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Conference Paper

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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2023: Growth and the "sociale Frage"

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Stern, Lennart (2023) : The Optimal Taxation of Air Travel under Monopolistic Dynamic Pricing, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2023: Growth and the "sociale Frage", ZBW - Leibniz Information Centre for Economics, Kiel, Hamburg

This Version is available at:

<https://hdl.handle.net/10419/277667>

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The optimal taxation of air travel under monopolistic dynamic pricing

Lennart Stern*

February 28, 2023

The latest version can be found [here](#).

Abstract

This article identifies a novel double dividend that countries would reap in the air travel sector from replacing ticket taxes with carbon taxes. I do so by showing that taxing vacant rather than occupied seats generates welfare gains through fuller planes, thereby calling into question previous Walrasian analysis that suggests a role for ticket taxes at the optimal tax system alongside carbon taxes. These results are obtained by revisiting the model from Gallego and van Ryzin (1994), in which a monopolistic airline chooses its dynamic pricing policy to sell tickets to randomly arriving consumers over a finite time horizon until the plane departs. In general, the profit maximizing policy differs from the welfare maximizing policy. However, for a certain class of demand functions that includes constant elasticity and exponential demand functions, a simple policy instrument, namely a tax on vacant seats, is sufficient to perfectly align profit maximization incentives with welfare maximization. Calibrating the model to predict a load factor of 80% (the current global average), the welfare maximizing tax on vacant seats leads to load factors of 97% for the constant elasticity demand function and 98% for the exponential demand function. These results suggest that club mechanisms for financing global public good institutions via aviation taxes will create stronger participation incentives if they do not constrain countries to use ticket taxes but instead allow them to use emission taxes and even taxes on vacant seats.

1 Introduction

In 2019, 19% of available seat kilometers on passenger flights remained vacant in the world. This figure rose to 45% in 2020 due to the coronavirus and stayed high at 32% in 2021. If the vacant seats were taxed, then airlines would adjust their dynamic pricing policies so as to fill more seats. Using a second instrument such as a tax/subsidy on occupied seats, one could achieve that the available seat kilometers would be unchanged but the planes would be filled with more passengers.¹ Would such a reform increase welfare?

This article studies this question in the monopolistic dynamic pricing model introduced by Gallego and van Ryzin (1994). In the model, an airline sells tickets for a single flight over a finite time horizon to consumers that arrive according to a Poisson point process. At each point in time, the airline posts a price given how much time and how many vacant seats remain. When a consumer appears, her valuation for the flight is randomly drawn from a time-invariant distribution D , independently of the other consumers. If her valuation exceeds the price posted by the airline, she purchases the ticket. Otherwise, she does not purchase it and disappears.

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For helpful suggestions and discussions, I thank Adrien Fabre, Thomas Douenne, Pierre Fleckinger, David Martimort, Jean-Christophe Mourrat, Jean-Christophe Poudou, Jérôme Pouyet, Tobias Rachidi and participants at the EEP Public Economics conference, the ITEA conference and the IIPF conference.

¹To see that this claim is plausible, consider the following algorithm: Suppose the status quo load factor (i.e. proportion of occupied seats) is Z and we aim to raise the load factor to Z^* whilst leaving the passenger kilometers unchanged. Start by introducing a small tax on vacant seats. This will increase the average load factor. It might also change the number of passenger kilometers. Now adjust the tax/subsidy on all seats so as to achieve that the passenger kilometers is like under the status quo. This adjustment might also affect the average load factor. However, presumably the load factor will still be higher than under the status quo.

In this model, the airline's problem is to maximize its expected profit. Its optimal pricing policy is Markovian in the sense that it conditions the posted price only on the number of remaining seats and the time remaining until departure.

The airline's optimal Markovian pricing policy depends on the tax and subsidy policy chosen by the government. In most of this paper I focus on the welfare implications of a particularly simple tax policy, namely a tax on vacant seats.

An increase in the tax on vacant seats causes the profit maximizing airline to lower its Markovian pricing policy everywhere (shown in proposition 7): In other words: For each remaining time and remaining number of vacant seats, the airline will set a lower price. For welfare, this has two effects: On the one hand, it will lead to additional passengers being taken on board, which yields welfare gains, as long as ticket prices are strictly positive. On the other hand, if it turns out that many consumers with high valuations appear later, some of them will end up not taking the flight. This constitutes foregone welfare.

In proposition 5 I show that for certain thick-tailed distributions of valuations the second effect dominates the first. However, for the distributions of valuations that are most commonly used in the literature, which correspond to constant elasticity and exponential demand functions, the first effect always dominates the second. In fact, for a class of demand functions that includes constant elasticity and exponential demand functions, I show with proposition 3 that a tax on vacant seats with a certain strictly positive rate achieves that the incentives of the profit maximizing airline will be perfectly aligned with the objective of social welfare maximization. In particular, under this optimal tax on vacant seats the profit-maximizing airline will choose the welfare maximizing pricing policy.

These results rationalize the UK's attempt in 2010 to replace its passenger duty with a per-plane tax. The UK government seems to have been convinced that a per-plane-tax could increase social welfare by increasing load factors (see Dresner (2010)). However, the UK abandoned the reform, recognizing that only distance-based air passenger taxes like their passenger duty are unambiguously legal under international law (see Larsson et al. (2019)).

In particular, the legality of pricing the carbon emissions of international flights remains contested. The EU included international aviation in its emissions trading scheme in 2012. However, other countries strongly opposed this measure which they considered to be illegal under international law. In the face a looming risk of retaliatory measures, the EU exempted international flights to and from non-EU states from November 2012 onward.

By default, the EU will again include international flights in its emissions trading scheme from 2024 onward (Larsson et al 2019). This could again lead to tensions as in 2012. To reduce the risk of opposition by other countries, the EU countries could use distance-based air passenger taxes², since these are legal under international law (Larsson et al 2019). However, an alternative approach could be for the EU to initiate a club mechanism that would require participating countries to tax aviation emission on all outgoing flights and also on all flights coming in from countries that do not participate in the club. The membership rules in the club would require countries to allocate a certain fraction of the tax revenue thus raised to Global Public Good Institutions (GPGIs) (see Stern (2019) for a comparative analysis of this and two alternative such proposals). The larger the proportion of the tax revenue that would be raised for GPGIs, the more beneficial the mechanisms would be for those who do not participate. If a sufficiently large proportion of the tax revenue was raised for GPGIs, then other countries would be unlikely to oppose the mechanism.

The results obtained in the current study weigh in favor of the use of emissions taxes instead of distance based air passenger duties for such a club mechanism. In fact, replacing a distance based air passenger duty with an emissions tax resembles the removal of a subsidy on vacant seats. Proposition 3 suggests that this would, under the assumptions identified in the proposition, increase social welfare for the countries to whose flights the measure is applied³. In fact, by further adding a tax on vacant seats such as a tax that is both proportional to emissions and to the proportion of seat kilometers left vacant, further welfare gains would be realized according to proposition 3. Thus a club mechanism allowing countries to use emissions taxes and even such taxes on vacant seats would increase the incentives for countries to participate.

²Taxes at the European level require unanimity, whilst for emissions trading scheme a simple majority suffices. Thus it would be more likely that the enthusiastic EU countries would individually set these taxes instead of hoping for consensus at the EU level.

³Proposition 3 shows that an appropriately chosen positive tax on vacant seats aligns profit maximization incentives with welfare maximization. I conjecture that social welfare is a single-peaked function in the tax on vacant seats.

The paper is organized as follows. Section 2 reviews the literature that this paper builds on. Section 3 defines the model and reproduces a standard heuristic derivation of the Bellman equation. Section 4 derives the main result about how a tax on vacant seats can for a certain family of demand functions perfectly align profit maximization incentives with welfare maximization. Section 5 gives the results from the numerical calibration. Section 6 explains a result that highlights that complementary regulations are potentially required to ensure that the introduction of a tax on vacant seats robustly increases social welfare. Section 7 discusses the implications of the results for the optimal design of international agreements. Section 8 concludes. Most proofs are relegated to the appendix.

2 Related Literature

Keen and Strand (2007,2012,2013). use Walrasian equilibrium analysis to study taxes on aviation fuel, sales taxes on tickets and trip taxes (i.e. passenger charges that depend on distance but not the ticket price). The Walrasian model abstracts away from dynamic pricing and endogenous load factors. As a result, ticket taxes have a role to play at the optimal tax system: In a Ramsey model, ticket taxes are useful for helping to raise revenue (Sandmo (1975)). In a model à la Atkinson & Stiglitz (1974), ticket taxes should be set at rates equal to the tax rates on other final goods. In either case, emission or, similarly, fuel taxes should be set to equal the external cost.

The results of the current paper call these results about the role of ticket taxes into question. This is because replacing ticket taxes by emission taxes will raise load factors and this, according to the model presented here, will increase social surplus, even in the absence of any externalities.

The current paper therefore aims to complement the Walrasian analyses by explicitly taking into account that the choice of taxes will affect load factors. To focus on this load factor dimension, I assume in the model that there is a single flight with an exogenous aircraft size. I focus particularly on the welfare effects of the introduction of a tax on vacant seats, or, equivalently in the model, the replacement of ticket taxes by emission taxes.

In the long term, such tax reforms will of course affect other dimensions chosen by the airlines such as the aircraft size. However, other tax instruments can be used to achieve any desired outcome under other dimensions. For example, a per-plane tax (subsidy) can be used to incentivize the use of larger (smaller) planes. Larger planes have lower costs per seat but, at given seat kilometers, decrease the flight frequency, thereby lowering convenience for travelers. Endogenizing the choice of aircraft size in a model with dynamic pricing like the one presented in the current paper seems computationally very difficult. Existing literature on the question such as Brueckner & Zhang (2010) and Zhou & Hansen (2012) study simpler models.

The task of finding the optimal tax system for aviation can roughly be decomposed into at least three dimensions: Firstly, using taxes to influence the type of aircraft that is used. Secondly, using taxes to influence the set of flights that are supplied. Thirdly, using taxes to influence for a given set of flights how the tickets are sold and in particular how many of them are sold. The Walrasian equilibrium analysis as done by Keen and Strand (2007,2012,2013). can inform the second dimension based on an estimate of the social cost of carbon. It is on the third dimension that I focus the attention in this paper.

For this, I use the model from Gallego and van Ryzin (1994), where a monopolistic airline sells tickets over a finite time horizon to consumers that arrive following a Poisson process. The airline chooses an entire Markovian pricing policy, specifying the price to set conditional on the number of remaining seats and time until departure. Taxes can influence the entire Markovian pricing policy. McAfee and te Velde (2006) apply this model to conduct welfare analysis for the case of an exponential distribution of consumer valuations. They do so by comparing the Bellman equation for the profit maximization problem and the welfare maximization problems. They find: “Thus the efficient solution is the solution a monopoly whose costs are reduced by the static monopoly profit would choose.” From this it follows immediately in the model that a tax on vacant seats is a sufficient policy instrument for inducing the profit maximizing airline to choose the welfare maximizing pricing policy. Proposition 3 generalizes this result to a large class of demand functions that includes the exponential functions and the constant elasticity functions.

McAfee and te Velde (2008) consider the case of the model where the consumers’ random valuations are distributed according to a constant elasticity function. Their paper studies exclusively the case without any taxes. They compute closed form solutions for the profit maximizing pricing policy and for the welfare maximizing policy and find that the two coincide.

However, I show (see proposition 5) that their version of the model, which assumes that the marginal cost of filling a seat is 0, predicts that the vacancy rate will be 0% with probability 1. This makes it intuitive that McAfee and te Velde (2008) find the same result as Stiglitz (1976) who established under constant price elasticity of demand that a profit-maximizing monopolist extracting a finite resource over time chooses a welfare maximizing extraction, or equivalently, pricing path.

However, once one allows for a strictly positive marginal cost of filling a seat, the analogy with the resource extraction problem with deterministic demand breaks down: At the optimal pricing policy, the expected vacancy rate is strictly positive.

I calibrate the model so as to reproduce in the absence of any taxes both an observed vacancy rate of 19% (the global average in 2019) and the observed marginal cost of filling a seat. I find that a tax on vacant seats that lowers the vacancy rate to 3% is required to achieve maximal social welfare (see section 5.2).

3 The model

The model that I use was introduced by Gallego and van Ryzin (1994). An airline considers a given flight with N seats in isolation and sells tickets over a sales horizon $[0, T]$ for a flight. For each small time interval of duration dt there is a probability λdt that a potential passenger will appear and consider buying a ticket. The passenger’s valuation v is defined to be the maximal price that she is willing to pay for the ticket.

