

# Stochastic frontier analysis by means of maximum likelihood and the method of moments

Andreas Behr

(University of Münster)

Sebastian Tente

(University of Münster)



Discussion Paper Series 2: Banking and Financial Studies No 19/2008

Discussion Papers represent the authors' personal opinions and do not necessarily reflect the views of the Deutsche Bundesbank or its staff.

Editorial Board: Heinz Herrmann

Thilo Liebig

Karl-Heinz Tödter

Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main, Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Telex within Germany 41227, telex from abroad 414431

Please address all orders in writing to: Deutsche Bundesbank, Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

Reproduction permitted only if source is stated.

ISBN 978-3-86558-463-2 (Printversion)

ISBN 978-3-86558-464-9 (Internetversion)

# Stochastic frontier analysis by means of maximum likelihood and the method of moments

Andreas Behr\*

Sebastian Tente

Institute for Econometrics

**Institute for Econometrics** 

University of Münster

University of Münster

September 2008

**Abstract** 

The stochastic frontier analysis (Aigner et al., 1977, Meeusen and van de Broeck, 1977) is widely used to estimate individual efficiency scores. The basic idea lies in the introduction of an additive error term consisting of a noise and an inefficiency term. Most often the assumption of a half-normal distributed inefficiency term is applied, but other distributions are also discussed in relevant literature. The natural estimation method seems to be Maximum Likelihood (ML) estimation because of the parametric assumptions. But simulation results obtained for the half normal model indicate that a method of moments approach (MOM) (Olson et al., 1980) is superior for small and medium sized samples in combination with inefficiency not strongly dominating noise (Coelli, 1995). In this paper we provide detailed simulation results comparing the two estimation approaches for both the half-normal and the exponential approach to inefficiency.

Based on the simulation results we obtain decision rules for the choice of the superior estimation approach. Both estimation methods, ML and MOM, are applied to a sample of German commercial banks based on the Bankscope database for estimation of cost efficiency scores.

**Keywords:** stochastic frontier, Maximum Likelihood, Method of moments, Bank efficiency

Jel-Classification: C13, D24

\*Corresponding author: Am Stadtgraben 9, 48143 Münster, Germany, andreas.behr@wiwi.uni-muenster.de, fax: 0049-251-22012, phone: 0049-251-22954

## Non-technical summary

Parametrical analyses of efficiency are based on the estimation of a frontier function (Aigner et al., 1977, Meeusen and van de Broeck, 1977). In the context of technical efficiency, it is about a production function indicating the maximum attainable output given the particular inputs. Any lower performances can be traced back to random noise – beyond the managers' control – as well as inefficiency.

The assumption of a certain inefficiency distribution as well as a normal noise distribution usually suggests the use of the maximum likelihood approach. In doing so, the log-likelihood function has to be maximised by numerical procedures.

A two-step method of moments approach turns out to be a noteworthy robust alternative: In a first step the parameters are estimated using ordinary least squares (OLS), disregarding the inefficiency term. In a second step the biased intercept is corrected by means of the moments of the OLS residuals.

This paper expands the simulation studies by Olson et al. (1980) and Coelli (1995) for the normal-half normal model and additionally compares the performance of MOM vs. ML for the normal-exponential model.

Large sample sizes and distinct sample inefficiency reveals the superiority of the ML-approach over MOM in terms of a smaller estimation error of efficiency scores. With respect to small sample sizes and/or a small ratio of inefficiency to noise, the method of moment estimation is favorable.

On the basis of the simulation results, parametrical decision rules are derived: They indicate the relative superiority of either ML- or MOM-estimation subject to sample size and inefficiency to noise-ratio.

Both ML- and MOM-techniques are applied to the estimation of a cost frontier for German commercial banks and to obtain bank-specific inefficiencies.

## Nichttechnische Zusammenfassung

Parametrischen Effizienzanalysen (*Stochastic Frontier Analysis*, SFA) liegt die Schätzung einer *Frontier*-Funktion zugrunde (Aigner et al., 1977, Meeusen and van de Broeck, 1977). Dabei handelt es sich im Untersuchungsrahmen technischer Effizienz um eine Art Produktionsfunktion, die für jedes Unternehmen den maximalen Output bei gegebenen Inputs abbildet. Abweichungen, bzw. Untererfüllung des Idealwertes können sowohl zurückgeführt werden auf zufällige Störungen jenseits der unternehmerischen Einflußspäre, sowie auf Ineffizienz.

Üblicherweise wird aufgrund der parametrischen Verteilungsannahmen die Maximum-Likelihood-Methode (ML) verwendet. Die logarithmierte Likelihoodfunktion muß dabei mittels numerischer Verfahren maximiert werden.

Ein deutlich robusteres Verfahren ist eine zweistufige Momentenschätzung (MOM): In einem ersten Schritt wird das Modell unter Vernachlässigung des Ineffizienzterms mit Hilfe der Methode der Kleinsten Quadrate (KQ) geschätzt. Im zweiten Schritt wird die Verzerrung des *Intercept* der KQ-Schätzung mit Hilfe der aus den KQ-Residuen geschätzten Momente korrigiert.

In der vorliegenden Arbeit werden die Simulationsstudien zur Vorteilhaftigkeit von Maximum-Likelihood- bzw. Momentenschätzung unter veränderlichen Rahmenbedingungen von Olson et al. (1980) und Coelli (1995) für das Normal-Halbnormal-Modell deutlich erweitert und zudem das Normal-Exponential-Modell analysiert. Es zeigt sich, dass für große Stichproben mit ausgeprägten Ineffizienzen ML dem Momentenansatz in Bezug auf die Präzision der mittleren geschätzten Effizienzen überlegen ist. Für kleine Stichproben und/oder geringe Ineffizienz in Relation zu zufälligen Störungen hingegen zeigt sich das Momentenverfahren überlegen.

Auf Basis der Simulationsergebnisse lassen sich parametrische Entscheidungsregeln für die Wahl des überlegenen Schätzverfahrens in Abhängigkeit von Stichprobengröße und Ausmaß der relativen Ineffizienz ableiten. Als Anwendung wird mit Hilfe der beiden diskutierten Verfahren eine *cost frontier* für deutsche Geschäftsbanken geschätzt und bankindividuelle Ineffizienzen ermittelt.

## **Contents**

1	Intr	oductio	n	1							
2	Effic	Efficiency and the stochastic frontier models									
	2.1	The co	oncept of output-based efficiency	2							
	2.2	The no	ormal-half normal model	3							
		2.2.1	The ML approach for the normal-half normal model	3							
		2.2.2	The MOM approach to the normal-half normal model	4							
		2.2.3	Estimates of individual inefficiencies	4							
	2.3	The ex	ponential model	5							
		2.3.1	The ML approach for the normal-exponential model	5							
		2.3.2	The MOM approach to the normal-exponential model	6							
		2.3.3	Estimates of individual inefficiencies	7							
3	Sin	nulation	l	7							
	3.1	Simula	ntion design	7							
	3.2	Simulation results in comparison									
		3.2.1	The normal-half normal case	8							
		3.2.2	The normal-exponential case	9							
		3.2.3	Misspecification scenarios	10							
		3.2.4	Rules of thumb	11							
4	An a	applicat	ion to German banks	12							
	4.1	Metho	dological issues	12							
		4.1.1	Inputs and Outputs	12							
		4.1.2	Shape of the cost function	14							
		4.1.3	Inefficiency distribution	15							
	4.2	Empiri	ical evidence	15							
5	Con	clusions	S	16							
6	App	endix		19							
	6.1	Deriva	tives of the log-likelihood: half-normal	19							
	6.2	Deriva	tives of the log-likelihood: exponential	19							
	6.3	Tables		20							

# **List of Tables**

1	Mean Average Error normal-halfnormal approach	8
2	Mean Average Error normal-exponential approach	10
3	Parameter estimates rules of thumb	12
4	Descriptive statistics of inputs, outputs, prices and bank size	14
5	Estimation results, all cases	16
6	Correlation table, cost efficiencies	16
7	Mean Square Error normal-halfnormal approach	20
8	Mean Square Error normal-exponential approach	21
9	Mean Average Error misspecification 1	22
10	Mean Average Error misspecification 2	23
11	Input and output-modelling in selected bank efficiency studies	24
List	of Figures	
1	Rules of thumb	11

## 1 Introduction

The stochastic frontier analysis is widely used to estimate individual efficiency scores. The stochastic frontier analysis (SFA) is based on the pioneering work of Aigner et al. (1977) and Meeusen and van de Broeck (1977). Kumbhakar and Lovell (2003) provide a comprehensive overview. The basic idea is the introduction of an additive error term consisting of a noise and an inefficiency term. For the error as well as the inefficiency term distributional assumptions are made. Most often the half normal assumption is applied, but the exponential, truncated normal and gamma cases are also discussed in specific literature. While the two-parameter distributions – the truncated normal and the gamma – potentially increase the flexibility of the model, in practical applications problems of identification seem to outweigh the potential gains for either distribution (Greene, 1997, p. 103 f.), (Ritter and Simar, 1997b,a).

The natural estimation method seems to be Maximum Likelihood (ML) estimation because of the parametric assumptions. But simulation results obtained for the normal-half normal model indicate that a method of moments approach (MOM) (Olson et al., 1980) is superior for small and medium sized samples in combination with inefficiency not strongly dominating noise (Coelli, 1995).

In this paper we provide detailed simulation results comparing the two estimation approaches for both the half-normal and the exponential approach to inefficiency. Furthermore, we compare the sensitivity of the estimation approaches towards misspecification. Our simulations extend those of Coelli (1995) and Olson et al. (1980) for the normal-half normal model as to sample size and comprise also the exponential model. The extensive simulation results allow formulation of rules of thumb for deciding on the estimation approach for normally and exponentially distributed inefficiency terms.

The paper is composed as follows: In section 2 we discuss the underlying concept of efficiency and the different approaches to detect inefficiency scores. In section 3 we lay down the results of our extensive simulation studies, especially the suggestions obtained for the choice of the competing estimation approaches in the form of parametrical rules of thumb. Section 4 introduces our field of application by first describing the well-established procedures to obtain efficiency scores for banking institutions in the relevant literature. Next we demonstrate an exemplifying application to a Bankscope dataset for German banks. Section 5 contains the conclusion.

## 2 Efficiency and the stochastic frontier models

In this section we briefly describe the concept of efficiency and stochastic frontier models based on the half-normal distribution for the inefficiency term, and alternatively the exponential model.

