

Ahorner, Solveig

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The Influence of Physician Rating Websites on the Quality of Primary Care

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The Influence of Physician Rating Websites on the Quality of Primary Care *

(Preliminary. Please do not cite without author's permission.)

Solveig Ahorner[†]

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Abstract

This paper examines the influence of ratings from former patients on the treatment quality chosen by a physician. I analyze a dynamic game in which a primary care physician can choose between a costly high treatment quality level or a free low quality level. Myopic patients with heterogeneous quality preferences can decide on whether to seek help from a physician. The quality level is not observable, and the patients have a belief whether the doctor is competent and able to offer the high quality. Patients rate the doctor on a Physician Rating Website and update their belief after every rating. By investing in high quality, the doctor can increase the patients' belief and thereby the expected demand. A negative rating decreases the patients' belief. In the pure strategy equilibrium, a doctor with a sufficiently high discount factor who is able to offer high quality will invest in providing costly high quality treatment instead of free low quality treatment for the patients if the belief of the patients is high enough and ratings are implemented. I furthermore analyze a mixed strategy equilibrium. If the discount factor is sufficiently high, the competent doctor exerts the high quality with positive probability depending on the belief. In equilibrium, the doctor can receive a finite number of negative ratings before instead exerting the low quality if the belief becomes too small.

Keywords: Asymmetric Information, Bayesian Games, Belief, Dynamic Games, Physician

JEL classification: C73, D83, I11

*The usual disclaimer applies.

[†]Mercator School of Management, University of Duisburg-Essen, Lotharstr. 65, 47057 Duisburg, Germany; solveig.ahorner@rgs-econ.de.

1 Introduction

How can the interaction between physicians and patients benefit from online platforms that allow patients to report their satisfaction with a physician's treatment through a rating system? A Physician Rating Website (PRW) provides the opportunity for a physician's former patients to rate their experience with this doctor and makes the resulting scores accessible for both doctors and patients. Ratings are accumulated into scores and physicians can be ranked by that score. Some examples of PRWs from Germany are [jameda.de](#), [sanego.de](#) or [aerzte.de](#) and [ratemds.com](#) provides ratings from several countries including Canada and the United States. Those platforms allow patients to report on satisfactory factors extending beyond clinical outcomes. And physicians who are offering a high quality of treatment that satisfies a lot of patients can build an attractive reputation. For example, [Ghose, Ipeirotis, and Li \(2014\)](#) show that subjects in an experiment click on top-ranked experience goods significantly more than on lower ranked ones. A relationship of trust is important between primary care physicians and their patients. Physicians are interested in customer-oriented health care management and patient satisfaction. The assessment of patients' experiences is seen as a powerful instrument in this procedure, as explored by [Bidmon et al. \(2020\)](#). The importance of interpersonal skills in an environment of limited time and budget is increasing: Including patients as a responsible and informed party into the process is the current goal in reforming the health care sector.

I analyze the effect of patients rating the treatment by a physician on a Physician Rating Website. In a dynamic setting where one doctor can be visited by different myopic patients, a situation without ratings is compared to a situation with ratings with respect to the quality with which patients are treated by the physician.

First, a basic model without a platform for rating physicians is introduced as a Benchmark in which patients cannot learn whether former patients were satisfied with the treatment they received from the physician. I then develop a dynamic model in which rating of physicians by former patients takes place. A physician can be either a competent or incompetent type doctor and patients form a belief about the physician's type based on the ratings. Depending on said type, the doctor can either strategically choose between offering high or low quality or always offers the low quality. The game is repeated infinitely and each period a patient who decided to visit the doctor informs future agents whether the treatment was satisfying by rating the doctor on a Physician Rating Website. Patients update their belief after every rating.

Patients take into account their belief when making their decision about getting treated. The expected demand of the doctor is dependent on the belief of the patients. By investing in the high quality, the doctor can positively influence the patients' belief and thereby increase the expected demand. On the other hand, the doctor can exploit

a patient for short-term profit by offering the low quality. This will result in a negative rating which negatively impacts the expected demand in future periods.

In the Benchmark model, both types of the doctor always exert the low quality. If the doctor exerts the costly high quality in one or more periods, patients in the following periods do not learn if this was the case. Investing in the high quality therefore has no positive effect on the expected demand. Patients also do not learn if the doctor exerted the low quality, which causes no cost for the doctor, in previous periods. It is therefore always profitable for the doctor to save cost and offer the low quality. Patients anticipate the doctor's strategy and never visit the doctor.

In the model with ratings, depending on the doctor's discount factor and if the belief of the patients is high enough, the competent doctor chooses to exert the high quality in every period. The doctor now faces a trade off between reducing the cost and investing in the belief. Patients rate the doctor after their visit and the belief is updated after every rating. Investing in the high quality increases the belief. This in turn has a positive effect on the expected demand. If the belief of the patients is high enough, the doctor has an incentive to invest in reputation. This constitutes a threshold value for the belief that determines the equilibrium region in which the doctor exerts high quality. Otherwise, the doctor exploits the first patient who visits for the immediate payoff and offers low quality to save cost. After this patient rates the doctor, the belief decreases and patients do not visit the doctor. Furthermore, the rating platform allows for patients to eventually learn the true quality of the doctor. This mechanism is an incentive for the competent type to not be mistaken for the incompetent type doctor. Thereby, the introduction of ratings ensures that there exists an equilibrium region in which the doctor exerts the high quality. The ex ante expected profit of the doctor and the ex ante expected utility of the patients is then higher in the model with ratings than in the Benchmark.

I furthermore analyze an equilibrium in mixed strategies. The competent doctor chooses to play the high quality with a certain probability and the low quality with complementary probability. Investing in the costly high quality with higher probability increases in turn the probability that the belief increases and results in an increased expected demand. If the doctor on the other hand exerts the free low quality with higher probability, this decreases the expected cost. But the resulting bad rating has a negative influence on the belief and thereby decreases the expected demand. In equilibrium, the doctor can exert the low quality for some periods before the belief decreases too much. If the belief is too small, the doctor will exert the low quality in every period. The patients will anticipate that they will be treated with the low quality in every period and no longer visit. The doctor chooses the probability to balance the cost of the high quality and the potential decrease in expected demand after a negative rating. For smaller values of the belief, the doctor exerts the high quality with a higher probability. Depending on the discount factor and the belief of the patients, there exists a finite number of equilibrium

regions in which the doctor exerts the high quality with a different positive probability in dependence of the patients' belief.

2 Related Literature

This paper is related to different strands of the research on reputation, health economics and medical literature.

Applied work finds that Physician Rating Websites play an increasingly important role for both patients and doctors' practices. Empirical work by [Emmert, Sander, and Pisch \(2013\)](#), [Okike et al. \(2016\)](#) and [Emmert, Meier, et al. \(2013\)](#) has found that PRWs become more important for the decisions of potential patients in the process of health care utilization. [Bidmon et al. \(2020\)](#) find that PRWs are also seen as a competitive advantage on the medical caretaker's side of the market. Due to the business model of PRWs, especially them possibly also functioning as advertising platforms or allowing for doctors to influence ranking in exchange for payment, their impact is not seen as unconditionally beneficial for health care utilization decisions according to [Rothenfluh and Schulz \(2018\)](#) and [Mulgund et al. \(2020\)](#). [Xu, Armony, and Ghose \(2021\)](#) and [Chen and Lee \(2021\)](#) show how social media platforms can help with understanding patient choices in health care and that user-generated physician ratings are positively associated with important conventional measures of clinical quality.

The interplay between quality of care and demand for health care providers and its implications for design of optimal health care policies is frequently discussed in theoretical and empirical research. [Gaynor, Ho, and Town \(2015\)](#) provide a literature overview that, among other factors, also focuses on quality determination with market determined prices. [Gravelle and Masiero \(2000\)](#) model general practitioners in a system adapting from the British National Health Service competing via imperfectly observed quality. They find that while more competition can lead to quality improvements, imperfect information negatively impacts quality. The potential impairment on the effect of quality differences on demand changes in information was for example as well studied by [Gravelle and Sivey \(2010\)](#). In [Godager, Iversen, and Ma \(2015\)](#) the interplay of intensified competition in the physician market on physician's behavior is modeled theoretically. The paper furthermore presents empirical results that suggest that competition has negligible or small positive effects. Without the emphasis on reputation, empirical evidence for example by [Dunn, Shapiro, et al. \(2012\)](#) or [Alexander \(2020\)](#) suggests that physicians react to competition and financial incentives.

In an experimental framework, [Reif, Hafner, and Seebauer \(2020\)](#) study the provision behavior of physicians under different payment systems and show that direct financial incentives have an influence on the physician's behavior. [Galizzi and Wiesen \(2018\)](#) present and discuss experimental literature in a comprehensive review of behavioral ex-

periments in health economics. Among other popular research areas, they also review research concerned with the influence of physician's payment or physicians' reputation among their peers have on physician performance, as well as misbehavior of physicians such as upcoding to a more lucrative diagnosis.

Such inefficiencies that may arise from information asymmetries in settings where customers heavily rely on the advice of experts, just as patients have to trust a physician, is more broadly studied in the credence goods literature. Those goods, for which the assessment of the value of credence qualities requires additional costly information even after the purchase, were introduced by [Darby and Karni \(1973\)](#). [Mimra, Rasch, and Waibel \(2016\)](#) experimentally investigate the role of reputation in credence goods markets under different intensities of price competition and different reputation mechanisms. They find that price competition undermines reputation-building and promotes under-treatment of patients. Furthermore they find that the level of overcharging is weakly significantly lower if information on prices, treatment and payoffs is publicly available compared to if patients only know their private history with a physician.

[Szech \(2011\)](#) finds that competition between doctors in quality and prices does not induce high quality treatment. In this theoretical approach, perfectly informed competing doctors choose their treatment quality and prices. Patients rely on anecdotes about a former patient's satisfaction with a doctor's quality and observe prices set by the doctors in the market. Doctors face a trade off between attracting fewer patients with a low quality and facing more intense price competition with high quality.

More generally, the influence of reputation on the quality of a product or service is frequently researched in different empirical and theoretical approaches. [Tadelis \(2016\)](#) gives an overview over research on different reputation and feedback systems for online marketplaces. [Elfenbein, Fisman, and McManus \(2015\)](#) analyze the effect of eBay quality certifications in a setting with fixed prices and public feedback. A comprehensive review of different extensions on a model with reputation formation using the repeated game setting can be found in [Mailath and Samuelson \(2006\)](#). In a model, in which good and bad firms choose their effort level and prices, [Horner \(2002\)](#) analyses the influence of reputations. Consumers observe the loyal consumer base and prices of a firm and form a belief over the types of firms. This model is then extended by allowing both entry and exit of firms. Good firms always exert high effort and consumers quit trading with a firm as soon as they experience a bad outcome. This can force good firms out of the market. [Mailath and Samuelson \(2001\)](#) study the influence of a good reputation on a firm's success and the value firms attribute to a good reputation. Long-lived firms face a trade off between short term profit from costless low effort or higher long term profit that requires costly high effort. Myopic consumers repeatedly purchase from the firm and receive noisy signals on the firm's effort choice. Prices are not fixed but tied to the consumers' expected utility. The key ingredient of their reputation model is the

possibility that a competent firm might be replaced by an inept firm that exploits the good reputation. They find that competent firms try to distinguish themselves from inept firms by exerting high effort. Competent firms buy an average reputation to build it up while inept firms purchase either low or high reputation. [Tadelis \(1999\)](#) analyses a competitive model of agents who differ in their probability of exerting high quality and who are employed by clients. Most agents can unobservably and costlessly change their name to clear a bad reputation or buy a reputable name of another firm that exits the market. The clients cannot determine whether an agent is responsible for that reputation or whether it was bought. Clients change their willingness to pay for the agent's service due to updated beliefs. He finds that both good and bad types of agents will buy reputable names and clients' beliefs are updated downwards after bad performance.

The analysis presented in my paper focuses on an environment in which the cost of a visit to the doctor, that patients have to bear, is covered by a health care insurance similarly to the German health insurance system. Whether or not a patient decides to visit a doctor therefore involves no additional cost. Consequently, prices are set administratively and there is no price competition between competing physicians. Furthermore, every patient who decides to visit the doctor generates the same revenue for the physician. Instead of implementing word of mouth or other forms of anecdotal evidence or noisy signals, information about the experiences of former patients become accessible through ratings that are publicly shared with both patients and the doctor. The platform or Physician Rating Website where those ratings are shared is not modeled as an additional player. Instead it is implemented only indirectly by making the ratings available to the players in the model.

Specifically, a setting with a monopolistic doctor who is able to treat one myopic patient per period is analyzed. Therefore, there also is no excess demand that the doctor may not be able to satisfy. Patients do not specifically face congestion cost. But they are drawn from a distribution over heterogeneous types which provides them with a different utility they receive without treatment from the doctor. The expected utility of the patients from getting treated by the doctor is dependent on their belief. Patients have a prior belief if the doctor is competent or incompetent. They update their belief after every period, taking into account the ratings if they are available. Patients who are unsatisfied with their treatment's quality give a negative rating and satisfied patients leave a positive rating. The belief increases with a positive rating and decreases with a negative rating. The expected demand for the doctor's treatment is depending on the patients' belief. Therefore, by investing in the high treatment quality that induces positive ratings, the doctor can influence the belief positively and increase the expected demand.

The contribution of this paper is in providing an analysis of the effects that ratings by former patients of a physician that are publicly displayed have on the treatment quality

provided by the doctors in a market with fixed prices.

The rest of paper is organized as follows. Section 3 introduces a Benchmark model in which patient do not have access to ratings of a physician’s former patients. In section 4 a model is analyzed in which former patients publicly report if they were satisfied with the treatment they received from a doctor through a rating platform. I analyze equilibria if the doctor plays a pure strategy (section 4) and a mixed strategy (section 5). For the latter case, a simulation of possible equilibrium regions and the resulting dynamics of the probability choice of the doctor induced by the belief updating is performed and evaluated. The two models are compared. Furthermore, the ex ante expected profit of the doctor and ex ante expected utility of the patients from the Benchmark and the rating platform model is compared for both the pure strategy and the mixed strategy case. In section 6, the theoretical results are discussed and compared to empirical findings from the literature on feedback systems and physician ratings in particularly. Sections 7 concludes the results and discusses further research proposals.

3 Benchmark

Consider an infinitely repeated game between one doctor and a myopic patient drawn every period from a distribution of heterogeneous types. The doctor can be either of type H who can choose between exerting a high quality level q_H at cost c or a low quality level q_L at no cost before every period. Or the doctor can be of type L who will always play the low quality level q_L in every period of the repeated game without costs. The type is private information to the doctor and drawn by nature before the beginning of the game. The doctor earns a revenue of w for every patient who decides to get treated, independent of whether the patient is satisfied or not. Period profits are discounted with $\delta \in (0, 1)$. Patients do not know the true type of the doctor, but instead have a belief π about the type of doctor.

A patient is characterized by parameter α , where α is continuous uniformly distributed on the $(0, 1]$ interval, $\alpha \sim U_{(0,1]}$. The parameter α describes the base utility a patient receives without treatment from a physician. Patients are sick by default and need medical treatment by a doctor. They cannot treat themselves. If the physician offers treatment quality q , patients derive utility q if they visit this doctor.

The utility of a representative patient i of type α_i seeking treatment of quality q is given by

$$U_i(\text{treatment}) = q \tag{1}$$

and the utility of a representative patient i of type α_i not seeking treatment is:

$$U_i(\text{no treatment}) = \alpha_i. \quad (2)$$

Utility maximizing patients choose to visit a doctor who offers treatment of quality q if $\alpha_i < q$. Patients who are indifferent between visiting a doctor or not, i.e. $\alpha_i = q$, are assumed to seek treatment from the doctor.¹

A monopolistic doctor who treats patients with quality q gains a revenue w from every patient who gets treated. Furthermore, the doctor has to pay costs c if $q = q_H$. If the doctor offers treatment quality q_L , there are no costs. The utility function of the patients is known to the doctor.

Table 1 summarizes the period utility of a patient of type α when getting treated by the doctor or when deciding against treatment and the period profit of a doctor from exerting q_H or q_L .

patient \ doctor	q_L	q_H
no treatment	$\alpha, 0$	$\alpha, -c$
treatment	q_L, w	$q_H, w - c$

Table 1: Period utility of a patient of type α and period profit of the doctor.

Every period, one patient is drawn from the distribution of types and decides whether or not to visit the doctor while taking into consideration his own type, i.e. his base quality in case of no treatment and the expected quality in case of treatment. Patients and the doctor know the probability distribution of patients' types but not the true type of potential future or former patients. Patients know the probability π_0 with which a doctor of type H is drawn by nature.

If patients believe that the type H doctor is exerting q_L in equilibrium, the expected utility from getting treated is q_L . Consequently, only patients of type $\alpha < q_L$ are visiting the doctor, independent of their belief π . The expected period profit of a monopolistic doctor is given by the difference between the expected revenue from w for every type of patient who is expected to visit and the cost induced by the quality choice. As exerting q_L does not induce cost, in this case the expected profit of the doctor is $q_L \cdot w$. It is not profitable for the doctor to exert q_H instead because the expected profit from deviating to q_H is $q_L w - c$ which is smaller than $q_L \cdot w$.

