n - 1)c_1(0, x_f)\{\lambda\mathbb{E}_F[b'(x_f + \tilde{z})] - c_{12}(0, x_f)\} < (\leq) 0. \quad (18)$$

Proposition 4 is proved in Appendix C. Condition (3) ensures that innovations are detrimental if (and only if) v_f is not increasing on $[x_0, \infty)$. Condition (18) describes the impact of a small-sized increment when the stock is close to, but below the cutoff x_f . Roughly speaking, the innovation increases the current payoff flow (first term), as well as future gross payoffs (first term inside braces), but increases the marginal cost of effort (second term inside braces). If the last force

dominates, the drop in effort following the innovation is large enough to counter the aforementioned payoff increase. In short, even though innovations increase the payoff flow, they reduce its growth rate as further increments are delayed, and may be harmful overall.

From the perspective of Proposition 4, increasing the number of agents exacerbates the risk of detrimental innovations. Formally, if (17) holds (strictly), it continues to hold (strictly) after an increase in n , as x_f does not depend on n .

Under natural regularity conditions, detrimental innovations will persist in the long run, provided innovations are sufficiently large and rare, and the population is large. Specifically, suppose that b is unbounded above and $b'(x)$ vanishes as x diverges, and that $c_1(0, 0) = 0$ and c_{12} is strictly positive. Then, $0 < x_f < \infty$ and, for any $\lambda' \leq \lambda$ and $n' \geq n$, there is an FOSD-shift F' of F such that replacing λ , F and n by λ' , F' and n' , respectively, leaves x_f unchanged. Moreover, given sufficiently small λ' and large n' , for any such F' , (18) holds after the substitution.³⁰

The next result characterises detrimental innovations for linear multiplicative payoffs. It is a simple consequence of Corollary 1, and is proved in Online Appendix K. Recall the definitions of M and y_f from Section 5, and denote by ‘ \vee ’ the ‘max’ operator.

Corollary 2. *Suppose that (3) holds and that payoffs are linear and multiplicative. Then, innovations are detrimental if and only if $x_0 < \lambda\mu$ and $M(x_0 \vee y_f) > \lambda$. In particular, payoffs drop with innovations raising the stock from any $x \geq y_f$ to any $\hat{x} \in (x, \lambda\mu)$ such that $M(\hat{x}) > \lambda$.*

Corollary 2 implies that, if innovations are sufficiently rare (that is, if $\lambda < 1$) and, given $x_0 \in (0, \lambda\mu)$, the population is sufficiently large so that $y_f \leq x_0$, all but the last innovation produced in equilibrium are harmful. It also implies that innovations are detrimental if, given any frequency λ of innovations and any distribution F of their size, the initial technology is sufficiently unproductive (i.e. x_0 is small enough), and the population is large enough.³¹

The expected share of innovations produced in equilibrium that are harmful may be arbitrarily large even if the number of agents is fixed and small increments are arbitrarily frequent, provided large ones are rare. This occurs in the illustrative example introduced in Section 3 (see Online Appendix K for details).

30. In detail, since b is unbounded above, given $\lambda' \in (0, \lambda)$, there exists an FOSD shift F' of F such that $\lambda' \mathbb{E}_{F'}[b(x_f + \tilde{z})] = \lambda \mathbb{E}_F[b(x_f + \tilde{z})]$. This guarantees that x_f is unchanged by the substitution. For the last part, note that $c_{12}(0, x_f) > 0$ and $c_1(0, x_f) > 0$ since c_{12} is strictly positive, and $\mathbb{E}_F[b'(x_f + \tilde{z})] > 0$ since b is unbounded above. Take $\lambda' < c_{12}(0, x_f) / \{2\mathbb{E}_F[b'(x_f + \tilde{z})]\}$ and $n' > 1 + 2b'(x_f)c_{11}(0, x_f) / [c_1(0, x_f)c_{12}(0, x_f)]$. Note that $\mathbb{E}_{F'}[b'(x_f + \tilde{z})] \leq \mathbb{E}_F[b'(x_f + \tilde{z})]$ for any FOSD-shift F' of F , since b is concave. Then, (18) holds with $\lambda = \lambda'$, $F = F'$ and $n = n'$, since both terms in the left-hand side are lower than $c_{12}(0, x_f)/2$.

31. Indeed, in this case, $M(x_0 \vee y_f) \approx M(0) > \lambda$, as $\lim_{n \rightarrow \infty} y_f = 0$.

7 Disposal

In this section, I extend the model by allowing each agent to freely dispose of the innovations that she produces, after observing their size. This raises effort and payoffs in the strongly symmetric equilibrium relative to the baseline setting.

7.1 Model

Enrich the model as follows. Whenever agent i obtains an innovation of size $z > 0$ at time $t \geq 0$, she (immediately) decides whether or not to reduce z to 0, after observing z . That is to say, agent i may either disclose the innovation (which results in its immediate adoption by all agents), or discard it. Assume that the arrival and size of innovations is private information, and that the disposal of innovations is unobserved. In particular, if an agent obtains and discards an innovation at some time t , her opponents will not be able to distinguish this event from the event that the agent does not obtain an innovation at time t . However, agents are immediately informed of any innovation that is disclosed, including its size and the identity of the agent disclosing it. As before, effort is hidden. I constrain agents to play pure strategies and, for simplicity, I rule out strategies that condition on innovations discarded in the past.³²

We recover the baseline model (Section 3) by restricting the agents' strategies so that all innovations produced are disclosed. I refer to the baseline model as the game *with forced disclosure*, and to this model as the game *with disposal*. As I argue at the end of Section 7.2, the analysis of strongly symmetric equilibria is unchanged if agents may conceal innovations instead of discarding them, but may not secretly refine the improvements that they hide.

Below is a brief description of histories, strategies and continuation payoffs. Formal definitions are in Online Appendix H.2. Each agent reaches a new private history whenever she either produces or discards an innovation, or any agent discloses. Thus (almost surely), agents reach finitely many private histories within any bounded interval of time. Public histories are formally unchanged (see (5) in Section 3), but they now only record *disclosed* innovations. A strategy $\xi^i := (\sigma^i, \chi^i)$ specifies, for each public history h_m , an effort schedule $\sigma^i(h_m) : (t_m, \infty) \rightarrow [0, 1]$ and a disclosure policy $\chi^i(h_m) : (t_m, \infty) \times (0, \infty) \rightarrow \{0, 1\}$. As before, agent i exerts effort $[\sigma^i(h_m)](t)$ at any time $t > t_m$ such that no innovation was disclosed within the time interval $[t_m, t)$. Moreover, if agent i produces an innovation of size z at such a time t , she discloses it if $[\chi^i(h_m)](t, z) = 1$, and discards it otherwise.

32. Admitting the latter strategies would complicate the definition of the game without affecting the results. In particular, the unique strongly symmetric equilibrium (in which, by definition, strategies cannot condition on discarded innovations), would continue to exist.

Note that it is not necessary to keep track of discarded innovations, due to the restriction to public strategies. If agents play a strategy profile $\xi := (\sigma^i, \chi^i)_{i=1}^n$, the continuation payoff $v_\xi^i(h_m)$ to agent i at a public history h_m may be expressed as the right-hand side of (6), with the random path $(\tilde{h}_l)_{l=m}^{\tilde{m}}$ obviously having a different distribution.

7.2 Analysis

The social-welfare benchmark (characterised in Proposition 1) is unaffected by the introduction of disposal. This is because, in the non-strategic setting, the effects of disposal on incentives can be ignored. Moreover, disposal of innovations hinders the growth of the payoff flow, since $b(x) - c(a, x)$ is increasing in x .

Given a public history h featuring $m \in \{0, 1, \dots\}$ (disclosed) innovations, write $h \circ (t, z, i)$ for the public history that features $m + 1$ innovations and extends h , and in which the last innovation is disclosed by agent i at time t , and has size z . Given a profile $\xi := (\sigma^i, \chi^i)_{i=1}^n$, let $\hat{v}_\xi^i(h)$ be the largest payoff that agent i can achieve, across all strategies, in the subgame after public history h , given that her opponents behave according to ξ . Given h leading to time $t_h \geq 0$, for all $t > t_h$, let $\hat{v}_{\xi, h}^i(t)$ be the largest payoff achievable by agent i at time t , assuming that no innovation was disclosed within the interval $[t_h, t)$, and that agent i does not produce an innovation at time t (and define $\hat{v}_{\xi, h}^i(t_h) := \hat{v}_\xi^i(h)$). Then, $\hat{v}_{\xi, h}^i : [t_h, \infty) \rightarrow \mathbb{R}$ is Lipschitz and, labelling x the stock that h leads to, for almost all $t > t_h$,³³

$$\begin{aligned} \hat{v}_{\xi, h}^i(t) = & \frac{d\hat{v}_{\xi, h}^i}{dt}(t) + b(x) + \max_{a \in [0, 1]} \{a\lambda \mathbb{E}_F[(\hat{v}_\xi^i(h \circ (t, \tilde{z}, i)) - \hat{v}_{\xi, h}^i(t)) \vee 0] - c(a, x)\} \\ & + \lambda \sum_{j \neq i}^n [\sigma^j(h)](t) \mathbb{E}_F[\chi^j(t, \tilde{z})(\hat{v}_\xi^i(h \circ (t, \tilde{z}, j)) - \hat{v}_{\xi, h}^i(t))]. \end{aligned} \quad (19)$$

The strategy ξ^i is a best response for agent i at every private history only if, given any public history h leading to any time t_h , both of the following conditions hold: for almost all $t > t_h$,

$$[\sigma^i(h)](t) \in \arg \max_{a \in [0, 1]} \{a\lambda \mathbb{E}_F[(\hat{v}_\xi^i(h \circ (t, \tilde{z}, i)) - \hat{v}_{\xi, h}^i(t)) \vee 0] - c(a, x)\} \quad (20)$$

and, furthermore, for all $t > t_h$ and $z > 0$,

$$[\chi^i(h)](t, z) \in \arg \max_{d \in \{0, 1\}} \{d(\hat{v}_\xi^i(h \circ (t, z, i)) - \hat{v}_{\xi, h}^i(t))\}. \quad (21)$$

33. See the end of Online Appendix H.2 for details on how to derive (19), (20), (22), (23) and (24). ‘ \vee ’ denotes the ‘max’ operator.

Condition (21) states that agent i optimally adopts or discards an innovation of size z obtained at time t following history h . A strategy $\xi := (\sigma, \chi)$ induces a *strongly symmetric equilibrium* (SSE) if, given that all opponents play ξ , ξ is a best response at every private history.

A *Markov strategy* is a pair $\pi := (\alpha, \delta)$ of (Borel measurable) maps $\alpha : \mathbb{R}_+ \rightarrow [0, 1]$ and $\delta : \mathbb{R}_+ \times (0, \infty) \rightarrow \{0, 1\}$ such that $a_t^i = \alpha(x_t)$ for all $t \geq 0$ and, if agent i obtains an innovation of size $z > 0$ at time t , she discloses it if $\delta(x_t, z) = 1$ and discards it otherwise. If the opponents of any given agent play a Markov strategy $\pi := (\alpha, \delta)$, the largest continuation payoff that this agent can achieve, across all strategies, is a (value) function \hat{v}_π of the current stock x_t , solving

$$\begin{aligned} \hat{v}(x) = & b(x) + \max_{a \in [0,1]} \{a\lambda \mathbb{E}_F[(\hat{v}(x + \tilde{z}) - \hat{v}(x)) \vee 0] - c(a, x)\} \\ & + \lambda(n-1)\alpha(x)\mathbb{E}_F[\delta(x, \tilde{z})(\hat{v}(x + \tilde{z}) - \hat{v}(x))]. \end{aligned} \quad (22)$$

for all $x \geq 0$.³⁴ Then, π is a best response, after any private history, against opponents playing π if and only if, for all $x \geq 0$, both of the following hold:

$$\alpha(x) \in \arg \max_{a \in [0,1]} \{a\lambda \mathbb{E}_F[(\hat{v}_\pi(x + \tilde{z}) - \hat{v}_\pi(x)) \vee 0] - c(a, x)\}, \quad (23)$$

$$\delta(x, z) \in \arg \max_{d \in \{0,1\}} \{d(\hat{v}_\pi(x + z) - \hat{v}_\pi(x))\} \quad \text{for all } z > 0. \quad (24)$$

The following result characterises the unique SSE of the game. The proof is in Appendix D. Recall from Section 5 the SSE α_f of the game with forced disclosure, and the notion of detrimental innovations.

Theorem 2. *The game with disposal admits an (essentially) unique SSE, and it is induced by a Markov strategy (α_d, δ_d) . In the absence of innovations at time t , and for any stock x_t , effort $\alpha_d(x_t)$ and continuation payoffs $v_d(x_t)$ are no lower than their analogues $\alpha_f(x_t)$ and $v_f(x_t)$ in the equilibrium with forced disclosure. Moreover, ex-ante payoffs $v_d(x_0)$ strictly exceed $v_f(x_0)$ if innovations are detrimental in α_f . Otherwise, no disposal occurs in (α_d, δ_d) and the equilibria coincide.*

Although there may exist multiple equilibria, α_d and v_d are uniquely pinned down. Multiplicity arises if agents are indifferent about the disposal of some innovations (i.e, if the objective in (24) admits multiple maximisers), and any way of breaking the indifferences yields an equilibrium. Moreover, the properties of α_f and v_f described in Theorem 1 are inherited by α_d and v_d . (I establish this in Appendix D).

34. This expression is valid if neither the agent nor one of her opponents produced an innovation at time t . If the agent obtains an innovation of size z at time t , the expression becomes $\hat{v}_\pi(x + z) \vee \hat{v}_\pi(x)$ (where \hat{v}_π is pinned down by (22)). Assuming an opponent obtains the innovation

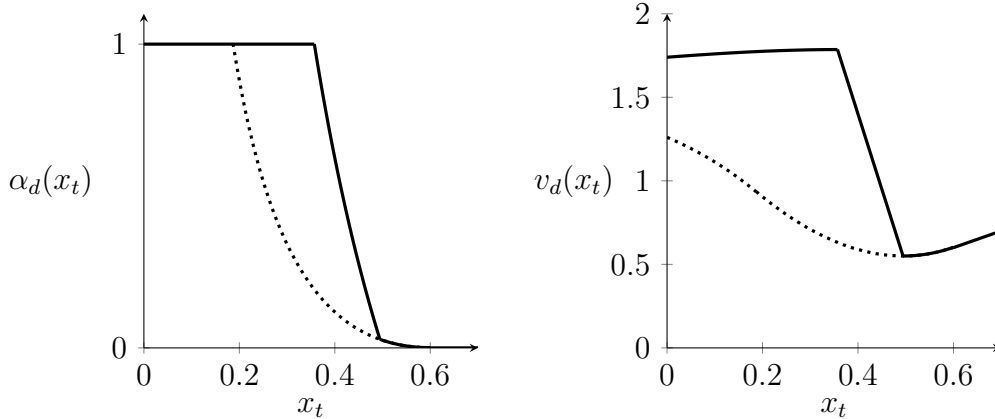


Figure 3: Effort (left) and payoffs (right) in the SSE of the game with disposal, as functions of the stock x_t . The dotted lines are effort α_f (left) and payoffs v_f (right) in the equilibrium of the game with forced disclosure. By (24), δ_d need not be a ‘cutoff’ strategy, since v_d is not quasi-concave. Parameter values are $b(x) = x$, $c(a, x) = ax$, $n = 5$, $F(z) = \rho \mathbf{1}_{z \geq \zeta} + (1 - \rho)(1 - e^{-z/\epsilon})$, $\zeta = 5$, $\lambda = 0.1/\rho$, and $\epsilon = \rho/(1 - \rho)$, where $\rho > 0$ is arbitrarily small.

Allowing agents to discard innovations increases equilibrium payoffs, and strictly so unless all innovations are guaranteed to be beneficial in the SSE with forced disclosure. This is because individual and social incentives for the disposal of innovations are aligned, since the equilibrium is symmetric. That is to say, whenever an agent discloses or discards an innovation in equilibrium, her opponents benefit from it. Moreover, the fact that future detrimental innovations will be discarded increases continuation payoffs at all stock values exceeding the current one and, therefore, the current incentive to exert effort. As a result, equilibrium effort is higher in the game with disposal. This strengthens the positive externality arising from the public nature of the good, increasing payoffs further.

