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Predator-Prey – An Alternative Model of Stock Market Bubbles and the Business Cycle

*Eduard Gracia**

For the last quarter of a century, the Real Business Cycle model has dominated the interpretation of business cycles in mainstream economics; yet a number of significant empirical objections to it justify exploring an alternative approach. This paper proposes to base such an approach on a predator-prey mechanism, along the lines of the classical Lotka-Volterra model for ecosystem dynamics, where agency costs play the role of the predatory activity, in a process very similar to the one proposed by the classical Austrian School interpretation of the cycle (Hayek/Schumpeter). The model is consistent with both Rational Expectations and the Efficient Markets Hypothesis, and predicts that stock market valuations will regularly present bubbles and crashes synchronised with the business cycle without this implying any irrational behaviour on the part of the investors.

JEL Classification: E32

»When in fact a community has overcome many and serious dangers and has reached unquestioned power and lordship, new factors come into play. Prosperity takes its seat in that community and life turns towards luxury. Men become ambitious in their rivalries to achieve magistracies and other distinctions. As this takes place, the aspirations to magistracies, the protests of those who see themselves rejected, the pride and luxury, will give rise to decadence.« (Polybius (ca. 140 B.C.), *Histories*, quoted by Mazarino (1959))

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1. Why another Model of the Business Cycle?

Interest in the business cycles tends to be cyclical itself. When the economy is booming, there are always voices raised to claim that the business cycle is a ghost of the past, a result of system imperfections that the most recent developments (be it Keynesian-style government intervention, as in the fifties and sixties, or the process-optimising capabilities of the »New Economy«, as in the nineties) will turn as obsolete as Richard Malthus' old demographical theories; conversely, when the downturn comes (and the point is that in the end it *always* comes), the public find it easier to accept that there may be a cyclical pattern of some sorts behind it and that, in the words of Peter Navarro (2004: 24),

»while the predictability of the business cycle remains very much a debate among both academics and managers, it seems quite beyond debate that the line between corporate success and failure is often defined by the decisions that are made around key turning points and movements in that cycle.«

Nowhere is this statement truer than in the financial markets, where the spectacular dive of 2002 helped to remind those who seemed to have forgotten, as well as many others who were just caught in the middle, that the stock market is also a lead indicator of the business cycle – although many would still agree with Paul Samuelson's (1966: 92) famous quip that »Wall Street indices predicted nine out of the last five recessions«.

To be sure, the ever-changing feelings triggered by our ephemeral economic fortunes do not prove anything by themselves. The fact is, nevertheless, that empirical evidence of the existence and relevance of business cycles has steadily accumulated at least since the late 19th century, and has been confirmed by numerous contemporary studies (see for example Zarnowitz 1992 or Diebold/Rudebusch 1999), although, at the same time, the evidence also strongly suggests that the cycle is, in the words of Dore (1993: 21), »recurrent but non-periodic« – i.e., albeit a cyclical pattern can be observed *on average* in the time series, this past information cannot be used to predict the timing of future swings with any acceptable degree of reliability. From this viewpoint, the business cycle looks strikingly similar to the stock-exchange mean-reversion patterns whose existence has been highlighted in a number of empirical works at least since those by Poterba/Summers (1988) and Fama/French (1988); furthermore, the stock market role as a lead indicator of the business cycle also suggests that there may well be a causal link, and not just a spurious structural similitude, between this mean-reversion and the cycle.

In this context, it is probably fair to state that, for the last quarter of a century or so, the Real Business Cycle (RBC) model has stood as the dominant interpretation of the business cycle in mainstream, neoclassical economics (see for example Long/Plosser 1983 or King et al. 1988a and 1988b). Its central idea is that, in a system where the production function presents decreasing returns of scale respective to the stock of capital, the long-term optimal level of capital investment can be modified by an unexpected technology shock, but then, as the move to the new »optimal« level requires a technical process of accumulation of »physical« or »real« means of production (hence its name), it cannot take place instantly.

The resulting model is compatible with rational expectations and market efficiency and, in addition, a number of papers have shown, primarily on the basis of calibration/simulation techniques (following the steps of Kydland/Prescott 1982), that the patterns of key variable co-movement the RBC model predicts are generally compatible with observed aggregate data. Yet there are also significant empirical objections to this model. In particular:

1. A number of studies have produced results that do not support the model's basic hypothesis that technology shocks are the primary factor driving the business cycle. The lack of empirical support for this assumption was already pointed out by Summers (1986), and the observation has subsequently been reinforced by a number of more recent empirical papers such as Galí (1996), according to whom »the pattern of economic fluctuations attributed to technology shocks seems to be largely unrelated to major post-war cyclical episodes«, or Shea (1998), who concludes that »technology shocks explain only a small fraction of input and TFP¹ volatility at business-cycle horizons«.
2. Similarly, recent papers aimed at verifying the presence of long-term memory in aggregate GDP series along the lines predicted by the RBC model (notably Haubrich/Lo 2001) have also yielded negative results² – in fact, Haubrich/Lo suggest in their paper that the greater power of the statistical tool they apply (the so-called »Modified R/S« test) may explain why their results contradict earlier work that purported to find long-range dependence.
3. Last but not least, a number of authors argue that, even at a stylised fact level, the RBC does not match the observable facts, and in particular that the business cycle is closely linked to market disequilibrium phenomena (most conspicuously involuntary unemployment and undesired accumulation of inventories) that the RBC model, to the extent it postulates a frictionless market, simply cannot explain, as the fluctuations it portrays are caused by a gradual adjustment to a desired stock of capital *along an equilibrium path*. Although additional explanatory hypothesis for these disequilibria have been proposed (based, for example, on the assumption of indivisibility of labour, as in Hansen 1985, or Rogerson 1988), they are neither supported by hard facts nor really consistent with the »frictionless« spirit of the RBC model itself (for a more extensive exposition of this critical view on the RBC model see for example Dore 1993). Some have stated this objection rather eloquently: for instance, Michael Mussa, former chief economist at the IMF and now at the Institute for International Economics, described it as »the theory according to which the 1930s should be known not as the Great Depression but the Great Vacation.«³

1 TFP: total factor productivity.

2 In their paper, Haubrich/Lo (2001) explicitly present a version of Long/Plosser (1983) as an example of the type of model specification they intend to test, and whose rejection their empirical results suggest.

3 Quoted in *The Economist* on 26 September 2002.

The evidence does not seem to be conclusive either way. On the one hand, the results of studies like Galí (1996) suggest that the cycle may be due to sticky prices, adaptative expectations and other autocorrelation phenomena that could be interpreted as incompatible with full market efficiency (and consequently strengthen the case for a Neo-Keynesian revival). On the other hand, works like Haubrich/Lo (2001) indicate that such inter-temporal autocorrelation phenomena may either not exist, or at least not have a significant explanatory weight over long periods of time, and would thus even lend support to the view that perhaps the business cycle is a statistical mirage – although this interpretation would, in turn, contradict the long-accumulated evidence suggesting that the business cycle, whatever its cause, is a real phenomenon. There is hence a case for exploring alternative models that may present a better fit to the observed facts; in the context of this debate, the purpose of this paper is to introduce an alternative model that may contribute to reconcile these disparate sources of evidence.

There is in economics a long tradition, going back at least to the glorious days of Alfred Marshall, of hiding mathematical developments in footnotes and appendices so that they do not scare away the non-specialist reader. I would like to think I am following so illustrious a precedent by relegating all the analytical development to a series of appendices to this paper. Commenting on Marshall's works, John Maynard Keynes wrote that the true economist would do well to study the footnotes and appendices while having no more than a cursory look at the main text; once again striving to humbly follow the Masters' steps, I would similarly like to encourage those readers with an analytical background whose appetite may have been opened by the following pages to have a look at these appendices at some point.