Suppose patients believe that type H in equilibrium always exerts q_H . The expected utility of visiting the doctor of a patient i of type $\alpha_i \sim U_{(0,1]}$ who believes that the doctor's

¹Note, that the exact appearance of the utility functions of the patients is less important than the demand for treatments induced by it, i.e. that the demand is linearly dependent on the belief. Other forms of the utility function which would yield the same results would be equivalent to the utility function chosen here.

type is H with probability π is then given by

$$\mathbb{E}U_i(\text{treatment}) = \pi q_H + (1 - \pi)q_L \quad (3)$$

and the utility from not visiting the doctor is again given by α_i . Consequently, type $\bar{\alpha}$ who is indifferent between getting treated by the doctor and not getting treated is characterized by

$$\bar{\alpha} = \pi(q_H - q_L) + q_L. \quad (4)$$

The expected period profit of the doctor is again given by the difference between the expected revenue from w for every type of patient $\alpha < \bar{\alpha}$ who is expected to visit when the belief is π and the cost induced by the quality choice:

$$\mathbb{E}\Pi(q) = \begin{cases} (\pi(q_H - q_L) + q_L)w - c, & \text{if } q = q_H \\ (\pi(q_H - q_L) + q_L)w, & \text{if } q = q_L. \end{cases} \quad (5)$$

The expected profit of playing q_H is smaller than the expected profit from playing q_L . Therefore, in equilibrium, the type H doctor will always exert the low quality and patients anticipate this.

Patients can not learn which quality the doctor is exerting. If the game is repeated, patients who were treated can not communicate to future patients, which quality the doctor did exert in previous periods. There is no incentive for the type H doctor to treat patients with the costly high quality. As patients anticipate that it is profitable for the doctor to deviate to the free low quality q_L , only patients who are satisfied with the low quality will be choosing to visit the doctor in equilibrium, independent of belief π . If $q_L = 0$, the doctor is never visited. The results derived above are summarized in the following proposition.

Proposition 1. *The unique equilibrium is a Pooling equilibrium. Both types of the doctor will exert the low quality in every period of the game.*

The results from section 3, specifically from Proposition 1, will from here on be referred to as the Benchmark model or Benchmark setting results.

4 Physician Rating Website

In this section, the influence of a Physician Rating Website is analyzed in an infinitely repeated game between one doctor and a patient drawn from a distribution of heterogeneous types every period who decides whether to visit the doctor. Patients do not know the doctor's true type. They rely on ratings former patients of the doctor left in previous

periods of the game to decide if they are going to visit the doctor and get treated by the doctor. Those ratings display for every former patient who got treated whether this patient was satisfied with the treatment they received or not. If the doctor is not visited in a period, there will be no rating for this period either. There is a monopolistic profit maximizing doctor whose type is either H or L . The doctor gains w for every patient who decides to get treated and potentially has to pay cost depending on the quality of treatment that is exerted. Period profits are discounted with $\delta \in (0, 1)$. A doctor of type L can only offer the smallest quality $q_L = 0$ (at no cost) with which no type of patient is satisfied and type H can choose between quality q_H at cost c and q_L at no cost. If $q_H w \leq c$, the type H doctor will never choose q_H . For the remainder of this paper assume $q_H w > c$. The type of the physician is again drawn at the beginning of the initial period and known only by the doctor. Patients prior belief is given by the probability π_0 with which type H is drawn. Patients' types are again distributed according to the uniform distribution function $U_{(0,1]}$, known to patients and doctor.

Every period, one patient is drawn from the distribution of types and decides whether or not to visit the doctor. Patients take into consideration their own type and the probability with which they believe that the doctor's type is H . Patients and the doctor know the probability distribution of patients' types but not the true type of potential future or former patients. By giving a rating, patients do not reveal the true quality with which they were treated to other patients. Furthermore they do not report by which type of doctor they got treated if they decided to visit. Patients instead report if they were satisfied with the treatment which is dependent on each patient's type. This means patients give a positive rating if their utility from being treated by the doctor was greater than or equal to the utility they would have received in the case of not getting treated. Depending on this rating, the patients' belief on the probability of the doctor's type being H is updated. The prior belief of the patients is given by the probability π_0 with which a doctor of type H is drawn by nature.

A rating in period t is a function of the patient's type and the doctor's exerted quality in period t , $R_t(\alpha_t, q_t) : (0, 1] \times [0, 1] \rightarrow \{-1, 1\}$. It takes the value 1 if the patient visited the doctor and was satisfied with the treatment, or -1 if the patient visited and was not satisfied with the treatment. If the doctor is not visited by a patient in period t , the doctor is not rated.

A patient in period t believes that the type of the doctor is H with probability π_t . The belief $\pi_t(\pi_0, h_t) : [0, 1] \times \{-1, 0, 1\}^t \rightarrow [0, 1]$ of the patient in period $t \geq 1$ is a function of the prior belief π_0 and the history of ratings h_t from all previous periods up until the most recent one. The history of ratings is a vector of all reviews up to period $t - 1$, and the rating R_t of period t , $h_t \in \{-1, 0, 1\}^t$. A value of 0 is assigned in every period when no rating was given.

The belief in period $t \geq 1$ that the doctor's type τ is H , given the history of ratings

h_{t-1} and the most recent rating $R_{t-1} = r$ in period $t - 1$ is given by Bayes' rule

$$\begin{aligned}
\pi_t(\pi_0, h_t) &= \pi_t(\pi_0, (h_{t-1}, r)) \\
&= P(\tau = H \mid R_{t-1} = r, h_{t-1}) \\
&= \frac{P(R_{t-1} = r \mid h_{t-1}, \tau = H)P(\tau = H \mid h_{t-1})}{P(R_{t-1} = r \mid h_{t-1}, \tau = H)P(\tau = H \mid h_{t-1}) + P(R_{t-1} = r \mid h_{t-1}, \tau = L)(1 - P(\tau = H \mid h_{t-1}))} \\
&= \frac{P(R_{t-1} = r \mid h_{t-1}, \tau = H)\pi_{t-1}(\pi_0, h_{t-1})}{P(R_{t-1} = r \mid h_{t-1}, \tau = H)\pi_{t-1}(\pi_0, h_{t-1}) + P(R_{t-1} = r \mid h_{t-1}, \tau = L)(1 - \pi_{t-1}(\pi_0, h_{t-1}))}. \quad (6)
\end{aligned}$$

The belief in period $t = 0$ is given by the prior belief π_0 , i.e. the probability with which nature draws type H . If the doctor does not treat a patient in period t , the belief is not updated, meaning $\pi_t = \pi_{t-1}$.

4.1 Persistent Quality Levels

The type H doctor can choose between exerting q_H at cost c or $q_L = 0$ without cost. Type L always exerts q_L . Patients anticipate the behavior of the different types of the doctor in the Perfect Bayesian equilibrium. There exists a threshold π^* such that if the belief of the patients that the doctor's type is H exceeds this threshold, a doctor of this type will exert the high quality q_H in every period of the game. The utility functions of the patients remain the same as in the Benchmark case in section 3.

The expected utility of being treated by the doctor is given by the utility derived from being treated by either type of the doctor, weighted with the probability that the doctor's type is either one (reflected in the belief). The expected utility of visiting the doctor of a patient i of type $\alpha_i \sim U_{(0,1]}$ who believes that the doctor's type is H with probability π is given by

$$\mathbb{E}U_i(\text{treatment}) = \begin{cases} \pi q_H + (1 - \pi) \cdot 0 & \text{if } \pi \geq \pi^* \\ 0 & \text{if } \pi < \pi^* \end{cases} \quad (7)$$

and the utility from not visiting the doctor is again given by α_i . Patients choose between treatment and no treatment to maximize their expected utility. Consequently, type $\bar{\alpha}$ who is indifferent between getting treated by the doctor and not getting treated is characterized by

$$\bar{\alpha} = \begin{cases} \pi q_H & \text{if } \pi \geq \pi^* \\ 0 & \text{if } \pi < \pi^*. \end{cases} \quad (8)$$

It follows, that the expected utility of a patient i whose type is $\alpha_i \geq q_H \geq \bar{\alpha}$ is always higher if the patient chooses to not get treated by the doctor. Those types will never visit the doctor, as they will not be satisfied with the service of either of the doctor's types. Therefore, in equilibrium, all types α of patients with $\alpha \leq \pi_t q_H$ will choose to visit the

doctor in period t if $\pi_t \geq \pi^*$ and the doctor will not be visited if $\pi_t < \pi^*$.

Patients update their belief that the doctor's type is H at the end of every period according to Bayes' rule as given in (6).

Depending on the patient's decision in period t , the belief that the doctor's type is H , whom patients anticipate to play the equilibrium strategy, evolves differently. Specifically, there are three possible scenarios in equilibrium. In scenario $S_{\pi_{t+1},1}$ the belief was updated after a recommendation by a satisfied patient. In scenario $S_{\pi_{t+1},2}$, the doctor is visited by a patient who is unsatisfied by the offered quality and the patients' belief is updated accordingly. In scenario $S_{\pi_{t+1},3}$, no patient visits the doctor, such that there is no new information to update the belief. In Figure 1, the change from period t to period $t + 1$ is illustrated. The transition probabilities from the current state to next period's state only depend on the current period's state and actions.

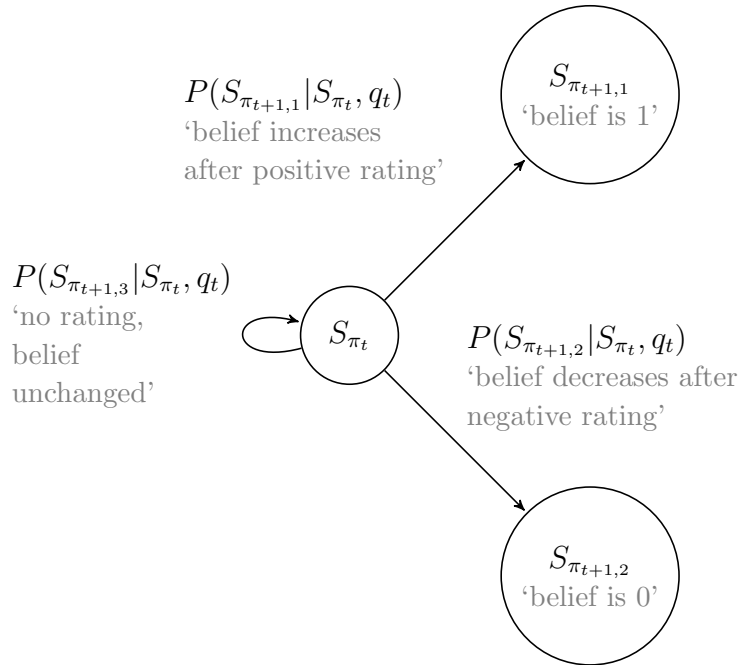


Figure 1: Mapping of the transition from one state to another given state S_{π_t} and action q_t in period t .

Specifically, in equilibrium, the belief $\pi_{t+1,i}$ that the doctor's type is H in scenario $i = 1, 2, 3$ at time $t + 1$ evolves only depending on last period's quality choice and last period's belief π_t according to Bayesian updating:

$$\begin{aligned}\pi_{t+1,1} &= 1 \\ \pi_{t+1,2} &= 0 \\ \pi_{t+1,3} &= \pi_t.\end{aligned}$$

As a patient who visits the doctor can only be satisfied with the treatment if the doctor

exerted the high quality q_H and only a doctor of type H is able to offer this treatment quality, patients know that the doctor's type is H after a satisfied patient leaves a positive rating. If a patient was unsatisfied, patients do also know that the treatment's quality must have been $q_L = 0$. And as the type H doctor's strategy is to play q_H in equilibrium, the unsatisfied patient must have been treated by the type L doctor. If no patient visited the doctor in a period, there is no new information to update the belief. This also implies that if the prior belief π_0 is smaller than π^* no patient will choose to get treated by the doctor in any period of the game, and the belief will not change. The transition probabilities from scenario S_{π_t} to scenario $S_{\pi_{t+1}}$, given that the type H doctor plays q_H , are given by the probabilities of each of the scenarios described above

$$\begin{aligned} P(S_{\pi_{t+1},1}|S_{\pi_t}, q_t) &= \pi_t q_H \\ P(S_{\pi_{t+1},2}|S_{\pi_t}, q_t) &= 0 \\ P(S_{\pi_{t+1},3}|S_{\pi_t}, q_t) &= 1 - \pi_t q_H. \end{aligned}$$

If the doctor would deviate to playing $q_L = 0$, the transition probabilities from scenario S_{π_t} to scenario $S_{\pi_{t+1}}$ are given by the probabilities

$$\begin{aligned} P(S_{\pi_{t+1},1}|S_{\pi_t}, q_t) &= 0 \\ P(S_{\pi_{t+1},2}|S_{\pi_t}, q_t) &= \pi_t q_H \\ P(S_{\pi_{t+1},3}|S_{\pi_t}, q_t) &= 1 - \pi_t q_H. \end{aligned}$$

The expected period profit of the doctor is given analogously to (5). Therefore, the equilibrium value function of a doctor playing q_H in every period, if the patient's belief is $\pi \geq \pi^*$, is given by

$$V^*(\pi) = \pi q_H w - c + \delta (\pi q_H V^*(1) + (1 - \pi q_H) V^*(\pi))$$

which can be simplified to

$$V^*(\pi) = \frac{\pi q_H w - c + \delta \pi q_H V^*(1)}{1 - \delta(1 - \pi q_H)} \quad (9)$$

where

$$V^*(1) = \frac{q_H w - c}{1 - \delta}$$

denotes the equilibrium value function if the belief is 1.

If it was profitable for the type H doctor to deviate from the equilibrium strategy and

play $q_L = 0$ instead, the value function is

$$V(\pi) = \pi q_H w + \delta (\pi q_H V(0) + (1 - \pi q_H) V(\pi))$$

which can be simplified to

$$V(\pi) = \frac{\pi q_H w + \delta \pi q_H V(0)}{1 - \delta(1 - \pi q_H)} \quad (10)$$

where $V(0) = 0$ denotes the value function when the belief is 0. Given these considerations, the following proposition is obtained.

Proposition 2. Denote $\pi^* = \frac{c(1 - \delta)}{\delta q_H (q_H w - c)}$ and $\delta_{min} = \frac{c}{q_H (q_H w - c) + c}$. If $\delta \geq \delta_{min}$, depending on the belief, there are two different types of equilibrium in pure strategies:

- (i) *Separating Equilibrium:* If the prior belief of the patients is $\pi_0 \geq \pi^*$ the type H doctor will exert the high quality in every period in the equilibrium.
- (ii) *Pooling Equilibrium:* If the prior belief $\pi_0 < \pi^*$ both types of the doctor will exert the low quality in every period in the equilibrium.

The formal proof is given in appendix A.

The threshold π^* describes the ratio between the cost of high quality and the discounted expected profit from offering high quality after patients know the doctor's true type weighted with the probability of being visited by a patient. The threshold is higher if cost c is higher. The intuition behind this is that the more costly the high quality is, the higher the belief must be to ensure that the high type doctor is offering the high quality in the equilibrium. The higher the belief, the higher the probability that the type H doctor is visited by a patient. A visiting patient is profitable for the doctor and furthermore patients learn the doctor's true type in this case and increase their belief to 1. If the high quality is more costly, the probability that the investment in the high quality pays off must be higher to ensure that the doctor has an incentive to make the investment.

The belief threshold π^* is decreasing in δ , meaning that the higher the discount factor, the smaller the belief can be for the type H doctor to be incentivized to play the high quality in every period in the equilibrium. Intuitively, a more far-sighted or more patient doctor is easier incentivized to invest in the high quality by the possibility of being recognized as the high type. The belief threshold is smaller the higher the value of q_H . A higher q_H increases the probability of being visited by a patient and therefore it is profitable for the high type doctor to invest in the costly high quality even for smaller values of the patients' prior belief. Similarly, the higher the revenue w from every visiting patient, the smaller the threshold.