As Figure 3 suggests, the equilibrium with disposal can improve substantially over the equilibrium with forced disclosure, both in terms of effort and of welfare. In fact, provided effort is costless if the stock is zero, and arbitrarily large innovations are possible, and gross benefits are unbounded, equilibrium payoffs at $x_0 = 0$ diverge as the number of agents grows.³⁵ This is because agents exert maximal effort prior to the first innovation and, if their number is large, they discard any innovation that would cause a meaningful reduction in effort, unless it is of considerable size. (As an illustration note that, given the parameters of Figure 3 and $x_0 = 0$, prior to the first innovation, each innovation is adopted if and only if its size lies either roughly below 0.36, or above 1.75.) Under these assumptions, the

instead, the expression is valid if $\delta(x, z) = 0$, and becomes $\hat{v}_\pi(x + z)$ if $\delta(x, z) = 1$.

35. In detail, if $c(1, 0) = 0$ then $v_d(0) = \{b(0) + \lambda n \mathbb{E}_F[v_d(z) \vee v_d(0)]\} / (1 + \lambda n)$, as $\alpha_d(0) = 1$. Then, if $\mathbb{E}_F[v_d(z) \vee v_d(0)]$ remained bounded as n grows, $v_d(0) \approx \mathbb{E}_F[v_d(z) \vee v_d(0)]$ for large n , which is not possible if F has unbounded support and $\lim_{x \rightarrow \infty} b(x) = \infty$, since $v_d \geq b$. Hence $\mathbb{E}_F[v_d(z) \vee v_d(0)]$ diverges with n and, therefore, so does $v_d(0)$, as $v_d - b$ is decreasing.

welfare improvement relative to any PPE of the game with forced disclosure is arbitrarily large provided (15) fails for some x , since this guarantees that payoffs in the latter equilibria remain bounded as the number of agents grows.

Theorem 2 remains valid if agents can conceal innovations instead of merely discarding them, provided they cannot covertly refine the improvements that they hide. Formally, the result extends to the following setting: each agent possesses a (time-varying) set of concealed increments $X_t^i \subset [0, \infty)$, initially empty. Whenever agent i obtains an innovation of size z at time t , the value $x_t + z$ is added to X_t^i (and the stock x_t does not grow). Moreover, each agent i picks a *disclosure* $d_t^i \in \{x \in X_t^i : x > x_t\} \cup \{x_t\}$ whenever she obtains an innovation,³⁶ and at any time $t \in \mathcal{T}$, where $\mathcal{T} \subset [0, \infty)$ is exogenous and $\mathcal{T} \cap [0, T]$ is finite for all $T > 0$. If a (non-empty) set of agents $I \subseteq \{1, \dots, n\}$ pick disclosures $(d_t^i)_{i \in I}$ at time t , the public stock x_t rises (weakly) to $\max\{d_t^i : i \in I\}$. Say that agent i *discloses* at time t if $d_t^i > x_t$ and *conceals* if $d_t^i = x_t$. Assume that the arrival and size of innovations as well as the sets X_t^i are private information, and the concealment of innovations is unobserved. However, agents are immediately informed of any innovation that is disclosed, including its size and the identity of the agent disclosing it. We recover the game with disposal if we constrain agents to choose $d_t^i \in \{x_t, x_t + z\}$ whenever producing an innovation of size z at time t , and $d_t^i = x_t$ at any other time $t \in \mathcal{T}$.

Theorem 2 carries over to this setting since strongly symmetric equilibria are necessarily public. Indeed, this rules out disclosing any innovation that was previously concealed, so that (α_d, δ_d) is the only candidate equilibrium; moreover, (α_d, δ_d) is an equilibrium since it is stationary, so that no agent has an incentive to delay disclosure.

This richer environment admits other symmetric perfect Bayesian equilibria that are not public, and that are closer in spirit to the equilibrium analysed in the next section.

8 Concealment

In this section, I extend the baseline model by allowing agents to conceal the innovations that they obtain for a period of time, after observing their size; moreover, agents may covertly refine the improvements that they hide. I show that the resulting game admits a relatively simple equilibrium which, under reasonable conditions, improves on all equilibria with forced disclosure. Throughout, I restrict attention to linear multiplicative payoffs, introduced in Section 3.

36. If agent i obtains an innovation of size z at time t then she can choose $d_t^i = x_t + z$.

8.1 Model

As before, x_t denotes the (public) stock of the good and, at any time $t \geq 0$, agent i exerts hidden effort $a_t^i \in [0, 1]$ and receives a flow payoff $r(1 - a_t^i)x_t$.

Each agent i possesses a *private stock* k_t^i of the good, initially equal to x_0 . Agent i produces increments in k_t^i (instead of x_t) at rate λa_t^i . The distribution F of the size of increments (i.e., of innovations) inherits the properties described in Section 3 and, for simplicity, is assumed to be Lipschitz continuous. Moreover, each agent i picks a *disclosure* $d_t^i \in [x_t, k_t^i]$ whenever (i) she obtains an innovation,³⁷ and (ii) at any time t such that $k_t^i > x_t$ and either $x_t = x_0$ or $t \in \mathcal{T}$, where $\mathcal{T} \subset [0, \infty)$ is exogenous and $\mathcal{T} \cap [0, T]$ is finite for all $T > 0$. If a (non-empty) set of agents $I \subseteq \{1, \dots, n\}$ pick disclosures $(d_t^i)_{i \in I}$ at time t , the public stock x_t rises (weakly) to $\max\{d_t^i : i \in I\}$, and the private stocks k_t^j that laid below this value rise to it, while the rest are unchanged. In particular, neither x_t nor any k_t^j increase if $d_t^i = x_t$ for all $i \in I$. Whenever agent i picks d_t^i at time t , say that she *conceals* if $d_t^i = x_t$, *discloses fully* if $d_t^i = k_t^i > x_t$, and *discloses partially* if $x_t < d_t^i < k_t^i$.³⁸

Assume that agents' effort, private stocks, as well as the production and size of innovations are private information, and that the concealment of innovations is not observed. In particular, if agent i produces an innovation at time t and conceals it, her opponents will not be able to distinguish this event from the event that agent i does not produce an increment at time t . If instead agent i chooses $d_t^i > x_t$, the opponents of agent i are immediately informed of her identity and of the value of d_t^i . However, as they do not observe k_t^i , they cannot distinguish partial from full disclosures.

I restrict attention to pure strategies. We recover the game with forced disclosure (Section 3) by constraining agents to always disclose fully. I refer to this model as the *game with concealment*.

Below is a brief description of histories, strategies, and continuation payoffs, which are defined in Online Appendix H.3. Each agent i reaches a new information set whenever one of the following events occurs: agent i produces an innovation, or one or more agents (including i) disclose, either partially or fully. The constraint on disclosures ensures that agents reach only finitely many information sets within

37. If agent i obtains an innovation at time t , k_t^i denotes her private stock *after* the innovation.

38. In any period t at which agent i does not obtain an increment, and such that $x_t > x_0$ and $t \notin \mathcal{T}$, agent i does *not* pick d_t^i . Thus, each agent can disclose at any point in time as long as no disclosure occurred but, after a disclosure occurs, she can only disclose at times in \mathcal{T} , or immediately after producing an innovation. This restriction guarantees that the game is well-defined. I impose no constraint on the first disclosure to simplify the construction of the equilibrium (Theorem 3).

any bounded period of time. A *private history* for agent i is a finite sequence

$$h := (x_0, (t_1, e_1), \dots, (t_m, e_m))$$

where $0 \leq t_1 \leq \dots \leq t_m$, and each e_l describes one of the two aforementioned events, occurring at time t_l . Private histories pin down the current time $t_m = T(h)$, the public stock $x_{t_m} = X(h)$, as well as agent i 's private stock $k_{t_m}^i = K^i(h)$. A strategy $\xi^i = (\sigma^i, \chi^i)$ specifies, for each h , an effort schedule $\sigma^i(h) : (T(h), \infty) \rightarrow [0, 1]$ and a disclosure policy $\chi^i(h) : [T(h), \infty) \times [0, \infty) \rightarrow [0, 1]$. Agent i exerts effort $[\sigma^i(h)](t)$ at any time $t > T(h)$ such that she produced no innovation within $[T(h), t)$, and no agent disclosed within this interval. Moreover, whenever agent i picks a disclosure d_t^i at any time $t \geq T(h)$ with the aforementioned property, $d_t^i = X(h) + [K^i(h) + z - X(h)][\chi^i(t)](t, z)$ if agent i produced an innovation of size $z > 0$ at time t , and $d_t^i = X(h) + [K^i(h) - X(h)][\chi^i(t)](t, 0)$ otherwise. An *information set* for agent i is a pair (h, b) , where h is a private history and b is a *belief*; that is, a distribution over the most recent private histories $(h^j)_{j \neq i}$ reached by the opponents of agent i . If agents play a profile $\xi := (\sigma^i, \chi^i)_{i=1}^n$, the continuation payoff $v_\xi^i(h, b)$ to agent i at information set (h, b) may be expressed as

$$v_\xi^i(h, b) := \mathbb{E} \left(\sum_{m=0}^{\tilde{m}} \int_{T(\tilde{h}_m)}^{T(\tilde{h}_{m+1})} e^{T(h)-t} (1 - [\sigma^i(\tilde{h}_l)](t)) X(\tilde{h}_l) dt \right)$$

where $\tilde{m} \in \{0, 1, \dots, \infty\}$ is the total number of information sets reached by agent i after time $T(h)$, \tilde{h}_l is the private history associated with the l th information set, $\tilde{h}_0 := h$ and, if $\tilde{m} < \infty$, then $T(\tilde{h}_{\tilde{m}+1}) := \infty$.

8.2 Discussion of the assumptions

Recall that, in the game *with disposal* (Section 7), agents may either discard or disclose the innovations that they produce. Concealing innovations is more profitable than discarding them, as it enables agents to refine the innovations that they hide. For instance, in the game with disposal, if an agent obtains an innovation of size z at time t and another of size z' at time $t' > t$, and discards the former while adopting the latter, the stock rises by z' at time t' . However, in the game with concealment, if the agent conceals at time t and discloses fully at time t' , the stock rises by $z + z'$ at time t' .³⁹

Concealing innovations leads to a stark form of redundancy. To see why, suppose that agent i obtains an innovation of size z at time t , and conceals it, while agent j obtains an innovation of size $z' > z$ at time $t' > t$, and discloses fully.

³⁹. Yet, the conditions described in Proposition 3, which are sufficient to prevent disposal in equilibrium, also suffice to prevent concealment. See Online Appendix L.1 for details.

Then, agent i 's private stock rises to z' at time t' , but no further: the innovations are perfect substitutes and, since agent j 's innovation is superior, agent i 's innovation is discarded at time t' . A more realistic model would feature some degree of complementarity between innovations. In this example, agent j 's disclosure at time t' would raise i 's stock to a value above z' . Despite this extreme form of redundancy, concealment may be beneficial (Proposition 5).

Concealed innovations cannot be used. Formally, the flow payoff of agent i depends on x_t instead of on k_t^i . This assumption is reasonable if (i) agents are firms developing a technology, and the technology is sold as part of the firms' products, or (ii) agents are non-profit organisations, since detailed information about the interventions they carry out is usually publicly available, or (iii) agents are workers who can observe and imitate each others' behaviour. A detailed study of the model in which concealed innovations can be used is beyond the scope of this paper, but I argue at the end of Section 8.4 that an analogue of Proposition 5 holds in this environment.

A simpler model would constrain the agents to either fully disclose or conceal their innovations. Formally, it would require that $d_t^i \in \{x_t, k_t^i\}$. With this restriction, agents may have an incentive to disclose fully as evidence to their opponents that they are not concealing more substantial innovations. Doing so would be beneficial, as it would raise the opponents' incentives to exert effort. Hence, this is likely to play an important role in equilibrium. I do not analyse this setting as it seems economically less relevant. Requiring instead that $d_t^i \in \{k_s^i : 0 \leq s \leq t\} \cap [x_t, \infty)$ would not affect the analysis.

8.3 Equilibrium

The social-welfare benchmark (Theorem 1) is unaffected by the introduction of concealment. This is because, in the non-strategic setting, the effects of delayed disclosure on incentives can be ignored. Moreover, concealing innovations hinders the growth of the payoff flow, and may lead to redundancy. In this section, I construct a symmetric perfect Bayesian equilibrium of the game with concealment.

A *perfect Bayesian equilibrium* (PBE) is composed of a profile of strategies ξ and a profile of beliefs β (see Online Appendix H.3 for the formal definition). The equilibrium α_f with forced disclosure (Theorem 1), coupled with the 'full-disclosure' policy, is a natural candidate equilibrium in the game with concealment. I begin by showing that it *is* a PBE (for some belief profile) if and only if the initial stock is so large that no effort is exerted. Since effort $\alpha_f(x_t)$ is decreasing in the current stock x_t , no effort is exerted in equilibrium if and only if $\alpha_f(x_0) = 0$, which is equivalent to $x_0 \geq \lambda\mu$, where μ is the average size of innovations.

Lemma 1. *Suppose that (3) holds. Then, the effort schedule α_f , coupled with the ‘full-disclosure’ policy, forms a symmetric PBE of the game with concealment (for some profile of beliefs) if and only if $x_0 \geq \lambda\mu$.*

Lemma 1 is proved in Appendix E. A comparison with Corollary 2 shows that agents may have an incentive to conceal innovations even if they are not detrimental in equilibrium (in the sense defined in Section 5). This is because innovations are detrimental if and only if agents have an incentive to discard them (see Section 7), and concealment is more profitable than disposal as it allows agents to covertly refine the innovations that they hide.

The next result describes a symmetric PBE of the game with concealment. The proof is in Appendix F.

Theorem 3. *There exists a symmetric PBE (ξ_c, β_c) such that, in any period t prior to which no disclosure occurred, agent i exerts effort $\alpha_t^c(k_t^i)$ and conceals (resp. discloses fully) if $k_t^i < (\geq) q(t)$, where $\alpha_t^c(k)$ is increasing in k and $\lambda\mu \leq q(t) \leq \lambda\mu n$. Moreover, agent i ’s continuation payoff in any such period t may be expressed as $v_t^c(k_t^i)$, where $v_t^c(k)$ is increasing in k . On the equilibrium path, no effort is exerted after a disclosure occurs.*

The equilibrium (ξ_c, β_c) is chosen so that no effort is exerted if $x_0 \geq \lambda\mu$. The rest of the discussion focuses on the case $x_0 < \lambda\mu$. In this case, agents conceal innovations as long as their respective stocks k_t^i lie below the common, time-varying cutoff $q(t)$. As soon as k_t^i reaches $q(t)$ for some i , agent i discloses fully. Moreover, at the time of the disclosure, $k_t^i \geq q(t) \geq k_t^j$ for all $j \neq i$, so that each k_t^j rises to k_t^i . That is to say, the first innovation disclosed is the best available at the time of disclosure. No effort is exerted past this time. From Lemma 1, this is incentive-compatible since the disclosure raises the public stock above $\lambda\mu$.

At any time t before the disclosure, agents believe that the private stocks of their opponents are i.i.d. with the same distribution (call it G_t). As time passes without a disclosure, agents need not become more optimistic about their opponents’ progress. Formally, G_t need not grow in the FOSD-sense over time and, therefore, the cutoff $q(t)$ need not be increasing in t . Hence, the disclosure may be due to a drop in $q(t)$ as well as to an innovation.

Equilibrium effort $\alpha_t^c(k_t^i)$ is increasing in k_t^i . This is because effort is beneficial ex-post if and only if it leads to a disclosure. The smaller the distance between k_t^i and the cutoff $q(t)$, the larger the impact of effort on the time to a disclosure, and thus, the higher the benefit of exerting effort. Moreover, the cost of effort does not grow as k_t^i grows, since it is determined by the public stock x_t , which is constant at x_0 until the disclosure.

The payoff $v_t^c(k_t^i)$ is increasing in the private stock k_t^i since a higher k_t^i brings agent i closer to a disclosure, but does not weaken her opponents' incentives for effort, as they do not observe k_t^i . Hence, when the disclosure occurs, all agents are better off. Indeed, if agent i discloses at time t , then she discloses $k_t^i \geq q(t)$, so that the payoff to any opponent j rises from $v_t^c(k_t^j)$ to $k_t^i = v_t^c(k_t^i)$, as $k_t^j \leq q(t)$.

The equilibrium (ξ_c, β_c) involves 'punishments' for any agent i disclosing a value $d_t^i < q(t)$ at any time t before which no disclosure occurred. If agent i does so, her opponents believe her private stock to be large and, as a consequence, exert no effort and never disclose after time t , no matter how agent i behaves.⁴⁰

8.4 The benefits of concealment

In this section, I derive a sufficient condition for the ex-ante payoffs in the equilibrium (ξ_c, β_c) of the game with concealment to exceed those of all equilibria of the game with forced disclosure. This is the case if there are sufficiently many agents, and the size of innovations is sufficiently 'dispersed'.

