

Cigno, Alessandro

**Working Paper**

## Can a ban on child labour be self-enforcing?

GLO Discussion Paper, No. 1325

**Provided in Cooperation with:**

Global Labor Organization (GLO)

*Suggested Citation:* Cigno, Alessandro (2023) : Can a ban on child labour be self-enforcing?, GLO Discussion Paper, No. 1325, Global Labor Organization (GLO), Essen

This Version is available at:

<https://hdl.handle.net/10419/276227>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Can a ban on child labour be self-enforcing?

Alessandro Cigno  
University of Florence and GLO

## Abstract

Basu and Van (1998) show that a ban on child labour may be self-enforcing if, above the subsistence level, no amount of consumption can compensate parents for the disutility of child labour. We show that a partial ban may be self-enforcing, but a total one never is, if education is available, and the disutility of child labour can be compensated by the expected utility of future consumption. If some of the work children do is not observable by the government, a ban may be only apparently self-enforcing, or actually counterproductive. If the government wants to reduce child labour and raise education to the efficient level, it can borrow from the international credit market to subsidize parents, and tax their children's future wages to pay the loan back with interests.

*Keywords:* Child labour, education, self-enforcing ban, norms.  
*JEL:* H31, J22, O12.

## 1 Introduction

A series of articles beginning with Basu and Van (1998) argue that, if an economy has a multiplicity of equilibria,<sup>1</sup> some with and some without child labour, a ban on the latter may be self-enforcing in the sense that, once an equilibrium without child labour is established, parents have no incentive to depart from it.<sup>2</sup> That is a valuable result, because it implies that a major obstacle to human and economic development can be eradicated without any need for coercion. However, the assumptions under

---

<sup>1</sup>See, on this, Basu (2002).

<sup>2</sup>The general idea, more fully developed in Basu (2002, 2011), is that, if more than one equilibrium exists, it may be possible to nudge the economy towards the socially more desirable one at (almost) no cost.

which it was originally obtained are rather extreme. One is that there are no opportunities for increasing a child's future earnings by investing in the child's education. Another is that, at or above the subsistence level, parents would rather see their children do nothing and consume the subsistence minimum, than work even a little and consume more than that minimum. The present paper investigates whether the proposition in question remains true if education is an alternative to work, and the disutility of child labour can be compensated by the expected utility of future consumption.

Baland and Robinson (2000) show that child labour may be inefficiently high even under conventional parental preferences, because children cannot commit to paying back a loan. As pointed out in Cigno and Rosati (2005), this implies not only that parents cannot make loans to their children, but also that parents cannot borrow on their children's behalf. Therefore, only sufficiently rich parents can give their children the efficient amount of education. Dessy and Pallage (2001) show that child labour may reflect a lack of coordination between parents investing in their children's education, and employers investing in skill-intensive technologies. Pouliot (2006) shows that child labour may be inefficiently high because the return to education is uncertain, and parents are risk averse.

We start by pointing out that a partial or total ban on child labour is self-enforcing under Basu-Van assumptions only if the demand for labour function is sufficiently inelastic for the adult wage rate to jump from below to above the level which would allow all family members to survive without the children's work. We then go on to show that, if education is an alternative to work, and parental preferences are such that the disutility of child labour can be compensated by the expected utility of future consumption, a partial ban may nudge the market from an equilibrium with more, to one with less child labour, but a total ban is never self-enforcing. If some of the work children do is not observable by the government because it takes place within the family, a ban may be apparently self-enforcing in the sense that children will not work in the market, but will do so within their families. In some cases, a ban may be actually counterproductive in the sense that it induces children to work more within their families than they would have done in the market.

As an alternative to imposing a partial or total ban on child labour, the government can borrow from the international credit market, use the loan to subsidize parents, and tax their children's future wages to pay the loan back with interests. In other words, the government can do what parents cannot, namely borrow on the children's behalf. We show

that this would reduce child labour in all its forms, and raise education to the efficient level.

## 2 The original model

We start by outlining the basic model in Basu and Van (1998) with some minimal and uninfluential changes that will make it easier to extend. There is a large number of identical firms demanding labour, and an equally large number of identical families supplying it. This allows us to reason in terms of a representative family, and a representative firm. Firm owners are neither workers, nor parents of potential workers, so that their profits do not end up in the workers' pockets. Adult labour, denoted by  $L_a$ , and child labour, denoted by  $L_c$ , are perfect substitutes in the production of the numeraire good at the constant rate  $\gamma$ . We may thus conduct the analysis in terms of a single market for adult-equivalent labour,

$$L = L_a + \gamma L_c, 0 < \gamma < 1,$$

instead of two interrelated ones.

The representative firm maximizes its profit

$$P = F(L_a + \gamma L_c) - wL_a - w_c L_c, \quad (1)$$

where  $w$  is the adult wage rate,  $w_c$  the child wage rate, and  $F(\cdot)$  the production function, increasing and concave. From the first-order conditions for an interior solution, we find

$$F'(L) = w \quad (2)$$

and

$$w_c = \gamma w.$$

The representative family is composed by the parental couple, and  $n$  school-age children. Each adult is endowed with one unit of time, and each child with  $\gamma$  units of adult-equivalent time, so that the family supplies  $L = 2 + \gamma n$  units of adult-equivalent labour if everybody works full time,  $L = 2$  if only the parents do. There are two critical values of  $w$ . One, denoted by  $\bar{w}$ , is defined as the minimum wage rate that would allow the entire family to barely survive if the parents worked full time, and the children did not work at all. The other, denoted by  $\underline{w}$ , is defined as the minimum wage rate that would allow the entire family to barely survive if all family members, children included, worked full time.<sup>3</sup> Clearly,

$$2\bar{w} = (2 + \gamma n)\underline{w}$$

---

<sup>3</sup>The original authors do not actually mention  $\underline{w}$ , but it matters.

is the subsistence level of family consumption.