Consumers arrive according to a homogeneous Poisson process. Each time a consumer arrives, the person’s valuation v is randomly determined. The probability that the person has a valuation of at least y is denoted by $D(y)$. If a person’s valuation is at least as high as the ticket price p , then she buys the ticket. Otherwise she does not buy it and disappears. Hence the probability that a potential passenger buys a ticket if faced with the price p is given by $D(p)$. Given this, I will call D the “demand function”. The cost caused by the sale of an additional ticket is given by a constant c . This cost consists partly of the increase in fuel consumption due to an additional person on board, which turns out to be around 10 percent of the fixed fuel cost per seat (Borenstein and Rose (2014)).

As a government policy instrument I consider a subsidy of $\varphi(p)$ that is given to the airline if a ticket is sold at the price p . A case of special interest will be where $\varphi(p)$ does not depend on p . By the following Lemma, in the model this creates the same incentives for the airline as a tax on vacant seats with the same rate.

Lemma 1. *Given the assumption of a fixed set of flights (a single one in our model), a tax on vacant seats induces the (profit maximizing) airline to choose the same dynamic pricing policy as a constant subsidy (of the same rate) for tickets sold.*

Proof. Consider a constant subsidy for tickets sold, i.e. a $\varphi(p) = s$, where s is a constant. Since we are assuming away any time discounting over the sales horizon, the airline does not care about when exactly it receives subsidy payments. Hence paying the airline s each time that it sells a ticket is equivalent to just waiting until the end of the sales horizon, i.e. until time T , and then paying the airline $s(N - n(T))$, where $n(T)$ denotes the number of seats remaining empty at the end of the sales horizon.

But since we are assuming that the aircraft size is exogenous, the airline’s policy does not affect N . Hence paying the airline $s(N - n(T))$ creates the same incentives as paying it $-sn(T)$, i.e. charging it a tax of s for each seat that remains empty. \square

The airline maximizes profits, without discounting the revenue it receives⁴. The airline’s profit maximization problem reduces to setting a pricing policy $p(n, t)$, which specifies the price posted at time t if n tickets remain at that time.

Let $v_n(t)$ be the expected net revenue from ticket sales (that is the money paid by passengers plus the subsidy minus the cost due to additional passengers) from time t until the end of the sales horizon at time T , given that n tickets are left at time t and that the optimal pricing policy is pursued from that time onward.

The probability that a passenger arrives in a small time interval of length dt is given by λdt ⁵. If the price is set at p then this person will with probability $D(p)$ buy a ticket. In that case the net revenue from that

⁴Neglecting discounting is justified since the sales horizon is less than a year in practice

⁵Here I am assuming that the Poisson arrival rate is constant over time. In the more general case, one can define the “effective time” as a function of the real time such that the arrival rate with respect to this new time is constant. Thereby, one can reduce the model to the case with a constant Poisson arrival date that I am studying here.

sale is given by $p - c + \varphi(p)$, because the airline receives the subsidy $\varphi(p)$ for the ticket and the cost c is incurred for every additional passenger on board. Moreover, in this case there are $n - 1$ tickets left to be sold from time $t + dt$ onward. With probability $(1 - D(p))$ the person does not buy a ticket and in that case there are n tickets left at time $t + dt$. Hence we have:

$$v_n(t) = \lambda dt (\max_p D(p) (p - c + \varphi(p) + v_{n-1}(t + dt)) + (1 - D(p)) v_n(t + dt)) + (1 - \lambda dt) v_n(t + dt)$$

Rearranging gives:

$$v_n(t) - v_n(t + dt) = \lambda dt \max_p D(p) (p - c + \varphi(p) - (v_n(t + dt) - v_{n-1}(t + dt)))$$

Dividing by dt yields:

$$\frac{v_n(t) - v_n(t + dt)}{dt} = \lambda \max_p D(p) (p - c + \varphi(p) - (v_n(t + dt) - v_{n-1}(t + dt)))$$

Taking the limit as dt goes to 0 yields:

$$-v'_n(t) = \lambda \max_p D(p) (p - c + \varphi(p) - (v_n(t) - v_{n-1}(t)))$$

This equation shows the trade-off that the airline faces at each point in time: Selling a ticket generates net revenue of $p - c + \varphi(p)$. On the other hand, there is a loss in the option value, $v_n(t) - v_{n-1}(t)$, that comes from having one less available seat thereafter.

4 Welfare analysis

Now consider the problem of choosing the prices p over time so as to maximize expected welfare. We obtain the welfare as the sum over all the people taking the flight of their valuation minus the marginal cost c that arises due to each additional person that is taken on board. Let $w_n(t)$ be the expected welfare accruing from time t until the end of the sales period at time T from the sale of tickets during that time interval $[t, T]$. Given that the price is p , the expected amount by which the valuation of a passenger who will buy a ticket at that price exceeds the price is given by:

$$\frac{1}{D(p)} \int_{s=p}^{\infty} D(s) ds$$

Similarly to the profit maximization problem we obtain:

$$w_n(t) = dt \lambda \max_p D(p) \left(p - c + \frac{1}{D(p)} \int_{s=p}^{\infty} D(s) ds + w_{n-1}(t + dt) \right) + (1 - dt \lambda) w_n(t + dt)$$

Rearranging yields:

$$w_n(t) - w_n(t + dt) = dt \lambda \max_p D(p) \left(p - c + \frac{1}{D(p)} \int_{s=p}^{\infty} D(s) ds + w_{n-1}(t + dt) - w_n(t + dt) \right)$$

Dividing by dt and letting dt go to 0 yields:

$$-w'_n(t) = \max_p D(p) \left(p - c + \frac{1}{D(p)} \int_{s=p}^{\infty} D(s) ds + w_{n-1}(t) - w_n(t) \right)$$

The boundary conditions are identical to those in the profit maximization problem:

$$w_n(T) = 0$$

$$w_0(t) = 0$$

We write the two Bellman equations to see the analogy:

$$-v'_n(t) = \max_p D(p) (p - c + \varphi(p) - (v_n(t) - v_{n-1}(t)))$$

$$-w'_n(t) = \max_p D(p) \left(p - c + \frac{1}{D(p)} \int_{s=p}^{\infty} D(s) ds - (w_n(t) - w_{n-1}(t)) \right)$$

We observe that if $\varphi = \frac{1}{D(p)} \int_{s=p}^{\infty} D(s) ds$ then the two Bellman equations are identical. We also note that the value function arising from welfare maximization satisfies the same boundary condition as the value function of the profit maximization problem, namely $w_n(T) = 0 \forall n$ and $w_0(t) = 0$. From this we can deduce that if $\varphi = \frac{1}{D(p)} \int_{s=p}^{\infty} D(s) ds$ then the value function arising from welfare maximization is the same as the value function arising from profit maximization and also that the set of pricing policies (typically a singleton) that maximize profit is equal to the set of pricing policies that maximize welfare. In other words: the social optimum can be achieved by setting $\varphi(p)$ equal to the expected consumer surplus of a consumer conditional on the event that her valuation exceeds the price p and then letting the airline pursue profit maximization. The consumer surplus is always positive. Hence this subsidy is positive for all prices.

I have assumed that the distribution of the valuations of the passengers considering at time t to buy a ticket does not change with the time t . Interestingly, if $D(p, t)$ is a function of time, then by setting $\varphi(p, t) = \frac{1}{D(p, t)} \int_{s=p}^{\infty} D(p, t) ds$, the profit maximizing policy is again welfare-maximizing. Let us write these general observations as a proposition:

Proposition 1. *Let $D(p, t)$ be the probability that a customer appearing at time t has a valuation of at least p . If a subsidy of $\varphi(p, t) = \frac{1}{D(p, t)} \int_{s=p}^{\infty} D(s, t) ds$ is given to the airline for each ticket that it sells at time t for the price p then this perfectly aligns the airline's profit maximization with welfare maximization.*

Consider the price-dependent subsidy $\varphi(p, t) = \frac{1}{D(p, t)} \int_{s=p}^{\infty} D(s, t) ds$ that according to proposition 1 can achieve that the profit maximising airline will end up maximizing expected social surplus. $\frac{1}{D(p, t)} \int_{s=p}^{\infty} D(s, t) ds$ is the expected consumer surplus of a customer, conditional on his valuation exceeding the price p . In particular, it is always positive. Now consider the following 3 questions:

- 1) Can a *constant* subsidy for occupied seats only ever increase the expected number of seats filled?
- 2) Can *any positive* price-dependent subsidy only ever increase the expected number of seats that it fills?
- 3) Can *the optimal* price-dependent subsidy only ever increase the expected number of seats that it fills?

The answers turn out to be: yes for 1), no for 2), and no for 3). Consider first question 1. This result is true in a much broader class of models than the one considered in this paper. To see why, denote by ρ the airline's pricing policy. In the setting of this paper, ρ is a Markovian pricing policy. Nor let us denote by $\pi(\rho)$ the airline's expected profit in the absence of any taxes or subsidies. Let $G(\rho)$ denote the expected number of occupied seats on the flight, i.e. the expected number of seats for which a ticket has been sold by the end of the sales horizon. In the presence of a constant subsidy of s per occupied seat, the airline's after-subsidy profit is $\pi(\rho) + sG(\rho)$.

This argument does not carry over to addressing question 2), since for general price-dependent subsidies, the mere *number* of seats filled is no longer a sufficient statistic for the computation of the airline's after-subsidy profit. Consider for instance a price-dependent subsidy that is some fixed number s^* for a ticket sold at a price exceeding some value p^* and 0 otherwise. Suppose p^* is larger than this prices that the airline ends up choosing in the absence of any subsidy. As long as $D(p^*) > 0$, then for sufficiently large values of s^* , it will become optimal for the airline to always set $p = s^*$ or even higher. This gives an example of a price-dependent subsidy that lowers the expected number of seats sold.

It turns out that this can happen even at the *optimal* price-dependent subsidy policy. This result follows from the constructions provided in the proof of proposition 5 in appendix A.5. The proof provided there treats the case where the government is (maybe for reasons of simplicity) restricted to only use constant tax/subsidy on tickets sold. However, it can be readily adapted to the case without this restriction.

Thus if do not make any assumption on the distribution of valuations $D(p)$, we cannot hope for any general results about how the optimal tax policy will affect the number of occupied seats. However, we will now study three classes of distributions where we can obtain clear results.

Example 1. Exponential demand function: $D(p) = e^{-ap}$

This demand function is commonly used in the literature on dynamic pricing in the airline industry (see e.g. McAfee and te Velde (2006)). Assuming this functional form, we compute:

$$\frac{1}{D(p)} \int_{s=p}^{\infty} D(s) ds = \frac{1}{a}$$

We observe that $\frac{1}{a}$ is the static monopoly price, i.e. the price that maximizes $pD(p) = pe^{-ap}$.

From our preceding discussion it follows that if a constant subsidy of $\frac{1}{a}$ for every ticket sold is given then profit maximization is equivalent to welfare maximization. Such a fixed subsidy is equivalent to a tax on the vacant seats at the rate $\frac{1}{a}$. We now state this result:

Corollary 1. *If the valuations are exponentially distributed with $D(p) = e^{-ap}$ then the socially optimal dynamic pricing policy is profit maximizing if a tax on vacant seat kilometers is introduced at a rate that equals the static monopoly price $\frac{1}{a}$.*

To prepare the analysis for constant elasticity demand functions, we now prove:

Lemma 2. *Suppose $\kappa > 0$. Consider any price-dependent subsidy $\varphi(p)$. The profit-maximising pricing policy under $\varphi(p)$ is the same as under the price dependent subsidy $(\kappa - 1)(p - c) + \kappa\varphi(p)$.*

Proof. This Lemma is a direct consequence of the fact that multiplying the objective function by a constant does not alter the optimal value of the choice variable. In our context, the choice variable is a (Markovian) pricing policy.