Recall that, if $x_0 > \lambda\mu$, no effort is exerted in either (ξ_c, β_c) or any equilibrium of the baseline game. If $x_0 < \lambda\mu$, the impact of concealment on equilibrium payoffs is less obvious than that of disposal, analysed in Theorem 2. This is because private and social incentives for the disposal of innovations are aligned in equilibrium (i.e. agents discard innovations whenever this benefits their opponents), whereas incentives for their concealment are misaligned. Indeed, in (ξ_c, β_c) , an agent with private stock $k_t^i < q(t)$ prefers to conceal than to disclose fully, even though full disclosure would benefit any opponent with a given stock $k_t^j < q(t)$, provided k_t^i is sufficiently close to $q(t)$.⁴¹ Despite the misaligned incentives, concealment is beneficial under general conditions, as the following result shows. Recall that ex-ante payoffs in (ξ_c, β_c) are $v_0^c(x_0)$, whereas agent i 's ex-ante payoff in a PPE σ of the baseline game is $v_\sigma^i(x_0)$. Since no effort is exerted in σ after the stock exceeds

40. To see why this punishment is sustainable, note that the opponents of agent i expect her to disclose fully as soon as she obtains an innovation and, in some cases, no later than some given time $t' \in \mathcal{T}$. If agent i discloses a value lower than expected (at any time after t), or does not disclose at time t' , her opponents will detect another deviation, and continue to believe her private stock to be large. This equilibrium does not satisfy the Intuitive Criterion (Cho and Kreps (1987)) as, if k_t^i were indeed large, agent i would be strictly better off if she disclosed k_t^i at time t than a value $d_t^i < q(t)$. An equilibrium that satisfies the Intuitive Criterion and features the 'cutoff' disclosure pattern of (ξ_c, β_c) is unlikely to exist, as no sustainable punishments could discourage agent i from disclosing at time t if $q(t)$ is large and $k_t^i > x_0$ is small compared to $q(t)$.

41. Since full disclosure makes the payoff of each agent j jump from $v_t^c(k_t^j)$ to k_t^i , it suffices to show that $k < v_t^c(k) < q(t)$ for any $k < q(t)$. The first inequality holds as q is lower semi-continuous (Proposition 7 in Appendix F). For the second, note that $v_t^c(k) < v_t^c(q(t)) = q(t)$ where the first inequality holds since $v_t^c(k)$ is increasing in k and, clearly, non-constant over $[k, q(t)]$, and the second follows from Proposition 7 and Lemma 3 (Appendix F).

$\lambda\mu$ (Proposition 2), $v_\sigma^i(x_0) \leq x_0 + V$, where

$$V = \mathbb{E} \left[\sum_{l=1}^{\tilde{m}} \tilde{z}_l \right],$$

z_1, z_2, \dots are i.i.d. draws from F , and $\tilde{m} = \min\{m \in \mathbb{N} : x_0 + \sum_{l=1}^m \tilde{z}_l \geq \lambda\mu\}$.

Proposition 5. *Suppose that $x_0 < \lambda\mu$ and*

$$x_0 < \lambda \mathbb{E}_F[(\tilde{z} - V) \vee 0]. \quad (25)$$

Then, if the number of agents n is sufficiently large, $v_0^c(x_0) > v_\sigma^i(x_0)$ for any PBE (ξ_c, β_c) satisfying the conditions of Theorem 3, any agent i , and any PPE σ of the baseline game with forced disclosure.

Proposition 5 is proved in Appendix G. Condition (25) holds if $F(V) < 1$ and x_0 is sufficiently small, or if the size of innovations is sufficiently ‘dispersed’. For instance, in the example introduced in Section 3, (25) holds if substantial innovations are arbitrarily rare, but drive progress in expectation (that is, if ρ and ϵ are small, keeping μ fixed).⁴²

Condition (25) ensures that $v_0^c(x_0) \geq x_0 + V$ provided interior effort is exerted at the outset in (ξ_c, β_c) . If instead effort is maximal at the outset in (ξ_c, β_c) , then $v_0^c(x_0) \geq x_0 + V$ for sufficiently large n . Combining these steps yields Proposition 5.

If the conditions of Proposition 5 are not met, agents may in principle be *worse off* in the equilibrium with concealment than in some equilibrium of the game with forced disclosure (including α_f). Among other things, this is because agents cannot build on each others’ innovations in (ξ_c, β_c) . Agents may be able to do so in other PBE that feature a richer disclosure pattern. Some (but not necessarily all) of these equilibria may induce higher payoffs than (ξ_c, β_c) .

If agents are able to use the innovations that they conceal, so that agent i ’s payoff flow is $r(1 - a_t^i)k_t^i$, there exists a symmetric PBE (ξ'_c, β'_c) inducing higher ex-ante payoffs than any equilibrium of the game with forced disclosure, provided arbitrarily large innovations are possible, and the number of agent is sufficiently large. In this equilibrium, all agents disclose (fully) at some exogenous time $T > 0$, and at no other times. Moreover, T is sufficiently large that each agent is willing to exert effort early on, as long as she does not obtain any innovation, no matter the expected increase in the stock at time T . If F has unbounded support and n is large, then the expected increase in the stock at time T will be large, so that

42. To see why note that, in the limit as ρ vanishes and ζ diverges while $\mu := \rho\zeta + (1 - \rho)\epsilon$ remains fixed, V is bounded and thus $\rho(\zeta - V)$ converges to $\mu - \epsilon$. Since $\mathbb{E}_F[(\tilde{z} - V) \vee 0] \geq \rho(\zeta - V)$, (25) holds for ρ sufficiently small, provided $\epsilon < \mu - x_0/\lambda$.

ex-ante payoffs will be large as well.⁴³

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43. The equilibrium (ξ'_c, β'_c) may be constructed as follows. Given $T > 0$, let R_T be the set of strategies involving full disclosure at time T if no one disclosed in the past, and (full) concealment at all other histories. Clearly, if the opponents of an agent all play some $\xi \in R_T$, the agent has a best response $\xi' \in R_T$. Then, we may construct a symmetric PBE (ξ'_c, β'_c) involving a strategy $\xi'_c \in R_T$, by a fixed-point argument. Moreover, given $x_0 < \lambda\mu$, we may choose T sufficiently large so that any best response $\xi' \in R_T$ to all opponents playing any given $\xi \in R_T$ involves full effort until either time 1 is reached, or a private increment obtains. In particular, given the chosen T , ξ'_c has this feature. Moreover, ex-ante payoffs in (ξ'_c, β'_c) are at least $x_0 + e^{-rT}(e - x_0)$, where e is the expected value of $\max\{k_1^i : i = 2, \dots, n\}$. Assuming that F has unbounded support, e diverges as n does, no matter the choice of (ξ'_c, β'_c) . Therefore, ex-ante payoffs in (ξ'_c, β'_c) exceed $x_0 + V$ for n sufficiently large.

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Appendices

A Proof of Theorem 1

The proof of Theorem 1 is based on two results (Theorem 4 and Proposition 6 below), proved at the end of this section. Given maps $v, w : \mathbb{R}_+ \rightarrow \mathbb{R}$, write ‘ $v \leq w$ ’ for ‘ $v(x) \leq w(x)$ for all $x \geq 0$ ’. Given a Borel $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ bounded below, define the maps $L_f v, L_d v : \mathbb{R}_+ \rightarrow \mathbb{R} \cup \{\infty\}$ by

$$L_f v(x) := \mathbb{E}_F[v(x + \tilde{z})] \quad \& \quad L_d v(x) := \mathbb{E}_F[v(x) \vee v(x + \tilde{z})]. \quad (26)$$

Given a map $w : \mathbb{R}_+ \rightarrow \mathbb{R}$ bounded below by $b(0)$, let B_w be the set of Borel $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ bounded below by $b(0)$, such that $v \leq w$. Recall the definition of v_* from Section 4. Let V_f (V_d) be the set of pairs (α, v) , where α is a Markov strategy of the game with forced disclosure and $v \in B_{v_*}$, such that (13) holds and (14) holds with $\hat{v}_\alpha = v$ (such that (22) holds for some δ satisfying (24) with $\hat{v}_\pi = v$ and (23) holds with $\hat{v}_\pi = v$), for all $x \geq 0$.

Theorem 4. *Given $k \in \{f, d\}$, $V_k = \{(\alpha_k, v_k)\}$. Moreover, α_k is continuous and decreasing, and lies below α_* ; v_k and $L_k v_k$ are continuous, and $v_k - b$ is decreasing.*

Proposition 6. *No strategy of the game with forced disclosure other than α_f induces a SSE. Moreover, a strategy $\xi := (\sigma, \chi)$ of the game with disposal induces a SSE only if $\sigma = \alpha_d$ and*

$$[\chi(h)](t, z) \in \arg \max_{d \in \{0,1\}} \{d[v_d(x+z) - v_d(x)]\} \quad \text{for all } t > 0 \text{ and } z > 0. \quad (27)$$

Proof of Theorem 1. Immediate from Theorem 4 and Proposition 6. □

The following two claims are used in the proofs of Theorem 4 and Proposition 6. Their easy proofs can be found in Online Appendix J.

Claim 1. Let $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ be Borel, bounded below, and such that $v - b$ is decreasing. Then $L_k v - b$ is decreasing for $k \in \{f, d\}$.

Claim 2. Let $\bar{v}, \underline{v} : \mathbb{R}_+ \rightarrow \mathbb{R}$ be Borel, bounded below, and such that $\bar{v} \geq \underline{v}$ and $\lim_{x \rightarrow \infty} \bar{v}(x) - \underline{v}(x) = 0$. Then $\bar{v} = \underline{v}$ if one of the following holds:

- $\bar{v} - \underline{v} \leq L_f \bar{v} - L_f \underline{v}$, or
- $\bar{v} - \underline{v} \leq L_d \bar{v} - L_d \underline{v}$ and $\lim_m \bar{v}(x_m) - \underline{v}(x_m) = 0$ for any bounded sequence $(x_m)_{m \in \mathbb{N}} \subset \mathbb{R}_+$ such that $\lim_m \Pr_F(\bar{v}(x_m + \tilde{z}) \leq \bar{v}(x_m)) = 1$.

Define $\Gamma : [0, 1] \times \mathbb{R}_+ \times \mathbb{R} \rightarrow \mathbb{R}$ by

$$\Gamma(a, x, l) := \frac{b(x) - c(a, x) + na\lambda l}{1 + na\lambda}.$$

Proof of Theorem 4. Step 1. There are $(\underline{\alpha}_k, \underline{v}_k), (\bar{\alpha}_k, \bar{v}_k) \in V_k$ such that $\bar{\alpha}_k \leq \alpha_*$ and, for all $(\alpha, v) \in V_k$, $\underline{\alpha}_k \leq \alpha \leq \bar{\alpha}_k \leq \alpha_*$ and $\underline{v}_k \leq v \leq \bar{v}_k$. Note that, for any $x \geq (>) 0$ and $l > (\geq) b(x)$, there is a unique $p(x, l) \in [0, 1]$ such that

$$p(x, l) \in \arg \max_{a \in [0, 1]} a\lambda \frac{l - [b(x) - c(p(x, l), x)]}{1 + \lambda np(x, l)} - c(a, x). \quad (28)$$

Indeed, the objective is continuously differentiable and concave in a , and its derivative has the same sign as $\gamma(p(x, l), x, l)$ where

$$\gamma(a, x, l) := l - [b(x) - c(a, x)] - \left(\frac{1}{\lambda} + na \right) c_1(a, x) \quad (29)$$

is decreasing in a , as $c(a, x)$ is convex in a . Moreover, p is continuous (when viewed as a map $(\{0\} \times (b(0), \infty)) \cup \{(x, l) \in (0, \infty) \times \mathbb{R} : l \geq b(x)\} \rightarrow [0, 1]$). Given $v \in B_{v_*}$, let $P_k v : \mathbb{R}_+ \rightarrow \mathbb{R}$ be given by

$$P_k v(x) := \Gamma(p(x, L_k v(x)), x, L_k v(x)).$$

Claim 3. $\Gamma(a, x, L_k v(x)) \leq P_k v(x)$ for all $x \geq 0$, $v \in B_{v_*}$, and $0 \leq a \leq p(x, L_k v(x))$.

Claim 3 is proved in Online Appendix J. I prove that P_k maps B_{v_*} to itself. Note that

$$L_k v \leq L_k v_* = L_f v_* \quad \text{for all } v \in B_{v_*} \quad (30)$$

where the inequality holds since $v \leq v_*$, and the equality since v_* is increasing (Proposition 1). Fix $v \in B_{v_*}$. Note that $L_k v$, p and, therefore, $P_k v$ are Borel. Fix $x \geq 0$. Since $p(x, L_k v(x)) \geq 0$, from Claim 3, $P_k v(x) \geq \Gamma(0, x, L_k v(x)) = b(x)$. Moreover,

$$P_k v(x) \leq \Gamma(p(x, L_k v(x)), x, L_f v_*(x)) \leq \max_{a \in [0, 1]} \Gamma(a, x, L_f v_*(x)) = v_*(x)$$

where the first inequality follows from (30) since $\Gamma(a, x, l)$ is increasing in l . Since $x \geq 0$ is arbitrary, $P_k v \in B_{v_*}$ as desired.

I show that P_k is increasing (in the pointwise order). Fix $v, w \in B_{v_*}$ with $w \leq v$, and $x \geq 0$. Then $L_k w(x) \leq L_k v(x)$. Then $P_k w(x) \leq \Gamma(p(x, L_k w(x)), x, L_k v(x))$ as $\Gamma(a, x, l)$ is increasing in l , and $p(x, L_k w(x)) \leq p(x, L_k v(x))$ as $p(x, l)$ is increasing

in l (since $\gamma(a, x, l)$ is). Together with Claim 3, the latter implies that

$$\Gamma(p(x, L_k w(x)), x, L_k v(x)) \leq P_k v(x).$$

Hence, $P_k w(x) \leq P_k v(x)$ and, since $x \geq 0$ is arbitrary, P_k is increasing.

Note that any countable set in B_{v_*} admits a supremum and an infimum (in the pointwise order) in B_{v_*} . Moreover, $P_k(\lim_m v_m) = \lim_m P_k(v_m)$ for any monotone sequence $(v_m)_{m \in \mathbb{N}} \subset B_{v_*}$, since $\lim_m L_k v_m = L_k(\lim_m v_m)$ by monotone convergence, and p and Γ_* are continuous. Then, P_k has smallest and largest fixed points $\underline{v}_k, \bar{v}_k \in B_{v_*}$. Define the Markov strategies $\underline{\alpha}_k$ and $\bar{\alpha}_k$ by $\underline{\alpha}_k(x) := p(x, L_k \underline{v}_k(x))$ and $\bar{\alpha}_k := p(x, L_k \bar{v}_k(x))$. Note that, for any Markov strategy α and any $v \in B_{v_*}$, $(\alpha, v) \in V_k$ if and only if $v = P_k v$ and $\alpha(x) = p(x, L_k v(x))$ for all $x \geq 0$.⁴⁴ Then $(\underline{\alpha}_k, \underline{v}_k), (\bar{\alpha}_k, \bar{v}_k) \in V_k$ and, for any $(\alpha, v) \in V_k$, $\underline{v}_k \leq v \leq \bar{v}_k$ so that $L_k \underline{v}_k \leq L_k v \leq L_k \bar{v}_k$ and therefore $\underline{\alpha}_k \leq \alpha \leq \bar{\alpha}_k$, as $p(x, l)$ is increasing in l .

It remains to prove that $\bar{\alpha}_k(x) \leq \alpha_*(x)$ for all $x \geq 0$. Fix $x \geq 0$ and assume without loss of generality that $\bar{\alpha}_k(x) > 0$ and $\alpha_*(x) < 1$. Then, from Step 2 of the proof of Proposition 1 (in Online Appendix I), it suffices to show that $\gamma^*(\bar{\alpha}_k(x), x) > 0$ where $\gamma^* : [0, 1] \times \mathbb{R}_+ \rightarrow \mathbb{R}$ is given by

$$\gamma^*(a, x) := L_f v_*(x) - [b(x) - c(a, x)] - \left(\frac{1}{n\lambda} + a \right) c_1(a, x).$$

Note that $\gamma^*(\bar{\alpha}_k(x), x) > \gamma(\bar{\alpha}_k(x), x, L_k \bar{v}_k(x)) \geq 0$, where the first inequality follows from (30) (as $\bar{v}_k \in B_{v_*}$) and the fact that $c_1(\bar{\alpha}_k(x), x) > 0$ (as $\bar{\alpha}_k(x) > 0$), and the second holds since $\bar{\alpha}_k(x) > 0$.

