Should We Trust the Empirical Evidence from Present Value Models of the Current Account?

Benoît Mercereau  
Sinopia Asset Management, Paris

Jacques Miniane  
International Monetary Fund, Washington

Abstract
The present value model of the current account has been very popular, as it provides an optimal benchmark to which actual current account series have often been compared. We show why persistence in observed current account data makes the estimated optimal series very sensitive to small-sample estimation error, making it almost impossible to determine whether the consumption-smoothing current account tracks the actual current account closely, or not closely at all. Moreover, the standard Wald test of the model will falsely accept or reject the model with substantial probability. Monte Carlo simulations and estimations using annual and quarterly data from five OECD countries strongly support our predictions. In particular, we conclude that two important consensus results in the literature – that the optimal series is highly correlated with the actual series, but substantially less volatile – are not statistically robust.

JEL: C11, C52, F32, F41  
Keywords: Current account; present value model; model evaluation

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Correspondence: Jacques Miniane, jminiane@imf.org.  
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1 Introduction

The intertemporal approach to the current account first popularized by Sachs (1981) views net accumulation of foreign assets as a way for domestic residents to smooth consumption intertemporally in the face of idiosyncratic income shocks.1 The intertemporal approach has been very popular over the last two decades. Under some simplifying assumptions and using a methodology developed by Campbell and Shiller (1987) in a different context, one can estimate the current account series that would have been optimal from a consumption smoothing perspective.

Starting with Sheffrin and Woo (1990), economists have been eager to compare actual current account data with this optimal benchmark.2 Several studies conducted at the International Monetary Fund have estimated this benchmark to assess the optimality of emerging economies’ external borrowing (see, e.g., Ostry, 1997 or Callen and Cashin, 2002). Numerous academic papers have looked at both emerging and industrial countries, and a consensus has emerged from this literature: while the model-predicted current account is positively correlated with the actual series, the latter is substantially more volatile, leading statistical tests to reject the model.3 Positive correlation has been interpreted as evidence that consumption-smoothing plays a role in the dynamics of the current account (Obstfeld and Rogoff, 1996). But given that the present value model assumes full capital mobility, the finding of excess current account volatility has been used as evidence against Feldstein and Horioka’s famous proposition of limited international capital mobility (see, e.g., Gosh, 1995). More recent papers have tried to “augment” the model in several directions to generate extra predicted volatility. Bergin and Sheffrin (2000) show that allowing for variable real exchange rates and interest rates improves the fit of the model for Australian, Canadian, and British data. Gruber (2004) generates extra volatility in the predicted series by way of habit formation and excess smoothness in consumption. Nason and Rogers (2006) test competing additions to the model and find that exogenous shocks to the world real interest rate best reconcile the extended model with Canadian data.

Our paper shows that none of the key results in the literature rests on robust statistical grounds. Specifically, we show that:

1) the dominant test in this literature - the Wald test of the cross-equation restrictions of the model - has very poor small-sample coverage, and hence inference based on this test can be very misleading.4 As we show in detail, the Wald test relies on a linear approximation of a variance-covariance matrix which is likely to be very imprecise in

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1 The intertemporal approach to the current account is surveyed in Obstfeld and Rogoff (1995, 1996).


3 Exceptions include Ghosh and Ostry (1995) who find that the model fits many developing countries.

4 Out of the 15 papers that we cite which test the present value model of the current account, 10 report the (non-linear) Wald test only, 1 the F-test only, and 4 report both. None of these papers follow the simple linearization of the Wald test that we use in this paper, and which has the advantage of being sufficient while not relying on the Delta-method linear approximation.
short samples when the observed current account is persistent, as it typically is.\textsuperscript{5,6} In our Monte Carlo simulations, Wald tests can erroneously reject the model at 95% confidence with a probability ranging between 11.7% and 28.3% depending on the data generating process assumed, implying an average rate of (false) rejection across DGP three and half times the proper rate. When we take the tests to quarterly and annual data for five different countries, we find that in four out of the ten samples the Wald tests accepts (rejects) the model when other tests with good coverage – a well-known F-test and a simple linear Wald test – reject (accept) the model.

2) persistence in actual current account series makes the model-predicted series excessively sensitive to small-sample estimation error. As such, the inference drawn in the literature from comparing actual and predicted series cannot and should not be considered robust. For instance, our estimated confidence bands around the optimal current account are very wide in all data samples, and easily encompass the observed current account.\textsuperscript{7} Empirical distributions of the variance ratio between actual and predicted current account that incorporate estimation uncertainty are strikingly dispersed in all our ten samples of data, and indicate substantial probability that the actual current account is either several times more or several times less volatile than the optimal series. Distributions of the correlation coefficient between the actual and predicted series are also very dispersed, with often substantial probability that the correlation is negative. These findings occur regardless of whether the more robust tests have accepted or rejected the model.

Our work is directly related to other papers. Bekaert and Hodrick (2001) show that Wald tests have the worst small-sample coverage of all the tests they consider for present value models of the term structure of interest rates, while Bouakez and Kano (2008) use Monte Carlo simulations based on UK quarterly data to document the size distortions of the Wald test of the present value model of the current account.\textsuperscript{8} Our paper goes one step further however, by explaining why the Wald test has such size distortions in the context of present value models. Beyond issues with the Wald test, we believe ours is the first paper to warn against and quantify the large small-sample uncertainty of the model-predicted current account and the concomitant unrobustness of the literature’s inference based on these series.

\textsuperscript{5} We note that when we talk about a “persistent” current account, we do not necessarily mean a non-stationary one. A stationary current account will be considered persistent if its process of mean reversion is slow. Clearly, if the current account is integrated of order one or higher then the model cannot be a correct representation of the data. What is crucial for our analysis is that the small sample problems we report in the paper can occur even if the current account is stationary but persistent, and there is no reason why a persistent but stationary current account cannot be model-consistent.

\textsuperscript{6} Elliot (1998) shows that cointegration tests have large size distortions if one of the variables is stationary but close to a unit root. While his paper is indirectly related to ours, the Wald test used in the present value literature is not a test of restrictions on a cointegrating vector but a test on the coefficients of a VAR, where one of the variables in the standard VAR is obviously stationary (income changes) and the other is persistent and may or may not have a unit root.