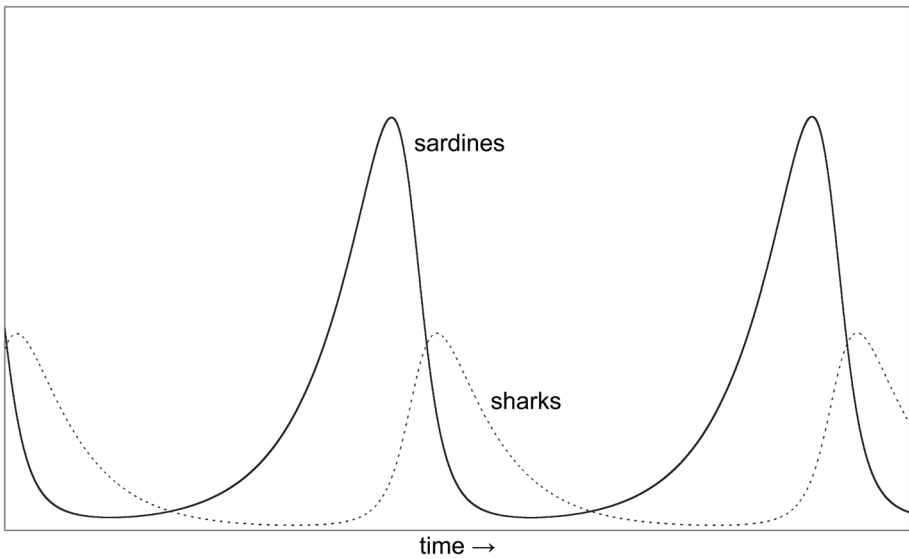
2. Predators and Preys

Originally put forward independently by Alfred Lotka and Vito Volterra in the 1920's, the so-called Lotka-Volterra predator-prey model is actually not difficult to understand from an intuitive viewpoint. As Volterra first proposed it to explain the observed dynamics of certain fish catches in the Adriatic, it probably makes sense to use the example of a marine ecosystem to illustrate it. Imagine an aquatic community with only two types of fish: sharks and sardines. The sardines eat plankton, whereas the sharks feed on the sardines. For simplicity, we assume the plankton supply to be unlimited; thus, in the absence of sharks, the sardine population would grow exponentially. In the presence of sharks, however, the sardine population will grow at a slower rate the higher the total population of sharks eating them. On the other hand, the population of sharks will grow faster the larger the proportion of sardines over sharks and, in the absence of sardines, will gradually starve away.⁴ Under these conditions, there should logically be a value for the shark and sardine populations such that they stay in equilibrium; unless both populations start with precisely the num-

4 Analytically, the model takes the following form:

bers that would be required to stay in equilibrium, however, their interaction will result in a cycle. Indeed, as the sardine population grows so will that of sharks and, as the number of sharks grows, their predatory activities will slow down the growth rate of the number of sardines until bending it down to nil and then to negative. But, at the same time, as the sardine population declines, the sharks themselves begins to starve and reduce their own numbers until, eventually, their numbers become small enough for the sardine population to recover its positive growth rate, thus resuming the cycle. Over time, therefore, the population of sharks and sardines will evolve as in figure 1:

Figure 1: Predator-Prey Dynamics over Time



For this particular example I have obviously chosen quite an extreme (and therefore also unrealistic) set of parameters in order to make the features of the model visually clearer. As we see, the predator-prey interaction not only results in a self-perpetuating cyclical motion, but also in a pattern where the prey population experiences relatively gradual growth periods punctuated by periodic, relatively spectacular »crashes«, whilst the predator popu-

$$\left. \begin{aligned} \frac{dx_t}{x_t} &= \alpha dt - \beta y_t dt \\ \frac{dy_t}{y_t} &= \gamma x_t dt - \delta dt \end{aligned} \right\}$$

Where, in this basic example, x_t would represent the population of sardines, y_t the population of sharks, and $\alpha, \beta, \gamma, \delta > 0$ would be positive parameters.

lation mirrors this pattern by having sudden increases coinciding with these crises, followed by relatively gradual periods of decline. If these »crises« look like the periodic burst of a sudden epidemic fever (where the »sardines« would now be the host population, be it cattle or human beings, and the »sharks« would be the disease germs), it is by no means a coincidence; indeed, since its inception, the Lotka-Volterra system (as well as its more sophisticated descendants) has proven very successful as a model of biological population patterns where one species predate on another – as it is the case, for example, of epidemic diseases.

One obvious objection could be raised here: as described so far, this is a deterministic model – so why are we not able to predict with perfect accuracy, for example, when the next cattle disease will burst? The simple answer is of course that, in the real world, there is no such thing as »other things being equal«, convenient as this qualification may be for modelling purposes; external, unexpected factors intervene by introducing changes in the population of predators and preys that may therefore radically change the timing of the next system crash.

We human beings are also biological entities, and have an uncanny ability to exploit each other. Save for a very low number of criminal exceptions, of course, human depredation on other humans does not usually take the brutal form of eating each other – yet this by no means makes human predatory activity less relevant as a mechanism of resource redistribution. In the words of the historian William McNeill (1982: 8),

»disease germs are the most important microparasites humans have to deal with. Our only significant macroparasites are other men who, by specializing in violence, are able to secure a living without themselves producing the food and other commodities they consume.«

For the purposes of this paper, we will define as »depredation« or »exploitation« a relationship between two economic agents such that one of them takes resources away from another without providing any goods on exchange as part of a mutually-agreed transaction. Basic, highly intuitive examples of this could be the burglar that steals from his neighbour or the security guard that runs away with the money; but other, somewhat more sophisticated examples are probably much more important from an economic viewpoint. Taxes, for example, constitute a predatory activity where the prey are those who pay them (and have to do so whether they like it or not) and the predators are those who benefit from the government's redistribution activities. Similarly, credit defaults also constitute a form of depredation of those who took the money and did not return it against those who lent it and never got it back; and equally predatory is the case of the employee who shirks his responsibilities, or the director that misleads his shareholders. The point is that, although the expected costs of depredation can of course be factored in their expectations by the economic players, and although the payments are often made voluntarily to avoid worse consequences (as in the man who pays taxes to avoid going to prison, or the shopkeeper who bribes the local gangsters to avoid having her shop vandalised), this type of economic transaction cannot be assimilated to a market exchange because, in the case of depredation,

the overall result is suboptimal respective to what would have been achieved in an ideal situation where information were perfect and the players were willing to cooperate for an agreed price (technically speaking, is »Pareto-inefficient«⁵). Armies, police forces, disciplinary rules of any kind, even the humble locks that we all put at the front doors of our homes are but a few among many examples of resources that could be devoted to more productive tasks if information were perfect, transaction costs did not exist and people were not opportunistic.⁶

Depredation mechanisms along these lines are actually quite common in economic theory, as well as in the wider realm of social sciences. Free-rider models, agency theory and many market failure theories, for instance, all fall within this group. Agency theory, in particular, deals with a number of depredation instances where what enables the predator to exploit the victim is not the threat of raw violence (as it is the case of government taxes) but the possession of privileged information by the »agent« that therefore enables him/her to extract resources from the »principal« – what is technically called an »information asymmetry« (for further background on modern agency theory see, for instance, Jensen 2000 or Williamson 1995). Thus, for example, the employee that shirks his responsibilities is able to do so because the principal (in this case, the shareholders of the company that pays his salary) has only an imperfect knowledge of his real productivity. To control this form of cheating, the principal can, for example, impose bureaucratic and disciplinary controls, and can also increase the employee's salary above market level so that the cost for the employee of being dismissed because of having caught shirking becomes higher –

5 The concept of Pareto-efficiency was introduced in 1906 by Vilfredo Pareto; according to it, we say the resource allocation in a system is »Pareto-efficient« if it is not possible to improve the welfare of at least one of the participants without reducing that of another one.

6 The logic underlying this idea is easy to see using what is probably the most famous device in non-cooperative game theory: the Prisoner's Dilemma. The classical exposition of this famous intellectual game is just like the plot of an old cops' movie. Two accomplices in crime go to jail and are interrogated by the police in separate cells. They know that, if no one confesses the crime, they will both go free because the police has no other evidence against them, whereas if they both accuse each other they will both stay in jail for, say, five years. But if one of them accuses the other and is not accused by the other one at the same time, then the accuser will go free whilst his »buddy« goes to prison for, say, ten years. Once isolated, and if the game is only going to be played once, the rational decision for each one is of course to betray the other because, regardless of whether the other criminal has betrayed him back or not, the outcome in either case is equal or better if he betrays – in other words, the rational decision is not to cooperate, even though the outcome of both being rational is to end up worse off than if they had both cooperated. In a world populated by rational agents, the result of this game will be Pareto-inefficient for, even if the prisoners could have agreed an exchange beforehand (in which the price for each one's loyalty would be that of the other), the agreement would become void simply because there would be no way to enforce compliance – in other words, because information is not perfect and, at the time of playing, no one can really know how the other is going to behave. It should now be easy to see that this conclusion can extend to the different forms of depredatory game we described above: the tax game, the default game, the shirking employee game, etc.

i.e., in order to provide an additional incentive to stay honest.⁷ From the viewpoint of the community as a whole, however, every one of these approaches leads to a form of inefficiency: bureaucratic controls not only represent extra costs, but also tend to slow down the production process as a whole, whilst increasing the salaries of those with a job above the market clearing wage level obviously generates unemployment, which will be larger the higher the difference between the wages actually paid and those that would equal labour demand and supply (see for example Shapiro/Stiglitz 1984 or Phelps 1994). Similarly, credit opens the door to a straightforward trick where an entrepreneur could simply set up a limited liability company, negotiate a credit, spend it carelessly and then declare bankruptcy and restart the cycle. Against this possibility, the creditors not only need to charge a premium on the basic interest rate to compensate for this risk, but also establish a system of (inevitably costly) bureaucratic controls to limit the ability of the debt issuers to cheat, or to increase the personal cost for them of doing so. Yet these bureaucratic controls also represent a cost of non-cooperation, and the risk premium above the basic interest rate means that there will also be legitimate investment opportunities that will not be pursued because the market cost of the funds to finance them is too high, particularly when the economy is in a downturn and thus all credit ratings tend to be lower (see for instance Lowe/Rohling 1993).