4.2 Non-Persistent Quality Levels

Consider now a type H doctor who does not choose between playing q_H or $q_L = 0$ before the start of the game but who plays a Markov strategy $\sigma(h_t, \pi_t)$, such that for any two ex post histories \tilde{h}_t and \tilde{h}_τ of the same length and terminating in the same state (and therefore the same belief value), $\sigma(\tilde{h}_t, \pi_t) = \sigma(\tilde{h}_\tau, \pi_t)$. A type L doctor will again play the low quality in every period of the game. Patients anticipate the doctor's strategy and maximize their expected utility. The expected utility from getting treated by the doctor if patients expect type H to exert $q_t \in \{q_L, q_H\}$ according to strategy $\sigma(h_t, \pi_t)$ is given by

$$U(\text{treatment}) = \begin{cases} \pi_t q_H, & \text{if } q_t = q_H \\ 0, & \text{if } q_t = q_L \end{cases}$$

Therefore, depending on the value of the belief, the value function in some period t of the type H who exerts $q_t \in \{q_L, q_H\}$ according to strategy $\sigma(h_t, \pi_t)$ doctor is given by

$$V(\sigma(h_t, \pi_t)) = \begin{cases} \frac{\pi_t q_H w - c + \delta \pi_t q_H V(\sigma(h_{t+1}, 1))}{1 - \delta + \delta \pi_t q_H}, & \text{if } q_t = q_H \\ 0, & \text{if } q_t = q_L \end{cases}$$

where

$$V(\sigma(h_t, 1)) = \frac{q_H w - c}{1 - \delta}$$

for all ex post histories h_t . As elaborated on below in Proposition 3, if the doctor chooses the treatment quality in every period of the game instead of before the beginning of the game, the equilibrium strategies resemble the ones from Proposition 2.

Proposition 3. Recall $\pi^* = \frac{c(1 - \delta)}{\delta(q_H w - c)}$ and $\delta_{min} = \frac{c}{q_H(q_H w - c) + c}$. If $\delta \geq \delta_{min}$, there are two different types of equilibrium:

- (i) *Separating Equilibrium:* If $\pi \geq \pi^*$, the type H doctor will exert the high quality in every period of the game.
- (ii) *Pooling Equilibrium:* If $\pi < \pi^*$, both types of the doctor will exert the low quality in every period of the game.

The formal proof is given in appendix A.

If type H chooses the quality in every period of the game, the doctor exerts the high quality in every period if the discount factor and the belief of the patients is high enough. The thresholds for the discount factor and the belief are the same as in Proposition 2. The intuition behind the result is that if patients for a given belief expect the doctor to

exert the low quality in equilibrium, no type of patient visits the doctor. Consequently, the belief does not change. The doctor does not receive any period revenue. If patients expect the type H doctor to exert the high quality in equilibrium but, because of exerting q_L , the doctor receives a negative rating, patients update their belief. Afterwards, again no type of patient visits the doctor. Therefore, type H will only deviate from exerting the high quality if the short term profit from deviating to q_L compensates for the decreased belief that results in not being visited anymore in the long term.

These results will from here on be referred to as the PRW setting or rating platform model results.

4.3 Comparison of Benchmark and Physician Rating Website

Below, the Benchmark setting from section 3 (with parameter choice $q_L = 0$) is compared to the rating platform model introduced above. In the Benchmark model, both types of the doctor will always offer the low quality treatment. In the rating platform model on the other hand, the type H doctor will exert the quality q_H in every period if the belief of the patients is greater than or equal to the ratio between period cost and the expected discounted future profit after patients have learned the true type of the doctor weighted with the probability of this event. The higher the discount factor, the more important future income is to the doctor compared to present income. A small discount factor implies that the doctor mostly cares about the current income this period and does not care much about future income in upcoming periods. The higher the discount factor, the more important the expected future profit becomes for the type H doctor. Therefore, investing in costly high quality is attractive. As the belief will be updated in the rating platform model after patients learn the true quality of the doctor, the type H doctor has an incentive to invest in the high quality.

Recall the discount factor threshold $\delta_{\min} = \frac{c}{q_H(q_H w - c) + c}$ and the belief threshold $\pi^* = \frac{c(1 - \delta)}{q_H(q_H w - c)\delta}$ from Proposition 2. The belief threshold π^* depends on the discount factor δ . In the rating platform model, the quality exerted by the doctor in one period does influence the probability with which patients learn the true quality of the doctor and thereby also the belief of the following period. In the Benchmark model, the belief of the patients does not change from one period to another. The experience of a patient in one period does not change the belief of patients in future periods. The following proposition compares the different equilibrium regions in the two models. It can be concluded directly from Proposition 1 and 2.

Proposition 4. *If $\delta \geq \delta_{\min}$, there are two equilibrium regions depending on the belief of the patients and the discount factor of the physician:*

- (i) *Pooling-Pooling (P-P) region : if $\pi_0 < \pi^*$, both types of the doctor play q_L in every*

period in equilibrium in both settings.

- (ii) *Separating-Pooling (S-P) region: if $\pi_0 \geq \pi^*$, the type H doctor plays q_H in every period in equilibrium in the rating platform model and both types of the doctor play q_L in every period in equilibrium in the Benchmark setting.*

The belief threshold $\pi^* = \frac{c(1-\delta)}{\delta q_H(q_H w - c)}$ is decreasing in q_H , w and δ and increasing in c . If $\pi \geq \pi^*$ in the PRW setting, the equilibrium is the Separating Equilibrium where the type H doctor always exerts the high quality q_H in the PRW model, but in the Benchmark model the equilibrium is the Pooling Equilibrium. The economic intuition behind this is that if the doctor cares a lot about future income, the ratings published by the rating platform are an incentive for the type H doctor to invest in a good reputation by exerting the costly high quality. Unlike in the Benchmark model, future potential patients can update their belief about the doctor's type after they learn about the satisfaction of a former patient through the rating platform. As soon as the doctor is visited by the first patient, all future patients know the doctor's type for sure. Therefore it is profitable for the type H doctor to exert the high quality in every period. In the Benchmark model, where patients will always continue to visit with the same probability as their belief does not change throughout the periods, the doctor has no incentive to exert q_H .

Yet, if $\frac{c(1-\delta)}{\delta q_H(q_H w - c)} > \pi_0$, the equilibrium in the Physician Rating Website model is the Pooling Equilibrium where both types of the doctor always play $q_L = 0$. Consequently, for a small prior belief, the Pooling Equilibrium will be the equilibrium in both settings.

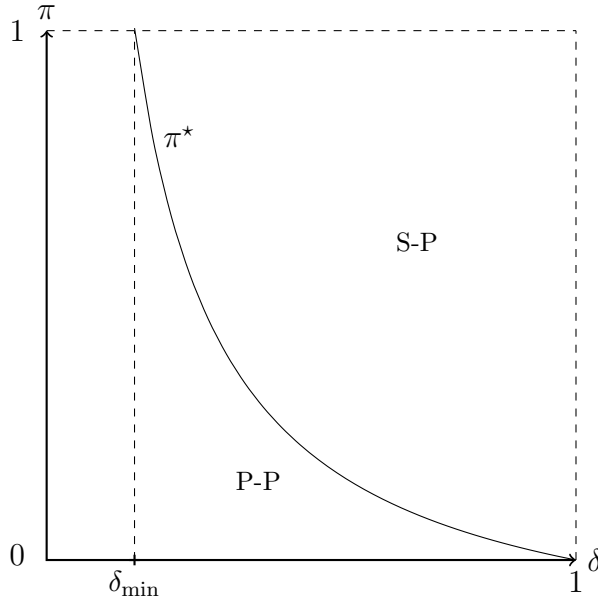


Figure 2: Comparison of Equilibrium Regions of Benchmark and Rating Platform Setting.

Figure 2 illustrates the different equilibrium regions. In the Pooling-Pooling region,

i.e. if $\pi < \pi^*$, both types of the doctor will play q_L in every period of the game in both the Physician Rating Website model as well as the Benchmark model. In the Separating-Pooling region, the type H doctor has an incentive in the Physician Rating Website model to instead invest in the high quality.

Consider the ex ante expected utility of the patients and the ex ante expected profit of the doctor. Assume that the ex ante expected utility is discounted with $\beta > 0$. With probability π_0 , the doctor's type is H . In the PRW setting, if the equilibrium is the Pooling Equilibrium, no patient will visit the doctor and all patients receive their base utility. The doctor does not make any profit. In the Benchmark model, patient's and doctor's ex ante expected profit and utility are the same as in the Pooling Equilibrium in the PRW setting. If the equilibrium in the rating platform model is the Separating Equilibrium and the prior belief is $\pi_0 \geq \pi^*$, if a patients of type $\alpha > \pi_0 q_H$ is drawn, this patient will receive the base utility α . If a patient of type $\alpha \leq \pi_0 q_H$ is drawn, this patient will visit the doctor and will be treated with q_H . After this, the belief increases to 1 and all patients of type $\alpha \leq q_H$ will receive q_H in the following periods. Patients of type $\alpha > q_H$ receive their base utility.

With probability $1 - \pi_0$, the doctor's type is L . Again, in the Separating Equilibrium of the PRW setting, patients of type $\alpha > \pi_0 q_H$ do not visit the doctor and receive their base utility. With probability $\pi_0 q_H$, a type of patient is drawn who visits and will be treated with $q_L = 0$. After this, the belief will be updated to 0 and no other patient will visit the doctor. Consequently, all patients will from then on receive their base utility.

If the belief is π_0 , with probability $\pi_0 q_H$, the doctor in the PRW model will be visited. If the doctor's type is H , the doctor receives w and pays c for the high quality. Furthermore, the belief increases to 1 and in every period, with probability q_H , the doctor receives a profit of $w - c$. With probability $1 - q_H$, the doctor is not visited in a period and still has to pay c . Compare for this the value function of the doctor as given in (9). If the doctor's type is L , with probability $\pi_0 q_H$, the doctor is visited and gains a profit of w . After that, the beliefs drops to 0 and the doctor will no longer be visited by any type of patient. But the doctor also does not have to bear any cost from offering the free low quality. From comparing the ex ante expected profit and utility, Proposition 5 follows.

Proposition 5.

- (i) *In the Separating-Pooling region, the ex ante expected profit of the doctor and the ex ante expected utility of the patients in the rating platform model are higher than in the Benchmark model.*
- (ii) *In the Pooling-Pooling region, the ex ante expected profit of the doctor and the ex ante expected utility of the patients in the Benchmark model are equal to the ex ante expected profit and utility in the rating platform model.*

The formal proof is given in appendix A.

The results show that the PRW setting is the favorable setting for both the patients with uniformly distributed types, and the doctor when comparing the ex ante expected doctor's profit and the ex ante expected patients' utility, respectively. In the Pooling Equilibrium, the ex ante expected profit of the doctor is the same in the Benchmark setting and in the PRW setting. In the Separating Equilibrium, because patients can learn the true type of the doctor, the type H doctor has an incentive to exert the high quality in equilibrium. If the doctor's type is L , one patient will potentially visit the doctor and will be dissatisfied with the doctor's quality. This patient then does not receive 0 in the PRW setting instead of the base utility compared to the Benchmark setting. All other patients receive the same utility in both models in this case. The type L doctor's expected profit is positive. However, with probability π_0 , the doctor's type is H . In this case, the first patient who visits gains a utility of q_H that is greater or equal compared to the base utility from the Benchmark model. The patients in the following periods also either receive q_H or their base utility if they are of a type that does not visit the doctor. The expected utility of each patient is therefore greater or equal to that from the Benchmark model. Despite the cost c the doctor pays in every period, the expected profit of the doctor is higher as in the Benchmark model. This follows from the fact that the doctor has no incentive to deviate to the low quality, and the expected profit from deviating is an upper bound for the ex ante expected profit of 0 in the Benchmark model.

5 Mixed Strategy Equilibrium

Consider now that the type H doctor can play a mixed strategy, i.e. chooses in every period the probability distribution used to randomly choose to exert either the high quality q_H or the low quality $q_L = 0$. Before, when the type H doctor was exerting a pure strategy, patients anticipated after the first bad rating that the doctor's type is L . They expected to be treated with the low quality and no patient would visit the doctor after a negative rating. If the competent doctor is playing the high quality with a positive probability, this allows for richer dynamics. If the doctor receives a negative rating, patients do no longer believe that the doctor's type is L right away. The patients' belief still decreases after a negative rating but does not instantly drop to zero. This allows for the type H doctor to get some negative ratings but still being able to be recognized as the high type after a positive rating. The doctor does not have to commit to the costly high quality in every period but exerts the quality with a positive probability which is dependent on the belief of the patients. This can also be interpreted as a generally competent doctor who makes effort to exert high quality but still does not perform consistently every day.

Patients anticipate the strategy of the doctor. If in one period the belief of the patients

is π_t and the doctor receives a positive rating, scenario $S_{\pi_t,1}$, or if the doctor is not visited and there is no rating, scenario $S_{\pi_t,3}$, the patients will update their belief in the same way as if the type H doctor plays a pure strategy. But if the doctor receives a negative rating, scenario $S_{\pi_t,2}$, patients will take into account that the previous patient may still have been treated by the type H doctor who was exerting the low quality with positive probability and update their belief accordingly. After a negative rating, according to Bayes' rule, the belief is given by $\pi_{t+1,2} = \frac{(1 - p(\pi_t))\pi_t}{1 - p(\pi_t)\pi_t}$. With probability $p(\pi_t)\pi_t q_H$, the doctor is visited by a patient in period t . With probability $p(\pi_t)$, the doctor will treat the patient with the high quality and will receive a positive rating. Therefore, with probability $(1 - p(\pi_t))p(\pi_t)\pi_t q_H$, the doctor will receive a negative rating. The transition probabilities from scenario S_{π_t} to scenario $S_{\pi_{t+1}}$, given that the type H doctor plays q_H , are given by the probabilities of each of the scenarios described above

$$\begin{aligned} P(S_{\pi_{t+1},1}|S_{\pi_t}, q_t) &= p(\pi_t)^2 \pi_t q_H \\ P(S_{\pi_{t+1},2}|S_{\pi_t}, q_t) &= (1 - p(\pi_t))p(\pi_t)\pi_t q_H \\ P(S_{\pi_{t+1},3}|S_{\pi_t}, q_t) &= 1 - p(\pi_t)\pi_t q_H. \end{aligned}$$

Figure 3 illustrates possible transitions in a given period in dependence of the current state.

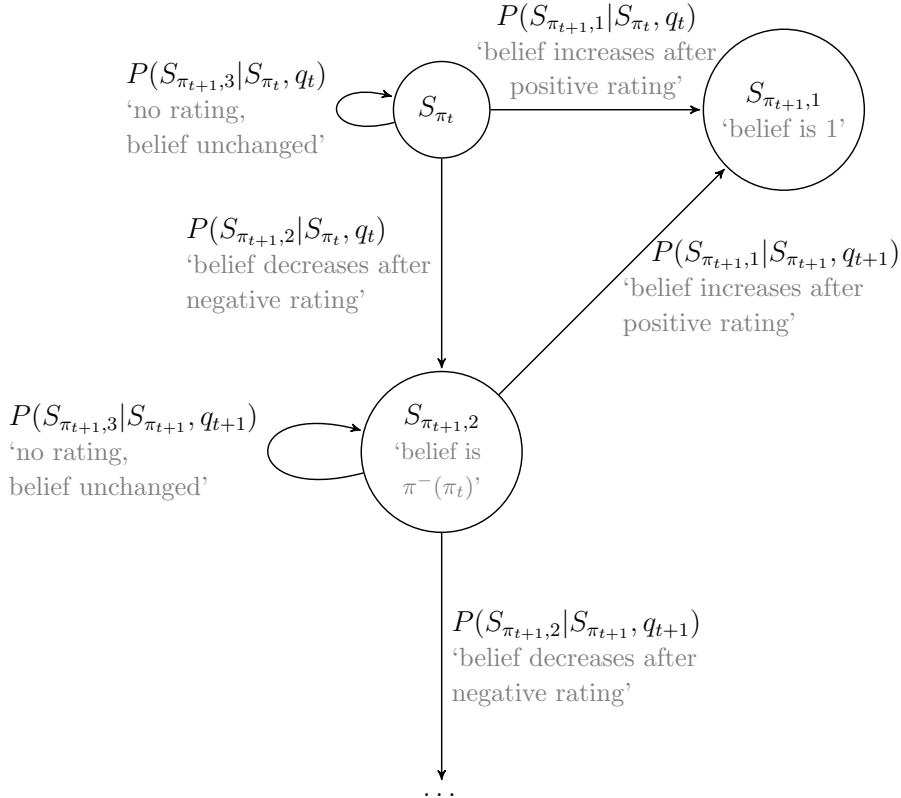


Figure 3: Mapping of the transition from one state to another given state S_{π_t} and action q_t in period t , where π^- is defined as in Proposition 7.

If the discount factor of the doctor is too small, there is only an equilibrium in pure strategies in which both types of the doctor always exert the low quality. The patients anticipate the strategy of the doctor and do not visit the doctor.