\textsuperscript{7} As we will see, few papers in the literature report measures of estimation uncertainty such as confidence intervals around the estimated series, or the standard error of the correlation and variance ratio estimates between actual and model-predicted series. Moreover, we will show that some of the methods used to account for this uncertainty may be inappropriate when the current account is persistent.

\textsuperscript{8} Bouakez and Kano (2008) was under review for publication in parallel to our paper, hence we only became aware of their work after we had submitted ours.
The paper is organized as follows. Section 2 explains how the present value model is tested and exposes the problems of the empirical methodology under current account persistence. Section 3 and Section 4 present the simulations and the empirical results based on OECD data. Section 5 concludes.

2 Assessing the Present Value Model

2.1 The Present Value Model

Let’s define $X_t$ as the actual current account, and $X_{p,t}$ as the current account predicted by the present value model. In the model’s simplest form, assuming quadratic utility, a constant real return on a single internationally traded bond, and a discount factor equal to the inverse of the (gross) return, the model-predicted current account is given by:

$$X_{p,t} = -E_t \sum_{i=t}^{\infty} \left( \frac{1}{1+r} \right)^i \left[ Y_t - Y_{t-i} \right]$$  

where $r$ is the (constant) real interest rate and $Y$ is income net of government spending and investment. Equation (1) characterizes the current account in an economy where a representative agent smooths her consumption by “saving for a rainy day.” That is, permanent shocks to income have no effect on the current account. Positive transitory shocks raise it on impact. Anticipated future income increases lower the current account.

2.2 Assessing the Present Value Model

A methodology developed by Campbell and Shiller (1987) is used to estimate the optimal current account benchmark given by equation (1). This methodology uses a VAR in income changes and the current account to estimate the expected income declines on the right hand of equation (1). Since the model implies that the current account should contain all the relevant information for agents to form their expectations of future income declines, the current account should be a sufficient variable in the VAR under the null to estimate these expected future income declines.\(^9\) Thus, following the literature we estimate the following $l$-order VAR:

$$Z_t = BZ_{t-1} + u_t$$

where $Z_t$ is the $2l$ vector containing actual data on income changes and current account ($Z_t = [\Delta Y_t, \ldots, \Delta Y_{t-l+1}, X_{t}, \ldots, X_{t-l+1}]$); $B$ is the companion matrix of the VAR; and $u_t$ is a $2l$ column vector of zero-mean homoskedastic errors.

In this setup, the expected income change $n$ periods ahead is given by:

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\(^9\) Kasa (2003) warns against plausible income processes for which the model-consistent current account may not reflect all relevant information to accurately forecast income changes, and hence to estimate optimal series. As the author acknowledges, however, such processes are plausible but very hard to distinguish empirically from processes for which the optimal series can be derived with no problem.
\[ E_t \Delta Y_{t+1} = VB^*Z_t \]

where \( V \) is the \( 2l \) row vector \([1 \ 0 \ 0 \ ... \ 0]\). Plugging these expressions into equation (1) yields the following expression for the current account predicted by the model:

\[ X_{p,t} = KZ_t \]  

(2)

where

\[ K = \frac{VB}{1+r} \left[ I - \frac{B}{1+r} \right]^{-1} \]

We then obtain the model’s estimated prediction \( \hat{X}_{p,t} \) by replacing \( B \) in (2) with its empirical estimate \( \hat{B} \). \( \hat{X}_{p,t} \) is the estimation of the country’s optimal current account from a consumption-smoothing perspective. We can compare the actual current account to this benchmark. This can be done in several ways.

First, we can simply plot the actual and predicted current account paths. The literature has drawn inference by comparing these paths. For example, some authors use this approach to try to identify the shortcomings of a consumption-smoothing approach to the current account. Sheffrin and Woo (1990) note that the model underestimates UK current account deficits generated by the first oil shock. Ghosh (1995) draws similar conclusions for Japan following the second oil shock. Also, the aforementioned IMF studies use this estimated benchmark to assess if a country’s external borrowing is excessive.

Second, we can informally assess the model by computing the correlation between the estimated and the actual current account, as well as the ratio of their (in sample) volatility. For example, the literature has often emphasized that actual current account series are typically more volatile than the model’s predictions (see among others Ghosh, 1995, Obstfeld and Rogoff, 1996, Nason and Rogers, 2006, and Gruber, 2004). In fact, Ghosh (1995) uses excess current account volatility as evidence against Feldstein and Horioka’s claim that international capital markets are not highly integrated. Also, the correlation between actual and predicted series tends to be quite high (see, e.g., Obstfeld and Rogoff, 1996 and Nason and Rogers, 2006), and some authors have taken this as evidence that consumption smoothing plays a role in the dynamics of the current account.

Third, we can run a formal Wald test of the model. Testing whether the model-predicted current account \( X_{p,t} \) and the actual current account \( X_t \) are equal is akin to testing whether the vector \( K \) in equation (2) equals the \( 2l \) vector \( Q \), where all elements of \( Q \) are zero except for the \((l+1)^{th}\) element, which equals one.\(^{10}\) To perform this test, we need an estimate of the variance-covariance of \( K \). Since \( K \) is a non-linear function of the VAR parameters, researchers approximate its variance-covariance by \([JVJ']\), where \( V \) is the variance-covariance matrix of the VAR parameters and \( J \) is the Jacobian of \( K \). This is the Delta method linear approximation of the variance-covariance. Then, the statistic:

\[^{10}\text{More precisely, this is a joint test of the model and of the assumption that the data generating process of income changes and the current account is given by the unrestricted VAR.}\]
\[
W = (K - Q) \ast \left[ JW J^\prime \right]^{-1} \ast (K - Q)^\prime
\]  
(3)

has an asymptotic chi-square distribution with 2l degrees of freedom.

The literature has sometimes used an alternative test. Let’s define
\[ R_t = X_t - (1 + r)X_{t-1} - \Delta Y_t \text{ and } I_{t-1} \]
as the information set containing all the lagged (ie. t-1 or previous) values of income changes and the current account, as well as the lagged values of any other variable. Since equation (1) implies that \( E_t(R_t \mid I_{t-1}) = 0 \), one can regress \( R_t \) on the variables in \( I_{t-1} \) and do a simple \( F \)-test on the joint nullity of the coefficients of all the regressors.