#### 2.1 The concept of output-based efficiency

Farrell (1957) introduced the idea of an empirical approach to efficiency by the firm specific quotient of observed production  $y_i$  to optimal production  $y_i^*$ . In conformity with microeconomic theory, production processes are technical relations of employed inputs to maximum attainable output. So when assuming cross sectional data for n units indexed by i (i = 1, ..., n) using K (k = 1, ..., K) different inputs contained in the input vector  $x_i$  to produce a single output  $y_i$ , we can formulate Farrell's idea of technical efficiency:

$$TE_i = \frac{y_i}{y_i^*} = \frac{y_i}{g(x_i; \beta)} \in [0, 1]$$

with  $g(x_i; \beta)$  as a deterministic production function. It is the aim of the stochastic frontier approach to estimate the underlying technology constituting the production possibilities of a set of firms. We allow a parametric form for the output including stochastic terms

$$y_i = g(x_i; \beta) \cdot e^{v_i} \cdot e^{-u_i}$$

which in logs is

$$\log(y_i) = \log(g(x_i; \beta)) + v_i - u_i$$

and  $v_i$  is considered as a normal error  $v_i \sim N(\mu_v; \sigma_v^2)$  and  $u_i$  is positive representing inefficiency.

In the following we assume a simple Cobb-Douglas production function

$$g(x_i; \beta) = e^{\beta_0} \prod_{k=1}^K x_{ik}^{\beta_k}$$

which in logs is

$$\log [g(x_i; \beta)] = \beta_0 + \sum_{k=1}^K \beta_k \log(x_{ik})$$

So the output model is given by

$$\log(y_i) = \beta_0 + \sum_{k=1}^{K} \beta_k \log(x_{ik}) + v_i - u_i$$

This leads to firm-specific efficiency scores in the Cobb-Douglas case

$$TE_i = \frac{g(x_i; \beta) \cdot e^{v_i} \cdot e^{-u_i}}{g(x_i; \beta) \cdot e^{v_i}} = e^{-u_i}$$

#### 2.2 The normal-half normal model

The component  $u_i$  is assumed to be positive representing production inefficiency. Most often  $u_i$  is assumed to be half-sided normal

$$u_i \stackrel{iid}{\sim} N^+(0,\sigma_u^2)$$

The density of u is given as

$$f(u) = \frac{2}{\sigma_u \sqrt{2\pi}} \exp\left(-\frac{u^2}{2\sigma_u^2}\right)$$

with the moments

$$E(u) = \frac{\sqrt{2}}{\sqrt{\pi}}\sigma_u$$
 and  $V(u) = \left(\frac{\pi - 2}{\pi}\right)\sigma_u^2$ 

#### 2.2.1 The ML approach for the normal-half normal model

Assuming independence of the error terms v und u the joint density function results as the product of individual density functions

$$f(u,v) = f(u) \cdot f(v) = \frac{2}{\sigma_u \sigma_v 2\pi} \exp\left(-\frac{u^2}{2\sigma_u^2} - \frac{v^2}{2\sigma_v^2}\right)$$

To obtain the density of the composed error term  $\varepsilon = v - u$ , we first obtain the joint density  $f(u, \varepsilon)$ . Integration over u results in

$$f(\varepsilon) = \int_{0}^{\infty} f(u, \varepsilon) du = \frac{2}{\sigma} \phi \left( \varepsilon \sigma^{-1} \right) \left[ 1 - \Phi \left( \varepsilon \lambda \sigma^{-1} \right) \right]$$

where  $\sigma^2 = \sigma_v^2 + \sigma_u^2$  and  $\lambda = \sigma_u/\sigma_v$ .

The density distribution of  $\varepsilon$  is asymmetric and characterized by

$$E(\varepsilon) = E(v - u) = E(-u) = -\frac{\sqrt{2}}{\sqrt{\pi}}\sigma_u$$

The variance of  $\varepsilon$  is given by

$$V(\varepsilon) = \sigma_{\varepsilon}^2 = V(u) + V(v) = \left(\frac{\pi - 2}{\pi}\right)\sigma_u^2 + \sigma_v^2$$

The log-likelihood is given by

$$\ln L(\varepsilon|\lambda,\sigma^2) = n \ln \left(\frac{\sqrt{2}}{\sqrt{\pi}}\right) + n \ln \left(\frac{1}{\sigma}\right) + \sum_{i=1}^n \ln \left[1 - \Phi\left(\varepsilon_i \lambda \sigma^{-1}\right)\right] - \frac{1}{2\sigma^2} \sum_{i=1}^n \varepsilon_i^2$$

using  $\varepsilon_i = \log y_i - \beta \log x_i$ .

Having obtained the estimates  $\hat{\beta}$ ,  $\hat{\sigma}^2 = \hat{\sigma}_u^2 + \hat{\sigma}_v^2$  and  $\hat{\lambda} = \hat{\sigma}_u/\hat{\sigma}_v$ , the estimates of the variance components can be recovered:

$$\hat{\sigma}_v^2 = rac{\hat{\sigma}^2}{1+\hat{\lambda}^2}$$
 and  $\hat{\sigma}_u^2 = \hat{\sigma}^2 - rac{\hat{\sigma}^2}{1+\hat{\lambda}^2}$ 

#### 2.2.2 The MOM approach to the normal-half normal model

Estimating the production model using OLS results in consistent estimates of the slope parameters  $\beta_1, ..., \beta_K$ , but biased estimate of the intercept  $\beta_0$ . As the assumed  $E(\varepsilon) = 0$  in OLS, the Bias is  $E(\varepsilon) = -E(u) = -\sigma_u \sqrt{\frac{2}{\pi}}$ . In the method of moments approach obtained OLS residuals are used to estimate the central moments  $m_2$  and  $m_3$  in order to shift the regression line.

$$m_2 = \frac{1}{n} \sum_{i=1}^{n} (\varepsilon_i - \bar{\varepsilon}_i)^2$$
 and  $m_3 = \frac{1}{n} \sum_{i=1}^{n} (\varepsilon_i - \bar{\varepsilon}_i)^3$ 

which correspond to the moment equations (Greene, 1997)

$$m_2 = \frac{\pi - 2}{\pi} \sigma_u^2 + \sigma_v^2$$
 and  $m_3 = \sqrt{\frac{2}{\pi}} \left( 1 - \frac{4}{\pi} \right) \sigma_u^3$ 

Solving for the variance components results in

$$\sigma_u^2 = \left(\frac{m_3}{\sqrt{\frac{2}{\pi}\left(1-\frac{4}{\pi}\right)}}\right)^{\frac{2}{3}}$$
 and  $\sigma_v^2 = m_2 - \frac{\pi-2}{\pi}\sigma_u^2$ 

The biased OLS estimate of the intercept  $\hat{\beta}_0^{OLS}$  can be adjusted on the basis of the estimate of the standard deviation of the inefficiency term

$$\hat{\beta}_0^{MOM} = \hat{\beta}_0^{OLS} + E(\hat{u}) = \hat{\beta}_0^{OLS} + \hat{\sigma}_u \sqrt{\frac{2}{\pi}}$$

Although the MOM-Estimator is easy to calculate, even without numerical optimization, Olson et al. (1980) note two types of errors occurring when either  $m_3$  is positive (Type I error) or  $m_2 \leq ((\pi-2)/\pi)\sigma_u^2$  (Type II error). A Type I error is likely to occur when  $\sigma_u$  is small  $(\lambda \to 0)$ . This immediately leads to the estimation of a negative variance  $\hat{\sigma}_u$  and prevents further calculations. In the latter case, a Type II error does not prohibit the estimation of  $\beta_0^{MOM}$ , but causes implausible values of  $\hat{\lambda} \to \pm \infty$ .

#### 2.2.3 Estimates of individual inefficiencies

As it is impossible to obtain for each individual firm i estimates for  $u_i$  and  $v_i$ , the inefficiency ratio  $TE_i$  is obtained as the exponential conditional expectation of -u given the composed error term  $\varepsilon$ :

$$\widehat{TE}_i = e^{E(-u_i|\varepsilon_i)}$$

The conditional density of u given  $\varepsilon$  is

$$f(u|\varepsilon) = \frac{f(u,\varepsilon)}{f(\varepsilon)} = \frac{1}{\sigma^* \sqrt{2\pi}} \exp\left(-\frac{(u-\mu^*)^2}{2\sigma^{*2}}\right) \left[1 - \Phi\left(-\frac{\mu^*}{\sigma^*}\right)\right]^{-1}$$

Hence, the distribution of u conditional on  $\varepsilon$  is  $N^+(\mu^*, \sigma^*)$ , where

$$\mu^* = -\frac{\varepsilon \sigma_u^2}{\sigma^2} = -\varepsilon \gamma$$

$$\sigma^{*2} = \frac{\sigma_u^2 \sigma_v^2}{\sigma^2} = \sigma^2 \frac{\sigma_u^2 (\sigma^2 - \sigma_u^2)}{\sigma^2 \sigma^2} = \sigma^2 \gamma (1 - \gamma)$$

using  $\gamma = \sigma_u^2/\sigma^2$ , the fraction of the variance of the inefficiency to the total variance.

Having obtained the distribution of  $u|\varepsilon$ , the expected value  $E(u|\varepsilon)$  can be used as point estimators for  $u_i$  (Jondrow et al., 1982):<sup>1</sup>

$$\hat{u}_{i} = E(u|\varepsilon) = \left(\frac{\sigma\lambda}{1+\lambda^{2}}\right) \left(z_{i} + \frac{\phi(z_{i})}{\Phi(z_{i})}\right)$$
$$z_{i} = \frac{-\varepsilon_{i}\lambda}{\sigma}$$

#### 2.3 The exponential model

The component  $u_i$  is assumed to follow the exponential distribution with density given in alternate parameterization  $\theta = 1/\sigma_u$  as

$$f(u) = \begin{cases} \frac{1}{\sigma_u} \exp\left(-\frac{u}{\sigma_u}\right) & u \ge 0\\ 0 & u < 0 \end{cases}$$

The moments are

$$E(u) = \sigma_u$$
 and  $V(u) = \sigma_u^2$ 

#### 2.3.1 The ML approach for the normal-exponential model

Assuming independence of the error terms v und u the joint density results in the product of individual density functions

$$f(u,v) = f(u)f(v) = \frac{2}{\sigma_u \sigma_v 2\pi} \exp\left(-\frac{u}{\sigma_u} - \frac{v^2}{2\sigma_v^2}\right)$$

$$\hat{TE}_i = E(\exp(-u_i)|\varepsilon_i) = \left[\Phi\left(\frac{u_i^*}{\sigma_*} - \sigma_*\right)/\Phi\left(\frac{u_i^*}{\sigma_*}\right)\right] \exp\left(\frac{\sigma_*^2}{2} - u_i^*\right)$$

where  $u_i^* = -(\log y_i - x'\beta)\sigma_u^2/\sigma^2$  and  $\sigma_*^2 = \sigma_v^2\sigma_u^2/\sigma^2$ . Note that in general  $\exp[-E(u|\varepsilon)] \neq E(\exp(-u_i)|\varepsilon_i)$ . Furthermore, both estimators are unbiased, but inconsistent estimators because  $Var(\hat{u}_i) \neq 0$  for  $N \to \infty$ .