Proposition 6. *If $\delta < \delta_{min}$, there is no equilibrium in mixed strategies. Both types of the doctor will play q_L in equilibrium.*

Intuitively, the type H doctor will choose to play the mixed strategy if it is not profitable to instead deviate to playing either $q_L = 0$ or q_H as pure strategy. By comparing the associated expected profits, an optimality condition depending on the expected profit after a negative rating can be derived. Depending on the discount factor and the belief, there exist finite many different regions for the belief in which the type H doctor exerts the high quality with a different positive probability. Proposition 7 describes the different equilibrium regions. Recall the thresholds $\delta_{min} = \frac{c}{q_H(q_H w - c) + c}$ and

$$\pi^* = \frac{c(1 - \delta)}{q_H(q_H w - c)\delta}.$$

Proposition 7. *Given belief π and probability $p(\pi)$ in a period where the type H doctor plays the high quality q_H with probability $p(\pi)$, denote $\pi^-(\pi) = \frac{(1 - p(\pi))\pi}{1 - p(\pi)\pi}$ the updated belief after a negative rating occurred in this period. Furthermore, $V(1) = \frac{q_H w - c}{1 - \delta}$. If $\delta \geq \delta_{min}$, the equilibrium strategy of the doctor is dependent on the belief π .*

- (i) *There exists a finite sequence of interval thresholds $(\pi^0, \pi^1, \dots, \pi^K, 1)$, with $\pi^0 = \pi^*$. Generally, for $1 \leq k \leq K$, if π lies in $[\pi^k, \pi^{k+1}]$, the belief after a negative rating decreases to π^- in $[\pi^{k-1}, \pi^k]$. If $\pi \in [\pi^0, \pi^1]$, the belief after a negative rating decreases to $\pi^- \in [0, \pi^0]$ and if $\pi \in (\pi^K, 1)$, the belief decreases to $\pi^- \in (\pi^{K-1}, \pi^K)$. For all $k \geq 1$ if π lies in (π^k, π^{k+1}) , $V(\pi^k) \leq V(1) - \frac{c}{\delta q_H}$ holds and the optimal mixing probability is then given by*

$$p(\pi) = \frac{c}{\delta \pi q_H (V(1) - V(\pi^{k-1}))}. \quad (11)$$

If π lies in $(\pi^K, 1)$, at π^K , $V(1) > V(\pi^K) > V(1) - \frac{c}{\delta q_H}$ holds and the optimal mixing probability is as well given by (11).

If π lies in (π^0, π^1) , the optimal mixing probability is given by

$$p(\pi) = \frac{c}{\delta \pi q_H V(1)}.$$

For all $k \geq 1$, at threshold π^k the optimal mixing probability is given by (11).

- (ii) *If $\pi < \pi^*$, the type H doctor will play q_L in every period in the equilibrium. If $\pi = \pi^*$ or $\pi = 1$, the type H doctor will play q_H in every period in the equilibrium.*

The formal proof is given in appendix A.

$V(\pi^k)$ is defined as in (30) and π^k is given according to (29), respectively. The intuition behind the recursive formula is as follows. If the belief is smaller than π^* , the doctor will exert the low quality and no patient will visit the doctor. The expected profit of the doctor is zero. There is a range of belief values that decrease to some $\pi < \pi^*$ after a negative rating. This range can be determined by rearranging the Bayesian updating formula to find the belief value that yields the desired value after a negative rating. So, for all beliefs in the determined range, the expected profit after a negative rating is zero. Using the optimality condition mentioned above that is depending on the expected profit after a negative rating, the probability p with which the doctor exerts the high quality can be derived for those belief values that lie in the determined range. Furthermore, if the belief is π^* , the doctor will exert the high quality. If the belief is given by π^1 and decreases to π^* after a negative rating, the expected profit after a negative rating when the belief decreases to π^* is again known. The doctor exerts q_H with probability $p = 1$. Using again the Bayesian updating formula, π^1 can be determined. Continuing in this way, the belief can be expressed as the recursive formula (29) and the expected discounted profit of the doctor as the recursive formula (30).

If the belief is given by $\pi \in (\pi^k, \pi^{k+1})$ for some $1 \leq k < K$ and the doctor receives a negative rating, the belief decreases to $\pi^- \in (\pi^{k-1}, \pi^k)$. This means that the doctor can receive k negative ratings before the belief decreases to $\pi < \pi^* = \pi^0$ and patients expect the doctor to exert the low quality. An illustration is given in Figure 4.

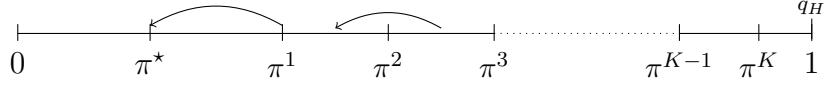


Figure 4: Illustration of K equilibrium regions. In each region, the doctor will exert q_H with a different positive probability. If the doctor receives a negative rating, the updated belief lies in the previous region that is closer to π^* .

In Figure 5, the different equilibrium strategies of the type H doctor for $\delta \geq \delta_{\min}$ depending on the belief are illustrated in case that the optimality condition is fulfilled for all $2\pi^* \leq \pi < 1$. If the discount factor is high enough but the belief is smaller than the threshold π^* , both types of the doctor exert the low quality in a pure strategy equilibrium. If the belief is equal to π^* , the type H doctor will play the pure strategy q_H in equilibrium. The intuition is that in this case, it is more profitable for the doctor to invest in the high quality which guaranties a positive rating as soon as a type of patient is drawn who visits the doctor. If the doctor would deviate to the low quality, patients would update their belief to zero after the first bad rating. After that, no type of patient visits the doctor in the following periods. As long as the value of the belief lies in the interval $(\pi^*, 2\pi^*)$, the doctor will mix with a probability that depends on the ratio between cost and discounted profit after patients learn the doctor's true type as well as the reciprocal of the belief

value. If the belief is higher than the threshold $2\pi^*$, the doctor will play a mixed strategy which then not only depends on the ratio between cost and the reciprocal of the belief value but as well on the difference in discounted profit after patients learn the doctor's true type and after the doctor receives a negative rating.

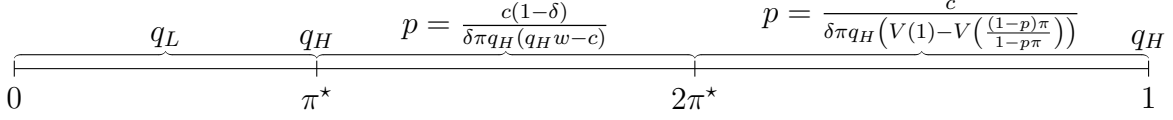


Figure 5: Equilibrium Regions if π^* and $2\pi^*$ are feasible.

Note threshold π^* , which determines the beginning of the first equilibrium region in which type H plays the mixed strategy. It is the same threshold that determines the Separating Equilibrium if type H plays a pure strategy in section 4. If $2\pi^* = 1$, the type H doctor will mix with probability $p = \frac{c(1-\delta)}{\delta\pi q_H(q_H w - c)}$ if the belief lies in the interval $(\pi^*, 1)$ and the expected profit from this is $V(\pi) = c \setminus (\delta q_H)$, i.e. the ratio between cost and discounted probability of being visited by a type of patient who demands a high quality. If $2\pi^* < 1$, there are two or more regions of the belief in which it is profitable for the type H doctor to play a mixed strategy.

There are two opposing effects at work regarding the influence of the belief on the probability p with which the doctor plays the high quality in the mixed strategy equilibrium. On the one hand, the smaller the patient's belief that the doctor's type is H , the higher the probability p that the doctor will invest in the high quality. On the other hand, the probability is decreasing with an increase in the term that captures the expected profit from the new belief value after a negative review, i.e. $V(\pi^-) = V\left(\frac{(1-p(\pi))\pi}{1-p(\pi)\pi}\right)$. The definition of the expected profit $V(\pi)$ in (30), implies that for higher π^- the expected profit $V(\pi^-)$ is also higher. And the updated belief after a negative rating, π^- , is smaller if π is smaller. To illustrate these effects, consider for example $\pi \in (2\pi^*, \bar{\pi})$ and $\bar{\pi}^- = 2\pi^*$. At the threshold $\bar{\pi}$, the expected profit is $V(\bar{\pi}) = \frac{c(w + \delta V(2\pi^*))}{\delta(q_H w - (1-\delta)V(2\pi^*))}$. If $\pi \in (2\pi^*, \bar{\pi})$, and the doctor received a negative rating, the belief decreases to $\pi^- \in (\pi^*, 2\pi^*)$ with $V(\pi^-) = V(\pi^*)$. The expected profit if $\pi \in (2\pi^*, \bar{\pi})$ is therefore given by $V(\pi) = \frac{c(w + \delta V(\pi^*))}{\delta(q_H w - (1-\delta)V(\pi^*))}$. Consequently, the probability of playing the high quality is given by $p(\pi) = \frac{c}{\delta\pi q_H(V(1) - V(\pi^*))}$ if $\pi \in (2\pi^*, \bar{\pi})$ and by $p(\bar{\pi}) = \frac{c}{\delta\bar{\pi} q_H(V(1) - V(2\pi^*))}$ if $\pi = \bar{\pi}$. The smaller the difference between $V(\pi^*)$ and $V(2\pi^*)$, the smaller the decreasing effect on the probability p . This dynamic is prevalent at the thresholds of all equilibrium regions. Intuitively, the more equilibrium regions there are, the smaller the difference in the expected profits in the different regions. Graphical illustrations of these dynamics are discussed in section 5.1. The results from Propositions 6 and 7, will from here on

be referred to as the PRW setting with mixed strategies or rating platform model with mixed strategies results.

5.1 Mixed Strategy Equilibrium - Simulation

In this section, I perform a simulation of different thresholds for the mixed strategy equilibrium regions depending on different choices of values for the exogenous variables δ , q_H , w and c .

I restrict the analysis to those parameter value choice combinations for which at least one region where the mixed strategy is played by the type H doctor exists. The parameter w is normalized to 1 and a grid of all parameter combinations for which $\frac{c(1-\delta)}{\delta q_H(q_H \cdot 1 - c)} \in (0, 1)$ is true, meaning that a feasible π^* exists, is created.²

Figure 11 illustrates for fixed choices of q_H and δ the according interval boundary π^* . The smaller q_H , the higher δ and the smaller c . This fits the intuition, that if q_H is smaller, the probability of being visited is smaller. For type H to have an incentive to exert the high quality with positive probability, this must be compensated by the doctor being more patient and interested in the future or by smaller cost of exerting the high quality. Accordingly, there are also more combinations of parameter value choices that induce the threshold π^* in dependence of c the higher q_H and δ . As also discussed in section 4.1, π^* is increasing in c and decreasing in δ . Intuitively, the higher the cost, the higher the belief of the patients must be for type H to be incentivized to invest in the high quality. A doctor who is more interested in long-term profit can be incentivized by a smaller belief to invest in the high quality. The smaller δ , the higher the thresholds induced by small value choices of c , respectively. Recall, that if $\pi^* \geq 0.5$, there exist only one region in which the doctor mixes with probability $p(\pi)$ in the equilibrium, if $\pi > \pi^*$. If π^* is smaller, there exist more regions that each induce a different probability in dependence of the belief.

Below, examples of equilibrium region thresholds for the belief and the mixing probabilities induced by specific choices of the exogenous parameters are examined.

There exist combinations of possible choices for the parameter values of δ , q_H , w and c , both π^* and $2\pi^*$ are feasible and there are two regions in which the type H doctor mixes with according mixing probabilities, as sketched before in Figure 5. Most parameter combinations induce more than two regions in which the competent doctor plays the

²Each parameter's value set starts at 0.09 with a 0.09 step size and ends at 1. Parameter c has an upper bound which is given by the according value of $q_H \cdot 1$ and the value set has a 0.01 step size. It is easy to see that two different parameter choices of c cannot yield the same threshold $\pi^* = \frac{c(1-\delta)}{\delta q_H(q_H \cdot 1 - c)}$, if the other parameters are held fixed. The same holds true for two different parameter choices of δ . If two different parameter choices q_1, q_2 for q_H would yield the same threshold, $q_1 + q_2 = c$ would hold. But as c is bound above by q_H , this cannot be fulfilled. Furthermore, the probability choice $p(\pi) = \frac{c(1-\delta)}{\delta \pi q_H(q_H w - c)}$ ensures that the expected probability of being visited by a patient, given by $p \cdot \pi \cdot q_H$ is held constant at $\frac{c(1-\delta)}{\delta q_H(q_H w - c)}$.

mixed strategy with positive probability. And a lot of parameter combinations induce more than 100 different equilibrium region thresholds. Figure 6 visualizes the probability

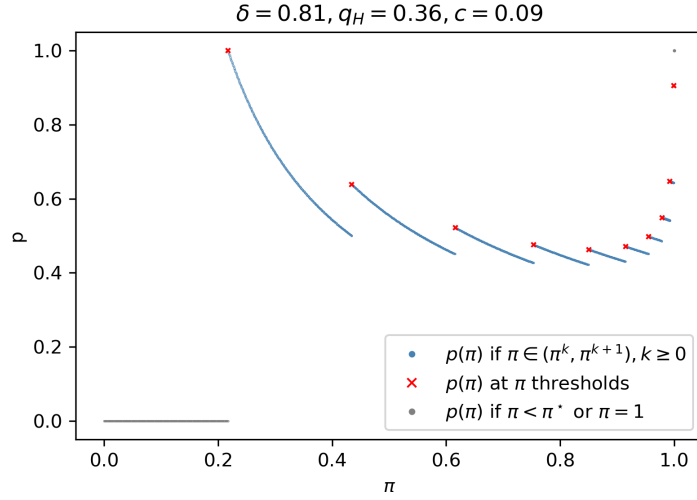


Figure 6: The mixing probability p depending on the belief π in 0.001 steps if $\delta = 0.81$, $q_H = 0.36$, $w = 1$ and $c = 0.09$, implying that there are 10 thresholds.

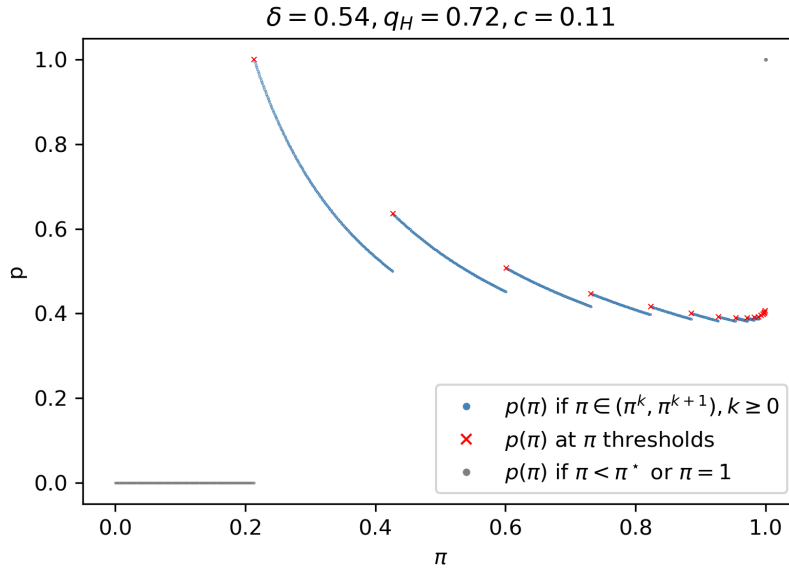


Figure 7: The mixing probability p depending on the belief π in 0.001 steps if $\delta = 0.54$, $q_H = 0.72$, $w = 1$ and $c = 0.11$.

with which the type H doctor mixes in dependence of the belief of the patients for a case with 10 different equilibrium thresholds. The thresholds that mark the beginning of a new equilibrium region in which the type H doctor mixes with a different probability are given in red. At every threshold π^k it holds true that if $\pi > \pi^k$, the belief after a negative rating, will always be smaller than π^k . Note how, in the neighborhood of the individual thresholds π^k , it holds that for $\pi - \varepsilon < \pi^k$, the mixing probability suddenly drops to a smaller value than $p(\pi^k)$ but then still overall increases in the interval (π^{k-1}, π^k) with decreasing π . These discontinuous jumps are caused by the change in the value function

of the doctor. The expected discounted profit in each of the intervals is different. The intuition behind this is that by entering a new equilibrium region which is nearer to the threshold π^* the doctor can receive one negative rating less before the belief of the patients drops below π^* . If the latter occurs, no patient will visit the doctor. The smaller the number of negative ratings that the doctor can receive before patients do no longer decide to get treated, the greater the difference in the value function in two neighboring intervals. This dynamic is not only prevalent for this specific parameter value combinations. It does occur for higher numbers of equilibrium thresholds as well, a second example is illustrated in Figure 7.

