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On the Relation between Dual-Rate Discounting and Substitutability

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Abstract

To justify substantial emission reductions, recent literature on cost-benefit analysis of climate change suggests discounting environment consumption with an environmental discount rate instead of a consumption discount rate that is usually used in cost-benefit analysis. The present study clarifies that whether or not this dual-rate discounting approach succeeds in justifying substantial emission reductions depends on whether or not environment and goods consumption are substitutes in the Hicks-Allen sense and in the Edgeworth-Pareto sense (substitutes in the Hicks-Allen sense implies the Hicksian goods demand to be increasing in the relative price of environmental goods, while substitutes in the Edgeworth-Pareto sense implies the marginal utility of goods consumption to be decreasing in environment consumption). Moreover, a low *intra*temporal elasticity of substitution between environment and goods consumption within a period contributes to a low environmental discount rate in comparison to the consumption discount rate, while a low *inter*temporal elasticity of substitution between composite consumption of different periods contributes to declining discount rates over time.

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1. Introduction

As of April 2008, 178 countries had signed and ratified the Kyoto Protocol, committing to reduce their emissions of greenhouse gases in an effort to mitigate climate change. Many people believe that the emission reductions of the Kyoto Protocol are not strong enough. Yet, standard economic cost-benefit analysis has difficulties in justifying the emission reductions of the Kyoto Protocol, not to speak about even higher emission reductions. The main reason for the justification problem is the discount rate. The costs of emission reductions are to be paid now, while the mitigation of damages from climate change will be enjoyed in the future and therefore proper cost-benefit analysis requires to discount them. However, the relevant time horizon for climate change damages is hundred years and more. For this reason, standard discount rates lead to dramatic reductions of the present value of damage mitigation and therefore standard cost-benefit analysis hardly ever suggests substantial emission reductions (see e.g. Nordhaus and Yang, 1996, and Nordhaus and Boyer, 2000). Many economists are unsatisfied with this conclusion and believe that standard cost-benefit analysis has to be modified to escape the justification problem of substantial emission reductions. One way out of this dilemma has been suggested in the very influential "Stern Review of the Economics of Climate change" for the British government, released on 30 October 2006, arguing that ethical considerations call for a very low discount rate. However, the justification of a very low discount rate with ethical considerations is heavily criticised by many economists as being paternalistic and disrespectful towards standard literature.[1]

Earlier, Yang (2003), assuming a model with utility from goods consumption and environmental amenities, proposed an alternative justification for the emission reductions of the Kyoto Protocol, namely dual-rate discounting. More specifically, he suggests that environment consumption should be discounted at a lower rate than goods consumption (see also Weitzman (1994) for a similar argument). This is important because standard economic literature usually uses for the value of the consumption discount rate the observed market rate of return to capital. An environmental discount rate could however be lower than this market rate. Applying dual-rate discounting to the RICE model of Nordhaus and Yang (1996) and Nordhaus and Boyer (2000), the author confirms that dual-rate discounting can justify the Kyoto Protocol obligations. However, Tol (2003), though expressing sympathy with Yang's reasoning, has some objections on Yang's methodology. More specifically, Tol objects that in Yang's framework the differential (i.e. the quantitative difference) between the consumption discount rate and the environmental discount rate is not explained within the model, that is, it is not endogenous. As a better alternative, he develops a model in which the marginal willingness-to-pay for environmental quality might grow with growing income levels. Within this model the differential between the consumption and the environmental discount rate is explained with growth in the marginal willingness-to-pay for environmental quality. For this reason, Tol recommends a model with fully endogenous dual-rate discounting as the preferred option when the growth rate of the marginal willingness-to-pay for environmental quality is known. In contrast, when this information is unavailable, then not fully endogenous dual-rate discounting, as in Yang (2003), might serve as a valid alternative.

[1] Among the critics are Nordhaus (2007), Yohe and Tol (2007) and Weitzman (2007).

In a follow-up paper, Weikard and Zhu (2005) contain in particular the following two important contributions to the literature. Firstly, they show that there is equivalence between use of dual-rate discounting and use of a single consumption discount rate, evaluated in consumption goods equivalents (where, in the latter approach, the evolution of consumption goods equivalents is measured with growth in the marginal willingness-to-pay for environmental quality).[2] Secondly, they recognised that Tol (2003) implicitly assumes the instantaneous utility function to be additively separable in environment and goods consumption. Relaxing the latter assumption and assuming for simplicity that the marginal willingness-to-pay for environmental quality is independent of the income level, the authors develop a model in which a rising relative price of environmental goods is the source of growing marginal willingness-to-pay for environmental quality. In related and independent work, Hoel and Sterner (2007) show numerically that under a range of plausible parameter constellations, in a model with the instantaneous utility function being non-separable in environment and goods consumption, growth in the relative price of environmental goods reduces the value of the environmental discount rate below the value of the conventional consumption discount rate. Furthermore, building on the latter work, Sterner and Persson (2008) show that a rising relative price of environmental goods can justify the drastic level of emission reductions, recommended in the Stern Review. Most importantly, it can do so without reliance on ethical considerations, suggested in the Stern Review, which have been criticised so heavily in the economic literature as being paternalistic and disrespectful towards standard literature. In turn, this brings us back to the motivation of Yang (2003) to propose dual-rate discounting.

