

Reimer, Julia; Doganoglu, Toker

Conference Paper

Optimal Operating Mode of a Platform

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2023: Growth and the "sociale Frage"

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Reimer, Julia; Doganoglu, Toker (2023) : Optimal Operating Mode of a Platform, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2023: Growth and the "sociale Frage", ZBW - Leibniz Information Centre for Economics, Kiel, Hamburg

This Version is available at:

<https://hdl.handle.net/10419/277683>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Optimal Operating Mode of a Platform

Toker Doganoglu* and Julia Reimer†

February 2023

PRELIMINARY AND INCOMPLETE! PLEASE DO NOT CITE!

Abstract

We provide a tractable framework to study the optimal operating mode of a platform. In our model, the monopoly platform can earn profits not only by collecting a proportional fee from sellers but also by possibly competing directly as a seller on its own marketplace. We find that the platform prefers to sell directly only if it benefits from a significant cost advantage over third-party sellers and its own marginal costs are low. On the other hand, we also find that the platform may prefer to become a seller and announce a price, even when it is comparably less efficient than independent sellers. This strategy is motivated by a desire to influence the equilibrium prices and bring them closer to a level that maximizes marketplace revenues. The platform finds it more profitable to operate as a pure marketplace only for a range of marginal costs close to the marginal cost of the independent sellers. Finally, we establish that the platform acting as an active seller always benefits consumers.

JEL Codes: D42, L12, L13, L40, L81

*University of Würzburg, toker.doganoglu@uni-wuerzburg.de

†University of Würzburg, julia.reimer@uni-wuerzburg.de

1 Introduction

At the New York Stock Exchange, the highest valued stocks are those of big tech companies, such as Amazon, Alphabet, Apple, and Meta. One reason is that they combine two features: the mechanics of online markets and the unparalleled availability of data. This combination has helped online platforms achieve significant market power, enabling them to control access to customers, data, and services. As a result, they act as gatekeepers of the online world. For instance, as stated in a recent report, for 37 percent out of the 2.3 million active third-party sellers on the Amazon marketplace, Amazon marketplace is the sole income source, showing the strong dependence of third-party sellers on Amazon.¹ For these sellers, Amazon is the gateway to reach consumers. Analogously, the app stores by Google and Apple are the gateways for app developers to reach end-users, and Meta is the door to the social media world.

Many of these online platforms use their own marketplaces to offer their own products. A prominent example is Amazon that offers many products itself, either as a first-party product seller or as a seller of products under its own label. So far, Amazon has introduced 243 thousand products under its private brands. Most strikingly, the private brand products resemble products of independent third-party sellers hosted on the online marketplace. There are a number of reports which observe these product imitations across multiple product categories, such as bags, car accessories, furniture and stuffed animals, to name a few.²

Similarly, there are other online marketplaces such as Zalando, one of the leading fashion online stores in Europe, and Walmart that not only act as marketplaces but also as sellers. These marketplaces operate in the so-called dual mode. Similarly, Google and Apple offer their own apps comparable to existing apps on their marketplace. The dual operating mode of platforms is becoming increasingly important.

It is crucial to understand how the dual operating mode works and which opportunities it creates as evidence suggests that platform owners systematically exploit data gained through their marketplace for their advantage.³ Prevailing public opinion accuses these firms of gathering non-public information on product sales. The inside information allows them to identify lucrative product spaces, and enter these markets. They use this information advantage to offer branded versions of popular third-party products and earn

¹See ‘The State of Amazon Seller 2020’, JungleScout, 2020, <https://www.junglescout.com/amazon-seller-report/>

²See ‘Competing With Amazon on Amazon’, The Wall Street Journal, June 27 2012, <https://www.wsj.com/articles/SB10001424052702304441404577482902055882264>, and ‘Amazon copied products and rigged search results to promote its own brands, documents show’, Reuters, October 13 2021, <https://www.reuters.com/investigates/special-report/amazon-india-rigging/>

³See ‘Amazon Scooped Up Data From its Own Sellers to Launch Competing Products’, The Wall Street Journal, April 23 2020, <https://www.wsj.com/articles/amazon-scooped-up-data-from-its-own-sellers-to-launch-competing-products-11587650015>

substantial profits without taking the risk of bringing new products to the market. This conflict of interest is ultimately a cause of concern, as it may discourage risky experiments by third-party sellers in bringing new products to the marketplace. On the other hand, competition between the platform itself and third-party sellers is still valuable competition. That is, such competition may likely result in lower prices and hence benefit consumers. Thus, it is becoming increasingly important to understand the functioning of dual mode platforms and sort out the consequences for competition in the marketplace.

These concerns regarding the potential negative implications of dual mode operation of platforms for market outcomes have led to a number of anti-trust investigations in various jurisdictions. In 2019, the EU Commission started an in-depth investigation against Amazon. The authorities suspect the online giant of infringing EU anti-trust laws by using sellers' data generated through its marketplace to benefit its own retail business. By investigating how Amazon uses proprietary seller data from its marketplace, U.S. authorities have placed a focus on digital competition as well. There are even calls for drastic measures to reorganize platform markets, such as Senator Warren's call to break up big tech companies such as Amazon.⁴ EU Commissioner Margrethe Vestager also threatens to break up the tech giants to protect competition.⁵ Such policy suggestions follow the example of India where, effectively, it is prohibited for platforms to operate in the dual mode.⁶

In this paper, building on recent research on online intermediaries, we contribute to the understanding of the optimal operating mode choice of an online platform. Our focus in this paper is on the short-term effects of dual mode operation of a platform and when it will arise as an equilibrium outcome. Intuitively, becoming active in product markets is reasonable whenever the platform can produce and distribute the products at lower costs, or offer a higher quality, which has been recognized by the existing literature.

We add to the prior literature by building a simple model in which a platform considers to adopt the dual mode. In our baseline model, the platform hosts a single third-party seller on its marketplace. The platform might sell products under its own label while simultaneously hosting sellers on its marketplace. That is, we allow for two operating modes: (1) the platform can either credibly commit not to become active in the product space, which we call the *pure marketplace mode*, or (2) become active in the product space as a seller and operate in the *dual mode* both as a seller and as a marketplace. After the platform has announced whether or not to become a seller, all sellers compete in prices. For every unit sold by the independent third-party seller, the platform collects

⁴See 'Elizabeth Warren says Amazon is 'like a monster' that must be fed every minute', CNN Business, October 15 2021, <https://edition.cnn.com/2021/10/15/business/amazon-elizabeth-warren/index.html>

⁵See 'Competition in a Digital Age: Changing Enforcement for Chaging Times', European Commission, June 26 2020, <https://europa.eu/!nB86wn>

⁶See 'India's ecommerce law forces Amazon and Flipkart to pull products', Financial Times, February 1 2019, <https://www.ft.com/content/29a96ff6-2615-11e9-8ce6-5db4543da632>

a fixed fraction of the price by means of an ad-valorem fee. When the platform decides to operate in the dual mode, depending on the prevailing prices, the platform may end up as the active seller of the product if it sets more attractive sales terms. However, even when the independent third-party makes the sales to the consumers, the platform ends up earning profits as a marketplace. Indeed, this very possibility introduces an incentive for the platform to become active in the product market in order to influence prevailing equilibrium prices and improving the marketplace revenues.

Our model yields some novel findings. First, we show that the platform prefers to be an active seller only if it benefits from a distinct cost advantage over third-party sellers. If not, the pure marketplace mode can be more profitable even when the platform is the most efficient firm. This captures that, instead of selling at a low price close to effective marginal costs, the platform prefers to collect a fraction of the seller revenues generated at a higher price.

Second, and more importantly, we show that if the platform has a clear cost disadvantage, it may nevertheless become active as a seller in the market solely to limit the market power of the independent seller. In these situations, the platform intends not to sell the good but to influence prevailing prices and move them closer to a level that maximizes platform revenues. Similarly, we also demonstrate that when the platform has a minor cost disadvantage and the costs are low, it can earn higher profits by operating in the pure marketplace mode.

We also show that the platform as a seller on its own marketplace always benefits consumers and reduces the profitability of the independent seller. Overall, the platform's decision to operate in the dual mode always increases total welfare. In fact, from a welfare perspective, dual mode operation is desired. In our baseline model, this result is not surprising as the dual mode changes the product market structure from a monopoly to duopoly.

We then consider an extension of our model, where an independent seller faces competition from a fringe of sellers as well as the platform itself. We confirm that our findings from the single seller case qualitative apply in this setting as well.

These insights give a better understanding of the optimal operating mode choice of online platforms and explain some reasons why we do not see a branded version of every product on the marketplace. We show that the cost range for which becoming active is profitable is much smaller than often assumed. On the other hand, for a wide range of cases, our model predicts that the platform will become active in the market not to sell but to discipline the price setting behavior of the independent seller(s).

The paper is organized as follows. In the next section, we discuss the related literature. In Section (3), we lay out the model and start the analyses by considering each operating mode, pure marketplace mode and dual mode, separately. Subsequently, we explore the platform's optimal operating mode choice. Section (5) looks at an extension in which

the platform competes with a competitive fringe of small third-party sellers. Finally, in Section (6) we conclude the paper. All proofs are in the appendix.

2 Related Literature

Our work relates to the existing literature on multi-sided markets, which focuses on platforms that operate as intermediaries facilitating the interactions between market sides in the presence of network effects. See Rochet and Tirole (2004) and Armstrong (2006) for seminal papers.

A growing body of this literature attempts to explore the business model choice of online platforms. Unlike earlier contributions, this strand departs by considering online (trade) platforms that are marketplaces while being active on one of their market sides at the same time.⁷ A recent and excellent survey of this burgeoning literature on the economics of a hybrid platform can be found in Etro (2022). We would like to therefore focus our attention below only on the specific literature dealing with the operating mode choice of a platform and its impact on consumers as well as the market as a whole.

Our analysis of the dual operation mode relates to the work by Anderson and Bedre-Defolie (2022), which illustrates the optimal fee set by a gatekeeper platform that can sell alongside other firms on its marketplace. They show that a platform can maximize its marketplace profits by charging a proportional or a per-transaction fee, while the former leads to lower consumer prices. By endogenizing the decision on the fee, the platform always prefers to sell the product itself unless it has a significant cost disadvantage. In these cases, the pure marketplace mode is more attractive. Our analysis differs from theirs in an important dimension. We model the seller fee exogenous and, thereby, we shut down the channel through which the platform can block a third-party firm from selling by setting its fee very high. Furthermore, they find that consumers benefit whenever the platform enters the dual mode if products are homogeneous. This result is driven by the fact that the platform in the dual mode functions as an additional competitor in the marketplace. This finding coincides with our results on consumer surplus.

Hagi, Teh, and Wright (2022) ask whether platforms such as Amazon should be allowed to sell on their marketplace. Their model incorporates three different product types involving different costs and allows for direct sales. Due to the linear seller fee charged by the platform, entering the dual mode is always more profitable than operating a pure marketplace. The hybrid platform intensifies competition in the marketplace, which, in turn, reduces prices and, thereby, increases marketplace profits. We show that with proportional fees, the operating mode of a platform is more nuanced. While for very

⁷We cannot draw parallels between online retailers and physical retailers. Physical retailers control the price and marketing rights of the third-party sellers they host. See Berges-Sennou, Bontems, and Réquillart (2004) for a literature survey of private label products of physical retailers.

low of high marginal costs, the platform prefers the dual mode, for an intermediate range of costs the platform in our mode prefers to operate as a pure marketplace, even when it is slightly more efficient in production. Their result regarding the positive effect of the dual mode on consumer surplus is consistent with our findings. They also show that the platform always adopts the dual mode unless it has a significant advantage. In this case, the pure reseller mode is more profitable. This result is driven by the fact that there exists an outside channel that creates competition to the platform. In contrast, we assume that the platform always hosts the independent firms.

