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## Article

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# What remains of the Cambridge critique of capital theory, if reswitching and reverse capital deepening are empirically rare and theoretically unlikely?

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*The paper summarises the main results of the Cambridge controversy on capital theory and discusses its actual relevance. The paradoxes that had first been regarded as most relevant (reswitching and reverse capital deepening) have turned out to be empirically rare, and this can be explained theoretically, but both neoclassical and anti-neoclassical Wicksell effects are ubiquitous. The number of efficient techniques that turn up on the envelope of the wage curves of a spectrum of techniques can be shown to be quite small both empirically and theoretically, which constitutes a new critique. It has implications for employment policies.*

**Keywords:** capital theory, production function, reswitching, Sraffa, employment

**JEL codes:** B24, C62, C67, D57

## 1 THE CAMBRIDGE CRITIQUE: A SHORT HISTORY

The Cambridge–Cambridge controversy was arguably the most important theoretical debate among economists in the second half of the twentieth century. It is difficult to imagine such a heated exchange about so abstract an argument today. As we shall see, the debate has not really been concluded, but it receives much less attention now and most younger economists know next to nothing about it. The purpose of this paper is to explain the core arguments once more, in a perspective that will lead to new conclusions. The original debate was entirely theoretical – pure armchair economics and incomplete *a priori* reasoning. Empirical analysis came later and brought results that did not correspond to the theoretical presuppositions of either side. Now the task is to explain these results by modifying the theory, and that affects the conclusions as well. We begin with a historical survey.

Capital theory has been controversial since Joan Robinson's (1953–1954 [1964]) essay of 1953, 'The production function and the theory of capital', which questions the marginal productivity theory of distribution. It implies that firms employ labour up to the point where the marginal product is equal to the wage, and this means, for all firms

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taken together, that the wage is equal to the marginal product at that given level of employment. But labour demand adapts to labour supply in this view through changes of technique – more labour-intensive methods must be introduced in order to absorb more labour – and this means that equipment changes and with it the amounts and prices of capital goods, while the concept of a marginal product of labour requires that capital be kept constant. The formula of the production function uses ‘capital’ as a value magnitude, but how is the value of the capital to be measured, if the prices change in the course of adaptation?

Two books appeared in 1960 that elevated the discussion following Joan Robinson’s challenge to a new level. Pierangelo Garegnani (1960) showed that the main exponents of the neoclassical theory of distribution, notably Walras and Böhm-Bawerk, used capital as a value concept in their analysis of distribution between wages and profit through supply and demand. If the capital goods were given in arbitrary proportions as the supply to be used in production, a uniform rate of profit could not be established on the value of the capital goods because of their different scarcities (Eatwell 1987). Hence, a uniform rate of profit could only be achieved by assuming that capital was given as a value magnitude, and the composition of capital had to be determined according to the final demand so that the endowments of capital goods had to be treated not as givens, but as unknowns to be determined in a long-period equilibrium (Petri 2004; Schefold 2018).

Sraffa’s book (1960) initiated a revival of classical thought by showing how long-period equilibria (with a uniform rate of profit on the capital goods advanced and measured in the long-run prices) could be derived, if the technique (the persistence of which is at the origin of the persistence of the long-run prices) is given together with the state of final demand. In this classical perspective, distribution does not result from demand and supply for the factors of production, but the level of the wage is given (in original classical theory as a minimum wage) so that profits follow as a residual, determined by the magnitude of the surplus, which remains if the means of reproduction and the wage goods are deducted from the gross product. Sraffa showed, in the spirit of Ricardo, how relative prices change with changes in the distribution variable (at first, as in the classical tradition, the wage, but then the rate of profit is regarded as given and wages become the residual).

At the time of Böhm-Bawerk, there were still Ricardian authors among the German academic economists (Schefold 2019a). Böhm-Bawerk argued his then-new version of neoclassical theory against them by first accepting this classical framework (which he did not derive formally), but he meant the same dependence of long-run prices on the structure of production, defined by the technique in use. He conceded that the workers needed the minimum wage – otherwise there could be no supply of labour. He objected that the same argument had to hold for capital, which was not forthcoming without an adequate return. Since profits are determined, if only one technique and a minimum wage is given, he postulated that there had to be many techniques, and the choice of technique had to be such that the claims of labour and capital could be fulfilled simultaneously. Both claims could be satisfied if the techniques could be ordered in such a way that a higher intensity of capital or capital–labour ratio was associated with a lower rate of profit and with higher wage rates. For if there was, for instance, unemployment of labour, the real wage would fall, the rate of profit would go up, the intensity of capital would fall, more labour-using techniques would be introduced and become profitable, and so the full employment equilibrium could be restored. As is well known, Böhm-Bawerk measured the amount of capital by the subsistence fund and the intensity of capital by the average period of production. Sraffa showed in his book, by means of an example, that the ordering of the techniques – necessary for Böhm-Bawerk’s construction – would not always obtain. It was possible that a technique with a high intensity of capital was

profitable at a low rate of profit, as Böhm-Bawerk would expect, and that, with a higher rate of profit, a technique with a lower intensity of capital and a shorter period of production would become profitable, but that, surprisingly for the neoclassical author, the first technique could reappear at a high rate of profit (Sraffa 1960: 38).

This was the true challenge for the neoclassical authors, for what is more fundamental in neoclassical theory than the hypothesis that the ratio of the rate of interest to the wage rate is inversely related to the capital–labour ratio? Sraffa’s hint was the tip of the iceberg. According to oral tradition, he deliberately left the exposure of the paradoxes in the theory of capital to a subsequent debate, in the belief that the impact of the discovery would be greater if many and diverse economists were involved in the investigation.

The main attempted answer came from Paul Samuelson (1962 [1971]). His paper on the ‘Surrogate production function’ showed how a production function with constant returns to scale and diminishing marginal products could be constructed from a system of wage curves for individual techniques, provided each technique could be characterised by an intensity of capital that was independent of distribution. The techniques could therefore be ordered in such a way that less and less capital-intensive techniques were profitable, the higher the rate of profit. But Sraffa had provided an example in which a technique with a high intensity of capital, which was profitable at a low rate of profit, returned to profitability at a high rate of profit, while techniques with a lower intensity of capital became profitable in between. In fact, not one but two paradoxes were involved here. On the one hand, it seemed paradoxical that the same technique could return; this was called ‘reswitching’. And the example implied that, with the rise in the rate of profit, a change from a lower intensity of capital to a higher intensity of capital was possible; this was called ‘reverse capital deepening’.

David Levhari, a PhD student of Paul Samuelson, published an article in which he tried to show that these phenomena were ruled out if the techniques were based on indecomposable matrices (so-called basic systems). Sraffa’s example was based on non-basics. But Levhari’s (1965) proof turned out to be mistaken, as was pointed out by Pasinetti, Garegnani and several other scholars, and the question now was what the paradoxes meant for the validity of neoclassical theory. Cambridge (UK) argued that neoclassical theory had been shown to be inconsistent and should be discarded. Cambridge (Massachusetts) preferred to say that an obstacle to the aggregation of capital had been found, that the production function was perhaps not a suitable instrument of rigorous theoretical reasoning, but that neoclassical theory in general was not affected. Individual statements were more differentiated, but we must here be content with a rough summary (Harcourt 1972; Kurz/Salvadori 1995).

The neoclassical production function would be used less for a while after this exchange, but it returned as if nothing had happened, especially in the context of the new theories of economic growth with endogenous technical progress. This state of affairs was unsatisfactory for those who had been impressed with the importance of the paradoxes. A student in Frankfurt, Zonghie Han, set out to prove empirically that reswitching and reverse capital deepening occurred often in reality. We shall show below the method of analysis. Suffice it to say here that we found in a joint paper (Han/Schefold 2006) that the opposite was true: reswitching and reverse capital deepening were rare. Many investigations into the shape of wage curves made earlier and later had raised doubts whether reality was really that much different from what Samuelson had assumed. The question then is: What remains of the Cambridge critique of capital?