Step 2: V_k is a singleton. From Step 1, it suffices to show that $(\underline{\alpha}_k, \underline{v}_k) = (\bar{\alpha}_k, \bar{v}_k)$. By definition of $\underline{\alpha}_k$ and $\bar{\alpha}_k$, it is enough to show that $\underline{v}_k = \bar{v}_k$. Note that $v_* \geq \bar{v}_k \geq \underline{v}_k \geq b$ since $\bar{v}_k, \underline{v}_k \in B_{v_*}$, so that $\lim_{x \rightarrow \infty} \bar{v}_k(x) - \underline{v}_k(x) = 0$ by Proposition 1. Moreover, if $k = d$, $\lim_m \bar{v}_d(x_m) - b(x_m) = 0$ for any bounded sequence $(x_m)_{m \in \mathbb{N}} \subset \mathbb{R}_+$ such that $\lim_m \Pr_F(\bar{v}_d(x_m + \tilde{z}) \leq \bar{v}_d(x_m)) = 1$, by (22). Then, in light of Claim 2, it suffices to show that

$$\bar{v}_k(x) - \underline{v}_k(x) \leq L_k \bar{v}_k(x) - L_k \underline{v}_k(x) \tag{31}$$

for all $x \geq 0$. Fix x and consider three cases. Suppose first that $\bar{\alpha}_k(x) = 0$. Then $\underline{\alpha}_k(x) = 0$ as $\bar{\alpha}_k \geq \underline{\alpha}_k$, so that $\bar{v}_k(x) = b(x) = \underline{v}_k(x)$ and, therefore, (31) holds since $\bar{v}_k \geq \underline{v}_k$. Suppose now that $\underline{\alpha}_k(x) = 1$. Then, $\bar{\alpha}_k(x) = 1$ so that, from (13)

44. Indeed, for $k = f$, given $(\alpha, v) \in V_f$ and $x \geq 0$, (13) is equivalent to $v(x) = b(x) - c(\alpha(x), x) + n\alpha(x)[L_f v(x) - v(x)]$, i.e. $v(x) = \Gamma(\alpha(x), x, L_f v(x))$. Given $v(x) = \Gamma(\alpha(x), x, L_k v(x))$, (14) with $\hat{v}_\alpha = v$ is equivalent to ‘(28) with $l = L_k v(x)$ and $p(x, l) = \alpha(x)$ ’, which is equivalent to $\alpha(x) = p(x, L_k v(x))$. Finally, given $\alpha(x) = p(x, L_k v(x))$, $v(x) = \Gamma(\alpha(x), x, L_k v(x))$ is equivalent to $v(x) = P_k v(x)$. The argument for $k = d$ is similar.

if $k = f$ and (22) if $k = d$,

$$[L_k \bar{v}_k(x) - \bar{v}_k(x)] - [L_k \underline{v}_k(x) - \underline{v}_k(x)] = \frac{\bar{v}_k(x) - \underline{v}_k(x)}{\lambda n} \geq 0$$

and thus (31) holds. Finally, assuming that $\bar{\alpha}_k(x) > 0$ and $\underline{\alpha}_k(x) < 1$, by (14) if $k = f$ and (23) if $k = d$,

$$L_k \bar{v}_k(x) - \bar{v}_k(x) \geq \frac{c(\bar{\alpha}_k(x), x)}{\lambda} \geq \frac{c(\underline{\alpha}_k(x), x)}{\lambda} \geq L_k \underline{v}_k(x) - \underline{v}_k(x)$$

where the first inequality holds since $\bar{\alpha}_k(x) > 0$, the second since $\bar{\alpha}_k(x) \geq \underline{\alpha}_k(x)$, and the third since $\underline{\alpha}_k(x) < 1$. Then, (31) holds.

Step 3: $v_k - b$ and α_k are decreasing. Let \widehat{V} be the set of $v \in B_{v_*}$ such that $v - b$ is decreasing. From Steps 1 and 2, it suffices to show that $p(x, L_k v(x))$ is decreasing in x for any $v \in B_{v_*}$, and to find a fixed point v of P_k in \widehat{V} . For the former, fix $v \in \widehat{V}$. From Claim 1, $L_k v(x) - b(x)$ is decreasing. Then, $\gamma(a, x, L_k v(x))$ is decreasing in x , as $c_1(a, x)$ and $c_{11}(a, x)$ are increasing in x . Hence $p(x, L_k v(x))$ is decreasing in x .

To show that P_k admits a fixed point in \widehat{V} , note that \widehat{V} is a complete lattice⁴⁵ and, from Step 1, P_k is increasing on \widehat{V} . Then, from Tarski's fixed-point theorem, it suffices to show that P_k maps \widehat{V} to itself. To this end, fix $v \in \widehat{V}$. From Step 1, $P_k v \in B_{v_*}$. Then, it suffices to show that $P_k v(x_2) - P_k v(x_1) \leq b(x_2) - b(x_1)$ for all $0 \leq x_1 \leq x_2$. Fix x_1 and x_2 , and let $p_i = p(x_i, L_k v_i(x_i))$ for $i = 1, 2$, so that $p_1 \geq p_2$ from the previous paragraph. Then

$$\begin{aligned} P_k v(x_2) - P_k v(x_1) &\leq \Gamma(p_2, x_2, L_k v(x_2)) - \Gamma(p_2, x_1, L_k v(x_1)) \\ &\leq \frac{b(x_2) - b(x_1) + \lambda n p_2 [L_k v(x_2) - L_k v(x_1)]}{1 + \lambda n p_2} \leq b(x_2) - b(x_1) \end{aligned}$$

where the first inequality follows from Claim 3 since $p_2 \leq p_1$, the second holds as $c(a, x)$ is increasing in x , and the third follows from Claim 1. Hence $P_k v \in \widehat{V}$.

Step 5. $v_k, L_k v_k$ and α_k are continuous. Given a map $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}$, define

$$\phi(x^+) := \lim_{y \downarrow x} \phi(y) \quad \& \quad \phi(x^-) := \begin{cases} \phi(0) & \text{if } x = 0 \\ \lim_{y \uparrow x} \phi(y) & \text{if } x > 0. \end{cases}$$

Let $\bar{v}, \underline{v} : \mathbb{R}_+ \rightarrow \mathbb{R}$ be given by $\bar{v}(x) := v_k(x^-)$ and $\underline{v}(x) := v_k(x^+)$, and note that \bar{v} and \underline{v} are well-defined and lie in B_{v_*} , since $v_k - b$ is decreasing and b is concave. Since α_k is decreasing, $\bar{\alpha}, \underline{\alpha} : \mathbb{R}_+ \rightarrow [0, 1]$ given by $\bar{\alpha}(x) := \alpha_k(x^-)$ and

45. The argument is similar to that of footnote 54 in Online Appendix I. Measurability follows from the fact that, clearly, the variation of any map $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ such that $b \leq v \leq v_*$ and $v - b$ is decreasing, is bounded by $|v_*(x) - b(0)| + 2|b(x) - b(0)|$ over $[0, x]$ for any $x \geq 0$.

$\underline{\alpha}(x) := \alpha_k(x^+)$ are well-defined Markov strategies.

Note that, for all $x \geq 0$, $L_k v_k(x^-) = L\bar{v}(x)$ and $L_k v_k(x^-) = L\underline{v}(x)$ by dominated convergence. Then

$$\begin{aligned}\bar{\alpha}(x) &= p(x, L\bar{v}(x)) \quad \& \quad \bar{v}(x) = \Gamma(\bar{\alpha}(x), x, L_k \bar{v}(x)) = P_k \bar{v}(x), \\ \underline{\alpha}(x) &= p(x, L\underline{v}(x)) \quad \& \quad \underline{v}(x) = \Gamma(\underline{\alpha}(x), x, L_k \underline{v}(x)) = P_k \underline{v}(x),\end{aligned}$$

since p and Γ are continuous, so that $(\bar{\alpha}, \bar{v}), (\underline{\alpha}, \underline{v}) \in V_k$ by the argument in footnote 44. Then, $(\bar{\alpha}, \bar{v}) = (\underline{\alpha}, \underline{v})$ from Step 2, and thus $v_k, L_k v$, and α_k are continuous. \square

Proof of Proposition 6. Let Σ (Ξ) be the set of strategies of the game with forced disclosure (disposal). For any $x_0 \geq 0$, let $H(x_0)$ be the set of public histories with initial stock $x_0 \geq 0$, and $\Xi_f^*(x_0)$ ($\Xi_d^*(x_0)$) be the set of $\sigma \in \Sigma$ ($\xi \in \Xi$) inducing SSE in the game with forced disclosure (disposal) and initial stock x_0 . Fix x_0 and $k \in \{f, d\}$.

I claim that it suffices to show that $v_{\xi, h}$ is constant with value $v_k(X(h))$ for all $h \in H(x_0)$ and $\xi \in \Xi_k^*(x_0)$. To see why note first that, given $x \geq 0$, $L_k v_k(x) \geq v_k(x)$ where the inequality is strict unless $x > 0$. Indeed, if $L_k v_k(x) \leq v_k(x)$ then $v_k(x) = b(x)$ by (13) if $k = f$ and (22) if $k = d$, so that $L_k v_k(x) = b(x)$ and b must be constant over $[x, \infty)$ since it is increasing and $v_k \geq b$, and thus $x > 0$. Now suppose that $v_{\xi, h}$ is constant with value $v_k(X(h))$ for all $h \in H(x_0)$ and $\xi \in \Xi_k^*(x_0)$. Fix $\xi := (\sigma, \chi) \in \Xi_k^*(x_0)$ and a history $h \in H(x_0)$. Note that (27) follows from (21) if $k = d$, so that it is enough to show that $\sigma(h) = \alpha_k(X(h))$ a.e. If $L_k v_k(X(h)) > v_k(X(h))$, then $\sigma(h) = \alpha_k(X(h))$ a.e. by (13) and (11) for $k = f$, and (22) and (19) for $k = d$. If instead $L_k v_k(X(h)) = v_k(X(h))$ then $X(h) > 0$, and thus $\sigma(h) \stackrel{\text{a.e.}}{=} 0 = \alpha_k(X(h))$ by (28) and (12) for $k = f$, (23) and (20) for $k = d$. Hence $\sigma(h) = \alpha_k(X(h))$ a.e.

To prove that $v_{\xi, h}$ is constant with value $v_k(X(h))$ for all $h \in H(x_0)$ and $\xi \in \Xi_k^*(x_0)$, define $\bar{v}_k, \underline{v}_k : \mathbb{R}_+ \rightarrow \mathbb{R}$ by

$$\begin{aligned}\bar{v}_k(x) &:= \inf_{\epsilon > 0} \sup \{v_{\xi}(y) : (x - \epsilon) \vee 0 \leq y \leq x + \epsilon, \xi \in \Sigma_k^*(y)\} \\ \underline{v}_k(x) &:= \sup_{\epsilon > 0} \inf \{v_{\xi}(y) : (x - \epsilon) \vee 0 \leq y \leq x + \epsilon, \xi \in \Sigma_k^*(y)\}.\end{aligned}$$

Note that $\bar{v}_k \geq \underline{v}_k$ and $v_{\xi, h}$ takes values in $[\underline{v}_k(X(h)), \bar{v}_k(X(h))]$ for all $\xi \in \Xi_k^*(x_0)$ and $h \in H(x_0)$. Then, it suffices to show that $\bar{v}_k = v_k = \underline{v}_k$.

I show that $\bar{v}_k = v_k$, relying on Claim 2. A similar reasoning yields that $v_k = \underline{v}_k$. Note first that \bar{v}_k is upper-semicontinuous and, hence, Borel. Also, $b \leq v_k \leq \bar{v}_k \leq v_*$ where the last inequality holds as v_* is continuous and $v_{\xi}(x) \leq v_*(x)$ for all $x \geq 0$ and $\xi \in \Xi_k^*(x)$, so that $\lim_{x \rightarrow \infty} \bar{v}_k - v_k = 0$ by Proposition 1.

I show that, if $k = d$, then $\lim_m \bar{v}_d(x^m) - b(x^m) = 0$ for any bounded sequence $(x^m)_{m \in \mathbb{N}} \subset \mathbb{R}_+$ such that $\lim_m \Pr_F(\bar{v}_d(x^m + \tilde{z}) \leq \bar{v}_d(x^m)) = 1$. Fix $(x^m)_{m \in \mathbb{N}} \subset \mathbb{R}_+$ and note that $\lim_m \mathbb{E}_F[0 \vee (\bar{v}_d(x^m + \tilde{z}) - \bar{v}_d(x^m))] = 0$ and that, for all $m \in \mathbb{N}$, $y \mapsto \mathbb{E}_F[0 \vee (\bar{v}_d(y + \tilde{z}) - \bar{v}_d(x_m))]$ is upper-semicontinuous at x_m , since \bar{v}_d is upper-semicontinuous. Then, it is clear that there exist sequences $(y_m)_{m \in \mathbb{N}} \subset \mathbb{R}_+$ and $(\xi^m)_{m \in \mathbb{N}} := (\sigma^m, \chi^m)_{m \in \mathbb{N}} \in \prod_{m \in \mathbb{N}} \Xi_d^*(y_m)$ such that $(y_m)_{m \in \mathbb{N}}$ is bounded and $b(y^m) - b(x^m)$, $\bar{v}_d(x^m) - v_{\xi^m}(y^m)$, and $\mathbb{E}_F[0 \vee (\bar{v}_d(y^m + \tilde{z}) - v_{\xi^m}(y^m))]$ all vanish as $m \rightarrow \infty$. Hence, it suffices to show that $\lim_m v_{\xi^m}(y^m) - b(y^m) = 0$. Since $(y_m)_{m \in \mathbb{N}}$ is bounded, $(v_{\xi^m, y^m})_{m \in \mathbb{N}}$ is uniformly l -Lipschitz for some $l > 0$. Also, for all $\epsilon > 0$ and $m \in \mathbb{N}$, as v_{ξ^m, y^m} is bounded, there is $t_m \geq \epsilon$ such that v_{ξ^m, y^m} is increasing over $[0, t_m - \epsilon]$, and differentiable at t_m with derivative lower than ϵ . Then

$$\begin{aligned} v_{\xi^m}(y^m) &= v_{\xi^m, y^m}(0) \leq v_{\xi^m, y^m}(t_m - \epsilon) \leq l\epsilon + v_{\xi^m, y^m}(t_m) \\ &\leq (l + 1)\epsilon + b(y^m) - c([\sigma^m(h)](t_m), y^m) \\ &\quad + \lambda[\sigma^m(h)](t_m) \sum_{i=1}^n \mathbb{E}_F[0 \vee (v_{\xi^m}(y^m \circ (t_m, \tilde{z}, i)) - v_{\xi^m, y^m}(t_m))] \\ &\leq [l(1 + \lambda n) + 1]\epsilon + b(y^m) + \lambda n \mathbb{E}_F[0 \vee (\bar{v}_d(y^m + \tilde{z}) - v_{\xi^m}(y^m))] \end{aligned}$$

where the third inequality follows from (19), (20), and (21), and the last inequality holds since $v_{\xi^m}(y^m \circ (t_m, z, i)) \leq \bar{v}_d(y^m + z)$ for all i . Letting $m \rightarrow \infty$ yields $\lim_m v_{\xi^m}(y^m) - b(y^m) \leq [l(1 + \lambda n) + 1]\epsilon$, and result follows as $\epsilon > 0$ is arbitrary.

In light of Claim 2, it remains to show that $\bar{v}_k - v_k \leq L_k \bar{v}_k - L_k v_k$. Since v_k is continuous, it is enough to show that, for all $x \geq 0$ and $\xi \in \Xi_k^*(x)$,

$$v_\xi(x) - v_k(x) \leq L_k \bar{v}_k(x) - L_k v_k(x). \quad (32)$$

I show that $L\bar{v}_k \geq \bar{v}_k$. Suppose that $L\bar{v}_k(\hat{x}) < \bar{v}_k(\hat{x})$ for some $\hat{x} \geq 0$ and seek a contradiction. If so, then $k = f$ and, since $L_f \bar{v}_f$ is upper-semicontinuous (as \bar{v}_f is), there exists $\epsilon > 0$ such that $L_f \bar{v}_f < \bar{v}_f(\hat{x})$ over $I := [(\hat{x} - \epsilon) \vee 0, \hat{x} + \epsilon]$. Then, there is $x \in I$ and $\sigma \in \Xi_f^*(x)$ such that $v_\sigma(x) > L_f \bar{v}_f(x)$. Let $t := \sup\{s \geq 0 : v_{\sigma, x} > L_f \bar{v}_f(x) \text{ over } [0, s]\}$ and note that $t > 0$ since $v_{\sigma, x}$ is continuous and $v_{\sigma, x}(0) = v_\sigma(x)$. But then $\mathbb{E}_F[v_\sigma(x \circ (s, \tilde{z}, i)) - v_{\sigma, x}(s)] < 0$ for all i and $s \in [0, t)$, since $v_\sigma(x \circ (s, z, i)) \leq \bar{v}_f(x + z)$ for all $z > 0$, and thus $\sigma(x) = 0$ a.e. over $(0, t)$ by (12). Hence $v_\sigma(x) = b(x)$ if $t = \infty$ and $v_\sigma(x) = (1 - e^{-t})b(x) + e^{-t}v_{\sigma, x}(t) = (1 - e^{-t})b(x) + e^{-t}L_f \bar{v}_f(x)$ otherwise (as $v_{\sigma, x}$ is continuous at t). This contradicts the fact that $v_\sigma(x) > L_f \bar{v}_f(x)$, since $L_f \bar{v}_f(x) \geq L_f b(x) \geq b(x)$.