3. *Rational Players*

So, if the concept of predatory behaviour is not really alien to standard economics, then, why is the predator-prey model not being considered as a potential model of the business cycle? The short answer is, of course, that it has already been considered: this is precisely what Richard Goodwin did in his famous paper, first presented at the First World Congress of the Econometric Society in 1965 under the beautifully simple title of »A Growth Cycle«, and subsequently published as Goodwin (1967). Today, nearly forty years thereafter, Goodwin's model is still held in very high regard; Dore (1993: 211), for instance, considers that »in the history so far of business cycle theory, the Goodwin model must be seen as a remarkable achievement.« The model manages to generate an endogenous, self-sustaining cycle primarily on the basis of assumptions that up to the mid-1970's were generally accepted as part of standard macroeconomics. The result is a Lotka-Volterra cycle like the one we saw for the sharks and sardines example, where it is assumed that all wages are consumed and all profits reinvested, and where wages grow at a faster rate the lower the market level of unemployment (the so-called Phillips curve), so that, in effect, wage growth plays the role of shark population growth, and production growth induced by profit reinvestment that of sardine population growth. From the viewpoint of modern standard economics, however, Goodwin's model is objectionable on three main grounds. First, it relies

7 This is the core argument of efficiency wage theory. As an introduction, I personally find that Ellingsen (1997) contains a particularly clear analytical development; a more general exposition of efficiency wage theory can be found in standard manuals like, for example, Milgrom/Roberts (1992).

on the assumption that all wages are consumed and all profits are reinvested, which is not consistent with empirical observation.⁸ Second, it also relies on the Phillips curve, a statistical relationship between the rate of unemployment and that of salary growth that became empirically discredited during the stagflation years in the 1970's, when salary growth accelerated in parallel with unemployment rates. And third, it does not postulate rational behaviour, for investors seem to be willing to reinvest the totality of their profits regardless of the expected rate of return.

The overall objection to non-rationality is by far the most serious from our viewpoint, as it represents a challenge not only for this specific model but also for any other portraying the cycle as a self-sustaining or otherwise predictable oscillation along the expected path. Indeed, as this argument goes, if investors were rational and they expected rates of return to experience an abnormal raise tomorrow, they would logically invest more until the abundance of supply cancelled this extraordinary profit opportunity by lowering the return – and vice versa, if they expected a fall they would divest (or sell short) until rates increased again. Thus, if people are rational and the capital markets they operate in are efficient (i.e., prices are always able to adjust instantly to clear supply against demand, so that they always end up reflecting the players' expectations at any point in time), they would arbitrage against expected future cyclical swings until ruling them out from the expected path. The strength of this argument relies primarily on the fact that there is very robust empirical evidence supporting the postulate that not only economic players develop their expectations rationally, but also that capital markets are generally efficient,⁹ to the point that, already in 1978, Jensen (1978: 95) felt entitled to state that »there is no other proposition in economics which has more solid empirical evidence supporting it than the Efficient Markets Hypothesis.« Thus, even if financial markets are actually not perfectly efficient and non-financial markets are even slower to adjust, over the long run one would expect the business cycle to be discounted out of the financial market valuations, instead of being reflected under the form of a mean-reversion pattern, as Poterba/Summers (1988) found; and if, as the »behavioural finance« school suggests, investors behaviour is non-rationally

8 Although it is also fair to say that this assumption was quite standard in macroeconomic models of the 1950's and 1960's: often referred to as the »golden rule«, it was essentially derived from the conditions of steady-state growth under the ordinary assumptions of a Solow-type growth model.

9 It should be noted, in this respect, that even the subsequently uncovered evidence highlighting instances of departure from the random walk does not necessarily lead to the logical rejection of the efficient markets hypothesis, except perhaps in the case of transaction costs and other forms of friction in the short run, for, as Lo/MacKinlay (1999: 5) rightly point out, »the random walk hypothesis need not be satisfied even if prices do fully reflect all available information«, as was already proven by LeRoy (1973) and Lucas (1978). The (rather common) view that market efficiency necessarily implies random walk prices and vice versa is thus a dangerous misperception. Indeed, in their comprehensive collection of previous articles, Lo/MacKinlay (1999) illustrate both the departures from the Efficient Markets Hypothesis observed in the short run (as one would anyway expect, for transaction costs and other forms of friction prevent market prices to adjust strictly instantaneously) and the fact that the hypothesis still holds in the long run (see particularly Lo 1991).

biased in a number of critical aspects, one must then wonder why would they nevertheless be rational in other areas, or why have these irrational players not yet been driven to extinction by smarter competition. Following this reasoning, the RBC model, despite its many shortcomings, logically trumps Goodwin's simply because it does not contradict the postulate of people's rationality.

Reasonable as the argument above may look, the fact is that there is a hole in it. Indeed, in a rational agents' world with an efficient financial market, any cyclical patterns or, for that matter, any forms of market disequilibrium should be ruled out from the future expected path; yet from this does not necessarily follow that systematic cyclical patterns or market disequilibria will also be excluded from the observed path, except in those particular cases where the observed path explicitly tracks the expected path (as it is the case, of course, in deterministic models). In particular, it is a well-known phenomenon that, in accumulative stochastic processes (such as the compounded accumulation of returns on an asset reinvested over time) where the accumulation rate follows a random walk, the path that empirical observations should be expected to track (technically, the path that minimises the tracking error respective to the observations) is the median, i.e., the path that leaves 50 percent of the distribution on either side, not the mean (that is, not the expected path); and these two paths, in asymmetric distributions, can be quite different.¹⁰

An example may help to understand this mechanism from an intuitive viewpoint. One such accumulative game is »double-or-nothing«, so widely popularised by television shows. Starting with an initial investment amount (say, \$ 1), the player multiplies it by two on the basis of a given random event (say, answering a question correctly) and loses the whole capital otherwise. If, for simplicity, we assume the random process is such that probabilities at every round are evenly distributed between the outcomes of »double« or »nothing« (say, as in the case of tossing a coin), it is easy to see that, regardless of whether the game is played once or a thousand times, the expected value at the end will obviously still be equal to the original investment, and thus the accumulated net return will be zero. It is also straightforward to see that, for the first round, a return of zero will also be the median. However, if the game is to be played more than once (say, ten times), the distribution of the outcomes changes: there is now less than a 0.098 percent probability of having accumulated \$ 1,024 over an uninterrupted sequence of successful games, and over a 99.9 percent probability of having lost the whole capital at any of those successive ten games, i.e., of a return of -100 percent. Hence, the mean value after ten games will still be \$ 1, but the median will be zero. Now, this means that, if an external observer, unaware of the internal rules of the game, simply tried to analyse empirically its results over one given sequence of observations, we should intuitively expect the outcome to be closer to the median than to the mean, for there is a very high probability that the final value of the game end up being zero.¹¹ Yet this result

10 For an example of utilisation of this principle in modern financial theory see, e.g., Roll (1992).

11 For a more rigorous, analytical development of this reasoning see either Appendix 1 or the technical version of this paper at <http://ssrn.com/abstract=549741>; for an example in the context of portfolio management theory, see Roll (1992).

would not imply that the player's expectations of a zero percent return (instead of a -100 percent) were irrational: simply, the expected value was an average calculated on a strongly asymmetrical probability distribution function.