Decisions are taken by parents. Their preferences are such, that the family will supply  $L = 2$  units of adult labour if  $w$  is at least equal to  $\bar{w}$ ,  $L = 2 + \gamma n$  units of adult-equivalent labour if  $w$  is less than  $\bar{w}$ . It is thus implicitly assumed that, at or above  $\bar{w}$ , no amount of the consumption good can compensate the parents for even a very small amount of the child labour bad. Below  $\bar{w}$ , child labour is not a bad. Given such preferences, it is not possible to draw indifference curves,<sup>4</sup> and no utility function exists. In Fig. 1, the labour supply schedule is represented by the broken **SS** curve. Below  $\underline{w}$ , there is no labour supply, because there are no workers. The value of  $n$  is not irrelevant. Even though a child cannot do as much work as an adult ( $\gamma < 1$ ), if  $n$  is 5 or 6, the effective labour force can be two or three times as large as the number of adults. Therefore, taking all the children out of the labour market could have a strong effect on the equilibrium wage rate.

If the demand schedule satisfying (2) is represented by the **DD** line,<sup>5</sup> the market has two possible equilibria, one at point  $A$ , where the wage rate is  $w_1^A$ , higher than  $\bar{w}$ , and children do not work, the other at point  $C$ , where the wage rate is  $w_1^C$ , lower than  $\bar{w}$ , and children work full time. If the equilibrium happened to be initially at point  $C$ , and child labour were banned, the only possible equilibrium left would be  $A$ . At that point, firms would have an interest in offering to employ children at a wage rate lower than  $\gamma w_1^A$ , but parents would reject the offer. The Basu-Van result thus applies. If the demand line were **DD'**, however, the only possible equilibrium would be  $C'$ , in which case a child labour ban would cause starvation. If the demand line were **DD''**, there would be no point in banning child labour, because there would be none. For the result of interest to apply, the labour demand function (hence, the production technology) must be such, that

$$\underline{w} \leq F'(2 + \gamma n) < \bar{w} \leq F'(2). \quad (3)$$

That is rather restrictive. Even a moderately elastic demand for labour function would in fact make a ban not self-enforcing. A sufficiently elastic one would make it either unenforceable, or irrelevant.

---

<sup>4</sup>Above the subsistence level of family consumption, parental preferences are lexicographic.

<sup>5</sup>Having assumed an equal number of families and firms, the quantity of labour demanded, and the quantity of labour supplied by the market are divided by that number.

### 3 An alternative formulation

Let us now examine a relatively more conventional setting, where children have access to education, and education raises the expected wage rate. For simplicity, we make the "equivalence-scale" assumption that the children's consumption is proportional to the parents'. There are two dates, indexed  $t = 1, 2$ . At date 1, there is a large number of firms, and an equally large number of families consisting of the parental couple (who do not live to date 2), and  $n$  school-age children (who will be adults at date 2). The representative couple take decisions affecting the family's date-1 consumption, denoted by  $c$ , and each of their children's date-2 income, denoted by  $y$ . The representative firm maximizes(1).

The date-1 wage rate, denoted by  $w_1$ , is known with certainty before parents take decisions. If parents invest in their children's education, the latter's date-2 wage rate, denoted by  $w_2$ , will be equal to  $w^S$  with probability  $\pi(e)$ , or to  $w^U < w^S$  with probability  $1 - \pi(e)$ , where  $e$  is the amount of adult-equivalent time that the child spends in education at date 1. The probability function  $\pi(\cdot)$  is increasing and concave, with  $\pi(0) = 0$ . Given that, for  $e = 0$ ,  $w_2 = w^U$  with certainty, we refer to  $w^U$  as the unskilled, and  $w^S$  as the skilled wage rate. A child's education has an opportunity-cost  $w_1 e$ , proportional to  $e$ , and an out-of-pocket cost  $p$  including the tuition fee and the cost of didactic material. At each date, each adult is endowed with one unit of time, and each child with  $\gamma$  units of adult-equivalent time. At date 1, the family supplies 2 units of adult labour, and

$$(\gamma - e)n \geq 0 \tag{4}$$

units of adult-equivalent labour.

Let  $s$  denote the amount saved by the parents at date 1 for each of their  $n$  children. As explained in Sec. 1, , so that, at date 2,

$$y = w_2 + rs, \tag{5}$$

where  $r$  is the interest factor practiced to families. As explained in Sec. 1,  $s$  cannot be negative, because parents cannot commit their children to honour a debt incurred on their behalf,

$$ns \geq 0. \tag{6}$$

Using (5), we represent parental preferences by the expected utility function

$$EU = u_1(c) + n \{ \pi(e) u_2(w^S + rs) + [1 - \pi(e)] u_2(w^U + rs) \}, \tag{7}$$

where  $u_t(\cdot)$  is increasing and concave (implying risk aversion),<sup>6</sup> with  $u'_t(2\bar{w}) = \infty$ .