To prove (32), fix x and $\xi := (\sigma, \chi)$ and assume without loss that $v_\xi(x) > v_k(x)$.

Let $t := \sup\{0\} \cup \{s > 0 : \sigma(x) \leq \alpha_k(x) \text{ a.e. on } (0, s)\}$ and define

$$\phi_* := \begin{cases} L_k \bar{v}_k(x) - L_k v_k(x) + v_k(x) & \text{if } t < \infty \\ 0 & \text{if } t = \infty \end{cases} \quad \& \quad \phi := \begin{cases} v_{\xi, x}(t) & \text{if } t < \infty \\ 0 & \text{if } t = \infty. \end{cases}$$

I claim that $\phi \leq \phi_*$. Indeed, if $t < \infty$ then $\alpha_k(x) < 1$ and, for a.e. $s \geq t$ such that $[\sigma(x)](s) > \alpha_k(x)$, and each i ,

$$\begin{aligned} L_k \bar{v}_k(x) - v_{\xi, x}(s) &\geq \mathbb{E}_F\{[\chi(x)](s, \tilde{z})(v_{\xi}(x \circ (s, \tilde{z}, i)) - v_{\xi, x}(s))\} \\ &\geq \frac{c_1([\sigma(x)](s), x)}{\lambda} \geq \frac{c_1(\alpha_k(x), x)}{\lambda} \geq L_k v_k(x) - v_k(x) \end{aligned}$$

where the first inequality holds since $\chi(x)$ is constant with value 1 if $k = f$, $L_k \bar{v}_k(x) \geq \bar{v}_k(x) \geq v_{\xi, x}(s)$ if $k = d$, and $\bar{v}_k(x + z) \geq v_{\xi}(x \circ (s, z, i))$ for all $z \geq 0$, the second follows from (12) if $k = f$, and from (20) and (21) if $k = d$, since $[\sigma(x)](s) > 0$, the third holds since $[\sigma(x)](s) \geq \alpha_k(x)$, and the last follows from (14) if $k = f$ and from (23) if $k = d$, since $\alpha_k(x) < 1$. Then, since $v_{\xi, x}$ is continuous at t , $\phi = v_{\xi, x}(t) \leq L_k \bar{v}_k(x) - L_k v_k(x) + v_k(x) = \phi_*$.

Let A_0 be the set of Borel $a : (0, \infty) \rightarrow [0, 1]$. Given $a \in A_0^n$, let $N(\cdot, a)$ be the CDF of the random time τ of the first innovation, if agents exert effort according to a (with $\tau = \infty$ if no innovation is produced). Let

$$\Phi(a, v, w) := \int \int_0^{s \wedge t} b(x) - c(a^1(r), x) dr + e^{-s \wedge t} (\mathbb{I}_{s \leq t} v + \mathbb{I}_{s > t} w) N(ds, a),$$

for all $v, w \in \mathbb{R}$, and $\hat{v} := \max_{a \in A_0} \Phi((a, \alpha_k(x)^{n-1}), L_k \bar{v}_k(x), \phi_*)$. Note that, writing $a^n := (a, \dots, a) \in A_0^n$ for any $a \in A_0$,

$$\begin{aligned} v_{\xi}(x) &= \begin{cases} \Phi(\sigma(x)^n, \frac{1}{n} \sum_{i=1}^n \mathbb{E}_F[v_{\xi}(x \circ (s, \tilde{z}, i))], \phi) & \text{if } k = f \\ \Phi(\sigma(x)^n, \frac{1}{n} \sum_{i=1}^n \mathbb{E}_F[v_{\xi, x}(s) \vee v_{\xi}(x \circ (s, \tilde{z}, i))], \phi) & \text{if } k = d, \text{ by (21)} \end{cases} \\ &\leq \Phi(\sigma(x)^n, L_k \bar{v}_k(x), \phi_*) \leq \hat{v} \end{aligned}$$

where the first inequality holds since $v_{\xi, x}(s) \leq \bar{v}_k(x)$ and $v_{\xi}(x \circ (s, z, i)) \leq \bar{v}_k(x + z)$ for all $s, z \geq 0$ and i , and $\phi \leq \phi_*$, and the second since $\sigma(x) \leq \alpha_k(x)$ over $[0, t]$, $L_k \bar{v}_k(x) \geq b(x)$, and $L_k \bar{v}_k(x) \geq \phi_*$ (as $L_k v_k \geq v_k$).

Therefore, it suffices to show that $\hat{v} - v_k(x) \leq L_k \bar{v}_k(x) - L_k v_k(x)$. To this end, let $\hat{a} \in A_0$ achieve \hat{v} , and note that

$$v_k(x) = \max_{a \in A} \Phi((a, \alpha_k(x)^{n-1}), L_k v_k(x), v_k(x)) \geq \Phi((\hat{a}, \alpha_k(x)^{n-1}), L_k v_k(x), v_k(x))$$

so that

$$\hat{v} - v_k(x) \leq [L_k \bar{v}_k(x) - L_k v_k(x)] \int e^{-s \wedge t} N(ds, (\hat{a}, \alpha_k(x)^{n-1})) \leq L_k \bar{v}_k(x) - L_k v_k(x).$$

Thus (32) holds, and therefore $\bar{v}_k = v_k$, by Claim 2. \square

B Proof of Proposition 2

Fix $\hat{x} \geq 0$ at which (15) fails. The map

$$\psi : x \mapsto \lambda \{ \mathbb{E}_F [b(x + \tilde{z}) - b(x)] \} - c_1(0, x)$$

is decreasing as b is concave and $c_1(0, x)$ is increasing. Then, there exist $\epsilon > 0$ and $\bar{x} > \hat{x}$ such that $\psi(x) + 2\lambda\epsilon < 0$ for all $x \geq \bar{x}$. By Proposition 1, we may choose \bar{x} sufficiently large so that $v_*(x) - b(x) \leq \epsilon/n$ for all $x \geq \bar{x}$.

Fix a PPE $\sigma = (\sigma^i)_{i=1}^n$. Note that, for any i and history h such that $X(h) \geq \bar{x}$, $v_\sigma^i(h) \leq v_*(X(h)) + \epsilon$, for otherwise $nv_*(X(h)) \geq \sum_{j=1}^n v_\sigma^j(h) > v_*(X(h)) + \epsilon + (n-1)b(X(h))$, contradicting $v_*(X(h)) \leq b(X(h)) + \epsilon/n$. Then, for any history h such that $X(h) \geq \bar{x}$, any i and $s \geq T(h)$,

$$\begin{aligned} \lambda \{ \mathbb{E}_F [v_\sigma^i(h \circ (s, \tilde{z}, i))] - v_{\sigma, h}(s) \} &\leq \lambda \{ \mathbb{E}_F [v_*(X(h) + \tilde{z})] + \epsilon - b(X(h)) \} \\ &\leq \lambda \{ \mathbb{E}_F [b_*(X(h) + \tilde{z})] + (1 + 1/n)\epsilon - b(X(h)) \} < c_1(0, X(h)) \end{aligned}$$

where the first inequality holds since $v_{\sigma, h}(s) \geq b(X(h))$ and, by the previous step, for all $z > 0$, $v_\sigma(h \circ (s, z, i)) \leq v_*(X(h) + z) + \epsilon$. Then no effort is exerted in σ at history h , by (12).

Assume without loss that $\sigma^i(x_0) > 0$ is non-null, for some i (so that $x_0 < \bar{x}$), and define:

$$x_\sigma := \sup \{ x \geq x_0 : \sigma^i(h) > 0 \text{ is non-null for some } i \text{ and } h \text{ with } X(h) \geq x \}.$$

Then, it suffices to show that $x_\sigma \leq \hat{x}$. From above, $x_\sigma \leq \bar{x} < \infty$. Fix $\hat{\epsilon} > 0$ and a history h such that $x_\sigma - \hat{\epsilon} \leq X(h) \leq x_\sigma$, and i such that $\sigma^i(h)$ is non-null. Then, there exists $t \geq T(h)$ such that $[\sigma^i(h)](t) > 0$ and (12) holds, and thus

$$\begin{aligned} c_1(0, X(h)) &\leq c_1([\sigma^i(h)](t), X(h)) \\ &\leq \lambda \{ \mathbb{E}_F [v_\sigma(h \circ (t, \tilde{z}, i))] - v_{\sigma, h}(t) \} \\ &\leq \lambda \{ \mathbb{E}_F [\mathbb{1}_{\tilde{z} \leq \hat{\epsilon}} \bar{w}(X(h) + \tilde{z}) + \mathbb{1}_{\tilde{z} > \hat{\epsilon}} b(X(h) + \tilde{z})] - b(X(h)) \} \end{aligned}$$

where $\bar{w}(x) := b(x) + n\lambda[b(\mu) - b(0)]$ is an upper bound on v_* (as established in the

proof of Proposition 1), the second inequality follows from (12) since $[\sigma^i(h)](t) > 0$, and the last since $v_{\sigma,h}(t) \geq b(X(h))$ and, for all $z > 0$, $v_{\sigma}(h \circ (t, z, i)) \leq \bar{w}(X(h)+z)$, and $v_{\sigma}(h \circ (t, z, i) = b(X(h) + z)$ for $z \geq x_{\sigma} - X(h)$. Since (15) holds with ‘<’ for $x > \hat{x}$, letting $\hat{\epsilon}$ tend to 0 yields $X(h) \leq \hat{x}$, so that $x_{\sigma} \leq \hat{x}$, as desired.

C Proofs of Propositions 3 and 4

In this appendix, I prove Propositions 3 and 4.

Claim 4. For any $x \geq 0$, $\mathbb{E}_F[v_f(x + \tilde{z})] \geq v_f(x)$.

Proof. Fix $x \geq 0$. If $\alpha_f(x) > 0$, from (14),

$$\lambda\{\mathbb{E}_F[v_f(x + \tilde{z})] - v_f(x)\} \geq c_1(\alpha_f(x), x) \geq 0.$$

If instead $\alpha_f(x) = 0$, $\alpha_f(y) = 0$ for all $y \geq x$, as α_f is decreasing (Theorem 1). Then

$$\mathbb{E}_F[v_f(x + \tilde{z})] = \mathbb{E}_F[b(x + \tilde{z})] \geq b(x) = v_f(x)$$

where the inequality holds since b is increasing. \square

Proof of Proposition 3. The first part is immediate from Claim 4. For the second part, suppose that $c(a, x) = c(a)$. We shall show that v_f is increasing. recall from the proofs of Theorems 1 and 4 that v_f is the unique fixed point of P_f in \widehat{V} , that P_f maps \widehat{V} to itself and is increasing, and that \widehat{V} is a complete lattice (with respect to the pointwise order). Let V' be the set of $v \in \widehat{V}$ that are increasing. It suffices to show that P_f admits a fixed point in V' . Note that V' is a complete lattice. Then, since P_f is increasing on \widehat{V} , by Tarski’s fixed-point theorem, it suffices to show that P_f maps V' to itself. Fix $v \in V'$. Note that $P_f v \in \widehat{V}$, so that it remains to show that $P_f v$ is increasing.

Since v is increasing and $v - b$ is decreasing, v and, thus, $L_f v$ are continuous. Then, $p(L_f v(x), x)$ and, therefore, $P_f v$, are continuous. Moreover, from Step 3 of the proof of Theorem 4, $p(x, L_f v(x))$ is decreasing in x . Let $I_a := \{x \geq 0 : p(x, L_f v(x)) = a\}$ for $a \in \{0, 1\}$ and $I := \mathbb{R}_+ \setminus (I_0 \cup I_1)$. If $p(x, L_f v(x)) = 0$ for some $x \geq 0$, then I_0 is an interval, and $P_f v = b$ over I_0 , so that it is increasing on I_0 . If $p(x, L_f v(x)) \in (0, 1)$ for some $x \geq 0$, then I is also an interval. Moreover, for any $x \in I$,

$$\lambda[L_f v(x) - P_f v(x)] = \lambda \frac{L_f v(x) - [b(x) - c(a, x)]}{1 + \lambda n a} = c'(a)$$

where $a := p(x, L_f v(x))$, the first equality follows by definition of P_f , and the second from (28). Then $P_f v$ is then increasing on I , as v is increasing and $p(x, L_f v(x))$

is decreasing in x . Finally, if $p(x, L_f v(x)) = 1$ for some $x \geq 0$, then I_1 is an interval, and

$$P_f v(x) = \frac{b(x) - c(1, x) + \lambda n L_f v(x)}{1 + \lambda n}$$

over I . Then, $P_f v$ is increasing on I_1 since $b(x) - c(1, x)$ and v are increasing. Therefore, $P_f v$ is decreasing since I_0, I_1 and I partition \mathbb{R}_+ , and $P_f v$ is continuous. \square

The proof of Proposition 4 relies on the following intuitive result, proved in Online Appendix K.

Lemma 2. *Suppose that (3) holds. Then, innovations are detrimental if and only if v_f is not increasing on $[x_0, \infty)$.*

Proof of Proposition 4. We may assume without loss of generality that $c_1(0, x_f) > 0$, for otherwise $b'(x_f) = 0$ by (15), so that (18) holds with equality and there is nothing to prove. By (15), $\alpha_f(x) = (>) 0$ for all $x \geq (<) x_f$. Then $v_f(x) = x$ for all $x \geq x_f$, and so innovations are detrimental for any initial stock $x_0 < x_f$ if and only if v_f is non-monotone on (x, x_f) for all $x < x_f$, by Lemma 2. Hence, it suffices to show that, for any $x < x_f$ sufficiently large and any $\hat{x} \in (x, x_f)$, $v_f(x) < (>) v_f(\hat{x})$ if (18) holds with ' $<$ ' (with ' $>$ ').

Since α_f is continuous (Theorem 1), $y_f := \max\{x \geq 0 : \alpha_f(x) = 1\} < x_f$. Then, as v_f is also continuous, for any $x \in [y_f, x_f]$,

$$\lambda\{\mathbb{E}_F[v_f(x + \tilde{z})] - v_f(x)\} = c_1(\alpha_f(x), x)$$

by (14). Hence, given $y_f > x > \hat{x} > x_f$, writing $\Delta v := v_f(x) - v_f(\hat{x})$,

$$\lambda(\Delta \mathbb{E} - \Delta v) = c_1(\alpha_f(x), x) - c_1(\alpha_f(\hat{x}), \hat{x}) = \Delta c_1 + c_1(\alpha_f(x), \hat{x}) - c_1(\alpha_f(\hat{x}), \hat{x}) \quad (33)$$

where $\Delta \mathbb{E} := \mathbb{E}_F[v_f(x + \tilde{z}) - b(\hat{x} + \tilde{z})]$ and $\Delta c_1 := c_1(\alpha_f(x), x) - c_1(\alpha_f(\hat{x}), \hat{x})$. Note that $v_f(x) - b(x) + c(\alpha_f(x), x) = n\alpha_f(x)c_1(\alpha_f(x), x)$ for any $x \in [y_f, x_f]$, by (7). Then, given $y_f < x < \hat{x} < x_f$ such that $c_1(\alpha_f(x), \hat{x}) > c_1(\alpha_f(\hat{x}), \hat{x})$, there are $a, \hat{a} \in [\alpha_f(\hat{x}), \alpha_f(x)]$ such that $c(\alpha_f(x), \hat{x}) - c(\alpha_f(\hat{x}), \hat{x}) = [\alpha_f(x) - \alpha_f(\hat{x})]c_1(a, \hat{x})$ and $c_1(\alpha_f(x), \hat{x}) - c_1(\alpha_f(\hat{x}), \hat{x}) = [\alpha_f(x) - \alpha_f(\hat{x})]c_{11}(\hat{a}, \hat{x})$. Hence, setting $\Delta c := c(\alpha_f(x), x) - c(\alpha_f(x), \hat{x})$,

$$\begin{aligned} \Delta v - b(x) + b(\hat{x}) + \Delta c &= n\alpha_f(x)[c_1(\alpha_f(x), x) - c_1(\alpha_f(\hat{x}), \hat{x})] \\ &\quad + [nc_1(\alpha_f(\hat{x}), \hat{x}) - c_1(a, \hat{x})][\alpha_f(x) - \alpha_f(\hat{x})] \\ &= n\alpha_f(x)\lambda(\Delta \mathbb{E} - \Delta v) + [nc_1(\alpha_f(\hat{x}), \hat{x}) - c_1(a, \hat{x})] \frac{\lambda(\Delta \mathbb{E} - \Delta v) - \Delta c_1}{c_{11}(\hat{a}, \hat{x})} \end{aligned}$$

where the second equality follows by (33). Then Δv has the sign of

$$\{b(x) - b(\hat{x}) - \Delta c + n\alpha_f(x)\lambda\Delta\mathbb{E}\}c_{11}(\hat{a}, \hat{x}) + [nc_1(\alpha_f(\hat{x}), \hat{x}) - c_1(a, \hat{x})](\lambda\Delta\mathbb{E} - \Delta c_1).$$

Note that, as $x \uparrow x_f$,

$$\frac{b(x) - b(\hat{x})}{x - \hat{x}} \rightarrow b'(x_f), \quad \frac{\Delta c}{x - \hat{x}} \rightarrow 0, \quad \frac{\Delta\mathbb{E}}{x - \hat{x}} \rightarrow \mathbb{E}_F[b'(x_f + \tilde{z})] \quad \& \quad \frac{\Delta c_1}{x - \hat{x}} \rightarrow c_{12}(0, x_f)$$

uniformly for all $\hat{x} \in (x, x_f)$, since $\alpha_f(x) \rightarrow 0$. Thus, for $x < x_f$ sufficiently large, $v_f(x) - v_f(\hat{x}) > (<) 0$ if (18) holds with ' $<$ ' (' $>$ ').