The airline is faced with the problem

$$\max_{\rho} E\left(\sum_{i \in I(\rho)} p_i(\rho) - c + \varphi(p_i(\rho))\right)$$

Where ρ is the pricing policy, $I(\rho)$ is the set of seats sold and $p_i(\rho)$ is the price at which seat i is sold (both are random variables). c is the marginal cost of filling a seat. Since any maximization problem is preserved under multiplication by a positive constant κ , this maximization problem is equivalent to

$$\max_{\rho} \kappa E\left(\sum_{i \in I(\rho)} p_i(\rho) - c + \varphi(p_i(\rho))\right)$$

Which can be rewritten as

$$\max_{\rho} E\left(\sum_{i \in I(\rho)} p_i(\rho) - c + (\kappa - 1)(p_i(\rho) - c) + \kappa\varphi(p_i(\rho))\right)$$

□

Example 2. Constant demand elasticity: $D(p) = p^{-\varepsilon}$

We compute the consumer surplus, conditional on a valuation exceeding p : $\frac{1}{D(p)} \int_{r=p}^{\infty} D(r) dr = \frac{p}{\varepsilon - 1}$

Proposition 2. *Suppose that valuations are distributed so as to give rise to constant demand elasticity. Then a constant tax on the vacant seats equal to $\frac{1}{\varepsilon}c$ achieves that welfare is maximized.*

Proof. By proposition 1 we know that a price dependent subsidy of $\frac{1}{D(p)} \int_{r=p}^{\infty} D(r) dr = \frac{p}{\varepsilon - 1}$ will induce the profit-maximising airline to maximize welfare. By Lemma 2, we know that for any $\kappa > 0$, the price dependent subsidy $\kappa \frac{p}{\varepsilon - 1} + (\kappa - 1)(p - c)$ will induce the airline to choose the same pricing policy as under the subsidy $\frac{p}{\varepsilon - 1}$.

Now let us choose κ such that the p disappears, i.e. let us pick κ such that $0 = \kappa \frac{1}{\varepsilon - 1} + (\kappa - 1)$, which just means $\kappa = \frac{\varepsilon - 1}{\varepsilon}$. For this value we obtain a subsidy on occupied seats of

$$\left(\frac{\varepsilon - 1}{\varepsilon} - 1\right)(-c) = \left(1 - \frac{\varepsilon - 1}{\varepsilon}\right)c = \frac{1}{\varepsilon}c$$

Equivalently, this is also achieved through a tax on the vacant seats at that same value $\frac{1}{\varepsilon}c$. □

In order for the constant demand elasticity model to be adequate, the elasticity ε has to be greater than 1, since otherwise expected profits could be increased without bounds by simply charging higher and higher prices. Therefore, the preceding proposition 2 implies that the optimal tax rate on vacant seats is smaller than the marginal cost for filling seats.

We can also deduce directly from the proposition 2 that if $c = 0$ then welfare is maximized without any taxes or subsidies. This fact is also proved by McAfee and te Velde (2008). They do this by explicitly finding the Markovian pricing policy for both the profit maximization problem and the welfare-maximization problem and showing that the two policies are identical. In appendix A.3I extend their calculations. This allows me to prove that under their assumption of 0 marginal costs the model predicts an expected vacancy rate of 0. Thus clearly the model with $c = 0$ is not appropriate for assessing whether a tax on vacant seats or other instruments affecting the vacancy rate can improve welfare. In the appendix A.3 I also calibrate the model for this case of constant elasticity.

We have seen above that for both the constant elasticity and the exponential demand functions the tax on vacant seats suffices as an instrument to achieve the equivalence of the profit maximization and the welfare maximization problem. These two classes of demand functions happen to be the ones that are most frequently used in the literature on dynamic pricing. The question arises as to whether there are any other demand functions with this property. The next proposition shows that there are in fact further such demand functions⁶:

Proposition 3. *For the following demand functions the tax on vacant seats is a sufficient instrument to achieve the perfect alignment between profit maximization and welfare maximization:*

demand function	optimal tax on vacant seats
$(a + p)^{-b}$ with $a \geq 0, b > 0$	$\chi = \frac{a+c}{b}$
e^{-gp} , with $g > 0$	$\chi = \frac{1}{g}$
$(\max(a - p, 0))^b$ with $a > c, b > 0$	$\chi = \frac{a-c}{b}$

Proof. see appendix A.1. □

These demand functions cover a large class. Linear demand functions are contained as a special case of the third class $(\max(a - p, 0))^b$. For the numerical simulations, we will focus on the first two specifications, since they are the ones used in the prior literature, the conclusions of which this paper purports to challenge.

To interpret the size of the optimal tax rates, it is helpful to compare them to the static monopoly price in both cases. In the case of the first specification, $D(p) = (a + p)^{-b}$, the static monopoly price is $\frac{a+cb}{b-1}$, so the ratio of the optimal tax rate on vacant seats to the static monopoly price is $\frac{a+c}{a+cb} \frac{(b-1)}{b}$. Since $b > 1$, this is always smaller than 1. It is close to 0 if b is close to 1 or if b is large.

In the case of the second specification, $D(p) = \exp(-ap)$, the static monopoly price is a , so the ratio of the optimal tax rate on vacant seats to the static monopoly price is 1.

Thus the predictions about the optimal tax on vacant seats (relative to the static monopoly prices) is sensitive to the choice of specification. However, it appears that conclusions about what the optimal vacancy rate will be under the optimal tax on vacant seats are more robust. This is suggested by the calibrations I will present in the next section.

5 Numerical calibrations

We have seen that for the class of demand functions identified in proposition 3 the tax on vacant seat is a sufficient instrument to achieve that welfare is maximized. The two classes of demand functions that

⁶The result presented here is partially reminiscent of proposition 1 in Karp & Livernois (1992). They study a monopolistic firm extracting a finite resource and a government that can choose a stock-dependent linear extraction subsidy. The government can always induce the monopolist to maximize social welfare through a (one-dimensional family of) stock dependent subsidy policies. However, these subsidy policies are time consistent if and only if they are stock-independent.

For the case of constant marginal extraction cost, they consider the optimal subsidy policies under full commitment and the Markov Perfect subsidy policies (i.e. the subsidy policies that only condition the (linear) extraction subsidy on the stock and that arise at a subgame perfect equilibrium in a game where the monopolist acts strategically). They show that these two sets overlap if and only if the demand function belongs to the first of the three classes shown in the table of proposition 3 below. Moreover, Karp & Livernois (1992) show that for this class of demand functions, the two sets of subsidy policies coincide.

have been studied in the literature on the welfare economics of aviation taxation, exponential and constant elasticity, are included in this class. It is for these functional forms that we now calibrate the model to compute the optimal tax rate on vacant seats. For these calibrations we assume that at the status quo there are no aviation taxes. Since in reality existing taxes are very low this assumption should not substantially distort the results.⁷

5.1 Exponential distribution: $D(p) = e^{-ap}$

In this case the following result shows that we can directly deduce the optimal load factor (i.e. proportion of occupied seats) from the load factor that is observed in the absence of any tax:

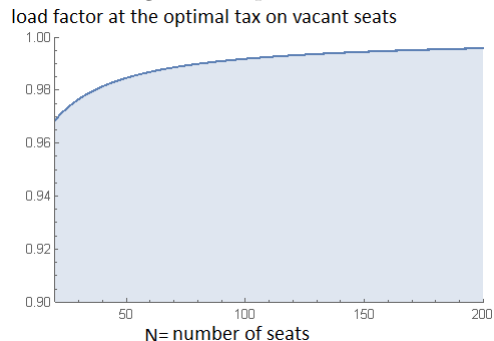
Proposition 4. *Suppose the demand function is exponential, $D(p) = e^{-ap}$. As always, denote by N the total number of seats on the plane. Let Z and Z^* be the expected proportion of seats occupied on the flight without any tax and with the optimal tax on vacant seats, respectively. Z^* can be deduced from Z according*

to the following formula $Z^ = \frac{q(Z)e^{\frac{\sum_{k=0}^{N-1} \frac{(q(Z)e)^k}{k!}}{\sum_{k=0}^N \frac{(q(Z)e)^k}{k!}}}}{N}$, where $q(z)$ is implicitly defined by $Z = \frac{q(Z)\sum_{k=0}^{N-1} \frac{q(Z)^k}{k!}}{\sum_{k=0}^N \frac{q(Z)^k}{k!}}$.*

Proof. see in appendix A.2. □

Results from calibration: Suppose that in the absence of any taxes there is a vacancy rate of 20% (which is approximately the global average according to IATA). Then we obtain the following results:

Figure 1: Optimal load factor if the load factor observed in the absence of taxes is 80%.

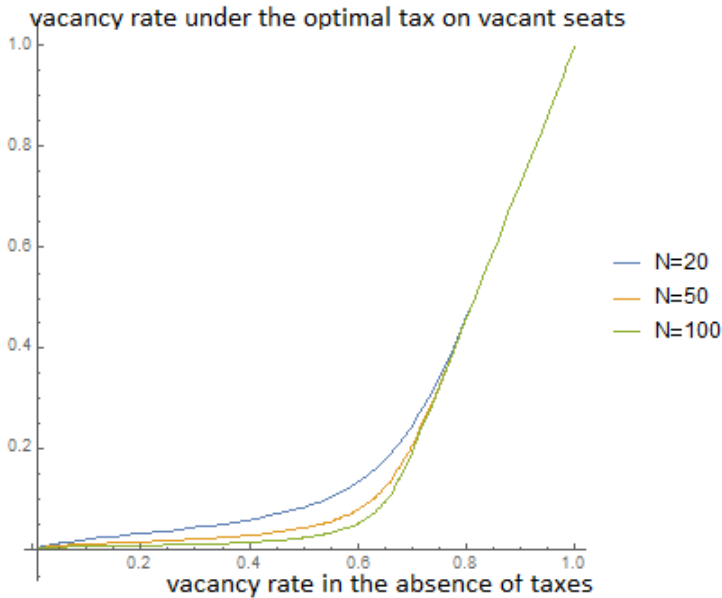


For example, suppose the number of seats is $N = 50$. Then at the optimal tax policy the optimal vacancy rate is 1.5%. In other words: According to the model with the exponential distribution of valuations the government should introduce a tax on vacant seats at a level that reduces the vacancy rate to 1.5%. $N = 50$ is the lower end for commercial passenger planes. For larger planes the optimal vacancy rate is even lower.

Commercial airlines today have between 20 and 900 seats. The following graph is obtained by applying proposition 4:

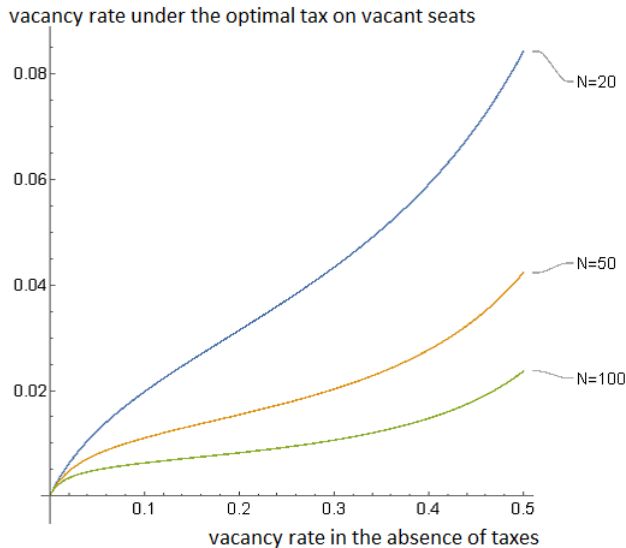
⁷It is clear that proposition 3 still holds if there are taxes on occupied seats in place: In the model, a ticket tax is equivalent to a subsidy on vacant seats. In fact, proposition 3 also still holds in the presence of sales taxes. This is shown in appendix A.4.

Figure 2: Optimal vacancy rate as a function of observed vacancy rate



Zooming into this graph gives:

Figure 3: Optimal vacancy rate as a function of observed vacancy rate



5.2 Constant elasticity distribution: $D(p) = p^{-\epsilon}$

I solved this version of the model numerically and calibrated it as explained in A.3.2.

Results from calibration: Suppose the parameters ϵ and c are chosen so as to produce the observed ratio of marginal costs of filling a seat to average ticket price and the observed vacancy rate (20 percent as global average) assuming the absence of taxes. Then for all realistic values of the number of seats per flight we obtain that at the socially optimal tax policy the vacancy rate is around 3 percent. Social welfare increases by 3.2 percent as a result of the introduction of the optimal tax on vacant seats.

6 Robustness

We have seen that in the absence of specific tax/subsidy policies profit maximization will typically not lead to welfare maximization. Moreover, in the cases where the distribution of valuations belongs to the family identified in proposition 3, we have found that what is required to align profit maximization incentives with the objective of welfare maximization is precisely a tax on vacant seats (or, equivalently in our model, a subsidy on occupied seats or a constant subsidy on ticket sales).