The \( F \)-test is a necessary but not sufficient test of the model, however. The reason for this was already noted by Campbell and Shiller (1987, p. 1065). While equation (1) defining the current account in the model implies \( E_t(R_t \mid I_{t-1}) = 0 \), the reverse is not true, because \( E_t(R_t \mid I_{t-1}) = 0 \) is consistent with a more general form of equation (1) that includes a “rational bubble”, a random variable \( b_t \) satisfying \( b_t = \delta E_t b_{t+1} \). Also, while both tests have been used in the literature, the non-linear Wald test has been more popular, probably because it focuses on the same equation (equation 2) that allows constructing the optimal path predicted by the model.\(^{11}\)

In short, the Campbell-Shiller methodology has been used to assess whether consumption-smoothing determines the dynamics of the current account, as well as to draw inference on capital mobility and, occasionally, on the optimality of a country’s external borrowing. We now show, however, that the methodology is problematic under near-singularity conditions commonly generated by current account data.

2.3 Problems Under Near-Singularity

From equation (2), we can see that \( CA_{p,j} \) is a linear function of the inverse of the matrix
\[
M = \left[ I - \frac{B}{1 + r} \right].
\]
When \( M \) has at least one eigenvalue close to zero (i.e. when \( B \) has at least one eigenvalue close to \( 1 + r \)), a small error in the estimated VAR parameters translates into potentially very large deviations in the inverse of \( M \) and hence on the model predicted current account (intuitively, one can think of the function \( 1/c \) as \( c \) is close to zero: a small variation in \( c \) translates into a large deviation in \( 1/c \)). The estimated optimal current account is therefore very imprecise, as are its (in sample) volatility and its correlation with the actual current account. In such circumstances, to draw inference from comparing the actual current account to the estimated optimal path as described above is dubious—a point we will illustrate in subsequent sections.

The non-linear Wald test of the model also becomes problematic, leading to false rejection and false acceptance of the model. In the near-singularity region mentioned above, the Delta method greatly distorts this variance-covariance in short samples. To see the problem, assume for simplicity that the element in \( K \) we are trying to test for is proportional to \( \frac{1}{c^2} \), where \( c \) is some parameter to be estimated. Figure 1 shows what

\(^{11}\) Out of the 15 papers that we cite which test the present value model, 10 report the (non-linear) Wald test only, 1 the \( F \)-test only, and 4 report both.
happens when the true value $c_0$ is close to zero. In this example, the small sample estimate $\hat{c}$ falls a bit further from zero but its probability interval still contains the true value. However, the interval $[\hat{K}_{L,\delta}, \hat{K}_{R,\delta}]$ computed by linear approximation will be “too small” given the steepness of the curve and may therefore not include the true value $K_0$. Clearly, the problem arises because the slope of $1/c$ changes rapidly in the neighborhood of 0. Also of note, distortion is a short sample issue since the interval around $\hat{c}$ shrinks as sample size increases. We see from equations (2) and (3) that the reasoning in Figure 2 can be extended to the Chi-square test of the present value model. Under near-singularity, the Delta method would produce a variance-covariance matrix which is “too small” and a $W$ statistic which is “too large”, leading to a false rejection of the model.

Figure 1: False Rejection in the Singularity Region

Note: $c_0$ is the true value of the parameter, $\hat{c}$ the empirical estimate, and $[\hat{c}_L, \hat{c}_R]$ the probability interval around $\hat{c}$. The Delta method yields the probability interval $[\hat{K}_{L,\delta}, \hat{K}_{R,\delta}]$ around $\hat{K}$ rather than the correct $[\hat{K}_L, \hat{K}_R]$.

Figure 2 shows the opposite problem. Here, the estimate $\hat{c}$ falls a bit closer to zero and its interval excludes $c_0$. However, the interval around $\hat{K}$ will be “too large” given the steepness of the curve and will include $K_0$. Translating this to the Chi-square test, the Delta method would produce a variance-covariance matrix which is “too large” and a $W$ statistic which is “too small”, leading to a false acceptance of the model. Figures 2 and 3 also show that the level of distortion in the test is not solely related to the distance to singularity: the width of the confidence interval or, more generally, the level of estimation uncertainty also matter.

While the F-test discussed in the previous section avoids the distortions created by the Delta method, we saw that it is not a sufficient test of the model. However, it is
Figure 2: False Acceptance in the Singularity Region

Note: \( c_0 \) is the true value of the parameter, \( \hat{c} \) the empirical estimate, and \([\hat{c}_L, \hat{c}_R]\) the probability interval around \( \hat{c} \). The Delta method yields the probability interval \([\hat{K}_L, \hat{K}_R]\) around \( \hat{K} \) which includes \( K_0 \) even though \([\hat{c}_L, \hat{c}_R]\) excludes \( c_0 \).

It is possible to obtain a sufficient test that does not rely on the Delta method by simply “linearizing” the non-linear Wald test. Strangely enough, linear Wald tests have not been used in this current account literature, but have been used to test present value models in finance (see for instance Campbell and Shiller, 1989). Rather than testing whether \( K = Q \) as in the standard test, one would simply postmultiply both sides by \( [I - \frac{B}{1+r}] \) and test whether \( \frac{-VB}{1+r} = Q[I - \frac{B}{1+r}] \).

While the \( F \)-test and the linear Wald test of the model do not rely on the problematic linear approximation, it is not straightforward to claim that these tests are more robust than the non-linear Wald test. Indeed, the estimated parameters have only an asymptotic justification. Moreover, if the unit root case is approached with sample size held fixed, the usual OLS intervals become less and less precise in the sense of coverage probability. It is therefore not theoretically impossible that the distortion created by the Delta method luckily offsets (rather than worsens) the other statistical issues. It is reasonable to think that this is unlikely to happen in practice, and our simulation results will confirm that the \( F \) and linear Wald tests have indeed much better coverage in small samples.