Finally, suppose that, for all $x < x_f$, there exists $\hat{x} \in (x, x_f)$ such that $c_1(\alpha_f(x), \hat{x}) = c_1(\alpha_f(\hat{x}), \hat{x})$. Then $c_{11}(0, x_f) = 0$ and, by (33), for $x < x_f$ sufficiently large and any $\hat{x} \in (x, x_f)$ such that $c_1(\alpha_f(x), \hat{x}) = c_1(\alpha_f(\hat{x}), \hat{x})$, $v_f(x) > (<) v_f(\hat{x})$ if (18) holds with ' $<$ ' (with ' $>$ '), as desired. \square

D Proof of Theorem 2

For the first part note that, by Theorem 4 and Proposition 6 (Appendix A), SSE are precisely the profiles induced by strategies (α_d, χ) such that χ solves (27), and that α_d and v_d inherit the properties of α_f and v_f described in Theorem 1.

I show that $v_d \geq v_f$. From Steps 1 and 2 of the proof of Theorem 4, it suffices to show that the map (P_f, P_d) admits a fixed point in the set \tilde{V} of pairs $(v, w) \in \hat{V} \times \hat{V}$ such that $v \leq w$. From Step 1, \tilde{V} is a complete lattice and (P_f, P_d) is increasing on \tilde{V} . Then, from Tarski's fixed-point theorem, it suffices to show that (P_f, P_d) maps \tilde{V} to itself.

Fix $(v, w) \in \tilde{V}$. From Step 1, $(P_f v, P_d w) \in \hat{V} \times \hat{V}$. Moreover, $L_f v \leq L_d w$ since $v \leq w$. Then, $p(x, L_f v(x)) \leq p(x, L_d w(x))$, since $p(x, l)$ is increasing in l (Step 1 of the proof of Theorem 4). Hence, for all $x \geq 0$,

$$\begin{aligned} P_f v(x) &= \Gamma(p(x, L_f v(x)), x, L_f v(x)) \leq \Gamma(p(x, L_f v(x)), x, L_d w(x)) \\ &\leq \Gamma(p(x, L_d w(x)), x, L_d w(x)) = P_d w(x) \end{aligned} \quad (34)$$

where the first inequality holds since $\Gamma(a, x, l)$ is increasing in l , and the second follows from Claim 3, as $p(x, L_f v(x)) \leq p(x, L_d w(x))$. Then $(P_f v, P_d w) \in \tilde{V}$.

To show that $\alpha_d \geq \alpha_f$, note that $L_d v_d \geq L_f v_f$ since $v_d \geq v_f$. Then, $\alpha_d \geq \alpha_f$ as $\alpha_k(x) = p(x, L_k v_k(x))$ for all $x \geq 0$ and $k \in \{e, d\}$, and $p(x, l)$ is increasing in l .

For the last part, suppose first that innovations are not detrimental. Then, it is clear that the game with disposal admits a SSE that coincides on path with α_f . Then, (α_d, δ_d) coincides with α_f since it is the unique SSE of the game.

Suppose now that innovations are detrimental. Let $X_0 = \{x \geq x_0 : \alpha_f(x) > 0 \text{ and } \Pr_F(v_f(x + \tilde{z}) < v_f(x)) > 0\}$ and define $(X_m)_{m=1}^\infty$ recursively by $X_m := \{x \geq x_0 : \alpha_f(x) > 0 \text{ and } \Pr_F(x + \tilde{z} \in X_{m-1}) > 0\}$ for all $m \geq 1$. By hypothesis, $x_0 \in X_m$ for some $m \geq 0$. Then, it suffices to show that $v_d > v_f$ over X_m for all $m \geq 0$. I proceed by backward induction on m . For the base case $m = 0$, fix $x \in X_0$ and note that $L_d v_d(x) \geq L_d v_f(x) > L_f v_f(x)$, where the first inequality holds since $v_d \geq v_f$. Then, $v_d(x) > v_f(x)$ by the argument used to derive (34), as $\alpha_f(x) > 0$. For the induction step, fix $m \geq 0$ and suppose that $v_d > v_f$ over X_m . Fix $x \in X_{m+1}$ and note that $L_d v_d \geq L_f v_d > L_f v_f$. Then, $v_d(x) > v_f(x)$ by the argument used to derive (34), as $\alpha_f(x) > 0$.

E Proof of Lemma 1

The ‘if’ part is immediate, since concealment has no value if opponents exert no effort. For the ‘only if’ part, fix an initial stock $x_0 < \lambda\mu$. It remains to show that α_f , coupled with the full-disclosure policy, does not form a PBE. Label this profile ξ_f . From (3), there exists $z < \lambda\mu - x_0$ in $\text{supp}(F)$.

Suppose that ξ_f is played at initial stock $\lambda\mu - z$ and the first innovation, obtained by some agent i at some time t , has size z . Then, agent i discloses fully at time t , and no agent exerts any effort thereafter. This raises the public stock to $\lambda\mu$, so that the time- t continuation payoff (to each agent) is $\lambda\mu$. Because $\lambda\mu$ solves (15), after time t , agent i would also earn $\lambda\mu$ if she exerted maximal effort until she obtained a further innovation, disclosed it fully, and exerted no effort thereafter. Consider the time- t deviation for agent i in which she does precisely this, *but* conceals fully at time t . Conditional on the opponents of agent i obtaining no innovation after time t , agent i ’s continuation payoff is again $\lambda\mu$. This is because exerting full effort yields no flow of payoff (i.e. $u(1, x_t) \equiv 0$), so the fact that the public stock x_t equals $\lambda\mu - z$ instead of $\lambda\mu$ after time t has no effect. Because $z > 0$, Corollary 3 implies that $\alpha_f(\lambda\mu - z) > 0$. Then, the above deviation is profitable, as the opponents of agent i will obtain an innovation before her with strictly positive probability.

By continuity, there are open neighbourhoods X of $\lambda\mu - z$ and Z of z such that, if ξ_f is played at x_0 , and an agent obtains an innovation of size within Z at some time t such that $x_t \in X$, agent i has a profitable deviation at time t . From (3), this occurs with positive probability, so that ξ_f does not form a PBE.

F Proof of Theorem 3

The proof of Theorem 3 has two parts. In this appendix, I show that there exist an effort schedule $\alpha_t^c(k)$ that is increasing in k , and a disclosure cutoff $q(t)$ taking values in $[\lambda\mu, \lambda\mu n]$, such that a family of strategy profiles matching on path the one described by Theorem 3 constitute Bayes-Nash equilibria. The formal statement of this intermediary result is Proposition 7 below. I complete the proof of Theorem 3 in Online Appendix L.4, by choosing a particular profile from the aforementioned family, and specifying off-path beliefs inducing a PBE.

The rest of the argument is structured as follows. I begin with several definitions, and state Proposition 7. I then state a dynamic-programming result (Lemma 3 below), and show that Proposition 7 holds as long as a given Bellman equation admits a solution (Lemma 4 below). In Appendix F.1, I define a discrete-time, finite-horizon approximation of this Bellman equation and construct a solution for any time length and horizon (Lemma 6). I then prove Proposition 7 in Online Appendix L.2, relying on continuity and compactness arguments.

As noted in Section 8.3, Theorem 3 is immediate from Lemma 1 if $x_0 \geq \lambda\mu$, so that we may assume without loss of generality that $x_0 < \lambda\mu$. Let \mathcal{S} be the quotient space of Borel $\alpha : \mathbb{R}_+^2 \rightarrow [0, 1]$ such that $\alpha_t : \mathbb{R}_+ \rightarrow [0, 1]$ is increasing for all $t \geq 0$, with respect to the equivalence relation $\sim_{\mathcal{S}}$, defined by $\alpha \sim_{\mathcal{S}} \alpha'$ if and only if $\int_0^t \alpha_s(k) ds = \int_0^t \alpha'_s(k) ds$ for each $t, k \geq 0$. Let \mathcal{Q} be the set of $q : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that $q(t) \leq \liminf_{s \downarrow t} q(s)$ for all $t \geq 0$, and \mathcal{Q}_0 be the set of lower-semicontinuous $q \in \mathcal{Q}$ with image in $[\lambda\mu, \lambda\mu n]$, such that $q(0) = \liminf_{t \downarrow 0} q(t)$. Recall from Online Appendix H.3 the formal definition of the game with concealment. Given $q \in \mathcal{Q}$, let H_q^i be the set of private histories $h \in H^i$ for agent i featuring at least one disclosure and such that either (i) $K^i(h) = X(h) \geq \lambda\mu$, or (ii) $K^i(h) \leq \lambda\mu n$, agent i did not disclose, and the first disclosure occurred at some time $t \leq T(h)$ and raised the stock to some value $x < q(t)$. Given $\alpha \in \mathcal{S}$, let $\Xi_{\alpha, q}$ be the set of symmetric strategy profiles ξ such that, (i) in any period t prior to which no disclosure occurred, agent i exerts effort $\alpha_t(k_t^i)$, discloses fully if $x_t < k_t^i \geq q(t)$ and conceals fully otherwise and (ii) agent i exerts no effort and does not disclose after any history in H_q^i . Suppose that the opponents of agent i play some $\xi \in \Xi_{\alpha, q}$. Note that, given $t > 0$, if no disclosure occurs within $[0, t)$, agent i believes that the private stocks $(k_t^j)_{j \neq i}$ of her opponents are i.i.d. with some CDF pinned down by t , α and q . Label it $G_t(\alpha, q)$, and let $G_0(\alpha, q)$ be degenerate at x_0 . Recall the definitions of beliefs β_{\bullet} and information sets (h, β_{\bullet}) from Online Appendix H.3. Given $t \geq 0$, let $B_{\alpha, q}^i(t)$ be the set of agent i 's beliefs $\beta_{\bullet} := \prod_{j \neq i} \beta_{\bullet}^j$ such that, for all $j \neq i$, the pushforward of β_{\bullet}^j by K^j is $G_t(\alpha, q)$.

Proposition 7. *If $x_0 < \lambda\mu$, there are $\alpha \in \mathcal{S}$ and $q \in \mathcal{Q}_0$ such that, for any*

$\xi \in \Xi_{\alpha,q}$ and any i , ξ^i is a best response against ξ^{-i} at any information set (h, β_\bullet) such that $X(h) = x_0$ and $\beta_\bullet \in B_{\alpha,q}^i(T(h))$.

Several definitions are needed to state the next result. For all $x \geq 0$, let $\underline{v}(x)$ be the value of the single-agent game with initial stock x . Given $t \geq 0$ and $k \geq x \geq \lambda\mu$, let $\hat{v}_t(x, k)$ be the time- t value of an agent i with $k_t^i = k$, assuming that she does not disclose at time t , that $x_t = x > x_0$, and that her opponents do not disclose for the remainder of the game. Let V be the space of 1-Lipschitz continuous, increasing and convex maps $v : \mathbb{R}_+ \rightarrow \mathbb{R}$, let V_0 be the set of $v \in V$ such that $v = \underline{v}$ on $[\lambda\mu, \infty)$ and $v - \underline{v}$ is decreasing, and let \mathcal{V}_0 be the set of maps $v : \mathbb{R}_+ \rightarrow V_0$ that are continuous in the supremum metric.

Let A^c be the quotient space of Borel $a : \mathbb{R}_+ \rightarrow [0, 1]$ by the equivalence relation identifying a and \hat{a} if and only if $\int_0^t a_s ds = \int_0^t \hat{a}_s ds$ for all $t > 0$. Given $\delta \in [0, \infty]$ and $a \in A^c$, let $H(a, \delta)$ be the CDF of the random time π of the first increment obtained by an agent who exerts effort a_s at any time $s < \delta$, and no effort after δ (with ‘ $\pi = \infty$ ’ if no increment arises). Given $\alpha \in \mathcal{S}$ and $q \in \mathcal{Q}$, suppose that the opponents of agent i play some $\xi \in \Xi_{\alpha,q}$. Note that, if no disclosure occurred by time $t \geq 0$, assuming that agent i does not disclose after time t , the joint distribution of (τ, κ) , where $t + \tau$ is the time of the first disclosure by any opponent of agent i after time t , and κ is the value disclosed (with ‘ $\tau = \infty$ ’ and $\kappa = 0$ if no opponent discloses), is pinned down by t , α , and q . Label it $J(t, \alpha, q)$.

Lemma 3. *Suppose that $x_0 < \lambda\mu$. Given $\alpha \in \mathcal{S}$, $q \in \mathcal{Q}_0$, and $v \in \mathcal{V}_0$, TFAE:*

(a) *For any $\xi \in \Xi_{\alpha,q}$, i , and information set (h, β_\bullet) for agent i such that $X(h) = x_0$ and $\beta_\bullet \in B_{\alpha,q}^i(T(h))$, ξ^i is a best response against ξ^{-i} at (h, b) and yields agent i continuation payoff $v_t(k_t^i)$.*

(b) *For each $t \geq 0$ and $k \geq x_0$,*

$$v_t(k) := \sup_{\substack{\delta \in [0, \infty] \\ a \in A^c}} \mathbb{E} \left[x_0 \int_0^{\tilde{T}} e^{-s} (1 - a_s) ds + e^{-\tilde{T}} \left(\mathbf{1}_{\tilde{\tau}=\tilde{T}} \hat{v}_{t+\tilde{T}}(\tilde{\kappa}, k \vee \tilde{\kappa}) + \mathbf{1}_{\tilde{\pi}=\tilde{T}} v_{t+\tilde{T}}(k + \tilde{z}) + \mathbf{1}_{\tilde{T}=\delta} \underline{v}(k) \right) \right] \quad (35)$$

where $(\tilde{\pi}, (\tilde{\tau}, \tilde{\kappa}), \tilde{z}) \sim H(a, \delta) \times J(t, \alpha, q) \times F$ and $\tilde{T} := \tilde{\pi} \wedge \tilde{\tau} \wedge \delta$. Moreover, the objective in (35) is maximised by $\delta \in [0, \infty]$ and $a \in A^c$ given by

$$\delta := \min \{s \geq 0 : k \geq q(t + s)\} \cup \{\infty\} \quad \& \quad a : s \mapsto \alpha_{t+s}(k). \quad (36)$$

I omit the proof of Lemma 3, as it is a standard dynamic-programming result. The Bellman equation (35) may be understood as follows. At any time t prior

to the first disclosure, an agent i with private stock $k_t^i = k$ picks a delay δ after which, if she obtained no increment and none of her opponents disclosed, she discloses fully (where $\delta = \infty$ if agent i does not disclose in this situation), and a schedule a describing her effort until either she obtains an increment, or an opponent discloses, or time $t + \delta$ is reached. Agent i 's value after she discloses fully is $\underline{v}(k)$ as, following such a disclosure, her opponents do not disclose for the remainder of the game. Moreover, given a realisation (τ, κ) of $(\tilde{\tau}, \tilde{\kappa})$, agent i 's value after an opponent discloses κ at time $t + \tau$ is $\hat{v}_{t+\tau}(\kappa, k \vee \kappa)$.