If the parents intend to send their children to school, they will pay  $np$ , and choose  $(c, e, s)$  so as to maximize (7) subject to (4) and (6),<sup>7</sup> and to the date-1 budget constraint

$$c + n(p + s) = w_1 [2 + n(\gamma - e)]. \quad (8)$$

Otherwise, they will not pay  $np$ . Then,  $e$  is zero,  $w_2$  will be  $w^U$  for certain, and (7) reduces to

$$U = u_1(c) + nu_2(w^U + rs). \quad (9)$$

The constraints are now (6), and (8) with  $e = p = 0$ .

The maximized value of (7) subject to (4), (6) and (8), denoted by  $v(w_1, p)$ , is an increasing function of  $w_1$ , and a decreasing function of  $p$ . That of (9) subject to (6) and (8), denoted by  $z(w_1)$ , is an increasing function of  $w_1$  alone. Given that, as  $w_1$  increases, the value of  $v(w_1, p)$  starts from a lower level, but rises faster than that of  $z(w_1)$ , there exists a threshold value of  $w_1$ , defined by

$$v(\tilde{w}, p) = z(\tilde{w}),$$

such that parents will enroll their children at school if and only if

$$w_1 \geq \tilde{w}. \quad (10)$$

Below  $\tilde{w}$ , children work full time. At or above  $\tilde{w}$ , children receive some (not necessarily full-time) education.

We show in Appendix that, either  $s$  is positive and the Marginal Rate of Substitution of  $c$  for  $y$ ,

$$MRS := \frac{u'_1(w_1 [2 + (\gamma - e)n] - (p + s)n)}{\pi(e) u'_2(w^S + rs) + [1 - \pi(e)] u'_2(w^U + rs)}, \quad (11)$$

is equated to  $r$ , or  $s$  is zero and  $MRS$  higher than  $r$ . The supply of labour has an adult component equal to 2, and a child component dependent on  $w_1$ . For  $w_1 \leq \tilde{w}$ , child labour is equal to  $\gamma n$ . For  $w_1 \geq \tilde{w}$ , either child labour is positive, in which case the expected Marginal Return to Education,

$$MRE := \pi'(e) \frac{u_2(w^S + rs) - u_2(w^U + rs)}{u'_1(w_1 [2 + (\gamma - e)n] - (p + s)n)}, \quad (12)$$

<sup>6</sup>The function  $u_1(\cdot)$  may differ from the function  $u_2(\cdot)$  due to time-preference or imperfect altruism.

<sup>7</sup>We do not need to impose a nonnegativity constraint on  $en$  because, having spent  $np$ , the parents will choose  $e$  positive.

is equated to  $w_1$ , or child labour is zero, in which case  $MRE$  is greater than  $w_1$ .  $MRE$  is a decreasing function of  $e$  and  $s$ .

The labour market equilibrium is illustrated in Fig. 2. The vertical axis measures  $w_1$  and  $MRE$ . The horizontal axis measures  $L$  from left to right, and  $en$  from right to left. The demand for  $L$  is represented by the **DD** line tracing the Marginal Product of this factor. This line is drawn under the assumption that the demand for  $L$  satisfies (3), and thus that a child labour ban would be self-enforcing under Basu-Van assumptions. The **MR** curve traces the  $MRE$  schedule under the assumption that (6) is binding, and  $s$  consequently equal to zero. We assume, in other words, that  $w_1$  cannot be so high that the representative couple will save, at date 1, in order to make their children presents (leave them bequests) at date 2. The couple would borrow on their children's behalf if the latter could be committed to honour the debt, but that is not possible. This assumption is realistic in a developing country context, where wage rates are low, and the vast majority of parents have no assets to peak of. As  $MRE$  is a decreasing function of  $s$ , if (6) were relaxed, the **MR** curve would shift upwards.

The market has two possible equilibria. One is again at point  $C$ , where the wage rate is  $w_1^C < \tilde{w}$  and the children work full time ( $L = 2 + \gamma n$ ). The other is at point  $B$ , where the wage rate, denoted by  $w_1^B$ , is again greater than  $\tilde{w}$ , but children now work some of the time ( $L = L^B < 2 + \gamma n$ ). If the equilibrium is initially at point  $C$ , the market can be nudged towards  $B$  by a relatively mild measure like a temporary ceiling on child labour equal to  $L^B - 2$ . Once the new equilibrium is established, there will be no further need for a ceiling. By contrast, a total ban would not be self-enforcing, because point  $A$  is not an equilibrium (the  $MRE$  is lower than  $w$ ). But, there is an efficient alternative.

Suppose that the government borrows from international money market at date 1, uses the money to subsidize parents, and taxes their children at date 2 to pay the loan back with interests. That will shift the curve representing the  $MRE$  schedule upwards. Given that the number of date-2 taxpayers is large, the date-2 tax base is determined with certainty by the date-1 choice of  $e$ . Suppose that

$$\pi(\gamma) w^S + [1 - \pi(\gamma)] w^U \geq r, \quad (13)$$

so that it is efficient for the children to study full time ( $e = \gamma$ ). If the government chose  $s$  so that  $MRS = r$  for  $e = \gamma$ , the curve representing the  $MRE$  schedule with the policy would cut the **DD** line at or above point  $A$ , like the **MR'** curve shown in Fig. 2. Otherwise, if (13) is not true, the efficient  $e$  is smaller than  $\gamma$ . The curve representing the  $MRE$  schedule if  $s$  is set at its efficient level is then lower than the



**MR'** curve, but still higher than the one which would represent the *MRE* schedule if the parents borrowed (i.e., if they could commit their children to honour the debts incurred on their behalf). That is because, unlike the government, parents face uncertainty. Being risk-averse, they would thus borrow, if they could, less than the efficient amount.