We shall now show more formally how the critique was expressed (in Sections 2 and 3), we shall summarise empirical results (in Section 4), and we shall then give theoretical reasons why the paradoxes are rare (in Section 5). We shall try to put the critique on a new

footing and shall end with conclusions that represent a middle-point between the original positions (in Section 6).

## 2 THE PRODUCTION FUNCTION AND LONG-RUN PRICES

Let the production function with positive diminishing marginal products be given in per capita form, with  $y = f(k)$ ,  $y = Y/L$  output per unit of employment and  $k = K/L$  capital intensity or capital per head. Constant returns to scale are assumed throughout. The rate of profit or interest is equal to the marginal product,

$$r = f'(k) \quad (1)$$

and the wage rate is equal to the marginal product of labour

$$w = f - kf'. \quad (2)$$

Equations (1) and (2) yield a parametric presentation of the wage curve  $w = w(r)$ , shown in Figure 1. The slope of the wage curve is negative and its absolute amount equal to the intensity of capital (equation (3)); it diminishes as the rate of profit rises (equation (4)).

$$\frac{dw}{dr} = \frac{dw}{dk} \frac{dk}{dr} = (f' - f' - kf'')/f'' = -k \quad (3)$$

$$\frac{d^2w}{dr^2} = \frac{d}{dr}(-k) = -\frac{dk}{dr} = -1/f'' > 0 \quad (4)$$

Figure 1 shows how output per head and the intensity of capital depend on distribution. It follows that, if a monotonically falling wage curve of diminishing slope is given, the production function can be constructed from its data, for one then has:

$$y = w(r) + kr \quad (5)$$

$$k = -w'(r). \quad (6)$$

Equations (5) and (6) are a parametric representation of a production function with parameter  $r$ . A short calculation analogous to equations (3) and (4) demonstrates that the requisite properties of the production function (positive diminishing marginal products) follow from the properties of the wage curve. Samuelson proposed in 1962 to reconstruct the production function from the techniques as given by their individual wage curves. The auxiliary line  $g$  in Figure 1 then is interpreted as the wage curve of an individual technique. If one moves away from  $r^*$ , it becomes profitable to use a more labour-intensive technique as the wage rate falls. The plough is abandoned and the spade reintroduced. But how does the wage rate change, if no other technique is adopted? One then follows the wage curve of the individual technique; it must be lower than the wage curve as it is drawn, representing the availability of all the techniques. Hence the wage curve of all the techniques must be the envelope of the wage curves of the individual techniques. If the wage curves of the individual techniques are straight lines, each is characterised by a unique intensity of capital, and with the assumptions made about this envelope, one does in fact obtain what Samuelson called a 'surrogate production function' – 'surrogate' because of the abstract nature of the construction.

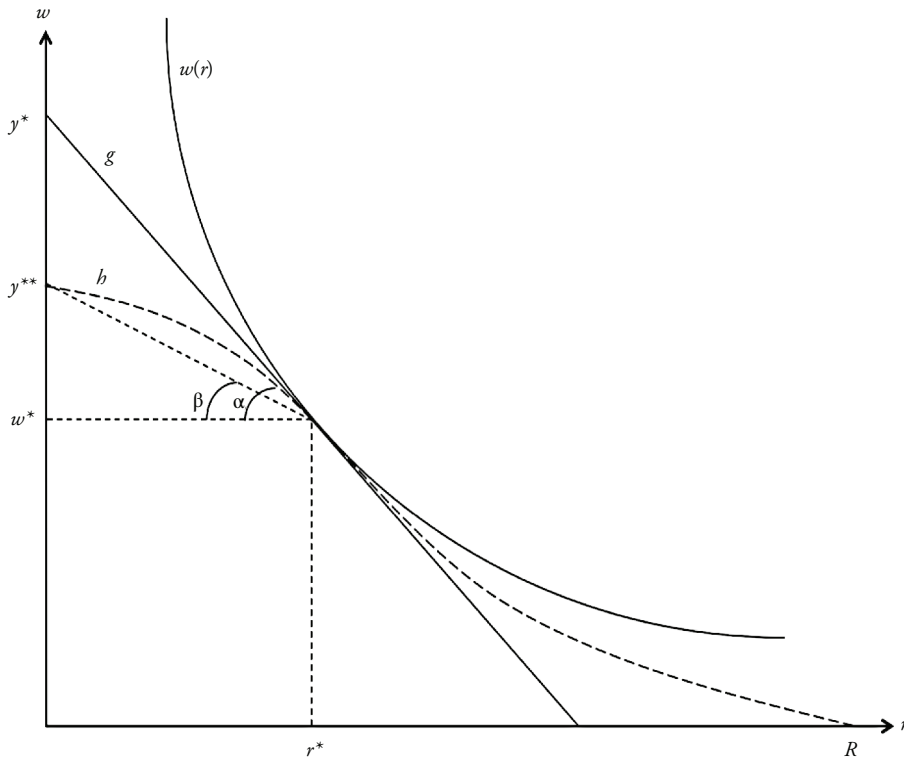


Figure 1 Each point on the wage curve represents a technique: output per head  $y^*$ , pertaining to the technique chosen at  $r^*$ , follows from equations (1) and (2) with  $k^* = \tan \alpha$ ; but if dashed line  $h$  represents the individual wage curve at  $r^*$ , we get output  $y^{**}$  and  $k^{**} = \tan \beta$

Samuelson represented the individual techniques as two-sector models with certain special properties. We choose Sraffa's systems of normal prices with a uniform rate of profit as our frame of reference, for they are the best representation of the conditions which must be fulfilled, if one is interested in long-run prices, which are uniform for each commodity, with a uniform wage rate and a uniform rate of profit. It is the essence of the long run that prices are the same on the output and the input side. The prices that prevail in the long run on average are used to evaluate the profitability of the individual processes, and the long run is characterised by the absence of incentives for capital to migrate from one sector to the other. Hence the rate of profit is uniform.

Regarding the structure of production and consumption as given, one obtains the following system of equations:

$$(1 + r)\mathbf{A}\mathbf{p} + w\mathbf{l} = \mathbf{p}, \quad (7)$$

where  $\mathbf{A}$  input–output matrix and  $\mathbf{l}$  labour vector represent an individual technique, where  $w$  is the wage rate,  $r$  the rate of profit and  $\mathbf{p}$  normal prices (or prices of production or natural prices). The system is assumed to be productive and indecomposable or basic. The givens are non-negative. Since equation (7) is homogeneous, prices can be normalised

by means of a numéraire. One finds that, whatever the numéraire, wages and the rate of profit are inversely related, but this relationship is in general (for arbitrarily given techniques) not linear; the wage falls monotonically from a maximum at  $r = 0$  to zero at a maximum rate of profit  $R$ , where prices are an eigenvector with

$$(1 + R)\mathbf{A}\mathbf{p} = \mathbf{p}. \quad (8)$$

The eigenvalue is  $1/(1 + R)$ . Considering the system now from the point of view of input–output analysis, we introduce activity levels  $\mathbf{q}$  and a vector of final demand  $\mathbf{s}$  with  $\mathbf{q}(\mathbf{I} - \mathbf{A}) = \mathbf{s}$ ;  $\mathbf{q}\mathbf{A}$  then is the vector of the totals of each capital good used in production;  $\mathbf{q}$  is not only the vector of activity levels but, since we have single-product industries, also the vector of gross production and  $\mathbf{s}$  is the net product.

Now we chose a special vector of activity levels such that gross and net output and the vector of capital goods  $\mathbf{q}$ ,  $\mathbf{q}(\mathbf{I} - \mathbf{A})$ ,  $\mathbf{q}\mathbf{A}$  are all proportional, and this will be the case, with economically meaningful (non-negative) solutions, if and only if

$$(1 + R)\mathbf{q}\mathbf{A} = \mathbf{q}, \quad (9)$$

that is, if and only if  $\mathbf{q}$  is the left-hand Frobenius eigenvector in (9), as  $\mathbf{p}$  is the right-hand Frobenius eigenvector in (8).