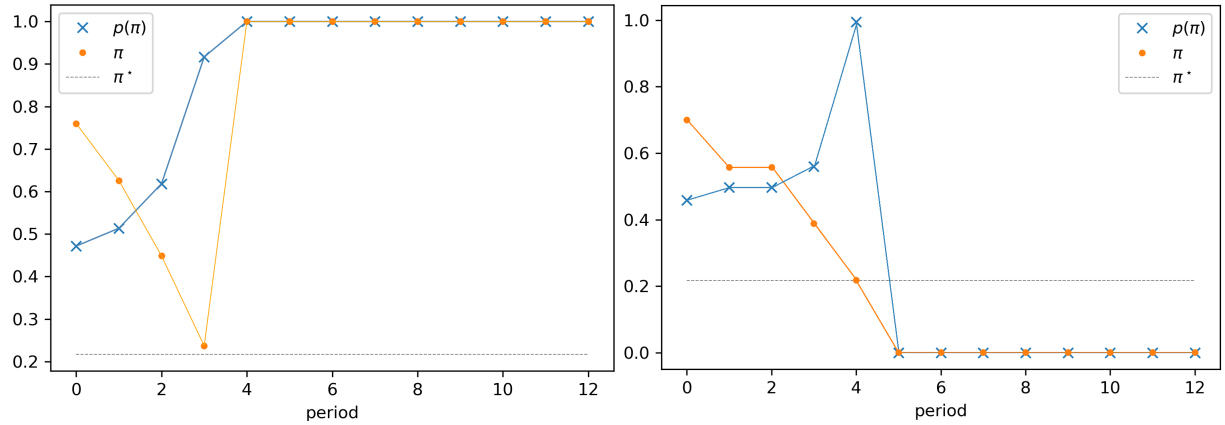


Figure 8: Evolution of the belief over 12 periods if the doctor gets 3 negative ratings and receives a positive rating after that which increase the belief to 1 (left). And if the doctor receives 4 negative ratings such that the belief decreases below the threshold π^* . When the doctor is not visited, the belief does not change (right).

In Figure 8, two examples for a possible evolution of the belief in an infinitely repeated game are illustrated. The depicted time frame is 12 periods. The threshold π^* is marked by the gray dotted horizontal line. The prior belief of the patients is greater than π^* . In both examples, the doctor is visited by a patient but did exert the low quality. Therefore, the dissatisfied patient left a negative rating and the belief decreases. The probability with which the doctor exerts the high quality increases with decreasing belief. On the left graph, the doctor eventually treats a patient with the high quality and the belief increases to 1. Afterwards, the doctor exerts q_H in every period instead of playing the mixed strategy. On the right graph, the doctor is not visited by a patient in one period, hence the belief does not change. Afterwards, a patient decides to get treated in the following periods, but all patients are dissatisfied. The belief decreases. Although the doctor increases the probability with which q_H is exerted, the patients are treated with the low quality and the belief drops below the threshold π^* . Afterwards, patients do not visit the doctor and the belief stays at the low level. Figure 9 illustrates how the mixing probability increases if the belief decreases after a negative rating. After the first negative rating the belief decreases. As the updated belief lies in a different equilibrium

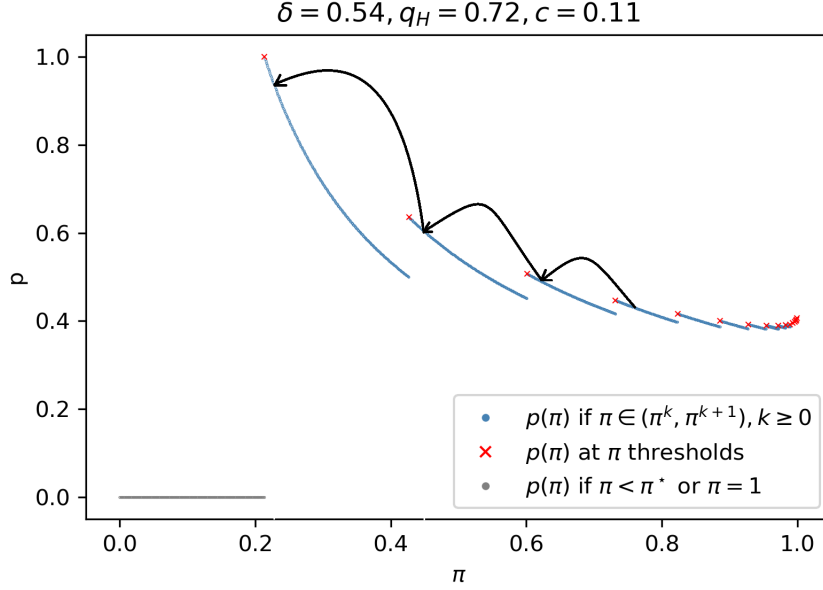


Figure 9: Illustration of the changing mixing probability if the belief decreases after a negative rating. Each arrow connects the probability induced by a certain belief value to the probability induced by the updated belief after a negative rating occurred.

region than before, the doctor mixes with a different probability. As the belief decreases, the mixing probability increases. The doctor exerts the high quality with the highest probability if the belief value lies in the equilibrium region nearest to the region in which patients expect the doctor to always exert the low quality.

Summarizing the simulation results, the probability with which the type H doctor plays the high quality is more smoothly decreasing with higher belief the more often the belief could possibly decrease after a negative rating without actually decreasing to $\pi < \pi^*$. Furthermore, a case with few regions in which the mixed strategy is played was analyzed in detail. As there are only few possible 'harmless' decreases of the belief, the mixing probability increases at the belief thresholds, but the increase is smaller the higher the belief is and the probability still decreases within those regions between one threshold and the next. The simulation illustrated the two opposing effects on the mixing probability through the influence of the belief π and the influence of the change in the expected profit after a negative rating discussed in the previous chapter.

A high number of equilibrium regions is induced, i.e. π^* is small, if the cost of the high quality is very small while the doctor's discount factor is high. The high discount factor can be interpreted such that the doctor values future profits. If furthermore the share of patient types who are satisfied with the high quality is big, i.e. q_H is high, there is only a small probability every period that a type of patient is drawn who will not visit the doctor. Because π^* is small, patients will expect only for very small π that both types of the doctor exert the low quality. But a high number of equilibrium regions also means that, if the prior belief of the patients is sufficiently high, the doctor can receive a

number of negative ratings, before the belief decreases below π^* . The high type doctor therefore chooses to exert the high quality with a very small probability. This also means that the probability of being visited is small and the probability of not being visited is large. If the doctor in one period exerts the low quality and is visited, the patient will be dissatisfied and give a negative rating. But if the belief is not already very small, the updated belief after the negative rating will not be significantly smaller than before receiving this rating. If the doctor in one period exerts the high quality and has to pay the cost for doing so but is not visited by a patient, the sunk cost are small due to c being very small in this scenario. On the other hand, if the doctor is visited while exerting the high quality with cost c , patients will learn the true type. The doctor does only invest in the high quality with a higher probability than in the low quality if the belief is close to the threshold π^* . In this scenario, the probability of a bad rating is small as is the impact of it. If the game is repeated infinitely, the patients will learn the doctor's true type with positive probability and after that the doctor will always exert the high quality. The doctor sacrifices short term profit by high quality treatment being unlikely. But this is compensated by the long term profit after being recognized as the high type.

Below, the equilibrium regions in the Benchmark model and the PRW model if type H plays a mixed strategy are compared. In particular, the ex ante expected profit of the doctor and the ex ante expected utility of the patients are analyzed.

5.2 Comparison of Benchmark and Physician Rating Website

The threshold π^* that marks the end of the Pooling-Pooling region in Proposition 4 also indicates the end of the region in which both types of the doctor exert the low quality if the doctor plays the mixed strategy. Compared to the analysis before, there arises now a different type of equilibrium in which the type H doctor exerts q_H with a different probability depending on the belief. From comparing Proposition 1 to Propositions 6 and 7, the proposition below is derived.

Proposition 8. *If $\delta \geq \delta_{min}$, there are three types of equilibrium regions depending on the belief π of the patients and the discount factor of the physician:*

- (i) *Pooling-Pooling (P-P) region: if $\pi < \pi^*$, both types of the doctor play q_L in every period in equilibrium in both settings.*
- (ii) *Separating-Pooling (S-P) region: if $\pi = \pi^*$ or if $\pi = 1$, the type H doctor plays q_H in every period in equilibrium in the rating platform model and both types of the doctor play q_L in every period in equilibrium in the Benchmark setting.*
- (iii) *Separating with Mixing-Pooling (S_M -P) region: if $\pi > \pi^*$, the type H doctor plays q_H with probability $p(\pi)$ in every period in equilibrium in the rating platform model.*

Both types of the doctor play q_L in every period in equilibrium in the Benchmark setting.

As illustrated in Figure 10, if $\pi < \pi^*$ or if $\pi = \pi^*$, the equilibrium regions arise analogously to Proposition 4. If $\pi = 1$, the type H doctor as well has no incentive to deviate from playing q_H in equilibrium in the PRW model. If $\pi > \pi^*$ however, depending on π^* there is a finite number of equilibrium regions for the PRW model in which the type H doctor exerts q_H with a different probability depending on π . In the Benchmark model, both types of the doctor always exert the low quality.

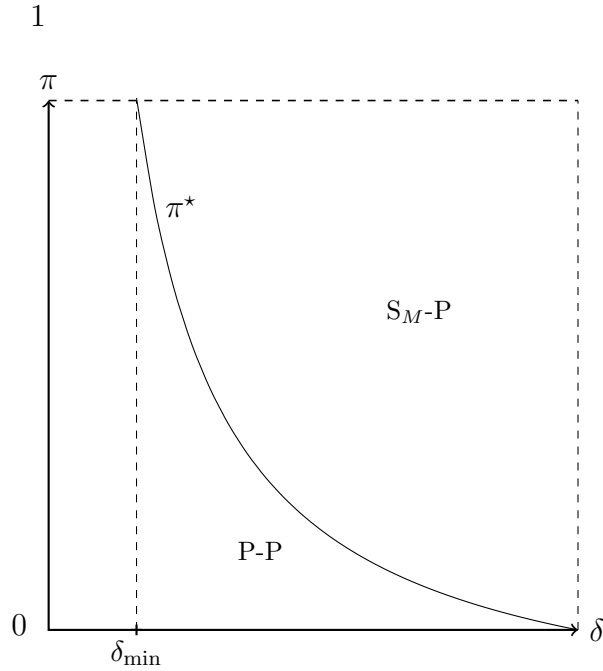


Figure 10: Comparison of Equilibrium Regions of Benchmark and Rating Platform Setting if the type H doctor can mix between q_H and $q_L = 0$. If $\pi = \pi^*$ or $\pi = 1$, type H always exerts q_H .

If $\delta < \delta_{min}$ or $\pi_0 < \pi^*$, both types of the doctor will exert q_L . Therefore, the ex ante expected profit of the doctor will be the same in the Benchmark model as in the extended model with mixed strategies. The same is true for the ex ante expected utility of the patients. If $\delta \geq \delta_{min}$ and $\pi_0 = \pi^*$ or $\pi_0 = 1$, the type H doctor will exert q_H and patients anticipate this. Therefore, the ex ante expected profit of the doctor and the ex ante expected utility of the patients will be the same as in Proposition 5 (i).

If $1 > \pi > \pi^*$, the type H doctor exerts the high quality with probability $p(\pi)$. With probability $p(\pi) \cdot \pi \cdot q_H$, a type of patient is drawn who visits the doctor. Consider first the case of a doctor of type L . After the first patient visits and rates the doctor, the belief decreases to π^- . The patient receives a utility of zero and the doctor receives a period profit of w . In the following period, the probability that a type of patient is drawn who visits the doctor is $p(\pi^-) \cdot \pi^- \cdot q_H$. In this case, the patient will again receive no utility

and the doctor receives a period profit of w . If another type of patient is drawn, the doctor is not visited and receives no period profit. The patient receives the base utility. If after some period t the belief decreases to $\pi_t^- < \pi^*$, in all following periods the patients will expect both types of the doctor to exert the low quality $q_L = 0$. All types of patients receive their base utility in every following period. The doctor does not make profit in the following periods.

If the doctor's type is H , with probability $p(\pi)$, the doctor exerted the high quality in the period when the first patient visited. In this case, the doctor's profit in this period is $w - c$. The patient receives utility q_H . After the patient rates the doctor, the belief increases to 1. In all following periods, the doctor exerts the high quality. All patients of type $\alpha \leq q_H$ visit the doctor and receive q_H . All remaining types receive their according base utility. In every period when the doctor is visited, the period profit is $w - c$. In all periods when the doctor is not visited, the period profit is $-c$. With probability $1 - p(\pi)$, the doctor exerted the low quality when visited by the first patient. In that case, the profit of the doctor is w and the utility of the patient is 0. After the patient rated the doctor, the belief decreases. This dynamic will then either repeat until the doctor exerts the high quality in a period when a patient visits. Then, the belief will increase to 1. Or, if the doctor exerts the low quality when visited, the dynamic repeats as long as the belief of the patients is still greater than π^* . All patients who visit in the meantime receive a utility of 0. All remaining types receive their base utility. If the belief decreases below π^* , every type of patient that is drawn receives the according base utility. If the belief increases to 1, all types of patients with $\alpha \leq q_H$ receive a utility of q_H instead. In all periods in which the doctor exerts q_H but no patient visits, the doctor's profit is $-c$ and the patient receives the base utility.

The ex ante expected profit of the doctor is higher in the PRW setting with mixed strategies than in the Benchmark model. It generally depends on the discount factor β if the ex ante expected utility of the patients is higher in the PRW setting with mixed strategies. However, if there is only one region in which type H exerts the mixed strategy, the ex ante expected utility of the patients is higher in the PRW setting than in the Benchmark setting independent of β .

The following proposition summarizes the results.

Proposition 9. Denote $\beta_{min} = \frac{1 - p(\pi_0)^2 \pi_0 q_H^2}{1 - (1 - q_H) p(\pi_0)^2 \pi_0^2 q_H^2}$.

- (i) If $\pi_0 < \pi^*$, the ex ante expected profit of the doctor and the ex ante expected utility of the patients in the Benchmark model are equal to the ex ante expected profit and utility in the rating platform model.
- (ii) If $\pi_0 = \pi^*$, the ex ante expected profit of the doctor and the ex ante expected utility of the patients in the rating platform model are higher than in the Benchmark model.

(iii) If $\pi_0 > \pi^*$ and the doctor plays a mixed strategy in the PRW model, the ex ante expected profit of the doctor in the rating platform model is higher than in the Benchmark model. If $\pi^* < 0.5$, the ex ante expected utility of the patients in the PRW model if the doctor plays a mixed strategy is higher compared to the Benchmark setting. If $\pi^* \geq 0.5$, the ex ante expected utility of the patients in the PRW model if the doctor plays a mixed strategy is higher compared to the Benchmark setting, if $\beta > \beta_{min}$.

The formal proof is given in Appendix A.

The difference between the ex ante expected utility of the patients if the doctor plays a pure strategy compared to if the doctor mixes between q_H and q_L , is that in the first case patients stop visiting after the first negative rating that occurs. They update their belief to zero and do not choose to get treated by the doctor. This results in patients receiving their according base utility instead. If the doctor plays a mixed strategy, patients take this into account when updating their belief after a negative rating. If the doctor can receive $K > 0$ negative ratings, before patients update their belief to some $\pi < \pi^*$, this means that K patients do not receive utility from being treated by the doctor before no patient will visit the doctor anymore. While if the doctor plays the pure strategy q_H , no patient is ever dissatisfied with the type H doctor's treatment, the same is not true if the doctor exerts a mixed strategy. Several patients may be dissatisfied with the treatment quality but the probability that a patient visits the doctor is still positive if the belief is still greater than or equal to π^* .

6 Discussion

Patients benefit if the high quality is offered by the type H doctor. Public ratings on a PRW are an incentive for competent physicians with a sufficiently high discount factor to invest in the costly high quality. The patients' belief that the doctor's true type is H has to be sufficiently high for this. Increasing the earnings per visiting patient w of the doctor or decreasing the cost c of offering the high quality will enlarge the range of belief values for which the type H doctor is offering the high quality in equilibrium. Those measures reduce the threshold on the discount factor δ and thereby offering q_H becomes the equilibrium strategy for physicians with a smaller discount factor as the threshold is exceeded for smaller values of the discount factor.

In the Benchmark model, neither type of the doctor offers the high quality in the equilibrium. Patients anticipate to be treated with the dissatisfying low quality and never visit the doctor. In the PRW setting, depending on the discount factor of the doctor, the high type will exert high quality in equilibrium. For small values of π^* , the discount factor has to be higher. In the case of a doctor with a small discount factor and

a small belief, the cost from offering high quality without being visited does not outweigh the discounted future revenue from patients after revealing that the doctor's type is H . The doctor instead exploits the first patient who visits. This patient will be unsatisfied with the low quality and therefore will expose the doctor as exerting low quality. The belief threshold that ensures that the competent doctor does not exert the low quality in equilibrium is the same, whether the competent doctor is exerting a pure strategy or mixes between the two strategies.

In the PRW setting with pure strategies, in equilibrium, one patient will be possibly left unsatisfied with the doctor's treatment if the doctor's type is L . If the doctor did not treat a patient before, with probability $\pi_0 q_H$ a patient may visit every period up until a patient does get treated. As patients update their beliefs in this setting, no other patient will visit the doctor after the first unsatisfied visitor. The patients learned the true type of the doctor. After the first visit, all other patients decide against getting treatment and receive the base utility as implied by their type. If the doctor's true type is H , patients will learn the true type after the first patient visited and no patient is unsatisfied.