## 4 Asymmetric information

Throughout the developing world, a large part of the work that children do is done within the children's own families,<sup>8</sup> where it is not observable by the government. Let  $f$  denote the amount of (adult-equivalent) time that a child spends in this activity, so that the nonnegativity constraint on the children's supply of market labour must now be written as

$$(\gamma - e - f)n \geq 0. \quad (14)$$

An analogous constraint applies to  $fn$ ,

$$fn \geq 0. \quad (15)$$

The date-1 budget constraint is now

$$c_1 + (p + s)n = w_1 [2 + n(\gamma - e - f)] + G(fn), \quad (16)$$

where  $G(fn)$  is the income that the children produce by working within the family, and  $G(\cdot)$  is another production function, increasing and concave like  $F(\cdot)$ . There is again a threshold value of  $w_1$ , call it  $\hat{w}$ , such that parents will enroll their children at school if and only if  $w_1 \geq \hat{w}$ . Given that the family has now an additional source of income, independent of  $w_1$ ,  $\hat{w}$  will be no higher than  $\tilde{w}$ .

If children are enrolled at school, the representative couple choose  $e$ ,  $f$  and  $s$  so as to maximize (7), subject to (6),(14), (15) and (16). If children are not enrolled,  $e = p = 0$ , and (7) reduces to (??). We show in Appendix that, as in the model without asymmetric information of the last section, if  $s$  were positive, the Marginal Rate of Substitution of  $c$  for  $y$ , now given by

$$MRS := \frac{u'_1([2 + (\gamma - e - f)n]w_1 + G(fn) - (p + s)n)}{\pi(e)u'_2(w^S) + [1 - \pi(e)]u'_2(w^U)}, \quad (17)$$

would be equated to  $r$ . Otherwise, if  $s$  is zero,  $MRS$  will be greater than  $r$ . Additionally, if children are enrolled at school, either they work

---

<sup>8</sup>See Cigno and Rosati (2005, Ch. 2).

(part time, or there would be no point in enrolling), and the expected Marginal Return to Education, now given by

$$MRE := \frac{\pi'(e) [u_2(w^S + rs) - u_2(w^U + rs)]}{u_1'(w_1 [2 + n(\gamma - e - f)] + G(fn) - n(p + s))}, \quad (18)$$

is equated to the higher of  $w_1$  and  $G'(fn)$ , or they do not work at all, and  $MRE$  is greater than both  $w_1$  and  $G'(fn)$ .

The introduction of a second form of child labour increases the number of equilibrium types. Some of them are illustrated in Fig. 3. The horizontal axis measures market and family work from left to right, time in education from right to left. The vertical axis measures the marginal return to the different time uses. The  $F'(L)$  schedule is again represented by the **DD** line. The  $G'(fn)$  schedule is represented by the **MP** line. The latter is drawn under the assumption that the Marginal Product of family work starts from a higher level, but falls faster than the Marginal Product of market work (we discuss what happens if the opposite is true). The  $MRE$  schedule is again represented by the **MR** curve. The market has two possible equilibria. One is again at point  $C$ , where the wage rate is  $w_1^C$ , lower than  $\hat{w}$ , but higher than the Marginal Product of family work. At this point, children work full time in the market. The other is at  $B$ , where the Marginal Product of family work is higher than the wage rate. At this equilibrium, children spend part of their time working in the family, and the rest studying. The wage rate is still  $w_1^A$ .

A ban on child labour leaving  $B$  as the only possible equilibrium would be only apparently self-enforcing, because children would go on working. If we switch the **DD** and **MP** labels, so that the former is now steeper than the latter, we find that, at point  $B$ , children spend part of their time working in the market, and the rest studying. A ban would then be actually counterproductive, because it would leave point  $C$ , where the Marginal Product of family work is greater than  $MRE$ , and children work within their families more than they would have done in the market, as the only possible equilibrium. As in the model without informational asymmetry, the efficient alternative is for the government to borrow on the international credit market, subsidize parents at date 1, and tax their children at date 2 to pay back the loan.

## 5 Discussion

We set out to investigate whether the proposition in Basu and Van (1998), that a ban on child labour may be self-enforcing if, above the subsistence level, no amount of current or future consumption can compensate parents for the disutility of child labour, remains true if education

is available, and the disutility of child labour can be compensated by the expected utility of future wages. We found that a partial ban (a ceiling higher than zero) may nudge the market from an equilibrium with more, to one with less child labour, but a total ban is never self-enforcing. If some of the work children do is not observable by the government because it takes place within the family, a ban may be only apparently self-enforcing in the sense that it would induce children to work in their families instead of the market. It may even be counterproductive in the sense that the children would work more than without the ban.