<sup>&</sup>lt;sup>1</sup>Instead of obtaining firm-specific efficiencies from  $\exp[-E(u|\varepsilon)]$ , Battese and Coelli (1988) propose the alternative estimator:

To obtain the density of the composed error term  $\varepsilon = v - u$ , we first obtain the joint density  $f(u, \varepsilon)$  and integrate out u from the joint density

$$f(\varepsilon) = \int_0^\infty f(u, \varepsilon) du = \frac{1}{\sigma_u} \Phi\left(-\frac{\varepsilon}{\sigma_v} - \frac{\sigma_v}{\sigma_u}\right) \exp\left(\frac{\varepsilon}{\sigma_u} + \frac{1}{2} \frac{\sigma_v^2}{\sigma_u^2}\right)$$

The density distribution of  $\varepsilon$  is asymmetric and characterized by

$$E(\varepsilon) = E(v - u) = E(-u) = -\sigma_u$$

The variance of  $\varepsilon$  is given by

$$V(\varepsilon) = \sigma_{\varepsilon}^2 = V(u) + V(v) = \sigma_u^2 + \sigma_v^2$$

Assuming independence across subjects i, the likelihood is the product of individual densities  $f(\varepsilon)$ :

$$L(\log y | \sigma_u^2, \sigma_v^2) = \frac{1}{\sigma_u^n} \exp\left(\frac{1}{2} \frac{\sigma_v^2}{\sigma_u^2}\right)^n \prod_{i=1}^n \left[\Phi\left(-\frac{\varepsilon_i}{\sigma_v} - \frac{\sigma_v}{\sigma_u}\right) \exp\left(\frac{\varepsilon_i}{\sigma_u}\right)\right]$$

The log-likelihood is given by

$$\ln L(\log y | \beta, \sigma_u^2, \sigma_v^2) = -n \log (\sigma_u) + n \frac{1}{2} \frac{\sigma_v^2}{\sigma_u^2} + \sum_{i=1}^n \left[ \log \Phi \left( -\frac{\log y_i - \log x_i' \beta}{\sigma_v} - \frac{\sigma_v}{\sigma_u} \right) + \frac{\log y_i - \log x_i' \beta}{\sigma_u} \right]$$

#### 2.3.2 The MOM approach to the normal-exponential model

Just as in the method of moments approach for the normal-half normal model discussed above, OLS residuals are used to estimate the central moments  $m_2$  and  $m_3$ , which correspond to (Greene, 1997)

$$m_2 = \sigma_u^2 + \sigma_v^2$$
 and  $m_3 = -2\sigma_u^3$ 

Solving for the variance components results in

$$\sigma_u^2 = \left(-\frac{m_3}{2}\right)^{\frac{2}{3}}$$
 and  $\sigma_v^2 = m_2 - \sigma_u^2$ 

Analogous to the half-normal case, a Type I error occurs when  $m_3 < 0$ , and a Type II error when  $m_2 < \sigma_u^2$  (virtually impossible). The biased OLS estimate of the intercept  $\hat{\beta}_0^{OLS}$  can be adjusted based on the estimate of the standard deviation (equal to mean value) of the inefficiency term

$$\hat{\beta}_0^{MOM} = \hat{\beta}_0^{OLS} + \hat{\sigma}_u$$

#### 2.3.3 Estimates of individual inefficiencies

As the conditional distribution  $f(u|\varepsilon)$  is distributed as  $N^+(\tilde{\mu}, \sigma_v^2)$  and given by

$$f(u|\varepsilon) = \frac{f(u,\varepsilon)}{f(\varepsilon)} = \frac{\exp[-(u-\tilde{\mu}^2)/2\sigma^2]}{\sqrt{2\pi}\sigma_v\Phi(-\tilde{\mu}/\sigma_v)}$$

with

$$\tilde{\mu} = -\varepsilon - \frac{\sigma_v^2}{\sigma_u}$$

the expected value of inefficiency u given estimated residual  $\varepsilon$  in the normal-exponential model can be taken as (Kumbhakar and Lovell, 2003):

$$E\left[u_{i}|\varepsilon_{i}\right] = \tilde{\mu}_{i} + \sigma_{v} \left[\frac{\phi(-\tilde{\mu}_{i}/\sigma_{v})}{\Phi(\tilde{\mu}_{i}/\sigma_{v})}\right]$$

## 3 Simulation

We apply the two estimation approaches outlined above to obtain estimates of individual efficiencies  $\widehat{TE}_i = \exp[E(-u_i|\varepsilon_i)]$ . To assess the performance of the efficiency score estimation we calculate the mean square and mean average error

$$mse = \frac{1}{n} \sum_{i=1}^{n} \left( \widehat{TE}_i - TE_i \right)^2 \qquad mae = \frac{1}{n} \sum_{i=1}^{n} |\widehat{TE}_i - TE_i|$$

between true and estimated efficiency scores.

In the simulation study we assess the relative performance of the ML and MOM estimators for different n. Additionally, we analyse the effect of  $\lambda$ , the relation of inefficiency variance to variance of the normal noise, on the appropriate choice of estimation method.

## 3.1 Simulation design

In this section we analyze the estimation of individual inefficiency scores by means of Monte Carlo simulations based on m=2000 replications using a standard simulation setting:

$$y_i = 1 + x_1 + x_2 + v_i - u_i$$

with  $v_i \sim N(0, \sigma_v)$  and  $u_i \sim N(0, \sigma_u)$  or  $u_i \sim Exp(\sigma_u)$ ;  $\sigma_u = \{0.283, 0.447, 0.526, 0.566, 0.587, 0.600, 0.614, 0.620\}$  and  $\sigma_v = \sqrt{0.4 - \sigma_u^2}$ . The inputs  $x_1, x_2$  were drawn independently from a uniform distribution on the interval (0, 1). Sample sizes are  $n = \{25, 50, 75, 100, 150, 250, 500, 1000\}$ . The computations were performed by means of the R Environment<sup>2</sup>.

<sup>&</sup>lt;sup>2</sup>Version 2.7.1; www.r-project.org.

To assess the robustness of the half normal and the exponential models towards misspecification, we add two misspecification scenarios. In scenario M1 we (falsly) apply the half normal model on data generated under the exponential assumption. Conversely in scenario M2 we (falsly) estimate the exponential model despite inefficiency terms  $u_i$  in fact drawn from the half normal distribution.

## 3.2 Simulation results in comparison

#### 3.2.1 The normal-half normal case

n	25	50	75	100	150	250	500	1000	
λ	method of moments								
0.5	0.1801	0.1742	0.1727	0.1686	0.1637	0.1568	0.1503	0.1426	
1.0	0.1964	0.1877	0.1780	0.1758	0.1685	0.1590	0.1459	0.1348	
1.5	0.1879	0.1697	0.1584	0.1521	0.1390	0.1298	0.1214	0.1183	
2.0	0.1752	0.1499	0.1368	0.1267	0.1188	0.1113	0.1073	0.1054	
2.5	0.1649	0.1308	0.1184	0.1092	0.1028	0.0981	0.0952	0.0935	
3.0	0.1531	0.1185	0.1049	0.0991	0.0931	0.0884	0.0853	0.0838	
4.0	0.1375	0.1028	0.0896	0.0841	0.0787	0.0741	0.0708	0.0686	
5.0	0.1253	0.0938	0.0808	0.0751	0.0697	0.0643	0.0604	0.0582	
λ				maximum	likelihood				
0.5	0.2440	0.2114	0.1962	0.1883	0.1774	0.1627	0.1496	0.1362	
1.0	0.2460	0.2123	0.1935	0.1883	0.1743	0.1584	0.1442	0.1330	
1.5	0.2172	0.1809	0.1645	0.1553	0.1395	0.1294	0.1212	0.1181	
2.0	0.1898	0.1535	0.1374	0.1259	0.1177	0.1101	0.1067	0.1052	
2.5	0.1724	0.1299	0.1152	0.1057	0.1000	0.0961	0.0944	0.0932	
3.0	0.1523	0.1141	0.0997	0.0939	0.0891	0.0857	0.0838	0.0833	
4.0	0.1290	0.0925	0.0804	0.0762	0.0726	0.0701	0.0687	0.0679	
5.0	0.1118	0.0788	0.0693	0.0653	0.0618	0.0598	0.0579	0.0576	
λ				adva	ntage				
0.5	MOM	MOM	MOM	MOM	MOM	MOM	MLE	MLE	
1.0	MOM	MOM	MOM	MOM	MOM	MLE	MLE	MLE	
1.5	MOM	MOM	MOM	MOM	MOM	MLE	MLE	MLE	
2.0	MOM	MOM	MOM	MLE	MLE	MLE	MLE	MLE	
2.5	MOM	MLE	MLE	MLE	MLE	MLE	MLE	MLE	
3.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE	
4.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE	
5.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE	

Table 1: Mean Average Error normal-halfnormal approach

To assess the quality of the estimation of the inefficiency terms, we calculate mean absolute differences between estimated and true efficiency scores (mae, corresponding mean squared deviations mse are reported in the appendix.) for all sample sizes n=(25,50,75,100,150,250,500,1000) and  $\lambda=(0.5,1.0,1.5,2.0,2.5,3.0,4.0,5.0)$ . Table 1 shows the results subdivided into three

parts: The first block gives the mae in the MOM-case, the second block in the ML-case and the third block indicates the superior estimation method in terms of a smaller error. Our findings confirm the results of Coelli (1995) and Olson et al. (1980). Due to improved computer capacity in the last decade, we were in a position to perform more extensive computations. Based on these extended simulation results including larger sample sizes and more simulation runs, we confirm their main findings: MOM-estimation is found strongly superior for rather small n and small  $\lambda$ . This is in conformity with intuition, because a small  $\lambda$  implies negligible inefficiency  $\sigma_u$  in comparison with dominating  $\sigma_v$ , which comes close to the classical OLS assumption. In this case, OLS provides the best linear unbiased estimators for  $\beta_1, ..., \beta_k$ , while  $\beta_0$  is just slightly biased. ML-estimation should be preferred for larger n and larger  $\lambda$ . But a look at the simulation results reveals only the smallest differences in performance for larger n, rendering the choice of estimation methods rather unimportant.

In general, the mean average error between estimated and true efficiency scores decreases with increasing sample sizes as well as with increasing  $\lambda$ . We find for small sample sizes n=25 and  $\lambda=0.5$  a mean average deviation about three times the size compared to the case n=1000 and  $\lambda=5.0$  for the MOM approach. In case of ML-estimation, which is found considerably inferior for small n, mean average deviations for small n,  $\lambda$ -combinations are about four times the value obtained for large  $n,\lambda$ .

#### 3.2.2 The normal-exponential case

The results of the normal-exponential model are illustrated in table 2. The findings of the normal-half normal and the normal-exponential model resemble each other. Again small sample sizes and a small variance ratio  $\lambda$  strongly suggest application of MOM-estimation. But obviously, we observe more n,  $\lambda$ -combinations for which ML-estimation is superior.

