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# A Model of Eco-Efficiency and Recycling

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## Abstract

This paper presents the model of an economy subject to the mass conservation principle. The economic system is related to the environment by a flow of virgin materials into the economy, and by the diffusion of waste into the environment. Eco-efficiency contributes to reducing material waste in all processes. Recycling can reduce the diffusion of waste by feeding it back into the economy. Human capital enhances productivity, eco-efficiency and the quality of all kinds of outputs. Recycling and human capital formation use productive factors and are rooted therefore, as all other activities, in the material basis of the economy. The paper studies an optimal material state of society.

**JEL:** D90, O30, O41, Q00

**Keywords:** Eco-Efficiency, Recycling, Materials Balances, Material Flows, Human Capital

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## 1. INTRODUCTION

The current anthropogenic pressure on the natural environment calls for effective control of the material dimensions of human activities in advanced industrial societies (van den Bergh 1996; Adriaanse et al. 1997; Fischer-Kowalski 1998; Fischer-Kowalski and Hüttler 1998; Ayres 1999b; Bouman et al. 2000). Different possible technological responses to the task of controlling anthropogenic material flows have been discussed in the literature as e.g.: dematerialisation, eco-efficiency, recycling, industrial ecology and industrial metabolism.

Although the analysis of physical constraints on economic activities (Ayres 1998, 1999a; Cleveland and Ruth 1997; Ruth 1993, 1999) is a much debated issue, the development of models combining physical insights with specific tools of economic analysis still remains a very fragmentary field of research, since existing economic-physical models make very different assumptions and focus on very different aspects of the complex interaction between economic and physical analysis.

Smith (1972) applies the law of mass conservation to recycling in a model without physical capital. He develops economic conditions for complete or zero recycling. Van den Bergh (1996) presents an experimental model, based on materials balances. Waste can be emitted, recycled or stored. Environmental quality depends on the stocks of renewables and on the stock of pollutants. Simulations are performed with exogenous scenarios. Di Vita (1997) studies the effects of recycling of imported raw materials on the balance of payments and on employment in an open economy. Huhtala (1999) investigates recycling as an alternative to resource extraction. Recycling reduces pollution, but labour is the only input and physical capital does not come into the picture. Nakamura (1999) applies an input-output approach to the study of waste recycling in a static setting without technical progress. Di Vita (2001) focuses on the effects of recycling on the rate of growth. His model includes technical progress, which is endogenously generated by research. Welfare is negatively affected by the dimension of the waste stock, but not by the scale of economic activities. Hosoda (2001) applies the corn-guano model of the post-Sraffian school (Bidard and Erreygers 2001) to an analysis of recycling. The residuals of a first process are used as inputs to a second

process. In this way, waste is completely absorbed and unlimited growth is possible, even after exhaustion of landfills. Eichner and Pethig (2001) present a labour-only model based on mass conservation with recycling activities and waste treatment before disposal. They investigate waste markets and market failures in the waste sector. Highfill and Mc Asey (2001) study recycling as an alternative to landfilling in the framework of a growing economy. Economic growth is exogenous and no interrelationship between physical capital, recycling and technical progress is therefore addressed. Eichner (2005) applies a labour-only partial equilibrium model to the analysis of imperfectly competitive markets in the recycling sector. André and Cerdá (2006) analyse the intertemporal shift in the proportion between recyclable and non-recyclable production inputs in a dynamic economy, not constrained by the mass conservation principle and without capital accumulation and technical progress. An overview of applied materials flows models is given in Bouman et al. (2000).

The present paper attempts to study material flows in an economic model, constrained by the mass conservation principle, and in which technology is determined by human capital. The flows considered are the flows between the natural environment and the economy and the material flows within the economy. The stocks are: the stock of accumulated emissions and the stock of materials temporarily frozen within the economy in the shape of physical capital. Physical capital occupies natural spaces and determines the material scale of the economy. If a recycling sector exists, materials are re-directed from the flow of waste back into the economy as a second kind of material inputs, which is added to the flow of virgin materials, directly extracted from the environment.

There is some ambiguity in the literature on the nature of this second kind of material inputs (Converse 1997; Ayres 1999a). Can waste be recycled after diffusion into the environment, or is a previous sequestration of waste before diffusion necessary for recycling activities? If diffused waste is the source of recycling, the difference between virgin materials and recycled materials disappears, since both would have to be extracted from the environment. I shall adopt the view of a combined sequestration-recycling activity, because I assume that capturing waste materials before dissipation is

economically more convenient than retrieving materials after dissipation. In this case, virgin materials and recycled materials are different inputs, since the first originate in nature, while the second originate from sequestration activities. This implies, that recycling is a two-stage process: sequestration comes first and recycling in the stricter sense follows, when materials are extracted, or 'mined' out of sequestered materials and transformed into inputs to final output.