The relevant question now is: how common is near-singularity in practice? Unfortunately, the answer is “very common.” Table 1 shows for each country sample one estimated eigenvalue of the VAR companion matrix. These eigenvalues are above 0.9 for all five countries in quarterly data, and in three out of five cases in annual data.
<table>
<thead>
<tr>
<th></th>
<th>Number of Lags</th>
<th>Eigenvalue</th>
<th>F-test (p-value)</th>
<th>Linear Wald test (p-value)</th>
<th>Non-linear Wald test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annual data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium 1953–1998</td>
<td>1</td>
<td>1.01</td>
<td>36.7%*</td>
<td>36.4%*</td>
<td>96.9%</td>
</tr>
<tr>
<td>Canada 1948–2002</td>
<td>1</td>
<td>0.61±0.2i</td>
<td>0.6%</td>
<td>0.9%</td>
<td>27.8%</td>
</tr>
<tr>
<td>Denmark 1966–2002</td>
<td>1</td>
<td>0.96</td>
<td>20%*</td>
<td>22.5%*</td>
<td>94.9%</td>
</tr>
<tr>
<td>Sweden 1950–2002</td>
<td>1</td>
<td>0.93</td>
<td>0.8%</td>
<td>0.7%</td>
<td>89.4%</td>
</tr>
<tr>
<td>United Kingdom 1948–2002</td>
<td>2</td>
<td>0.67</td>
<td>0.1%</td>
<td>0%</td>
<td>2.8%</td>
</tr>
<tr>
<td><strong>Quarterly data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium 1980–1998</td>
<td>3</td>
<td>0.98</td>
<td>43.7%*</td>
<td>10.7%*</td>
<td>0.7%</td>
</tr>
<tr>
<td>Canada 1948–2002</td>
<td>8</td>
<td>0.94</td>
<td>38.1%*</td>
<td>31%*</td>
<td>2%</td>
</tr>
<tr>
<td>Denmark 1988–2002</td>
<td>4</td>
<td>0.93</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Sweden 1990–2002</td>
<td>4</td>
<td>0.97</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>United Kingdom 1955–2002</td>
<td>4</td>
<td>0.94</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Notes:** A star denotes model acceptance by the F-test at the 95 percent confidence level. For each sample the number of lags was selected using the Akaike criterion.

In fact, the eigenvalues are often close to \(1 + r\) which is the critical value for singularity. In our VAR estimations the coefficients on lagged income changes are often small (and insignificant.) It is current account persistence that generates near-singularity in our data.\(^{12}\)

We illustrate the problems discussed in this section in two steps. First, we do Monte-Carlo simulations assuming that the “observed” current account is fully consistent with

\(^{12}\) This does not necessarily mean that the current account is non-stationary. Note from our previous discussion that near-singularity can easily arise with a persistent but stationary current account.
the model. Then, we turn to actual rather than simulated data to show how problematic the methodology is in typical situations faced by practitioners.

3 Simulations

3.1 Simulation Set Up

We use the following Monte-Carlo procedure to test the unrobustness of the literature’s results:

Step 1: we generate data for income changes $\Delta Y$ and for a variable arbitrarily called $G$ (see below for more details on this second variable) using a bivariate VAR. The coefficients used in this bivariate VAR come from the standard bivariate VAR in income changes and the current account, estimated on our quarterly samples of actual data. The shocks to each bivariate VAR process used to generate $\Delta Y$ and $G$ are drawn from a bivariate normal distribution using the variance-covariance matrix of the empirical quarterly VARs. Note, and this is the key point, that we do not use $G$ as our generated current account, even though $G$ should look very similar to observed quarterly current account data given the procedure used in step 1. The reason we do not use $G$ as our simulated current account is simple: we want our simulated current account to be 100% model-consistent, and there is no reason *a priori* that the simulated $G$ should be. And the reason we want our simulated current account data to be fully model-consistent is to insure that the simulated current account and the model-predicted current account estimated from simulated data will track each other very closely. If they don’t track each other, then there is a clear problem with the methodology to estimate model-predicted series (whereas if the simulated current account is not fully model-consistent, then the two series can diverge for no particular methodological reason).

Step 2: we generate our simulated current account $X_{s,t}$ by plugging our simulated data for $\Delta Y$ and $G$ together with the VAR coefficients used in step 1 into equation (2) in the paper. This ensures that the simulated current account equals the present discounted value of all expected income declines, where expectations are consistent with the DGP used to generate the income declines. In other words, our simulated current account is fully model-consistent.

Step 3: we estimate a bivariate VAR on the simulated data on income changes and current account from steps 1 and 2 and then plug these simulated data together with the just estimated VAR coefficients onto equation 2 in the paper, to obtain the model-predicted current account, $X_{p,t}$.

Step 4: we compute the correlation coefficient and variance ratio between $X_{p,t}$ and $X_{s,t}$. As previously mentioned, $X_{p,t}$ and $X_{s,t}$ should track each other closely if the

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13 Bouakez and Kano (2008) use a similar method, but only based on quarterly UK data, whereas we use five distinct quarterly samples (Belgium, Canada, Denmark, Sweden, and UK).

14 This is why we arbitrarily called it $G$. We could have called it something else, but the key is that $G$ is not our simulated current account. The only reason we generate values of $G$ is to be able to generate values of income changes following a DGP similar to what is assumed in the literature.

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methodology to estimate $X_{p,t}$ isn’t flawed, implying that the correlation coefficient and variance ratio should be close to one. We also test whether the model is a good representation of the simulated current account $X_{s,t}$ using the F-test, as well as the linear and non-linear Wald tests. We the econometricians know that the model is a good (in fact, perfect) representation of $X_{s,t}$, so the tests should reject the model with probability $1-\alpha$, where $\alpha$ is the confidence level of the test. Note that, by construction, our simulated current account is stationary and the estimation residuals from step 3 are Gaussian, hence the Wald test is well-defined in the simulations, as are the correlation coefficients and variance ratios.

Step 5: repeat steps 1 – 4 ten thousand times.\(^\text{15}\)

3.2 Simulation Results

Figure 3\(^\text{16}\) shows the distributions of the correlation coefficient and the variance ratio obtained in our five sets of simulations. It is worth emphasizing that the country names as used in this figure and this section identify the different simulations and not the true country data, which is the purpose of our next section.