Again, the mean average deviation between estimated and true efficiency scores decreases with increasing sample sizes as well as with increasing  $\lambda$  similarly found for the half normal model. We also find for small sample sizes n=25 and  $\lambda=0.5$  a mean average deviation about three times the size compared to n=1000 and  $\lambda=5.0$ . Just as in the normal-half normal case MOM strongly outperforms ML-estimation for very small  $\lambda$ , while the preferability of ML is based on very small performance differences only.

n	25	50	75	100	150	250	500	1000
$\lambda$	method of moments							
0.5	0.1630	0.1598	0.1566	0.1535	0.1509	0.1443	0.1378	0.1324
1.0	0.1806	0.1622	0.1503	0.1453	0.1412	0.1370	0.1349	0.1338
1.5	0.1669	0.1422	0.1341	0.1291	0.1250	0.1217	0.1195	0.1180
2.0	0.1551	0.1305	0.1209	0.1169	0.1125	0.1086	0.1053	0.1032
2.5	0.1489	0.1197	0.1118	0.1075	0.1022	0.0977	0.0938	0.0913
3.0	0.1419	0.1137	0.1051	0.1002	0.0950	0.0894	0.0850	0.0820
4.0	0.1355	0.1069	0.0972	0.0909	0.0843	0.0779	0.0722	0.0683
5.0	0.1355	0.1023	0.0930	0.0857	0.0781	0.0709	0.0640	0.0594
λ				maximum	likelihood			
0.5	0.1888	0.1729	0.1655	0.1616	0.1557	0.1470	0.1385	0.1326
1.0	0.2006	0.1695	0.1539	0.1471	0.1410	0.1366	0.1343	0.1336
1.5	0.1729	0.1408	0.1303	0.1246	0.1210	0.1188	0.1175	0.1170
2.0	0.1534	0.1211	0.1111	0.1078	0.1047	0.1030	0.1017	0.1012
2.5	0.1376	0.1057	0.0976	0.0945	0.0920	0.0901	0.0888	0.0883
3.0	0.1234	0.0948	0.0878	0.0845	0.0820	0.0800	0.0785	0.0780
4.0	0.1076	0.0793	0.0728	0.0698	0.0666	0.0648	0.0636	0.0630
5.0	0.0980	0.0694	0.0630	0.0598	0.0566	0.0547	0.0534	0.0527
$\lambda$				adva	ntage			
0.5	MOM	MOM	MOM	MOM	MOM	MOM	MOM	MOM
1.0	MOM	MOM	MOM	MOM	MLE	MLE	MLE	MLE
1.5	MOM	MLE	MLE	MLE	MLE	MLE	MLE	MLE
2.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE
2.5	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE
3.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE
4.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE
5.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE

Table 2: Mean Average Error normal-exponential approach

#### 3.2.3 Misspecification scenarios

As it is an unfeasible task to determine any real inefficiency distribution across an industry, we can basically assume an underlying misspecification in every applied Stochastic Frontier Analysis. To exemplify the impacts of misspecification against the background of MOM vs. ML-estimation, we interchanged the data generating processes of the normal-exponential and normal-halfnormal case. So table 9 given in the appendix shows the mae of an exponential model estimated as halfnormal (misspecification scenario M1), and table 10 in the appendix obtains the results of a halfnormal model estimated as exponential (misspecification M2).

The findings are straightforward: The exponential model M1 shows the predominant advantage of ML-estimation, even more clearly than in the correctly specified case. Obviously, the mae of MOM-estimation improves with increasing n or  $\lambda$ , but does not decrease in jointly larger  $n,\lambda$ -combinations. The indications in the halfnormal model M2 suggest slight advantages of MOM-estimation facing an overall lower error.

#### 3.2.4 Rules of thumb

To summarize the particular advantages of ML- or MOM-estimation, we estimated multiple linear regressions based on tables 1 and 2 for the normal and exponential cases, respectively:

$$y_k = \beta_0 + \beta_{1k} \cdot \lambda + \beta_{2k} \cdot n + \mu_k$$

with

$$y = \begin{cases} 0 & \text{if MOM has smaller error} \\ 1 & \text{if MLE has smaller error} \end{cases}$$

and  $\mu_k$  as normal error in all k possible combinations of sample size n and  $\lambda$ . Predicted values  $\hat{y_k} > 0.5$  imply an advantage of ML- over MOM-estimation. Figure 1 illustrates the separating line between both approaches in the half normal and exponential case. The corresponding parameter estimates are shown in table 3.

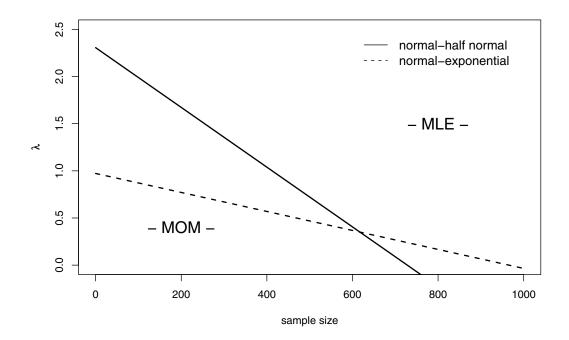


Figure 1: Rules of thumb

	Estimate	Std. Error	t value	Pr(> t )					
normal-half normal									
Intercept	0.0602	0.0900	0.67	0.5062					
$\lambda$	0.1908	0.0292	6.53	0.0000					
n	0.0006	0.0001	4.52	0.0000					
	norma	al-exponentia	al						
Intercept	0.3338	0.0874	3.82	0.0003					
$\lambda$	0.1710	0.0284	6.02	0.0000					
n	0.0002	0.0001	1.33	0.1897					

Table 3: Parameter estimates rules of thumb

## 4 An application to German banks

### 4.1 Methodological issues

#### 4.1.1 Inputs and Outputs

We review the most relevant literature covering bank efficiency estimation by means of stochastic frontier analysis. One of the most cited articles in recent literature is Mester (1996). She laid out the main features of current efficiency analyses via frontier cost functions. Resorting to cost functions instead of production functions (*technologies*) has several advantages.

Beside the methodical problems discussed above, we find another key question of empirical bank efficiency estimation in modelling inputs and outputs of the production process. As Girardone et al. (2004) state: While the multiproduct nature of the banking firm is widely recognized, there is still no agreement as to the explicit definition and measurement of banks' inputs and outputs. So it is common practice to operationalise bank production according to the fundamental idea of the Intermediation Approach proposed by Sealey and Lindley (1977). They had in mind a multistage production process of the financial firm, using capital, labour and material to acquire customer deposits in a first step. Lending these funds in a (virtual) second step to deficit spending units and issuing securities and other earning assets involve in general an interest profit. So financial production for intermediation purposes is about adding value to deposits.

Obviously, the use of multiple outputs does not apply to the single-output production functions described above. But, referring to Duality Theory<sup>3</sup>, one can prove under certain regularity conditions<sup>4</sup> the equivalence of indirect cost functions  $tc = tc(\mathbf{y}, \mathbf{c})$  and the underlying techno-

<sup>&</sup>lt;sup>3</sup>Cp. Beatti and Taylor (1985), chapter 6.

<sup>&</sup>lt;sup>4</sup>In particular, linear homogeneity and weak concavity in input prices if the implicit production technology is

logy  $F(\mathbf{y}, \mathbf{x}) = 0$  with tc total operating costs, y' a vector of outputs, and c' a vector of prices of the inputs x'. Estimating restricted stochastic cost frontiers is virtually the same as production frontiers, as the lower stochastic frontier of the 'data cloud' is simply defined by turning the sign of  $u_i$ , using the symmetry of  $v_i$ :

$$\log(tc_i) = \log(g(y_i, c_i); \beta) + v_i + u_i \quad |v_i| \text{ symmetric}$$
  
$$\Leftrightarrow -\log(tc_i) = -\log(g(y_i, c_i); \beta) + v_i - u_i$$

In the context of cost functions efficiency is definded in terms of cost efficiency CE instead of technical efficiency TE. An advantage over consideration of 'pure' technologies is the possibility to evaluate scale economies, i.e. the existence of decreasing average costs in conjunction with financial firm growth.

Thus, Mester proposes a basic cost function with outputs being real estate loans, commercial/industrial and other loans as well as loans to individuals. Inputs, input-prices respectively, are prices of labour, physical capital and deposits (borrowed money). Furthermore, she included two bank quality proxies: The average volume of nonperforming loans and the average volume of equity capital. Because of a highly homogeneous dataset no further specific regional/economic distinction had to be drawn. Bos and Kool (2006) investigate small cooperative banks in the Netherlands, so they also can confine themselves to just control for a bank-specific solvency measure provided by Rabobank Netherlands added to the cost function. The authors' interpretation of the Intermediation approach imply inputs such as public relations, labour, physical capital and financial capital. Outputs are retail loans, wholesale loans, mortgages and provisions. This is a slight modification of the intermediation idea, but Lang and Welzel (1996) go even further in modelling the outputs short-term and long-term loans to non-banks, loans to banks, bonds, cash and real estate investments, fees and commissions, and revenue from sales of commodities. Obviously, some studies justify more or less distinctive alterations of the value-adding idea by Sealey and Lindley. On the other hand, authors like Altunbas et al. (2000), Perera et al. (2007), Girardone et al. (2004) are able to adopt the classical idea of banking intermediation. Table 11 in the appendix shows the different approaches at a glance.

Based on this discussion we put forward the opinion that an adequate image of banking activity in the course of a simulation study can be deduced by the basic intermediation approach, without adding any control variables.

So we accessed a Bankscope<sup>5</sup> cross section dataset of n=56 German commercial banks with

strictly quasi-convex.

<sup>&</sup>lt;sup>5</sup>Bureau van Dijk, www.bvdep.com.

positive and feasible values for all variables we are interested in. The annual statements of account refer to the end of 2005.

In line with the above-mentioned literature we assumed banks to minimize total operating costs, and set up a basic cost function with outputs  $y_1$  interbank loans,  $y_2$  commercial loans and  $y_3$  securities. Inputs are  $x_1$  fixed assets,  $x_2$  number of employees and  $x_3$  borrowed funds (deposits). Input prices  $c_k$  can be approximated by the ratio of the costs of the inputs  $x_k$  to the amount of the particular input. In the case of  $x_1$ ,  $x_2$  we obtain percentaged values, while  $c_3$  is average cost per employee per year. Table 4 shows the descriptive statistics of bank size in terms of total assets ta, inputs x, input prices x and outputs y.

Variable	Description	Mean	St.Dev.	Median
ta	Total assets (BEUR)	3894.079	9043.181	1017.800
tc	Total operating costs (BEUR)	246.205	615.106	58.150
$y_1$	Interbank loans (BEUR)	614.568	1374.506	145.650
$y_2$	Commercial loans (BEUR)	2646.856	7734.673	374.150
$y_3$	Securities (BEUR)	179.694	561.050	10.200
$x_1$	Fixed assets (BEUR)	54.552	264.650	5.950
$x_2$	Employees	645.39	1888.22	142.50
$x_3$	Borrowed funds (BEUR)	2107.203	5420.554	486.800
$c_1$	Cost of fixed assets (% depreciation)	0.160	0.089	0.141
$c_2$	Cost of labour (TEUR/employee)	80.671	64.654	65.621
$c_3$	Price of funds (% interest expenses)	0.049	0.058	0.031

Table 4: Descriptive statistics of inputs, outputs, prices and bank size

#### 4.1.2 Shape of the cost function

As for the formal issues, one can state that most authors apply a 'regular' translog cost frontier. In most cases the translog form offers an appropriate balance between flexibility (in price and output elasticities), parameters to estimate and global fit. The exceptions among the reviewed literature are Altunbas et al. (2000), Altunbas and Chakravarty (2001), Girardone et al. (2004), and Weill (2004), using a Fourier Flexible form with additional trigonometric terms. Otherwise, Fitzpatrick and McQuinn (2005) had to restrict themselves to a simple Cobb-Douglas form due to an insufficient number of observations.