Weikard and Zhu (2005) further conclude from their analysis that the aforementioned equivalence between use of dual-rate discounting and use of the consumption discount rate (evaluated in consumption goods equivalents) breaks down in case of non-substitutability between environment and goods consumption. This is where the present paper joins the debate. In section 2, the paper shows that the equivalence does actually survive in case of non-substitutability. By showing this result, the paper also provides a clarification that one can distinguish between substitutes in the Hicks-Allen sense and substitutes in the Edgeworth-Pareto sense, where substitutes in the Hicks-Allen sense implies the Hicksian goods demand to be increasing in the relative price of environmental goods, while substitutes in the Edgeworth-Pareto sense implies the marginal utility of goods consumption to be decreasing in environment consumption.

Section 3 aims to connect the issue of substitutability to the motivation of Yang (2003) to propose dual-rate discounting for solving the justification problem of substantial emission reductions. As a result of that effort, section 3 shows that the environmental discount rate is lower than the consumption discount rate if environment and goods consumption are not complements in the Hicks-Allen sense (i.e. if the Hicksian goods demand is not decreasing in the relative price of environmental goods). If this condition is fulfilled, then the differential between the consumption discount rate and the environmental discount rate is the larger the lower the value of the intratemporal elasticity of substitution between environment and goods consumption within a period. As a consequence, section 3 challenges to some extent the narrative argument of Neumayer (1999), who argues that the discount rate debate regarding optimal climate change policy would miss the point, as substitutability rather than discounting would be the real issue. In contrast to

[2] See for a similar argument also Horowitz (2002) and related Perman et al. (2003, pp. 369-373), Fisher and Krutilla (1975) and Arrow et al. (1995).

Neumayer's argument, the present paper shows that the degree of substitutability between environment and goods consumption affects the size of the environmental discount rate and therefore substitutability and discounting are issues that are closely related to each other.

An alternative way to escape the justification problem of substantial emission reductions is offered by a related literature, arguing that the consumption discount rate declines over time. Most of this literature abstracts from utility from environmental amenities.[3] However, a recent literature uses the same framework as the present paper and investigates whether, due to limited substitutability between environment and goods consumption, economic growth causes the environmental discount rate and the consumption discount rate both to decline over time. In particular, Traeger (2007) challenges the argument of Neumayer (1999) that limited substitutability calls for substantial emission reductions. Traeger shows that instead an intratemporal elasticity of substitution between environment and goods consumption that is lower than one (implying weak substitutability) implies economic growth to cause over time *rising* discount rates. However, building on the mathematical framework of Gollier (2008), Guesnerie (2004) and Hoel and Sterner (2007), section 4 of the present paper shows that this is only true if environment and goods consumption are complements in the Edgeworth-Pareto sense (which means that the marginal utility of goods consumption is increasing in environment consumption). It is shown that the latter implies the intertemporal elasticity of substitution between composite consumption of different periods to be larger than the intratemporal elasticity of substitution between environment and goods consumption within a period. If one assumes instead environment and goods consumption to be substitutes in the Edgeworth-Pareto sense and therefore the aforementioned intertemporal elasticity of substitution to be lower than the aforementioned intratemporal elasticity of substitution, then economic growth causes over time declining discount rates, provided the intratemporal elasticity of substitution is lower than one.

2. A model with non-substitutability

The starting point of the analysis is the assumption of a simple two-period model in discrete time, with periods 0 and 1. Suppose a project leads to an increase in period 1 goods consumption by ΔC at the expense of a reduction in period 1 environment consumption by ΔE. Further, assume that in case of absence of the assumed project, there would be no growth in environment and goods consumption and that the project is large enough to influence the discount rates. Equivalence between use of dual-rate discounting and use of a single consumption discount rate, evaluated in consumption goods equivalents, implies equivalence of the following two equations:[4]

$$NPV = \frac{\Delta C}{1 + r_{C_0,C_1}} + \frac{p_0 \Delta E}{1 + r_{E_0,E_1}}, \tag{1}$$

$$NPV = \frac{\Delta C}{1 + r_{C_0,C_1}} + \frac{\left(1 + g_p\right) p_0 \Delta E}{\left(1 + r_{C_0,C_1}\right)}, \tag{2}$$

[3] See Groom et al. (2005) for an excellent survey on the literature on declining discount rates.
[4] See similarly in continuous time in Perman et al. (2003, pp. 375-377).

where NPV is the net present value of the assumed project, p_0 represents the relative price of environmental goods in period 0 (consumption goods being the numéraire) and g_p denotes the growth rate of this relative price between period 0 and 1. In (1) the NPV is calculated, using dual-rate discounting, with the consumption discount rate, r_{C_0,C_1}, and the environmental discount rate, r_{E_0,E_1}.[5] In (2) the NPV is calculated with the single consumption discount rate, r_{C_0,C_1}, accounting for growth in the relative price of environmental goods, g_p. In turn, efficient allocation requires the relative price of environmental goods to be equal to the marginal rate of substitution between environment and goods consumption. For this reason, growth in the relative price of environmental goods measures growth in the marginal willingness-to-pay for environment consumption. Further, $\left(1+g_p\right)$ converts the flow of environmental goods into period 1 consumption goods equivalents. The decision maker should go ahead with the project if the NPV is positive. (1) and (2) are equivalent because, as shown in Appendix B:

$$1+r_{E_0,E_1}=\frac{1+r_{C_0,C_1}}{1+g_p}. \qquad (3)$$

Weikard and Zhu (2005) argue that a marginal rate of substitution between environment and goods consumption would not exist in case of non-substitutability between environment and goods consumption and therefore in this case the equivalence between (1) and (2) would break down. The latter argument is reconsidered in this paper.