(9) implies  $R\mathbf{q}\mathbf{A} = \mathbf{q}(\mathbf{I} - \mathbf{A})$ . With this, and considering that  $r$  is the ratio of profit to capital, we can write for the wage rate  $w(r)$ :

$$w = (1/\mathbf{q}\mathbf{l})(\mathbf{q}(\mathbf{I} - \mathbf{A})\mathbf{p} - r\mathbf{q}\mathbf{A}\mathbf{p}) = (1/\mathbf{q}\mathbf{l})\left(1 - \frac{r}{R}\right)\mathbf{q}(\mathbf{I} - \mathbf{A})\mathbf{p}.$$

Sraffa compared the actual systems, in which all activity levels are equal to unity, with a system in ‘balanced proportions’ that are defined by equation (9). These balanced proportions define an eigenvector, the scale of which can be normalised. He used the normalisation  $\mathbf{q}\mathbf{l} = 1$ . And he then took the net product, which would be produced if the activity levels were equal to these balanced proportions,  $\mathbf{q}(\mathbf{I} - \mathbf{A})$ , as the numéraire, therefore defining  $\mathbf{q}(\mathbf{I} - \mathbf{A})\mathbf{p} = 1$ . He called this numéraire the ‘standard commodity’. It follows that the wage rate, expressed in the standard commodity, is a linear function of the rate of profit. It is zero for  $r = R$ , economically the maximum rate of profit:

$$w = 1 - \frac{r}{R}. \quad (10)$$

If prices are expressed in terms of this standard commodity and the linear wage function is used, formula (7) can be solved for the prices, and this leads to the following expression by expanding the inverted matrix into a generalised geometric series, which converges for rates of profit between zero and all  $r$  smaller than the maximum  $R$ :

$$\mathbf{p} = w(\mathbf{I} - (1 + r)\mathbf{A})^{-1}\mathbf{l} = \left(1 - \frac{r}{R}\right) \sum_{t=0}^{\infty} (1 + r)^t \mathbf{A}^t \mathbf{l}. \quad (11)$$

The baffling elegance of this formula, with which Sraffa’s exposition of single product systems culminates (Sraffa 1960: 34–37), allows a clear economic interpretation. The vectors  $\mathbf{A}^t \mathbf{l}$ ;  $t = 0, 1, 2, \dots$ ; are the amounts of labour entering directly ( $t = 0$ ) or indirectly ( $t = 1, 2, \dots$ ) present production by being embodied successively in present production, in the means of production for present production, in the means of production for the

means of production of present production, etc., and the index  $t$  indicates by how many periods an indirect labour input lies back in the past. It must be multiplied by  $(1 + r)^t$ , because the means of production are capital that was advanced repeatedly so that profit is earned on them as if the capital had been put into a bank and earned compound interest, at a rate of interest equal to the rate of profit. The interest cost rises with  $t$ , and the labour inputs  $\mathbf{A}\mathbf{l}$  similarly diminish with  $t$ . Prices  $\mathbf{p}$  increase with  $r$ , to the extent that profits earned on past labour cost increase, but they diminish insofar as the wage rate falls. The composite effect is such that the price of the standard commodity stays constant by definition. Prices are equal to total labour cost, therefore equal to labour values in the traditional sense, if  $r = 0$ . As the rate of profit rises, some prices go up and others down, and relative prices stay constant only if they are equal to relative labour values for all rates of profit, and this will happen, as one can show, if and only if the labour vector happens to be a right-hand-side eigenvector of matrix  $\mathbf{A}$  according to equation (12):

$$(1 + R)\mathbf{A}\mathbf{l} = \mathbf{l}. \quad (12)$$

We have found that wage curves are linear if either the numéraire is special and equal to the standard commodity (10) or if the labour vector stands in a special relationship to the input–output matrix and is a right-hand-side eigenvector (12). We shall see later that linear wage curves may also be obtained for a more general set of numéraires or for a less special relationship of the labour vector to the input–output matrix, if the input–output matrix itself is special.

### 3 THE PARADOXES

Let's go back to Figure 1 and verify what conclusions have to be drawn for the existence of a production function, if the wage curves are not straight lines as was assumed by Samuelson, but monotonically falling curves. The same numéraire must be used for all the techniques in order to compare them. The standard commodity is specific for the technique; it therefore cannot be used here. The usual assumption, then, is to assume the same net output for all the techniques, for it is being asked which technique is best for fulfilling the same task in a given situation (here defined by the state of distribution). This net output is taken as the numéraire. This implies that net output per head is equal to the wage rate, if the rate of profit is zero. Net output, then, does not change with distribution by definition. And so we find at once the first critique of the Samuelsonian construction of the production function. According to the construction, based on equations (5) and (6), net output is equal to  $y^*$  which is the net output per head found by drawing the auxiliary line  $g$  as a tangent to the envelope of the wage curves at  $r^*$ . But if the wage curve is given by the dashed line  $h$  in Figure 1, net output per head equals

$$y^{**} = (\mathbf{d}\mathbf{p}(r^*)) / \mathbf{q}^* \mathbf{l} = 1 / \mathbf{q}^* \mathbf{l},$$

where  $\mathbf{d}$  is the net output, equal to the numéraire,  $\mathbf{p}(r^*)$  prices expressed in this numéraire at rate of profit  $r^*$ , and  $\mathbf{q}^*$  the activity levels such that  $\mathbf{d}$  results as net output, therefore  $\mathbf{q}^* = \mathbf{d}(\mathbf{I} - \mathbf{A})^{-1}$ . This, as opposed to the auxiliary line  $g$ , now 'true' individual wage curve  $h$  is below or on the envelope  $w(r)$ . Wage curve  $h$  has been drawn with an inflection point in  $r^*$  which is a special case, chosen to render the geometrical construction more transparent. It is clear that  $y^*$  and  $y^{**}$  will differ in general, if wage curves are curves and not straight. This deviation was called 'declination' in Han/Schefold (2006); it implies that



constructing the production function from (5) and (6) according to Samuelson leads into error. Declination is a phenomenon that seems on the whole to have been neglected during the original Cambridge debate.

Paul Samuelson, by drawing the individual wage curves as straight lines, implicitly assumed that the labour theory of value held for each technique. This was taken up by the critics; it was thought to be ironical that Samuelson made such a Marxian assumption (Marx assumes that prices are proportional to labour values in the first two volumes of *Das Kapital* (Marx 1867; 1885)). Only later was it pointed out by Salvadori/Steedman (1988) that even this assumption could not consistently be made. For if two adjacent techniques on the envelope of the wage curves fulfil condition (12) so that the labour theory of value holds for each, it will in general be possible to select different methods of production from both and to combine them to form a new technique which will be better than either at the apparent switchpoint, and the wage curve of which will not be a straight line.

Next, one may ask how the intensity of capital changes. We denote the intensity of capital, to be read off from wage curve  $h$  in Figure 1, by  $k^{**}$ . We have at  $r^*$

$$w^* + r^* k^{**} = y^{**} \quad (13)$$

hence

$$k^{**} = \frac{y^{**} - w^*}{r^*}; \quad (14)$$

the intensity of capital is not given by the slope of the envelope of the wage curves any more, and this intensity of capital changes along the individual wage curve as distribution changes, along curve  $h$ , for the same technique, and along the envelope, as techniques change.

The change of the capital–labour ratio for the same technique along the wage curve is called the Wicksell effect. Samuelson assumed that the Wicksell effects were zero by assuming that the wage curves were straight. One may speak of a neoclassical Wicksell effect, if the intensity of capital falls as the rate of profit rises for a given wage curve, and of an anti-neoclassical effect if, paradoxically from the neoclassical point of view, the rate of profit and the capital–labour ratio increase together. As we shall see, the occurrence of Wicksell effects is the rule, not the exception, and neoclassical and anti-neoclassical Wicksell effects are, in a sense, equally likely. Their relevance is easily explained.