As we, too, work on a small-sized dataset (n=56), we encountered severe multicollinearity in the flexible translog form. So we fall back to a simple log linear Cobb-Douglas cost function (Kumbhakar and Tsionas, 2008). To ensure linear homogeneity in input prices  $tc(y, k \cdot c) =$ 

 $k^1 \cdot tc(y, c)$  with k > 0, we normalize total costs and input prices by the price of labour  $c_2$  (Lang and Welzel, 1996).

$$\log tc(y,c) = \beta_0 + \sum_{i=1}^{3} \beta_i \log y_i + \sum_{j=1}^{3} \gamma_j \log c_j + v + u \qquad \text{s.t.} \qquad \sum_{i=j}^{3} \gamma_j = 1$$

The homogeneity-constrained cost frontier results in:

$$\log \frac{tc(y,c)}{c_2} = \beta_0 + \sum_{i=1}^{3} \beta_i \log y_i + \sum_{j=1,3} \gamma_j \log \frac{c_j}{c_2} + v + u$$

#### 4.1.3 Inefficiency distribution

In the course of the simulation part, we applied the half normal and exponential assumption of inefficiency distribution. A closer look at the relevant literature reveals another 'truncated' approach (Bos and Kool, 2006, Battese et al., 2000, Fitzpatrick and McQuinn, 2005):  $u_i \sim N^+(\mu, \sigma_u)$  with  $\mu \geq 0$ . Referring to Greene (1990), Weill (2004) is the only one using a gamma-distributed inefficiency term. But the selection of an adequate distribution of  $u_i$  does not have to be overvalued, as Greene (1990) reportet extremely high rank correlations in the efficieny estimates between half normal, truncated, gamma and exponential model. So obviously there is no need to make use of two-parameter distributions.

## 4.2 Empirical evidence

To our knowledge there is not a single bank efficiency study applying the method of moments estimator we discussed. Conventionally, authors prefer Maximum Likelihood Estimation with the Jondrow et al. (1982)  $\exp[-E(u|\varepsilon)]$  estimator mentioned above.

As our sample is rather small-sized, we expect the method of moments approach to deliver highly reliable efficiency scores. Table 5 shows the results of all scenarios in discussion. Our 'rules of thumb'-indicator gives rather ambivalent recommendations: Values greater than 0.5 point at MLE application. But especially in the normal-exponential case we are facing values  $\approx 0.5$ . So we should reckon with equivalent results in both MOM and ML-estimation. In fact, the correlation table 6 confirms a  $\rho(\widehat{CE}_{MOM}, \widehat{CE}_{MLE}) \approx 0.99$ . Mean efficiencies  $\frac{1}{n} \sum_{i} \widehat{CE}_{i}$  differ only between varying inefficiency distribution assumptions. By the way, just like Greene (1990), we can still report externely high correlations  $\rho > 0.95$  between the half normal and exponential approach to inefficiency.

We observe slight differences in the estimated output and price elasticities (note that  $\gamma_2 = 1 - \gamma_1 - \gamma_3$ ). The overall scale economies are calculated as  $\epsilon_c = \left(\sum_i \frac{\partial \log tc(y,c)}{\partial \log \beta_i}\right)^{-1}$ . As  $\epsilon_c > 1$  in all cases, the results hint at increasing returns to scale in the German banking industry.

	normal-l	nalf normal	normal-exponential		
	MLE	MOM	MLE	MOM	
Intercept	0.760	1.526	2.096	1.819	
$y_1$	0.107	0.157	0.117	0.157	
$y_2$	0.543	0.505	0.552	0.505	
$y_3$	0.055	0.060	0.050	0.060	
$c_1$	0.208	0.233	0.254	0.233	
$c_3$	0.289	0.388	0.418	0.388	
$\lambda$	2.696	2.804	1.158	0.933	
$\sigma_v$	0.326	0.325	0.416	0.467	
$\sigma_u$	0.880	0.913	0.481	0.436	
$\operatorname{mean} \widehat{CE}$	0.537	0.531	0.655	0.673	
rule of thumb	0.608	0.629	0.541	0.503	

Table 5: Estimation results, all cases

	nhn-mle	nhn-mom	exp-mle	exp-mom
nhn-mle	1.000			
nhn-mom	0.987	1.000		
exp-mle	0.957	0.952	1.000	
exp-mom	0.951	0.952	0.988	1.000

Table 6: Correlation table, cost efficiencies

## 5 Conclusions

We put forward the MOM-approach to stochastic frontiers in bank efficiency analysis. An extensive simulation study confirmed the findings of Coelli (1995) and Olson et al. (1980): Rules of thumb suggest that the MOM-estimation of parametrical frontiers assuming a half normal inefficiency distribution can be favourable in terms of  $mse(\widehat{TE},TE)$  and  $mae(\widehat{TE},TE)$  if the sample size is medium scale ( $\leq 700$  observations) and inefficiency does not strongly dominate noise ( $\lambda < 2$ ), i.e. the bias of  $\beta_0^{OLS}$  is small.

So we propose that method of moment estimation should be considered an alternative to maximum likelihood estimation. We do so especially in cases focusing on a small number of homogeneous banks (Fitzpatrick and McQuinn, 2005). Applying MOM-estimation additionally to the ML-procedure even in larger samples could shed new light on the significance of the findings.

Another practical advantage of MOM-estimation is obvious: Whenever Newton-like numerical optimization is unavailable or fails due to awkward data structure, MOM provides a robust and easy to implement loophole. A simple two-step procedure (OLS-fitting and bias correction based on estimated residuals) is available within every statistical environment.

### References

- Aigner, D., Lovell, C. K. and Schmidt, P.: 1977, Formulation and estimation of stochastic frontier production function models, *Journal of Econometrics* **6**, 21–37.
- Altunbas, Y. and Chakravarty, S. P.: 2001, Frontier cost functions and bank efficiency, *Economics Letters* **72**, 233–240.
- Altunbas, Y., Liu, M.-H., Molyneux, P. and Seth, R.: 2000, Efficiency and risk in japanese banking, *Journal of Banking & Finance* **24**, 1605–1628.
- Battese, G. E. and Coelli, T. J.: 1988, Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data, *Journal of Econometrics* **38**, 387–399.
- Battese, G. E., Heshmati, A. and Hjalmarsson, L.: 2000, Efficiency in labour use in swedish banking industry: a stochastic frontier approach, *Empirical Economics* **25**, 623–640.
- Beatti, B. R. and Taylor, C. R.: 1985, The economics of production, Wiley, New York.
- Bos, J. and Kool, C.: 2006, Bank efficiency: The role of bank strategy and local market conditions, *Journal of Banking and Finance* **30**, 1953–1974.
- Coelli, T. J.: 1995, Estimators and hypothesis tests for a stochastic frontier function: A monte carlo analysis, *The Journal of Productivity Analysis* **6**, 247–268.
- Farrell, M. J.: 1957, The measurement of productive efficiency, *Journal of the Royal Statistical Society* **120**, 253–281.
- Ferrier, G. D. and Lovell, C. K.: 1990, Measuring cost efficiency in banking, *Journal of Econometrics* **46**, 229–245.
- Fitzpatrick, T. and McQuinn, K.: 2005, Cost efficiency in uk and irish credit institutions, *The Economic and Social Review* **36**(1), 45–66.

- Girardone, C., Molyneux, P. and Gardener, E. P.: 2004, Analysing the determinants of bank efficiency: the case of italian banks, *Applied Economics* **36**, 215–227.
- Greene, W. H.: 1990, A gamma-distributed stochastic frontier model, *Journal of Econometrics* **46**, 141–163.
- Greene, W. H.: 1997, Frontier production functions, *in* M. Pesaran and P. Schmidt (eds), *Hand-book of applied econometrics*, Blackwell, pp. 81–166.
- Jondrow, J., Lovell, C., Materov, I. and Schmidt, P.: 1982, On the estimation of technical inefficiency in the stochastic frontier production function model, *Journal of Econometrics* **19**, 233–238.
- Kumbhakar, S. C. and Lovell, C. K.: 2003, *Stochastic Frontier Analysis*, Cambridge University Press.
- Kumbhakar, S. C. and Tsionas, E. G.: 2008, Scale and efficiency measurement using a semi-parametric stochastic frontier model: evidence from the u.s. commercial banks, *Empirical Economics* **34**, 585–602.
- Lang, G. and Welzel, P.: 1996, Efficiency and technical progress in banking empirical results for a panel of german cooperative banks, *Journal of Banking & Finance* **20**, 1003–1023.
- Meeusen, W. and van de Broeck, J.: 1977, Efficiency estimation from cobb-douglas production functions with composed errors, *International Economic Review* **18**, 435–444.
- Mester, L. J.: 1996, A study of bank efficiency taking into account risk-preferences, *Journal of Banking & Finance* **20**, 1025–1045.
- Olson, J. A., Schmidt, P. and Waldman, D. M.: 1980, A monte carlo study of estimators of stochastic frontier production functions, *Journal of Econometrics* **13**, 67–82.
- Perera, S., Skully, M. and Wickramanyake, J.: 2007, Cost efficiency in south asian banking: The impact of bank size, state ownership and stock exchange listings, *International Review of Finance* **7**(1-2), 35–60.
- Ritter, C. and Simar, L.: 1997a, Another look at the american electrical utility data. CORE and Institute de Statistique, Université Catholique de Louvain, Working Paper.
- Ritter, C. and Simar, L.: 1997b, Pitfalls of normal-gamma stochastic frontier models, *Journal of Productivity Analysis* **8**, 168–182.

Sealey, S. j. and Lindley, J.: 1977, Inputs, outputs and a theory of production and cost at depository financial institutions, *Journal of Finance* **32**, 1251–1266.

Weill, L.: 2004, Measuring cost efficiency in european banking: A comparison of frontier techniques, *Journal of Productivity Analysis* **21**, 133–152.