Hervas-Drane and Shelegia (2022b) account for capacity and information constraints a platform faces when deciding its operating mode. The platform controls the fees and influences the consumers' purchase decisions through a recommendation system in their setup. Both mechanisms soften price competition in the marketplace. They find that the platform sells the most profitable products, whereas products of low profitability are only sold directly if capacity constraints permit. Since a platform that employs self-preferencing through a recommendation system can always divert demand for its benefit, we do not consider steering in our model.

Tremblay (2022) consider how the extent of fee discrimination impacts fee and platform retail entry decisions in a model of Cournot competition. They present cases where the platform with a small cost disadvantage enters the dual mode to increase competition in the marketplace, then the platform sets a lower linear seller fee as well. In these cases, the platform's entry decision aligns with consumer and seller surplus. Whenever the platform benefits from a cost advantage, it becomes a monopolistic seller in the market.

Further implications of the dual mode are studied by Muthers and Wismer (2013). In their setup, the platform only becomes a seller if the cost advantage is sufficiently large. For small cost advantages, the platform prefers the pure marketplace mode. Muthers and Wismer (2013) assume a fixed consumer demand with a maximum willingness to pay. Thus, a price reduction does not increase the platform's marketplace profits. By allowing for a demand that decreases in price, we can investigate the incentives of a platform with a cost disadvantage to become a competitor in its marketplace with the sole purpose of disciplining the seller's pricing.

Concurrent with this paper, Hervas-Drane and Shelegia (2022a) capture the impact of dual operation mode on competition in the marketplace. They analyze cases where the platform charges a proportional fee to the seller it hosts while competing with the seller in its own marketplace. They show that the platform prices less aggressively as it considers the opportunity costs from the marketplace unit. Our model builds on this idea and should be considered complementary. While Hervas-Drane and Shelegia (2022a) only focus on the price effect given that the intermediary has already announced to become active, we endogenize the decision of the platform to become active as a seller in its own marketplace.

Furthermore, Hagiu, Jullien, and Wright (2020) address a research question that is somehow similar to ours. In their setup, a seller decides whether to become a platform and host its rivals while we investigate the opposite. We examine the incentives of a platform to become a seller while hosting rivals. Their model shows that joint profits may increase under the dual mode if the platform can extract a variable fee from the sellers in its marketplace.

There is an ongoing debate regarding the welfare implications of an intermediary's dual mode. Etro (2021) focuses on the profit-maximizing operating mode choice of a platform and whether this aligns with the social optimum. In the case of competitive sellers, the platform prefers to become a seller if its cost advantage exceeds the marketing advantage of the sellers. There exists a maximum threshold on sellers' shipping costs for which it is profitable for the platform to become active. Under competitive sellers, entry is socially desired for common demand functions.

Anderson and Bedre-Defolie (2021) study hybrid trade platforms with fluid free entry of third-party sellers of heterogeneous products. In their model, the platform can adjust the fee. When the platform faces high costs, it prefers to operate a pure marketplace. It turns out that the pure marketplace mode results in a lower fee charged to sellers and more sellers in the market. As a result, the platform in the dual mode harms consumer surplus and total welfare.

Finally, our paper is also related to the empirical literature studying online platforms that operate in the dual mode. Most recently, Crawford, Courthoud, Seibel, and Zuzek (2022) investigate the effects of Amazon being active on its marketplace using proprietary data by Amazon. They conclude that Amazon becoming active is motivated by intensifying competition in the marketplace and mild market expansion. They find that Amazon entering the dual mode was associated with slightly lower third-party prices, sometimes slightly lower prices compared to pre-entry third-party average prices, and no relevant effect on third-party revenues. The picture emerging regarding the price effects of Amazon's dual mode supports our findings on the optimal operating mode. The platform may become active to restrict the pricing of independent sellers, which benefits consumers.

In conclusion, while all these papers mirror our work in that they include an analysis of the operation mode choice of a platform, they consider an endogenous seller fee, independently of whether it is of linear or proportional type. Instead, our work assumes that the fee is exogenous. This allows us to explore a different perspective and to contrast some of the results in the existing literature which suggest that the platform prefers to become active and sell the good itself only if it has a significant cost advantage. More importantly, we demonstrate that there are cases where the platform opts for becoming active as a seller to strengthen competition.

3 Model

We consider a platform M which hosts an independent seller S in its marketplace.⁸ In our setting, M can also start selling the *same* product as the independent seller, and compete with her. When M also offers the product for sale, we refer to the situation as the *dual mode* of operation. In this mode, depending on the prices of the both firms, either one of them may end up making the sale. On the other hand, if M refrains from becoming a seller and commits to not offering the product for sale and just acts as a marketplace, we refer to the situation as the *pure marketplace mode*.

In our setting, we will make the rather strong assumption that S cannot sell to consumers directly and hence has to join the marketplace to generate any sales at all. As mentioned in the introduction, for many third-party sellers this approximates the reality well. Ultimately, this assumption amounts to adjusting the outside option of S to zero and hence qualitatively will yield similar results to the case where the independent seller can sell to consumers on its own, albeit at different conditions.

We assume that the independent seller has already joined the platform in both operating modes, and focus on the decision of the platform to whether operate in the pure marketplace mode or dual mode. In the latter case, namely under the dual mode, the seller's product and the platform's product compete with one another. In this case, we assume that both products are perceived as perfect substitutes by the consumers.

The consumer demand for this homogeneous product is characterized by the demand function $D(p)$, where p is the lowest price offered. We assume that the demand function is log concave so that it will yield a single peaked profit function and a unique profit maximizing price. Furthermore, let $D(p) \geq 0$. Where relevant, we will introduce a simple linear demand function, namely, $D(p) = 1 - p$, to illustrate our results.

We assume that the platform does not incur any costs for providing the online marketplace services. However, in the dual mode, the platform pays a cost of production $c_M > 0$, for each unit sold. The independent seller, similarly, faces a marginal cost of $c_S > 0$ per unit. Moreover, in order to keep the exposition simple, we assume that the platform and the independent seller have no fixed costs of production.⁹

Typically, online platforms charge a commission for each sale carried out through their marketplaces. Although theoretically, the commission fee can be adjusted for every single product on the marketplace, this is hardly feasible when considering platforms such as Amazon that sell millions of products. Instead, it is observed that such platforms charge a common fee which applies to all products in a given category. On its German platform, Amazon charges a referral fee as a fraction of the final consumer price which varies between

⁸After presenting our main intuitions regarding the economic mechanisms guiding the operation mode choice of the platform in this baseline model, we will consider an environment with many sellers below.

⁹However, this is not without loss of generality. Including fixed production costs may change some of our results. We leave the analysis of this more general case for future research.

7 % and 45 % based on the product category. For example, the ad-valorem fee for the product category “consumer electronics” amounts to 7 %. Hence, a firm selling a product in this category pays 7 % of the final consumer price to Amazon for each unit sold, no matter the total amount sold on the marketplace, the costs of the platform, or the costs of the independent seller.¹⁰

To reflect the difficulty of adjusting the fees, we assume in our model that a seller has to pay a fixed ad-valorem fee $\tau \in [0, 1]$ for each transaction carried out through the platform. We assume that τ is determined earlier, and cannot be easily adjusted. This is justified considering the fact that the objectives of the platform in determining τ depends on a multitude of products, and the sales of a single product has very little impact on the overall profitability of a product category. Thus, for our analysis below, τ , the ad-valorem fee charged by the platform is assumed to be exogenous.¹¹

If M decides to operate as a pure marketplace (PMP), S would be the only seller of the good and thus, will be in a position to charge the monopoly price to consumers. Consequently, M makes profits through the ad-valorem fee τ . On the other hand, if M adopts the dual mode, the platform hosts the seller and offers the good to consumers simultaneously. In this case, the two firms selling products which are perceived to be homogeneous by consumers compete according to standard Bertrand logic. There exist two possible outcomes, either M is able to set a lower price and is the seller (DM) or S sets a lower price and sells the good (DS).

Throughout the paper, we make two tie breaking assumptions. We assume that whenever M and S are active sellers of the product, then consumers break the tie in favor of the low-cost firm’s product whenever they are indifferent between M and S . In addition, whenever M is indifferent between operating as a pure marketplace or in the dual mode, M prefers the marketplace mode over the dual mode.

3.1 Pure Marketplace Mode

We start our analysis by characterizing the equilibrium in which the platform commits to operate as a pure marketplace (PMP). While the platform will be passive and collect fraction τ of the seller revenue, the independent seller can set her profit-maximizing price as she is the sole seller of the good. This price considers the ad-valorem fee the seller has to pay to the platform. Before we present the Proposition, let us make a few definitions. We denote p^R as the ratio of the demand and the negative of the demand derivative, $\frac{D(p)}{-D'(p)}$.

¹⁰Amazon has introduced a per-item minimum referral fee of 0.30. However, this further supports our assumption that the ad-valorem fee is exogenous, since almost all products have the same per-item minimum fee of 0.30, only a few products have no fee at all.

¹¹In the simple model above, if the platform could adjust the ad-valorem fee, it would be possible for the platform to obtain monopoly profits by rendering it unprofitable for the independent seller to operate. For a detailed discussion, see Anderson and Bedre-Defolie (2022).

Proposition 1 (Pure Marketplace Equilibrium)

In equilibrium, seller S sets the monopoly price $p^{PMP}(c_S)$, where p^{PMP} solves the equation $p = \frac{c_S}{1-\tau} + p^R$, and earns $\pi_S^{PMP} = (p^{PMP}(1-\tau) - c_S)D(p^{PMP})$. As a result, the intermediary M only collects commission profits which are given by $\pi_M^{PMP} = \tau p^{PMP} D(p^{PMP})$.

Given the demand function, $D(p)$, it is necessary that we have $D(p^{PMP}) > 0$ so that production takes place. This is an assumption we make for the rest of the paper.

Suppose platform M was to maximize its marketplace profits, $\tau p D(p)$. Then, the optimal price from the perspective of M would have been p^R . Comparing this to the function describing the monopoly price of seller S , it is immediate that the closer c_S is to zero, the closer $p^{PMP}(c_S)$ is to the price that maximizes marketplace profits. For larger values of c_S , the optimal price from the perspective of M , when serving as a marketplace, deviates significantly from what the independent seller will set. Even though the product will not be sold by the platform, it may want to influence the pricing behavior of the seller. This deviation between the optimal price from the perspective of the platform and the seller creates a possibility that the platform may want to become active not to make sales but to align the price of the independent seller with a price that maximizes its own marketplace profits. This observation has important implications for the following analysis when we consider the choice of the platform on operating as a pure marketplace.

3.2 Dual Mode

In this section, we characterize the price equilibria when the platform has become active and operates in the dual mode. That is, it has announced to become active as a seller on its own marketplace while hosting the independent seller S . Thus, the platform directly competes with seller S in the marketplace. We assume that both sellers, M and S , set their prices simultaneously and compete à la Bertrand. Given our assumption of product homogeneity, consumers buy from the firm that offers the lowest price. Which firm makes the sales in equilibrium depends on the effective marginal costs of S and M . After prices are announced, the platform might end up either operating as the seller of the good (DM) or as a marketplace (DS).

The payoff function of the platform is composed of two orthogonal components. When S sets a lower price and makes the sales to the consumers, M makes zero profits from selling but earns a commission profit through the ad-valorem fee τ . In contrast, if M offers the lowest price, it earns a profit from selling to consumers but obtains no profit from the marketplace. In this case, the independent seller earns zero profit.