As demonstrated by Baland and Robinson (2000), education may be inefficiently low, and child labour inefficiently high, because children cannot commit to pay back a loan. As demonstrated by Pouliot (2006), education may be inefficiently low also because, where parents are concerned, education is a risky investment, and parents are risk-averse. We have shown that, as the government faces no uncertainty, because the number of taxpayers is large and the tax base consequently certain, and can exact repayment in the form of a tax, an alternative to a partial or total ban on child labour is for the government to borrow from the international credit market, use the loan to subsidize parents, and tax their children's future wages to pay the loan back with interests. That will reduce child labour, and raise education, to the efficient level. This policy gets round both the Baland-Robinson and the Pouliot problem. There remains the problem, raised in Dessy and Pallage (2001), that the Marginal Product of market labour may be lower than it would be if firms and families could coordinate their technological and educational investments.

Cigno (1993, 2006) demonstrates the possible existence (in a three overlapping generations framework) of a self-enforcing, renegotiation-proof family norm, such that adults will transfer a specified amount of income to their elderly parents and young children.<sup>9</sup> Such a norm gets round the problem that children cannot commit to pay back a debt incurred on their behalf, but not the problem that parents cannot borrow on their children's behalf, because the implicit credit parents acquire towards their children by fulfilling their family obligations cannot be legally passed on to a third party. The government does not face this restriction, because it can claw back any subsidy it gives the parents by taxing their children. Therefore, the government can effectively do what the parents cannot, borrow on the children's behalf to pay for educational investment. That is an instance where "specialized enforcement" (in the case in point, by the government) in the sense of Acemoglu and Wolitzky

---

<sup>9</sup>Cigno et al. (2021) demonstrate the persistence of such a rule in the face of mixed marriages (unions of individuals with different characteristics).

(2020) beats community (in the present case, family) enforcement.

## 6 References

Acemoglu, Daron and Alexander Wolitzky (2020), "Sustaining cooperation: community enforcement versus specialized enforcement", *Journal of the European Economic Association* 18(2), 1078-1122

Baland, Jean-Marie and James A. Robinson (2000), "Is child labor inefficient?", *Journal of Political Economy* 108, 663-679

Basu, Kaushik (2002), "A note on multiple general equilibria with child labor", *Economics Letters* 74:301-308

————— (2011), *Beyond the Invisible Hand*, Princeton and Oxford: Princeton University Press

————— and Pham Hoang Van (1998), "The economics of child labor", *American Economic Review* 88:412-427

Cigno, Alessandro (1993), "Intergenerational transfers without altruism: family, market and state", *European Journal of Political Economy* 9, 505-518

————— (2006), "A constitutional theory of the family", *Journal of Population Economics* 19, 259-283

—————, Alessandro Gioffré and Annalisa Luporini (2021), "Evolution of individual preferences and persistence of family rules," *Review of Economics of the Household* 19, 935-958

————— and Furio C. Rosati (2005), *The Economics of Child Labour*, Oxford and New York: Oxford University Press

Dessy, Sylvain E. and Stéphane Pallage (2001), "Child labor and coordination failures", *Journal of Development Economics* 65, 469-476

Pouliot, William (2006), "Introducing uncertainty into Baland and Robinson's model of child labour", *Journal of Development Economics* 79 (1), 264-272

## 7 Appendix

### 7.1 All child labour is observable

If (10) holds, parents choose  $(e, s)$  so as to maximize the Lagrangian expression obtained by substituting (8) into (7), and appending (4) and (6),

$$E\Lambda = u_1(w_1[2 + n(\gamma - e)] - n(p + s)) + n\{\pi(e)u_2(w^S + rs) + [1 - \pi(e)]u_2(w^U + rs)\} + \lambda(\gamma - e)$$

where  $\lambda$  is the Lagrange-multiplier of (4), and  $\mu$  that of (6). The FOCs,

$$-nw_1u_1'(w_1[2 + (\gamma - e)n] - n(p + s)) + n\pi'(e)[u_2(w^S + rs) - u_2(w^U + rs)] - \lambda n = 0 \quad (19)$$

and

$$-nu'_1(w_1[2 + (\gamma - e)n] - (p + s)n) + nr\{\pi(e)u'_2(w^S + rs) + [1 - \pi(e)]u'_2(w^U + rs)\} + \mu n = 0, \quad (20)$$

may be re-written as

$$MRE - \frac{\lambda}{u'_1(w_1[2 + (\gamma - e)n] - (p + s)n)} = w_1 \quad (21)$$

and

$$MRS = r + \frac{\mu}{\pi(e)u'_2(w^S + rs) + [1 - \pi(e)]u'_2(w^U + rs)}. \quad (22)$$

For complementary slackness, either an inequality constraint is binding and its multiplier positive, or it is slack and its multiplier equal to zero. Therefore, either  $(\gamma - e)n > 0$  and  $MRE = w_1$ , or  $(\gamma - e)n = 0$  and  $MRE > w_1$ . By the same principle, either  $ns > 0$  and  $MRS = r$ , or  $ns = 0$  and  $MRS > r$ .