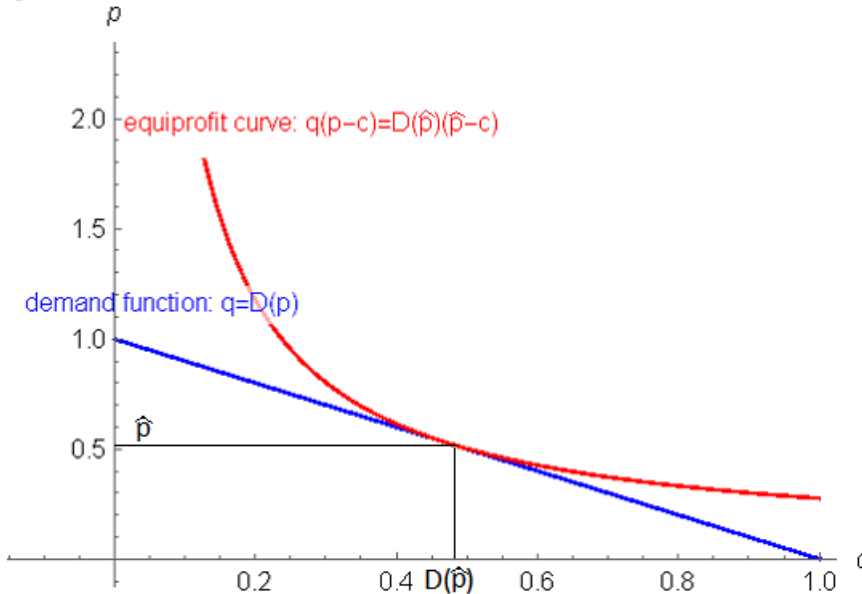
For other distributions of valuations more complicated instruments would be needed, as shown in proposition 1. However, the question arises as to what the optimal tax on vacant seats would be under different distributions of valuations under the restriction of only using this simple instrument of a tax on vacant seats (thus excluding the price dependent subsidy that would generally be required to achieve maximal welfare). A natural question is then whether the optimal tax rate on vacant seats is always positive. It turns out that it can be negative. In fact, we even have in proposition 5 a stronger result. It states that as long as there is a strictly positive probability that all tickets are sold out, one can always rationalize a policy change that consists of lowering the tax on vacant seats. In particular, this proposition implies that one can always rationalize subsidizing vacant seats.

Proposition 5. *Consider a continuously differentiable demand function D and a tax rate α for the tax on the vacant seats. Suppose that under α and D there is a strictly positive probability that all tickets will be sold. Then for any $\alpha' < \alpha$ there exists a demand function $D^\#$ that satisfies the following conditions:*

- 1) *the model with $D^\#$ implies that welfare will be strictly increased if α is lowered to α'*
- 2) *the model with $D^\#$ generates the same predictions as D for both the tax rate α and the tax rate α' . (i.e. the profit maximizing pricing policies are the same for $D^\#$ and D under both α and α').*

Sketch of the proof:

Roughly speaking, in the proof of proposition 5 the demand function $D^\#$ is obtained from D by thickening the upper tail of the distribution of valuations without making it profitable for the airline to increase prices. To illustrate this, consider a discrete one period version of the model with just one seat, where the demand function $D(p)$ now denotes the probability that there is a customer willing to buy the ticket, given that the price is p . Let \hat{p} be an expected-profit maximizing price. The red curve in the following diagram shows the equiprofit curve:



If the demand function was actually the equiprofit curve, $q = \frac{D(\hat{p})(\hat{p}-c)}{p-c}$, then by definition \hat{p} would still be expected-profit maximizing. However, the expected valuation for a consumer conditional on his valuation exceeding \hat{p} would then be infinite, since it is given by $\frac{1}{D(\hat{p})} \int_{p=\hat{p}}^{\infty} \frac{D(\hat{p})(\hat{p}-c)}{p-c} = (\hat{p}-c)[\log(p-c)]_{p=\hat{p}}^{\infty} = \infty$.

Building on this observation, I show in appendix A.5 that also in the full dynamic model that is the object studied in this paper, one can thicken the upper tail further and further such that the expected

valuation of a potential passenger goes to infinity, whilst the predictions of the model are unchanged for given tax rates. It thus becomes an overwhelming priority to make sure that no person is ever refused the possibility to buy a ticket. Subsidizing vacant seats is one way to induce the airline to make sure that such refusals are less likely to happen. \square

The proof of proposition 5 shows that it is possible for the introduction of a tax on vacant seats to reduce social surplus if the distribution of valuations is sufficiently thick so that in expectation a lot of surplus is lost when the flight runs out of tickets and people are refused the possibility to buy one. However, there are other public policy instruments that could be used to reduce the probability of such refusals. For example, a government could enable a secondary market for tickets (see Love (2019)). It could impose by regulation that all airline tickets must be transferable⁸. If people with very high valuations appear, they would then be able to buy tickets from other customers with lower valuations.

One can analyze such a regulation to make all tickets transferable in extensions of the model used above. Consider the following sketch of a version of such an extension: Suppose that once a consumer has appeared and has his valuation realized, this valuation does not change. Thus if given the chance to sell his ticket at a price exceeding his valuation, he will want to do so. Suppose there is no friction in the resale market and all mutually beneficial trades are realized. Typically, the unique socially optimal dynamic pricing policy for the airline to use to sell its tickets would be to always sell tickets at their marginal cost. If and when the airline runs out of tickets, the resale market would ensure that the consumers with the highest valuations would get them.

In such a model the social optimum could be implemented by combining the obligation to make all tickets transferable with a sufficiently high tax on vacant seats and a prohibition to sell tickets below their marginal cost. If introduced without an accompanying tax on vacant seats, the obligation to make all tickets transferable might increase the incentives for airlines to raise their dynamic pricing policy: If the airline sells tickets at a low price then consumers with low valuations might later resell their tickets, thus undermining the price that the airline can charge then. The results from sections 4 and 5 suggest that this could lower welfare, as more seats will be left empty as a result. However, the introduction of sufficiently high tax on vacant seats would counteract this effect. We have no guarantee that the introduction of the obligation to make all tickets transferable will increase welfare (by the arguments just given) nor that the introduction of a tax on vacant seats will increase welfare (by proposition 5). However, in the model just sketched, the joint introduction of these two government policies (combined with a prohibition of sales below marginal cost) allows for the social optimum to be implemented.

A proper analysis will have to take into account that people make decisions with lock-in when they purchase flight tickets: They decide on when to take days off work, book accommodation etc., which are decisions that are potentially costly to reverse. In a richer model taking into account such effects it will no longer be straightforward to find socially optimal sales mechanisms, let alone government regulations that could cause such socially optimal sales mechanisms to be implemented. However, the simplistic analysis just given should at least prevent us from drawing too pessimistic conclusions from proposition 5: This proposition should not lead us to conclude that taxes on vacant seats are not promising instruments for increasing welfare. Instead, proposition 5 should motivate us to pursue the following research question: How can we best design regulations (such as an obligation to make tickets transferable) as complements to a tax on vacant seats in such a way that more seats will be filled whilst still ensuring that people with high valuations will always get a ticket?

7 Implications for the design of mechanisms to fund Global Public Good Institutions via taxes on international aviation

In 2016, the member states of the International Civil Aviation Organization (ICAO) adopted the Carbon Offsetting and Reduction Scheme for International Aviation (CORSIA). 81 States, representing 77% of international aviation activity, intend to voluntarily participate in CORSIA from its outset. Starting in 2021, the participating countries have obliged airlines to offset a part of their emissions by buying certificates from projects in developing countries that have been certified to have reduced emissions.

⁸Currently, only a small fraction of tickets is transferable. \$7 billion worth of tickets remain unused per year.

However, since the depth of this agreement is expected to end up being quite shallow (the carbon price that airlines will face might correspond to only about 0.3% of ticket prices (see Stern (2019) for a discussion based on Warnecke et al. (2019)), the EU is considering taking further unilateral action to achieve higher carbon pricing. In 2016 the EU held a 'consultation on market-based measures to reduce the climate change impact from international aviation'. A common suggestion was to tax all international flights to and from countries that do not have equivalent taxes in place. Moreover, many submissions suggested to allocate a part of the tax revenue to a Global Public Good Institution (GPGI)⁹ for climate change mitigation such as the Green Climate Fund. Based on these suggestions, one can define the following proposal for a club mechanism that the EU could initiate¹⁰. The name MGF Mechanism stands for "Multiple GPGI Funding Mechanism":

Definition 1. The Basic MGF Mechanism

The Basic MGF Mechanism for a given set of recognized Global Public Good Institutions (GPGIs) defines a club through the following two obligations that participating countries have to comply with:

the taxation obligation (version 1): Each participating country is required to levy *an emissions tax at a fixed rate* τ on all outgoing international flights and also on incoming international flights arriving from countries that do not participate.

the allocation obligation: Each participating country must allocate all the tax revenue collected to GPGIs.

The set of recognized GPGIs would be determined and updated regularly through some voting mechanism in which all countries could participate.

Alternatively, the taxation obligation could restrict countries to use ticket taxes:

Definition 2. the taxation obligation (version 2): Each participating country is required to levy a *distance-based ticket tax at a fixed rate* τ *per kilometre* on all outgoing international flights and also on incoming international flights arriving from countries that do not participate.

To assess which of these two versions of the Basic MGF Mechanism countries should best initiate, there are several considerations to be taken into account. Firstly, the version with ticket taxes instead of carbon taxes is less likely to cause opposition by other countries, given that ticket taxes are unambiguously legal under international law. On the other hand, the results from sections 4 and 5 suggest that countries' incentives to participate in the Basic MGF Mechanism would be stronger in versions of this mechanism where countries would be allowed to use carbon taxes, due to the welfare gains through fuller planes. In fact, by this logic, participation incentives could be further improved if countries were allowed to use taxes on vacant seats to fulfil the taxation obligation.

I will now argue that it is of critical importance that countries abstain from unilaterally taxing emissions from international aviation if they keep the revenue for themselves. Doing so could undermine the current norm against such policies and lead to a stable state where the opportunity to establish the Basic MGF Mechanism is lost. In fact, I will argue that it would be beneficial to extend the norm against taxation of international flights with revenue retention to include all kinds of taxes, including the ticket taxes that are currently exempt. One reason for this is given by the results from sections 4 and 5: Switching away from ticket taxes to carbon taxes yields welfare gains through fuller planes, in addition to more efficient emission reductions. A second reason is that such an extended norm could help lock in the Basic MGF Mechanism as a substantial continuous source of funding for GPGIs. For the rest of this section, I will argue in favor of this claim.

It follows directly from the definition of the Basic MGF Mechanism that if countries act individually and all countries participate then no country can gain from withdrawing from the mechanism: The tax burden would be unchanged, given that by the allocation obligation, the other countries would switch to taxing all flights to and from the country that withdraws. Moreover, by participating the country would be able to decide on how to allocate the tax revenue that it collects to GPGIs. Thus the country would strictly gain from participating. Hence there would be a Nash equilibrium with full participation.

⁹A GPGI is defined to be any international institution that uses its available budget to contribute to a particular global public good. See Stern (2020) for a more precise definition and for a list of all existing GPGIs.

¹⁰Stern (2023) proposes two alternative versions of MGF Mechanisms. Simulation results reported there suggest that these alternative mechanisms would greatly outperform the Basic MGF Mechanism that I discuss below. I discuss the Basic MGF Mechanism here for ease of exposition. However, the discussion applies analogously to the alternative MGF Mechanisms from Stern (2019).

This suggests that the following scenario is a plausible possibility:

Scenario 1:

The Basic MGF Mechanism gets initiated by a subset of countries. Some countries oppose it on the grounds that carbon pricing of international flights is not legal under international law. Other countries point out that contrary to past instances like the EU's 2012 carbon pricing of international flights, the participants in the Basic MGF Mechanism do not retain any of the carbon pricing revenue for themselves. Instead, all the carbon pricing revenue is used to fund GPGIs which in the aggregate benefits all countries. Eventually, all countries join the Basic MGF Mechanism.