In order to identify the equilibrium outcomes whenever M operates in the dual mode, we first derive the lowest price for which each market participant is willing to sell the product. Seller S prefers to make sales whenever she earns a nonnegative profit. Thus,

the seller's price p_S must satisfy

$$((1 - \tau) p_S - c_S) D(p_S) \geq 0. \quad (1)$$

The term on the left-hand side refers to the profit from selling, whereas the right-hand term corresponds to the earnings of S when she does not make the sales. Solving (3) for p_S yields

$$p_S \geq \frac{c_S}{1 - \tau}. \quad (2)$$

The lowest price for which S is willing to sell the good is $\frac{c_S}{1 - \tau}$. This price makes her indifferent between making the sales or not. Of course, S makes the sales only if her price p_S is lower than the price of M p_M . We assume for the rest of the paper that $D(\frac{c_S}{1 - \tau}) > 0$ so that a sale occurs.¹²

We consider next the pricing decision of the platform. Under the dual mode, M either becomes the seller of the good or operates as a marketplace. Suppose seller S sets price $p_S = p$. Then, whenever M sets a price that is slightly below p , it will earn approximately the sales profits corresponding to the price p . On the other hand, whenever M sets a price that is larger than the p , it will earn marketplace profits of $\tau p D(p)$. Thus, M prefers to sell the good itself over collecting marketplace profits if

$$(p - c_M) D(p) \geq \tau p D(p) \quad (3)$$

holds. The left-hand side of (3) is the profit M earns when it sells the product itself at price p , which does not depend on the ad-valorem fee, τ . The right-hand side of (3) corresponds to the marketplace profit M earns when S sells the product at price p . The platform has to balance a trade-off between making sales itself and acting as a marketplace. In (3), the right-hand side represents the marketplace profit the platform would earn when prices are fixed at p , and thus, (3) constitutes the opportunity costs the platform faces in the dual mode. This trade-off yields interesting considerations. Solving (3) for p leads to

$$p \geq \frac{c_M}{1 - \tau}. \quad (4)$$

For any price set by S that is above $\frac{c_M}{1 - \tau}$, it is more profitable for M to sell the product itself. For M to constitute a credible competitive threat to S , the monopoly price, $p^{PMP}(c_S)$, must exceed the competitive price, $\frac{c_M}{1 - \tau}$. We assume this for the rest of the

¹²Moreover, this price, $\frac{c_S}{1 - \tau}$, has to be lower than the price that maximizes the monopoly selling profits of M , $c_M + \eta(p)$. We assume this to be the case. Otherwise, M faces no competitive pressure from the seller.

paper.

By Bertrand logic, the firm that can offer the homogeneous product at the lowest price will make the sales in equilibrium. When comparing the two price limits, we see that the identity of the firm which will make sales in equilibrium crucially depends on the marginal costs, c_S and c_M .

Now, we identify the equilibria if M operates in the dual mode. Given that our predictions differ based on the marginal cost of both firms, we perform this analysis by considering two mutually exclusive cases. Note that when both firms have different marginal costs, as it is well known in traditional price competition models with homogeneous goods, there exist multiple equilibria. Nevertheless, it is customary to select the equilibrium where the low marginal cost firm makes the sales at a price equal to the marginal cost of the less efficient firm. This equilibrium is selected by the appealing fact that the high marginal cost firm does not follow dominated strategies. In the propositions below, we will similarly rule out equilibria which rely on weakly dominated strategies and select the equilibrium with undominated strategies as our equilibrium prediction.

CASE 1: $\frac{c_M}{1-\tau} \leq \frac{c_S}{1-\tau}$

From our earlier arguments it is clear that whenever the independent seller sets the lowest price at which she is willing to make a sale, which we identified in (??), the platform with its lower marginal cost can make the sales. In this case, it earns higher profits than when operating as a marketplace. The next proposition characterizes our preferred equilibrium outcome when the platform is more efficient than the independent seller and no firm employs dominated strategies.

Proposition 2 (Dual Mode Equilibrium, $\frac{c_M}{1-\tau} < \frac{c_S}{1-\tau}$)

If $c_M \leq c_S$, M makes all the sales at an equilibrium price of $p^{DM} = \frac{c_S}{1-\tau}$ and earns a profit of $\pi_M^{DM} = (\frac{c_S}{1-\tau} - c_M)D(\frac{c_S}{1-\tau})$. The seller, S , in this case, earns zero profit in equilibrium.

Note that under this cost structure, there is a continuum of equilibria for $p \in (\frac{c_M}{1-\tau}, \frac{c_S}{1-\tau}]$. But, only $p = \frac{c_S}{1-\tau}$ involves undominated strategies. Hence, in line with standard treatment of price competition between firms with different marginal costs, we select $\frac{c_S}{1-\tau}$ as our prediction. The equilibrium in this case is straightforward, because there is no price S can select, for which the platform prefers operating as a marketplace instead of selling. The next case we study demonstrates the opposite case where the platform will ultimately operate as a marketplace after competition took place.

CASE 2: $\frac{c_S}{1-\tau} < \frac{c_M}{1-\tau} < p^{PMP}(c_S)$

Now suppose S is more efficient than M . However, also suppose that M has sufficiently low cost so that S perceives it as a competitive threat. Namely, the effective marginal

cost of the platform is below the monopoly price of the independent seller. It is reasonable to expect that, in this case, the independent seller will be in an advantageous position to make the sales to the consumers and the platform will end up acting as a marketplace. Nevertheless, platform M in the dual mode can influence the prevailing equilibrium price in the market. Our findings are summarized in the following proposition.

Proposition 3 (Dual Mode Equilibrium, $c_S < c_M < p^{PMP}(c_S)$)

If $c_S < c_M < p^{PMP}(c_S)$, S makes all the sales at an equilibrium price of $p^{DS} = \frac{c_M}{1-\tau}$ and earns a profit of $\pi_S^{DS} = (c_M - c_S)D(\frac{c_M}{1-\tau})$. In this case, the platform earns a profit of $\pi_M^{DS} = \tau \frac{c_M}{1-\tau} D(\frac{c_M}{1-\tau})$ through the commission fee.

In analyzing pricing behavior, we assumed that the platform is already active. The prevailing equilibrium price, when the platform is less efficient, may be closer to the price which maximizes the marketplace profits of the platform, in comparison to pure marketplace model. Thus, the platform may consider becoming an active seller even when it has no hope of making sales. In general, M entering the dual mode forces S to deviate from her optimal monopoly price, namely $p^{PMP}(c_S)$. Depending on the respective distances of p^{DS} and $p^{PMP}(c_S)$ from p^R , the platform may have been better off not becoming active as a seller at all. On the other hand, there may also be cases where the platform can improve its marketplace profits by entering as a seller and influencing the pricing choice of S . To the best of our knowledge, this possibility is a novel finding. We will analyze this trade-off further when we investigate the optimal operating mode choice of the platform in the next section.

4 Optimal Operating Mode

Equipped with predictions regarding the outcomes in possible situations that may arise in the marketplace depending on different cost constellations, we can now investigate whether the platform will prefer to remain only a marketplace or to operate in the dual mode, where it also offers the product for sale itself in its marketplace. Recall that we assume that S is already in the market. Therefore, in our setting, first, the platform decides its operating mode, and then, S and if applicable M , set prices simultaneously. Accordingly, the platform chooses between two possible strategies in the first stage. It chooses either to operate in the *pure marketplace* mode (PMP), or to operate in *dual* mode (D). Operating in the dual mode does not necessarily mean that the goal of the platform is to make sales to consumers. Since M being active may ultimately entail acting as a marketplace as well, the platform may have the possibility to influence equilibrium prices to increase marketplace profits. Indeed, one of our important contributions in this paper is to highlight that the platform can influence equilibrium prices by acting as a

seller and announcing a sale price, although it primarily intends to earn its profits by acting as a marketplace in the dual mode.

4.1 Efficient Platform

We start our analysis assuming that the platform is more efficient, i.e., $c_M \leq c_S$. With such low cost, if platform M becomes active, it will make the sales in equilibrium earning π_M^{DM} . However, if the platform opts for the pure marketplace mode, S will sell at the monopoly price, and the platform will obtain a profit of π_M^{PMP} due to its marketplace fee τ . Let \bar{c}_M be defined as $\frac{c_S}{1-\tau} - \tau p^{PMP} D(p^{PMP}) \frac{1}{D(\frac{c_S}{1-\tau})}$. It turns out that π_M^{PMP} exceeds π_M^{DM} whenever $\bar{c}_M < c_M$.

Proposition 4 (Optimal Operating Mode, $c_M \leq c_S$)

Whenever $\max(0, \bar{c}_M) < c_M \leq c_S$, M operates in the pure marketplace mode. Otherwise, for all other values of $c_M \leq c_S$, M operates as the active seller of the good.

Of course, it is difficult to further characterize these results due to the general nature of the demand function. Later in our analysis, we will return to the same problem with a linear demand specification, which allows us to provide further insights.

4.2 Inefficient Platform

When the platform has a higher marginal cost relative to seller S , namely whenever $c_S < c_M$, S makes the sales in any situation. It is, then, tempting to think that the platform will opt to remain a pure marketplace. As we will argue below, this is not the case in general.

By comparing the profits of the platform under two possible operation modes, we find that although in some cases, the platform prefers to remain a pure marketplace, in many others, it chooses to operate in the dual mode. However, this choice is not motivated by making sales to consumers. Instead, when operating in the dual mode, the platform influences the equilibrium price chosen by the independent seller, which in turn influences its profits as a marketplace.

Recall that marketplace profits of M , when S sets a price equal to p , $\tau p D(p)$, are maximized at a price described by $p = p^R$. In Case 2, we restricted our attention to the situation where the possible equilibria of the price competition results in prices below $p^{PMP}(c_S)$, the price that maximizes the monopoly profits of S . Namely, we assume that $\frac{c_S}{1-\tau} < \frac{c_M}{1-\tau} < p^{PMP}$.

As such, recognizing that the platform is active in the marketplace as a seller implies that it may have an effect on the prices that prevail in equilibrium by means of its strategic choices.

Proposition 5 (Optimal Operating Mode $c_S < c_M$)

Whenever $\max(c_S, \hat{c}_M) < c_M < p^{PMP}(1 - \tau)$, M becomes a seller and operates in the dual marketplace mode, where $\hat{c}_M \equiv (1 - \tau) \frac{D(p^{PMP})}{D(\frac{c_M}{1-\tau})} p^{PMP}$. Otherwise, for all other values of $c_S < c_M$, M prefers to operate in the pure marketplace mode. S makes all the sales.

If $c_S < c_M \leq \hat{c}_M$ and the platform had decided to become active, it would not have made the sales. It turns out that, in this case, M prefers to remain a pure marketplace and not to exert pressure on the seller's price. Recall that if c_S is small, the monopoly price posted by S , p^{PMP} , defined by $p = \frac{c_S}{1-\tau} + p^R$, is close to p^R . Comparing the monopoly price that S charges if M is in the pure marketplace mode, p^{PMP} , against the competitive price that would be charged after M announced to become active, $\frac{c_M}{1-\tau}$, we find that p^{PMP} is closer to the revenue-maximizing price, p^R . Hence, the platform is better off operating in the pure marketplace mode even though it could, in principle, discipline the seller's price. If costs are similar and low and M has become active, competitive pressure keeps the price low. This price is too low compared to p^{PMP} from the perspective of M operating a marketplace. Hence, M in such a case prefers to remain a pure marketplace, and collect marketplace profits based on the higher retail price set by the independent seller.