This is critical to cycle theory because, even if prices do indeed adjust automatically to preclude market disequilibria and extraordinary profit opportunities along the expected (i.e., mean) path as soon as they appear, there is no reason to assume they would at the same time cancel them along the median path and thus, to the extent the observed path tracks the median and not the mean, they would appear on the observations. Of course, every time unexpected disequilibria appear, prices will realign to preclude them going forward *along the expected path* – but, to the extent the path observed over time is closer to the median, such disequilibria will still consistently appear. In such a system, what would tell us that, despite the presence of cyclical patterns and persistent disequilibria, the economic players are still placing their bets rationally would be that autocorrelation tests would nevertheless tend to reject the hypothesis of long-term dependency, and that attempts to predict these patterns with enough accuracy to arbitrage against them would ultimately be bound to fail. This is, indeed, what has been observed in the real-life business cycle, whose oscillations, as we have seen, can be statistically detected and measured, but cannot be traced to any underlying long-term dependency mechanism or analogous predictable pattern, neither in the stock market (Lo 1991) nor in the GDP series (Haubrich/Lo 2001). Furthermore, to the extent the median path presents a cycle whereas the mean does not, for many (albeit not necessarily all) functional specification, one should also expect empirical data to display cyclical patterns in their sample variance and skewedness (i.e., the degree of asymmetry), simply because the mean path is not cyclical, whereas the median (as well as the other key statistical paths) is, so that, when the median path is close to the mean (i.e., in the expansion part of the cycle) the whole distribution is compressed, with the corresponding reduction of on variance and skewedness – and vice versa during the downturn of the cycle. This phenomenon, incidentally, also seems to be consistent with the observations in most of the literature on stock market variability (see for example Granger/Poon 2003).

So it should be possible to devise a predator-prey system such that a cyclical pattern appears on the median path but not on the mean; in fact, one could conceive not one but many such models. The following section simply describes one of them from a discursive, intuitive viewpoint.¹²

It should be noted, before we proceed, that the primary purpose of model described below is to portray how financial market valuations in an efficient market with fully rational players could follow, along their median path, a predator-prey dynamic process where agency costs played the part of the predatory activity. It has therefore been conceived on the basis of essentially microeconomic assumptions (focusing on the behaviour of »a firm in a market« instead of »a closed economy«) and thus, strictly speaking, a number of ad-

12 An analytical development can be found either in Appendix 2 or in the technical version of this paper at <http://ssrn.com/abstract=549741>.

ditional assumptions would be needed to turn this model into a proper macroeconomic expression. Doing so, however, would go beyond the scope of this paper.

4. A Predator-Prey Model of the Financial Cycle

Imagine a market with two kinds of individuals: some people who lend funds to firms without directly participating in the production process (whom we will call «creditors» or «debt holders») and others who, in addition to investing their own resources as equity in the company, have direct control of the productive process (and hence will be referred to as «producers» or «entrepreneurs»). Of course, in the absence of any form of control mechanism by the creditors on the producers, this information asymmetry would open an easy route for the producers to exploit the creditors simply by raising the money, spending it for their personal purposes or transferring it to their own private accounts and then declaring bankruptcy. Against this danger, the creditors implement a system of punishments and rewards, i.e., a set of rules against the producers' shirking (e.g., legal regulation against fraud, a set of internal bureaucratic controls, the request that the producers provide collateral guarantees against their personal assets, the threat that a bad credit history may represent on their future funds raising capabilities, etc.) whose purpose is to, if not completely prevent, at least make it more difficult for the producers to appropriate the resources of the company. To the extent these deterrents are not 100 percent effective, though, deprecation will take place under the form of a «rent» (which we will call a «quasi-rent» to distinguish it from rents resulting from the mere market price of a good) composed of earnings received by the entrepreneurs above and beyond the market clearing price of the services they provide to the firm (i.e., of the productive resources they invest in it). These quasi-rents will be higher the greater the degree of control enjoyed by the producers on the assets of the firm (for, in order to act as an effective deterrent of shirking activities, the quasi-rent must be such that the net present value of the future income it generates be equal to the expected profit the producer would obtain from shirking and running the risk of being caught and fired), i.e., there will be a trade off between external controls imposed by the creditors and quasi-rents earned by the entrepreneurs. We will also assume that the establishment and maintenance of these surveillance and control mechanisms against shirking is not cost-free for the creditors and, therefore, they will only impose them up to the point where their marginal costs equal their marginal benefits (i.e., the expected reduction of future default costs).

The appropriation of company resources by the producers takes place as follows. For a given production process structure, the entrepreneur has a certain degree of control that translates into a given level of quasi-rents paid as a percentage of the output. At every point in time, a certain number of opportunities to modify this productive structure will randomly appear. Other things being equal, the producer's decisions will of course be biased in favour of those alternatives that generate a higher level of quasi-rents, but their level of discretionality in this respect is limited by the controls imposed by the creditors. As the

purpose of these controls is to offset the probability of default, they will be stronger for those companies whose solvency ratio (i.e., the total asset value divided by external debt, which in the case of this model is equal to the part of the invested capital that is not owned as equity by the entrepreneurs) is lower. Thus, other things being equal, the speed at which the entrepreneurial quasi-rents increase over time will be faster the higher the firm's solvency ratio, and vice versa. Note, therefore, that this mechanism is playing the role of the growth of the shark population in the predator-prey model.

As we have already discussed, under market efficiency the future expected (or »mean«) path of the firm's value will not be cyclical, for the market is assumed to be able to discount the expected future depredation quasi-rents upfront. Yet if we assume that the asset rate of return is stochastic, and its distribution function is such that the observed path is closer to the median than to the mean (as it is the case in the most widely used asset return models used in finance¹³), then there is nothing to prevent more complex patterns from appearing on this observed path. Along the median path (or simply along any path different from the mean) the agents' expectations consistently fail to be met, not because they were not set rationally and on the basis of all the information previously available, but simply because dice rarely, if ever, produce their »expected« value. Hence, along the median path, asset return expectations will consistently be proven wrong and thus need to be continuously revised. The impact of the market inefficiencies caused by the depredation mechanism between creditors and producers will thus also fluctuate with the unexpected circumstances and, as the market prices will, at every new change in expectations, realign instantly so that the future *mean* path remains cycle-free, there is nothing to prevent the predator-prey cycle from appearing on the *median* path.

Thus, in this model, when the economy grows at a rate lower than expected (and therefore moves away from the equilibrium path), the solvency ratio of companies falls, thus increasing the risk of default, which makes it more cost-effective for creditors to strengthen the control measures they impose on the firms' producers. This curbs the level of discretionality these enjoy over future changes in the productive process, and therefore also the growth rate of the percentage of the output they subtract under the form of quasi-rents, until the positive effect of this on the finances of the firm overturns the crisis and re-ignites growth. Then, of course, the process repeats itself, as the solvency ratio improves and the incentive for the creditors to keep tight control mechanisms reduces, which also gradually allows the producers to increase their control on the assets on the firm until the weight of their increased quasi-rents starts to bring the solvency ratio down again, thus restarting the cycle.

Note that the perturbation that explains the difference between mean and median, and is thus responsible for fuelling the cycle, could in principle come from any source, including technology shocks as well as many others. For the analytical development of this model, for example, precisely in order to emphasise its independence from technology considerations, the assumptions were purposefully devised so that the only source of perturbation in it would be the variability of the money market interest rate. Using the interest

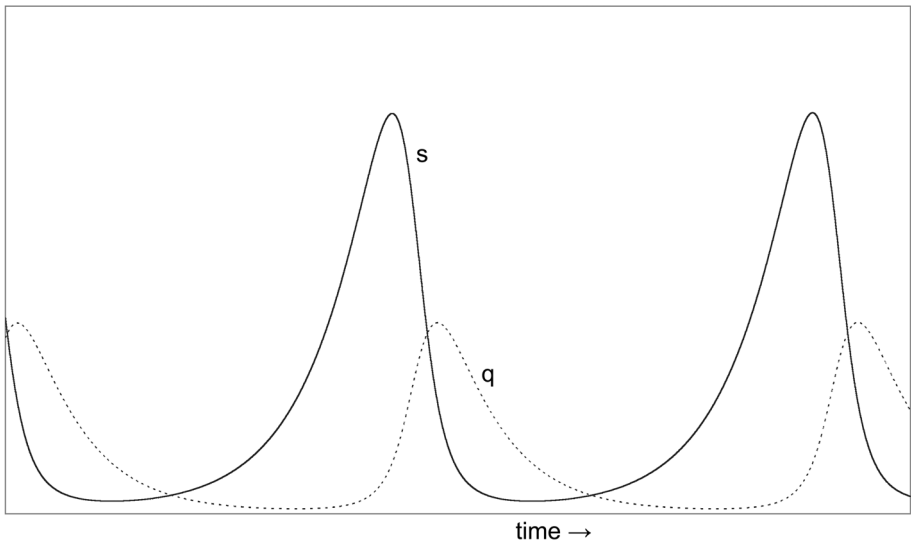
13 Including the most basic of them all: the geometric Wiener diffusion process.

rate as the source of uncertainty also has the advantage that, as it constitutes a price that is visible to all players in the market, it also provides an example of how the market could induce synchronicity on the cycles experienced by different firms and industries – for all the firms in a country are simultaneously exposed to the same fluctuations in the market interest rate. Nevertheless, there is no necessary assumption in this model that this be the only cycle-inducing source of uncertainty in the real world or that the risk premium would need to be constant; more complex and probably more realistic models could no doubt be developed.