One expects correlation coefficients to display the smallest dispersion away from a unit value. Correlations are not very demanding tests: all that is required for a high value is that the estimate of the optimal current account move in the same direction as the “actual” (model-consistent) series. Since the estimate of the optimal current account and the “actual” (model consistent) series are both linear combinations of the same variables, one should expect the correlations to be high (this point is further discussed in the next section). However, distributions of the correlation coefficient shown in Figure 3 are a bit dispersed in the case of Belgium and the United Kingdom, and very dispersed in the case of Canada. In the case of Belgium, there is a 9.1% probability that the correlation will be below 0.8. For the UK, this probability jumps to 19.6%, and for Canada to 78.4%. In fact, there is a 12.8% probability for Canada that the correlation will actually be negative.

The distributions of the variance ratio, also shown in Figure 3, show even more starkly the problems with estimating model-predicted series, the distributions being very dispersed for Belgium, Canada, the UK and to a lesser degree Denmark. For Belgium, there is a 24.5% probability that the estimated series is less than half as volatile as the “observed” series, and over 10% probability that it is at least twice as volatile. For Canada, these figures are 10.6% and 32.7% respectively; for the UK, 11.2% and 13.6%. In all three cases, the probability that the variance ratio is between 0.75 and 1.25 is less than 1/3. At the other extreme, this probability is 100% for Sweden. Dispersion as we see for Canada, Belgium, and the UK is startling and makes standard claims in the literature about excess volatility of the actual current account series – or lack of – dubious at best.

\(^\text{15}\) In each repetition, we generate 620 observations and discard the first 500 to eliminate the effect of arbitrary starting values, all set to zero. The resulting 120 quarterly observations are close to the average sample length across the different countries in our sample, and is what one typically finds in the literature.

\(^\text{16}\) See Appendix for Figures 3–16.
We also computed the frequency of (false) model rejection at 95% confidence, using the $F$-test as well as the linear and non-linear Wald tests. Table 2 summarizes the results. For the $F$ and the linear Wald tests, this frequency is always very close to 5%, showing that these tests have good coverage in small samples even though they only have an asymptotic justification. But as predicted by the previous section, the performance of the non-linear Wald test is substantially worse. Rejection probabilities are 11.7% for Belgium, 28.3% for Canada, 15.1% for Denmark, 13% for Sweden, and 16.8% for the UK.\textsuperscript{17} Deviations from the 5% benchmark are not trivial if one considers that the “observed” current account was assumed perfectly model consistent.

To verify that these are small sample issues as argued in the previous section, we redid our simulations assuming five hundred years of quarterly observations instead. Test performance improves significantly, while correlation and variance ratio distributions tighten around one. For practical applications it is fair to ask if the problems would also “go away” with a sample size longer than thirty years but still realistically short. The answer is no. When sample size is set at sixty years of quarterly observations – more than can be expected in the near future for most countries - significant problems persist. Finally, when one assumes a sample size of forty observations instead – the typical size of annual data sets in the literature – all problems are greatly magnified.\textsuperscript{18}

Table 2: Simulations: Probability of Model Rejection at 95% Confidence (in percent)

<table>
<thead>
<tr>
<th></th>
<th>$F$-test</th>
<th>Linear Wald Test</th>
<th>Non-linear Wald Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>5.3</td>
<td>6.2</td>
<td>11.7</td>
</tr>
<tr>
<td>Canada</td>
<td>5.1</td>
<td>7.2</td>
<td>28.3</td>
</tr>
<tr>
<td>Denmark</td>
<td>5.3</td>
<td>6.3</td>
<td>15.1</td>
</tr>
<tr>
<td>Sweden</td>
<td>5.0</td>
<td>6.3</td>
<td>13.0</td>
</tr>
<tr>
<td>UK</td>
<td>4.8</td>
<td>6.3</td>
<td>16.8</td>
</tr>
</tbody>
</table>

4 Empirical Results

We now turn to actual data to show how problematic the methodology is in typical situations faced by practitioners. We use annual and quarterly data from the same five countries as above, which are the ones chosen by Obstfeld and Rogoff (1996) to survey the literature. The data appendix details data construction and estimation issues. As mentioned before, the eigenvalues of the variance-covariance matrix in Table 1 show that near-singularity is a real issue in most country samples considered. In this context, we study each aspect of the Campbell-Shiller methodology as used in this literature.

\textsuperscript{17} Bouakez and Kano (2008) find very similar rejection rates (15.1%) in their simulations based on UK quarterly data covering a similar but not identical sample period.

\textsuperscript{18} We do not present detailed simulation results for all these alternative specifications to be concise. The results can be requested from the authors.
4.1 Tests of the Model

Table 1 gives the $p$-value of the Wald and $F$-tests. We see that the difference between the non-linear Wald and the $F$ and linear Wald tests can be large. The non-linear Wald statistic suggests significance levels that are up to eighty-eight percentage points different from the level given by the $F$ or linear Wald tests. Most strikingly perhaps, in four out of ten cases the non-linear Wald test leads to the inference opposite to that of the $F$- and linear Wald tests at the traditional 95 percent level of confidence (the $F$- and linear Wald tests lead to the same inference in all 10 cases). In particular, the non-linear Wald test always rejects the model with quarterly data while the $F$- and linear Wald tests accept it for two out of five countries (Belgium and Canada). It should also be noted that in three of the four papers cited that use both the $F$ and non-linear Wald tests (Sheffrin and Woo 1992, Cashin and McDermott 1998a, Gruber 2004) the two tests frequently lead to opposite conclusions regarding the validity of the model.

4.2 Graphical Analysis

We noted that the literature has often drawn inference by comparing the paths of the actual and predicted current account during economically significant periods. To evaluate the robustness of such inference, we construct the empirical distribution of the predicted current account. For each country sample, we generate 10,000 draws from the multivariate Normal distribution given by the estimated VAR parameters and their associated variance-covariance matrix.\(^{19}\) For each of these draws we compute the associated $K$ vector and the corresponding predicted current account. Figures 5 and 7 plot the 25th and 97.5th percentiles of the empirical distribution of the predicted current account at each point in time. These confidence bands are typically very wide compared with the actual series, often dramatically so.\(^{20}\) This has important implications. Consider the case of Sheffrin and Woo (1990), who analyze the UK current account after the first oil shock and conclude that actual deficits in the UK did exceed the model predicted series following the first oil shock (see Figures 4 or 6). Yet particularly in quarterly data the confidence bands easily encompass those deficits, showing that the conclusion that the model underestimated the deficits is unwarranted. Also, the imprecision of the estimated optimal current account casts doubt on the conclusions of empirical studies that used this estimated current account as a benchmark to assess the optimality of some emerging countries’ external borrowing (Ostry 1997, Callen and Cashin, 2002).