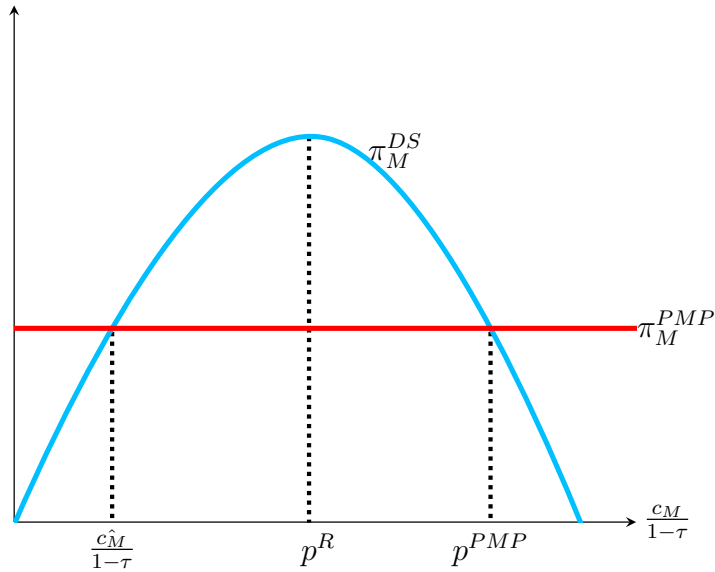


Figure 1: Dual marketplace (DS) profit vs. pure marketplace (PMP) profit of M

Operating in the dual mode becomes an attractive strategy if the platform has a considerably higher marginal cost than the independent seller. Becoming active and applying competitive pressure to the independent seller S brings her price closer to the value that maximizes marketplace revenues. In such a situation, M anticipates that it will not sell the good itself, but it has a chance to constrain the pricing behavior of the independent seller. By becoming active, the platform does not want to really compete with the independent seller but rather push her price to a level that will increase

marketplace profits. Nevertheless, the effect from a consumer perspective is a lower price when compared with the option of purchasing from a monopolistic independent seller.

4.3 Linear Demand Function

Although the general demand function we used previously is sufficient to derive the conditions characterizing the equilibrium operation mode choices, we need a more restrictive demand formulation in order to quantify the corresponding welfare implications. To this end, we do away with the generic demand system and exemplify our analysis employing a linear demand specification. Specifically, we assume that $D(p) = 1 - p$, where p is the lowest price offered.

Before we present the equilibrium welfare result, we establish the equilibrium prices and profits under all possible operating modes. Our findings are summarized below.

Proposition 6 (Equilibrium Profits, Linear Demand)

- (i) Whenever M operates in the pure marketplace mode, seller S sets the monopoly price $p^{PMP}(c_S) = \frac{1}{2} + \frac{1}{2} \frac{c_S}{1-\tau}$ and earns $\pi_S^{PMP} = \frac{(1-\tau-c_S)^2}{4(1-\tau)}$ in equilibrium. As a result, the intermediary M only collects commission profits which are given by $\pi_M^{PMP} = \tau \frac{(1-\tau)^2 - c_S^2}{4(1-\tau)^2}$ in equilibrium.
- (ii) Whenever platform M enters the dual mode, and $c_M < c_S$, M makes all the sales at an equilibrium price of $p^{DM} = \frac{c_S}{1-\tau}$ and earns a profit of $\pi_M^{DM} = \frac{(c_M\tau + c_S - c_M)(1-\tau - c_S)}{(1-\tau)^2}$. The seller, S , in this case, earns zero profit in equilibrium.
- (iii) Whenever M enters the dual mode, and $c_S < c_M$, S makes all the sales at an equilibrium price of $p^{DS} = \frac{c_M}{1-\tau}$ and earns a profit of $\pi_S^{DS} = (1 - \frac{c_M}{1-\tau})(c_M - c_S)$. In this case, the platform earns a profit of $\pi_M^{DS} = \tau \frac{c_M(1-\tau - c_M)}{(1-\tau)^2}$ through the commission fee.

At all these prices, it is necessary that we have $D(p) \geq 0$. Whenever M decides to operate as a pure marketplace, seller S charges the monopoly price, $p^{PMP}(c_S)$, to consumers. However, there might be a mismatch between the monopoly price set by S and the price that maximizes the marketplace profits of M , $\frac{1}{2}$. In accordance with our prior results, we find that for small values of c_S , the monopoly price set by seller S in equilibrium, $p^{PMP}(c_S)$, is close to the price that maximizes M 's marketplace profits, $\frac{1}{2}$. This creates a possibility for M to align the price set by seller S with $\frac{1}{2}$ by becoming active.

On the other hand, whenever platform M has announced to become a seller on its own marketplace, our equilibrium prediction depends on the cost constellation. Whenever $c_S < c_M$, M as an additional competitor disciplines the price that seller S charges. We, therefore, expect lower prices. Since $c_S < c_M$, S still sells the good in this case, but her

profits decrease. Increasing competitive pressure compels her to deviate from the optimal price $p^{PMP}(c_S)$.

Next, suppose $c_M < c_S$, and platform M has adopted the dual mode. In this case, M makes all the sales at a rather low price, namely $\frac{c_S}{1-\tau}$.

Applying Propositions (4) and (5), we characterize the cost constellations that identify the operation mode choice of platform M in equilibrium. We obtain the following results.

Proposition 7 (Optimal Operating Mode, Linear Demand)

- (i) *Whenever $c_S < c_M \leq \frac{1}{2}(1 - \tau - c_S)$, and $c_S < \frac{1}{3}(1 - \tau)$, M operates in the pure marketplace mode. Otherwise, for all other values of $c_S < c_M$, M prefers to become a seller and operate in the dual marketplace mode.*
- (ii) *Whenever $c_S < \frac{1}{3}(1 - \tau)$, and $\max(0, \overline{c_M}) < c_M \leq c_S$, M operates in the pure marketplace mode, where $c_S - \frac{\tau}{4} + \frac{3\tau c_S}{4(1-\tau)} \equiv \overline{c_M} < c_M$. Otherwise, for all other values of $c_M \leq c_S$, M operates as the active seller of the good.*

Figure 2 illustrates the equilibrium outcomes characterized in Proposition (7). The 45°-line represents $c_M = c_S$. We identify three cases by focusing on the area below the 45°-line where $c_M \leq c_S$. First, for all cost combinations covered by the pink area, platform M finds it more profitable to remain a pure marketplace than to become active and sell the good itself. In the pink area, the costs of the firms are similar and low. With such costs, competitive pressure would force M to charge a rather low price if the platform chose to become active. This price would be close to its effective marginal cost. Instead, the platform could also remain a pure marketplace. In such cases, the independent seller sells at her monopoly price, and M earns marketplace profits through τ . Despite its cost advantage, the platform opts for the pure marketplace mode for all cost constellations summarized by the pink area. In these cases, a direct commitment by the platform not to become active is not required. All cost constellations of the pink area endogenously yield the pure marketplace mode.

Second, whenever the constellation of c_M and c_S fall into the yellow area, M is much more efficient than S . Due to its cost advantage, the platform prefers to become active and make the sales itself.

Finally, the gray area indicates all constellations where seller S cannot affect the pricing of the active platform, i.e., the competitive price $\frac{c_S}{1-\tau}$ exceeds the price that maximizes M 's selling profit, $\frac{1}{2}(1 + c_M)$. We do not want to focus on such situations.

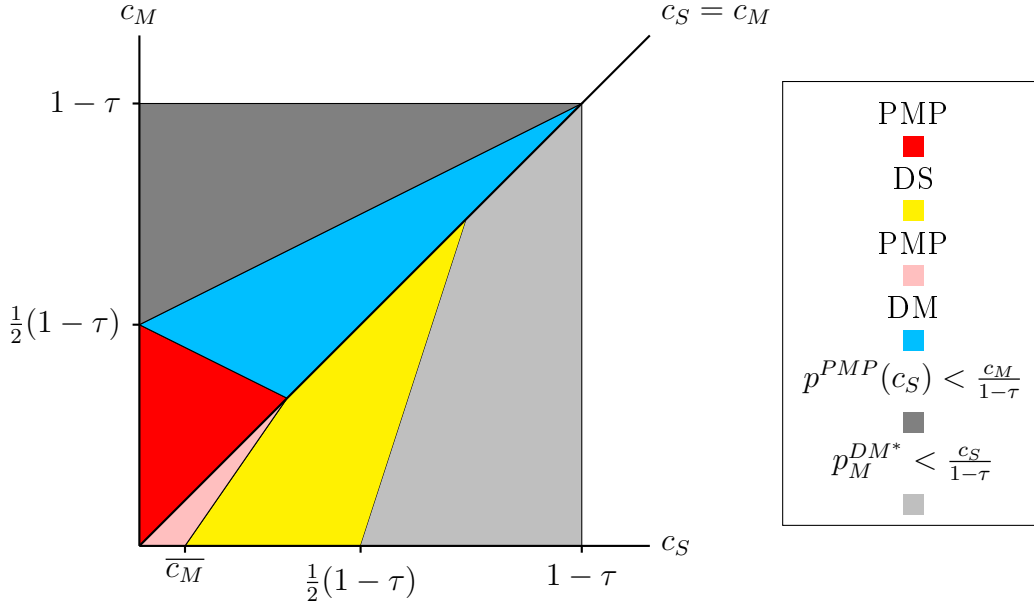


Figure 2: Optimal Operating Mode of M under the linear demand specification $D(p) = 1 - p$.

We consider next all cost constellations where $c_S < c_M$. In all these cases, S will be the seller of the good in any situation. Nevertheless, M can take the possibility and influence prevailing prices when becoming active. Our findings are summarized in three cases.

First, the red area illustrates all cost constellations where the platform opts for the pure marketplace mode. With such costs, the monopoly price charged by seller S leads to consumer prices that are closer to the price that maximizes marketplace profits, namely $p^R = \frac{1}{2}$. Accordingly, the monopoly price leads to higher marketplace profits. Note that it is not necessary for the platform to specifically commit on its operating mode to act credible in these cases.

Second, the blue area illustrates the opposite case. Here, the platform finds it more profitable to become active and discipline the seller's pricing.

Finally, in the dark gray area, the platform is incapable of applying competitive pressure on S when entering the market. Such situations are not of interest in our analysis.

Given the results of Proposition (7), we can evaluate the welfare implications of the dual mode. To do so, we take the pure marketplace mode as a benchmark. Thus, technically, there is no welfare loss whenever the platform operates in the pure marketplace mode.

Corollary 1 (Welfare Comparison, Linear Demand)

Total welfare increases whenever M operates in the dual mode.

From a welfare perspective, dual operation by the platform should be welcomed. And indeed, the operating mode of the platform generally agrees with the socially optimal choice, apart from two cases. In particular, if the platform is only marginally more

efficient relative to the independent seller, the profits it could obtain as a seller are too low compared to the profits it can obtain through the ad-valorem fee as a pure marketplace where seller S sets her price as a monopolist. Technically, in such cases, there is no welfare loss. However, from a social perspective, having the platform as a seller is desirable, given its cost advantage. Second, whenever both firms have low cost while M slightly less efficient, the platform can decrease the price when operating in the dual mode. From this viewpoint, it is clear that consumers benefit from dual operation mode even if seller S still sells the good.

Thus, we can conclude that there are instances where the platform's choice to operate as a pure marketplace is inefficient. Rather, it would have been welfare improving if it had chosen to operate in the dual mode.

5 Competition with Fringe Sellers

So far, our results have been derived for situations where the platform in the dual mode competes with only one seller in its marketplace. In this section, we explore what can happen if many active sellers compete for the same homogeneous product in the marketplace. By doing so, we confirm that our main results go through largely unchanged. We focus on the main results; detailed derivations can be found in the appendix.