5. *Stylised Implications*

When this model is explicitly built and resolved analytically, it yields a Lotka-Volterra median path (figure 2):

Figure 2: Financial Cycle Model – Median Path over Time



Where q represents the ratio of total producers' quasi-rents divided by investors' returns and s represents the solvency ratio, i.e., total capital invested divided by total creditor debt, always at market values. To those acquainted with the behaviour of financial markets and national economies over the business cycle, the patterns represented in this diagram should look fairly familiar. Indeed, on the one hand, the solvency ratio s increases gradually with good times (the »bubble«) and then falls down much quicker than it went up (the »crash«). This, incidentally, is consistent with the empirical evidence available (for a recent study see, for example, Koopman/Lucas 2003). On the other, the quasi-rent ratio q shoots

up precisely at the time of the crash, and then only gradually falls down again; although measuring what portion of the income is distributed as quasi-rent as opposed to prices at market-clearing levels is a tricky business, we can use other metrics as a proxy, such as the unemployment rates (because, if the quasi-rents paid to workers¹⁴ above market-clearing level are very high, the market will remain further away from equilibrium, i.e., will present a higher level of unemployment). This, again, is consistent with well-known stylised facts.

Note that this is, to a large extent, a cycle of market expectations. Even if the series is deflated to constant prices of goods and services, the value of the stock of capital (and therefore also of its first differential respective to time, net investment) is impacted by future expectations. It is thus on the basis of these expectations that the solvency ratio is calculated by the creditors, who then use it to decide how much effort should be devoted to contain the producers' ability to gradually change the productive process so that it results in an increase of their quasi-rents. Because of this, the cycle cannot take place on the mean (i.e., expected) path, and thus cannot be forecasted at any point in time, even though the model allows us to predict an average length of wave that could be observed over time. At every given point in time, the current prices are in equilibrium vis-à-vis the most current expectations of future returns (i.e., in respect to the mean path from that instant onwards), but in disequilibrium respective to the median path. What path will be followed afterwards is, of course, unknown; what we know, however, is that an analysis of the series of observations it generates will be closer to the median than to the mean path and, therefore, will be different from that mean, equilibrium path. To the extent the cycle is fuelled by unexpected shocks reflected by the market in its pricing and discount rate structure, many firms that have not suffered the impact directly on their productive processes would experience it anyway through the impact of the market, and therefore we should also expect the cycles experienced by different companies to synchronise with each other (in other words, we should expect to observe simultaneous market valuation bubbles and crashes of large groups of stocks) even when the relationship between their business activities is small or non-existent – in other words, even when the co-movement is not justified by the firms' fundamentals.

Furthermore, although one could in principle estimate the future median path more or less the same way one would estimate the mean, this path changes its shape at every next point in time so that it always begins at the same starting position – in other words, one cannot determine with any degree of accuracy how long it is going to take to clean out the »parasitical structures« off the system before growth can be resumed, simply because the most likely path has already been discounted from the current market valuation so that the expected future path remains cycle-free. The cycle in this model thus postulates the

14 The model as defined does not establish a distinction between the resources invested in a company under the form of labour or of any other input. Thus, in using unemployment as a proxy of the quasi-rent ratio we are implicitly assuming that workers have some degree of control on the well-functioning of the business (even if it is only because they can threaten to interrupt it by going on strike), as efficiency wage theory postulates, and therefore also get their share of quasi-rents.

existence of an average cycle wavelength to be found in past time series, but does not allow to predict the timing of its future behaviour with any useful degree of accuracy.

Computer simulations on the model indicate that, given a constant basic asset rate of return, the frequency of the cycle can be reduced by increasing the rate of growth of the credit available (in macroeconomic terms, the money supply) and/or the portion of the returns of the companies that is paid as net cash to investors¹⁵ as opposed to being reinvested (in macroeconomic terms, consumption as opposed to investment). Nonetheless, this reduction in frequency comes together with an increase in the amplitude, i.e., in the difference between cycle highs and lows, until, if the credit burden grows faster than the rate of earnings retention (which is of course the difference between the assets rate of return and the portion of it that is distributed as cash), the system collapses at the first crisis never to recover again. It is intuitively easy to see why: the difference between the rate of earnings retention and the debt growth rate represents the rate of accumulation of equity the company would experience if there were no quasi-rents; therefore, a negative value implies that it would never be able to restore its solvency after the first shock and thus, in the long run, it would inevitably end up in bankruptcy. Taken strictly at face value, this would also suggest that policies of credit injection and stimulation of demand would not so much work by avoiding the crisis as by postponing it, as they would essentially push the system into a regime where crises are less frequent but more serious. The reason becomes readily apparent as we inspect the assumptions this model is based on: in essence, it postulates that *a growing economy gradually accumulates inefficient internal depredation mechanisms, and the crisis is the way the system cleans itself of these parasitical structures* – thus, by postponing the crisis one such system keeps feeding these parasites and thus implicitly sets the scenario for a more violent adjustment when the crisis eventually arrives. The association of cycle swings with unexpected events such as technology innovations or political events would thus simply be due to the fact that, had these events been expected, an efficient market would already have discounted them away. One could say the unexpected event does not, strictly speaking, »cause« the crisis – it just triggers it.

6. Final Considerations

As we have already pointed out, Goodwin (1967) had already used a predator-prey system to model the business cycle in an analytical development that, with its reliance on the Phillips curve and its assumption of a labour market with a very rigid clearing mechanism, can safely be classified as closely related to the Keynesian family. However, as we have also seen throughout this paper, a predator-prey model can be specified so that it becomes compatible with market efficiency, as long as we recognise that the cycle will be absent from the expected path but not necessarily from the observed one. The inefficiencies can take any

¹⁵ Once again, remember that the model admits an interpretation where workers are just investors that contribute an input called labour.

or all of the forms identified by the labour market literature: unemployment above frictional level as in Shapiro/Stiglitz (1984), influence-seeking activities as in Milgrom/Roberts (1988) or simply the cost of strikes and trade union activism – at least in its present, highly simplified form, the model does not need to make any specific assumptions about how the inefficiency will manifest itself. This would thus seem to support a Keynesian view of the cycle, while remaining compatible with rational expectations. Yet in the critical area of public intervention, the model, as we have seen, invites to draw rather gloomier conclusions than in traditional Keynesian thought: intervention can indeed be used to reduce the frequency of the cycle, but only at the expense of making the crisis deeper once it arrives. One can delay Judgement Day, but only by accumulating interests on the debt that will then need to be paid.

From this viewpoint, the model we have described in this paper can also be interpreted as providing a rational expectations basis to the so-called Austrian School interpretation of the business cycle. Indeed, the model that Friedrich von Hayek, Ludwig von Mises and later Joseph Schumpeter put forward in the early 20th century was based on the postulate that during growth periods companies tended to over-invest and that, when it eventually became evident that the return on these investments was going to be lower than initially expected, an overproduction crisis would ensue – what Schumpeter, in an expression that would equally fit the model we have presented in this paper, called »the eternal gale of creative destruction«. The problem with this theory has traditionally been that, unless there is a source of inefficiency operating somewhere in the background, the Austrian model is in principle no more compatible with market efficiency than Goodwin's: in a rational expectations world, one would expect the investors to rationally forecast the likely return on their investments and then act accordingly. Schumpeter himself rejected »rationality« as an explanation for co-ordination (which was due to routine) as well as for entrepreneurship. Yet empirical evidence suggests that over-investment or »empire building« can also constitute a form of depredation: for instance, an empirical study by Hennessy/Levy (2002) found »strong evidence in favour of empire building incentives, with the effect being strongest when founder status is used as a proxy for empire references«, which is what one would expect if the reason for over-investment were a form of depredation related to the implicit moral hazard that is always associated to credit (i.e., when the game is such that, if the investment goes well, the upside is for the equity holder whereas, if it goes badly, the result is a default to the creditor). Thus, a manager-owner would logically feel more tempted to over-invest than a hired CEO whose participation in the firm's profit is limited.