In light of Lemma 3, to prove Proposition 7, it suffices to exhibit $\alpha \in \mathcal{S}$, $q \in \mathcal{Q}_0$, and $v \in \mathcal{V}_0$ such that (b) holds. I end this section by deriving a strengthening of (b) (Lemma 4), expressed in a language that allows us to formulate a discrete-time approximation of (35) for any time length $\Delta > 0$. Several definitions are necessary. Given $\Delta > 0$, for all $a \in [0, 1]$, define the CDF $F^{\Delta, a}$ with support within \mathbb{R}_+ by

$$F^{\Delta, a}(z) := (1 - e^{-a\Delta\lambda})F(z) + e^{-a\Delta\lambda} \quad \text{for all } z \geq 0.$$

Let $\underline{v}_\Delta : \mathbb{R}_+ \rightarrow \mathbb{R}$ be the unique element of V solving the Bellman equation

$$\underline{v}_\Delta(x) = \max_{a \in [0, 1]} x(1 - e^{-\Delta})(1 - a) + e^{-\Delta} \mathbb{E}_{F^{\Delta, a}}[\underline{v}_\Delta(x + \tilde{z})], \quad (37)$$

and $\underline{v}_0 := \underline{v}$.⁴⁶ Let \mathcal{V} be the set of Borel $v : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ such that $v_t \in V$ for all $t \geq 0$,

$$Y' := [0, \infty] \times A^c \quad \& \quad \Theta_0 := \mathbb{R}_+^3 \times \mathcal{S} \times \mathcal{Q} \times \mathcal{V}$$

with typical elements $y := (\delta, a)$ and $\theta := (\Delta, t, k, \alpha, q, v)$, respectively. Let $I : Y' \times \Theta_0 \rightarrow \mathbb{R}$ be given by

$$I(y, \theta) := \mathbb{E} \left\{ x_0 \int_0^{\tilde{T}} e^{-s} (1 - a_s) ds + e^{-\tilde{T}} [\mathbf{1}_{\tilde{\tau} > \tilde{\pi} \leq \delta} v_{t+\tilde{T}}(k + \tilde{z}) + (1 - \mathbf{1}_{\tilde{\tau} > \tilde{\pi} \leq \delta}) \underline{v}_\Delta((k + \mathbf{1}_{\tilde{\pi} = \tilde{T}} \tilde{z}) \vee \mathbf{1}_{\tilde{\tau} = \tilde{T}} \tilde{\kappa})] \right\}$$

where $y := (\delta, a)$, $\theta := (\Delta, t, k, \alpha, q, v)$, $\tilde{T} := \tilde{\tau} \wedge \tilde{\pi} \wedge \delta$,

$$(\tilde{\tau}, \tilde{\pi}) := \begin{cases} (\Delta \lceil \tilde{\tau}_0 / \Delta \rceil, \Delta \lceil \tilde{\pi}_0 / \Delta \rceil) & \text{if } \Delta > 0 \\ (\tilde{\tau}_0, \tilde{\pi}_0) & \text{if } \Delta = 0, \end{cases} \quad (38)$$

and $(\tilde{\pi}_0, (\tilde{\tau}_0, \tilde{\kappa}), \tilde{z}) \sim H(a, \delta) \times J(t, \alpha, q) \times F$. Given $\Delta > 0$, let $T_\Delta := \{0, \Delta, 2\Delta, \dots\}$, $\bar{T}_\Delta := T_\Delta \cup \{\infty\}$, A_Δ^c be the set of $a \in A^c$ that are constant over $[(m-1)\Delta, m\Delta)$

46. We may view \underline{v}_Δ as a discrete-time approximation of the map \underline{v} for time length $\Delta > 0$. In particular, $\underline{v}_\Delta \rightarrow \underline{v}$ uniformly as $\Delta \rightarrow 0$.

for all $m \in \mathbb{N}$. Let $T_0 := \mathbb{R}_+$, $\bar{T}_0 := [0, \infty]$, and $A_0^c := A$. Define $I^* : \Theta_0 \rightarrow \mathbb{R}$ by

$$I^*(\theta) := \sup_{\bar{T}_\Delta \times A_\Delta^c} I(\cdot, \theta) \quad \text{for any } \theta := (\Delta, t, k, \alpha, q, v).$$

Lemma 4. *Let $\alpha \in \mathcal{S}$, $q \in \mathcal{Q}_0$, and $v \in \mathcal{V}_0$ be such that, for all $t \geq 0$ and $k \geq x_0$,*

$$v_t(k) = I^*(\theta) = I((\delta, a), \theta) \quad \text{where } \delta \text{ and } a \text{ are given by (36),} \quad (39)$$

holds with $\theta := (0, t, k, \alpha, q, v)$. Then, any $\xi \in \Xi_{\alpha, q}$ is a Bayes-Nash equilibrium.

Proof. In light of Lemma 3, it suffices to show that (b) holds. Fix $t \geq 0$ and $k \geq x_0$, let $\theta := (0, t, k, \alpha, q, v)$, and label $I_0(\delta, a)$ the objective on the right-hand side of (35) for any $\delta \in [0, \infty]$ and $a \in A^c$. Let δ^* and a^* be the values of δ and a in (36). Then, it suffices to show that $I_0(\delta, a) \leq I((\delta^*, a^*), \theta)$ for any $\delta \in [0, \infty]$ and $a \in A^c$, where equality holds for $(\delta, a) = (\delta^*, a^*)$. Fix δ and a and suppose without loss that $\delta > 0$. It suffices to show that, given $(\tilde{\kappa}, \tilde{\tau}) \sim J(t, \alpha, q)$, a.s., $\hat{v}_{t+\tilde{\tau}}(\tilde{\kappa}, k \vee \tilde{\kappa}) \leq \underline{v}(k \vee \tilde{\kappa})$, and that equality holds if $(\delta, a) = (\delta^*, a^*)$. For the former note that, a.s., $\tilde{\kappa} \geq q(\tilde{\tau}) \geq \lambda\mu$ so that $\hat{v}_{t+\tilde{\tau}}(\tilde{\kappa}, k \vee \tilde{\kappa}) \leq k \vee \tilde{\kappa} = \underline{v}(k \vee \tilde{\kappa})$. For the latter, note that $\tilde{\kappa} \geq k$ a.s. if $0 < \delta = \delta^*$, so that $\hat{v}_{t+\tilde{\tau}}(\tilde{\kappa}, k \vee \tilde{\kappa}) = k \vee \tilde{\kappa}$. \square

F.1 Approximate Bellman equation

In this section, I define a discrete-time, finite-horizon approximation of the Bellman equation ‘ $v_t(k) = I^*(\theta)$ ’ (Lemma 4), and show that it admits a solution with the desired property (36) for any small enough time length $\Delta > 0$ and any horizon $M \in \mathbb{N}$ (Lemma 6 below). As part of the analysis, I establish bounds for the disclosure cutoff q that is part of the solution, and show that they converge to the desired values as the approximation becomes more accurate (Lemma 5 below).

I begin with several definitions. Recall from Appendix F the definitions of \mathcal{S} , \mathcal{Q} , V , J , $F^{\Delta, a}$ and \underline{v}_Δ . Let S be the set of increasing $\alpha : \mathbb{R}_+ \rightarrow [0, 1]$, and

$$Y := \{0, 1\} \times [0, 1] \quad \& \quad \Theta_m := \mathbb{R}_+ \times S^m \times \mathbb{R}_+^m \times V \quad \text{for all } m \in \mathbb{N},$$

with typical elements $y := (\delta, a)$ and $\theta := (k, \alpha, q, v)$, respectively. Let $I_{\Delta, m} : Y \times \Theta_m \rightarrow \mathbb{R}$ be given by

$$I_{\Delta, m}(y, \theta) = \begin{cases} \underline{v}_\Delta(k) & \text{if } \delta = 0 \\ x_0(1 - e^{-\Delta})(1 - a) \\ \quad + e^{-\Delta} \mathbb{E}[\mathbf{1}_{\tilde{\tau} \leq \Delta} \underline{v}_\Delta(\tilde{\kappa} \vee (k + \tilde{z})) + \mathbf{1}_{\tilde{\tau} > \Delta} v(k + \tilde{z})] & \text{if } \delta = 1 \end{cases}$$

where $y := (\delta, a)$, $\theta := (k, \alpha, q, v)$, $(\tilde{z}, (\tilde{\tau}, \tilde{\kappa})) \sim F^{\Delta, a} \times J((m-1)\Delta, \hat{\alpha}, \hat{q})$, and

$\hat{\alpha} \in \mathcal{S}$ and $\hat{q} \in \mathcal{Q}$ are such that $\hat{\alpha}_t = \alpha_l$ and $\hat{q}_t = q_l$ for all $l \in \{1, \dots, m\}$ and $t \in [(l-1)\Delta, l\Delta)$. Also let $I_{\Delta, m}^* : \Theta_m \rightarrow \mathbb{R}$ be given by

$$I_{\Delta, m}^*(\theta) := \max_Y I_{\Delta, m}(\cdot, \theta).$$

I define bounds q_{Δ}^+ and q_{Δ}^- for the disclosure cutoff q for each $\Delta > 0$, and bound their values as Δ vanishes (Lemma 5 below). Given $\Delta > 0$, let q_{Δ}^- be the largest $x \geq 0$ such that the objective in (37) is maximised by $a = 1$, and

$$q_{\Delta}^+ := \sup \left\{ k \in \mathbb{R}_+ : \arg \max_{a \in [0, 1]} I_{\Delta, m}((1, a), (k, \alpha, q, v)) \geq k \geq \max q \text{ for } m \in \mathbb{N}, \right. \\ \left. \alpha \in S^m, q \in \mathbb{R}_+^m \text{ and } v \in V, \text{ such that } v(l) = l \text{ for all } l \geq k \right\}.$$

Lemma 5. $q_{\Delta}^- \leq q_{\Delta}^+ < \infty$ for all $\Delta > 0$. Moreover, $\liminf_{\Delta \rightarrow 0} q_{\Delta}^- \geq \lambda\mu$ and $\limsup_{\Delta \rightarrow 0} q_{\Delta}^+ \leq n\lambda\mu$.

Lemma 5 is proved in Online Appendix L.3. Next, I restate (39) in a language that allows to formulate a finite-horizon approximation for any given time length $\Delta > 0$ (Remark 1 below). Given $\Delta > 0$, let $Q_{\Delta} := [q_{\Delta}^-, q_{\Delta}^+]$, and V_{Δ} be the set of $v \in V$ such that $v - \underline{v}_{\Delta}$ is decreasing and $v(k) = \underline{v}_{\Delta}(k)$ for $k \geq q_{\Delta}^+$. Let \mathcal{Q}_{Δ} be the set of $q \in \mathcal{Q}$ that have image in Q_{Δ} and are constant on $[(m-1)\Delta, m\Delta)$ for all $m \in \mathbb{N}$. Similarly, let \mathcal{S}_{Δ} (\mathcal{V}_{Δ}) be the set of $\alpha \in \mathcal{S}$ ($v \in \mathcal{V}$) such that, for all $k \geq 0$, $t \mapsto \alpha_t(k)$ ($t \mapsto v_t(k)$) is constant over $[(m-1)\Delta, m\Delta)$ for all $m \in \mathbb{N}$.

Remark 1. Let $\Delta > 0$ and $(\alpha_m, q_m, v_m)_{m \in \mathbb{N}} \subset S \times Q_{\Delta} \times V_{\Delta}$ be such that, for all $m \in \mathbb{N}$ and $k \geq x_0$,

$$v_m(k) = I_{\Delta, m}^*(k, \theta_m) = I_{\Delta, m}((\mathbf{1}_{k < q_m}, \alpha_m), \theta_m) \quad (40)$$

where $\theta_m := (k, (\alpha_l)_{l=1}^m, (q_l)_{l=1}^m, v_{m+1})$. Define $(\hat{\alpha}, \hat{q}, \hat{v}) \in \mathcal{S}_{\Delta} \times \mathcal{Q}_{\Delta} \times \mathcal{V}_{\Delta}$ by

$$(\hat{\alpha}_t, \hat{q}_t, \hat{v}_t) = (\alpha_m, q_m, v_m) \quad \text{for all } m \in \mathbb{N} \text{ and } t \in [(m-1)\Delta, m\Delta). \quad (41)$$

Then (39) holds with $v = \hat{v}$ and $\theta := (\Delta, t, k, \hat{\alpha}, \hat{q}, \hat{v})$ for any $t \in T_{\Delta}$ and $k \geq x_0$.

Remark 1 is proved in Online Appendix L.3. I end this section by showing that there are (well-behaved) sequences of arbitrary length in $S \times Q_{\Delta} \times V_{\Delta}$ that satisfy (40) (Lemma 6 below). Recall the definition of $G_t(\alpha, q)$ from page 45. Note that $G_t(\alpha, q) = G_t(\hat{\alpha}, \hat{q})$ for any $\hat{\alpha} \in \mathcal{S}$, and $\hat{q} \in \mathcal{Q}$ such that $\alpha = \hat{\alpha}$ and $q = \hat{q}$ on $[0, t)$. Given $\Delta > 0$, $m \in \mathbb{N}$, $\alpha \in S^m$, and $q \in \mathbb{R}_+^m$, define

$$G^{\Delta, m}(\alpha, q) := G_{m\Delta}(\hat{\alpha}, \hat{q}) \quad (42)$$

where $\hat{\alpha} \in \mathcal{S}$ and $\hat{q} \in \mathcal{Q}$ are given by $\hat{\alpha}_t = \alpha_l$ and $\hat{q}_t = q_l$ for all $l \in \{1, \dots, m\}$ and $t \in [(l-1)\Delta, l\Delta)$.

Lemma 6. *There are $\Delta_0 > 0$ and $\gamma : (0, \infty) \rightarrow (0, \infty)$ such that, for any $\Delta \in (0, \Delta_0)$ and $M \in \mathbb{N}$, there are $\alpha \in S^M$, $q \in Q_\Delta^M$, and $v \in V_\Delta^{M+1}$ such that, given any $m = 1, \dots, M$, (40) holds for all $k \geq x_0$ and, moreover,*

$$\sup|v_{m+1} - v_m| \vee |g_m - g_{m-1}| \leq \gamma(m\Delta)\Delta, \quad (43)$$

where $g_m := G^{\Delta, m}((\alpha_l)_{l=1}^m, (q_l)_{l=1}^m)$ for $m = 1, \dots, M$, and g_0 is degenerate at x_0 .

Endow S with the topology of pointwise convergence. The proof of Lemma 6 relies on the following remark, Online Appendix L.3.

Remark 2. Given $\Delta > 0$ and $m \in \mathbb{N}$,

$$(\alpha, q, v) \mapsto \left(k \mapsto \max \arg \max_{a \in [0,1]} I_{\Delta, m}((1, a), (k, \alpha, q, v)) \right)$$

is a well-defined and continuous map $S^m \times Q_\Delta^m \times V_\Delta \rightarrow S$, and

$$(\alpha, q, v) \mapsto (k \mapsto I_{\Delta, m}^*(k, \alpha, q, v))$$

is a well-defined and continuous map $S^m \times Q_\Delta^m \times V_\Delta \rightarrow V_\Delta$.