The model differs from the one presented above in that we now introduce many small third-party sellers F . Specifically, we design them as a competitive fringe. In addition, as before, there is a strategic seller S that has joined the marketplace.

As in the baseline model, the consumer demand for the homogeneous product is characterized by the generalized demand function $D(p)$. Platform M decides between operating as a pure marketplace (PMP) or operating the dual mode, competing with S and the fringe of sellers in its own marketplace (D). The presence of the fringe already induces some degree of competition in the marketplace in any situation. This model modification entails that the M 's decision to become active in the dual mode is no longer based on the comparison to monopoly prices but on competitive prices.

We assume that seller S and the fringe sellers always join the marketplace. Therefore, the fringe and the independent seller, and if applicable also M , compete in prices in the marketplace. Each fringe retailer can produce the homogeneous product at a constant marginal cost c_F . We assume that each fringe seller is a price taker in the market earning zero profit. Accordingly, their joint profits must satisfy

$$((1 - \tau)p_F - c_F)D(p_F) = 0. \quad (5)$$

Solving (5) for p_F , we obtain $p_F = \frac{c_F}{1-\tau}$. This means that independently of the prices announced by S and M , each fringe seller sets its price equal to its effective marginal

cost. Note that the lower price limit of M , $\frac{c_M}{1-\tau}$, and seller S , $\frac{c_S}{1-\tau}$, derived in Section (3.2), are unaffected by the introduction of a competitive fringe. By standard Bertrand, the firm that can offer the homogeneous product at the lowest price will make the sales in equilibrium. To avoid situations where neither the fringe nor platform M can exert competitive pressure on the pricing of S , it is necessary that $\max(\frac{c_F}{1-\tau}, \frac{c_M}{1-\tau}) < p^{PMP}(c_S)$. Moreover, the Bertrand undercutting mechanism comes into effect only if $\frac{c_S}{1-\tau}$ is less than the price that maximizes M 's profit when selling the good directly, $p = c_M + p^R$.

Our results stay qualitatively the same. The operation mode choice of M depends on its own efficiency and that of its competitors in the marketplace. To show this, we now characterize the equilibrium outcomes under any cost constellation.

Whenever the platform is a pure marketplace, S and the fringe compete in prices in the marketplace. Recall that each fringe seller is a price taker earning zero profit. Thus, their price, $p_F = \frac{c_F}{1-\tau}$, is independent of what S sets as a price. Conversely, the best response of S depends on p_F . Given this, we can summarize our results as follows.

- (i) If $c_S < c_F$, S makes the sales at an equilibrium price of $\frac{c_F}{1-\tau}$ and earns a profit of $D(\frac{c_F}{1-\tau})(c_F - c_S)$. The platform collects pure marketplace profits of $\tau \frac{c_F}{1-\tau} D(\frac{c_F}{1-\tau})$. The competitive fringe obtains zero profit.
- (ii) If $c_F < c_S$, the competitive fringe makes the sales at an equilibrium price of $\frac{c_F}{1-\tau}$ and earns a profit of zero. The platform collects pure marketplace profits of $\tau \frac{c_F}{1-\tau} D(\frac{c_F}{1-\tau})$. Seller S obtains zero profit.

Next, suppose that M has announced to become active on its marketplace. M operating in the dual mode implies that it competes with S and the fringe in its own marketplace. We obtain the following outcomes in equilibrium.

cost structure	seller	price	operation mode platform	π_M	π_S	π_F
$c_M < c_S < c_F$	M	$\frac{c_S}{1-\tau}$	seller	$(\frac{c_S}{1-\tau} - c_M)D(\frac{c_S}{1-\tau})$	0	0
$c_M < c_F < c_S$	M	$\frac{c_F}{1-\tau}$	seller	$(\frac{c_F}{1-\tau} - c_M)D(\frac{c_F}{1-\tau})$	0	0
$c_S < c_M < c_F$	S	$\frac{c_M}{1-\tau}$	marketplace	$\tau \frac{c_M}{1-\tau} D(\frac{c_M}{1-\tau})$	$(c_M - c_S)D(\frac{c_M}{1-\tau})$	0
$c_S < c_F < c_M$	S	$\frac{c_F}{1-\tau}$	marketplace	$\tau \frac{c_F}{1-\tau} D(\frac{c_F}{1-\tau})$	$(c_F - c_S)D(\frac{c_F}{1-\tau})$	0
$c_F < \min(c_S, c_M)$	F	$\frac{c_F}{1-\tau}$	marketplace	$\tau \frac{c_F}{1-\tau} D(\frac{c_F}{1-\tau})$	0	0

Table 1: Dual Mode Equilibrium

Now that we have established the outcomes given that M has already decided on its operating mode, we will characterize the optimal operating mode choice. To this end,

we identify the optimal operating mode given that the platform is the most efficient firm in the marketplace before assuming the opposite, M being not the most efficient firm. Whenever $c_M < \min(c_S, c_F)$, M decides between selling the good directly in the dual mode, leaving S and the fringe with zero profit, and operating in the pure marketplace mode, such that S and the fringe compete in prices. When becoming active, competitive pressure induces M to make the sales at a rather low price. On the other hand, when operating as a pure marketplace, M collects fraction τ of the seller revenue based on competitive prices. We find that only if M benefits from a distinct cost advantage, it prefers to sell the good itself.

Proposition 8 (Optimal Operating Mode, $c_M < \min(c_S, c_F)$)

If $c_M^* < c_M < c_S$, and $c_S < c_F$, M operates in the pure marketplace mode, where $c_M^* \equiv \frac{c_S}{1-\tau} - \tau \frac{c_F}{1-\tau} D(\frac{c_F}{1-\tau}) \frac{1}{D(\frac{c_S}{1-\tau})}$. Otherwise, for all values of $c_M \leq \min(c_S, c_F)$, M operates as the active seller of the good.

Now, suppose that M is not the most efficient firm in the market, namely, $c_S < c_M$, $c_F < c_M$, or both holds. As a result, the platform would operate in the marketplace mode after it has announced to become a seller. Even though M would not make the sales under this cost structure when becoming active, it may be profitable to operate in the dual mode nevertheless. By announcing to operate in the dual mode, the platform can influence prevailing prices which may increase its marketplace profits. As argued before, marketplace profits of M are $\tau p D(p)$ when price equals p . These are maximized when $p = p^R$. Thus, whenever the price after M has announced to become active is closer to p^R compared to the price that occurs when M operates in the pure marketplace mode, M will operate in the dual mode.

Proposition 9 (Optimal Operating Mode, $\min(c_S, c_F) < c_M$)

Whenever $c_S < c_M < c_F$, and $\hat{c}_M < c_M$, M prefers to become a seller and operates in the dual marketplace mode, where $\hat{c}_M \equiv c_F \frac{D(\frac{c_F}{1-\tau})}{D(\frac{c_M}{1-\tau})}$. Otherwise, for all other cases where $\min(c_S, c_F) < c_M$, M operates in the pure marketplace mode.

We find that whenever M has a significant cost disadvantage, M operates in the dual mode. In doing so, the platform can align the prevailing prices with p^R and, thereby, increase its marketplace profits. On the other hand, for all other values of c_M , c_S , and c_F given that $\min(c_S, c_F) < c_M$, M prefers to operate in the pure marketplace mode. Proposition (7) comprises two cost situations where M decides on operating in the pure marketplace mode, namely $c_M < c_F$ and $c_F < c_M$, which we will briefly discuss in the following.

Let us start with $c_M < c_F$. Note that $\min(c_S, c_F) < c_M$ and $c_M < c_F$ lead to $c_S < c_M < c_F$. Under this cost structure, whenever M operates as a pure marketplace, S sells to consumers at price $\frac{c_F}{1-\tau}$. On the other hand, in the dual mode, M induces S to lower

the price to $\frac{c_M}{1-\tau}$. In accordance with the results from the baseline model, we find that whenever M has a relatively large cost disadvantage compared to S , it will operate in the dual mode. Moreover, a comparison to the baseline model reveals that under this cost structure, $\frac{c_F}{1-\tau}$ acts as the monopoly price p^{PMP} .

Next, suppose $c_F < c_M$. Under this cost structure, the equilibrium price is $\frac{c_F}{1-\tau}$ in any situation. See Table (1) for more insights. Since the platform in these cases cannot induce a lower price by operating in the dual mode, it obtains the same marketplace profit under both operation mode. Therefore, M is indifferent between operating in the pure marketplace mode and the dual mode where it ultimately serves as a marketplace. According to our tie breaking assumption, M will operate in the pure marketplace mode in these cases.

Finally, we are aware that most products are sold by more than one independent seller. It is important, therefore, to show that our findings can be extended to cases where many sellers offer the same homogeneous product. In this section, we have shown that adding many sellers to the model, designed as a competitive fringe of small third-party sellers, gives qualitatively identical results as in the baseline model with only one independent seller.

6 Conclusion

It is undeniable that many of the interesting high tech markets are dominated by a few firms. In many cases, a single platform controls a large chunk of the market infrastructure. There has been increasing concerns regarding the dual mode operation of these platforms where they not only host some users but also actively compete with some of them.

In view of this, we have analyzed the operating mode choice of online platforms such as Amazon. We employed a stylized model to study the optimal operating mode of a platform and characterized the corresponding welfare effects.

Our first finding indicates that the platform prefers to sell only if it benefits from a significant cost advantage. Otherwise, the pure marketplace mode is more profitable even if the platform is the most efficient firm in the marketplace. It turns out that, if the efficiency advantage of the independent is relatively small, the platform earns higher profits by remaining a pure marketplace. Second, and most importantly, we show that if the platform faces a clear cost disadvantage, it may nevertheless become active in the market as a seller solely with the intention not to sell the good but to discipline the seller's price. By influencing the pricing behavior of the independent seller, entry in the dual mode can help the platform to maximize marketplace profits.

In an extension of the model, we introduce an additional competitive fringe of small third-party sellers, and confirm that our results from the single seller case do not change qualitatively. Finally, the platform in the dual mode benefits consumers and reduces the

surplus of the independent seller.

In light of our results, it appears that a ban on the dual mode is not an appropriate policy reaction, at least when we only consider short term effects of such a ban. Clearly, a more complete policy response will take dynamic innovation incentives into account as well. Although total welfare increases and entry in the dual mode is underprovided from a social perspective, decreasing profitability of independent sellers that are the driving forces of innovative activity may have severe consequences on new product creations in a dynamic setting. However, we leave this aspect to future research.

References

- Anderson, S., & Bedre-Defolie, Ö. (2021). Hybrid platform model.
- Anderson, S., & Bedre-Defolie, Ö. (2022). Online trade platforms: hosting, selling, or both? *Available at SSRN 4056493*.
- Armstrong, M. (2006). Competition in two-sided markets. *The RAND journal of economics*, 37(3), 668–691.
- Berges-Sennou, F., Bontems, P., & Réquillart, V. (2004). Economics of private labels: A survey of literature. *Journal of Agricultural & Food Industrial Organization*, 2(1).
- Crawford, G., Courthoud, M., Seibel, R., & Zuzek, S. (2022). Amazon entry on amazon marketplace. *CEPR DP*, 17531.
- Etro, F. (2021). Product selection in online marketplaces. *Journal of Economics & Management Strategy*, 30(3), 614–637.
- Etro, F. (2022). The economics of amazon. *Available at SSRN 4307213*.
- Hagiu, A., Jullien, B., & Wright, J. (2020). Creating platforms by hosting rivals. *Management Science*, 66(7), 3234–3248.
- Hagiu, A., Teh, T.-H., & Wright, J. (2022). Should platforms be allowed to sell on their own marketplaces? *The RAND Journal of Economics*, 53(2), 297–327.
- Hervas-Drane, A., & Shelegia, S. (2022a). Price competition with a stake in your rival.
- Hervas-Drane, A., & Shelegia, S. (2022b). Retailer-led marketplaces.
- Muthers, J., & Wismer, S. (2013). Why do platforms charge proportional fees? commitment and seller participation. *Commitment and Seller Participation (January 21, 2013)*.
- Rochet, J.-C., & Tirole, J. (2004). Two-sided markets: An overview. *Institut d’Economie Industrielle working paper*.
- Tremblay, M. (2022). Fee discrimination by hybrid platforms. *Available at SSRN 4195017*.