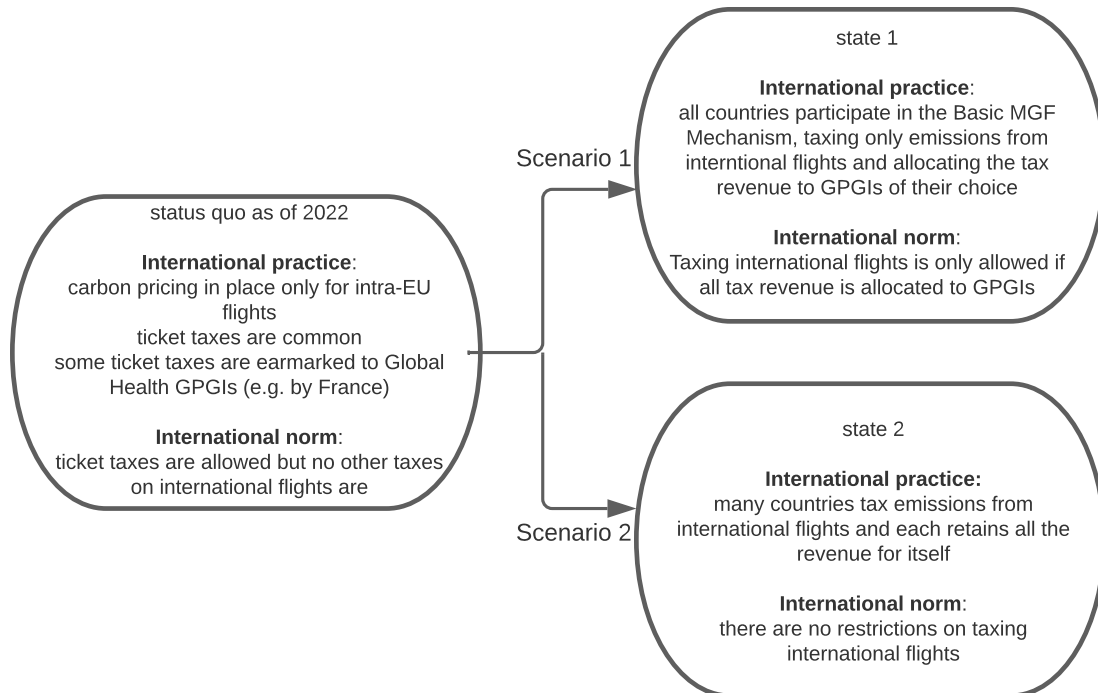
The norm against taxing international flights and retaining the revenue gets strengthened: The current exemption for ticket taxes gets abandoned. Thus all taxes on international flights become illegal unless all the revenue is allocated to GPGIs. All countries with existing ticket taxes on international flights replace them with carbon taxes that they allocated to GPGIs.

Another plausible scenario is the following:

Scenario 2:

The EU reintroduces carbon pricing for all international flights (by extending its emissions trading scheme to include all flights) as it did in 2012. As in 2012, it retains all the carbon pricing revenue for itself and many countries oppose it. Instead of yielding to this opposition as it did in 2012, the EU maintains the policy. After some time, other countries adopt analogous policies to the EU's policies. Like the EU, they exempt flights arriving from countries having imposed at least an equivalent carbon price on these flights (EU directive 2008/101/EC). All countries retain the carbon pricing revenue for themselves. As more and more countries adopt such policies, the incentives for the remaining countries to do so as well increase. Eventually, even the countries that initially opposed the EU adopt analogous policies. The norm against taxing emissions from international aviation gets eroded. This further increases the incentives for countries to continue taxing international flights and retaining the tax revenue.

Scenarios 1 and 2 sketched above are ways in which the world could transition to states 1 and 2, respectively, as described here:



States 1 and 2 would likely be stable and could persist for many decades. Our current status quo (as of 2022), on the other hand, seems unstable. The EU and the UK have revealed through their unilateral initiatives that they would prefer to switch away from the ticket taxes and to use carbon pricing and per plane taxes instead. The results presented in the preceding sections 4 and 5 can help rationalise these policy preferences as being grounded in the welfare gains from fuller planes that these countries would reap, in

addition to the climate change mitigation benefits.

A major force that has prevented countries from switching away from ticket taxes to emission taxes or other taxes is the current international norm according to which only ticket taxes are permitted to be levied unilaterally on international flights. This exemption of ticket taxes is historically contingent and lacks any rationale. In fact, according to the results presented in sections 4 and 5, it runs counter to economic efficiency, even without taking into account climate change externalities.

As countries' concern for reducing emissions increases, it seems likely that the world will move away from the status quo to some other state such as state 1 or 2 shown in the above diagram. How do states 1 and 2 compare in terms of global welfare? Scenario 1 has the advantage of generating a stable flow of funding for Global Public Good Institutions (GPGIs). This benefit will be much greater than the benefit from the emission reductions induced by the aviation taxes (Stern (2023)). The main opportunity for furthering global welfare therefore seems to lie in moving probability mass away from scenarios where the world ends up in state 2 (like the specific scenario 2 sketched above) and towards scenarios where the world ends up in state 1 (like the specific scenario 1 sketched above).

The observation that the main opportunity for furthering global welfare lies in mobilising tax revenue for GPGIs implies the claim, asserted above, that it would be beneficial to extend the current norm against taxes on international flights with revenue retention to include all taxes, including the ticket taxes that are currently exempted. This is the case for two reasons.

Firstly, such an extended norm would likely be more stable than the current norm that arbitrarily distinguishes between ticket taxes on the one hand and all other taxes on the other hand.

Secondly, given the convexity of the cost from taxation, countries' incentives to join the Basic MGF Mechanism would be stronger if they do not have ticket taxes in place. Therefore, the Basic MGF Mechanism could involve a higher tax rate whilst still generating sufficient participation incentives.

Overall, an extended norm that would only allow countries to tax international flights if they allocate the money to GPGIs¹¹ would therefore both increase the expected time during which money gets mobilized for GPGIs and increase the expected size of the flow of this money.

8 Conclusion

For a class of demand functions that includes the constant elasticity and the exponential functions this study has shown in a monopolistic dynamic pricing model that a tax on vacant seats is required to achieve maximal social surplus. The illustrative calibrations suggest that at the optimal tax on vacant seats the load factor might be substantially higher than it is today.

Further work could explore how to use complementary government policies to soften the trade-off mentioned above. In other words: what government policies could ensure that people with high valuations will always be able to get a ticket even in the presence of a large tax on vacant seats? Proposition 5 shows that this is potentially very important. One candidate government policy for ensuring that people with high valuations can always get a ticket could be to oblige all airlines to make their tickets transferable (Love (2019)). This could enable the emergence of a resale market in which people with low valuations or more flexibility would be able to sell tickets to people with high valuations. To adequately analyze such regulations, one would have to model the fact that people make decisions when they plan a trip such as taking days off work and booking accommodation. These decisions are costly to reverse. An interesting mechanisms design question arises from this: What sales mechanism would be socially optimally given that purchase timing will be endogenous? Moreover, which government policies would induce airlines to use socially optimal sales mechanisms?

A useful next step could be to extend the analysis for to oligopolistic models with multiple competing airlines. A natural solution concept for such models could be Markov Perfect Equilibrium, since airlines can estimate competitors remaining vacant seats in real time from seat maps (see e.g. Williams (2018)). An

¹¹This simple norm would be appropriate in the presence of the Basic MGF Mechanism. In the presence of alternative MGF Mechanisms like the ones discussed in Stern (2023), a slightly different norm would be appropriate. In fact, in these mechanisms participating countries would be allowed to retain a certain fraction (e.g. 40%) of their tax revenue in which case the remaining fraction would match the other countries' allocations to GPGIs, thereby increasing increasing the latter's effective influence over the overall allocation. In the presence of such mechanisms, the appropriate norm would be to only allow countries to tax international flights if this is done in accordance to the rules of such mechanisms but not outside of that.

important trade-off that is captured in the monopolistic model of this study would again be present in the oligopolistic models: For social welfare, it is valuable if additional people take flights. On the other hand, leaving more seats empty reduces the expected number of people who will not be able to take the flights even though they have high valuations. The present study suggests that monopolistic airlines will leave too many seats empty. This effect could be called the “load factor distortion”. Importantly, this distortion is distinct from the usual “output level distortion” that occurs as monopolists restrict output below the socially optimal level. In particular, whilst increasing the number of firms will mitigate the standard output distortion, the arguments demonstrating this result do not establish that the “load factor distortion” will disappear with sufficiently many firms. Further modeling work is required to find out how the “load factor distortion” is affected by the number of competing airlines.

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A Appendix

A.1 Proof of proposition 3 for the sufficiency of the tax on vacant seats

In order for profit maximization to be equivalent to welfare maximization under the tax on vacant seats with rate χ it is sufficient that there exists some $\mu > 0$ such that $D(p)$ satisfies on the domain $[0, \sup\{p : D(p) > 0\}]$:

$$\mu(p - c + \chi) = E_t(v|v \geq p) - c$$

This is because maximizing a function is equivalent to maximizing a positive multiple of that function. Rearranging this condition yields:

$$E_t(v - p|v \geq p) = (1 - \mu)c + (\mu - 1)p + \mu\chi$$

We have

$$E_t(v - p|v \geq p) = \frac{\int_{x=p}^{\infty} D(x) dx}{D(p)}$$

Plugging this in yields:

$$\frac{\int_{x=p}^{\infty} D(x) dx}{D(p)} = (1 - \mu)c + (\mu - 1)p + \mu\chi$$

Multiplying through by $D(p)$ and then differentiating yields:

$$-D(p) = ((1 - \mu)c + \mu\chi + (\mu - 1)p) D'(p) + (\mu - 1) D(p)$$

Again rearranging:

$$-((1 - \mu)c + \mu\chi + (\mu - 1)p) D'(p) = \mu D(p)$$

Using separation of variables yields:

$$\frac{dD}{D} = -\frac{\mu}{((1 - \mu)c + \mu\chi + (\mu - 1)p)} dp$$

Now we can distinguish the possible cases:

Case 1: $\mu = 1$

In this case we get:

$$\frac{dD}{D} = -\frac{1}{\chi} dp$$

$$D = B \exp\left(-\frac{1}{\chi} p\right)$$

where B is a positive constant. This is the exponential demand function.

Case 2: $\mu > 1$

$$\frac{dD}{D} = -\frac{\mu}{((1-\mu)c + \mu\chi + (\mu-1)p)} dp$$

$$\log(D) = -\frac{\mu}{(\mu-1)} \log((1-\mu)c + \mu\chi + (\mu-1)p) + const$$

$$D = B((1-\mu)c + \mu\chi + (\mu-1)p)^{-\frac{\mu}{(\mu-1)}}$$

where $B > 0$ is a constant. Denoting $\hat{B} := B(1-\mu)^{-\frac{\mu}{(\mu-1)}} > 0$, we can write this as:

$$D = \hat{B} \left(-c + \frac{\mu}{(\mu-1)}\chi + p\right)^{-\frac{\mu}{(\mu-1)}}$$

The set of pairs (μ, χ) such that $-c + \frac{\mu}{(\mu-1)}\chi = 0$ corresponds to the constant elasticity demand functions. However, we now also see another class of demand functions. Denoting $a = -c + \frac{\mu}{(\mu-1)}\chi$ and $b = \frac{\mu}{(\mu-1)}$, we have:

$$D = \hat{B} (a + p)^{-b}$$

To find the optimal tax rate on the vacant seats in this case we solve:

$$a = -c + b\chi$$

To obtain:

$$\chi = \frac{a + c}{b}$$

If $a > 0$ then $D(p) = \hat{B} (a + p)^{-b}$ defines a continuous decreasing function on the entire domain $p \geq 0$. We note that in this case $\chi > 0$.

If, on the other hand, $a = -c + \frac{\mu}{(\mu-1)}\chi$ were negative, then the expression $D = B(a + p)^{-\frac{\mu}{(\mu-1)}}$ would have a vertical asymptote at $p = -a$. This means in particular that such a function defined by the expression $D(p) = B(a + p)^{-\frac{\mu}{(\mu-1)}}$ on the domain where $p > -a$ cannot be continuously extended to the domain defined by $p \geq 0$.