## 6 Appendix

#### 6.1 Derivatives of the log-likelihood: half-normal

$$\ln L(y|\beta, \lambda, \sigma^2) = n \ln \left( \frac{\sqrt{2}}{\sqrt{\pi}} \right) + n \ln \left( \sigma^{-1} \right) + \sum_{i=1}^{n} \ln \left[ 1 - \Phi \left( [y_i - x_i'\beta] \lambda \sigma^{-1} \right) \right] - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - x_i'\beta)^2$$

The derivatives are given by

$$\frac{\partial \ln L}{\partial \beta} = -\frac{n}{\sigma^2} \sum_{i=1}^{n} (y_i - x_i' \beta) x_i + \frac{\lambda}{\sigma} \sum_{i=1}^{n} \frac{\phi_i^*}{(1 - F_i^*)} x_i$$

$$\frac{\partial \ln L}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^n (y_i - x_i'\beta)^2 + \frac{1}{2\sigma^3} \sum_{i=1}^n \frac{\phi_i^*}{(1 - \Phi_i^*)} (y_i - x_i'\beta)$$

$$\frac{\partial \ln L}{\partial \lambda} = -\frac{1}{\sigma} + \frac{1}{2\sigma^4} \sum_{i=1}^n \frac{\phi_i^*}{(1 - \Phi_i^*)} (y_i - x_i'\beta)$$

where

$$\phi_i^* = \phi([\ln y_i - x_i'\beta] \lambda \sigma^{-1})$$
  
$$\Phi_i^* = \Phi([\ln y_i - x_i'\beta] \lambda \sigma^{-1})$$

## 6.2 Derivatives of the log-likelihood: exponential

$$\ln L(y|\beta, \sigma_u^2, \sigma_v^2) = -n \ln(\sigma_u) + n \frac{1}{2} \frac{\sigma_v^2}{\sigma_u^2} + \sum_{i=1}^n \ln \Phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right) + \sum_{i=1}^n \frac{1}{\sigma_u}(y_i - x_i'\beta)$$

$$\frac{d}{d\beta} \sum_{i=1}^n \ln \Phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)$$

$$\frac{\partial \ln L}{\partial \beta} = \sum_{i=1}^n \frac{x_i'/\sigma_v \phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)}{\Phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)} - \sum_{i=1}^n x_i'/\sigma_u$$

$$\frac{\partial \ln L}{\partial \sigma_u} = -\frac{n}{\sigma_u} - n \frac{\sigma_v^2}{\sigma_u^3} + \sum_{i=1}^n \frac{\sigma_v \phi \left( -\frac{1}{\sigma_v} (y_i - x_i' \beta) - \frac{\sigma_v}{\sigma_u} \right)}{\sigma_u^2 \Phi \left( -\frac{1}{\sigma_v} (y_i - x_i' \beta) - \frac{\sigma_v}{\sigma_u} \right)} - \sum_{i=1}^n \frac{1}{\sigma_u^2} (y_i - x_i' \beta)$$

$$\frac{\partial \ln L}{\partial \sigma_v} = n \frac{\sigma_v}{\sigma_u^2} + \sum_{i=1}^n \left( \sigma_v^{-2} (y_i - x_i' \beta) - \frac{1}{\sigma_u} \right) \frac{\phi \left( -\frac{1}{\sigma_v} (y_i - x_i' \beta) - \frac{\sigma_v}{\sigma_u} \right)}{\Phi \left( -\frac{1}{\sigma_v} (y_i - x_i' \beta) - \frac{\sigma_v}{\sigma_u} \right)}$$

## 6.3 Tables

n	25	50	75	100	150	250	500	1000
λ	method of moments							
0.5	0.0516	0.0475	0.0464	0.0441	0.0416	0.0383	0.0351	0.0318
1.0	0.0612	0.0565	0.0513	0.0502	0.0464	0.0412	0.0343	0.0284
1.5	0.0588	0.0485	0.0425	0.0390	0.0319	0.0269	0.0226	0.0212
2.0	0.0531	0.0391	0.0319	0.0269	0.0228	0.0192	0.0177	0.0171
2.5	0.0480	0.0299	0.0235	0.0192	0.0166	0.0150	0.0142	0.0137
3.0	0.0424	0.0246	0.0181	0.0158	0.0136	0.0123	0.0115	0.0111
4.0	0.0345	0.0182	0.0131	0.0112	0.0098	0.0087	0.0080	0.0076
5.0	0.0291	0.0150	0.0106	0.0090	0.0077	0.0066	0.0059	0.0055
$\lambda$				maximum	likelihood			
0.5	0.1370	0.0928	0.0746	0.0639	0.0546	0.0455	0.0359	0.0294
1.0	0.1191	0.0800	0.0667	0.0614	0.0520	0.0414	0.0333	0.0274
1.5	0.0891	0.0578	0.0471	0.0417	0.0323	0.0268	0.0225	0.0211
2.0	0.0654	0.0426	0.0331	0.0269	0.0226	0.0188	0.0176	0.0171
2.5	0.0571	0.0308	0.0229	0.0184	0.0159	0.0145	0.0140	0.0136
3.0	0.0464	0.0240	0.0171	0.0145	0.0127	0.0116	0.0111	0.0110
4.0	0.0348	0.0159	0.0110	0.0095	0.0085	0.0079	0.0076	0.0075
5.0	0.0274	0.0118	0.0082	0.0071	0.0062	0.0059	0.0055	0.0054
λ				adva	ntage			
0.5	MOM	MOM	MOM	MOM	MOM	MOM	MOM	MLE
1.0	MOM	MOM	MOM	MOM	MOM	MOM	MLE	MLE
1.5	MOM	MOM	MOM	MOM	MOM	MLE	MLE	MLE
2.0	MOM	MOM	MOM	MOM	MLE	MLE	MLE	MOM
2.5	MOM	MOM	MLE	MLE	MLE	MLE	MLE	MLE
3.0	MOM	MLE	MLE	MLE	MLE	MLE	MLE	MLE
4.0	MOM	MLE	MLE	MLE	MLE	MLE	MLE	MLE
5.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE

Table 7: Mean Square Error normal-halfnormal approach

n	25	50	75	100	150	250	500	1000
$\lambda$								
0.5	0.0461	0.0441	0.0421	0.0403	0.0387	0.0348	0.0307	0.0274
1.0	0.0566	0.0443	0.0368	0.0339	0.0313	0.0288	0.0278	0.0272
1.5	0.0478	0.0331	0.0287	0.0262	0.0245	0.0232	0.0224	0.0218
2.0	0.0407	0.0274	0.0233	0.0216	0.0201	0.0188	0.0178	0.0172
2.5	0.0372	0.0229	0.0198	0.0184	0.0167	0.0154	0.0143	0.0136
3.0	0.0338	0.0207	0.0176	0.0160	0.0146	0.0129	0.0119	0.0111
4.0	0.0308	0.0182	0.0150	0.0132	0.0115	0.0099	0.0087	0.0078
5.0	0.0305	0.0168	0.0138	0.0116	0.0098	0.0082	0.0068	0.0059
$\lambda$				maximum	likelihood			
0.5	0.0612	0.0515	0.0469	0.0446	0.0413	0.0363	0.0311	0.0275
1.0	0.0692	0.0490	0.0393	0.0353	0.0314	0.0288	0.0275	0.0272
1.5	0.0535	0.0339	0.0279	0.0249	0.0232	0.0222	0.0217	0.0215
2.0	0.0434	0.0250	0.0204	0.0189	0.0177	0.0171	0.0167	0.0165
2.5	0.0358	0.0193	0.0159	0.0148	0.0139	0.0134	0.0130	0.0128
3.0	0.0297	0.0156	0.0130	0.0120	0.0113	0.0107	0.0103	0.0102
4.0	0.0239	0.0111	0.0091	0.0084	0.0076	0.0072	0.0069	0.0068
5.0	0.0206	0.0088	0.0069	0.0062	0.0055	0.0052	0.0049	0.0048
$\lambda$				adva	ntage			
0.5	MOM	MOM	MOM	MOM	MOM	MOM	MOM	MOM
1.0	MOM	MOM	MOM	MOM	MOM	MOM	MLE	MOM
1.5	MOM	MOM	MLE	MLE	MLE	MLE	MLE	MLE
2.0	MOM	MLE	MLE	MLE	MLE	MLE	MLE	MLE
2.5	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE
3.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE
4.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE
5.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE

Table 8: Mean Square Error normal-exponential approach

21

λ         method of moments           0.5         0.2035         0.2022         0.1992         0.1963         0.1933         0.1874         0.1850         0.18           1.0         0.2100         0.2013         0.1931         0.1923         0.1938         0.1934         0.1965         0.19           1.5         0.1896         0.1817         0.1841         0.1845         0.1895         0.1931         0.1967         0.19           2.0         0.1748         0.1745         0.1775         0.1787         0.1858         0.1910         0.1953         0.19           2.5         0.1680         0.1685         0.1734         0.1775         0.1830         0.1871         0.1935         0.19           3.0         0.1639         0.1670         0.1724         0.1746         0.1821         0.1875         0.1945         0.19           4.0         0.1554         0.1622         0.1665         0.1744         0.1829         0.1894         0.1969         0.19           5.0         0.1532         0.1592         0.1660         0.1703         0.1816         0.1906         0.1980         0.20           λ         maximum likelihood           0.5         0	1000
0.5         0.2035         0.2022         0.1992         0.1963         0.1933         0.1874         0.1850         0.18           1.0         0.2100         0.2013         0.1931         0.1923         0.1938         0.1934         0.1965         0.19           1.5         0.1896         0.1817         0.1841         0.1845         0.1895         0.1931         0.1967         0.19           2.0         0.1748         0.1745         0.1775         0.1787         0.1858         0.1910         0.1953         0.19           2.5         0.1680         0.1685         0.1734         0.1775         0.1830         0.1871         0.1935         0.19           3.0         0.1639         0.1670         0.1724         0.1746         0.1821         0.1875         0.1945         0.19           4.0         0.1554         0.1622         0.1665         0.1744         0.1829         0.1894         0.1969         0.19           5.0         0.1532         0.1592         0.1660         0.1703         0.1816         0.1906         0.1980         0.20 $\lambda$ maximum likelihood           0.5         0.2537         0.2280         0.2139         0	
1.0         0.2100         0.2013         0.1931         0.1923         0.1938         0.1934         0.1965         0.191           1.5         0.1896         0.1817         0.1841         0.1845         0.1895         0.1931         0.1967         0.19           2.0         0.1748         0.1745         0.1775         0.1787         0.1858         0.1910         0.1953         0.19           2.5         0.1680         0.1685         0.1734         0.1775         0.1830         0.1871         0.1935         0.19           3.0         0.1639         0.1670         0.1724         0.1746         0.1821         0.1875         0.1945         0.19           4.0         0.1554         0.1622         0.1665         0.1744         0.1829         0.1894         0.1969         0.19           5.0         0.1532         0.1592         0.1660         0.1703         0.1816         0.1906         0.1980         0.20 $\lambda$ maximum likelihood           0.5         0.2537         0.2280         0.2139         0.2078         0.1934         0.1846         0.1738         0.17           1.0         0.2291         0.1960         0.1840	0.1801
1.5         0.1896         0.1817         0.1841         0.1845         0.1895         0.1931         0.1967         0.1932           2.0         0.1748         0.1745         0.1775         0.1787         0.1858         0.1910         0.1953         0.1932           2.5         0.1680         0.1685         0.1734         0.1775         0.1830         0.1871         0.1935         0.193           3.0         0.1639         0.1670         0.1724         0.1746         0.1821         0.1875         0.1945         0.1945           4.0         0.1554         0.1622         0.1665         0.1744         0.1829         0.1894         0.1969         0.19           5.0         0.1532         0.1592         0.1660         0.1703         0.1816         0.1906         0.1980         0.20 $\lambda$ maximum likelihood           0.5         0.2537         0.2280         0.2139         0.2078         0.1934         0.1846         0.1738         0.17           1.0         0.2291         0.1960         0.1840         0.1800         0.1763         0.1736         0.1738         0.15           1.5         0.1834         0.1614         0.1555	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
2.5     0.1680     0.1685     0.1734     0.1775     0.1830     0.1871     0.1935     0.1935       3.0     0.1639     0.1670     0.1724     0.1746     0.1821     0.1875     0.1945     0.1945       4.0     0.1554     0.1622     0.1665     0.1744     0.1829     0.1894     0.1969     0.19       5.0     0.1532     0.1592     0.1660     0.1703     0.1816     0.1906     0.1980     0.20       λ     maximum likelihood       0.5     0.2537     0.2280     0.2139     0.2078     0.1934     0.1846     0.1785     0.173       1.0     0.2291     0.1960     0.1840     0.1800     0.1763     0.1736     0.1738     0.15       1.5     0.1834     0.1614     0.1555     0.1536     0.1513     0.1508     0.1503     0.15       2.0     0.1533     0.1381     0.1343     0.1317     0.1309     0.1303     0.1297     0.12	
4.0     0.1554     0.1622     0.1665     0.1744     0.1829     0.1894     0.1969     0.19       5.0     0.1532     0.1592     0.1660     0.1703     0.1816     0.1906     0.1980     0.20       λ     maximum likelihood       0.5     0.2537     0.2280     0.2139     0.2078     0.1934     0.1846     0.1785     0.17       1.0     0.2291     0.1960     0.1840     0.1800     0.1763     0.1736     0.1738     0.17       1.5     0.1834     0.1614     0.1555     0.1536     0.1513     0.1508     0.1503     0.15       2.0     0.1533     0.1381     0.1343     0.1317     0.1309     0.1303     0.1297     0.12	0.1945
5.0         0.1532         0.1592         0.1660         0.1703         0.1816         0.1906         0.1980         0.20           λ         maximum likelihood           0.5         0.2537         0.2280         0.2139         0.2078         0.1934         0.1846         0.1785         0.17           1.0         0.2291         0.1960         0.1840         0.1800         0.1763         0.1736         0.1738         0.17           1.5         0.1834         0.1614         0.1555         0.1536         0.1513         0.1508         0.1503         0.12           2.0         0.1533         0.1381         0.1343         0.1317         0.1309         0.1303         0.1297         0.12	0.1975
λ         maximum likelihood           0.5         0.2537         0.2280         0.2139         0.2078         0.1934         0.1846         0.1785         0.1785           1.0         0.2291         0.1960         0.1840         0.1800         0.1763         0.1736         0.1738         0.173           1.5         0.1834         0.1614         0.1555         0.1536         0.1513         0.1508         0.1503         0.15           2.0         0.1533         0.1381         0.1343         0.1317         0.1309         0.1303         0.1297         0.12	0.1995
0.5         0.2537         0.2280         0.2139         0.2078         0.1934         0.1846         0.1785         0.1785           1.0         0.2291         0.1960         0.1840         0.1800         0.1763         0.1736         0.1738         0.173           1.5         0.1834         0.1614         0.1555         0.1536         0.1513         0.1508         0.1503         0.15           2.0         0.1533         0.1381         0.1343         0.1317         0.1309         0.1303         0.1297         0.12	0.2011
1.0     0.2291     0.1960     0.1840     0.1800     0.1763     0.1736     0.1738     0.1731       1.5     0.1834     0.1614     0.1555     0.1536     0.1513     0.1508     0.1503     0.15       2.0     0.1533     0.1381     0.1343     0.1317     0.1309     0.1303     0.1297     0.12	
1.5     0.1834     0.1614     0.1555     0.1536     0.1513     0.1508     0.1503     0.15       2.0     0.1533     0.1381     0.1343     0.1317     0.1309     0.1303     0.1297     0.12	0.1756
2.0 0.1533 0.1381 0.1343 0.1317 0.1309 0.1303 0.1297 0.12	0.1736
	0.1502
2.5 0.1321 0.1197 0.1170 0.1165 0.1142 0.1136 0.1128 0.11	0.1295
	0.1124
3.0 0.1176 0.1058 0.1049 0.1032 0.1018 0.1005 0.0997 0.09	0.0995
4.0 0.0964 0.0861 0.0841 0.0841 0.0825 0.0815 0.0808 0.08	0.0804
5.0 0.0858 0.0724 0.0713 0.0707 0.0700 0.0689 0.0679 0.06	0.0674
$\lambda$ advantage	
0.5 MOM MOM MOM MOM MLE MLE ML	MLE
1.0 MOM MLE MLE MLE MLE MLE ML	MLE
1.5 MLE MLE MLE MLE MLE MLE ML	MLE
2.0 MLE MLE MLE MLE MLE MLE ML	MLE
2.5 MLE MLE MLE MLE MLE MLE MLE ML	MLE
3.0 MLE MLE MLE MLE MLE MLE ML	MLE
4.0 MLE MLE MLE MLE MLE MLE ML	MLE
5.0 MLE MLE MLE MLE MLE MLE MLE	MLE

Table 9: Mean Average Error misspecification 1

22

n	25	50	75	100	150	250	500	1000
$\overline{\lambda}$	method of moments							
0.5	0.1451	0.1409	0.1382	0.1377	0.1351	0.1331	0.1300	0.1281
1.0	0.1884	0.1813	0.1742	0.1728	0.1690	0.1638	0.1559	0.1479
1.5	0.1964	0.1823	0.1739	0.1698	0.1608	0.1538	0.1474	0.1450
2.0	0.1962	0.1758	0.1664	0.1587	0.1525	0.1466	0.1433	0.1412
2.5	0.1935	0.1677	0.1590	0.1510	0.1454	0.1423	0.1392	0.1379
3.0	0.1902	0.1628	0.1514	0.1472	0.1434	0.1391	0.1361	0.1352
4.0	0.1829	0.1571	0.1475	0.1430	0.1389	0.1355	0.1327	0.1320
5.0	0.1773	0.1537	0.1445	0.1419	0.1367	0.1342	0.1316	0.1304
$\lambda$				maximum	likelihood			
0.5	0.1688	0.1548	0.1488	0.1469	0.1425	0.1392	0.1351	0.1322
1.0	0.2117	0.1959	0.1865	0.1837	0.1780	0.1706	0.1612	0.1510
1.5	0.2124	0.1921	0.1798	0.1749	0.1644	0.1554	0.1472	0.1440
2.0	0.2044	0.1781	0.1666	0.1554	0.1485	0.1410	0.1363	0.1338
2.5	0.1954	0.1612	0.1511	0.1408	0.1322	0.1289	0.1252	0.1235
3.0	0.1838	0.1489	0.1348	0.1284	0.1234	0.1180	0.1152	0.1140
4.0	0.1669	0.1312	0.1168	0.1107	0.1056	0.1022	0.0994	0.0982
5.0	0.1525	0.1168	0.1036	0.0982	0.0932	0.0903	0.0877	0.0862
λ				adva	ntage			
0.5	MOM	MOM	MOM	MOM	MOM	MOM	MOM	MOM
1.0	MOM	MOM	MOM	MOM	MOM	MOM	MOM	MOM
1.5	MOM	MOM	MOM	MOM	MOM	MOM	MLE	MLE
2.0	MOM	MOM	MOM	MLE	MLE	MLE	MLE	MLE
2.5	MOM	MLE	MLE	MLE	MLE	MLE	MLE	MLE
3.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE
4.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE
5.0	MLE	MLE	MLE	MLE	MLE	MLE	MLE	MLE

Table 10: Mean Average Error misspecification 2

23

Inputs	Outputs	Reference					
	Altunbas et al. (2000)						
labour, funds, physical capital	total loans, securities, off-balance sheet items	Intermediation					
	(contingent liabilities, acceptances,	(varied)					
	guarantees						
Altunbas and Chakravarty (2001)							
labour, total funds, physical capital	all types of loans, total aggregate securities,	Intermediation					
	off-balance sheet activities	(varied)					
Battese et al. (2000)							
public loans, guarantees, deposits,	costs of labour use	Input-requirement					
number of branches, value of		model					
inventories							
	Bos and Kool (2006)						
public relations, labour, housing,	retail loans, wholesale loans, mortgages,	Intermediation					
physical capital	provisions	(varied)					
	Ferrier and Lovell (1990)						
employees, occupancy costs,	demand deposit accounts, time deposit	Production					
materials	accounts, real estate loans, real estate loans,						
	installment loans, commercial loans						
	Fitzpatrick and McQuinn (2005)						
labour, physical capital, financial	consumer/commercial/other loans,	Intermediation					
capital	non-interest revenue	(varied)					
	Girardone et al. (2004)						
employees, total customer deposits,	total customer loans, other earning assets	Intermediation					
total fixed assets	I 1W1 1/1000						
	Lang and Welzel (1996)	T . 11 .:					
employees, fixed assets, deposits	short-term and long-term loans to non-banks,	Intermediation					
	loans to banks, bonds/cash/real estate	(varied)					
	investments, fees and commissions, revenue						
	from sales of commodities						
	Mester (1996)						
labour, physical capital, deposits	real estate loans,	Intermediation					
	commercial/industrial/government/ loans,						
	loans to individuals						
	Perera et al. (2007)						
funds, labour, capital	net total loans, other earning assets	Intermediation					
	Weill (2004)						
labour, physical capital, borrowed	loans, investment assets	Intermediation					
funds							

Table 11: Input and output-modelling in selected bank efficiency studies

## The following Discussion Papers have been published since 2007:

## **Series 1: Economic Studies**

01	2007	The effect of FDI on job separation	Sascha O. Becker Marc-Andreas Mündler
02	2007	Threshold dynamics of short-term interest rates: empirical evidence and implications for the term structure	Theofanis Archontakis Wolfgang Lemke
03	2007	Price setting in the euro area: some stylised facts from individual producer price data	Dias, Dossche, Gautier Hernando, Sabbatini Stahl, Vermeulen
04	2007	Unemployment and employment protection in a unionized economy with search frictions	Nikolai Stähler
05	2007	End-user order flow and exchange rate dynamics	S. Reitz, M. A. Schmidt M. P. Taylor
06	2007	Money-based interest rate rules: lessons from German data	C. Gerberding F. Seitz, A. Worms
07	2007	Moral hazard and bail-out in fiscal federations: evidence for the German Länder	Kirsten H. Heppke-Falk Guntram B. Wolff
08	2007	An assessment of the trends in international price competitiveness among EMU countries	Christoph Fischer
09	2007	Reconsidering the role of monetary indicators for euro area inflation from a Bayesian perspective using group inclusion probabilities	Michael Scharnagl Christian Schumacher
10	2007	A note on the coefficient of determination in regression models with infinite-variance variables	Jeong-Ryeol Kurz-Kim Mico Loretan