Appendix

Proof of Proposition (1).

The profit function of seller S in the pure marketplace case is given by

$$((1 - \tau)p - c_S)D(p),$$

which attains a maximum at the price that solves the following equation:

$$(1 - \tau)D(p) + ((1 - \tau)p - c_S)D'(p) = 0$$

$$\frac{c_S}{1 - \tau} + \eta(p) = p$$

with $\eta = \frac{D(p)}{-D'(p)}$.

Proof of Proposition (2).

In order to demonstrate that the equilibrium outcome presented in Proposition (2) indeed constitutes a Nash equilibrium, we first need to establish the best response functions faced by both firms.

The best response correspondence of S given the price of platform M , p_M , is

$$p_S(p_M) = \begin{cases} p^{PMP} & , \quad p^{PMP} < p_M \\ p_M - \delta & , \quad \frac{c_S}{1-\tau} < p_M \leq p^{PMP} \\ p_S \geq \frac{c_S}{1-\tau} & , \quad p_M = \frac{c_S}{1-\tau} \\ p_S > p_M & , \quad p_M < \frac{c_S}{1-\tau} \end{cases}$$

The best response correspondence of M when S charges p_S is given by

$$p_M(p_S) = \begin{cases} p_M = c_M + \eta(p_M) & , \quad p_M = c_M + \eta(p_M) < p_S \\ p_S - \delta & , \quad \frac{c_M}{1-\tau} < p_S \leq c_M + \eta(p_M) \\ p_M \geq \frac{c_M}{1-\tau} & , \quad p_S = \frac{c_M}{1-\tau} \\ p_M \geq p_S & , \quad p_S < \frac{c_M}{1-\tau} \end{cases}$$

Notice that if M were to be the sole seller of the good, the price that maximizes its monopoly profits, $(p - c_M)D(p)$, is defined by $p = c_M + \eta(p)$. We have already established in Proposition (1) that S maximizes her monopoly profits at p^{PMP} .

M strictly prefers to sell the good directly over collecting the proportional fee whenever $p > \frac{c_M}{1-\tau}$. When S sets a price larger than $\frac{c_M}{1-\tau}$, M formulates its best response in order to make the sales. On the contrary, when the independent seller chooses a price lower than $\frac{c_M}{1-\tau}$, M has no interest in making the sale, and is content with the marketplace profits it earns via the ad-valorem fee. On the other hand, S is willing to sell the good for any price above $\frac{c_S}{1-\tau}$ as established in Section (3.2).

For any price set by S , p_S , M is making all sales if it posts $p_S - \delta$. Analogously, S serves the market only if she slightly undercuts M 's price, and sets her price equal to $p_M - \delta$.

We will now argue that $p_M \in (\frac{c_M}{1-\tau}, \frac{c_S}{1-\tau}]$ where M makes the sales at this price can be supported as an equilibrium outcome when $c_M < c_S$ which in turn implies $\frac{c_M}{1-\tau} < \frac{c_S}{1-\tau}$. If M sets a price in this interval, namely in $(\frac{c_M}{1-\tau}, \frac{c_S}{1-\tau}]$, S cannot profitably undercut p_M .

Suppose, M sets a price, \bar{p} , in the interval $(\frac{c_M}{1-\tau}, \frac{c_S}{1-\tau}]$. In the analysis of Section (3.2) we have shown that S is not willing to set a price below $\frac{c_S}{1-\tau}$ and capture the sales. Thus, any price of S that exceeds \bar{p} is a best response. In particular, setting $p_S = \bar{p}$, is a best response, given our tie breaking rule.

Next, consider the situation where S sets her price equal to $\bar{p} \in (\frac{c_M}{1-\tau}, \frac{c_S}{1-\tau}]$. Given that M prefers to sell the good directly over collecting the ad-valorem fee for any price above $\frac{c_M}{1-\tau}$, M makes higher profits compared to the marketplace mode for all price in the interval $(\frac{c_M}{1-\tau}, \bar{p}]$. M will not charge a price below $\frac{c_M}{1-\tau}$ as this yields to losses. Furthermore, if M posts $p_M > \frac{c_S}{1-\tau}$ and S sets $\frac{c_S}{1-\tau}$, S will be the seller of the good and M will operate in the marketplace mode, which yields lower profits compared to selling the good itself under this cost structure.

We select $\frac{c_S}{1-\tau}$ as our preferred price prediction given our equilibrium selection rule.

Proof of Proposition (3).

We will now show that $p_S \in [\frac{c_S}{1-\tau}, \frac{c_M}{1-\tau}]$ where S makes the sales at this price can be supported as an equilibrium outcome. Case 2, $c_S < c_M$, implies that $\frac{c_S}{1-\tau} < \frac{c_M}{1-\tau}$ from which we can conclude that S can profitably undercut any price set by M for which the platform is willing to sell the good directly. For any price set by S that is in the interval $[\frac{c_S}{1-\tau}, \frac{c_M}{1-\tau}]$, M prefers to operate as marketplace and collect the ad-valorem fee over undercutting p_S and making the sales.

To fix ideas, suppose, S sets a price, \bar{p} , in the interval $[\frac{c_S}{1-\tau}, \frac{c_M}{1-\tau}]$. In general, M is not willing to make the sales by setting a price below $\frac{c_M}{1-\tau}$. Therefore, M 's best response to \bar{p} is to post $p_M \geq \frac{c_M}{1-\tau}$ which ensures that it does not make the sales, given our tie breaking rule.

Next, assume that M posts a price equal to $\bar{p} = \frac{c_M}{1-\tau}$. Given our prior analysis, we know that S is willing to sell the good for any price that at least covers her perceived marginal costs, i.e., $\frac{c_S}{1-\tau}$, and does not compete if price is below this threshold. In response to the price set by M , \bar{p} , S will post a price in the interval $[\frac{c_S}{1-\tau}, \bar{p}]$. Thus, S earns at least as much when undercutting M than in the situation, in which M makes the sales.

Our preferred price prediction is $\frac{c_M}{1-\tau}$ given our equilibrium selection rule.

Proof of Proposition (4).

Let us start by establishing the profit comparison. When $c_M < c_S$, platform M will either be the active seller of the good whenever it enters in the dual mode or it will operate as a pure marketplace. We have derived the corresponding profit functions in Proposition (1), $\tau p^{PMP} D(p^{PMP})$, and (2), $(\frac{c_S}{1-\tau} - c_M) D(\frac{c_S}{1-\tau})$. We obtain

$$\begin{aligned} \tau p^{PMP} D(p^{PMP}) &\geq (\frac{c_S}{1-\tau} - c_M) D(\frac{c_S}{1-\tau}) \\ c_M &\geq \frac{c_S}{1-\tau} - \tau p^{PMP} D(p^{PMP}) \frac{1}{D(\frac{c_S}{1-\tau})} \equiv \bar{c}_M \end{aligned}$$

These arguments suggest that when $c_M < c_S$, and c_M is relatively large, the platform prefers to operate in the pure marketplace mode.

Proof of Proposition (5).

First note that whenever $c_S < c_M$, the platform in the dual mode will operate a marketplace while the independent seller serves the demand at $\frac{c_M}{1-\tau}$. Thus, the profit comparison considered by the platform when deciding on its operating mode involves π_M^{DS} and π_M^{PMP} . It follows that if

$$\begin{aligned} \pi_M^{PMP} &\geq \pi_M^{DS} \\ \tau p^{PMP} D(p^{PMP}) &\geq \tau \frac{c_M}{1-\tau} D(\frac{c_M}{1-\tau}) \\ \hat{c}_M \equiv (1-\tau) p^{PMP} D(p^{PMP}) \frac{1}{D(\frac{c_M}{1-\tau})} &\geq c_M \end{aligned}$$

the platform operates in the pure marketplace.

Proof of Proposition (6).

Pure Marketplace Mode. S is the sole seller of the good maximizing $\pi_S^{PMP} = ((1 - \tau)p - c_S)(1 - p)$. The first order condition equals $(1 - \tau)(1 - p) - p(1 - \tau) + c_S = 0$ yielding $p^{PMP}(c_S) = \frac{1}{2} + \frac{1}{2} \frac{c_S}{1 - \tau}$.

Best Response Correspondences. In order to demonstrate that the equilibrium outcome presented in Proposition (6) indeed constitute a Nash equilibrium, we first need to characterize the best response correspondences faced by both firms.

The best response correspondence of S given the price of platform M , p_M , is

$$p_S(p_M) = \begin{cases} \frac{1}{2} + \frac{1}{2} \frac{c_S}{1 - \tau} & , \quad \frac{1}{2} + \frac{1}{2} \frac{c_S}{1 - \tau} < p_M \\ p_M - \delta & , \quad \frac{c_S}{1 - \tau} < p_M \leq \frac{1}{2} + \frac{1}{2} \frac{c_S}{1 - \tau} \\ p_S \geq \frac{c_S}{1 - \tau} & , \quad p_M = \frac{c_S}{1 - \tau} \\ p_S > p_M & , \quad p_M < \frac{c_S}{1 - \tau} \end{cases}$$

The best response correspondence of M when S charges p_S is given by

$$p_M(p_S) = \begin{cases} \frac{1}{2} + \frac{c_M}{2} & , \quad \frac{1}{2} + \frac{c_M}{2} < p_S \\ p_S - \delta & , \quad \frac{c_M}{1 - \tau} < p_S \leq \frac{1}{2} + \frac{c_M}{2} \\ p_M \geq \frac{c_M}{1 - \tau} & , \quad p_S = \frac{c_M}{1 - \tau} \\ p_M > p_S & , \quad p_S < \frac{c_M}{1 - \tau} \end{cases}$$

Notice that if M were to be the sole seller of the good, the price that maximizes its monopoly profits, $(1 - p)(p - c_M)$, is given by $p = \frac{1}{2} + \frac{c_M}{2}$. We have already established in Proposition (1) that S maximizes her monopoly profits at $p = \frac{1}{2} + \frac{1}{2} \frac{c_S}{1 - \tau}$.

Dual Mode, $c_M < c_S$. In this case, our arguments follow the Proof of Proposition (2).

Dual Mode, $c_S < c_M$. The logic is similar to that in Proof of Proposition (3).

Proof of Proposition (7). The proof is long, so we present it in two steps. The first step involves characterizing the equilibrium operating mode assuming $c_M \leq c_S$. The second step involves doing the same thing assuming $c_S < c_M$.

1. $c_M \leq c_S$.

PURE MARKETPLACE EQUILIBRIUM.