The partial derivatives of  $MRE = H(e, s)$ ,

$$H_e = \pi''(e) \frac{u_2(w^S + rs) - u_2(w^U + rs)}{u'_1(w_1[2 + (\gamma - e - f)n] - (p + s)n)} + u''_1(w_1[2 + (\gamma - e)n] - n(p + s)) \frac{\pi'(e)[u_2(w^S + rs) - u_2(w^U + rs)]}{\{u'_1(w_1[2 + (\gamma - e)n] - (p + s)n)\}^2}$$

and

$$H_s = \pi'(e) \left[ r \frac{u'_2(w^S + rs) - u'_2(w^U + rs)}{u'_1(w_1[2 + (\gamma - e)n] - (p + s)n)} + nu''_1(w_1[2 + (\gamma - e)n] - (p + s)n) \frac{u_2(w^S + rs) - u_2(w^U + rs)}{\{u'_1(w_1[2 + (\gamma - e)n] - (p + s)n)\}^2} \right]$$

are both negative for concavity of  $\pi(\cdot)$  and  $u_t(\cdot)$ .

If (10) does not hold,  $p = e = 0$ . Parents then choose  $s$  so as to maximize the Lagrangian expression obtained by substituting (??) into (9), and appending (6),

$$\Lambda = u_1(w_1(2 + n\gamma) - ns) + nu_2(w^U + rs) + \mu ns. \quad (23)$$

The FOC,

$$-nu'_1(w_1(2 + n\gamma) - ns) + nru'_2(w^U + rs) + \mu n = 0, \quad (24)$$

may be re-written as

$$MRS = r + \frac{\mu}{u'_2(w^U + rs)}. \quad (25)$$

Hence, it remains true that either  $sn > 0$  and  $MRS = r$ , or  $sn = 0$  and  $MRS > r$ .

## 7.2 Not all child labour is observable

If children are enrolled at school, the representative couple choose  $e$ ,  $f$  and  $s$  so as to maximize the Lagrangean expression obtained by substituting (16) into (7), and appending (14), (6) and (15),

$$E\Lambda = u_1 (w_1 [2 + n(\gamma - e - f)] + G(nf) - n(p + s)) + n \{ \pi(e) u_2 (w^S + rs) + [1 - \pi(e)] u_2 (w^U + rs) \} - \lambda n \quad (26)$$

where  $\varphi$  is the multiplier of (15). The FOCs are

$$-w_1 n u_1' (w_1 [2 + n(\gamma - e - f)] + G(nf) - n(p + s)) + n \pi'(e) \{ u_2 (w^S + rs) - u_2 (w^U + rs) \} - \lambda n = 0 \quad (27)$$

$$[-w_1 + G'(nf)] n u_1' (w_1 [2 + n(\gamma - e - f)] + G(nf) - n(p + s)) + n(\varphi - \lambda) n = 0 \quad (28)$$

and

$$-n u_1' (w_1 [2 + n(\gamma - e - f)] + G(nf) - n(p + s)) + n r \{ \pi(e) u_2' (w^S + rs) + [1 - \pi(e)] u_2' (w^U + rs) \} = 0 \quad (29)$$

Conditions (27) and (29) may be re-written as

$$MRE = w_1 + \frac{\lambda}{u_1' (w_1 [2 + n(\gamma - e - f)] + G(nf) - n(p + s))} -$$

and

$$MRS = r + \frac{\mu}{\pi(e) u_2' (w^S + rs) + [1 - \pi(e)] u_2' (w^U + rs)}. \quad (30)$$

For complementary slackness, therefore, in light of (27), either  $(\gamma - e - f) n > 0$  and  $MRE = w_1$ , or  $(\gamma - e - f) n = 0$  and  $MRE > w_1$ . Similarly, in light of (29), either  $sn > 0$  and  $MRS = r$ , or  $sn = 0$  and  $MRS > r$ . By the same principle, in light of (28), as either or both  $(\gamma - e - f) n$  and  $fn$  can be positive or zero,  $G'(fn)$  may be greater than, equal to, or smaller than  $w_1$ .

If children are not enrolled at school, parents choose  $(f, s)$  so as to maximize the Lagrangean expression obtained substituting (16) into (9), and appending (14), (6) and (15) with  $p = e = 0$ ,

$$\Lambda = u_1 (w_1 [2 + n(\gamma - f)] + G(fn) - ns) + n u_2 (w^U + rs) + \lambda (\gamma - f) n + \mu ns + \varphi fn. \quad (31)$$

The FOCs are now

$$n [-w_1 + G'(fn)] u_1' (w_1 [2 + n(\gamma - f)] + G(fn) - ns) + (\varphi - \lambda) n = 0 \quad (32)$$

and

$$-n u_1' (w_1 [2 + n(\gamma - f)] + G(fn) - ns) + n r u_2' (w^U + rs) + \mu n = 0. \quad (33)$$

Except for the fact that  $e$  is held at zero, the solution has the same properties as if children were enrolled at school.