Consider the following CES function of instantaneous utility function with environmental amenities (see Hoel and Sterner, 2007):

$$U_t=\left(\frac{1}{1-\alpha}\right)\left[(1-\gamma)C_t^{\frac{\sigma-1}{\sigma}}+\gamma E_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{\sigma-1}}, \quad \text{with } \sigma\geq 0, \quad \forall \text{ t=0,1,} \qquad (4)$$

where γ is a constant parameter, σ is the intratemporal elasticity of substitution between environment and goods consumption within a period and α is the coefficient of relative risk aversion. In (4), σ and α are assumed to be constant. Moreover, α can be shown to be the inverse of the intertemporal elasticity of substitution between composite consumption of different periods, where the composite consumption index of period t=0,1 is defined as:

$$\widetilde{C}_t=\left[(1-\gamma)C_t^{\frac{\sigma-1}{\sigma}}+\gamma E_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

[5] As a matter of fact, since in (1) ΔE is multiplied with p_0, the NPV in (1) is in principle evaluated in consumption goods equivalents. However, it is evaluated in period 0 consumption goods equivalents rather than in period 1 consumption goods equivalents and therefore, for given discount rate values, it is not accounting for growth in the relative price of environmental goods.

implying that in (4) $U_t = \widetilde{C}_t^{1-\alpha}/(1-\alpha)$.

As shown in Appendix C, from (4) the consumption discount rate and the environmental discount rate can be derived to be approximately equal to:[6]

$$r_{C_0,C_1} = \rho + \bar{\eta}_{CC}\frac{\Delta C}{C_0} - \bar{\eta}_{CE}\frac{\Delta E}{E_0} = \rho + [\,(1-\bar{\gamma}^*)\alpha + \bar{\gamma}^*\frac{1}{\sigma}]\frac{\Delta C}{C_0} - \bar{\gamma}^*(\frac{1}{\sigma}-\alpha)\frac{\Delta E}{E_0}, \qquad (5)$$

$$r_{E_0,E_1} = \rho + \bar{\eta}_{EE}\frac{\Delta E}{E_0} - \bar{\eta}_{EC}\frac{\Delta C}{C_0} = \rho + [\bar{\gamma}^*\alpha + (1-\bar{\gamma}^*)\frac{1}{\sigma}]\frac{\Delta E}{E_0} - \left(1-\bar{\gamma}^*\right)(\frac{1}{\sigma}-\alpha)\frac{\Delta C}{C_0}, \qquad (6)$$

with $\bar{\eta}_{CC} \equiv -\frac{\bar{C}\bar{U}_{CC}}{\bar{U}_{C,1}}, \bar{\eta}_{CE} \equiv \frac{\bar{E}\bar{U}_{CE}}{\bar{U}_C}$, $\bar{\eta}_{EE} \equiv -\frac{\bar{E}\bar{U}_{EE}}{\bar{U}_E}$, $\bar{\eta}_{EC} \equiv \frac{\bar{C}\bar{U}_{EC}}{\bar{U}_E}$

and $\bar{\gamma}^* \equiv \frac{\gamma\bar{E}^{\frac{\sigma-1}{\sigma}}}{(1-\gamma)\bar{C}^{\frac{\sigma-1}{\sigma}} + \gamma\bar{E}^{\frac{\sigma-1}{\sigma}}}$,

where ρ denotes the utility discount rate, $\Delta X = X_1 - X_0$, $\bar{X} = (X_1 - X_0)/2$, $\bar{U}_X \equiv \partial\bar{U}/\partial X$ for any variable X and $U_{XY} \equiv \partial^2 U/\partial X\partial Y$ for all X and Y. Further, $\bar{\gamma}^*$ denotes the value share of environmental goods (see Gerlagh and van der Zwaan, 2002, and Hoel and Sterner 2007).

As is shown in Appendix A, from (4) and a social planner's constraint, the price elasticity of Hicksian goods demand can be derived as:

$$\frac{\partial C_1^{\,h}}{\partial p_1}\frac{p_1}{C_1^h} = \sigma\theta_{E,1}, \qquad (7)$$

where $\theta_{E,1}$ denotes the expenditure share of environmental goods and the index h represents Hicksian demand. From (7) follows that environment and goods consumption are not substitutes in the Hicks-Allen sense if $\sigma \to 0$, as in this case the price elasticity of Hicksian goods demand approaches the value zero.[7] In contrast, environment and goods consumption are substitutes in the Hicks-Allen sense if $\sigma > 0$ and complements in the Hicks-Allen sense if $\sigma < 0$ (note however that in (4) the possibility of $\sigma < 0$ was excluded, but $\sigma < 0$ is possible in case of more general utility functions – getting analytical solutions from more general utility functions is however more difficult). In case $\sigma \to 0$, (4) approaches the Leontief utility function:[8]

$$U_t = \min[(1-\gamma)C_t, \gamma E_t], \quad \forall \text{ t=0,1.}$$

This is the case Weikard and Zhu (2005) have in mind when they consider non-substitutability between environment and goods consumption. However, this is only one case of non-substitutability. In addition, goods consumption and environmental consumption are no substitutes in the Edgeworth-Pareto sense if:[9]

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[6] The need of approximation is due to the fact that the model is in discrete time.
[7] See, e.g., Samuelson (1974, page 1256, footnote 3).
[8] See, e.g., Tohamy and Mixon (2003).
[9] See, e.g., Samuelson (1974, page 1256, footnote 2). Related, see also Amano and Wirjanto (1998).

$$\bar{\eta}_{CE} = \bar{\gamma}^{*}(\frac{1}{\sigma} - \alpha) = 0. \qquad (8)$$

In contrast, environment and goods consumption are substitutes in the Edgeworth-Pareto sense if $\bar{\eta}_{CE} < 0$ and complements in the Hicks-Allen sense if $\bar{\eta}_{CE} > 0.$ As (8) shows, $\bar{\eta}_{CE} = 0$ is fulfilled if $(1/\sigma)=\alpha,$ in which case (4) reduces to:

$$U_t = (1-\gamma)\frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \gamma\frac{E_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \qquad \forall \text{ t=0,1,}$$

where the instantaneous utility function is additively separable in environment and goods consumption.