Case 3: $\mu < 1$

$$\frac{dD}{D} = -\frac{\mu}{((1-\mu)c + \mu\chi + (\mu-1)p)} dp$$

$$\frac{dD}{D} = -\frac{\frac{\mu}{(1-\mu)}}{\left(c + \frac{\mu}{(1-\mu)}\chi - p\right)} dp$$

Let us define $g = \frac{\mu}{(1-\mu)}$

$$\frac{dD}{D} = -\frac{g}{(c + g\chi - p)} dp$$

$$\log(D) = g \log(c + g\chi - p) + const$$

$$D = B(c + g\chi - p)^g$$

Where $B > 0$ is a constant. We see that if $c + g\chi > 0$ then we obtain a demand function that is defined by $D(p) = B(c + g\chi - p)^g$ on the domain $[0, c + g\chi]$ and $D(p) = 0$ for $p > c + g\chi$. Denoting $a = c + g\chi$ implies $\chi = \frac{a-c}{g}$.

This means that for $B > 0, a > 0$ the function that is defined by $D(p) = B(a - p)^g$ on the domain $[0, a]$ and $D(p) = 0$ for $p > a$ is a demand function such that with the tax on the vacant seats with tax rate of $\chi = \frac{a-c}{g}$ the profit maximization problem is transformed into the welfare maximization problem. If $a < c$ then the airline could only sell tickets at a loss. Thus $a \geq c$ is the only relevant case. In that case the optimal tax rate on the vacant seats, $\chi = \frac{a-c}{g}$, is always positive. \square

A.2 Analytical solution for the case of an exponential distribution of valuations

Now let us consider the special case with exponential distribution of valuations:

$$D(p) = e^{-ap}$$

As I showed in proposition 1, the constant tax rate of $\varphi(p) = \frac{1}{a}$ achieves that profit maximization is equivalent to welfare maximization.

We need to solve the model for the two cases: Firstly, we need to solve it without any tax, i.e. with $\varphi(p) = 0$, so that we can calibrate it to predict the current vacancy rate and the current ratio of the marginal cost of taking an additional passenger on board to the average ticket price. Secondly, we need to solve for the case of the tax rate on vacant seats that achieves that profit maximization leads to welfare maximization, $\varphi(p) = \frac{1}{a}$. Let us denote by $c^* = c - \varphi(p)$ for the two cases, which is hence always a constant. In other words, $c^* = c$ in the case without any tax and $c^* = c - \frac{1}{a}$ in the case where the optimal tax rate has been implemented, but we will use the notation c^* to do the computations for the two cases at once.

The general equation

$$-v'_n(t) = \max_p D(p) (p - c^* - (v_n(t) - v_{n-1}(t)))$$

Now becomes

$$-v'_n(t) = \max_p \lambda e^{-ap} (p - c^* - (v_n(t) - v_{n-1}(t)))$$

Gallego and van Ryzin (1994) provide the solution to this system of differential equations, which I list below. It is straightforward to verify that the functions provided below do indeed solve the system of differential equations. An inductive derivation of these solutions is available upon request.

Let us denote $\beta := \lambda \exp(-1 - ac^*)$.

Lemma 3. (Gallego and van Ryzin (1994)) $v_n = \frac{1}{a} \log(\sum_{k=0}^n \frac{\beta^k (T-t)^k}{k!})$

Let us define $B_n(t) := \sum_{k=0}^n \frac{\beta^k (T-t)^k}{k!}$

Corollary 2. (Gallego and van Ryzin (1994)) $p_n^*(t) = \frac{1}{a} \log\left(\frac{\lambda B_n(t)}{\beta B_{n-1}(t)}\right)$

Let $g(t)$ denote the expected number of seats sold at time t .

Corollary 3. (McAfee and te Velde (2006)) $g(t) = \beta t \frac{B_{N-1}(0)}{B_N(0)}$

In section 5 I already stated the following result:

Proposition. 4 As always, denote by N the total number of seats on the plane. Let Z and Z^* be the expected proportion of seats occupied on the flight without any tax and with the optimal tax, respectively. Z^* can be

deduced from Z according to the following formula $Z^* = \frac{q(Z) e^{\sum_{k=0}^{N-1} \frac{(q(Z)e)^k}{k!}}}{\sum_{k=0}^N \frac{(q(Z)e)^k}{k!}}$, where $q(z)$ is implicitly defined

$$\text{by } Z = \frac{q(Z) \sum_{k=0}^{N-1} \frac{q(Z)^k}{k!}}{\sum_{k=0}^N \frac{q(Z)^k}{k!}}$$

proof of proposition 4:
Using corollary 3, we obtain:

$$Z = \frac{\beta T \frac{B_{N-1}(0)}{B_N(0)}}{N}$$

Now using that $B_n(0) := \sum_{k=0}^n \frac{\beta^k T^k}{k!}$ we obtain:

$$Z = \frac{\beta T \frac{\sum_{k=0}^{N-1} \frac{\beta^k T^k}{k!}}{\sum_{k=0}^N \frac{\beta^k T^k}{k!}}}{N}$$

Let $\beta^* = \lambda \exp(-1 - ac^*)$ denote β under the optimal tax on vacant seats. We have:

$$Z^* = \frac{\beta^* T \frac{\sum_{k=0}^{N-1} \frac{\beta^{*k} T^k}{k!}}{\sum_{k=0}^N \frac{\beta^{*k} T^k}{k!}}}{N}$$

By Lemma 7 we know that there is a unique welfare maximizing dynamic pricing policy. We also know by Lemma 9 that the load factor at the welfare maximizing pricing policy, z^* , must strictly increase in T . Moreover $\lim_{T \rightarrow 0} z^* = 0$ and $\lim_{T \rightarrow \infty} z^* = 1$. In particular, this holds for the expected load factor, Z^* . We

have thereby established the following algebraic fact: The function $q \mapsto \frac{\sum_{k=0}^{N-1} \frac{q^k}{k!}}{\sum_{k=0}^N \frac{q^k}{k!}}$ defined on the domain $[0, \infty)$ is strictly increasing and has as its image $[0, 1)$.

This algebraic fact, applied to the analogous expression $Z = \frac{\beta T \frac{\sum_{k=0}^{N-1} \frac{\beta^k T^k}{k!}}{\sum_{k=0}^N \frac{\beta^k T^k}{k!}}}{N}$ implies that the function $q(z)$

is well-defined on $[0, 1)$ via the condition $Z = \frac{q(Z) \frac{\sum_{k=0}^{N-1} \frac{q(Z)^k}{k!}}{\sum_{k=0}^N \frac{q(Z)^k}{k!}}}{N}$. Thus for any load factor from $[0, 1)$ there is a unique value for βT such that the model predicts that load factor.

We have:

$$\beta = \lambda \exp(-1 - ac)$$

Now since by corollary 1 the optimal tax rate on vacant seats is $\frac{1}{a}$, we get:

$$\beta^* = \lambda \exp\left(-1 - a\left(c - \frac{1}{a}\right)\right) = \beta e$$

This implies that $Z^* = \frac{q(Z) e \frac{\sum_{k=0}^{N-1} \frac{(q(Z)e)^k}{k!}}{\sum_{k=0}^N \frac{(q(Z)e)^k}{k!}}}{N}$. \square

A.3 Appendix 2: The case of constant elasticity distributions of valuations

A.3.1 Proofs

In the case of constant elasticity with

$$D(p) = p^{-\epsilon}$$

The general equation

$$-v'_n(t) = \max_p \lambda D(p) (p - c^* - (v_n(t) - v_{n-1}(t)))$$

now becomes:

$$-v'_n(t) = \max_p \lambda p^{-\epsilon} (p - c^* - (v_n(t) - v_{n-1}(t)))$$

Proposition 6. (McAfee and te Velde (2006)) Suppose $c = 0$. Then the value function is given by $v_n(t) = \beta_n (T - t)^x$ where $\beta_0 = 0$ and β_n is given inductively by

$$\beta_n (\beta_n - \beta_{n-1})^{\epsilon-1} = \lambda \frac{\epsilon^{\epsilon+1}}{(\epsilon-1)^{\epsilon-1}}$$

Moreover, the profit maximizing pricing policy is given by $p_n(t) = \beta_n^{-\frac{1}{\epsilon-1}} (\lambda(T-t))^{\frac{1}{\epsilon}}$.

Corollary 4. Let $\eta_m(t)$ denote the probability that there are m vacant seats at time t given that there were N vacant seats at time t . We have $\eta_m(t) = \left(\frac{T-t}{T}\right)^{\beta_N^{\frac{\epsilon}{\epsilon-1}}}$, where β_N is determined as in proposition 6.

Proof. Now let us find the time evolution of the vacancy rate. We have for $m \leq N - 1$:

$$\frac{d}{dt} \eta_m = -\eta_m \lambda D(p_m(t), t) + \eta_{m+1} \lambda D(p_{m+1}(t), t)$$

And for $m = N$:

$$\frac{d}{dt} \eta_N = -\eta_N \lambda D(p_N(t), t)$$

Using the solution from proposition 6, we obtain:

$$\begin{aligned} \frac{d}{dt} \eta_N &= -\eta_N \lambda \left(\beta_N^{-\frac{1}{\epsilon-1}} (\lambda(T-t))^{\frac{1}{\epsilon}} \right)^{-\epsilon} \\ \frac{d}{dt} \eta_N + \eta_N \beta_N^{-\frac{\epsilon}{\epsilon-1}} (T-t)^{-1} &= 0 \end{aligned}$$

We compute the integrating factor:

$$\begin{aligned} \int \beta_n^{\frac{\epsilon}{\epsilon-1}} (T-t)^{-1} dt &= -\beta_n^{\frac{\epsilon}{\epsilon-1}} \log(T-t) \\ \exp\left(\int \beta_n^{\frac{\epsilon}{\epsilon-1}} (T-t)^{-1} dt\right) &= (T-t)^{-\beta_n^{\frac{\epsilon}{\epsilon-1}}} \end{aligned}$$

so we get:

$$\begin{aligned} \frac{d}{dt} \left(\eta_N (T-t)^{-\beta_n^{\frac{\epsilon}{\epsilon-1}}} \right) &= 0 \\ \eta_N (T-t)^{-\beta_n^{\frac{\epsilon}{\epsilon-1}}} &= b \end{aligned}$$

Where b is some constant. But we have the initial condition $\eta_N(0) = 1$, so we have:

$$\eta_N = \left(\frac{T-t}{T} \right)^{\beta_N^{\frac{\epsilon}{\epsilon-1}}}$$

□

We note in particular that $\eta_N(T) = 0$ which is the key observation allowing us to prove:

Corollary 5. With constant demand elasticity and 0 marginal cost per occupied seat the flight sells out completely with probability 1.

Proof. Let $f(t)$ be the random variable that is the number of free seats at time t . Let \tilde{t} be the time of the first ticket sale. Since $\eta_N(T) = 0$ by corollary 4, we know that $\tilde{t} \in [0, T]$ with probability 1.

The proposition is equivalent to the statement that the expected number of vacant seats is 0 at time T . We use the law of iterated expectations:

$$E(f(T)|f(0) = N) = E_{\tilde{t}}(E(f(T)|f(0) = N \text{ and } \tilde{t}))$$

But since we are dealing with a Poisson process we know that

$$E(f(T)|f(0) = N \text{ and } \tilde{t} = t) = E(f(T)|f(t) = N - 1)$$

Denote $h(t, n) := E(f(T)|f(t) = n)$. With this notation, we obtain:

$$h(0, N) = E_{\tilde{t}}(h(\tilde{t}, N - 1))$$

But by definition we have $h(t, 0) = 0 \forall t \in [0, T]$. Hence we deduce that $h(0, n) = 0 \forall n$. \square

A.3.2 Numerical calibration

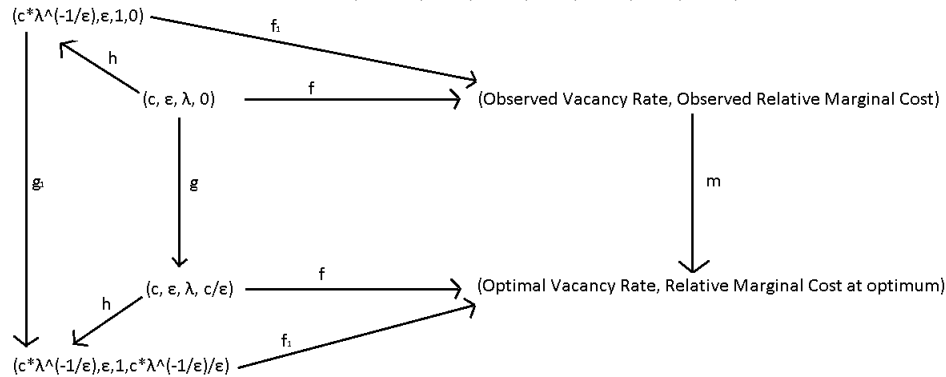
Without loss of generality we can normalize $T = 1$. There are three parameters in the model, namely the marginal cost per occupied seat c , the elasticity ϵ and the intensity λ of the Poisson arrival process. It seems a natural approach to try to calibrate the model using the observed vacancy rate and the relative marginal cost, i.e. the ratio between the additional cost incurred through an additional seat being occupied to the average ticket price paid by passengers. It turns out that for the constant elasticity case it is actually possible to calibrate the model solely with these two numbers, despite the fact that there are three parameters.