Consider now a type L doctor in the PRW setting if the belief of the patients is sufficiently high and the competent doctor chooses the probability with which q_H is exerted. In the mixed strategy equilibrium, there will as well always be one dissatisfied patient before patients learn the true type of the doctor. There can potentially be up to K dissatisfied patients, if the doctor can receive K negative ratings before the patients' belief is no longer high enough that they choose to visit. Furthermore, if $1 > \pi > \pi^*$ the competent doctor in this case exerts q_H with positive probability smaller than one. Therefore, patients can even be dissatisfied with the competent doctor's treatment in the PRW setting with sufficiently high belief. Nevertheless, the ex ante expected utility of the patients is higher in both variants of the PRW setting than in the Benchmark setting. This is because, although with positive probability some patients are dissatisfied in the PRW setting with mixed strategies, this is compensated by the ex ante expected utility from being treated with the high quality by the type H doctor instead of all patients receiving their base utility from not getting treated.

Findings from applied work show that Physician Rating Websites in Germany were frequently updated with new ratings over the last years. Furthermore, the number of ratings a physician received on the platform is positively correlated with the ratings being positive. An increasing number of physicians, including specialists but also general practitioners, in Germany have been rated on one of the important PRW in Germany. In [Emmert and Meier \(2013\)](#) it is reported that 30 percent of general practitioners (and overall 37 percent physicians of varying specialties in primary care) in Germany by 2012 were rated on the Physician Rating Website [jameda.de](#). General practitioners furthermore belong to the highest rated physicians on the platform. Those findings were confirmed

in a second study building on more data from the years 2016 to 2021 in [Emmert and McLennan \(2021\)](#). Ratings on PRWs used to be rare and positive, which could have negatively affected validity and usefulness of information displayed on PRWs. But over the past years, the number of ratings on PRWs has increased while trends regarding positive ratings have been stable. By 2021, more than half of all German physicians have been rated on [jameda.de](#), although the number of ratings declines over the last 2 years. Physicians with a higher number of ratings overall receive significantly better ratings.

The relationships of health care providers and patients usually involve a combination of both repeated interactions and first-time interactions. In the models presented in section 4 and section 5, the analysis was restricted to the influence of public ratings published on a PRW in a market where patients and the physician meet for the first time. [Cai et al. \(2014\)](#) show that feedback systems can be especially beneficial in situations where there is no established relationship between buyers and sellers and trade occurs only once as opposed to repeated interactions and word-of-mouth reputation. It is safe to assume that most physicians usually have a number of regular patients. Those patients have repeated interactions with their physician and may benefit less from a public rating system as provided through a PRW as they have their own experiences. On the other hand, for example because of different demographics, other types of patients are more prone to search for and compare different physicians before deciding to visit one. [López et al. \(2012\)](#) among other findings point out that in their sample a great share of online reviewing patients for which demographics were available is young, well-educated and healthy. This can be an indication that doctors with more and higher ratings may receive these good reputations from types of patients who are more actively engaging in the choice process. On the other hand, physicians with only a small number of patients and more varying rating scores may be visited more by their regular patients and less frequently by new patients. Furthermore, it is still important to note that by far not all patients of a physician rate their experience on some Physician Rating Website. As there exist different platforms, physicians may also be rated by different patients on different platforms.

Ratings are to a certain extent always based on the perception a patient has of the success of a treatment rather than objective quality measures. Nevertheless, they prove useful for physicians and patients. [Emmert, Meszmer, and Sander \(2016\)](#) find in an online-based cross-sectional study by surveying physicians and other health care providers that physicians show interest in responding to online ratings with measures to improve (perceived) patient care. Physicians preferred scaled-survey ratings over narrative comments of former patients. In [Greaves et al. \(2012\)](#), the authors examined the associations between web-based patient ratings on the government's English family practices rating website 'NHS Choices' and conventional measures of patient experience as well as clinical quality in primary care. They find moderate associations between patients' ratings and feedback from conventional surveys of patient experiences but only a weak

association between ratings and clinical outcomes. [Okike et al. \(2016\)](#) find that patients are increasingly using PRWs when choosing their doctor. However, they found no correlation between online ratings of physicians from five US states and 30-day risk-adjusted mortality rate following coronary artery bypass graft surgery. In a more recent analysis using machine learning and data from both Yelp and Medicare, [Chen and Lee \(2021\)](#) find that online physician rating platforms can promote efficiency by disseminating important quality information to patients and directing patients to higher-quality physicians. Although the analyzed reviews primarily describe physicians' service quality and interpersonal skills, ratings are positively associated with important conventional measures of clinical quality, including physicians' credentials, their adherence to clinical guidelines, and their patients' risk-adjusted health outcomes. To capture the effects of online ratings on patients' physician choices, the effects of ratings on patient flow, measured by physicians' patient revenue and patient volume, are examined. They find that a 1-star increase in a physician's average rating increases physicians' patient revenue and volume by 1.9% and 1.2%, respectively.

7 Conclusion

A Benchmark model was introduced in which in an infinitely repeated game one doctor chooses the quality of a treatment. A myopic patient drawn from a distribution of heterogeneous types every period decides whether or not to get treated by the doctor. The doctor gains revenue from every patient who decides to get treated. Patients take into consideration their base utility induced by their type in case of no treatment and the expected utility in case of treatment. In equilibrium, both types of the doctor will exert the low quality $q_L = 0$ and no patient will visit the doctor.

Next, I analyzed the influence of public ratings on a rating platform on the quality choice of the doctor. When making their decision, patients now rely on ratings former patients of the doctor left in previous periods of the game. The expected utility of the patients is dependent on their belief of the doctor's type. The belief is updated according to the ratings of the doctor. Two types of equilibria are identified. In the Separating Equilibrium, a type H doctor will exert the costly high quality in every period while a type L doctor treats patients with low quality. In the Pooling Equilibrium, both types of the doctor exert the low quality. In the Benchmark model, the Pooling equilibrium is the unique equilibrium. In the PRW setting on the other hand, the Separating Equilibrium takes place if the belief is greater than or equal to the ratio between period cost and the expected discounted future profit after patients have learned the true type of the doctor weighted with the probability of this event. As the belief will be updated in the rating platform model after the true quality of the doctor is exposed through a positive rating, the type H doctor has an incentive to invest in the high quality if the discount

factor exceeds a determined threshold. Intuitively, in the PRW setting, a type H doctor whose discount factor is too small does in this case not invest in a good reputation by exerting the high quality but instead exploits the prior belief of the patients to earn a short-term profit before being exposed as playing q_L . The results hold if the doctor can choose the quality every period as well as if the doctor chooses before the beginning of the game. Patients and the doctor are never worse off in the rating platform model compared to the Benchmark model independent of the discount factors δ and β . In the Separating-Pooling region, the doctor's ex ante expected profit and patients' ex ante expected utility in the rating platform model are higher than in the Benchmark model. In the Pooling-Pooling region, doctor's ex ante expected profit and patients' ex ante expected utility in the Benchmark model are equal to the according profit and utility in the rating platform model. The analysis was furthermore extended by allowing the type H doctor to play a mixed strategy. If the belief of the patients is sufficiently high, the competent doctor chooses a positive probability with which q_H is exerted in dependence of the belief. Depending on the belief threshold value, the doctor can receive a finite positive number of negative ratings before patients expect the doctor to always exert the low quality. For smaller values of the belief, if the belief decreases after a negative rating, implying the number of negative ratings the doctor can still receive also decreased, the type H doctor will play the high quality with higher probability than before. If the patients' belief is equal to the belief threshold that separates the equilibrium regions, the competent doctor exerts the high quality with probability one. It is possible that a patient is dissatisfied with the competent doctor's treatment.

The doctor is never worse off in the PRW setting with mixed strategies compared to the Benchmark setting. The patients are not worse off if their discount factor β is sufficiently high. In the Pooling-Pooling region, the ex ante expected profit of the doctor and the ex ante expected utility of the patients in the Benchmark model are equal to the ex ante expected profit and utility in the rating platform model. In the Separating-Pooling region, the ex ante expected profit of the doctor and the ex ante expected utility of the patients in the Benchmark model are higher compared to the ex ante expected profit and utility in the rating platform model. In the Separating with Mixing-Pooling region, the ex ante expected profit of the doctor in the rating platform model is higher than in the Benchmark model. The ex ante expected utility of the patients in the PRW model if the doctor plays a mixed strategy is higher compared to the Benchmark setting, if β is high enough.

In future research it may be interesting to explore other kinds of patients. For example, as explored before, ratings on physician rating platforms are not only based on the quality of medical treatment but several other factors. Therefore, the perceived quality can be highly subjective. This means that information regarding the quality of medical treatment that is provided through ratings can be noisy. In the PRW models from section

4 and 5, patients do not report the true treatment quality but they give feedback if they were satisfied with the provided treatment quality which is dependent on each patient's type. Further increasing the degree of noise in the rating information will leave the basic relationship of the expected demand being an increasing function of the patients' belief untouched but will most likely result in a more complex expected demand function.

My analysis was also restricted to a monopolistic doctor who does not face competition. Furthermore, in the presented models, the PRW is not included as an additional agent. Including the rating platform as a third player in the game may give rise to interesting dynamics and further insights. This is another promising research plan. Lastly, I compared my findings in this paper to results from applied research, specifically on PRWs in Germany. Insights were mostly limited to the number of ratings for each physician and the overall numbers of 'good' or 'bad' ratings a physician received. A more thorough analysis of data sets including the history of ratings for different physicians may provide promising insights and comparison for current and future results.

A Proofs

Proof of Proposition 2. To find the threshold π^* it is sufficient to check for which values of π it would be profitable for the type H doctor to deviate from the equilibrium strategy and play $q_L = 0$ instead. Then, $V(0) = 0$ and $V(\pi)$ would have to satisfy

$$\begin{aligned} V(\pi) &> V^*(\pi) \\ \pi q_H w &> \pi q_H w - c + \delta \pi q_H \frac{q_H w - c}{1 - \delta} \\ \frac{c(1 - \delta)}{\delta q_H (q_H w - c)} &> \pi. \end{aligned} \tag{12}$$

The same is true when evaluating an one-shot deviation to playing q_L where the doctor returns to playing q_H after deviating to q_L for one period. A deviation in the first period is profitable if

$$\begin{aligned} \pi q_H w + \delta \pi q_H V(0) + \delta(1 - \pi q_H) V^*(\pi) &> V^*(\pi) \\ \pi q_H w + \delta \pi q_H V(0) &> (1 - \delta(1 - \pi q_H)) V^*(\pi) \\ \pi q_H w + \delta \pi q_H V(0) &> \pi q_H w - c + \delta \pi q_H V^*(1) \\ 0 &> -c + \delta \pi q_H V^*(1) \\ \frac{c(1 - \delta)}{\delta q_H (q_H w - c)} &> \pi \end{aligned}$$

This reasoning holds analogously for a deviation in any other period.

Define $\pi^* := \frac{c(1-\delta)}{\delta q_H (q_H w - c)}$. Because π^* is non negative, for it to be feasible it remains to

be ensured that $\pi^* \leq 1$:

$$\begin{aligned} \frac{c(1-\delta)}{\delta q_H(q_H w - c)} &\leq 1 \\ c &\leq \delta q_H(q_H w - c) + \delta c \\ \delta &\geq \frac{c}{q_H(q_H w - c) + c}. \end{aligned} \tag{13}$$

It is easy to see that if (13) holds true, it is always profitable for the type H doctor to play q_H every period if the belief is 1 and not profitable to deviate to playing q_L .

Defining $\delta_{\min} := \frac{c}{q_H(q_H w - c) + c}$ concludes the proof. \square

Proof of Proposition 3. It is easy to see that if the type H doctor's strategy $\sigma(h_t, \pi_t)$ is to play $q_L = 0$ in some given period t with a history of ratings h_t , it is not profitable for the doctor to deviate to playing q_H instead. The patients anticipate the doctor's strategy and the patient who is drawn in period t therefore expects the treatment quality to be zero, independent of the type of the doctor. Regardless of his own type, the patient will therefore choose to not get treated by the doctor. This also means that the belief and number of ratings will not change.

If patients anticipate that the type H doctor's strategy $\sigma(h_t, \pi_t)$ is to play q_H in some given period t with a history of ratings h_t , the probability that the doctor will be visited by a patient is given by $\pi_t q_H$. Therefore, the value function of a one-shot deviation to q_L is given by

$$V(\tilde{\sigma}(h_t, \pi_t)) = \pi_t q_H w + \delta \pi_t q_H V(\sigma(h_{t+1}, 0)) + \delta(1 - \pi_t q_H) V(\sigma(h_t, \pi_t)).$$

If the type H doctor chooses to exert the low quality and is visited by a patient, this patient will be unsatisfied with the treatment which results in a bad rating. As patients anticipate the doctor's strategy to be $\sigma(h_t, \pi_t)$ and expect the type H doctor to exert high quality in the given state, they will update their belief to 0 after a bad rating is reported. Note that if the belief is $\pi = 1$ the type H doctor would deviate to exerting the low quality if the belief of the patients is $\pi = 1$ and is then visited by a patient who will be unsatisfied with the rating, patients will recognize this out-of-equilibrium behavior and update their belief to $\pi = 0$. The value function in this case is given by $V(\sigma(h_t, 0)) = 0$ for all ex post histories h_t .

Deviating from strategy $\sigma(h_t, \pi_t)$ is profitable if $V(\tilde{\sigma}(h_t, \pi_t)) > V(\sigma(h_t, \pi_t))$:

$$\begin{aligned}
\pi_t q_H w + \delta \pi_t q_H V(\sigma(h_{t+1}, 0)) + \delta(1 - \pi_t q_H) V(\sigma(h_t, \pi_t)) &> V(\sigma(h_t, \pi_t)) \\
\pi_t q_H w + \delta \pi_t q_H V(\sigma(h_{t+1}, 0)) &> (1 - \delta(1 - \pi_t q_H)) V(\sigma(h_t, \pi_t)) \\
\pi_t q_H w &> \pi_t q_H w - c + \delta \pi_t q_H V(\sigma(h_{t+1}, 1)) \\
c &> \delta \pi_t q_H \frac{q_H w - c}{1 - \delta} \\
\frac{c(1 - \delta)}{\delta q_H (q_H w - c)} &> \pi_t.
\end{aligned}$$

Therefore, deviating from playing q_H is not profitable if $\pi \geq \pi^*$ and deviating from q_L is never profitable. It is easy to see that if $\delta < \delta_{\min}$, no feasible π_t exists. \square

Proof of Proposition 5. Consider first the ex ante expected profit of the doctor in both equilibrium settings for both models. If $\pi_0 \geq \pi^*$ (ensuring the Separating-Pooling region), the ex ante expected profit of the doctor in the rating platform model, PR_{SP}^{PRW} , is given by

$$\pi_0 \cdot \frac{\pi_0 q_H w - c + \delta \pi_0 q_H \frac{q_H w - c}{1 - \delta}}{1 - \delta + \delta \pi_0 q_H} + (1 - \pi_0) \cdot \frac{\pi_0 q_H w}{1 - \delta + \delta \pi_0 q_H} = \frac{\pi_0 (q_H w - c) + \delta \pi_0^2 q_H \frac{q_H w - c}{1 - \delta}}{1 - \delta + \delta \pi_0 q_H} \quad (14)$$

and the ex ante expected profit of the doctor in the benchmark model, PR_{SP}^B , is zero. Therefore, $PR_{SP}^B < PR_{SP}^{PRW}$ and the doctor's ex ante expected profit is higher in rating platform model than in Benchmark model in the Separating-Pooling region.

If $\pi_0 < \pi^*$, in the Pooling-Pooling region, patients anticipate that both types of the doctor exert the low quality and no type of patient visits the doctor. The doctor's ex ante expected profit is the same in the Benchmark model and in the rating platform model: $PR_{PP}^B = PR_{PP}^{PRW} = 0$.

The ex ante expected utility of the patients is equal as well: Because patients expect the doctor to always exert the low quality, no type of patient will get treated. In every period, a type of patient is drawn from the uniform distribution who will not get treated by the doctor and receives the base utility equal to the patient's type. The period utilities are discounted with β :

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 \alpha \, d\alpha = \frac{1}{1 - \beta} \int_0^1 \alpha \, d\alpha = \frac{1}{2(1 - \beta)}. \quad (15)$$

In the Separating-Pooling region in the rating platform model, after the first patient who received treatment from the doctor, future patients learn the doctor's true type. This happens with probability $\pi_0 q_H$. Otherwise, the belief of the patients stays the same.

Patients can only update their belief after a type of patient is drawn who decides to visit the doctor. Every period, either a type of patient is drawn from the uniform distribution who decides to visit the doctor and receives the utility according to the doctor's type. Then, the belief is updated. Or, if a type is drawn who decides to not get treated by the doctor, this type will receive the according base utility and the belief again stays the same. The value function of the patients is given by

$$U_H(\pi_0) = \int_0^{\pi_0 q_H} q_H d\alpha + \int_{\pi_0 q_H}^1 \alpha d\alpha + \beta \pi_0 q_H \frac{\int_0^{q_H} q_H d\alpha + \int_{q_H}^1 \alpha d\alpha}{1 - \beta} + \beta(1 - \pi_0 q_H)U_H(\pi_0), \quad (16)$$

if the doctor's type is H and

$$U_L(\pi_0) = \int_{\pi_0 q_H}^1 \alpha d\alpha + \beta \pi_0 q_H \frac{\int_0^1 \alpha d\alpha}{1 - \beta} + \beta(1 - \pi_0 q_H)U_L(\pi_0), \quad (17)$$

if the doctor's type is L .