From these exercises it should be clear that the wording non-substitutability and being no substitutes are interchangeable and mean that environment and goods consumption are neither substitutes nor complements. In case of non-substitutability in the Hicks-Allen sense, implying (4) to approach the Leontief utility function, the decision maker is however unwilling to sacrifice some units environment consumption for some additional units goods consumption. In contrast, in case of non-substitutability in the Edgeworth-Pareto sense, implying (4) to be reduced to a utility function that is additively separable in environment and goods consumption, the decision maker's utility gain from increase in goods consumption is unaffected from a reduction of environment consumption.

Suppose $\sigma \to 0,$ (that is, suppose non-substitutability in the Hicks-Allen sense) and hence $(1/\sigma) \to \infty$. Using the fact that the assumed project implies $\Delta C > 0$ and $\Delta E < 0,$ (3), (5), and (6) therefore give rise to:

$$\lim_{\sigma\to 0}\left(1+r_{C_0,C_1}\right) = \infty \quad \wedge \quad \lim_{\sigma\to 0}\left(1+r_{E_0,E_1}\right) = \lim_{\sigma\to 0}\left(\frac{1+r_{C_0,C_1}}{1+g_{p,1}}\right) = -\infty.$$

Hence, when $\sigma \to 0,$ then equation (1) and (2) are equivalent. Since $\Delta C > 0$ and $\Delta E < 0,$ in this case, no matter whether the NPV is calculated using formula (1) or formula (2), the NPV of the assumed project always equals zero. This is due to the fact that the decision maker is unwilling to sacrifice some units environment consumption for some additional units goods consumption. Therefore, the decision maker never goes ahead with the assumed project.

Alternatively, suppose $(1/\sigma) = \alpha$ (that is, suppose non-substitutability in the Edgeworth-Pareto sense). In this case (3), (5) and (6) yield:

$$1+r_{C_0,C_1} = 1+\rho+\left(\frac{1}{\sigma}\right)\frac{\Delta C}{C_0} \quad \wedge \quad 1+r_{E_0,E_1} = \frac{1+r_{C_0,C_1}}{1+g_p} = 1+\rho+\left(\frac{1}{\sigma}\right)\frac{\Delta E}{E_0}.$$

Hence, again equation (1) and (2) are equivalent. In this case, the consumption discount rate is independent of the reduction in environment consumption, as is the standard result in the Ramsey model of optimal consumption growth. Analogously, the environmental discount rate is independent of the increase in goods consumption. Nevertheless, the NPV is the same no matter whether one uses dual-discounting or a single consumption discount rate, evaluated in consumption goods equivalents.

3. The role of substitutability for the discount rate differential

The purpose of this section is to bring us back to the motivation of Yang (2003) to propose dual-rate discounting, namely to ask whether or not the paper's model provides conditions under which substantial emission reductions to mitigate climate change can be justified. Since in the last section's model the project implied changes of environment and goods consumption in the same period, the model could not approximate cost-benefit analysis of future climate change. Doing so requires a slight change of the model assumptions. For this purpose, assume now a T period model and assume that a project leads to a reduction of period 0 goods consumption by $\Delta C'$ and to an increase in period T environment consumption by $\Delta E'$. This approximates in a simple way current abatement costs from emission reductions and future mitigation of damages from climate change. Contrary to the last sections model, in this section's model it is assumed that the project is so small that it cannot influence the discount rates.

Using dual-rate discounting, the project's NPV is now:

$$NPV = \Delta C' + \frac{p_0 \Delta E'}{\prod_{t=1}^{T}\left(1+r_{E_{t-1},E_t}\right)}, \tag{9}$$

Again, the decision maker should go ahead with the project if the NPV is positive. Similar to the last section it holds that:

$$1+r_{E_{t-1},E_t} = \frac{1+r_{C_{t-1},C_t}}{1+g_{p,t}}, \tag{3'}$$

where $g_{p,t}$ denotes the growth rate of the relative price of environmental goods from period t-1 to period t. From (3') follows (after use of the fact that $\ln(1+x) \cong x$, since x is close to zero):

$$\ln\left(1+r_{E_{t-1},E_t}\right) = \ln\left(1+r_{C_{t-1},C_t}\right) - \ln\left(1+g_{p,t}\right) \;\Rightarrow\; r_{E_{t-1},E_t} \cong r_{C_{t-1},C_t} - g_{p,t} \tag{3''}$$

Noting that, as mentioned before, efficient allocation requires that $p_t = MRS_{E_t,C_t} = U_{E_t}/U_{C_t}$, using (4), straightforward calculation yields:

$$p_t = \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{C_t}{E_t}\right)^{\frac{1}{\sigma}} \;\Rightarrow\; \ln p_t = \frac{1}{\sigma}\left(\ln C_t - \ln E_t\right),\ \forall\ t=0,1.$$

and therefore:[10]

$$\ln\left(\frac{p_t}{p_{t-1}}\right)=\frac{1}{\sigma}\left[\ln\left(\frac{C_t}{C_{t-1}}\right)-\ln\left(\frac{E_t}{E_{t-1}}\right)\right] \Rightarrow g_{p,t}=\frac{1}{\sigma}\left(\frac{\Delta C_t}{C_{t-1}}-\frac{\Delta E_t}{E_{t-1}}\right) \tag{10}$$

where $\Delta X_t = X_t - X_{t-1}$ for any variable X (and now use was made of $\mathbf{\ln(x')\cong x'-1}$, since $x'-1$ is close to zero).