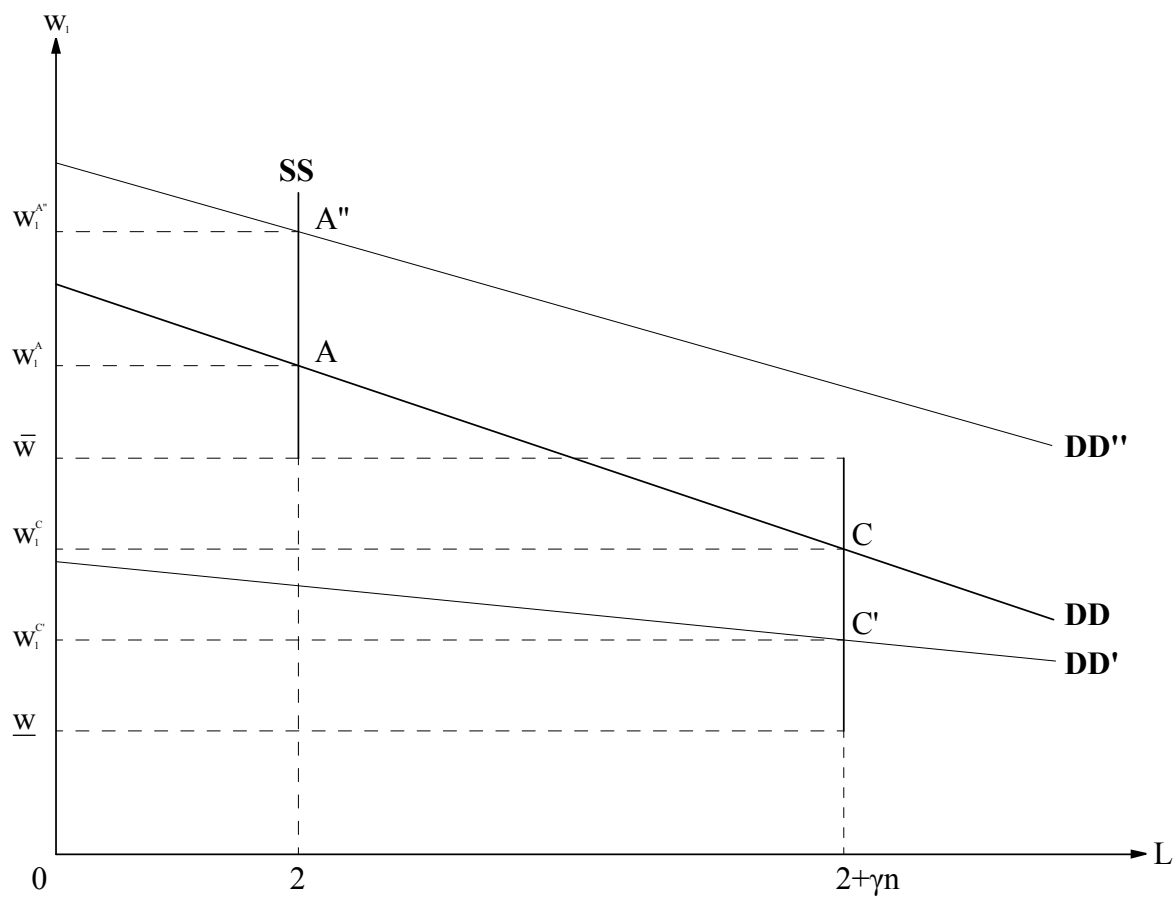


Fig. 1

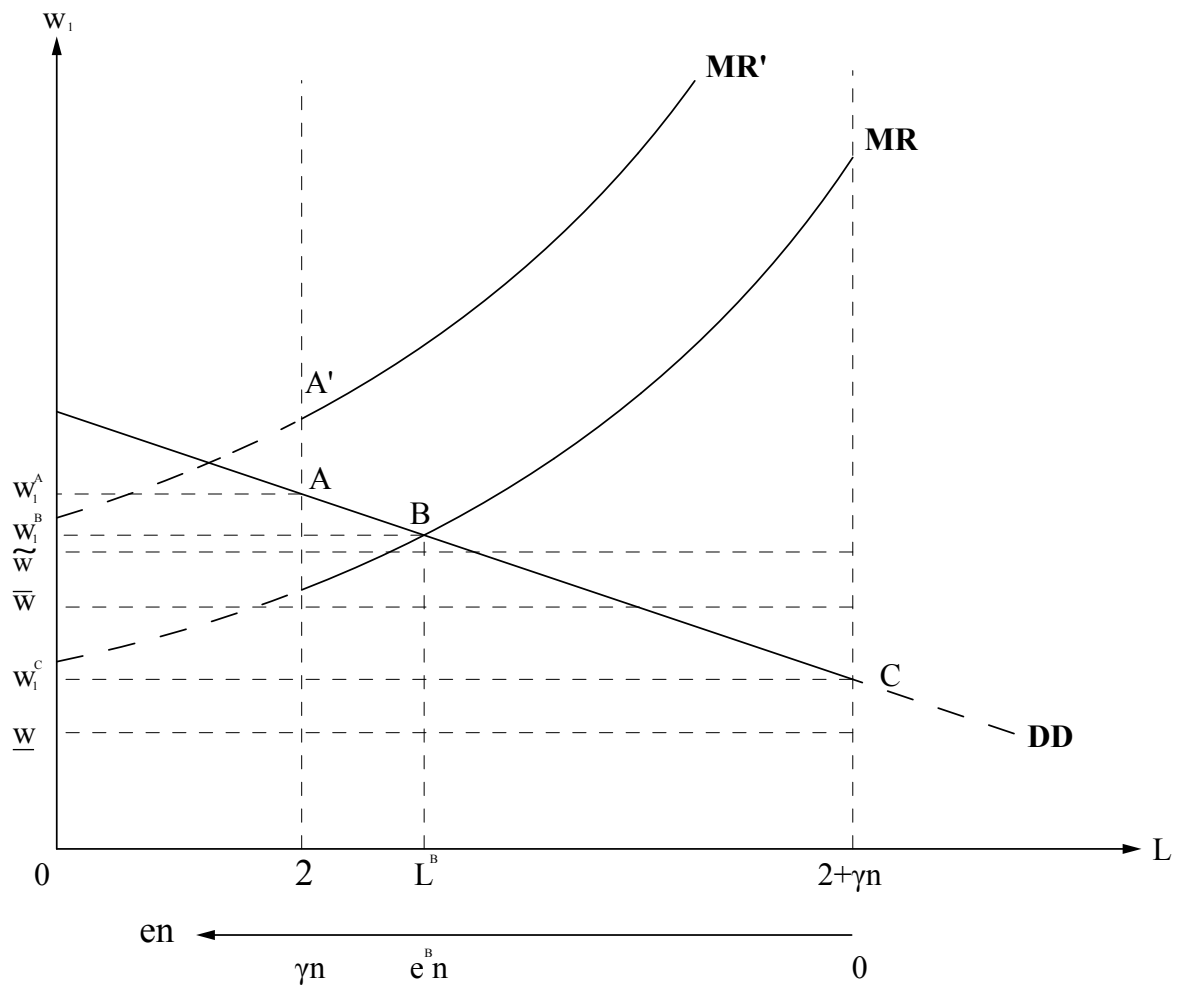