A neoclassical Wicksell effect results if the wage curve is curved towards the origin (for example, curve  $h$  between  $r^*$  and  $R$  in Figure 1). Suppose that the economy is in equilibrium, in that capital and labour are fully employed, in a given state of distribution. Suppose that an immigration disturbs this equilibrium. The wage rate will fall. The neoclassical Wicksell effect implies that the capital–labour ratio will fall, and this means that, with the same amount of capital, more labour can be employed, so that a tendency towards an improvement of unemployment results; the technique need not change. But the converse happens in the case of an anti-neoclassical Wicksell effect with the opposite curvature (curve  $h$  between 0 and  $r^*$  in Figure 1). The fall of the wage rate, due to the exogenous increase in the labour force, leads to a higher capital–labour ratio, because of the change of relative prices of the capital goods, so that, if the value of capital is given and constant, fewer labourers can be employed and the employment situation gets worse. The result is instability. The anti-neoclassical Wicksell effect implies that the factor prices and the factor quantities move in the same direction and not in opposite directions, as the stability of the supply–demand mechanism requires. The paradox of the anti-neoclassical Wicksell effect does not question the existence of equilibrium, but its stability; this has often not been understood.

These criticisms (declination, the capital–labour ratio determined by (10) and not by (6), and the anti-neoclassical Wicksell effect) were disturbing in the Cambridge debate, but two other effects, reswitching and reverse capital deepening (mentioned in Section 1) made an even greater impression.

Consider Figure 2. Intersections of wage curves on the envelope are called switch-points. It can be shown that generically only one method changes in only one industry at a switchpoint; the other methods in the other industries are used on both sides of this point. Technical change is ‘piecemeal’. If technique  $\gamma$  was not there, we should have a case of reswitching, in that wage curves  $\alpha$  and  $\beta$  would intersect twice on the envelope; first  $\beta$  would be used, then  $\alpha$  and then  $\beta$  would become profitable again. This return of the technique at switchpoint  $r_2$  is the first paradox here to be observed (reswitching). The second consists of the fact that the capital–labour ratio increases at  $r_2$  with an increase in the rate of profit. This is called reverse capital deepening. (The reader is advised to draw the auxiliary lines in Figure 2 according to Figure 1, in order to see it.) The introduction of another technique, represented by wage curve  $\gamma$ , causes the switchpoint at  $r_1$  between  $\alpha$  and  $\beta$  to disappear from the envelope and a new switchpoint appears at  $r_3$ . The former switchpoint at  $r_1$  and the new switchpoint at  $r_3$  have in common that the change of technique, induced by the corresponding rise of the rate of profit, causes the capital–labour ratio to fall, as corresponds to neoclassical theory. If the agents in the economy only perceive what happens on the envelope, which expresses the result of profit maximisation at each rate of profit, they will now not notice a return of technique any more at  $r_2$ , but the

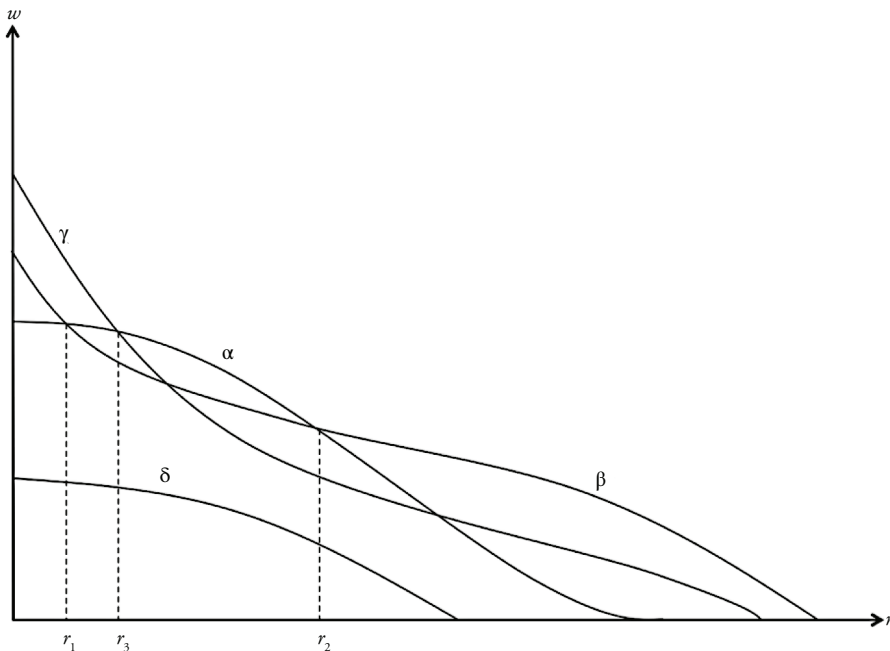


Figure 2 Four wage curves  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , with  $\gamma$ ,  $\alpha$  and  $\beta$  appearing successively on the envelope; without  $\gamma$ ,  $\alpha$  and  $\beta$  exhibit reswitching, with capital per head increasing at the second switch (reverse capital deepening); the latter paradox remains, if  $\gamma$  is introduced – it turns out that most wage curves remain below the envelope like  $\delta$

phenomenon of reverse capital deepening will remain and will seem all the more surprising. It is possible only because the wage curves are not straight lines; there are Wicksell effects on the envelope between the switchpoints. Many wage curves will not appear on the envelope anywhere like wage curve  $\delta$  in the diagram.

Wage curves were calculated by hand at the time of the Cambridge debate for two-sector models and had no empirical foundation. In part this was due to the lack of calculating equipment, in part to the lack of data. Moreover, the critics in the debate were convinced that this important theoretical matter should be resolved by pure theory alone. There was some evidence that empirical wage curves, derived from input–output systems, were rather straighter than the critics expected. I for one thought that switches with reverse capital deepening might occur just as often as ‘normal’ switches, since, in theory, even more than two switches are possible between any two techniques, depending on the degree of the polynomials. Neoclassical authors felt unconcerned and said that reswitching had never been seen.

#### 4 EMPIRICAL FINDINGS AND THEIR EXPLANATIONS

The empirical investigation by Han/Schefold (2006) uncovered one case of reswitching, but more than 96 per cent of the switchpoints observed were neoclassical. The method was based on the derivation of the wage curves pertaining to pairs of input–output tables. Thirty-two comparable input–output tables were at hand; these taken in pairs gave rise to 496 envelopes of wage curves, with a total of 4389 switchpoints. The challenge now posed was to explain why reswitching and reverse capital deepening, if they existed, were not more frequent and why, given these figures, only about ten wage curves of ten techniques appeared on average on each envelope. This second observation was for me as surprising as the first, for the input–output tables contained the data for 33 relevant industries. Comparing the tables in pairs meant that two methods were available in each industry so that  $2^{33}$  wage curves could be derived for each pair, which means that only one in  $10^7$  wage curves showed up on the envelope on average.

The envelopes were calculated by means of linear programming. In principle, one envelope for all the techniques given by 32 tables taken together should have been derived as the expression of the total knowledge available. This seemed not to be feasible, but Stefano Zambelli (Zambelli et al. 2017) has found an algorithm which is based on a different method and is theoretically less satisfactory than linear programming because the algorithm that singles out the wage curves on the envelope could theoretically fail in special cases, but it is effective in practice. He compared the input–output systems of 30 countries, with 31 sectors of production in each. Hence his algorithm finds the envelope from  $31^{30}$  wage curves. All but 63 curves are below the envelope. His diagram exhibits Wicksell effects of both kinds, but there is no reverse capital deepening at all. The anti-neoclassical Wicksell effects are strong enough to produce capital reversals, in that the capital–labour ratio increases not only between switchpoints but also across switchpoints, insofar as capital per head falls at the switchpoint itself, but rises afterwards to the extent that capital per head is then higher than it had been on the left of the switchpoint. The range of the rate of profit in his example extends from zero to more than 250 per cent, because the input–output tables reflect circulating capital only (ibid.: 41).