Definition 3. Let us denote by f the correspondence that assigns to the quadruple $(c, \epsilon, \lambda, s)$ of marginal cost per occupied seat c , elasticity ϵ , intensity λ of the Poisson arrival process and subsidy for sold tickets s the pairs (V, C) of vacancy rates V and relative marginal costs C (defined as the ratio between the marginal cost per occupied seat and the average ticket price).

Lemma 4. $f(\gamma c, \epsilon, \gamma^\epsilon \lambda, \gamma s) = f(c, \epsilon, \lambda, s)$

Proof. We first note that for any given Markovian pricing policy if we multiply it by some factor γ , multiply the marginal cost c and the subsidy s by that same factor γ and multiply λ by the factor γ^ϵ then the resulting demand processes are isomorphic and, the resulting profit is multiplied by the factor γ . Hence multiplying c and s by any positive factor γ and multiplying λ by the factor γ^ϵ results in analogous optimal pricing policies, simply scaled by γ , yielding identical flows of ticket sales. Hence in particular the resulting vacancy rates are identical. Moreover, since both c and the average ticket price are multiplied by the factor γ , their ratio, i.e. the relative marginal cost, is also unaffected. \square

A solution to the ‘calibration problem’ should be defined to be a map m such that the inner rectangular diagram below commutes for all $(c, \epsilon, \lambda) \in (0, \infty) \times (1, \infty) \times (0, \infty)$.



f maps any combination of the parameter values to the values of the vacancy rate and the relative marginal cost that result

g is the function that introduces the optimal tax rate on vacant seats

h is a scaling function defined by $(c, \epsilon, \lambda, s) \rightarrow (c \cdot \lambda^{-1/\epsilon}, \epsilon, 1, s \cdot \lambda^{-1/\epsilon})$

f_1 is the restriction of f to the set of parameter values such that $\lambda=1$

g_1 is the restriction of g to the set of parameter values such that $\lambda=1$

Lemma 5. *The inner rectangular part of the diagram commutes iff the outer part commutes.*

Proof. The inner diagram commutes iff

$$f^\circ g = m^\circ f$$

By Lemma 4 we have that $f = f_1 \circ h$ so this is equivalent to

$$f_1^\circ h^\circ g = m^\circ f_1^\circ h$$

But algebraically we can see that $g_1^\circ h = h^\circ g$ so this is equivalent to

$$f_1^\circ g_1^\circ h = m^\circ f_1^\circ h$$

Hence we see that if the outer diagram commutes (i.e. if $f_1^\circ g_1 = m^\circ f_1$), then the inner diagram commutes (i.e. $f_1^\circ g_1^\circ h = m^\circ f_1^\circ h$).

Conversely, if the inner diagram commutes then the outer diagram must commute, since it is a restriction of the inner diagram to the set of parameter values such that $\lambda = 1$. (In category language, we could draw another diagram where h is replaced by an inclusion map going into the opposite direction and then we can deduce the result as above through identities of maps.) \square

Lemma 5 implies that the calibration is equivalent to finding a map m such that the outer diagram commutes.

Conjecture 1. *f_1 is injective.*

If conjecture 11 is true then the calibration problem simply amounts to inverting f_1 because once we have found such a function k that satisfies $k^\circ f_1 = \text{identity}$ then we can define $m := f_1^\circ g_1^\circ k$. This task is carried out by a Matlab program downloadable here, which computes m assuming conjecture 11. It is done by approximating the continuous time problem by the discrete time analogue, which we describe there.

In order to carry out the calibration we need an estimate for the relative marginal cost. To get this, we first note that from Gillen et al. (1990) we have to very good approximation:

$$\text{total cost} = \text{fixed cost} + \text{number of people on board} \times c$$

Where c is the marginal cost. Taking the expectation of this equation yields:

$$\text{average total cost} = \text{fixed cost} + \text{average number of people on board} \times c$$

Borenstein and Rose (2014) makes the rough estimate that “80% of costs are assumed to be invariant to changes in the load factor”. This means

$$\frac{\text{fixed cost}}{\text{average number of people on board} \times c} = \frac{0.8}{0.2} = 4$$

$$\text{fixed cost} = 4 \times \text{average number of people on board} \times c$$

So that we get:

$$\text{average total cost} = 5 \times \text{average number of people on board} \times c$$

Let us assume that profits are zero so that average total cost = average revenue. Using this assumption we obtain:

$$\text{average total cost} = \text{average revenue} = \text{average number of people on board} \times \text{average price}$$

Putting these two equations together yields:

$$5 \times \text{average number of people on board} \times c = \text{average number of people on board} \times \text{average price}$$

$5 \times c = \text{average price}$

$$\text{relative marginal cost} := \frac{c}{\text{average price}} = 0.2$$

Globally, the average vacancy rate is around 20% (see IATA). Using this number together with the relative marginal cost estimate of 0.2 we compute the optimal vacancy rate for planes with 50 seats using a backward induction algorithm. It turns out to be 0.0295.

To summarize: Under the assumption of constant elasticity for the distribution of valuations a tax on vacant seats can perfectly align profit maximization with welfare maximization. If the observed vacancy rate in the absence of the tax is 20% then the optimal tax rate on vacant seats will lower the vacancy rate to about 3% .

A.4 Optimal tax rates on vacant seats in the presence of a sales tax

Now we consider the case where there is a sales tax in place¹² that is a fixed proportion of ticket prices.

Lemma 6. *For the monopolistic profit maximization problem the tax policy pair (η, χ) , meaning a sales tax rate of η and a tax on vacant seats of χ , is equivalent to the tax policy pair $(0, \chi - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi)$.*

Proof. Consider the case where there is a sales tax at rate η and a fixed subsidy of χ per occupied seat. The net revenue generated for the airline through a sale of a ticket at price p is given by

$$p(1 - \eta) - c + \chi$$

We can rewrite this as follows:

$$p(1 - \eta) - c + \chi = (1 - \eta) \left(p - c + \chi - \frac{\eta}{(1 - \eta)}c + \frac{\eta}{(1 - \eta)}\chi \right)$$

But since an optimization problem is not changed under multiplication by a positive constant, this creates the same incentive for the airlines as when the net revenue generated through a sale of a ticket at price p is

$$p - c + \chi - \frac{\eta}{(1 - \eta)}c + \frac{\eta}{(1 - \eta)}\chi$$

This is case when there is no sales tax and a subsidy on occupied seats (or equivalently, a tax on vacant seats) of $\chi - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi$. \square

Lemma 6 implies that Lemma 3 still holds in the presence of a sales tax.

We see from Lemma 6 that the effect of a change in the sales tax rate on the vacancy rate depends on the rate of the tax on the vacant seats that we are at. When $\chi > c$, i.e. when the tax on the vacant seats exceeds the marginal cost, then the sales tax increases the incentives for decreasing the vacancy rate.

Corollary 6. *Let $\chi^*(\eta)$ denote the optimal tax rate on vacant seats, given that the sales tax rate is η . Then we have*

$$\chi^*(\eta) = \chi(0) + \eta(c - \chi(0))$$

Proof. Lemma 6 establishes the equivalence of (η, χ) and $(0, \chi - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi)$, so in particular we have

$$\chi(0) = \chi^*(\eta) - \frac{\eta}{(1-\eta)}c + \frac{\eta}{(1-\eta)}\chi^*(\eta) = -\frac{\eta}{(1-\eta)}c + \frac{1}{(1-\eta)}\chi^*(\eta)$$

which implies

$$\chi^*(\eta) = \chi(0) + \eta(c - \chi(0))$$

\square

¹²This is the case for flights between the US, Mexico and in Canada.

Corollary 7. *For the case of constant demand elasticity we have*

$$\chi^*(\eta) = \frac{1}{\varepsilon}c + \eta \frac{\varepsilon - 1}{\varepsilon}c$$

Proof. This follows directly from 2 and 6 since

$$\chi^*(\eta) = \frac{1}{\varepsilon}c + \eta \left(c - \frac{1}{\varepsilon}c \right) = \frac{1}{\varepsilon}c + \eta \frac{\varepsilon - 1}{\varepsilon}c$$

□

Corollary 8. *For the case of exponential demand we have*

$$\chi^*(\eta) = \frac{1}{a} + \eta \left(c - \frac{1}{a} \right)$$

Proof. This follows directly from 1 and 6

□

We see that the optimal tax rate on vacant seats is a linear function of the sales tax rate. However, whereas in the constant elasticity case the optimal tax rate on the vacant seats is an increasing function of the sales tax rate, this relationship holds in the exponential case only if $a < 1$. If $a > 1$, on the other hand, then the optimal tax rate on vacant seats is a decreasing function of the sales tax rate.

A.5 Proof of proposition 5

To prepare the proof of proposition 5, we will here prove some results that are also of some interest in themselves.

Lemma 7. *Suppose that the demand function D is differentiable and $D'(p) > 0 \forall p$. Then there exists a unique pricing policy that maximizes welfare.*

Proof. Consider the Bellman equation:

$$-w'_n(t) = \max_p D(p) \left(p - c + \frac{1}{D(p)} \int_{s=p}^{\infty} D(s) ds - (w_n(t) - w_{n-1}(t)) \right)$$

The first order condition for the optimality of p is:

$$D'(p) (p - c + (w_n(t) - w_{n-1}(t))) + D(p) - D(p) = 0$$

As long as $D'(p) > 0 \forall p$, this has the unique solution given by:

$$p = c - (w_n(t) - w_{n-1}(t))$$

We also have $w_n(T) = 0 \forall n$. Thus by the continuous analogue of backward induction a unique welfare maximizing pricing policy is determined. □

Assumption 1. *Given any tax policy, there exists a unique profit maximizing dynamic pricing policy.*

Justification:

This assumption holds for “regular” demand functions, as defined and discussed in Gallego and van Ryzin (1994). It simplifies the exposition of the results and proofs leading up to proposition 5. From the proof provided below it will become clear that an appropriately reformulated version of proposition 5 still holds if assumption 1 is relaxed. □

Definition 4. A complete history h is specified by the arrivals over time of passengers characterized by their valuation for the flight. For example, one history is specified by saying that a first consumer arrived at time t_1 and had valuation v_1 , a second arrived at time t_2 and had a valuation of v_2 and so on.

Definition 5. Let $f(t, n, \rho, h)$ be the number of seats that will remain empty at departure if at the time t the airline has n seats left and given that the airline chooses from then onwards the pricing policy ρ .

Since we are assuming that the probability measure on the set of histories is such that it gives rise to a Poisson arrival process of potential passengers we can throughout assume that the airline chooses a Markovian pricing policy. We have:

Lemma 8. *Let h be any complete history from time t onward and let ρ be a Markovian pricing policy. Then $f(t, n, \rho, h)$ is increasing in n .*

Proof. Let $m(n, \rho, s, h)$ be number of remaining seats at time s , given that at time t there were n remaining seats.

Let $t^* := \sup\{s : m(n+1, \rho, s, h) > m(n, \rho, s, h)\}$. If $t^* = T$ then $f(t, n+1, \rho, h) = m(n+1, \rho, T, h) > m(n, \rho, T, h) = f(t, n, \rho, h)$. If $t^* < T$ then we have: $m(n+1, \rho, s, h) = m(n, \rho, s, h) \forall t^* \in [s, T]$, so $f(t, n+1, \rho, h) = m(n+1, \rho, T, h) = m(n, \rho, T, h) = f(t, n, \rho, h)$ \square

Corollary 9. *Suppose that the demand function D is differentiable and $D'(p) > 0 \forall p$. Let z^* denote the load factor (i.e. the proportion of occupied seats) at the unique (by Lemma 7) welfare maximizing dynamic pricing policy. Then z^* is a strictly increasing function in T with $\lim_{T \rightarrow 0} z^* = 0$ and $\lim_{T \rightarrow \infty} z^* = 1$.*

Proof. Suppose $\tilde{T} > T$. Let $\tilde{\rho}$ denote the welfare maximizing pricing policy when the time horizon is \tilde{T} and let ρ denote the welfare maximizing pricing policy when the time horizon is T . From time $t = \tilde{T} - T$, $\tilde{\rho}$ must be equivalent to ρ applied to the time horizon remaining then. Hence the result follows by Lemma 8. \square

Definition 6. Let $F(t, n, \rho)$ denote the expected number of seats that will remain empty, given that at time t there are n remaining tickets and given that the airline chooses the complete pricing policy ρ from time t onward.