With probability π_0 , patients play against type H . Combining and simplifying the equations above, the ex ante expected utility of the patients is then given by

$$\begin{aligned} \pi_0 \left(\frac{\pi_0 q_H^2 + \frac{1}{2} \left(1 - \pi_0^2 q_H^2 + \beta \pi_0 q_H \frac{1 + q_H^2}{1 - \beta} \right)}{1 - \beta(1 - \pi_0 q_H)} \right) + (1 - \pi_0) \left(\frac{\frac{1}{2} \left(1 - \pi_0^2 q_H^2 + \beta \pi_0 q_H \frac{1}{1 - \beta} \right)}{1 - \beta(1 - \pi_0 q_H)} \right) \\ = \frac{(1 + \pi_0^2 q_H^2)(1 - \beta) + \beta \pi_0 q_H (1 + \pi_0 q_H^2)}{2(1 - \beta)(1 - \beta + \beta \pi_0 q_H)}. \end{aligned} \quad (18)$$

From comparing the ex ante expected profit and utility, Proposition 5 follows. \square

Proof of Proposition 7. Given belief π , for mixing between playing q_L and q_H to be profitable for the type H doctor in equilibrium, it must hold that deviating from mixing with some probability $p > 0$ to either playing the pure strategy q_H or the pure strategy q_L in a period is not profitable.

Note first, that if the belief is $\pi = 1$, i.e. patients know that the true type of the doctor is H , the value function is given by

$$V(1) = \frac{p(q_H w - c)}{1 - \delta}$$

which is maximized for $p = 1$ and the type H doctor will therefore always play q_H in equilibrium. Furthermore, a one-shot deviation to mixing with probability $p \in (0, 1)$ is

not profitable for the doctor as the value function

$$V^p(1) = q_H w - pc + \delta p q_H V(1) + \delta(1-p)q_H V(0) + \delta(1-q_H)V(1)$$

is only greater than $V(1)$ if $p > 1$ for $\delta \geq \delta_{\min}$.

Suppose the belief in some period is π and $V(\pi^-)$ denotes the value function which captures the expected profit after a bad review in this period. The value function from mixing with probability p is given by :

$$V(\pi) = p(\pi q_H w - c) + \delta p^2 \pi q_H V(1) + \delta p(1-p)\pi q_H V(\pi^-) + \delta(1-p\pi q_H)V(\pi)$$

which can be simplified to

$$V(\pi) = \frac{p(\pi q_H w - c) + \delta p^2 \pi q_H V(1) + \delta p(1-p)\pi q_H V(\pi^-)}{1 - \delta(1-p\pi q_H)}. \quad (19)$$

On the other hand, the value function from deviating to playing the pure strategy q_H is given by

$$V^{q_H} = p\pi q_H w - c + \delta p\pi q_H V(1) + \delta(1-p\pi q_H)V^{q_H}$$

which can be simplified to

$$V^{q_H} = \frac{p\pi q_H w - c + \delta p\pi q_H V(1)}{1 - \delta(1-p\pi q_H)} \quad (20)$$

and analogously the simplified value function from deviating to the pure strategy q_L is given by

$$V^{q_H} = \frac{p\pi q_H w + \delta p\pi q_H V(0)}{1 - \delta(1-p\pi q_H)}. \quad (21)$$

Combining (20) and (21), $V(\pi^-)$ must fulfill

$$\begin{aligned} p\pi q_H w - pc + \delta p^2 \pi q_H V(1) + \delta p(1-p)\pi q_H V(\pi^-) &\geq p\pi q_H w - c + \delta p\pi q_H V(1) \\ (1-p)c + \delta p(1-p)\pi q_H V(\pi^-) &\geq \delta p(1-p)\pi q_H V(1) \\ V(\pi^-) &\geq V(1) - \frac{c}{\delta p\pi q_H} \end{aligned} \quad (22)$$

and at the same time

$$\begin{aligned}
p\pi q_H w - pc + \delta p^2 \pi q_H V(1) + \delta p(1-p)\pi q_H V(\pi^-) &\geq p\pi q_H w + \delta p\pi q_H V(\pi^-) \\
\delta p^2 \pi q_H V(1) - pc &\geq \delta p^2 \pi q_H V(\pi^-) \\
V(1) - \frac{c}{\delta p \pi q_H} &\geq V(\pi^-).
\end{aligned} \tag{23}$$

From this follows the optimality condition

$$V(\pi^-) = V(1) - \frac{c}{\delta p \pi q_H}. \tag{24}$$

To verify that the same condition holds when evaluating one-shot deviations note that

$$\begin{aligned}
p\pi q_H w - pc + \delta p^2 \pi q_H V(1) + \delta p(1-p)\pi q_H V(\pi^-) + \delta(1-p\pi q_H)V(\pi) \\
\geq p\pi q_H w - c + \delta p\pi q_H V(1) + \delta(1-p\pi q_H)V(\pi) \\
p\pi q_H w - pc + \delta p^2 \pi q_H V(1) + \delta p(1-p)\pi q_H V(\pi^-) \geq p\pi q_H w - c + \delta p\pi q_H V(1)
\end{aligned} \tag{25}$$

and

$$\begin{aligned}
p\pi q_H w - pc + \delta p^2 \pi q_H V(1) + \delta p(1-p)\pi q_H V(\pi^-) + \delta(1-p\pi q_H)V(\pi) \\
\geq p\pi q_H w + \delta p\pi q_H V(\pi^-) + \delta(1-p\pi q_H)V(\pi) \\
p\pi q_H w - pc + \delta p^2 \pi q_H V(1) + \delta p(1-p)\pi q_H V(\pi^-) \geq p\pi q_H w + \delta p\pi q_H V(\pi^-)
\end{aligned} \tag{26}$$

yield the same equations used above for deriving the optimality condition.

Suppose the value of the belief is π where it is profitable to play the mixed strategy $(p(\pi), (1-p(\pi)))$ for the type H doctor and after a bad review the belief decreases to π^- , where it is no longer profitable to mix but instead the best answer of the type H doctor is to play q_L . Then, from (24) and using the definitions of $V(0)$ and $V(1)$, it follows that

$$V(0) = V(1) - \frac{c}{\delta p(\pi)\pi q_H} \Leftrightarrow p(\pi) = \frac{c(1-\delta)}{\delta \pi q_H (q_H w - c)}. \tag{27}$$

For (27) to be feasible, it must hold that

$$\begin{aligned}
\frac{c(1-\delta)}{\delta \pi q_H (q_H w - c)} &\leq 1 \\
\frac{c(1-\delta)}{\delta q_H (q_H w - c)} &\leq \pi,
\end{aligned}$$

implying the condition $\pi \geq \pi^*$ on the belief for mixing to be the equilibrium strategy in this case.

Furthermore, suppose there exists a threshold $\tilde{\pi}$, where for all values of the belief $\pi < \tilde{\pi}$ the updated belief after a negative rating is $\pi^- < \pi^*$, for all values $\pi' > \tilde{\pi}$ the

updated belief after a negative rating is higher than π^* and $\tilde{\pi}^- = \pi^*$.

Rearranging equation (24) yields that for $\pi \geq \tilde{\pi}$ the optimal probability p is given by $p(\tilde{\pi}) = \frac{c}{\delta \tilde{\pi} q_H (V(1) - V(\pi^*))}$. Furthermore, $p(\pi) = 1$ if $\pi = \pi^*$. Using

$$V(1) - V(\pi^*) = V(1) - \frac{\pi^* q_H w - c + \delta \pi^* q_H V(1)}{1 - \delta + \delta \pi^* q_H} = \frac{q_H w (1 - \pi^*)}{1 - \delta + \delta q_H \pi^*},$$

implies that

$$p(\tilde{\pi}) \tilde{\pi} = \frac{c(1 - \delta + \delta \pi^* q_H)}{\delta q_H^2 w (1 - \pi^*)}.$$

It follows that

$$\begin{aligned} \pi^* &= \frac{\tilde{\pi}(1 - p(\tilde{\pi}))}{1 - p(\tilde{\pi})\tilde{\pi}} \\ \pi^* &= \frac{\tilde{\pi} \delta q_H^2 w (1 - \pi^*) - c(1 - \delta + \delta \pi^* q_H)}{\delta q_H^2 w (1 - \pi^*) - c(1 - \delta + \delta \pi^* q_H)} \\ \tilde{\pi} \delta q_H^2 w (1 - \pi^*) &= \pi^* (\delta q_H^2 w (1 - \pi^*) - c(1 - \delta + \delta \pi^* q_H)) + c(1 - \delta + \delta \pi^* q_H) \\ \tilde{\pi} &= \frac{\pi^* \delta q_H^2 w (1 - \pi^*) + (1 - \pi^*) c(1 - \delta + \delta \pi^* q_H)}{\delta q_H^2 w (1 - \pi^*)} \\ \tilde{\pi} &= \pi^* + c \frac{1 - \delta + \delta \pi^* q_H}{\delta q_H^2 w} \\ \tilde{\pi} &= \pi^* \left(1 + \frac{c}{q_H w} \right) + c \frac{1 - \delta}{\delta q_H^2 w} \end{aligned}$$

and using $\pi^* = \frac{c(1 - \delta)}{\delta q_H (q_H w - c)}$

$$\begin{aligned} \tilde{\pi} &= \frac{c(1 - \delta)}{\delta q_H (q_H w - c)} \frac{q_H w + c}{q_H w} + c \frac{1 - \delta}{\delta q_H^2 w} \\ \tilde{\pi} &= \frac{c(1 - \delta)}{\delta q_H^2 w} \left(\frac{q_H w + c}{q_H w - c} + 1 \right) \\ \tilde{\pi} &= \frac{c(1 - \delta)}{\delta q_H^2 w} \left(\frac{2q_H w}{q_H w - c} \right) \\ \tilde{\pi} &= \frac{2c(1 - \delta)}{\delta q_H (q_H w - c)} \end{aligned}$$

Note, that $\tilde{\pi} = 2\pi^*$.

Furthermore, for all values of the belief in the range $(\pi^*, \tilde{\pi})$, the value function of the type H doctor is given by $V(\pi) = c \setminus (\delta q_H)$. The same is true for the value function if the

belief is π^* . This can be verified by using the fact that if the belief drops to a value in the range $(0, \pi^*)$ after a negative rating, the value function of the type H doctor in this case is zero. Plugging this into the expression for the value function if $\pi \in (\pi^*, \tilde{\pi})$ and if $\pi = \pi^*$, respectively, yields the result.

Lastly, note that equation (19) if the doctor is mixing with the optimal probability p can be simplified to

$$\begin{aligned}
V(\pi) &= \frac{\frac{c(\pi q_H w - c + \delta \pi q_H V(\pi^-))}{\delta \pi q_H (V(1) - V(\pi^-))} + \frac{c^2 \delta \pi q_H ((V(1) - V(\pi^-))}{\delta^2 \pi^2 q_H^2 (V(1) - V(\pi^-))^2}}{\frac{(1-\delta)\delta \pi q_H (V(1) - V(\pi^-)) + c \delta \pi q_H}{\delta \pi q_H (V(1) - V(\pi^-))}} \\
&= \frac{c(\pi q_H w - c + \delta \pi q_H V(\pi^-)) + c^2}{(1-\delta)\delta \pi q_H (V(1) - V(\pi^-)) + c \delta \pi q_H} \\
&= \frac{c \pi q_H (w + \delta V(\pi^-))}{\delta \pi q_H (q_H w - c - (1-\delta)V(\pi^-) + c)} \\
&= \frac{c(w + \delta V(\pi^-))}{\delta (q_H w - (1-\delta)V(\pi^-))}. \tag{28}
\end{aligned}$$

Rearranging equation (24) for p , plugging the result into the formula of π^- and rearranging for π yields the recursive formula for π : For $k \geq 1$, the interval boundaries are defined by the recursive formula

$$\pi^k = \frac{\pi^{k-1} (\delta q_H (V(1) - V(\pi^{k-1})) - c) + c}{\delta q_H (V(1) - V(\pi^{k-1}))} \tag{29}$$

and $\pi^0 = \pi^*$.

From (28), the recursive formula for the value function is obtained: For $k \geq 1$, the value function of the type H doctor is defined by

$$V(\pi^k) = \frac{c(w + \delta V(\pi^{k-1}))}{\delta (q_H w - (1-\delta)V(\pi^{k-1}))} \tag{30}$$

and $V(\pi^0) = \frac{c}{\delta q_H}$.

To finish the proof of case (i) assume that $1 \geq \pi^{k+1} \geq \pi^k$ for all k such that the limit of (π^k) exists and $\lim_{k \rightarrow \infty} \pi^k = \bar{\pi} \leq 1$. If $\pi^k = 1$ would hold, the type H doctor would not mix but play the pure strategy q_H . Therefore, from the recursive formula of π^k in

(29) follows for $\pi^{k+1} \leq 1$ and $\pi^k < 1$ that

$$\begin{aligned}
\frac{\pi^k (\delta q_H (V(1) - V(\pi^k)) - c) + c - \delta q_H (V(1) - V(\pi^k))}{\delta q_H (V(1) - V(\pi^k))} &\leq 0 \\
\frac{c - \delta q_H (V(1) - V(\pi^k))}{\delta q_H (V(1) - V(\pi^k))} (1 - \pi^k) &\leq 0 \\
\frac{c}{\delta q_H (V(1) - V(\pi^k))} &\leq 1 \\
V(\pi^k) &\leq V(1) - \frac{c}{\delta q_H} \quad (31)
\end{aligned}$$

must then hold for all k .

The value function V is monotone increasing and bounded above, therefore $\lim_{k \rightarrow \infty} V(\pi^k) = \bar{V}$ exists. From the recursive formula of π^k follows that

$$\begin{aligned}
\bar{\pi} &= \frac{\bar{\pi} (\delta q_H (V(1) - \bar{V}) - c) + c}{\delta q_H (V(1) - \bar{V})} \\
\bar{\pi} \delta q_H (V(1) - \bar{V}) &= \bar{\pi} (\delta q_H (V(1) - \bar{V}) - c) + c \\
\bar{\pi} &= 1.
\end{aligned}$$

Therefore $\lim_{k \rightarrow \infty} \pi^k = \bar{\pi} = 1$. But from this follows $\lim_{k \rightarrow \infty} V(\pi^k) = V(1)$, contradicting equation (31).

If for some K , $V(1) > V(\pi^K) > V(1) - \frac{c}{\delta q_H}$ from which follows that

$$\begin{aligned}
\frac{c}{\delta q_H} &> V(1) - V(\pi^K) \\
\frac{1}{V(1) - V(\pi^K)} &> \frac{\delta q_H}{c}.
\end{aligned}$$

If the doctor could choose an optimal mixing probability $p(\pi) < 1$, the optimality condition for $\pi > \pi^K$ would imply a contradiction

$$\pi > \frac{c}{\delta q_H} \cdot \frac{1}{V(1) - V(\pi^K)} > \frac{c}{\delta q_H} \cdot \frac{\delta q_H}{c}.$$

It is easy to see that it is not profitable for the type H doctor to deviate from playing the pure strategy q_H in equilibrium, if $\pi = \pi^*$. \square

Proof of Proposition 9. If $\pi_0 > \pi^* \geq 0.5$, the ex ante expected profit of the type H doctor is given by $c \setminus (\delta q_H)$. The same is true for the type L doctor. If $\pi > \pi^*$, the type H doctor exerts q_H with probability $p(\pi) = \frac{c}{\delta \pi q_H V(1)}$. After a negative rating, the belief decreases to $\pi^- < \pi^*$ and patients anticipate that both types of the doctor exert $q_L = 0$. Consider the case of type L . In the first period, a patient is drawn who decides to visit the doctor

with probability $p(\pi_0)\pi_0$. After a visit, the doctor will receive a negative rating and the belief falls to $\pi_0^- < \pi^*$. In all following periods, the patients will expect both types of the doctor to exert the low quality $q_L = 0$. Therefore, the ex ante expected profit of the type L doctor is given by

$$\frac{p(\pi_0)\pi_0q_Hw}{1 - \delta + \delta p(\pi_0)\pi_0q_H} = \frac{\frac{c}{\delta\pi_0q_HV(1)}\pi_0q_Hw}{1 - \delta + \delta\frac{c}{\delta\pi_0q_HV(1)}\pi_0q_H} = \frac{\frac{cw}{\delta V(1)}}{1 - \delta + \frac{c}{V(1)}} = \frac{cw}{\delta(q_Hw - c) + \delta c} = \frac{c}{\delta q_H}.$$

It follows that the ex ante expected profit of the doctor is given by $PR^{PRW} = \frac{c}{\delta q_H}$.