\(^{19}\) We cannot reject the null of joint normality of the residuals at a 95% confidence level in any of the annual samples. In quarterly data, we can only reject it for Sweden. We do not detect serial correlation in any of the residual series. In the case of the UK quarterly data we generate 5,000 draws for computational reasons.

\(^{20}\) As the referees pointed out to us, the annual data for Belgium has a clear outlier, while the Swedish quarterly data show a seasonal pattern. However, all our results carry through when we reestimate the VAR setting the value of the outlier to 0 in the case of Belgium, or after applying seasonal-adjustment to the Swedish data. If anything, the confidence bands are even wider after these adjustments. For proof, see our responses to the referee reports at: http://www.economics-ejournal.org/economics/discussionpapers/2008-10/?searchterm=miniane
A limited number of papers in the literature have constructed confidence bands around the predicted series (see Cashin and McDermott 1998a and Hall et al. 2001 among others). Their bands are typically much narrower than ours, sometimes an order of magnitude so. Because the main claim in our paper is that the model predicted series is extremely sensitive to small sample estimation error, it is important to understand why our bands are so large. As it turns out, confidence bands in the above cited papers are built using standard bootstrapping techniques, as proposed by Runkle (1987). As Killian (1998) has shown however, the distribution of bootstrap VAR coefficients is biased towards stationarity, possibly severely so. In such a case, the bootstrap distribution of the $M = \left[ I - \frac{B}{1 + r} \right]$ matrix can be biased away from the singularity region, resulting in less extreme values for the vector $K$ and bands that are too narrow. To see this, Figure 8 shows bootstrap bands for our annual data but adjusted for stationarity bias following the method proposed by Killian. These bands are similar to ours: remarkably close in the case of Canada, slightly narrower but still very wide for Sweden and the UK, and wider for Belgium and Denmark. In all cases, they are of similar order of magnitude. In other words, our bands are representative of the true underlying uncertainty around the model predicted series, while confidence bands using non-bias adjusted bootstrap techniques may severely underestimate this uncertainty.

The imprecision of the estimated optimal current account is in line with our theoretical discussion. In the presence of singularity created by persistence, the coefficients of the $K$ vector in equation (2) can be very imprecisely estimated, leading to an unstable estimated optimal current account path (by construction). To further illustrate the source of instability of the estimated optimal current account, Figures 9 and 10 present the distribution of the $(l+1)^{th}$ coefficient of $K$, which is supposed to be equal to one under the null. For all country samples, the variance of the coefficient is very large. Even when the model is consistent with the data as determined by the $F$-test, there is a high probability that the coefficient will be far from its theoretical value. The coefficient can easily be negative.$^{21}$

Some papers start the analysis by informally comparing the estimated $K$ vector with its theoretical value (see Obstfeld and Rogoff 1995, 1996). Our discussion suggests, however, that the point estimate of the $K$ vector is unlikely to be very informative.

The large variance of the $K$ vector and the associated imprecision of the predicted current account also has strong implications for the variance of the predicted current account and its correlation with actual data, as we will now discuss.

### 4.3 Variance Analysis

We saw that the literature has often emphasized that actual current account series are typically more volatile than the model’s predictions. The literature typically reports only the variance of the predicted current account, without any indication on how precise this estimate is.$^{22}$ Figures 11 and 12 plot the distributions of the intertemporal variance

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$^{21}$ To see why a large and negative coefficient can occur, note in Figure 2 that the probability interval around $\hat{c}$ can encompass small and negative values and hence imply large and negative values of $K$.

$^{22}$ Gosh (1995), Gosh and Ostry (1995), and Hall et al. (2006) are exceptions.
Variance ratios are often very dispersed. Even when the model is consistent with the data the predicted current account can still be much more or much less volatile than the actual. For example, the $F$-test suggests that the model is strongly consistent with Belgian annual data, yet there is a 40 percent probability that the predicted current account is over four times as volatile as the actual. There is also a 20 percent probability that the predicted current account displays less than a fourth of the volatility of the actual series. Similar observations hold for the other data sets which failed to reject the model. Also, the data does not support claims that the current account is excessively volatile. In our samples, the probability that the predicted series is more volatile than the actual is often large. It averages 44 percent over our 10 samples, ranging from 11 percent for Belgian quarterly data to over 97 percent for Swedish quarterly data. Note that one cannot conclude that the current account is less volatile than the model predictions either. These results cast doubt on the literature’s finding that actual current accounts are more volatile than predicted by the model, and by extension on the interpretation that excess current account volatility is evidence against Feldstein and Horioka’s claim of limited international capital mobility.

### 4.4 Correlation Analysis

Despite the supposed failure of the model to match current account volatility, some authors have claimed that the model has explanatory value in that the correlation between actual and predicted series tends to be quite high (see, e.g., Obstfeld and Rogoff 1996). Figures 13 and 14 show the distributions of the in-sample correlation between actual and predicted series. These distributions are once again very wide reflecting dispersion in the $K$ vector, casting doubt on the above claim. Also, correlation values are in no way indicative of the model’s statistical validity. Most strikingly in the case of Belgian and Danish annual data the $F$-test accepts the model, yet there is over 45 percent probability that the correlation lies between $-1$ and $-0.9$. Conversely, for Swedish annual data there is a 37 percent probability that the correlation will exceed 0.95 even though the test has rejected the model. Finally, the distributions often cluster around one and minus one which can also be explained by the behavior of $K$ under near-singularity. To see why, consider the case of one lag in estimation. Then:

$$X_{p,t} = k_1 \Delta Y_t + k_2 X_t$$

23 For each draw $i$ from the multivariate Normal, we construct the predicted current account $X'_{p,t} = K'Z_t$ for all $t$ in our time range, where $Z_t$ is the time $t$ vector of data. For each draw $i$ the variance ratio is calculated as the intertemporal variance of $X'_{p,t}$ over the $t$ range divided by the intertemporal variance of the actual series.