11	2007	Exchange rate dynamics in a target zone - a heterogeneous expectations approach	Christian Bauer Paul De Grauwe, Stefan Reitz
12	2007	Money and housing - evidence for the euro area and the US	Claus Greiber Ralph Setzer
13	2007	An affine macro-finance term structure model for the euro area	Wolfgang Lemke
14	2007	Does anticipation of government spending matter? Evidence from an expectation augmented VAR	Jörn Tenhofen Guntram B. Wolff
15	2007	On-the-job search and the cyclical dynamics of the labor market	Michael Krause Thomas Lubik
16	2007	Heterogeneous expectations, learning and European inflation dynamics	Anke Weber
17	2007	Does intra-firm bargaining matter for business cycle dynamics?	Michael Krause Thomas Lubik
18	2007	Uncertainty about perceived inflation target and monetary policy	Kosuke Aoki Takeshi Kimura
19	2007	The rationality and reliability of expectations reported by British households: micro evidence from the British household panel survey	James Mitchell Martin Weale
20	2007	Money in monetary policy design under uncertainty: the Two-Pillar Phillips Curve versus ECB-style cross-checking	Günter W. Beck Volker Wieland
21	2007	Corporate marginal tax rate, tax loss carryforwards and investment functions – empirical analysis using a large German panel data set	Fred Ramb

22	2007	Volatile multinationals? Evidence from the labor demand of German firms	Claudia M. Buch Alexander Lipponer
23	2007	International investment positions and exchange rate dynamics: a dynamic panel analysis	Michael Binder Christian J. Offermanns
24	2007	Testing for contemporary fiscal policy discretion with real time data	Ulf von Kalckreuth Guntram B. Wolff
25	2007	Quantifying risk and uncertainty in macroeconomic forecasts	Malte Knüppel Karl-Heinz Tödter
26	2007	Taxing deficits to restrain government spending and foster capital accumulation	Nikolai Stähler
27	2007	Spill-over effects of monetary policy – a progress report on interest rate convergence in Europe	Michael Flad
28	2007	The timing and magnitude of exchange rate overshooting	Hoffmann Sondergaard, Westelius
29	2007	The timeless perspective vs. discretion: theory and monetary policy implications for an open economy	Alfred V. Guender
30	2007	International cooperation on innovation: empirical evidence for German and Portuguese firms	Pedro Faria Tobias Schmidt
31	2007	Simple interest rate rules with a role for money	M. Scharnagl C. Gerberding, F. Seitz
32	2007	Does Benford's law hold in economic research and forecasting?	Stefan Günnel Karl-Heinz Tödter
33	2007	The welfare effects of inflation: a cost-benefit perspective	Karl-Heinz Tödter Bernhard Manzke

34	2007	Factor-MIDAS for now- and forecasting with	
		ragged-edge data: a model comparison for	Massimiliano Marcellino
		German GDP	Christian Schumacher
35	2007	Monetary policy and core inflation	Michele Lenza
0.1	2000		
01	2008	Can capacity constraints explain	N. 1. 17 1
		asymmetries of the business cycle?	Malte Knüppel
02	2008	Communication, decision-making and the	
02	2000	optimal degree of transparency of monetary	
		policy committees	Anke Weber
		poncy committees	Alike Webel
03	2008	The impact of thin-capitalization rules on	Buettner, Overesch
		multinationals' financing and investment decisions	•
04	2008	Comparing the DSGE model with the factor model:	
		an out-of-sample forecasting experiment	Mu-Chun Wang
			_
05	2008	Financial markets and the current account –	Sabine Herrmann
		emerging Europe versus emerging Asia	Adalbert Winkler
06	2008	The German sub-national government bond	Alexander Schulz
		market: evolution, yields and liquidity	Guntram B. Wolff
07	2008	Integration of financial markets and national	Mathias Hoffmann
		price levels: the role of exchange rate volatility	Peter Tillmann
08	2008	Business cycle evidence on firm entry	Vivien Lewis
09	2008	Panel estimation of state dependent adjustment	
		when the target is unobserved	Ulf von Kalckreuth
10	2006		
10	2008	Nonlinear oil price dynamics –	Stefan Reitz
		a tale of heterogeneous speculators?	Ulf Slopek

11	2008	Financing constraints, firm level adjustment of capital and aggregate implications	Ulf von Kalckreuth
12	2008	Sovereign bond market integration: the euro, trading platforms and globalization	Alexander Schulz Guntram B. Wolff
13	2008	Great moderation at the firm level? Unconditional versus conditional output volatility	Claudia M. Buch Jörg Döpke Kerstin Stahn
14	2008	How informative are macroeconomic risk forecasts? An examination of the Bank of England's inflation forecasts	Malte Knüppel Guido Schultefrankenfeld
15	2008	Foreign (in)direct investment and corporate taxation	Georg Wamser
16	2008	The global dimension of inflation – evidence from factor-augmented Phillips curves	Sandra Eickmeier Katharina Moll
17	2008	Global business cycles: convergence or decoupling?	M. Ayhan Kose Christopher Otrok, Ewar Prasad
18	2008	Restrictive immigration policy in Germany: pains and gains foregone?	Gabriel Felbermayr Wido Geis Wilhelm Kohler
19	2008	International portfolios, capital accumulation and foreign assets dynamics	Nicolas Coeurdacier Robert Kollmann Philippe Martin
20	2008	Financial globalization and monetary policy	Michael B. Devereux Alan Sutherland
21	2008	Banking globalization, monetary transmission and the lending channel	Nicola Cetorelli Linda S. Goldberg

22	2008	Financial exchange rates and international currency exposures	Philip R. Lane Jay C. Shambaugh
23	2008	Financial integration, specialization and systemic risk	F. Fecht, H. P. Grüner P. Hartmann
24	2008	Sectoral differences in wage freezes and wage cuts: evidence from a new firm survey	Daniel Radowski Holger Bonin

## **Series 2: Banking and Financial Studies**

01	2007	Granularity adjustment for Basel II	Michael B. Gordy Eva Lütkebohmert
02	2007	Efficient, profitable and safe banking: an oxymoron? Evidence from a panel VAR approach	Michael Koetter Daniel Porath
03	2007	Slippery slopes of stress: ordered failure events in German banking	Thomas Kick Michael Koetter
04	2007	Open-end real estate funds in Germany – genesis and crisis	C. E. Bannier F. Fecht, M. Tyrell
05	2007	Diversification and the banks' risk-return-characteristics – evidence from loan portfolios of German banks	A. Behr, A. Kamp C. Memmel, A. Pfingsten
06	2007	How do banks adjust their capital ratios? Evidence from Germany	Christoph Memmel Peter Raupach
07	2007	Modelling dynamic portfolio risk using risk drivers of elliptical processes	Rafael Schmidt Christian Schmieder
08	2007	Time-varying contributions by the corporate bond and CDS markets to credit risk price discovery	Niko Dötz
09	2007	Banking consolidation and small business finance – empirical evidence for Germany	K. Marsch, C. Schmieder K. Forster-van Aerssen
10	2007	The quality of banking and regional growth	Hasan, Koetter, Wedow
11	2007	Welfare effects of financial integration	Fecht, Grüner, Hartmann
12	2007	The marketability of bank assets and managerial rents: implications for financial stability	Falko Fecht Wolf Wagner

13	2007	Asset correlations and credit portfolio risk – an empirical analysis	K. Düllmann, M. Scheicher C. Schmieder
14	2007	Relationship lending – empirical evidence for Germany	C. Memmel C. Schmieder, I. Stein
15	2007	Creditor concentration: an empirical investigation	S. Ongena, G.Tümer-Alkan N. von Westernhagen
16	2007	Endogenous credit derivatives and bank behaviour	Thilo Pausch
17	2007	Profitability of Western European banking systems: panel evidence on structural and cyclical determinants	Rainer Beckmann
18	2007	Estimating probabilities of default with support vector machines	W. K. Härdle R. A. Moro, D. Schäfer
01	2008	Analyzing the interest rate risk of banks using time series of accounting-based data: evidence from Germany	O. Entrop, C. Memmel M. Wilkens, A. Zeisler
02	2008	Bank mergers and the dynamics of deposit interest rates	Ben R. Craig Valeriya Dinger
03	2008	Monetary policy and bank distress: an integrated micro-macro approach	F. de Graeve T. Kick, M. Koetter
04	2008	Estimating asset correlations from stock prices or default rates – which method is superior?	K. Düllmann J. Küll, M. Kunisch
05	2008	Rollover risk in commercial paper markets and firms' debt maturity choice	Felix Thierfelder
06	2008	The success of bank mergers revisited – an assessment based on a matching strategy	Andreas Behr Frank Heid

07	2008	Which interest rate scenario is the worst one for a bank? Evidence from a tracking bank approach	
		for German savings and cooperative banks	Christoph Memmel
08	2008	Market conditions, default risk and credit spreads	Dragon Yongjun Tang Hong Yan
09	2008	The pricing of correlated default risk: evidence from the credit derivatives market	Nikola Tarashev Haibin Zhu
10	2008	Determinants of European banks' engagement in loan securitization	Christina E. Bannier Dennis N. Hänsel
11	2008	Interaction of market and credit risk: an analysis of inter-risk correlation and risk aggregation	Klaus Böcker Martin Hillebrand
12	2008	A value at risk analysis of credit default swaps	B. Raunig, M. Scheicher
13	2008	Systemic bank risk in Brazil: an assessment of correlated market, credit, sovereign and interbank risk in an environment with stochastic volatilities and correlations	Theodore M. Barnhill, Jr. Marcos Rietti Souto
14	2008	Regulatory capital for market and credit risk interaction: is current regulation always conservative?	•
15	2008	The implications of latent technology regimes for competition and efficiency in banking	Michael Koetter Tigran Poghosyan
16	2008	The impact of downward rating momentum on credit portfolio risk	André Güttler Peter Raupach
17	2008	Stress testing of real credit portfolios	F. Mager, C. Schmieder
18	2008	Real estate markets and bank distress	M. Koetter, T. Poghosyan

19 2008 Stochastic frontier analysis by means of maxi- Andreas Behr mum likelihood and the method of moments Sebastian Tente

## Visiting researcher at the Deutsche Bundesbank

The Deutsche Bundesbank in Frankfurt is looking for a visiting researcher. Among others under certain conditions visiting researchers have access to a wide range of data in the Bundesbank. They include micro data on firms and banks not available in the public. Visitors should prepare a research project during their stay at the Bundesbank. Candidates must hold a PhD and be engaged in the field of either macroeconomics and monetary economics, financial markets or international economics. Proposed research projects should be from these fields. The visiting term will be from 3 to 6 months. Salary is commensurate with experience.

Applicants are requested to send a CV, copies of recent papers, letters of reference and a proposal for a research project to:

Deutsche Bundesbank Personalabteilung Wilhelm-Epstein-Str. 14

60431 Frankfurt GERMANY