The material structure of the economy is determined by knowledge and technology, and the level of society's technological capabilities is measured in this paper by human capital. I shall consider three channels through which human capital influences the material basis of human activities.

First of all, human capital contributes to an increase in productivity. This role of human capital has been extensively investigated in economic theory (Lucas 1988; Romer 1989) and will also be considered here. Productivity gains imply however a rising throughput of materials, which has to be accounted for, both backwards, as an increase in material requirements, and also forwards, as a growing production of waste.

A second effect of knowledge is to contribute to a more efficient use of materials in the economic process. Human capital can contribute to reduce material losses in the process of transformation of material inputs into useful outputs (Reijnders 1998; Schmidt-Bleek 1993, 1997; Weizsäcker et al. 1997). In this way, human capital enhances eco-efficiency and reduces the environmental impacts of economic activities.

A third effect is on consumption. Consumption is not only a material, but also a cultural, aesthetic and social process and is therefore fundamentally influenced by the general level of accumulated social knowledge (Cogoy 1999).

In this paper I shall study the optimal material state of the economy in stock-flow equilibrium. This means, that the social planner chooses the optimal level of stocks, human capital included. The optimal level of knowledge is not infinite, since human capital requires a material infrastructure supporting its operations, and is therefore constrained by environmental considerations, as are all physical stocks. For this reason, the optimal level of knowledge is endogenous in this paper. The description of a

transitional dynamic path, leading to the optimal material state of the economy is outside the scope of the present paper. Sections 2 and 3 describe the model and derive first-order conditions for optimality. Sections 4 and 5 investigate a simplified reference case and offers an analytical discussion of the solution. The final section concludes and formulates some warnings.

## 2. THE MODEL

### 2.1. *Material stocks and flows*

The relationship between the environment and the economy is described in this paper by focussing on two material stocks, and on the flows between them.  $K$  is the physical capital stock, and  $D$  are accumulated emissions.  $S$  is the amount of materials extracted by human activities from the natural environment. By definition:

$$S = K + D \tag{1}$$

Although  $S$  is nothing more than the negative image of  $K + D$ , it is given a positive sign in (1). Equation (1) states that human activities displace materials from the natural environment into two types of sinks: physical capital and accumulated anthropogenic emissions.

Now consider flows. (Upper case notation denotes stocks, lower case denotes flows.)  $v$  is the flow of virgin materials currently extracted from the natural environment in order to be processed by the economic system,  $e$  is the flow of emissions, and  $a$  is the flow of materials reabsorbed by natural regeneration from the stock of discharged materials back into natural processes. In stock equilibrium (constant stocks):

$$v = a = e \tag{2}$$

Equation (2) states that in stock equilibrium virgin materials inputs into the economy must be equal to dispersed waste and to natural absorption.

I assume that absorption is a linear function of the stock of pollutants:

$$a = \tau D \qquad 0 < \tau < 1 \tag{3}$$

where  $\tau$  is the rate of absorption. From (2) and (3):

$$e = \tau D \quad (4)$$

$$v = \tau D \quad (5)$$

A constant capital stock requires:

$$y - c = \delta_K K \quad (6)$$

where  $y$  are materials embodied in final output,  $c$  is aggregate physical consumption, and  $\delta_K K$  is linear capital depreciation. This is the familiar equation, stating that net investment is zero if gross investment is equal to capital depreciation. It has to be interpreted here as a materials balance equation, since all quantities are expressed in mass units. From the point of view of materials balances the role of capital on consumption is a negative one, since a larger capital stock will require a larger flow of materials replacing worn-out capital. Capital is not only a container of materials however, but also a productive factor, and this role will be studied in the next section. Virgin materials are extracted from the environment and processed up to the point that they may serve as inputs to the production of final output. From the materials balance point of view, the output of processed and refined virgin materials is a fraction of the amount of materials extracted from the environment:

$$m_v = \eta v \quad 0 \leq \eta \leq 1 \quad (7)$$

where  $m_v$  are useful materials, transformed and refined out of the flow of virgin materials  $v$ , and  $\eta$  is an eco-efficiency coefficient, depending on human capital. If  $\eta$  is equal to one, the process is perfectly efficient and no materials are lost in transformation. If  $\eta$  is equal to zero, the process is perfectly inefficient, no output comes out of the process, and all inputs are transformed into waste. Waste from extraction is obviously:  $(1 - \eta) v$ .