Eq. (10) shows that, provided $(\Delta C_t / C_{t-1}) > (\Delta E_t / E_{t-1})$, then $\sigma \geq 0$ ensures a rising relative price of environmental goods (i.e. then environment and goods consumption not to be complements in the Hicks-Allen sense, cf. (7), ensures a rising relative price of environmental goods). Therefore (3') implies, in this case, the environmental discount rate to be smaller than the consumption discount rate. Moreover, (10) and (3') imply that, provided $\sigma \geq 0$, then the difference between the value of the consumption discount rate and the value of the environmental discount rate is the larger, the smaller σ, that is the weaker substitutes in the Hicks-Allen sense environment and goods consumption are.

To better understand the mechanisms at work, assume for simplicity that $(\Delta E_t / E_{t-1}) = 0$, while $(\Delta C_t / C_{t-1}) > 0$. In this case, upon use of (5) and (6) the discount rates become:

$$r_{C_{t-1},C_t} = \rho + \left[(1-\bar{\gamma}_t^*)\alpha + \bar{\gamma}_t^* \frac{1}{\sigma}\right]\frac{\Delta C_t}{C_{t-1}} \text{ or } r_{C_{t-1},C_t} = \rho + \left[\frac{1}{\sigma} - (1-\bar{\gamma}_t^*)\left(\frac{1}{\sigma}-\alpha\right)\right]\frac{\Delta C_t}{C_{t-1}} \tag{5'}$$

$$r_{E_{t-1},E_t} = \rho - (1-\bar{\gamma}_t^*)\left(\frac{1}{\sigma}-\alpha\right)\frac{\Delta C_t}{C_{t-1}}, \tag{6'}$$

$$\textit{where } \bar{\gamma}_t^* \equiv \frac{\gamma \bar{E}_t^{\frac{\sigma-1}{\sigma}}}{(1-\gamma)\bar{C}_t^{\frac{\sigma-1}{\sigma}} + \gamma \bar{E}_t^{\frac{\sigma-1}{\sigma}}},$$

with $\bar{X}_t = (X_t - X_{t-1})/2$ for any variable X.

According to the first equation of (5'), growth in goods consumption unambiguously increases r_{C_{t-1},C_t}, while according to (6') the impact of growth in goods consumption on r_{E_{t-1},E_t} is either an increase (if $(1/\sigma) > \alpha$), a decrease (if $(1/\sigma) < \alpha$) or no change (if $(1/\sigma) = \alpha$). Most importantly, the second equation in (5') and (6') imply for the discount rate differential that we have $r_{C_{t-1},C_t} - r_{E_{t-1},E_t} = (1/\sigma)(\Delta C_t / C_{t-1})$. Clearly, the discount rate differential is only positive if $\sigma \geq 0$ (i.e. if environment and goods consumption are not complements in the Hicks-Allen sense). Further, provided $\sigma \geq 0$, then the discount rate differential is the larger the smaller the value of σ. It is straightforward to confirm that the same mechanisms work if we relax the simplifying assumption $(\Delta E_t / E_{t-1}) = 0$.

[10] See Hoel M., Sterner, T (2007, p. 271) in a model in continuous time.

The bottom line of this exercise is that dual-rate discounting might offer an escape from the justification problem of substantial emission reductions if environment and goods consumption are weak substitutes in the Hicks-Allen sense. Therefore, the aforementioned narrative argument of Neumayer (1999) is challenged with the analytical result that weak substitutability might actually be reflected in a larger discount rate differential.

4. The role of substitutability for the change of discount rates over time

Traeger (2007) argues that weak substitutability in the Hicks-Allen sense (i.e. $\sigma < 1$) implies over time rising discount rates. With this he challenges the argument of Neumayer (1999) that weak substitutability calls for substantial emission reductions. However, Traeger 's result is driven by his seemingly innocent assumption, in the instantaneous utility function, the coefficient of relative risk aversion, α, to be equal to zero. To show this point, assume the same model as in the last section (i.e. with the NPV calculated according to (9) and the project being so small that it cannot influence the discount rates). Assuming $\alpha = 0$, upon use of (5) and (6) the discount rates become:

$$r_{C_{t-1},C_t} = \rho + \overline{\gamma}_t^* \frac{1}{\sigma}\left(\frac{\Delta C_t}{C_{t-1}} - \frac{\Delta E_t}{E_{t-1}}\right), \qquad (5")$$

$$r_{E_{t-1},E_t} = \rho - (1-\overline{\gamma}_t^*)\frac{1}{\sigma}\left(\frac{\Delta C_t}{C_{t-1}} - \frac{\Delta E_t}{E_{t-1}}\right). \qquad (6")$$

As shown in Gerlagh and van der Zwaan (2002), when $(\Delta C_t / C_{t-1}) > (\Delta E_t / E_{t-1})$, then the value share of environmental goods, $\overline{\gamma}_t^*$, increases if environment and goods consumption are weak substitutes in the Hicks-Allen sense, i.e. if $\sigma < 1$ (see also Hoel and Sterner, 2007, and Guesnerie, 2004). As argued in Traeger, $(\Delta C_t / C_{t-1}) > (\Delta E_t / E_{t-1})$ therefore implies that, in case $\sigma < 1$, according to (5"), r_{C_{t-1},C_t} rises because of an increasing value of $\overline{\gamma}_t^*$, while, according to (6"), r_{E_{t-1},E_t} rises because of a decreasing value of $(1-\overline{\gamma}_t^*)$.