One is tantalised at this point to explore other potential applications, particularly in reference to longer-range cycles related to political phenomena. After all, we have already seen that taxes constitute a form of depredation supported by the government's control of the main instruments of violence in a country, and we have also quoted McNeill's (1982: 8) view that

»our only significant macroparasites are other men who, by specializing in violence, are able to secure a living without themselves producing the food and other commodities they consume.«

History is full of examples where a predator-prey mechanism seemed to operate behind the rise and fall of states and societies. For instance, the parasitic dependency of the Central Asian steppe nomad kingdoms on the Chinese Empire, where the former would grow strongly as the Chinese economy developed and the emperors could afford paying them tribute, and then would precipitate the crisis by plundering the Empire like starving wolves when the bad times came, is already quite well understood. Similarly, the interpretation of the fall of the Roman Empire as the consequence of the gradually increased weight of a bureaucratic system that ended up weakening that formidable government structure until it was too feeble to resist the pressure of the barbarians is also well established now (see for example Grant 1990). It is therefore tempting to conjecture that perhaps both the «classical», Austrian-school over-production cycle and the «Keynesian» market-rigidity crisis model may simply constitute different manifestations of the same type of underlying depredation mechanism – a process belonging to the same family as the crisis of pre-industrial empires or the periodic reappearance of epidemic infections in human and animal populations.

Appendix 1

The purpose of this appendix is to show how an empirical analysis of a time series generated by a geometric Gauss-Wiener diffusion process (i.e., a «geometric Brownian motion») should be expected to yield an observed average growth rate closer to the median, not the mean, path of the distribution.

Consider an asset P_t whose market value follows a geometric Wiener diffusion process such as:

$$\frac{dP_t}{P_t} = \mu dt + \sigma dZ_t$$

Where μ, σ are parameters and Z_t is a linear Brownian motion $dZ_t = \varepsilon_t \sqrt{dt}$, where $Z_0 \equiv 0$ and where the white noise variable ε_t is normally-distributed, i.e., $\varepsilon_t \sim N[0,1]$. Then of course the expected (or «mean») growth rate of the asset's market value is:

$$E_0 \left[\frac{dP_t}{P_t dt} \right] = \mu + \sigma E_0 \left[\frac{dZ_t}{dt} \right] = \mu$$

We can now use Itô's lemma to obtain the general expression of P_t :

$$d(\ln P_t) = \frac{dP_t}{P_t} - \frac{1}{2} \left(\frac{dP_t}{P_t} \right)^2 = \frac{dP_t}{P_t} - \frac{(\mu dt + \sigma dZ_t)^2}{2} = \frac{dP_t}{P_t} - \frac{\sigma^2}{2} dt$$

$$\frac{dP_t}{P_t} = d(\ln P_t) + \frac{\sigma^2}{2} dt = \mu dt + \sigma dZ_t$$

$$d(\ln P_t) = \mu dt - \frac{\sigma^2}{2} dt + \sigma dZ_t \Leftrightarrow \ln P_t = \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma Z_t + ct$$

$$P_t = P_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma Z_t}$$

Since Z_t is a normally-distributed and therefore symmetrical variable, it is easy to see that the growth rate of the median path (i.e., the one that would cut across the distribution leaving 50 percent of the probability on each side) is $\mu - \sigma^2 / 2$, i.e., different from that of the mean path.

If we now had a sufficiently long time series of empirical observations of P_t , then we could calculate its average growth rate through a logarithmic regression under the following specification:

$$\ln P_t = a + bt + u_t$$

Where a and b are the regression parameters and u_t the series of residuals. By inspection it can thus immediately be seen that the results of this regression should be expected to approximate the following:

$$\begin{aligned} a &= \ln P_0 \\ b &= \mu - \frac{\sigma^2}{2} \\ u_t &= \sigma Z_t \end{aligned}$$

In other words, the result of the empirical analysis should be expected to be a growth rate equal to that of the median, not the mean.

Furthermore, it is also straightforward to see that the result should be expected to be the same if, instead of estimating the parameters of a logarithmic regression, we calculated the average continuous growth rate along the time series. Indeed, if, given a time interval $t \in [0, T]$, the average continuous growth rate g is defined as a magnitude such that:

$$e^{gT} \equiv \frac{P_T}{P_0} \Leftrightarrow g \equiv \frac{\ln P_T - \ln P_0}{T}$$

Then, in our case, it yields the following expected result:

$$\begin{aligned} g &\equiv \frac{\ln P_T - \ln P_0}{T} = \frac{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma Z_T}{T} \\ E_0[g] &= \left(\mu - \frac{\sigma^2}{2}\right) + \frac{\sigma}{T} E_0[Z_T] = \left(\mu - \frac{\sigma^2}{2}\right) \end{aligned}$$

Which is, once again, the median growth rate of the distribution, not the mean.

The conclusion is thus that, to the extent the valuation of this asset takes place in an efficient market, the analysis of the time series it generates could present average results significantly removed from this expected equilibrium without this necessarily challenging the efficiency of the underlying market.

Appendix 2

The purpose of this appendix is to develop analytically the reasoning that in Section 4 of the main text was presented under an intuitive, discursive form.

Definitions and Assumptions

Consider a firm with a stock of productive resources whose total asset market value at instant t is K_t , of which an amount D_t has been financed through debt while the rest belongs to the producers that also run and control the business. The net value added Y_t generated by this entity is then allocated between a cash flow C_t paid to the investors (including both debt and equity holders) and a quasi-rent flow Q_t paid to the producers that control the production process in addition to the market return on the equity resources they have invested in it, while the rest remains in the company as retained earnings to be reinvested, i.e.:

$$Y_t \equiv Q_t + C_t + \frac{dK_t}{dt} \equiv Q_t + C_t + \dot{K}_t \quad (\text{i})$$

We designate as \tilde{K}_t the market value of the producer's control of the productive process, i.e., of the »asset« that the entrepreneur's ability to perceive quasi-rents represents (or, what is the same, the value of the resources he would be willing to divert from productive activities and invest in those required to maintain this position of control), and the firm's rate of return r_t as:

$$r_t \equiv \frac{Y_t}{K_t + \tilde{K}_t} \quad (\text{ii})$$

We represent by ρ_t the money market interest rate, and we define the risk premium π_t as:

$$\pi_t \equiv r_t - \rho_t \quad (\text{iii})$$

Finally, we define the solvency ratio as:

$$s_t \equiv \frac{K_t}{D_t} \quad (\text{iv})$$

And the producer control ratio as:

$$q_t \equiv \frac{\tilde{K}_t}{K_t} \quad (\text{v})$$

In this context, we introduce the following assumptions:

1. Market efficiency

The asset market valuation is such that the future return on investment is always expected to be equal to the market rate of return,¹ i.e.:

$$\forall t \geq T \quad ; \quad E_T [r_t K_t] = E_T [Y_t - Q_t] \quad (1)$$

Where the operator $E_t[\circ]$ indicates expected value according to the information available at instant t .

Comment Note the implicit assumption that $E_T[r_t]$ is equal to the expected market rate of return.

Note as well that, per definition (ii), this implies that:

$$\forall t \geq T \quad ; \quad E_T [r_t \tilde{K}_t] = E_T [Q_t]$$

2. Wiener perturbation

The money market rate of return follows a normally-distributed Wiener diffusion process with drift,² i.e.:

$$\rho_t dt = \bar{\rho} dt + \sigma dZ_t \quad (2)$$

Where $\bar{\rho}, \sigma > 0$ represent positive parameters and Z_t is a Brownian motion $dZ_t = \varepsilon_t \sqrt{dt}$, where $Z_0 \equiv 0$ and where the white noise variable ε_t is normally-distributed, i.e., $\varepsilon_t \sim N[0,1]$. We also assume the firm's risk premium $\bar{\pi}$ to be a constant, so that, per definition (iii), the asset rate of return is:

$$r_t dt = (\bar{\rho} + \bar{\pi}) dt + \sigma dZ_t = \bar{r} dt + \sigma dZ_t$$

Where, for convenience, we define $\bar{r} \equiv \bar{\rho} + \bar{\pi}$.

Comment Note that we implicitly follow here the usual assumption that information is never lost over time, so that, $\forall T < t$, $I_T \subseteq I_t$ or, what is the same, $E_T [E_t[\circ]] = E_T[\circ]$.