Proof of Lemma 6. From Lemma 5, we can choose $\Delta_0 > 0$ such that

$$\bar{q} := \frac{e^{\Delta_0} - 1}{\Delta_0} \times \sup\{q_\Delta^+ : \Delta \in (0, \Delta_0)\} \cup \{1, \lambda\mu\} < \infty.$$

To define γ , note that there exists $\epsilon > \lambda$ such that, for all $m \in \mathbb{N}$, $\Delta \in (0, \Delta_0)$, $a \in [0, 1]$, $k \leq q_\Delta^+$, $\alpha \in S^m$, $q \in Q_\Delta^m$, and $v \in V_\Delta$,

$$\left| \int_{q_m}^{\infty} \text{Id} g_{m-1}^{n-1}(l) + g_{m-1}^{n-1}(q_m)v(k) - I_{\Delta, m}((1, a), (k, \alpha, q, v)) \right| < \epsilon\Delta$$

where, recall, g_0 is degenerate with value x_0 , and $g_{m-1} := G^{\Delta, m-1}((\alpha_l)_{l=1}^{m-1}, (q_l)_{l=1}^{m-1})$ for $m > 1$. Moreover, as F is Lipschitz, there is an increasing map $\gamma_0 : \mathbb{R}_+ \rightarrow [1, \infty)$ such that $G_t(\alpha, q)$ is $\gamma_0(t)$ -Lipschitz on $[x_0, \infty)$ for all $\alpha \in \mathcal{S}$, $q \in \mathcal{Q}$, and $t \geq 0$. Define $\gamma : (0, \infty) \rightarrow (0, \infty)$ by

$$\gamma(t) := 3\epsilon n \bar{q} \gamma_0(t).$$

Fix $\Delta \in (0, \Delta_0)$ and $M \in \mathbb{N}$. To construct $\alpha \in S^M$, $q \in Q_\Delta^M$, and $v \in V_\Delta^{M+1}$,

let $w : S^M \times Q^M \rightarrow V_\Delta^{M+1}$ be given by $w_{M+1}(\alpha, q) := \underline{v}_\Delta$, and

$$[w_m(\alpha, q)](k) := I_{\Delta, m}^*(k, \alpha^m, q^m, w_{m+1}(\alpha, q))$$

for each $m \in \{1, \dots, M\}$, where $\alpha^m = (\alpha_l)_{l=1}^m$ and $q^m := (q_l)_{l=1}^m$. Note that w_{M+1} has image in V_Δ , and it is constant. Then Remark 2, coupled with an induction argument, implies that w is well-defined and continuous. Define the map $\phi : S^M \times Q_\Delta^M \times V_\Delta^{M+1} \rightarrow S^M \times Q_\Delta^M \times V_\Delta^{M+1}$ by

$$\begin{aligned} \phi(\alpha, q, v) := & \left((k \mapsto \max \arg \max_{a \in [0,1]} I_{\Delta, m}((1, a), (\alpha^m, q^m, w_{m+1}(\alpha, q))))_{m=1}^M, \right. \\ & \left. ([w_m(\alpha, q)](q_m))_{m=1}^M, w(\alpha, q) \right). \end{aligned}$$

Since w is continuous and V is endowed with the topology of uniform convergence, ϕ is well-defined and continuous by Remark 2. Then ϕ admits a fixed point (α, q, v) by Brower's fixed-point theorem, as S is compact by Helly's selection theorem, and V_Δ is compact by the Arzela-Ascoli theorem.

Fix $m \in \{1, \dots, M\}$. To prove (40) note that, by definition of ϕ , it suffices to prove that $v_m(k) \leq (\geq) \underline{v}_\Delta(k)$ for all $k \geq (\leq) q_m$. This holds since $v_m(q_m) = q_m \leq \underline{v}_\Delta(q_m)$, and $v_m - \underline{v}_\Delta$ is positive and decreasing (as $v_m \in V_\Delta$).

It remains to prove (43). Let $g^* := g_{m-1}^{n-1}$.

Claim 5. $1 - g^*(q_m) \leq 2\Delta\epsilon n\gamma_0(m\Delta)$.

Proof. Assume without loss that $g_{m-1}(q_m) < 1$. Note that g_{m-1} is $\gamma_0((m-1)\Delta)$ -Lipschitz and, therefore, $\gamma_0(m\Delta)$ -Lipschitz on $[x_0, \infty)$, as γ_0 is increasing. Then g^* is $n\gamma_0(m\Delta)$ -Lipschitz on $[x_0, \infty)$ so that g^* conditioned on the event (q_m, ∞) FOSD-dominates the uniform distribution over $[q_m, q_m + 1/(n\gamma_0(m\Delta))]$. Hence

$$q_m + \frac{1}{2n\gamma_0(m\Delta)} \leq \frac{\int_{q_m}^{\infty} l dg^*}{1 - g^*(q_m)} \Leftrightarrow 1 - g^*(q_m) \leq 2n\gamma_0(m\Delta) \int_{q_m}^{\infty} l - q_m dg^*.$$

Then, $1 - g^*(q_m) \leq 2\Delta\epsilon n\gamma_0(m\Delta)$ since

$$\begin{aligned} \int_{q_m}^{\infty} l - q_m dg^* & \leq \int_{q_m}^{\infty} l - q_m dg^* + g^*(q_m)[v_{m+1}(q_m) - q_m] \\ & \leq \int_{q_m}^{\infty} l dg^* + g^*(q_m)v_{m+1}(q_m) - I_{\Delta, m}((1, \alpha_m), (q_m, \alpha^m, q^m, v_{m+1})) \leq \epsilon\Delta \end{aligned}$$

where the first inequality holds as $v_{m+1}(q_m) \geq q_m$ (as $v_{m+1} \in V_\Delta$), and the second as $q_m = v_m(q_m) = I_{\Delta, m}^*(q_m, \alpha^m, q^m, v_{m+1}) \geq I_{\Delta, m}((1, \alpha_m), (q_m, \alpha^m, q^m, v_{m+1}))$. \square

To prove that $\sup |v_{m+1} - v_m| \leq \gamma(m\Delta)\Delta$, let $k_0 := \min\{k \geq 0 : v_m = \underline{v}_\Delta \text{ over } [k, \infty)\}$ and note that $k_0 \leq q_m$. Since $v_{m+1} - \underline{v}_\Delta$ is positive and decreasing,

$v_{m+1} - v_m$ is positive and decreasing on $[k_0, \infty)$. Hence, we may assume without loss of generality that $k_0 > 0$ and restrict attention to $k \in [0, k_0)$, so that

$$\begin{aligned} |v_{m+1}(k) - v_m(k)| &\leq \left| \int_{q_m}^{\infty} v_{m+1}(k) - ldg^*(l) \right| + \left| \int_{q_m}^{\infty} ldg^*(l) + g^*(q_m)v_{m+1}(k) \right. \\ &\quad \left. - I_{\Delta, m}((1, \alpha_m), (k, \alpha^m, q^m, v_{m+1})) \right| \leq 2\epsilon n \gamma_0(m\Delta)\Delta \times \bar{q} + \epsilon\Delta \leq \gamma(m\Delta)\Delta \end{aligned}$$

where the second inequality follows from Claim 5 since $|v_{m+1}(k) - l| \leq \bar{q}$ for all $l \geq q_m$ in the support of g^* , as $g^*(\bar{q}) = 1$ and $k \leq v_{m+1}(k) \leq v_{m+1}(\bar{q}) = \underline{v}_{\Delta}(\bar{q}) = \bar{q}$.

It remains to prove $\sup |g_m - g_{m-1}| \leq \gamma(m\Delta)\Delta$. To this end note that, for $k > q_m$, $g_m(k) = 1$ so that

$$|g_m(k) - g_{m-1}(k)| \leq 1 - g^*(q_m) \leq 2\Delta\epsilon n \gamma_0(m\Delta) \leq \gamma(m\Delta)\Delta$$

where the second inequality follows from Claim 5. Fix $k \in [x_0, q_m]$. If agent i plays ξ^i for some $\xi \in \Xi_{\hat{\alpha}, \hat{q}}$ where $\hat{\alpha} \in \mathcal{S}$ and $\hat{q} \in \mathcal{Q}$ are such that $\hat{\alpha}_t = \alpha_l$ and $\hat{q}_t = q_l$ for all $t \in [(l-1)\Delta, l\Delta)$ and $l = 1, \dots, m$, and her opponents never disclose,

$$\begin{aligned} \Pr\left(\tilde{k}_{(m-1)\Delta}^i \leq l \ \&\ \tilde{x}_{(m-1)\Delta} = x_0\right) &\geq \Pr\left(\tilde{k}_{m\Delta}^i \leq l \ \&\ \tilde{x}_{m\Delta} = x_0\right) \\ &\geq \Pr\left(\tilde{k}_{(m-1)\Delta}^i \leq l \ \&\ q_m \ \&\ \tilde{x}_{(m-1)\Delta} = x_0\right) e^{-\Delta\lambda} \end{aligned}$$

for any $l \geq x_0$. Suppose that $g_m(k) \geq g_{m-1}(k)$. Note that

$$\begin{aligned} g_m(k) &= \Pr\left(\tilde{k}_{m\Delta}^i \leq k \mid \tilde{x}_{m\Delta} = x_0\right) \\ &\leq \frac{\Pr\left(\tilde{k}_{(m-1)\Delta}^i \leq k \ \&\ \tilde{x}_{(m-1)\Delta} = x_0\right)}{\Pr\left(\tilde{k}_{(m-1)\Delta}^i \leq q_m \ \&\ \tilde{x}_{(m-1)\Delta} = x_0\right)} e^{\Delta\lambda} = \frac{g_{m-1}(k)}{g_{m-1}(q_m)} e^{\Delta\lambda} \end{aligned}$$

so that, since $k \leq q_m$,

$$g_m(k) - g_{m-1}(k) \leq e^{\lambda\Delta} - g_{m-1}(q_m) \leq e^{\lambda\Delta} - 1 + 2\Delta\epsilon n \gamma_0(m\Delta) \leq \gamma(m\Delta)\Delta$$

where the second inequality follows from Claim 5. Finally, suppose that $g_m(k) < g_{m-1}(k)$. Note that, since $k \leq q_m$,

$$g_m(k) \geq \frac{\Pr\left(\tilde{k}_{(m-1)\Delta}^i \leq k \ \&\ q_m \ \&\ \tilde{x}_{(m-1)\Delta} = x_0\right)}{\Pr\left(\tilde{x}_{(m-1)\Delta} = x_0\right)} e^{-\Delta\lambda} = g_{m-1}(k) e^{-\Delta\lambda}$$

so that $g_{m-1}(k) - g_m(k) \leq 1 - e^{-\Delta\lambda} \leq \Delta\lambda \leq \gamma(m\Delta)\Delta$. \square

G Proof of Proposition 5

In this appendix, I prove Proposition 5. Given ξ_c satisfying the conditions of Theorem 3, let $A(\xi_c)$ be the set of $t \geq 0$ such that, given some $T > t$, $\alpha_s^c(x_0) = 1$ for a.e. $s \in (t, T)$.

Claim 6. If (25) holds, there exists $w > x_0 + V$ such that $v_t^c(x_0) \geq w$ for any $n \geq 2$ and any α^c and v^c that satisfy the conditions of Theorem 3 for n agents, and any $t \notin A(\xi_c)$.

Claim 6 is proved at the end of this appendix. Given ξ_c satisfying the conditions of Theorem 3, let $\pi_t^\epsilon(\xi_c)$ be the probability that, conditional on no disclosure having occurred prior to time t when ξ_c is played, a disclosure occurs before time $t + \epsilon$.

Remark 3. For all $w \geq 0$ and $\epsilon > 0$, there is $N \geq 2$ such that, for any $n \geq N$ and ξ_c satisfying the conditions of Theorem 3 for n agents, and any $t \geq 0$ with $(t, t + \epsilon) \subset A(\xi_c)$ and $\pi_t^\epsilon(\xi_c) \leq 1 - \epsilon$, $v_t^c(x_0) \geq w$.

The logic behind Remark 3 is the following: if $(t, t + \epsilon) \subset A(\xi_c)$ and $\pi_{t,\epsilon}(\xi_c) \leq 1 - \epsilon$, and n is large, then $\inf_{[t, t+\epsilon]} q$ is large as well. Hence $k \mapsto v_t^c(k)$ is approximately flat over $[x_0, \hat{w}]$ for any fixed $\hat{w} \geq x_0$, and choosing $\hat{w} > w$ yields $v_t^c(x_0) \geq w$.

Proof of Proposition 5. Fix $x_0 < \lambda\mu$ and let w satisfy the hypothesis of Claim 6. Note that there exists $\hat{z} > 0$ such that, for any $n \in \mathbb{N}$, any ξ_c and q satisfying the conditions of Theorem 3 for n agents, and any $t \geq 0$ and $\epsilon > 0$ such that $q(t) \leq w$ and $q(t) \leq q(s)$ for all $s \in (t, t + \epsilon)$, conditional on the first disclosure occurring within $(t, t + \epsilon)$ when ξ_c is played, its value is at least $q(t) + \hat{z}$. This is because F is not degenerate and, if a disclosure occurs in this case, the agent who discloses must have produced a private increment after time t . Since $\hat{z} > 0$ and $w > x_0 + V$, we may choose $\epsilon > 0$ such that $e^{-2\epsilon}(1 - \epsilon)^2 > [(x_0 + V)/w] \vee [w/(w + \hat{z})]$.

Given these ϵ and w , choose $N \in \mathbb{N}$ such that Remark 3 holds. Fix $n \geq N$ and ξ_c satisfying the conditions of Theorem 3 for n agents. Since $v_\sigma^i(x_0) \leq x_0 + V$ for any i and any PPE σ of the game with forced disclosure, it suffices to show that $v_0^c(x_0) \geq x_0 + V$. To this end, let

$$t' = \sup\{0\} \cup A(\xi_c) \quad \& \quad t'' = \sup\{0\} \cup \{t > 0 : \pi_s^\epsilon(\xi_c) \geq 1 - \epsilon \text{ for all } s \in [0, t]\}$$

and $T = t' \wedge t''$. Note that $T < \infty$ and let t minimise q over $[0, T]$ (the minimum is achieved since q is lower semi-continuous, by Proposition 7 in Appendix F). Note that

$$q(t) = v_t^c(q(t)) \geq e^{-\epsilon}(1 - \epsilon)[(q(t) + \hat{z}) \wedge w] \tag{44}$$

as the rightmost expression is lower than the time- t continuation payoff to an agent who never discloses given that her opponents play ξ_c , conditional on no disclosure having occurred prior to time t . The latter claim holds since $v_T^c(x_0) \geq w$ if $t + \epsilon \geq T$, and since $\pi_t^\epsilon(\xi_c) \geq 1 - \epsilon$ if $t + \epsilon < T$ and $q(t) \geq w$; If instead $t + \epsilon < T$ and $q(t) < w$, then the claim holds since $\pi_t^\epsilon(\xi_c) \geq 1 - \epsilon$ and, conditional on an opponent disclosing within $(t, t + \epsilon)$, the expected value disclosed is at least $q(t) + \hat{z}$. Then $q(t) \geq e^{-\epsilon}(1 - \epsilon)w$ by definition of ϵ and thus, since $q(t) = \min_{[0, T]} q$, by the logic used to obtain (44), $v_0^c(x_0) \geq e^{-2\epsilon}(1 - \epsilon)^2 w \geq x_0 + V$, where the last inequality holds by definition of ϵ . \square

Proof of Claim 6. Since (25) holds and F is atomless, there exists a lowest $w \geq 0$ such that

$$x_0 = \lambda \mathbb{E}_F[(x_0 + \tilde{z} - w) \vee 0],$$

and, moreover, $w > x_0 + V$. Fix $n \geq 2$ and α^c and v^c satisfying the conditions of Theorem 3 for n agents, and $t \notin A(\xi_c)$. As $s \mapsto v_s^c(x_0)$ is continuous, it suffices to show that, for all $\epsilon > 0$, there is $s \in [t, t + \epsilon)$ such that $v_s^c(x_0) \geq w$. Fix ϵ . Recall from the proof of Theorem 3 that $v_t^c(x_0)$ satisfies (35), and that q and α^c satisfy (36). Then (35) is maximised by $\delta = \infty$, since $x_0 < \lambda\mu \leq q$. Then,

$$\alpha_s^c(x_0) \in \arg \max_{a \in [0, 1]} \{a\lambda\{\mathbb{E}_F[v_s^c(x_0 + \tilde{z})] - v_s^c(x_0)\} - ax_0\} \quad \text{for almost all } s > t.$$

(This follows from Lemma 7 in Online Appendix H.1 with $b(x) = x$, $c(a, x) = ax$, $\hat{w}(s) := \mathbb{E}_F[v_s^c(x_0 + \tilde{z})]$, $\check{w}(s) := \mathbb{E}[\tilde{\kappa} | \tilde{\tau} - t = s]$, and G being the CDF of $t + \tilde{\tau}$, where $(\tilde{\tau}, \tilde{\kappa}) \sim J(t, \alpha^c, q)$.) Hence, for some $s \in [t, t + \epsilon)$,

$$\begin{aligned} x_0 &\geq \lambda\{\mathbb{E}_F[v_s^c(x_0 + \tilde{z})] - v_s^c(x_0)\} \\ &\geq \lambda\{\mathbb{E}_F[(x_0 + \tilde{z}) \vee v_s^c(x_0)] - v_s^c(x_0)\} = \lambda\mathbb{E}_F[(x_0 + \tilde{z} - v_s^c(x_0)) \vee 0] \end{aligned}$$

where the second inequality holds since $k \mapsto v_s^c(k)$ is increasing and $v_s^c(k) \geq k$ for all $k \geq x_0$. Hence $v_s^c(x_0) \geq w$, by definition of w . \square