24 Gosh (1995) and Gosh and Ostry (1995) often reject the null that the variance ratio equals one in favor of the alternative hypothesis that the variance of the predicted series is higher. The fact that they can reject the null implies that their estimated variance of the variance ratio is much smaller than ours. The authors do not specify how they compute this variance, and hence we cannot account for the discrepancy. Two things should be noted however: (i) the numerical expression of the variance of the variance ratio is a function of the variance of the cross-equation coefficients, hence if the latter is approximated by the Delta method then the results for the former will not be reliable; (ii) if the variance of the variance ratio is obtained by standard bootstrap simulations, the result may be strongly biased downwards, as discussed previously in the context of confidence intervals.
Intuitively, if near-singularity makes $\hat{k}_2$ positive (negative) and large in absolute value relative to $\hat{k}_1$, then $\hat{X}_p$ will be mostly driven by $k_2 X_t$ and the correlation will tend to one (minus one). Figure 15 plots the value of the $\frac{\hat{k}_2}{\hat{k}_1}$ ratio obtained over 10,000 draws of the empirical distribution for each country. The graphs fully confirm that the ratio will tend to be large in absolute value under near-singularity (note that the graphs were censored at -1000 and +1000 for presentation purposes, but the ratio is sometimes higher than this in the data).

4.5 Robustness Checks

Are the assumptions underlying the Wald test verified in the data?

As our and Bouakez and Kano’s Monte-Carlo simulations show, the Wald test has large size distortions in small samples even if the conditions of the test are met by the simulated data. But could the problems that we document on actual data be due not to the test itself but to the fact that the data does not conform to the assumptions underlying the test, in particular stationarity and iid residuals?

To start, we tested whether $X$ and $\Delta Y$ have a unit root using a standard Philipps-Perron test. As can be seen Table 3, we reject the null of a unit root for all $\Delta Y$ samples. As for the current account, we reject the null of a unit root at the 10 percent level for all quarterly samples except Belgium. At the annual frequency, we reject the null hypothesis for Canada and the U.K but not for Belgium, Denmark, and Sweden. At the same time, the auto-regressive coefficient is high but always below 0.9 which, combined with the low power of the tests given the relatively short quarterly period, suggests that the series may still be stationary. Here, it would be worth pointing out that if current accounts are truly non-stationary, attempts by the literature to estimate a model-predicted series are futile, because the present-value model implies a stationary current account.

Regarding whether the estimation residuals are iid, we cannot reject the null of joint normality of the residuals at a 95% confidence level in any of the annual samples. In quarterly data, we can only reject it for Sweden, but this could be due to data seasonality (see Footnote 18). Moreover, we do not detect serial correlation in any of the residual series.

In short, it does not appear that the problems we document in the paper are due to the data not conforming with underlying assumptions in the test or in the model.
Table 3: Unit Root Tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Country</th>
<th>AR coefficient</th>
<th>Philipps-Perron Test: Significance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta Y ) (quarterly)</td>
<td>Belgium</td>
<td>-0.50</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>-0.37</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Denmark</td>
<td>-0.62</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Sweden</td>
<td>-0.71</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>-0.21</td>
<td>0.00</td>
</tr>
<tr>
<td>( \Delta Y ) (annual)</td>
<td>Belgium</td>
<td>-0.06</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>0.47</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Denmark</td>
<td>-0.36</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Sweden</td>
<td>-0.05</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>Current Account</td>
<td>Belgium</td>
<td>0.86</td>
<td>0.48</td>
</tr>
<tr>
<td>(quarterly)</td>
<td>Canada</td>
<td>0.91</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Denmark</td>
<td>0.43</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>Sweden</td>
<td>0.55</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>0.83</td>
<td>0.00</td>
</tr>
<tr>
<td>Current Account</td>
<td>Belgium</td>
<td>0.88</td>
<td>0.97</td>
</tr>
<tr>
<td>(annual)</td>
<td>Canada</td>
<td>0.73</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>Denmark</td>
<td>0.83</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>Sweden</td>
<td>0.79</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>0.75</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Note: we reject the null hypothesis of a unit root if the significance level is below 0.1.

Does the bivariate VAR contain enough information to properly compute expected income changes?

In order to estimate the model-predicted series, we estimated a bivariate VAR on the current account and net income changes and used it to compute expected values of future income declines. Remember that the goal of our paper is, first and foremost, to question the validity of the statistical inference in this literature, and the literature has overwhelmingly used such bivariate VARs. In other words, we followed the same procedure that most papers in the literature have, and our findings that the literature’s inference is not robust cannot then be attributed to us using different methods. This being said, it is worth checking whether the assumption underpinning the Campbell-Shiller methodology (ie. that the current account is a sufficient predictor of future income changes) is verified in the data. We checked this assumption by regressing

\[
R_t = X_t - (1 + r)X_{t-1} - \Delta Y_t
\]

not only on past values of income changes and the current account but other lagged variables as well, and testing whether their coefficients are significant. We added the annual budget balance to our annual data estimations, as it is sometimes argued that fiscal deficits lead to a “twin” current account deficit. In quarterly data we also added energy prices with up to four lags, as commodity prices may contain
information about future income changes resulting from the terms of trade.25 As Table 4 makes clear, these variables are very seldom significant at 90% confidence and never at 95% confidence, showing that in our data the current account appears to contain sufficient information to predict future income changes.

Table 4: Testing Whether the Current Account is a Sufficient Predictor of Income Changes

<table>
<thead>
<tr>
<th></th>
<th>Significance level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Budget Balance</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.72</td>
</tr>
<tr>
<td>Canada</td>
<td>0.86</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.09</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.82</td>
</tr>
<tr>
<td>UK</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Note: We reject the null that these variables do not help predict future income changes if the significance level is below 0.1.

**Heteroscedasticity in the data**

Because our income and current account data is in levels rather than log levels, it may grow exponentially, leading to problems in estimation in particular through heteroscedasticity. On this, it should be noted that only a few papers in the literature correct for potential heteroscedasticity. Bergin and Sheffrin (2000) and Gruber (2004) express their data in percent of GDP, while Ghosh (1995) corrects the VAR residuals for heteroscedasticity using White’s method.