Fig. 2



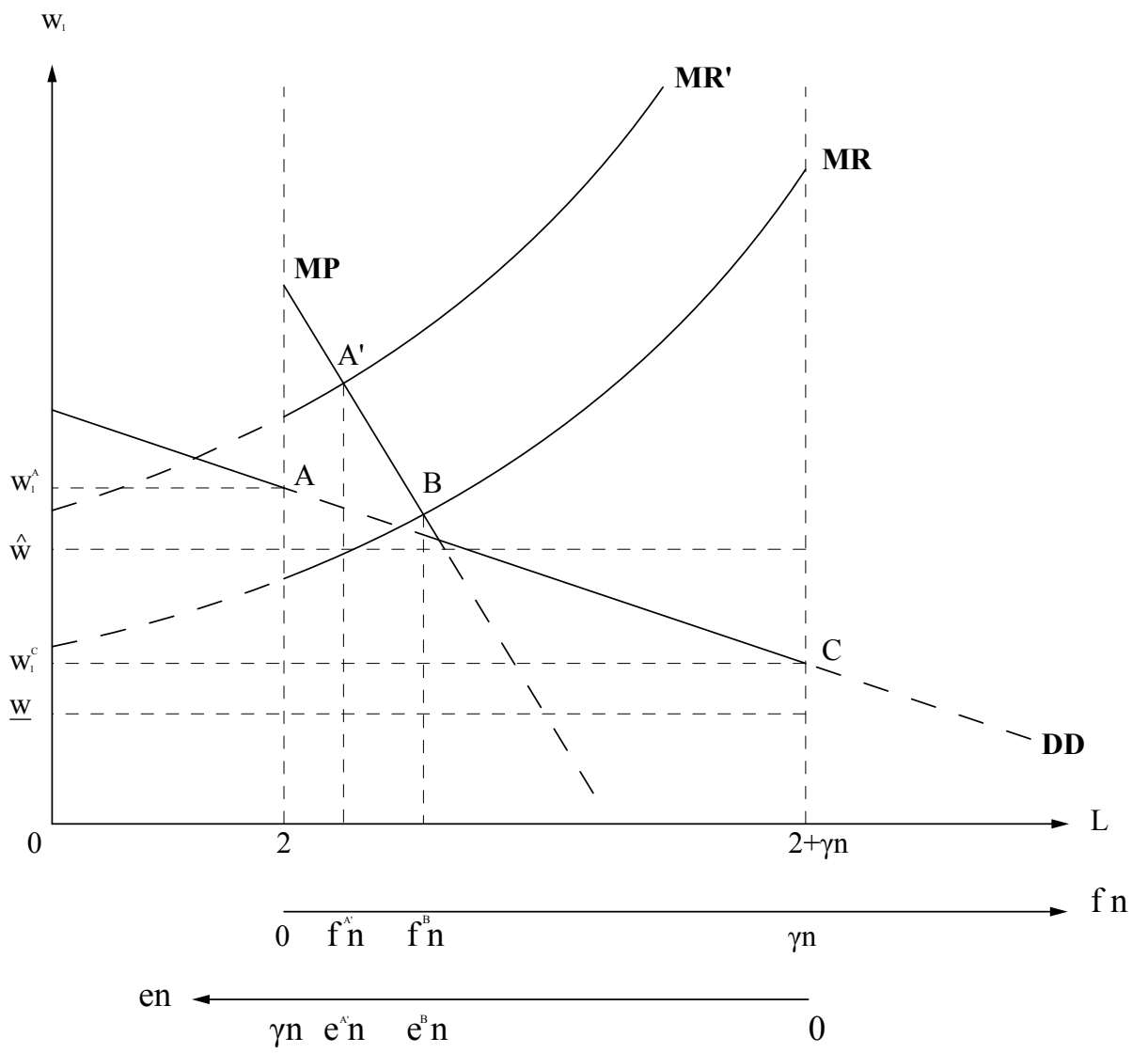


Fig. 3