With probability π_0 , the doctor's type is H and the ex ante expected utility of the patients in this case is given by

$$\begin{aligned} & \frac{p(\pi_0) \int_0^{p(\pi_0)\pi_0q_H} q_H d\alpha + \int_{p(\pi_0)\pi_0q_H}^1 \alpha d\alpha}{1 - \beta(1 - p(\pi_0)\pi_0q_H)} \\ & + \frac{p(\pi_0)^2\pi_0q_H\beta \frac{\int_0^{q_H} q_H d\alpha + \int_{q_H}^1 \alpha d\alpha}{1 - \beta} + (1 - p(\pi_0))p(\pi_0)\pi_0q_H\beta \frac{\int_0^1 \alpha d\alpha}{1 - \beta}}{1 - \beta(1 - p(\pi_0)\pi_0q_H)} \end{aligned} \quad (32)$$

And with probability $1 - \pi_0$, the doctor's type is L and the ex ante expected utility in this case is given by

$$\frac{\int_{p(\pi_0)\pi_0q_H}^1 \alpha d\alpha + p(\pi_0)\pi_0q_H\beta \frac{\int_0^1 \alpha d\alpha}{1 - \beta}}{1 - \beta(1 - p(\pi_0)\pi_0q_H)}. \quad (33)$$

Combining the results from above, the ex ante expected utility of the patients is given by

$$\frac{p(\pi_0)^2\pi_0^2q_H^2(1 - \beta) + \beta p(\pi_0)\pi_0q_H(1 + p(\pi_0)\pi_0q_H^2)}{2(1 - \beta)(1 - \beta(1 - p(\pi_0)\pi_0q_H))}. \quad (34)$$

The ex ante utility of the patients exceeds the ex ante expected utility in the Benchmark model, if $\beta > \frac{1 - p(\pi_0)^2\pi_0q_H^2}{1 - (1 - q_H)p(\pi_0)^2\pi_0^2q_H^2}$.

If $\pi_0 = \pi^*$, the type H doctor will play the pure strategy q_H . The ex ante expected profit of the doctor and the ex ante expected utility of the patients are the same as in the Separating-Pooling region in the Physician Rating Website model for $\pi_0 = \pi^*$. The ex ante utility of the patients if the belief is $\pi = \pi^*$ is

$$\frac{\int_{\pi q_H}^1 \alpha d\alpha + \beta\pi q_H \frac{\int_0^1 \alpha d\alpha}{1 - \beta}}{1 - \beta(1 - \pi q_H)}, \quad (35)$$

if the doctor's type is L . The intuition is as follows. If a type of patient is drawn, who does not visit the doctor, the patient receives the base utility and the belief does not change. If a type of patient is drawn who visits the doctor, the patient receives a utility

of zero. The belief is updated to zero from the next period on, all patients do not visit the doctor and receive their base utility α according to their type. The ex ante utility of the patients is

$$\frac{\int_0^{\pi q_H} q_H d\alpha + \int_{\pi q_H}^1 \alpha d\alpha + \beta \pi q_H \frac{\int_0^{q_H} q_H d\alpha + \int_{q_H}^1 \alpha d\alpha}{1-\beta}}{1 - \beta(1 - \pi q_H)}, \quad (36)$$

if the doctor's type is H . This is because, if a type of patient is drawn who does not visit the doctor, the patient receives the base utility. if the doctor is visited, the patient will receive a utility of q_H and the belief increases to 1. Afterwards, the doctor always exerts the high quality and all visiting patients receive a utility of q_H .

Analogously to the pure strategy case, the doctor's type is H with probability π_0 which in this case then equals π^* . The ex ante expected utility of the patients is given by (18) with $\pi_0 = \pi^* = \frac{c(1-\delta)}{\delta q_H w (q_H w - c)}$:

$$\frac{(1 + (\pi^*)^2 q_H^2)(1 - \beta) + \beta \pi^* q_H (1 + \pi^* q_H^2)}{2(1 - \beta)(1 - \beta + \beta \pi^* q_H)}. \quad (37)$$

Consider $1 > \pi_0 > \pi^*$ and $\pi^* < 0.5$. The ex ante expected profit of the type H doctor in the mixed strategy equilibrium if $\pi_0 > \pi^*$ is given by

$$\frac{c(w + \delta V(\pi_0^-))}{\delta(q_H w - (1 - \delta)V(\pi_0^-))}$$

and the ex ante expected profit of the type L doctor is given by

$$\frac{p(\pi_0)\pi_0 q_H w}{1 - \delta + \delta p(\pi_0)\pi_0 q_H} = \frac{\frac{c\pi_0 q_H w}{\delta\pi_0 q_H (V(1) - V(\pi_0^-))}}{\frac{(1-\delta)\delta\pi_0 q_H (V(1) - V(\pi_0^-)) + \delta c\pi_0 q_H}{\delta\pi_0 q_H (V(1) - V(\pi_0^-))}} = \frac{cw}{\delta(q_H w - (1 - \delta)V(\pi_0^-))}.$$

Therefore, the ex ante expected doctor's rent is given by the recursive formula

$$\pi_0 \frac{c(w + \delta V(\pi_0^-))}{\delta(q_H w - (1 - \delta)V(\pi_0^-))} + (1 - \pi_0) \frac{cw}{\delta(q_H w - (1 - \delta)V(\pi_0^-))} = \frac{c\delta\pi_0 V(\pi_0^-) + cw}{\delta(q_H w - (1 - \delta)V(\pi_0^-))}.$$

The ex ante profit of the doctor is increasing in $V(\pi_0^-)$ and $\pi_0 > \pi^*$, this implies $V(\pi_0^-) \geq V(0)$. Therefore, the ex ante expected profit of the doctor is bound below by $\frac{c}{\delta q_H} > 0$.

In every period a type of patient is drawn from the uniform distribution. The period utilities are discounted with β . With probability of π^0 , the doctor's type is H . If a type of patient is drawn who visits the doctor, with probability of $p(\pi)$, this patient receives a

utility of q_H and the belief is updated to 1. In the following periods, all types of patients whose base utility is smaller or equal to q_H visit the doctor and receive a utility of q_H . With probability $1 - p(\pi)$, the patient is treated with $q_L = 0$, gives a negative rating and the belief decreases. All remaining types receive their base utility according to their type α . If the doctor's type is L and a type of patient is drawn who visits the doctor, the patient receives a utility of 0. The patient gives a negative rating and the belief decreases. If the doctor is not visited, the belief does not change. If $\pi > \pi^*$, the value function of the patients is given by the recursive formulas

$$\begin{aligned} & \frac{p(\pi) \cdot \int_0^{p(\pi)\pi q_H} q_H d\alpha + \int_{p(\pi)\pi q_H}^1 \alpha d\alpha}{1 - \beta(1 - p(\pi)\pi q_H)} \\ & + \frac{\beta p(\pi)^2 \pi q_H \cdot \frac{\int_0^{q_H} q_H d\alpha + \int_{q_H}^1 \alpha d\alpha}{1 - \beta} + \beta(1 - p(\pi))p(\pi)\pi q_H U(\pi^-)}{1 - \beta(1 - p(\pi)\pi q_H)} \end{aligned} \quad (38)$$

if the doctor's type is H and by

$$\frac{\int_{p(\pi)\pi q_H}^1 \alpha d\alpha + \beta p(\pi)\pi q_H U(\pi^-)}{1 - \beta(1 - p(\pi)\pi q_H)} \quad (39)$$

if the doctor's type is L . Furthermore, $p(\pi) = \frac{c}{\delta \pi q_H (V(1) - V(\pi^-))}$. If $\pi = \pi^*$, the value functions are given as in (35) and (36), respectively. If $\pi < \pi^*$, all patients receive their base utility, as described in the beginning of the section.

The doctor's type is H with probability π_0 . Therefore, the ex ante expected utility of the patients is given by the recursive formula

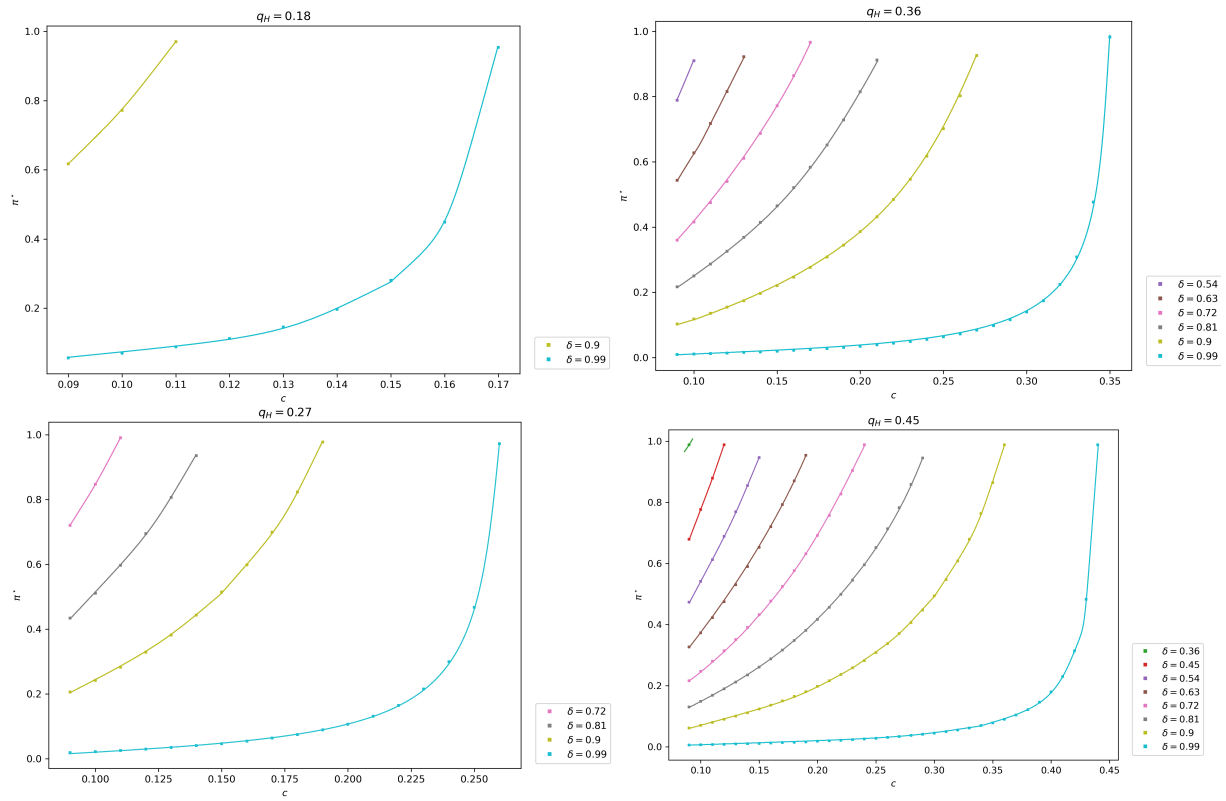
$$\begin{aligned} & \frac{p(\pi_0) \cdot \int_0^{p(\pi_0)\pi_0 q_H} q_H d\alpha + \int_{p(\pi_0)\pi_0 q_H}^1 \alpha d\alpha}{1 - \beta(1 - p(\pi_0)\pi_0 q_H)} \\ & + \frac{\beta p(\pi_0)^2 \pi_0^2 q_H \frac{\int_0^{q_H} q_H d\alpha + \int_{q_H}^1 \alpha d\alpha}{1 - \beta} + \beta p(\pi_0)(1 - p(\pi_0)\pi_0)\pi_0 q_H U(\pi_0^-)}{1 - \beta(1 - p(\pi_0)\pi_0 q_H)}. \end{aligned} \quad (40)$$

As the formula is increasing in $U(\pi_0^-)$ and $U(\pi_0^-) \geq \frac{\int_0^1 \alpha d\alpha}{1 - \beta} = \frac{1}{2(1 - \beta)}$, the ex ante utility of the patients is bound below by

$$\begin{aligned} & \frac{p(\pi_0) \cdot \int_0^{p(\pi_0)\pi_0 q_H} q_H d\alpha + \int_{p(\pi_0)\pi_0 q_H}^1 \alpha d\alpha}{1 - \beta(1 - p(\pi_0)\pi_0 q_H)} \\ & + \frac{\beta p(\pi_0)\pi_0 q_H \left(p(\pi_0)\pi_0 \frac{\int_0^{q_H} q_H d\alpha + \int_{q_H}^1 \alpha d\alpha}{1 - \beta} + (1 - p(\pi_0)\pi_0) \frac{\int_0^1 \alpha d\alpha}{1 - \beta} \right)}{1 - \beta(1 - p(\pi_0)\pi_0 q_H)} \\ & = \frac{(1 - p(\pi_0)^2 \pi_0^2 q_H^2)(1 - \beta) + \beta p(\pi_0)\pi_0 q_H (1 + p(\pi_0)\pi_0 q_H^2)}{2(1 - \beta)(1 - \beta(1 - p(\pi_0)\pi_0 q_H))}. \end{aligned} \quad (41)$$

The ex ante expected utility patients in the Benchmark model is not greater than (41). \square

B Graphs



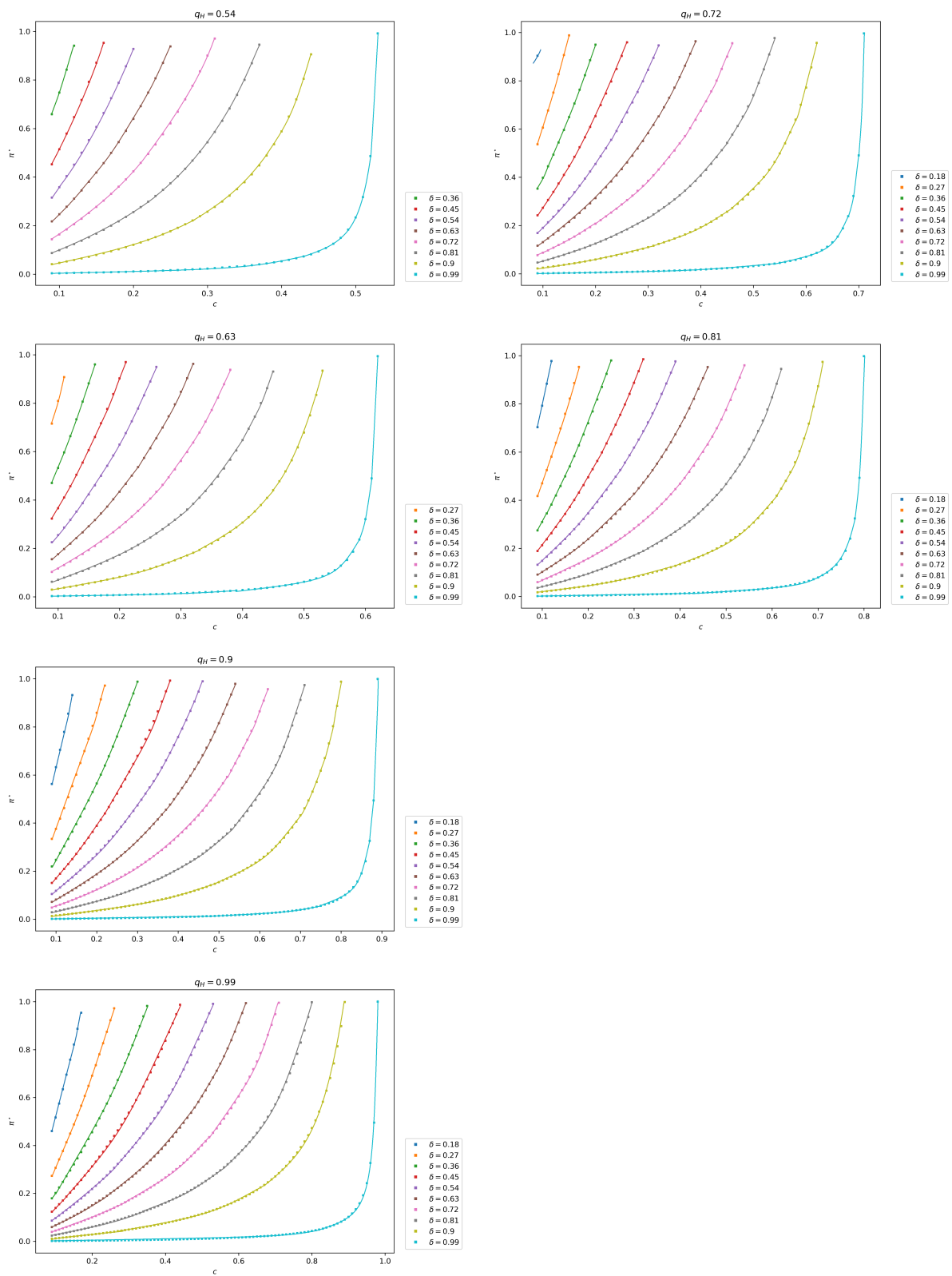


Figure 11: The threshold π^* in dependence of c if q_H and δ are fixed.

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