It now needs to be asked what this means from the point of view of neoclassical theory and also post-Keynesian theory, for at least the post-Keynesian models of economic growth are based on the stylised facts of growth theory and are, in the case of Kaldor (1956 [1970]: 88), based on the explicit assumption that the capital–output ratio does not change strongly with changes in distribution. Zambelli’s Wicksell effects come in

groups. As one moves along the envelope, there is little substitution between zero and a rate of profit of about 50 per cent. Thereafter, a series of techniques and wage curves follow upon each other which all exhibit anti-neoclassical Wicksell effects, and neoclassical Wicksell effects predominate in sequence at very high rates of profit. None of the wage curves shows a very strong curvature, and this corresponds to the results found by other observers. The task is to explain these findings.

The entire debate about capital theory is really about the character of the change of relative prices in long-run conditions. Equation (11), the reduction to dated quantities of labour, renders transparent the influence of the distribution of past labour inputs over time, of the compound profits and of the wage on the cost of production, including normal profit. But it is not obvious from the reduction whether the changes of relative prices with distribution are compatible with a return to the same relative prices, whether the price vector moves perhaps in a sub-space or whether relative prices keep changing in all directions. That relative prices return or are confined to a sub-space is excluded according to a mathematical proposition, which one might call the fundamental Neo-Ricardian Lemma (Schefold 1971, published as a then novel result in a mathematical journal as Schefold 1976b). If any sequence of  $n$  different rates of profit is given, for instance in ascending order as  $0 \leq r_1 < r_2 < \dots < r_n < R$ , the  $n$  vectors  $\mathbf{p}(r_i)$ ;  $i = 1, \dots, n$ ; will be linearly independent if the system is 'regular', and this means essentially that the labour theory of value does not hold, therefore that condition (12) does not hold, and that the characteristic equation of the indecomposable input-output matrix does not have multiple roots. Similarly, it can be shown that the vectors

$$\tilde{\mathbf{p}}(r) = \begin{pmatrix} (1+r)\mathbf{p}(r) \\ w \end{pmatrix} \quad (15)$$

take  $n+1$  linearly independent values for any  $n+1$  distinct rates of profit  $r_i$ ;  $i = 1, \dots, n+1$ ; if the system is regular. Since multiple roots are mathematically and the labour theory of value economically exceptional, the fundamental Neo-Ricardian Lemma appears to be very general, and I trusted the conclusions drawn from it, once I had discovered it, and began to use it for my work. As we shall see, the economic application requires care, but let us first look at some of the conclusions.

The mathematical point is that the price vector of regular systems twists in space as a function of the rate of profit in such a way that it never stays in any sub-space, and this condition turns out to be necessary for reswitching and reverse capital deepening to have a positive likelihood, as we shall now show following Schefold (1976a). Let a regular system (7) be given; it uses the method  $(\mathbf{a}_1, l_1)$  in the first industry. Now consider the simplex

$$M(r) = \{(\mathbf{a}_0, l_0) \geq 0 \mid (\mathbf{a}_0, l_0)\tilde{\mathbf{p}}(r) = (\mathbf{a}_1, l_1)\tilde{\mathbf{p}}(r)\}.$$

We interpret  $M(r)$  as the set of potential methods of production for producing the first commodity that would be equally profitable as the actual existing method of production  $(\mathbf{a}_1, l_1)$  at the given rate of profit. Whether one or several potential methods of this production set actually exist, and whether they are perhaps being used in another country, are open questions. The set of the potential techniques  $(\mathbf{a}_0, l_0)$  is the non-negative orthant of  $(n+1)$ -dimensional space.

Now let us choose a rate of profit  $r_1$  at which our system actually is. The intersection  $M(r_1) \cap M(r_2)$ ;  $r_1 \neq r_2$ ; is the set of potential techniques that are equally profitable at

$r_1$  and  $r_2$  and it contains  $(\mathbf{a}_1, l_1)$ .  $M(r_1)$  is of dimension  $n$  and  $M(r_1) \cap M(r_2)$  is of dimension  $n - 1$ . Hence it is unlikely that any actual alternative technique will be found in that intersection. This means that it is unlikely that an alternative method of production can be found that leads to preassigned switchpoints at  $r_1$  and  $r_2$ . But the intersection will move, given  $r_1$  if we let  $r_2$  vary; and if  $r_2$  varies between 0 and  $R$ , the intersection will cover a sub-set of  $M(r_1)$  containing  $(\mathbf{a}_1, l_1)$ . The sub-set, denoted by  $M^*$ , will be of dimension  $n - 1$ , if the labour theory of value holds and the system is not regular, because the price vector then fails to twist and the intersection does not move. But in the case of regularity,  $M^*$  covers a full  $n$ -dimensional neighbourhood so that the likelihood of reswitching is not zero. The reader will find diagrams and more extensive explanations in Schefold (1976a). I was content with having proved that reswitching is not exceptional in 1976, but the empirical results have caused me to ask more recently why reswitching is so difficult to encounter in actual large systems, while numerical examples with reswitching are not difficult to construct in the case of two- or three-sector models. Such examples were produced in the reswitching debate and shown to be not so infrequent by others (Petri 2011). Related examples, based on the work of Ian Steedman primarily, have been discussed by Heinz Kurz in the parallel paper in this symposium.

The proof that the likelihood of reswitching and reverse capital deepening tends to zero as the dimension of the matrix increases is difficult and requires assumptions. It cannot be discussed here (Schefold 2016a), but a very simple intuitive reason may be provided.

Suppose our consideration could be limited to the analysis of the amounts of the commodity inputs. Suppose the input–output coefficient of each of the commodities could be anywhere between 0 and 1. Suppose that, in order to have reswitching, these input–output coefficients had to be anywhere between 0 and  $\frac{1}{2}$ . It might seem that the likelihood of reswitching would be  $\frac{1}{2}$ , but that would not be correct. The set of all potential techniques would be a cube with edges of length 1. The volume of the cube therefore would be equal to 1. The potential techniques leading to reswitching would be in a cube with edges of length  $\frac{1}{2}$ . Its volume would be  $(\frac{1}{2})^n$  and it would tend to zero as  $n$  the number of commodities, tended to infinity. It is an effect of this type that causes reswitching to be irrelevant in large economic systems.

Reverse capital deepening also disappears, if one introduces a rate of balanced growth (so far we have been dealing with stationary states) and assumes that the rate of growth is equal to the rate of profit, according to the ‘golden rule’. Neoclassical authors working with the golden rule assumption regard reverse capital deepening as unimportant for this reason (compare C.C. von Weizsäcker’s contribution to this symposium).

Wicksell effects, by contrast, are ubiquitous, and they go both ways. To show this, we exclude wage curves with inflection points, for if there is an inflection point as in Figure 1, nothing needs to be proved. Let a technique with its associated standard system be given and consider the set of numéraires (baskets of goods) that assign the same value as the standard commodity at  $r = 0$ . This set  $D$  therefore is given by

$$D = \{\mathbf{d} \geq 0 \mid \mathbf{dp}(0) = 1\}. \quad (16)$$

$D$  is a simplex and the standard commodity is an inner point of  $D$ , because the standard commodity is a positive vector. We suppose that the technique is regular. We choose a rate of profit  $r_1$  between zero and the maximum on a wage curve which is linear because the standard commodity is the numéraire. If we increase the rate of profit a little, starting from  $r_1$  some prices will go up and the prices of other commodities will go down. If we change the numéraire given by the standard commodity a little, without violating (16), by increasing the amounts of commodities for which prices go up and diminishing the

amounts of commodities for which prices go down, the value of the wage, expressed in this modified standard, will diminish. This means that the capital–labour ratio will, with the modified numéraire, be larger because of (16); hence one has engendered a neo-classical Wicksell effect, and the opposite will happen, if the composition of the numéraire is changed in the opposite direction. The  $n - 1$ -dimensional simplex  $D$  will thus be divided into two sets, separated by an  $n - 2$ -dimensional simplex of numéraires for which the wage does not change at  $r_1$ , remaining equal to the standard wage. In this sense, anti-neoclassical and neoclassical Wicksell effects may be said to be equally likely. The numéraire that is used in the comparison of wage curves along an envelope for a spectrum of techniques is the net product to be produced. If one compares the wage curves of different input–output systems for different countries that have different net products, it is best to choose for the comparison the net product that is an average of the products of the different countries, thus simulating that they all produce the same. This numéraire will then differ somewhat from the standard commodities pertaining to those countries, and it is *a priori* just as likely that this deviation causes a neoclassical as that it causes an anti-neoclassical Wicksell effect for any of the wage curves under consideration.