To insure that our key results do not stem from heteroscedastic data, we reestimated the model using data scaled by GDP. Our results are unaffected in all cases. To witness, Figure 16 shows the actual and model-predicted current accounts estimated with our quarterly data. For all five countries, the confidence bands remain very wide, as wide as with non-scaled data in Figure 7. The associated distributions of the correlation coefficient and variance ratio – not plotted – remain highly dispersed. In short, the large small sample uncertainty of the estimated model-predicted series, and concomitant unrobustness of any inference based on it, do not appear to be driven by heteroscedasticity in the data.

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25 We use the Standard and Poor’s Goldman Sachs Energy Index. The index being in U.S. dollars, we convert it into local currency. We also express it in real terms as we do for other variables.
5 Concluding Remarks

Our discussion challenges the results found in the large empirical literature on present-value models of the current account. Our discussion also suggests ways to revisit this large empirical literature. First, priority should be given to the $F$- and linear Wald tests rather than the non-linear Wald test when assessing the relevance of a consumption-smoothing model in explaining current account fluctuations. Second, confidence bands around the predicted optimal current account that account for the true underlying estimation uncertainty are needed, as are empirical distributions of the correlation coefficient and the variance ratio. Methods to account for estimation uncertainty should avoid biasing results away from the singularity region, or rely on Delta-method approximations. However, uncertainty surrounding estimates of future changes in output is likely to be such as to compromise present value estimates. If future research confirms this, one of the workhorse models of the literature would have proved of little empirical use. It would then be time to devote more attention to other models of the current account.

References


Appendix

Data

All our data are from the International Financial Statistics of the International Monetary Fund. The periods covered are indicated in the table below, noting that for each country we use the longest available sample in IFS.

<table>
<thead>
<tr>
<th>Country</th>
<th>Annual Data</th>
<th>Quarterly Data</th>
</tr>
</thead>
</table>

Net output and current account are defined as: \( Y_t = GDP_t - G_t - I_t \), and \( X_t = GNP_t - C_t - I_t - G_t \), and are expressed in real, per capita terms. The corresponding IFS series used to construct \( Y_t \) and \( X_t \) are as follows: GNP: gross national income (line 99a); G: government consumption (line 91f); I: sum of private gross fixed capital formation (line 93e) and increase/decrease in stocks (line 93i); C: household consumption (line 96f); and GDP volume (line 99b). For conversion into real, per capita terms we use GDP volume (line 99b, with base year 1995 or 1996 depending on the country) together with base year nominal GDP to construct a GDP deflator. We then use the GDP deflator to convert all other IFS series (which are nominal) into real terms. Dividing by population leads to real variables per capita. To give a sense of the data, we plot at the end of this appendix the constructed real, per-capita annual income changes and current accounts.

Estimation

Our VAR in changes in net output and current account is absolutely standard. The lag number is selected using the Akaike information criterion. To estimate the predicted current account, we set annual and quarterly real interest rates to 4 percent and 1 percent respectively. Our results are robust to changes in the number of lags or in the value of the real interest rate—results under alternative assumptions are available upon request.

The present value model as expressed in equation 1 implies that the mean of the current account should equal the negative of the mean of net output changes divided by the real interest rate. However, when we test this equality we find that it is strongly rejected in our data. Hence, as has been standard practice in the literature (see, e.g., Campbell, 1987, or Sheffrin and Woo, 1990), we demean the current account and the

\[ \text{for Belgium we cut the sample in 1998 as there is a break in the data following the adoption of the euro in 1999.} \]

\[ \text{The data has been demeaned (see below why), hence data units are hard to interpret.} \]
first difference in net output in all our estimations and only test the dynamic restrictions of the theory.\textsuperscript{28}

Finally, some authors have assumed that the discount factor is not equal to the inverse of the gross real interest rate (see Ghosh, 1995). In such a case, the current account equation includes a consumption-tilting parameter which needs to be estimated. We followed this procedure as a robustness check. The estimated consumption-tilting parameters are usually close to one (their value when the discount factor is equal to the inverse of the gross real interest rate). Our results remain robust to this specification—again, results under this alternative specification are available upon request.

\textsuperscript{28} We did not demean our series in the simulations because there the equality of means is verified by construction.
Figure 3: Simulated Distributions

Note: The variance ratio is expressed as the variance of the predicted series over that of the actual.
Figure 4: Actual (-) and Predicted (--\textsuperscript{--}) Series: Annual Data

Note: A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 5: Actual (-), Predicted (--), and Confidence Bands (Bold): Annual Data

Note: Bold lines correspond to the 2.5th and 97.5th percentiles of the distribution of the predicted series at each point in time. A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 6: Actual (-) and Predicted (--) Series: Quarterly Data

Note: A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 7: Actual (-), Predicted (--), and Confidence Bands (Bold): Quarterly Data

Notes: Bold lines correspond to the 2.5th and 97.5th percentiles of the distribution of the predicted series at each point in time. A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 8: Actual (-), Predicted (--), and Killian-Adjusted Bands (Bold): Annual Data

Note: A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 9: Distributions of the \((l+1)^{th}\) Coefficient of the \(K\) Vector: Annual Data

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure9}
\caption{Distributions of the \((l+1)^{th}\) Coefficient of the \(K\) Vector: Annual Data}
\end{figure}

\textit{Note}: A star denotes model acceptance by the \(F\)- and linear Wald tests at the 95 percent confidence level.
Figure 10: Distributions of the \((l+1)\text{th}\) Coefficient of the \(K\) Vector: Quarterly Data

Note: A star denotes model acceptance by the \(F\)- and linear Wald tests at the 95 percent confidence level.
Notes: The variance ratio is expressed as the variance of the predicted series over that of the actual. A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 12: Distributions of the Variance Ratio: Quarterly Data

Notes: The variance ratio is expressed as the variance of the predicted series over that of the actual. A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 13: Distributions of the Correlation Coefficient: Annual Data

Note: A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 14: Distributions of the Correlation Coefficient: Quarterly Data

Note: A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 15: Empirical Values of the $\frac{K_2}{K_1}$ Ratio

Belgium

Canada

Denmark

Sweden
**Figure 16:** Actual and Model-Predicted Current Accounts, Data Scaled by GDP
Figure 16: (continued)

![Graph of Sweden's Actual vs. Predicted data with 2.5% and 97.5% bands]

![Graph of United Kingdom's Actual vs. Predicted data with 2.5% and 97.5% bands]
Please note:

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The Editor