1 There is an implicit assumption here that »predatory/influence« activities are subject to the same level of non-diversifiable risk as productive ones, and therefore their risk premium is also π_t . An alternative, more general assumption would have been to assume that the market return of these activities is a rate \tilde{r}_t , so that we would turn identity (ii) into $Y_t \equiv r_t K_t + \tilde{r}_t \tilde{K}_t$. It is easy to see, however, that this formulation would lead to exactly the same result, so little insight would be gained from this additional layer of complexity. Note as well that assuming predatory activities to be subject to non-diversifiable risk from the point of view of each one of the individual producers does not contradict the assumption postulated in (4), according to which the total cash flow paid to the producers as quasi-rents Q_t is known at the start of the period, i.e., $\lim_{T \rightarrow t} E_T [Q_t] = Q_t$.

2 The assumption of a normally-distributed Wiener diffusion process with drift (also known as »Gauss-Wiener process« or »Brownian motion«) as the distribution function of the asset returns demanded in an efficient market is a very common assumption in modern financial theory. For a good sample of mainstream economic and financial models developed under this assumption see for example Malliaris/Brock (1988) or Merton (1992).

Note also that, if $\bar{\pi} > 0$, there is no implicit assumption that ρ_t be risk-free, simply that the non-diversifiable risk associated to r_t is larger than that of ρ_t .

3. Observable market valuation

The market values K_t , \tilde{K}_t and D_t are observable at the instant t in which they take place, i.e., $K_t, D_t \in I_t$ (where I_t represents the set of information available at time t) or, what is the same:

$$\begin{aligned} \lim_{T \rightarrow t} E_T [K_t] &= K_t \quad \text{and} \quad \lim_{T \rightarrow t} E_T [\tilde{K}_t] = \tilde{K}_t \quad \text{and} \\ \lim_{T \rightarrow t} E_T [D_t] &= D_t \end{aligned} \quad (3)$$

4. Pre-determined cash flows

Both the cash flow paid to investors C_t and the one paid to producers under the form of the quasi-rent Q_t are known at the start of the instant t in which they take place, i.e., $C_t, Q_t \in I_t$ (where I_t represents the set of information available at time t) or, what is the same:

$$\lim_{T \rightarrow t} E_T [C_t] = C_t \quad \text{and} \quad \lim_{T \rightarrow t} E_T [Q_t] = Q_t \quad (4)$$

Comment In other words, the risk of the return being different from expected is supported by the retained earnings dK_t / dt .

5. Constant cash distribution rate

The overall cash distribution rate is a constant, i.e.:

$$\frac{C_t}{K_t} = \bar{c} \rightarrow \text{const.} \quad (5)$$

And the same ratio is applied to those cash payments that are specific to debt holders (i.e., $\bar{c} D_t$ would be the cash payment to creditors).

Comment This assumption has been introduced for simplicity purposes, as there are a number of alternative formulations that would yield the same result. To illustrate this, in Appendix 3 we show how it can be derived from a fairly standard representative consumer's utility function form – namely a time-additive von Neumann-Morgenstern discounted expected utility function with unity time elasticity. Other reasonable sets of assumptions could equally be used to justify this assumption.

6. Constant expected debt growth rate

We assume the debt growth rate to follow the distribution:

$$\frac{dD_t}{D_t} = \bar{\gamma} dt + \sigma dZ_t \quad (6)$$

Where $\bar{\gamma}$ is a constant and the component σdZ_t represents the Brownian perturbation defined in Assumption 2.

Comment We are thus implicitly assuming that the perturbation of the money market interest rate impacts the debt growth rate simply because the portion of the debt inter-

ests that has not been paid as a cash flow to creditors $\bar{c} D_t$ is accumulated as higher value of the debt itself.

7. Linear correlation between solvency and quasi-rent growth

The higher the solvency ratio (i.e., the lower the probability of default), the lower the obstacles the creditors will impose on the producers to prevent their gradually diverting resources under their control. For simplicity, we will assume this relationship to be linear, i.e.:

$$\frac{d\tilde{K}_t}{\tilde{K}_t} = \frac{dK_t}{K_t} + \theta(s_t - \bar{s})dt \quad (7)$$

Where $\theta, \bar{s} > 0$ represent positive parameters.

Comment In other words, the degree of freedom enjoyed by the producers to gradually reorganise the productive process so as to increase the value of their »slice of the pie« (i.e., their ability to perceive quasi-rents) grows linearly with the degree of solvency of the firm, assuming that there is a positive level of solvency $s = \bar{s} > 0$ such that, if sustained, would make their slice of the pie remain constant as a proportion of the total value of the firm.

Analytical Development

Step 1

By combining assumption (I) with definition (i) we obtain that:

$$E_T [r_t K_t] = E_T [Y_t - Q_t] \Leftrightarrow E_T [r_t \tilde{K}_t] = E_T [Q_t] \quad (8)$$

Which, according to assumptions (3) and (4), becomes for the special case $T = t$:

$$E_t [r_t] \tilde{K}_t = Q_t \quad (9)$$

If we now combine definitions (i), (ii) and (v) we obtain:

$$Q_t + C_t + \frac{dK_t}{dt} \equiv r_t (K_t + \tilde{K}_t) \equiv r_t (1 + q_t) K_t \quad (10)$$

Which, combined with assumptions (4) and (5) as well as expression (9), becomes:

$$\begin{aligned} E_t [r_t] q_t K_t + \bar{c} K_t + \frac{dK_t}{dt} &= r_t (1 + q_t) K_t \\ \frac{dK_t}{dt} &= (r_t - \bar{c}) K_t + \underbrace{(E_t [r_t] - E_t [r_t])}_{=0} q_t K_t \end{aligned} \quad (11)$$

By simple inspection, we can see that, along the expected path, the impact of q_t will be fully discounted out, for, if we write the expected value of (11) at instant t and then apply assumption (I), we obtain:

$$E_t \left[\frac{dK_t}{dt} \right] = (E_t [r_t] - \bar{c}) K_t + \underbrace{(E_t [r_t] - E_t [r_t])}_{=0} q_t K_t = (\bar{\rho} + \bar{\pi} - \bar{c}) K_t \quad (12)$$

Thus, by applying the expected value for any reference point in time T such that $T \geq t$ and then following a reasoning like the one presented in Appendix 1 we can conclude that the mean or »expected« path of the stock of capital follows a simple, exponential path:

$$E_0 \left[\frac{dK_t}{K_t} \right] = (\bar{r} - \bar{c}) dt \Leftrightarrow E_0 [K_t] = K_0 e^{(\bar{r} - \bar{c})t} \quad (13)$$

In other words, the mean path of K_t is an exponential trajectory growing at a constant rate equal to $\bar{r} - \bar{c}$. This, to be sure, does not imply that the percentage of the output that is paid out under the form of quasi-rents does not have any impact on the asset value, but simply that, just as one would expect in an efficient market, this expected impact has already been discounted from the asset market value at instant $t = 0$ and therefore, as long as the observed path matches the initial expectations, no further adjustment is necessary.

Step 2

Let's now divide K_t everywhere by D_t in order to express these equations in terms of the solvency ratio s_t as defined in (iv). Then (11) becomes:

$$\begin{aligned} \frac{dK_t}{D_t} &= (r_t - \bar{c}) \frac{K_t}{D_t} + (r_t - E_t[r_t]) q_t \frac{K_t}{D_t} \\ \frac{ds_t}{s_t} + \frac{dD_t}{D_t} &= (r_t - \bar{c}) dt + (r_t - E_t[r_t]) q_t dt \end{aligned} \quad (14)$$

Which, if we now replace according to assumptions (2) and (6), becomes:

$$\frac{ds_t}{s_t} = (\bar{r} - \bar{\gamma} - \bar{c}) dt + (r_t - E_t[r_t]) q_t dt \quad (15)$$

Thus, following the same process as in Step 1, we can express the mean path as independent from q_t , namely:

$$E_0 \left[\frac{ds_t}{s_t} \right] = (\bar{r} - \bar{\gamma} - \bar{c}) dt \Leftrightarrow E_0 [s_t] = s_0 e^{(\bar{r} - \bar{\gamma} - \bar{c})t} \quad (16)$$