The assumptions that have been made about the numéraire in different empirical investigations differ. The following generalisations seem both plausible from the point of view of the theory and supported by empirical facts:

1. The curvatures of the wage curves are more pronounced if numéraires consisting of single commodities are chosen than if composites are used.
2. If net output is the numéraire, the wage curves are nearly linear over considerable stretches. For instance, Zambelli et al.'s (2017: 41) diagram exhibits wage curves between  $r = 0$  and  $r = 50$  per cent that are nearly linear in that interval according to visual inspection.
3. As one moves down the envelope of wage curves engendered by a spectrum of techniques, the curvatures of the individual wage curves making up the envelope will alternate, but not regularly. Usually, several switchpoints must be passed before the sign of the Wicksell effect changes, because the sign change does not depend upon the change of method in one sector only, but on all the sectors; since the curvature is a property of the system as a whole, it usually changes only after several method changes. Zambelli et al.'s diagram confirms this supposition.
4. If the Wicksell effects followed upon each other closely and alternating, they would lose importance for the critique, for they might cancel on average, but this does not seem to be the case; entire groups of subsequent wage curves exhibit either neoclassical or anti-neoclassical Wicksell effects.

## 5 RANDOM SYSTEMS: WHY WAGE CURVES MAY TEND TO BE LINEAR AND WHY ONLY A FEW APPEAR ON THE ENVELOPE

We now come to the main analytical innovation. It had been thought that wage curves could be linear only because either the numéraire was equal to the standard commodity or the labour theory of value held (equation (12)). Input–output systems have a random character insofar as the coefficients unpredictably go up and down with innovations. By contrast, the productivity of labour develops more steadily and the components of the labour vector diminish with a certain regularity. If one now assumes that the input–output matrices are approximately random, one can, with certain additional assumptions, conclude that the system tends to be not regular in that the absolute values of all the non-dominant eigenvalues of

the characteristic equation will tend to zero. In order again to cut a long story short, we now assume that  $\mathbf{A}$  is an indecomposable random matrix fulfilling the conditions of the Goldberg–Neumann theorem (see Schefold 2013a). This means that the inputs of the industries have a distribution that is independent and identically distributed (i.i.d.) with a mean specific for the industry. This distribution does not exclude that many input coefficients are zero. The assumptions imply that the non-dominant eigenvalues tend to zero for large matrices. We assume these small eigenvalues to be different so that the matrix is diagonalisable with  $n$  left-hand-row eigenvectors  $\mathbf{q}_1, \dots, \mathbf{q}_n$  and right-hand-column eigenvectors  $\mathbf{x}^1, \dots, \mathbf{x}^n$ , which we here assume to be normalised in an unusual way, namely such that the linear combination of the left-hand-side eigenvectors is equal to the net output of the system  $\mathbf{d}$ , and the linear combination of the right-hand-side eigenvectors equals the labour vector. Hence

$$\mathbf{d} = \mathbf{q}_1 + \dots + \mathbf{q}_n; \mathbf{l} = \mathbf{x}^1 + \dots + \mathbf{x}^n.$$

The vector  $\mathbf{q}_1$  is here proportional to Sraffa's standard commodity, since it is a Frobenius eigenvector, and  $\mathbf{x}^1$  is equal to the vector which, according to formula (12) would cause prices to be equal to labour values, if  $\mathbf{x}^1$  were the actual labour vector of the system.  $\mathbf{q}_1$  is called the Sraffa vector,  $\mathbf{x}^1$  the Marx vector. We shall be interested in the deviations of the numéraire vector from the Sraffa vector, hence in vector  $\mathbf{m} = \mathbf{d} - \mathbf{q}_1 = \mathbf{q}_2 + \dots + \mathbf{q}_n$  and in the deviations of the labour vector from the Marx vector, hence in  $\mathbf{v} = \mathbf{l} - \mathbf{x}^1 = \mathbf{x}^2 + \dots + \mathbf{x}^n$ .

Using the representation of the labour vector by the right-hand-side eigenvectors, we get for the prices with  $p = 1 + r$ :

$$\mathbf{p} = w(\mathbf{I} - (1 + r)\mathbf{A})^{-1}\mathbf{l} = w \sum_{i=1}^n \frac{\mathbf{x}^i}{1 - \rho\mu_i},$$

where  $\mu_i$  are the eigenvalues. Next we use the representation of the net output or numéraire vector  $\mathbf{d}$  by the left-hand-side eigenvalues and the orthogonality of the eigenvectors pertaining to different eigenvalues to obtain

$$1/w = \mathbf{d}\mathbf{p}/w = \frac{\mathbf{q}_1\mathbf{x}^1}{1 - \rho\mu_1} + \sum_{i,j=2}^n \mathbf{q}_i\mathbf{x}^j = \frac{\mathbf{q}_1\mathbf{x}^1}{1 - \rho\mu_1} + \mathbf{m}\mathbf{v},$$

where we have set  $\mu_2 = \dots = \mu_n = 0$  as an approximation.

We now make a further assumption: The vectors of deviations  $\mathbf{m}$  and  $\mathbf{v}$  are not correlated. So we have for the covariance  $\text{cov}(\mathbf{m}, \mathbf{v}) = 0$ , and this means according to a standard formula that  $\mathbf{m}\mathbf{v} = n\bar{m}\bar{v}$ , where  $\bar{m}$  and  $\bar{v}$  are the ordinary averages of the coefficients of vectors  $\mathbf{m}$  and  $\mathbf{v}$ . Now the summation vector  $\mathbf{e}$  tends to be the Frobenius eigenvector of  $\mathbf{A}$ , because  $\mathbf{A}$  is random, and one gets in the limit

$$n\bar{v} = n\mathbf{e}\mathbf{v}/n = \mathbf{e}(\mathbf{x}^2 + \dots + \mathbf{x}^n) \rightarrow 0,$$

because of the orthogonality conditions. So we have  $\mathbf{m}\mathbf{v} = 0$ , tendentially, and get a linear wage curve

$$w = (1 - \rho\mu_1)/\mathbf{q}_1\mathbf{x}^1 \quad (17)$$

and prices also become a linear function

$$\mathbf{p} = (1 - \rho \mathbf{q}_1 \mathbf{x}^1) [\mathbf{x}^1 + (1 - \rho \mu_1) \mathbf{v}]. \quad (18)$$

As we said, these are approximations, valid if the systems are random, and the very assumption of the randomness of systems is itself an approximation. Nonetheless, the exercise shows that the quasi-linearity of wage curves of real systems is capable of a theoretical explanation. One can move in two directions from here. It can be shown that an additional assumption allows us to justify the Marxian transformation of values into prices. Profits can be shown to be equal to total surplus value, redistributed in proportion to capital advanced (Schefold 2016b; 2019b). We are not interested in this connection here, but pursue the implications for neoclassical theory. We only note with respect to the Marxian tradition that here prices appear to be equal to labour values on average. For prices deviate from values according to formula (18) linearly with  $(1 - \rho \mu_1) \mathbf{v}$  and the average  $\bar{v}$  tends to zero, so that the average of prices is equal to the average of values at all levels of the rate of profit.