We follow Proof of Proposition (4). In case of the linear demand specification $D(p) = 1-p$, we obtain that

$$\begin{aligned} c_M &\geq \frac{c_S}{1-\tau} - \tau p^{PMP} D(p^{PMP}) \frac{1}{D(\frac{c_S}{1-\tau})} \\ c_M &\geq \frac{c_S}{1-\tau} - \tau \left(\frac{1}{2} + \frac{1}{2} \frac{c_S}{1-\tau} \right) \left(1 - \frac{1}{2} - \frac{1}{2} \frac{c_S}{1-\tau} \right) \frac{1}{1 - \frac{c_S}{1-\tau}} \\ c_M &\geq c_S - \frac{\tau}{4} + \frac{3\tau c_S}{4(1-\tau)} \equiv \bar{c}_M \end{aligned}$$

Thus, whenever $\bar{c}_M < c_M$ and $c_M < c_S$ the platform prefers to operate in the pure marketplace mode over selling the good directly. Note that this cost structure implies that $\bar{c}_M < c_S$, which is only feasible if $c_S < \frac{1}{3}(1-\tau)$. To complete our analysis, we check whether the price M charges to consumers when becoming active, namely $\frac{c_S}{1-\tau}$, is below the price that maximizes its profits from selling, $\frac{1}{2} + \frac{c_M}{2}$. It is easy to verify that $\frac{c_S}{1-\tau} < \frac{1}{2} + \frac{c_M}{2}$ if $c_S < \frac{(1+c_M)(1-\tau)}{2}$. When comparing $c_S < \frac{(1+c_M)(1-\tau)}{2}$ and $c_S < \frac{1}{3}(1-\tau)$, which ensures $\hat{c}_M < c_S$, we find that whenever $c_S < \frac{1}{3}(1-\tau)$, $c_S < \frac{(1+c_M)(1-\tau)}{2}$ holds by definition.

To summarize, whenever $c_S < \frac{1}{3}(1-\tau)$, and $\max(0, \bar{c}_M) < c_M < c_S$, M operates in the pure marketplace mode in equilibrium.

DUAL MODE SELLING EQUILIBRIUM.

Now, we investigate the cost ranges under which the platform prefers to become active and to sell the good over collecting pure marketplace profits. We can use the previous results from PURE MARKETPLACE EQUILIBRIUM, where we have shown that $\pi_M^{PMP} > \pi_M^{DM}$ if $c_S - \frac{\tau}{4} + \frac{3\tau c_S}{4(1-\tau)} \equiv \bar{c}_M < c_M$. Notice that by implication this yields $\pi_M^{PMP} < \pi_M^{DM}$ if $c_M < \bar{c}_M$. Unfortunately, there are cases where \bar{c}_M is negative. We find that if $\tau \frac{1-\tau}{4-\tau} < c_S$ holds, $0 < \bar{c}_M$ so that $c_M < \bar{c}_M$ is a feasible cost structure. A second aspect we have to consider is that the competitive price charged by M to be less than the price that maximizes its monopoly selling profits. Formally, we need $\frac{c_S}{1-\tau} < \frac{1}{2} + \frac{c_M}{2}$.

To summarize, the cost structure so that the platform prefers to become active and sell the good over collecting pure marketplace profit is $c_M < \bar{c}_M$ and $c_M < c_S$. As discussed before this requires $\tau \frac{1-\tau}{4-\tau} < c_S$ (so that \bar{c}_M is positive) and $\frac{c_S}{1-\tau} < \frac{1}{2} + \frac{c_M}{2}$ (so that M constitutes a competitive threat to S).

We examine two subcases. First, we consider all cost combinations yielding $c_M < \bar{c}_M < c_S$. Subsequently, we look at all cost constellations where $c_M < c_S < \bar{c}_M$ is true.

Case A. $\tau \frac{1-\tau}{4-\tau} < c_S \leq \frac{1}{3}(1-\tau)$, and $c_M < \bar{c}_M$ which gives us $0 < c_M < \bar{c}_M \leq c_S$. As argued before, $\tau \frac{1-\tau}{4-\tau} < c_S$ ensures that \bar{c}_M is positive. Moreover, let $c_S \leq \frac{1}{3}(1-\tau)$ so that $\bar{c}_M \leq c_S$. We consider to opposite case in the next paragraph. Note that whenever $c_S \leq \frac{1}{3}(1-\tau)$, the competitive price $\frac{c_S}{1-\tau}$ is always lower than the price that maximizes the selling profits of platform M , $\frac{1}{2} + \frac{c_M}{2}$.

Case B. $\frac{1}{3}(1-\tau) < c_S < \frac{1-\tau}{1+\tau}$, and $\frac{\tau+2c_S-1}{1-\tau} < c_M < c_S$ which leads to $c_M < c_S < \bar{c}_M$. First note that $\frac{1}{3}(1-\tau) < c_S$ leads to $c_S < \bar{c}_M$. Moreover, we want to focus on cost constellations where the competitive price, $\frac{c_S}{1-\tau}$ is lower than the price that maximizes the selling profits of the platform, $\frac{1}{2} + \frac{c_M}{2}$. Solving this for c_M gives us $\frac{\tau+2c_S-1}{1-\tau} < c_M$. Recall that we are looking at cost structures where $c_M < c_S$. Thus, it is necessary that $\frac{\tau+2c_S-1}{1-\tau} < c_M < c_S$ holds. It turns out that $c_S < \frac{1-\tau}{1+\tau}$ implies $\frac{\tau+2c_S-1}{1-\tau} < c_S$. We conclude that whenever $c_S < \frac{1-\tau}{1+\tau}$, $\frac{\tau+2c_S-1}{1-\tau} < c_M < c_S$ holds which implies that M the competitive price charged by M is below the price that maximizes its selling profits.

Notice that $\frac{\tau+2c_S-1}{1-\tau}$ is negative if $c_S < \frac{1}{2}(1-\tau)$. In these cases, we do not need to worry about the condition $\frac{\tau+2c_S-1}{1-\tau} < c_M$ as it is always true. However, in the opposite case, if $c_S > \frac{1}{2}(1-\tau)$, we need $c_S < \frac{1-\tau}{1+\tau}$ to ensure that c_M can be in the respective interval, namely $(\frac{\tau+2c_S-1}{1-\tau}, c_S)$ to ensure that $\frac{c_S}{1-\tau} < \frac{1}{2} + \frac{c_M}{2} = (p^{DM})^*$.

Let us summarize the conditions under **Case B.** We obtain that whenever $\frac{1}{3}(1-\tau) < c_S < \frac{1-\tau}{1+\tau}$, $\frac{\tau+2c_S-1}{1-\tau} < c_M < c_S$, and $c_S < \hat{c}_M$.

Now, taking **Case A.** and **Case B.** together leads to $c_S < \frac{1-\tau}{1+\tau}$, and $c_M < \min(\hat{c}_M, c_S)$.

2. $c_S < c_M$.

PURE MARKETPLACE EQUILIBRIUM.

Following Proof of Proposition (5), it is easily checked that

$$(1-\tau)p^{PMP}D(p^{PMP})\frac{1}{D(\frac{c_M}{1-\tau})} \geq c_M$$

$$\hat{c}_M \equiv \frac{(1-\tau-c_S)(1-\tau+c_S)}{4(1-\tau-c_M)} \geq c_M$$

Solving this for c_M gives $z_1 \equiv \frac{1}{2}(1-\tau-c_S)$ and $z_2 \equiv \frac{1}{2}(1-\tau+c_S)$. Notice that we are looking at the difference of $\pi_M^{PMP} - \pi_M^{DS}$ which is an upwards opened parabola in c_M . Thus, $\pi_M^{PMP} > \pi_M^{DS}$ if $c_M \leq z_1$ or $c_M \geq z_2$.

As before, we want to restrict our attention to cases where the competitive price,

$\frac{c_M}{1-\tau}$, is below the monopoly price of S , $p^{PMP}(c_S)$. Solving $\frac{c_M}{1-\tau} < p^{PMP}(c_S)$ gives us $c_M < \frac{1}{2}(1-\tau+c_S) = z_2$. Thus, we can never have $c_M \geq z_2$.

Combining these conditions, namely $c_M \leq z_1$ and $\frac{c_M}{1-\tau} < \frac{1}{2} + \frac{1}{2}\frac{c_S}{1-\tau}$, we find that whenever $c_S < c_M \leq \frac{1}{2}(1-\tau-c_S)$, and $c_S < \frac{1}{3}(1-\tau)$, M prefers to operate in the pure marketplace mode. Note that it is necessary that $c_S < c_M$ and $c_M < c_S < \frac{1}{2}(1-\tau-c_S)$. We find that only if $c_S < \frac{1}{3}(1-\tau)$, $c_S < \frac{1}{2}(1-\tau-c_S)$.

DUAL MODE MARKETPLACE EQUILIBRIUM.

We know from the profit comparison that the platform prefers the dual marketplace mode over the pure marketplace mode if $\frac{1}{2}(1-\tau-c_S) < c_M < \frac{1}{2}(1-\tau+c_S)$. As argued before, it is necessary that $\frac{c_M}{1-\tau} < \frac{1}{2} + \frac{1}{2}\frac{c_S}{1-\tau} = p^{PMP}(c_S)$ so that M pursues competitive pressure on S when entering in the dual mode. Summarizing these conditions, we find that M operates in the dual marketplace mode whenever $\max(c_S, \frac{1}{2}(1-\tau-c_S)) < c_M < \frac{1}{2}(1-\tau+c_S)$.

Proof of Corollary (1). We divide the proof into three parts. The first step involves characterizing total welfare, assuming that M operates as a pure marketplace. The pure marketplace mode is the reference point of our welfare consideration as it is the default mode of the platform in our model. Thus, technically, there is no change in welfare whenever M operates in the pure marketplace mode. In the second step, we consider the welfare effect whenever the platform switches from the pure marketplace mode to the dual mode, given that $c_M \leq c_S$. In the opposite case, whenever $c_S < c_M$, we have to deal with the welfare effects whenever the platform enters the dual mode operating a marketplace compared to the situation when remaining in the pure marketplace mode. This is part three.

1. Pure Marketplace Mode.

Whenever M decides to operate in the pure marketplace mode, S charges the monopoly price, $p^{PMP}(c_S) = \frac{1}{2} + \frac{1}{2}\frac{c_S}{1-\tau}$, to consumers (See Proposition (6)). The resulting consumer surplus amounts to

$$CS^{PMP} = \frac{1}{8} \frac{(1-\tau-c_S)^2}{(1-\tau)^2}.$$

When M operates in the pure marketplace mode, it earns $\pi_M^{PMP} = \tau \frac{(1-\tau)^2 - c_S^2}{4(1-\tau)^2}$ while seller S obtains selling profits of $\pi_S^{PMP} = \frac{(1-\tau-c_S)^2}{4(1-\tau)}$. Thus, producer surplus defined as the sum of the profits obtained by seller S and platform M is given by

$$PS^{PMP} = \frac{(1-\tau-c_S)(2\tau c_S - c_S - \tau + 1)}{4(1-\tau)^2}.$$

2. Dual Selling Mode.

Suppose $c_M \leq c_S$. In this case, when M decides to become active, it will be the seller of the good charging $\frac{c_S}{1-\tau}$ (See Proposition (6)). Accordingly, consumer surplus is

$$CS^{DM} = \frac{1}{2} \frac{(1 - \tau - c_S)^2}{(1 - \tau)^2}.$$

From this it is immediate that consumers benefit whenever M announces to become active compared to the pure marketplace mode.