The median path (or any other different from the mean), conversely, is not necessarily independent from q_t , for the rate of return may be different from its expected value. Thus, although under the particular conditions of this problem it is not possible to obtain an explicit analytical expression of the median path of the solvency ratio, one can identify a function \hat{s}_t (which we will call a »generator«) such that its deterministic integral would be the median path we are looking for. This »generator« can be obtained if we replace the rate of return in expression (15) with the discount rate corresponding to the median path. Now, as we have seen in Appendix 1, for an asset P_t whose rate of return was a geometric Brownian motion such as the one postulated in assumption 2 the growth rate would be:

$$\frac{dP_t}{P_t} = \bar{r} dt + \sigma dZ_t \Leftrightarrow P_t = P_0 e^{\left(\bar{r} - \frac{\sigma^2}{2} \right) t + \sigma Z_t} \quad (17)$$

Which, as the Wiener process Z_t is normally-distributed and therefore symmetrical, means that the median rate would be:

$$\text{Median}_0 [P_t] \rightarrow \frac{d\hat{P}_t}{\hat{P}_t} = \left(\bar{r} - \frac{\sigma^2}{2} \right) dt \tag{18}$$

Where \hat{P}_t is the function whose deterministic integral yields the median path. Therefore, if P_t represents the market reference asset against which all other assets are discounted (i.e., a money-market deposit plus an appropriate risk premium), then:

$$\text{Median}_0 [s_t] \rightarrow \frac{d\hat{s}_t}{\hat{s}_t} = (\bar{r} - \bar{\gamma} - \bar{c}) dt - \frac{\sigma^2}{2} q_t dt \tag{19}$$

Step 3

Now, in order to close the system, we need to introduce assumption (7), as along this path the observed values do not match the initial expectations and, hence, there are also unexpected changes of the rate of depreddation represented by q_t that need to be taken into account. If we now combine definition (v) with assumption (7), we obtain:

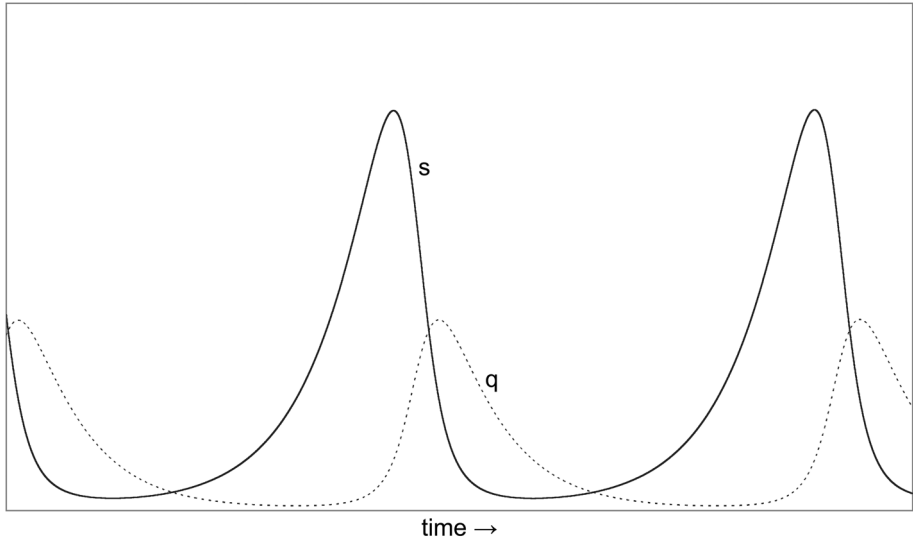
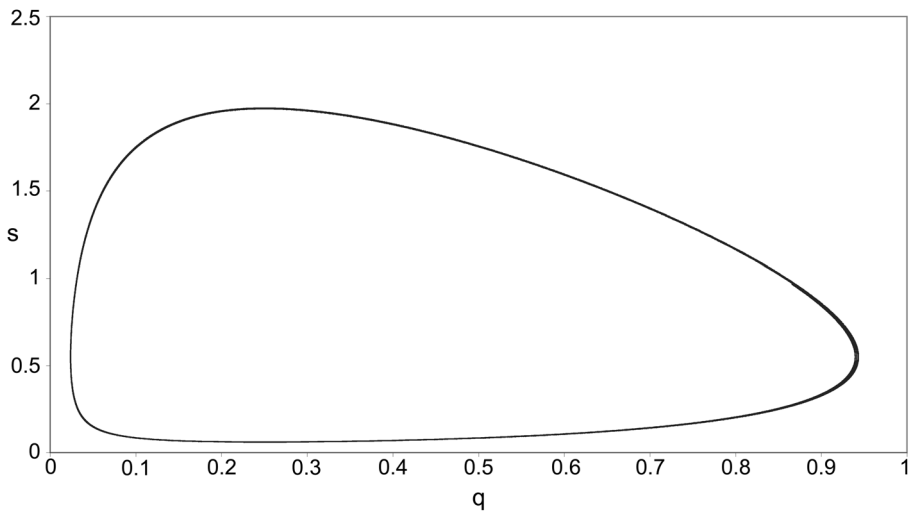
$$\frac{dq_t}{q_t} = \theta (s_t - \bar{s}) dt \tag{20}$$

It is thus possible now to close the system representing the median path by combining (19) with (20) in the final expression:

$$\left. \begin{aligned} \frac{ds_t}{s_t} &= (\bar{r} - \bar{\gamma} - \bar{c}) dt - \frac{\sigma^2}{2} q_t dt \\ \frac{dq_t}{q_t} &= \theta (s_t - \bar{s}) dt \end{aligned} \right\} \tag{21}$$

Which belongs to the family of Lotka-Volterra predator-prey dynamic systems and thus, although it cannot be analytically reduced to an explicit formula where each one of the variables is expressed as dependent only on t , has well understood behaviour patterns, as plotted in the simulation in figures 3 and 4 on page 102 (for which I have purposefully chosen a combination of variables yielding an obviously unrealistically wide wave bandwidth precisely in order to visually highlight the cyclical nature of the result).

Note that the system in expression (21) will only display a cyclical pattern like the one represented in these figures as long as the value of $\bar{r} - \bar{\gamma} - \bar{c}$ is positive. In other words, the cycle will only take place in a system with a positive savings rate – and, as a matter of fact, the frequency of its ups and downs will be faster the higher this expected retained earnings rate is.

Figure 3: Median Path over Time*Figure 4: Median Path Phase Diagram*

Appendix 3

The purpose of this appendix is to show how assumption 5 in Appendix 2 can be derived from a very simple representative consumer utility function within the parameters most usually applied in the standard literature.

In the following example, we will assume that the representative consumer tries to maximise a von Neumann-Morgenstern time-additive discounted utility function with unity inter-temporal elasticity of substitution, i.e.:

$$\max E_0 \left[\int_0^\infty (\ln C_t) e^{-\beta t} dt \right]$$

Where $\beta > 0$ is constant.

This maximization is then subject to the budgetary restriction:

$$K_0 = E_0 \left[\int_0^\infty C_t \frac{M_t}{M_0} dt \right]$$

Where M_t represents a martingale³ such that:

$$\frac{dM_t}{M_t} \equiv -(\bar{r}dt + \sigma dZ_t) \Leftrightarrow M_t \equiv e^{-\left(\bar{r} - \frac{\sigma^2}{2}\right)t - \sigma Z_t}$$

Thus, the first-order condition for the resolution of this problem is, $\forall t \geq 0$:

$$\begin{aligned} \frac{\partial U(C_t)}{\partial C_t} e^{-\beta t} &= \frac{e^{-\beta t}}{C_t} = \lambda e^{-\left(\bar{r} - \frac{\sigma^2}{2}\right)t - \sigma Z_t} \\ \lambda &= C_t^{-1} e^{\left(\bar{r} - \frac{\sigma^2}{2} - \beta\right)t + \sigma Z_t} \end{aligned}$$

Where λ represents the Lagrange multiplier. As this applies $\forall t \geq 0$, then:

$$\frac{C_t}{C_0} = e^{\left(\bar{r} - \frac{\sigma^2}{2} - \beta\right)t + \sigma Z_t} \Leftrightarrow \frac{dC_t}{C_t} = (\bar{r} - \beta)dt + \sigma dZ_t$$

If we now use this to replace into the budgetary restriction, we obtain:

$$K_0 = E_0 \left[\int_0^\infty C_t \frac{M_t}{M_0} dt \right] = E_0 \left[\int_0^\infty C_0 e^{-\beta t} dt \right] = \frac{C_0}{\beta}$$

Thus, if we define $\beta \equiv \bar{r}$, we have precisely reached assumption 5.

3 There are several valid ways to resolve this type of stochastic optimisation problem; here, for the most part, we follow the methodology of Cox/Huang (1989).

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