If we regard the approximation (17) as an explanation of the quasi-linearity of empirical wage curves, we lend some support to the Samuelsonian idea of the construction of a surrogate production function. If techniques are random, the combinations of processes chosen from different techniques of this sort will also tend to be random, and an envelope of many linear or quasi-linear wage curves according to (17) could give rise to a surrogate production function with constant returns to scale and diminishing marginal products (no constancy of the elasticity of substitution or a particular value of that elasticity would be implied). The surrogate production function is less absurd than I once thought.

However, here another critical element comes in. We have not yet tried to explain the empirical finding that most wage curves are below the envelope and that the number of wage curves actually appearing on it, and consequently the number of switchpoints, is surprisingly small. In order to explain the phenomenon, I propose the following argument (Schefold 2013b). Suppose the wage curves are really straight lines. There are, for instance, 10 countries with 100 industries in each so that we shall have  $10^{100}$  wage curves. How many of them can we expect to appear on the envelope? We assume that the wage curves  $w_\sigma$  where  $\sigma$  runs from 1 to  $10^{100}$ , can be ordered according to the values on the ordinate so that  $w_1(0) > w_2(0) > w_3(0) > \dots$ . For reasons discussed in greater detail elsewhere, I assume that the  $s$  wage curves (in the example  $s = 10^{100}$ ) have distinct maximum rates of profit  $R_\sigma$ , and that for any selection of wage curves  $(\sigma_1, \dots, \sigma_t)$ , with  $\sigma_1 > \sigma_2 > \dots > \sigma_t$  and hence  $w_{\sigma_1}(0) > w_{\sigma_2}(0) > w_{\sigma_t}(0)$ , the corresponding  $R_{\sigma_1}, \dots, R_{\sigma_t}$  are ordered randomly.

Hence  $w_1(r)$  will appear on the envelope with probability 1, since  $w_1(0) > w_2(0) > \dots$ . Wage curve  $w_2(r)$  will appear on the envelope with probability  $1/2$ , for it will appear if  $R_2 > R_1$ , but not if  $R_2 < R_1$ . By assumption, both possibilities are equally likely.  $w_3(r)$  will appear, if either  $R_3 > R_2 > R_1$  or  $R_3 > R_1 > R_2$ , which are two possibilities out of six, hence it will appear with probability  $1/3$  and so, going on, we conclude that  $w_\sigma(r)$  will appear with probability  $1/\sigma$ , if we look at the likelihood of appearance moving down the ladder. This consideration leads us to estimate the expected number of wage curves on the envelope  $\Omega$  by the formula

$$\Omega = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{s} \cong \ln s,$$

where  $\ln s$  denotes the natural logarithm.



Formula (19) only gives an upper limit for our expectation, for a wage curve which comes later in our ordering can have a maximum wage and a maximum rate of profit sufficiently high to prevent an earlier wage curve from getting on the envelope, as is exemplified in Figure 3. It is curiously difficult to calculate the likelihood of this possibility of later domination exactly. Numerical experiments based on the generation of random numbers by Anastasia Biermann indicate that the value given by formula (19) roughly is not quite halved by this effect.<sup>1</sup>

The phenomenon of later dominance need not detain us further; formula (19) suffices to show that the substitution possibilities are likely to be quite low on an envelope, even if

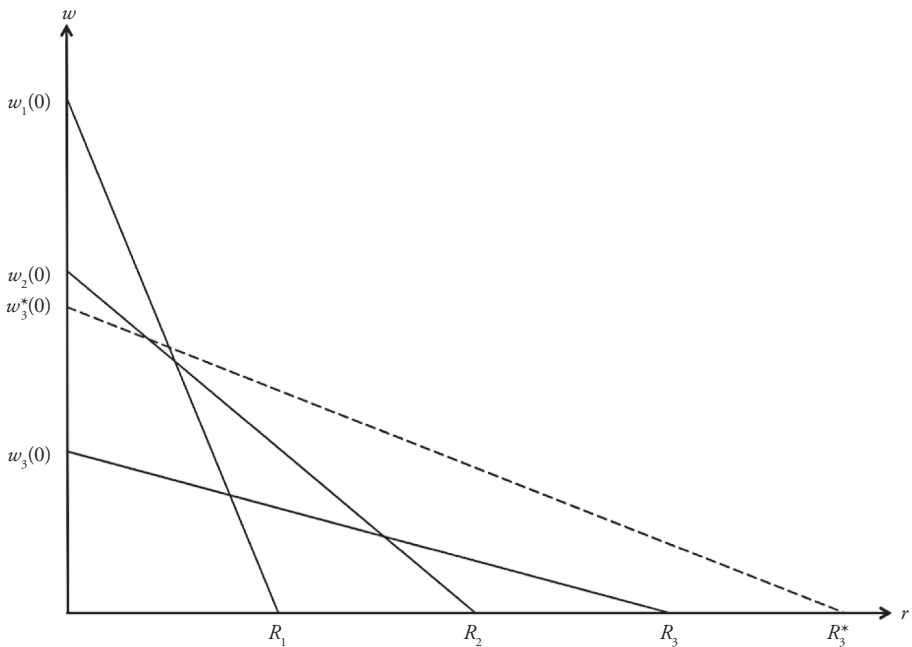


Figure 3 Three techniques with three straight wage curves: all three wage curves appear on the envelope, because the ordering of the maximum rates of profit happens to be inverse to that of the wage rate at  $r = 0$ ; but if the third wage curve is given by the dashed line,  $w_3^*(0)$  is so close to the second wage curve and so large that the first switch point on the envelope is dominated and only the first and the dashed wage curve appear on it

1. Five-hundred simulations with different numbers of wage curves were made, ranging from  $10^4$  to  $10^8$ . The mean number of wage curves observed on the envelopes on average rose from 7 in the case of  $10^4$  wage curves to 13 in the case of  $10^8$  wage curves. If this mean is denoted by  $\Theta$ , the ratio  $\Theta/\Omega$  fell monotonically from 0.80 to 0.71. It is not known whether  $\Theta/\Omega$  approaches a positive limit as  $\iota$  tends to infinity. In the case analysed by Han/Schefold (2006), there was a total 4389 of switch-points on 496 envelopes, therefore on average  $(4389 + 496)/496 \cong 9.8$ . Formula (19) yields here  $\Omega = \ln 2^{33} = 33 \ln 2 \cong 22.7$ . The corresponding ratio is somewhat lower than one would expect from the numerical experiment; the discrepancy is perhaps to be explained by the fact that the wage curves are not exactly straight lines. Finally, we may recall that Zambelli et al. (2017) have 62 wage curves on their envelope, and we obtain in their case  $\Omega = \ln 31^{30} \cong 102$ .

the individual wage curves are straight lines, so that not only reverse capital deepening but also Wickssell effects are excluded. The assumptions imply that a kind of non-differentiable surrogate production function exists with discrete substitution possibilities, which are, however, presumably too sparse to rely on the substitution mechanism for preserving full employment.