In equilibrium, producer surplus in case M adopts the dual mode and $c_M \leq c_S$ amounts to

$$\begin{aligned} PS^{DM} &= \pi_M^{DM} + \pi_S^{DM} \\ &= \left(\frac{c_S}{1-\tau} - c_M\right) \left(1 - \frac{c_S}{1-\tau}\right) - 0 \\ &= \left(\frac{c_S}{1-\tau} - c_M\right) \left(1 - \frac{c_S}{1-\tau}\right) \end{aligned}$$

Now, let us compare PS^{PMP} and PS^{DM} which yields

$$\begin{aligned} &PS^{PMP} - PS^{DM} \\ &\frac{(1 - \tau - c_S)(2\tau c_S - c_S - \tau + 1)}{4(1 - \tau)^2} - \left(\frac{c_S}{1 - \tau} - c_M\right) \left(1 - \frac{c_S}{1 - \tau}\right) \\ &\frac{(1 - \tau - c_S)(2\tau c_S - 4c_M\tau - 5c_S + 4c_M + 1 - \tau)}{4(1 - \tau)^2} \end{aligned}$$

When comparing PS^{PMP} with PS^{DM} , we find that whenever c_M sufficiently large, producer surplus increases when M enters the dual mode. More specifically, whenever

$$\frac{2c_S^2\tau + 2c_S\tau^2 - 5c_S^2 - 4c_S\tau + 3\tau^2 + 2c_S - 6\tau + 3}{4c_S(\tau - 1)} < c_M$$

holds, total producer surplus increases under the dual selling mode compared to the pure marketplace mode. It turns out that the left-hand side of this inequation is negative for $0 < c_S < 1 - \tau$. Thus, we conclude that producer surplus improves when M operates in the dual mode and $c_M \leq c_S$.

Hence, total welfare increases whenever platform M is the active seller of the good after it announced to operate in the dual mode compared to the benchmark where it operates as a sole marketplace.

3. Dual Marketplace Mode.

Let us assume that $c_S < c_M$ so that M operates in the marketplace mode after announcing

to become an active seller. In this case, seller S sells to consumers at $p^{DS} = \frac{c_M}{1-\tau}$. On the other hand, whenever M operates in the pure marketplace mode, S is a monopolist charging $p^{PMP}(c_S) = \frac{1}{2} + \frac{1}{2} \frac{c_S}{1-\tau}$. Since $p^{DS} < p^{PMP}(c_S)$ holds throughout our analysis (recall that otherwise M cannot influence the pricing of the seller by entering in the dual mode). This requires c_M to be sufficiently small, in particular, we need $c_M < \frac{1}{2}(1-\tau+c_S)$. Given the assumption, it is intuitive to conclude that consumer surplus increases whenever $c_S < c_M$ and M announces to become active in the product space.

The profit of seller S always decreases whenever M announces to become a seller of the good. By entering the dual mode M forces S to deviate from the monopoly price, p^{PMP} , to a lower price, namely $\frac{c_M}{1-\tau}$. We can show that the difference in profits of S ,

$$\pi_S^{PMP} - \pi_S^{DS},$$

equals

$$\frac{(1-\tau+c_S-2c_M)^2}{4(1-\tau)} > 0.$$

Unfortunately, the difference in producer surplus of M when operating in the dual mode compared to the benchmark is not straightforward. Whether the profit of M increases or decreases after it announced to become an active seller depends on the relative efficiencies of c_M and c_S as discussed in Proposition (7),

$$\begin{aligned} & \pi_M^{PMP} - \pi_M^{DS} \\ & \tau \frac{(1-\tau)^2 - c_S^2}{4(1-\tau)^2} - \tau \frac{c_M(1-\tau-c_M)}{(1-\tau)^2} \\ & \tau \frac{(1-\tau+c_S-2c_M)(1-\tau-c_S-2c_M)}{4(1-\tau)^2} \begin{matrix} \leq \\ \geq \end{matrix} 0. \end{aligned}$$

Therefore, we will now focus on total welfare showing that the increase in consumer surplus always recovers potential losses in total surplus when M becomes active and $c_S < c_M$.

$$\pi_M^{PMP} + \pi_S^{PMP} + CS^{PMP} - \pi_M^{DS} - \pi_S^{DS} - CS^{DS} \geq 0.$$

The difference of welfare expressions is an upwards opened parabola in c_M . The zeros are $c_{M,1} \equiv \frac{1}{2}(3c_S + \tau - 4c_S\tau - 1)$ and $c_{M,2} \equiv \frac{1}{2}(1-\tau+c_S)$. In the following we will show that total welfare increases under the dual mode only if $c_{M,1} < c_M < c_{M,2}$.

First, note that $c_{M,1} < c_{M,2}$ is always true. To show this, we need to make a case distinction depending on $\tau \in (0, 1)$. After we have established that $c_{M,1} < c_{M,2}$ holds, we will show that c_M is always the range that leads to an increase in total welfare. Formally, this means we claim that $c_{M,1} < c_M < c_{M,2}$ holds. This follows in the second part of this proof.

Case A. Suppose $\tau < \frac{1}{2}$. In this case, we find that $c_{M,1} < c_{M,2}$ if $c_S < \frac{1-\tau}{1-2\tau}$. Recall that it is necessary that $c_S < 1 - \tau$ so that demand, $D(p) = 1 - p$, is positive. We can verify that $1 < \frac{1-\tau}{1-2\tau}$. Thus, it turns out that $c_S < \frac{1-\tau}{1-2\tau}$ when $\tau < \frac{1}{2}$ and, therefore, $c_{M,1} < c_{M,2}$ holds.

Case B. Suppose $\tau = \frac{1}{2}$ from which $c_{M,1} < c_{M,2}$ follows directly.

Case C. Suppose $\tau > \frac{1}{2}$. In this case, if we want $c_{M,1} < c_{M,2}$, it is necessary that $c_S > \frac{1-\tau}{1-2\tau}$. Notice that in this case the lower limit of c_S , $\frac{1-\tau}{1-2\tau}$, is negative given that $\tau > \frac{1}{2}$. Thus, $c_S > \frac{1-\tau}{1-2\tau}$ is true, which, in turn, leads to $c_{M,1} < c_{M,2}$.

Thus, $c_{M,1} < c_{M,2}$ holds for any τ .

Once again, recall that only if $c_{M,1} < c_M < c_{M,2}$, total welfare increases whenever M enters the dual mode and $c_S < c_M$ compared to the situation where M operates in the pure marketplace mode. First, we present that $c_M < c_{M,2}$ holds. Note that in order to discipline the pricing of the independent seller when $c_S < c_M$, the competitive price, $\frac{c_M}{1-\tau}$, must be lower than the price that maximizes the selling profits of S , p^{PMP} . Formally, this implies

$$\begin{aligned} \frac{c_M}{1-\tau} &< \frac{1}{2} + \frac{1}{2} \frac{c_S}{1-\tau} = p^{PMP} \\ c_M &< \frac{1}{2}(1-\tau + c_S) = c_{M,2} \end{aligned}$$

Thus, it turns out that $c_M < c_{M,2}$ is true.

Now, we focus on the lower limit, $c_{M,1} < c_M$. It is enough to show that $c_{M,1} < c_S$, since $c_S < c_M$ holds in this part of the analysis. To prove that $c_{M,1} < c_S$ we need to consider three distinct cases, namely,

Case A. Suppose $\tau < \frac{1}{4}$. In this case, $c_{M,1} < c_S$ if $c_S < \frac{1-\tau}{1-4\tau}$. It is easy to verify that $\frac{1-\tau}{1-4\tau} > 1$. As a result, $c_S < \frac{1-\tau}{1-4\tau}$ holds which implies $c_{M,1} < c_S$.

Case B. Suppose $\tau = \frac{1}{2}$. From this $c_{M,1} < c_S$ follows directly.

Case C. Suppose $\tau > \frac{1}{2}$. Under this assumption, $c_{M,1} < c_S$ if $c_S > \frac{1-\tau}{1-4\tau}$. Note that $\frac{1-\tau}{1-4\tau} < 0$ whenever $\tau > \frac{1}{2}$. As a result, $c_S > 0 > \frac{1-\tau}{1-4\tau}$ implying $c_{M,1} < c_S$.

Thus, it becomes clear that $c_{M,1} < c_S$. Once again, total welfare increases only if

$c_{M,1} < c_M < c_{M,2}$ and $c_M < c_S$. Summarizing our results, we obtain that $c_{M,1} < c_S < c_M < c_{M,2}$. Consequently, total welfare increases whenever M enters in the dual mode operating a marketplace compared to the benchmark where it operates as a pure marketplace.

Proof of Proposition (8). Let $c_M < \min(c_S, c_F)$ so that when being active, M will be the most efficient seller in the marketplace. Consider two cases (i) $c_M < c_S < c_F$ and (ii) $c_M < c_F < c_S$.

(i) Suppose $c_M < c_S < c_F$. In this case, M compares $\pi_M^{DM} = (\frac{c_S}{1-\tau} - c_M)D(\frac{c_S}{1-\tau})$ and $\pi_M^{PMP} = \tau \frac{c_F}{1-\tau} D(\frac{c_F}{1-\tau})$. We obtain

$$\begin{aligned} \pi_M^{PMP} &> \pi_M^{DM} \\ \tau \frac{c_F}{1-\tau} D(\frac{c_F}{1-\tau}) &> (\frac{c_S}{1-\tau} - c_M) D(\frac{c_S}{1-\tau}) \\ c_M &> \frac{c_S}{1-\tau} - \tau \frac{c_F}{1-\tau} \frac{D(\frac{c_F}{1-\tau})}{D(\frac{c_S}{1-\tau})} \equiv c_M^* \end{aligned}$$

The platform makes a higher profit if it operates in the pure marketplace mode whenever $c_M > c_M^*$. On the other hand, whenever $c_M < c_M^*$, M yields higher profits by operating under the dual mode and selling the good directly.

Next, let us consider (ii) $c_M < c_F < c_S$. Under this cost structure, M prefers to sell the good directly over collecting pure marketplace profits

$$\begin{aligned} \pi_M^{PMP} &> \pi_M^{DM} \\ \tau \frac{c_F}{1-\tau} D(\frac{c_F}{1-\tau}) &> (\frac{c_F}{1-\tau} - c_M) D(\frac{c_F}{1-\tau}) \\ c_F &> c_M, \end{aligned}$$

which is a contradiction to our assumption (ii) $c_M < c_F < c_S$. Hence, it turns out that M always prefers to sell the good directly over collecting pure marketplace profits whenever $c_M < c_F < c_S$.

Proof of Proposition (9). In order to perform the analysis, we define two distinct cases, namely (i) $c_M < c_F$ and, the opposite case, (ii) $c_F < c_M$.

First, we consider (i) $c_M < c_F$. Recall that in this proof, we analyze the parameter space $\min(c_S, c_F) < c_M$. Thus, whenever we have $c_M < c_F$, it is necessary that $c_S < c_M$. Consequently, M decides between operating in the pure marketplace mode or entering

the dual mode earning marketplace profits. It is easily checked that the pure marketplace mode is more lucrative whenever

$$\begin{aligned} \pi_M^{PMP} &> \pi_M^{DS} \\ \tau \frac{c_F}{1-\tau} D\left(\frac{c_F}{1-\tau}\right) &> \tau \frac{c_M}{1-\tau} D\left(\frac{c_M}{1-\tau}\right) \\ \hat{c}_M \equiv c_F \frac{D\left(\frac{c_F}{1-\tau}\right)}{D\left(\frac{c_M}{1-\tau}\right)} &> c_M \end{aligned}$$

On the other hand, whenever $\hat{c}_M < c_M$, entering the dual mode and operating a marketplace is the preferred mode of operation of platform M whenever $c_S < c_M < c_F$.

Next, we consider all cost combinations involving (ii) $c_F < c_M$. Notice that whether M is more or less efficient than the independent seller does not change the equilibrium outcome.

In all cases that comprise $c_F < c_M$, M can not influence prevailing prices by entering the dual mode. It obtains the same profit in the dual marketplace mode as in the pure marketplace mode, independently of the seller's efficiency. According to our tie breaking assumption, in all these cases M operates in the pure marketplace mode.