However, suppose the more general case $\alpha \neq 0$ and assume $(\Delta E_t / E_{t-1}) = 0$ and $(\Delta C_t / C_{t-1}) = g = \text{constant}>0$, which gives upon use of (5) and (6) the discount rates as:[11]

$$r_{C_{t-1},C_t} = \rho + \left[(1-\overline{\gamma}_t^*)\alpha + \overline{\gamma}_t^* \frac{1}{\sigma}\right] g = \rho + \left[\frac{1}{\sigma} - (1-\overline{\gamma}_t^*)\left(\frac{1}{\sigma} - \alpha\right)\right] g, \qquad (5''')$$

$$r_{E_{t-1},E_t} = \rho - \left(1-\overline{\gamma}_t^*\right)\left(\frac{1}{\sigma} - \alpha\right) g. \qquad (6''')$$

$$\text{where } \overline{\gamma}_t^* = \frac{\gamma}{(1-\gamma)\overline{\kappa}_t^{\frac{\sigma-1}{\sigma}} + \gamma}, \quad \text{with } \overline{\kappa}_t \equiv \frac{\overline{C}_t}{\overline{E}_t}. \qquad (11)$$

[11] See similar in Gollier (2008).

In turn, using (5"'), (6"') and (11) it is straightforward to derive that:

$$\frac{\partial r_{C_{t-1},C_t}}{\partial \bar{\kappa}_t} = \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\bar{\gamma}_t^*(1-\bar{\gamma}_t^*)}{\bar{\kappa}_t}\right)\left(\frac{1}{\sigma}-\alpha\right)g, \tag{12}$$

$$\frac{\partial r_{E_{t-1},E_t}}{\partial \bar{\kappa}_t} = \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\bar{\gamma}_t^*(1-\bar{\gamma}_t^*)}{\bar{\kappa}_t}\right)\left(\frac{1}{\sigma}-\alpha\right)g. \tag{13}$$

Obviously, (12) and (13) describe the change of the discount rates over time, as $\bar{\kappa}_t$ rises over time because $\left(\Delta C_t / C_{t-1}\right) = g > 0$, while $\left(\Delta E_t / E_{t-1}\right) = 0$. Eq. (12) and (13) show that, provided $\sigma < 1$ (i.e. provided the intratemporal elasticity of substitution between environment and goods consumption is smaller than one), then both discount rates only rise if $\left(1/\sigma\right) > \alpha$, that is if environment and goods consumption are complements in the Edgeworth-Pareto sense (cf. (8)). The latter is indeed fulfilled in Traeger's model, provided environment and goods consumption are not complements in the Hicks-Allen sense, as he assumes $\alpha = 0,$ and therefore $0 \le \sigma < 1$ implies $\left(1/\sigma\right) > 0,$ that is implies environment and goods consumption to be complements in the Edgeworth-Pareto sense. However, $\alpha = 0$ is an unrealistic assumption, as this implies risk neutral consumers, while it seems more realistic to assume risk avers consumers. As (12) and (13) show, in case $\sigma < 1,$ if instead $\left(1/\sigma\right) < \alpha$, that is if instead environment and goods consumption are substitutes in the Edgeworth-Pareto sense (cf (8)), then both discount rates decline. In turn, eq. (8) implies that environment and goods consumption are substitutes in the Edgeworth-Pareto sense if the intertemporal elasticity of substitution between composite consumption of different periods is lower than the intratemporal elasticity of substitution between environment and goods consumption within a period.

The bottom line of this section is that, for the direction of the change of the discount rates over time, it is relative weak *inter*temporal substitutability that matters rather than relative weak *intra*temporal substitutability. This makes also intuitively sense, as declining discount rates is an intertemporal change. Clearly, the policy implication is that relatively weak intertemporal substitutability can also contribute to justify substantial emission reductions.

5. Conclusion

Recent literature on cost-benefit analysis of climate change suggests dual-rate discounting, where goods consumption is discounted with a consumption discount rate and environment consumption is discounted with an environmental discount rate. The motivation for dual-rate discounting is to justify substantial emission reductions, as possibly in this framework the environmental discount rate might be lower than the consumption discount rate and possibly in a model with endogenous dual-rate discounting both discount rates might decline over time. The present study showed that, provided environment and goods consumption are not complements in the Hicks-Allen sense, then the environmental discount rate is lower than the consumption discount rate (not complements in the Hicks-Allen sense implies the Hicksian goods demand not to be decreasing in the relative price of environmental

goods). Further, if environment and goods consumption are not complements in the Hicks-Allen sense, then the quantitative difference between the consumption discount rate and the environmental discount rate is the larger, the lower the value of the intratemporal elasticity of substitution between environment and goods consumption within a period. In addition, the present paper showed that, provided the intratemporal elasticity of substitution between environment and goods consumption within a period is lower than one, then environment and goods consumption to be substitutes in the Edgeworth-Pareto sense ensures both discount rates to decline over time (substitutes in the Edgeworth-Pareto sense implies the marginal utility of goods consumption to be decreasing in environment consumption). Moreover, it is shown that environment and goods consumption are substitutes in the Edgeworth-Pareto sense if the intertemporal elasticity of substitution between composite consumption of different periods is lower than the intratemporal elasticity of substitution between environment and goods consumption within a period.

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Appendixes

Appendix A: Derivation of eq. (7)

The price elasticity of Hicksian consumption demand can be derived in three steps. Step 1: Derive the instantaneous expenditure function. Step 2: Derive the Hicksian goods demand function upon application of Shepard's lemma to the instantaneous expenditure function. Step 3: Derive the price elasticity of Hicksian goods demand.