## 6 FUTURE RESEARCH AND CONCLUSIONS

We introduce a last transformation in order to indicate in which direction research is moving, before getting to the conclusions. We propose to explain the quasi-linearity of the wage curve by assuming that the input–output matrices are approximately random and that the deviations of the numéraire from the Sraffa vector and of the labour vector from the Marx vector are uncorrelated. A rough way to express randomness is to write

$$\mathbf{A} = \mathbf{c}\mathbf{e} + \mathbf{P}. \quad (20)$$

The matrix is represented as the sum of a matrix of rank one  $\mathbf{c}\mathbf{e}$  (column vector  $\mathbf{c} > 0$ ) and a perturbation matrix  $\mathbf{P}$ ; it must be such that  $\mathbf{A}$  fulfils the randomness assumptions. On the other hand, it is known that  $\mathbf{A}$  can be written as

$$\mathbf{A} = \sum_{i=1}^n \mu_i \mathbf{M}_i \quad (21)$$

with  $\mathbf{M}_i = \mathbf{x}^i \mathbf{q}_i$  where  $\mathbf{q}_i$  and  $\mathbf{x}^i$  are the left-hand-side and right-hand-side eigenvectors, which we now normalise in the usual fashion such that  $\mathbf{q}_i \mathbf{x}^j = \delta_{ij}$ . This is possible if  $\mathbf{A}$  is diagonalisable. The matrices  $\mathbf{M}_i$  are idempotent, therefore  $(\mathbf{M}_i)^2 = \mathbf{M}_i$ , and orthogonal in that  $\mathbf{M}_i \mathbf{M}_j = 0$  for  $i \neq j$ . One finds that the trace of each  $\mathbf{M}_i$  equals one:  $tr(\mathbf{M}_i) = 1$ . In this sense, the  $\mathbf{M}_i$  are normalised, and one finds that the trace of  $\mathbf{A}$  is equal to the sum of the eigenvalues:

$$tr(\mathbf{A}) = \sum_{i=1}^n \mu_i.$$

This formula also follows from the characteristic equation. Some eigenvalues may be complex and not real, so that the same holds for the eigenvectors. But a conjugate complex root and conjugate complex eigenvectors are associated with each complex root so that the imaginary parts in (21) will cancel, and (21) can be interpreted as a representation of  $\mathbf{A}$  by means of a sum of real matrices, weighted by the real parts of the eigenvalues, which begins with  $\mathbf{M}_1 = \mathbf{x}^1 \mathbf{q}_1$ ,  $\mathbf{x}^1$  and  $\mathbf{q}_1$  being the Frobenius positive eigenvectors, and  $\mu_1 = \text{dom } \mathbf{A}$ .

We can now assess the degree of the randomness of  $\mathbf{A}$  by comparing (20) and (21). More generally: We have seen that linear individual wage curves are necessary for the neo-classical construction of an aggregate production function. The main Neo-Ricardian Lemma demonstrated that wage curves will be curves, not straight, in the regular case. We have here focused on three kinds of irregularity that lead to linear curves: The numéraire may be proportional to the standard commodity, the labour theory of value may hold according to equation (12) or near linearity is possible if the matrix has random properties and the covariance condition holds, for this leads to small non-dominant eigenvalues. This analysis may be extended to consider intermediate cases. Empirical analyses confirm that

most eigenvalues are indeed close to zero, according to a distribution of the eigenvalues that falls off rapidly (Mariolis/Tsoufidis 2014) but the second or third and maybe a few more eigenvalues will usually not be so small and will give rise to deviations from linearity, even if the covariance condition holds.

This critical perspective should be complemented by a positive analysis of the evolution of the structure of input–output systems (Shaikh 2016). Equation (21) suggests that this structure is primarily described by  $\mathbf{M}_1 = \mathbf{x}^1 \mathbf{q}_1$ , and this implies that the processes have a basic similarity, modified by the real parts of  $\mathbf{M}_2, \mathbf{M}_3$  and perhaps a few others. The structure of the input–output table of a large economy such as that of the USA seems to be persistent, as Torres Gonzàles (2018) has shown in a seminal analysis; it is proposed to represent its evolution in terms of the decomposition of formula (21). The importance of  $\mathbf{M}_1$  follows, by the way, from the fact that  $\mathbf{A}$  is a primitive matrix (for it is necessary for a matrix to be imprimitive that all diagonal elements vanish, but many industries, not only corn as in Ricardo, need some input of the commodity they produce for reproduction). It is a known result that the dominant root of a primitive indecomposable semi-positive matrix is strictly greater than the moduli of all other roots. The theorem about random matrices reinforces this drastically, in that the second eigenvalue tends to zero, if  $n$  increases.

We may now conclude that the Cambridge critique has been transformed considerably because reswitching, which once constituted the main objection against neoclassical theory, has turned out to be irrelevant for large systems. The Wicksell effects, by contrast, have remained and are now explained at a deeper level. Finally, the substitution possibilities are far fewer than expected. We expand the conclusion in three directions:

1. We have here dealt only with the neoclassical theory in its macro form, based on the aggregate production function. It can be shown (Scheffold 2016a) that this critique applies also to what I call the old neoclassical equilibrium, as is found in Walras, Böhm-Bawerk, Jevons and others, where a uniform rate of profit is assumed and made possible because the endowment of capital is introduced as a value magnitude (Clark 1899) and the endowments needed for reproduction are endogenously determined. The specific kind of instability, which we observed here in the presence of non-neoclassical Wicksell effects, also shows in modern intertemporal equilibrium systems, where the endowment consists of a vector of heterogeneous factors and commodities, as can be shown using the method explained in Scheffold (1997: 477–482).
2. We need a more differentiated extension of the critique than the simple observation: ‘Neoclassical theory is flawed, a different theory is called for’, without saying what exactly should be changed. The two main applications concern, in my view, the theory of accumulation and the theory of employment. We have here concentrated on the latter. It is not enough to say that, if the neoclassical mechanism to establish full employment fails, Keynesian measures must be right. On the one hand, it has to be admitted that lowering wages improves employment in an open economy by strengthening competitiveness. On the other hand, we have shown that the possibilities of substitution on the envelope to safeguard a high level of employment are more limited than usually thought, but that does not rule out the possibility of maintaining employment by using inefficient methods. In the extreme, as was observed already in antiquity, if we abandoned the plough, almost all of us would have to take the spade in order to survive. It seems that all countries rely on the use of some inefficient methods in order to avoid drastic forms of unemployment – no country seems to be on the envelope of the techniques that are available internationally.<sup>2</sup> The sparseness of

2. This was observed by Zambelli et al. (2017: 6).

efficient techniques that we must expect in consequence of (19) shows in reality also in the coexistence of different methods that are used in the same industry. The plough, now drawn by a tractor, and the spade coexist; subsistence farming complements modern farm production but the analysis of such phenomena requires a different level of abstraction. There is not only one way to improve employment.

3. At any rate, I hope to have shown that the discussion about the Cambridge critique is alive and well. It is broader than what I was able to discuss here; I have concentrated on the aspects that are connected with the theory of employment. The link between the theories of employment and of capital has been approached from different angles. In the first part of his book on this matter, Pasinetti (2007) has created a monument to the post-Keynesian economists at Cambridge (in particular Sraffa, Robinson and Kaldor); he explains the deep historical and conceptual difficulties that have so far obstructed the way to a true synthesis of the revival of classical thought and Keynesianism. The third part contains Pasinetti's structural analyses of the conditions for full employment. Other endeavours in this direction included the famous Trieste summer schools, 1980–1990 (Birolo 2010). Given the approach of this paper, we only refer to the Keynesian investment function, where the marginal efficiency of capital represents the internal rates of return of alternative investment projects, each characterised by the expected yields. Keynes took the expected yields as given in his short-run perspective, but if the investment function is seen in the long run, the investment projects must be described as alternative techniques, to be evaluated in terms of long-run prices, with the rate of interest being equal to the rate of profit. The investment function then will resemble the demand function for capital of neoclassical theory and will therefore be subject to the same critique. It follows that anti-neoclassical Wicksell effects may lead to investment functions such that the volume of investment falls, if the rate of interest is lowered. Or the investment function will be inelastic because only a few techniques turn out to be efficient. The economy will then not respond with an increase of investment that would be due to a substitution of techniques, if one tries to stimulate activity by monetary means. Whether the easing of finance leads to more investment in techniques that are already in actual use or whether it causes savings to be reduced are other questions. Our findings for the long run are not contradicted by the fact that short-run increases in interest rates force firms to curtail existing investment plans. The converse is not true: lowering interest rates does not create, but only perhaps induces, more investment. The Cambridge critique thus turns out to question not only the surrogate production function, but also the Keynesian marginal efficiency of capital schedule. There was no such schedule in the classical theory of accumulation of Smith, Ricardo, Mill or Marx.

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