Lemma 9. *Suppose that $D(p)$ is bounded above. Then $F(t, n, \rho)$ is a strictly increasing function in n for any ρ .*

Proof. From Lemma 8 we deduce that $F(t, n, \rho)$ is increasing. We also know that there is a strictly positive probability of at least $\exp(-\sup_p D(p)T)$ that no ticket is ever sold, which implies that $F(t, n, \rho)$ is strictly increasing in n . \square

Definition 7. Let $p(t, n, \alpha)$ denote the price that the (profit maximizing) airline sets for time t , given that the rate of the tax on vacant seats is α and given that n seats are vacant at the time t .

Proposition 7. ¹³

Suppose $D(p)$ is continuously differentiable with $D'(p) < 0 \forall p$. Then $p(t, n, \alpha)$ is strictly decreasing in α for all n and t .

Proof. In section 4, we assumed that the subsidy on tickets sold are paid to the airline immediately at the time of sale. In the model, a constant subsidy on tickets sold is equivalent to a tax on vacant seats that is paid at time T . It is this latter perspective that will turn out to be the most convenient for the purposes of this proof.

Let $R(t, m, \rho)$ be the expected sales revenue net of marginal costs that the airline will make from period $i+1$ until departure if it employs the complete pricing strategy ρ , given that there are m tickets left at the beginning of period $i+1$. Let $V(t, m, \rho, \alpha)$ be the expected net revenue that the airline will make from time t until departure, given that it employs the pricing strategy ρ and given that there is a tax on vacant seats with rate α . It follows from the definitions that

$$V(t, m, \rho, \alpha) = R(t, m, \rho) - \alpha F(t, m, \rho)$$

¹³This result parallels proposition 3 from Gershkov and Moldovanu (2009), which establishes that under the assumption that $-\frac{F'(y)}{F(y)}$ is increasing, we have: for any time t and any number n of remaining tickets, the welfare maximizing price policy is lower than the profit maximizing price policy. In a sense, “the changes in the pricing policy induced by the tax on vacant seats point in the right direction”. This suggest the conjecture that the optimal tax on vacant seats is always positive if $-\frac{F'(y)}{F(y)}$ is increasing. Proposition 5 does not refute this conjecture, since the demand functions constructed violate the condition that $-\frac{F'(y)}{F(y)}$ be increasing.

Let $v_n(t, \alpha)$ the airline's value function at time t , given that n seats remain then and given that there is a tax on vacant seats with rate α . By definition, we have:

$$v_n(t, \alpha) = \sup_{\rho} V(t, m, \rho, \alpha)$$

Let $\rho(\alpha)$ denote the profit maximizing pricing policy, given the tax rate α on the vacant seats. By the envelope theorem, we have:

$$\frac{d}{d\alpha}(v_n(t, \alpha)) = \frac{d}{d\alpha}V(t, m, \rho(\alpha), \alpha) = -F(t, m, \rho(\alpha))$$

As in section 3 we can derive the Bellman equation: ¹⁴:

$$-v'_n(t) = \lambda \max_p D(p) (p - c - (v_n(t, \alpha) - v_{n-1}(t, \alpha)))$$

Let us denote: $\pi(p, \alpha) := D(p) (p - c - (v_n(t, \alpha) - v_{n-1}(t, \alpha)))$ and $p^*(\alpha) = \operatorname{argmax}_p \pi(p, \alpha)$.

$$\frac{\partial}{\partial \alpha} \left(\frac{\partial \pi}{\partial p} \right) = \frac{\partial}{\partial \alpha} (D'(p) (p - c - (v_n(t, \alpha) - v_{n-1}(t, \alpha))) + D(p))$$

$$= D'(p) (F(t, n, \rho(\alpha)) - F(t, n-1, \rho(\alpha)))$$

where $\rho(\alpha)$ denotes the optimal pricing policy, given the tax on vacant seats with rate α . By Lemma 9, we can deduce that $\frac{\partial}{\partial \alpha} \left(\frac{\partial \pi}{\partial p} \right) < 0$. Now the result follows from the strict monotonicity theorem 1 from Edlin and Shannon (1998). \square

Lemma 10. *Consider the probability distribution $H(\alpha, t)$ over the number of seats remaining empty at time t , given that the tax rate on vacant seats is α and given that the airline chooses the expected profit maximizing dynamic pricing policy. Then if $\alpha' > \alpha$ then $H(\alpha', t)$ can be obtained from $H(\alpha, t)$ by shifting probability weight downward from higher numbers of seats remaining empty to lower ones. Moreover, if $D(p)$ is continuously differentiable with $D'(p) > 0 \forall p$ then the probability weight shifted downwards is strictly positive.*

Proof. We proceed by a sample path argument. Given a history h , we can compare what happens in the case with α' to that with α . We start in both cases with N available seats at time 0. By proposition 7 we know that the price charged at time 0 cannot be higher under the policy α' . Now there are two possibilities: Either in both cases the same sales occur or at some point there is a divergence in the sense that a sale that does not occur under α does occur under α' . From this point onward there exists the possibility that a sale that occurs under α does not occur under α' , since by theorem 1 from Gallego and van Ryzin (1994) the price under α' might be higher. The probability of there ever being more tickets sold under α at any point in time is 0. Hence all that could happen is that we revert back to the case where under both policies the same number of tickets have been sold, so we are back to the case discussed before. This establishes the fact that for each sample path an increase in the tax on vacant seat kilometers decreases the number of seats remaining empty at any point in time. In particular, the probability distribution $H(\alpha)$ over the number of seats remaining empty at time t is obtained from $H(\alpha', t)$ by shifting probability weight downward from higher numbers of seats remaining empty to lower ones. \square

Lemma 11. *The expected number of tickets sold converges to 0 as $\alpha \rightarrow -\infty$.*

Proof. From Lemma 10 it follows that the expected number of tickets sold increases in α . Thus as $\alpha \rightarrow -\infty$, the expected number of tickets sold has to decrease. Suppose it does not converge to 0. Then there is some number $f > 0$ such that the expected number of tickets sold exceeds f for arbitrarily negative values for α . But the revenue that can be extracted from consumers is finite, so this would mean that the airline would for some values α make negative profits at its optimal policy, in contradiction to the fact that not selling any tickets and making 0 profit is an option. \square

Definition 8. Let $z(\alpha)$ be the expected number of potential passengers arriving during the period $[0, T]$ that face a plane that is not entirely filled.

Lemma 12. *$\lim_{\alpha \rightarrow -\infty} z(\alpha) = \lambda T$ and $\lim_{\alpha \rightarrow \infty} z(\alpha) = \lambda T$. Moreover, $z(\alpha)$ is increasing on $[\sup\{\alpha : z(\alpha) > 0\}, \inf\{\alpha : z(\alpha) < \lambda T\}]$ and if $D(p)$ is continuously differentiable with $D'(p) > 0 \forall p$ then $z(\alpha)$ is strictly increasing on $[\sup\{\alpha : z(\alpha) > 0\}, \inf\{\alpha : z(\alpha) < \lambda T\}]$.*

¹⁴Note that $v_n(t, \alpha)$ here includes the fact that at time T the tax on vacant seats will have to be paid. In other words, there is a negative scrap value for the airline. This timing is different in section 3, where the subsidy is paid to the airline when the sales are made rather than at the end of the sales horizon.

Proof. By definition $z(\alpha) = \int_{r=0}^T \lambda \text{Probability}(\text{some seats left at time } r) dr$

It follows from Lemma 10 that *Probability(some seats left at time r)* is increasing in α for every r and strictly increasing if $D(p)$ is continuously differentiable with $D'(p) > 0 \forall p$.

The fact that $\lim_{\alpha \rightarrow -\infty} z(\alpha, 0) = 0$ follows from Lemma 11 □

Now we are finally ready for:

Proof of proposition 5:

Suppose that there is a tax on vacant seats with tax rate α . Suppose that under α and D there is a strictly positive probability that all tickets will be sold. Now consider any $\alpha' < \alpha$. By 12, there will be strictly fewer refusals, i.e. $z(\alpha') > z(\alpha)$

Consider the situation under α' : Let us denote by $p_{n,t}$ the airline's optimal price at time t given that there are n tickets left to be sold. Let as before $V_n(t)$ denote the value function for the airline and $\Delta_{n,t}V := V_n(t) - V_{n-1}(t)$. From theorem 1 from Gallego and van Ryzin (1994) we know that $\Delta_{n,t}V$ is decreasing in n . Also, it is clear that $\Delta_{n,t}V$ is decreasing in t : It can only be valuable to have more time to sell tickets. Thus in particular we have $\Delta_{1,0}V \geq \Delta_{n,t}V \forall n, t$. Also from theorem 1 from Gallego and van Ryzin (1994) we know that $p_{n,t}$ is decreasing in n and t . In particular, we have $p_{1,0} \geq p_{n,t} \forall n, t$.

Fix a small $\delta > 0$. Fix a $\bar{p} > p_{1,0}$ and consider the demand function $D^\#$ defined by $D^\#(p) = D(p)$ for $p \leq p_{1,0}$ and $D^\#(p) := \frac{D(p_{1,0})(p_{1,0}-c+s)}{(p(1+\delta)-\delta p_{1,0}-c+s)}$ for $p \in (p_{1,0}, \bar{p}]$ and $D^\#(p) = 0$ for $p \in (\bar{p}, \infty)$. Then we have:

$$D^\#(p)(p-c+s-\Delta_{n,t}V) = \frac{D(p_{1,0})(p_{1,0}-c+s)}{(p(1+\delta)-\delta p_{1,0}-c+s)}(p-c+s-\Delta_{n,t}V)$$

Hence for $p \geq p_{1,0}$ we have:

$$D^\#(p)(p-c+s-\Delta_{n,t}V) < D(p_{1,0})(p_{1,0}-c+s-\Delta_{n,t}V)$$

which implies that no price $p \geq p_{1,0}$ can ever be chosen at a profit maximizing pricing policy. Hence the profit maximizing pricing policy is unchanged by the modification.

Now let us compute the expected value of the valuation of a consumer:

$$\begin{aligned} E_{D^\#}[v] &= \int_{v=0}^{\infty} D^\#(v) dv = \int_{v=0}^{p_{1,0}} D(v) dv + \int_{v=p_{1,0}}^{\infty} D^\#(v) dv \\ &= \int_{v=0}^{p_{1,0}} D(v) dv + \int_{v=p_{1,0}}^{\bar{p}} \frac{D(p_{1,0})(p_{1,0}-c+s)}{(v(1+\delta)-\delta p_{1,0}-c+s)} dv \\ &= \int_{v=0}^{p_{1,0}} D(v) dv + D(p_{1,0}) \frac{p_{1,0}-c+s}{1+\delta} \log \left(\frac{\bar{p}(1+\delta)-\delta p_{1,0}-c+s}{p_{1,0}-c-s} \right) \end{aligned}$$

We see that this converges to infinity as $\bar{p} \rightarrow \infty$.

Now consider the welfare consequences. Let $w(D^\#, \alpha')$ denote the welfare given that the demand function is $D^\#$ and given that the pricing policy is chosen so as to maximize profits under α' . Since $D^\#$ is constructed so as to lead to the same profit maximizing pricing policy under α' as D , we have:

$$w(D^\#, \alpha') = w(D, \alpha') + z(\alpha')(E_{D^\#}[v] - E_D[v])$$

$$\begin{aligned} w(D^\#, \alpha') - w(D^\#, \alpha) &= w(D, \alpha') + z(\alpha')(E_{D^\#}[v] - E_D[v]) - w(D, \alpha) - z(\alpha)(E_{D^\#}[v] - E_D[v]) \\ &= w(D, \alpha') - w(D, \alpha) + (z(\alpha') - z(\alpha))(E_{D^\#}[v] - E_D[v]) \end{aligned}$$

Since $z(\alpha') > z(\alpha)$ we conclude that $w(D^\#, \alpha') > w(D^\#, \alpha)$ as long as \bar{p} is sufficiently large. □