Step 1: Derivation of the instantaneous expenditure function

Suppose a social planner has to solve the following two period optimisation problem:

$$\max V = \left(\frac{1}{1-\alpha}\right)\left[(1-\gamma)C_0^{\frac{\sigma-1}{\sigma}} + \gamma E_0^{\frac{\sigma-1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{\sigma-1}} + \left(\frac{1}{1+\rho}\right)\left(\frac{1}{1-\alpha}\right)\left[(1-\gamma)C_1^{\frac{\sigma-1}{\sigma}} + \gamma E_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{\sigma-1}}$$

$$\text{s.t. } K_0 + Y_0(K_0) = K_1 + C_0, \quad K_1 + Y_1(K_1) = C_1 \ \wedge \ \bar{S} = E_0 + E_1.$$

Hence, the social planner maximises life-time utility, V, subject to goods consumption constraints in period 0 and 1 and an intertemporal constraint of environment consumption. Physical capital can be used for output production, Y. Physical capital and output can be transformed into consumption goods. The sum of physical capital

and output that has not been consumed in period 0 equals the physical capital stock at the begin of period 1, K_1. In period 1, the physical capital stock and all output is entirely consumed. For simplicity neither labour nor natural resources are used in production. The economy's initial stock of natural resources, $\bar{S}$, is assumed to be consumed by household's as environmental amenities.

Setting a Lagrangian gives rise to the following first order conditions:

$$\frac{\partial L}{\partial C_1} = 0 \;\Rightarrow\; (1-\gamma)\frac{C_1^{-\frac{1}{\sigma}}}{1+\rho}\left[(1-\gamma)C_0^{\frac{\sigma-1}{\sigma}} + \gamma E_0^{\frac{\sigma-1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{(\sigma-1)}-1} = \lambda_1, \quad \text{(A.1)}$$

$$\frac{\partial L}{\partial E_1} = 0 \;\Rightarrow\; \gamma\frac{E_1^{-\frac{1}{\sigma}}}{1+\rho}\left[(1-\gamma)C_0^{\frac{\sigma-1}{\sigma}} + \gamma E_0^{\frac{\sigma-1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{(\sigma-1)}-1} = \mu, \quad \text{(A.2)}$$

$$\frac{\partial L}{\partial K_1} = 0 \;\Rightarrow\; \frac{\lambda_0}{\lambda_1} = 1 + Y_{K_1}, \quad \text{(A.3)}$$

where λ_0, respectively, λ_1 are the Lagrangian multipliers of the goods consumption constraint in period 0, respectively, period 1 and μ is the Lagrangian multiplier of the intertemporal environment consumption constraint. Further, applying the Euler theorem to the production function, which is assumed to be homogenous of degree one, gives: $Y_{K_1}K_1 = Y_1$. Combining the latter expression with the goods consumption constraint in period 1 yields: $K_1 = C_1/\left(1+Y_{K_1}\right)$. Substituting the latter expression in the goods consumption constraint in period 0 gives rise to $C_0 + C_1/\left(1+Y_{K_1}\right) = K_0 + Y_0(K_0)$. In turn, combining the latter equation with the intertemporal constraint of environment consumption gives the overall intertemporal budget constraint as: $C_0 + C_1/\left(1+Y_{K_1}\right) + \mu E_0 + \mu E_1 = W_0$, *with* $W_0 \equiv K_0 + Y_0(K_0) + \mu\bar{S}$. Obviously from this constraint it follows that:

$$\frac{C_1}{1+Y_{K_1}} + \mu E_1 = M_1, \quad \text{with } M_1 \equiv W_0 - C_0 - \mu E_0, \quad \text{(A.4)}$$

where M_1 is expenditure in period 1. Combing (A.1) with (A.2) yields:

$$C_1 = \gamma^{-\sigma}(1-\gamma)^{\sigma}\lambda_1^{-\sigma}\mu^{\sigma}E_1. \quad \text{(A.5)}$$

Upon combining (A.5) with (A.4) and (A.3) and normalising λ_0 equal to one, we get:

$$C_1 = \lambda_1^{-1}\left(1-\theta_{E,1}\right)M_1, \tag{A.6}$$

$$E_1 = \mu^{-1}\theta_{E,1}M_1, \tag{A.7}$$

with $\theta_{E,1} \equiv \dfrac{\gamma^{\sigma}\mu^{1-\sigma}}{(1-\gamma)^{\sigma}\lambda_1^{1-\sigma}+\gamma^{\sigma}\mu^{1-\sigma}}$.

Substituting (A.6) and (A.7) in eq. (4) in the text for t=1 gives the indirect instantaneous utility function for period 1 as:

$$U_1 = \left(\frac{1}{1-\alpha}\right)\left[(1-\gamma)^{\sigma}\lambda_1^{1-\sigma}+\gamma^{\sigma}\mu^{1-\sigma}\right]^{\frac{1-\alpha}{\sigma-1}}M_1^{1-\alpha}. \tag{A.8}$$

Since the instantaneous expenditure function for period 1 is the inverse of the indirect instantaneous utility function, (A.8) implies the instantaneous expenditure function for period 1 to be:

$$M_1 = (1-\alpha)^{\frac{1}{1-\alpha}}\left[(1-\gamma)^{\sigma}\lambda_1^{1-\sigma}+\gamma^{\sigma}\mu^{1-\sigma}\right]^{\frac{1}{\sigma-1}}U_1^{\frac{1}{1-\alpha}}. \tag{A.9}$$

Step 2: Derivation of the Hicksian goods demand function

Application of Shepard's lemma to (A.9) yields:

$$C_1^h = \frac{\partial M_1}{\partial \lambda_1} = \left(\frac{1}{1-\sigma}\right)(1-\alpha)^{\frac{1}{1-\alpha}}(1-\gamma)^{\sigma}\lambda_1^{-\sigma}\left[(1-\gamma)^{\sigma}\lambda_1^{1-\sigma}+\gamma^{\sigma}\mu^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}U_1^{\frac{1}{1-\alpha}}, \tag{A.10}$$

where C_1^h stands for Hicksian goods demand**.** Re-arranging (A.10) gives the Hicksian goods demand function as:

$$C_1^h = \left(\frac{1}{1-\sigma}\right)(1-\alpha)^{\frac{1}{1-\alpha}}(1-\gamma)^{\sigma}\left[(1-\gamma)^{\sigma}+\gamma^{\sigma}p_1^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}U_1^{\frac{1}{1-\alpha}}, \quad \text{with } p_1 \equiv \frac{\mu}{\lambda_1}, \tag{A.11}$$

where p_1 denotes the relative price of environmental goods.

Step 3: Derivation of the price elasticity of Hicksian goods demand

Calculating the price elasticity from (A.11) gives eq. (7) in the text from:

$$\frac{\partial C_1^h}{\partial p_1}\frac{p_1}{C_1^h} = \sigma\left[\frac{\gamma^{\sigma}p_1^{1-\sigma}}{(1-\gamma)^{\sigma}+\gamma^{\sigma}p_1^{1-\sigma}}\right] \equiv \sigma\theta_{E,1},$$

Appendix B: Derivation of eq. (3)

As mentioned in the text, efficient allocation requires the relative price of environmental goods in period 1 to be equal to the marginal rate of substitution between environment and goods consumption, MRS_{E_1,C_1}. In turn, using the definition of life-time utility in Appendix A:

$$p_1 = MRS_{E_1,C_1} = \frac{V_{E_1}}{V_{C_1}} \tag{B.1}$$

Further, from (B.1) follows:

$$1+g_p \equiv 1+\frac{p_1-p_o}{p_0} = \frac{p_1}{p_0} = \frac{V_{E_1}/V_{C_1}}{V_{E_0}/V_{C_0}} = \frac{V_{C_0}/V_{C_1}}{V_{E_0}/V_{E_0}} = \frac{MRS_{C_0,C_1}}{MRS_{E_0,E_1}}. \tag{B.2}$$

In turn, the consumption discount rate and the environmental discount rate are defined as:

$$r_{C_0,C_1} \equiv MRS_{C_0,C_1} - 1\,, \tag{B.3}$$

$$r_{E_0,E_1} \equiv MRS_{E_0,E_1} - 1\,. \tag{B.4}$$

Combining (B.3) and (B.4) with (B.2) yields eq. (3) in the text.

Appendix C: Derivation of eq. (5) and (6)

From $V = U(C_0,E_0) + \left[1/(1+\rho)\right]U(C_1,E_1)$ follows:

$$dV = U_{C_0}\Delta C_0 + \left(\frac{U_{C_1}}{1+\rho}\right)\Delta C_1 \;\Rightarrow\; -\frac{\Delta C_1}{\Delta C_0} = \frac{U_{C_0}}{\left[1/(1+\rho)U_{C_1}\right]}, \tag{C.1}$$

$$dV = U_{E_0}\Delta E_0 + \left(\frac{U_{E_1}}{1+\rho}\right)\Delta E_1 \;\Rightarrow\; -\frac{\Delta E_1}{\Delta E_0} = \frac{U_{E_0}}{\left[1/(1+\rho)U_{E_1}\right]}. \tag{C.2}$$

Further, the discount rates are defined as:

$$r_{C_0,C_1} \equiv MRS_{C_0,C_1} - 1 = -\frac{\Delta C_1}{\Delta C_0} - 1, \qquad \text{(C.3)}$$

$$r_{E_0,E_1} \equiv MRS_{E_0,E_1} - 1 = -\frac{\Delta E_1}{\Delta E_0} - 1. \qquad \text{(C.4)}$$

In turn, combining (C.3) and (C.4) with (C.1) and (C.2) yields:

$$r_{C_0,C_1} = \frac{U_{C_0} - \frac{U_{C_1}}{1+\rho}}{\frac{U_{C_1}}{1+\rho}} = \frac{\left(\frac{1+\rho-1}{1+\rho}\right)U_{C_1}}{\frac{U_{C_1}}{1+\rho}} - \frac{U_{C_1} - U_{C_0}}{\frac{U_{C_1}}{1+\rho}} = \rho - \frac{\Delta U_C}{\left[1/(1+\rho)\right]U_{C_1}}, \qquad \text{(C.5)}$$

$$r_{E_0,E_1} = \frac{U_{E_0} - \frac{U_{E_1}}{1+\rho}}{\frac{U_{C_1}}{1+\rho}} = \frac{\left(\frac{1+\rho-1}{1+\rho}\right)U_{E_1}}{\frac{U_{E_1}}{1+\rho}} - \frac{U_{E_1} - U_{E_0}}{\frac{U_{E_1}}{1+\rho}} = \rho - \frac{\Delta U_E}{\left[1/(1+\rho)\right]U_{E_1}}, \qquad \text{(C.6)}$$

where $\Delta U_C = U_{C_1} - U_{C_0}$ and $\Delta U_E = U_{E_1} - U_{E_0}$. In turn, using $\Delta U_C = \bar{U}_{CC}\Delta C + \bar{U}_{CE}\Delta E$, $\Delta U_E = \bar{U}_{EE}\Delta E + \bar{U}_{EC}\Delta C$ and eq. (4) in the text, (C.5) and (C.6) can be approximated with eq. (5) and (6) in the text.

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