$\eta$  is a complex measure of eco-efficiency, since it evaluates more than one aspect of materials efficiency with one variable only. If fewer materials are lost in transformation,  $\eta$  will rise. But  $\eta$  will also rise, if process waste is directly channelled as input to other

processes. In other words,  $\eta$  measures materials efficiency both at plant level and also at the level of interconnected plants (industrial ecology and industrial metabolism, cf. Ayres 1989; Ayres and Simonis 1994; Ayres and Ayres 1996; Erkman 1997). If materials are discarded from one production plant and directly channelled to another, they cannot be considered as waste in a strict sense. The definition of waste is confined in this paper to those materials which are either emitted into the environment, or sequestered for future recycling. If industrial ecology were perfectly successful, process waste would be reduced to zero and  $\eta$  would be equal to one. Depreciating capital and consumption would then be the only sources of waste.

If a recycling activity exists, one fraction of total waste is sequestered and directed to the recycling sector, in a similar way, as virgin material inputs are extracted from the environment and submitted to the process of refining and transformation. Useful recycled materials are a fraction of the materials “mined” from sequestered waste:

$$m_R = \eta r \quad (8)$$

where  $m_R$  are recycled materials, suitable for serving as inputs to the final output sector and  $r$  is the flow of materials sequestered from waste. I assume that the eco-efficiency coefficient  $\eta$  is the same in all processes. Waste from the recycling process is therefore:  $(1 - \eta)r$ .

I assume that society chooses how to divide waste between dispersion and sequestration. Such a choice is a choice under technological constraint however, since waste materials can only be sequestered, if capital and labour are allocated to this purpose. Recycling substitutes virgin material inputs. At the same time the scale of the economy is affected, since recycling capital will be required. If society chooses to recycle, waste of different types will enter the sequestration process. It is certainly not meaningful to mix all sorts of waste and have a uniform mixture of materials out of which recycled materials can be “mined”. It will be probably more reasonable to have separate storage facilities for different types of materials (Craig 2001). Bent nails will have to be straightened, and not ground and mixed with sand before extracting iron out of the mixture. The recycling sector, as it is modelled in the present paper, uses capital



and labour in order to reprocess waste of different types, collected in different sequestration facilities. I assume that these operations deliver processed materials of the same quality as the output of the virgin materials sector, so that processed virgin materials and processed waste are perfect substitutes.

Complete recycling implies:  $v = a = e = 0$  and therefore  $D = 0$ . It will be shown in section 5, that this option is unlikely to be economically optimal. A consequence of this is that the issue of complete recycling (Bianciardi et al. 1993, 1996; Converse 1996, 1997; Washida 1998) turns out to be of little economic relevance. Even if complete recycling were technically possible, it can hardly be economically meaningful.

The same reasoning as before also applies to final output, where refined materials (virgin and recycled) are transformed to useful goods:

$$y = \eta (m_v + m_r) \quad (9)$$

Waste in the final output sector is:  $(1 - \eta)(m_v + m_r)$ .

From (7) to (9) we get:

$$y = \eta^2 (v + r) \quad (10)$$

Equation (10) defines output in terms of eco-efficiency and the material flows into the economy. The quadratic exponent of  $\eta$  for final output obviously follows from the assumption, that two stages are required: materials refinement (virgin and/or recycled) and final production. At each of these stages some materials are lost.

The relationship between material stocks and flows between the economy and the natural environment and within the economy is represented in Figure 1:

Figure 1

It can be seen from Figure 1 that there are in this model three sources of waste. A first source of waste is in the transformation process, where materials are moulded into the desired shape: if some materials are 'lost', while others are given a useful economic shape, these losses represent a source of waste.  $v + r - y$  is waste from transformation, since  $v + r$  are materials (virgin and recycled) entering the transformation process, and

$y$  is the result of this process. The difference has been ‘lost’ in transformation. A second source of waste is consumption. Consumption is a materials processing sector of a particular kind: it transforms one fraction of final output into waste and yields welfare as its specific immaterial output<sup>1</sup>. Therefore, eco-efficiency in consumption is zero by definition. A third source is from physical capital depreciation ( $\delta_K K$ ).

The materials balance equation for waste is therefore:

$$r + e = v + r - y + c + \delta_K K \quad (11)$$

It can be easily seen, that this equation can be directly derived from (2) and (6). This implies, that the transformation of materials is completely accounted for: no piece of matter disappears or comes in unexplained at any point in the model.

## 2.2. *Materials processing sectors*

I consider three materials processing sectors: a) extraction and refining of virgin materials, b) sequestration and recycling of waste and c) production of final output<sup>2</sup>.

From the economic point of view, processes are described by production functions, which establish a relationship between material flows and productive factors.

Production functions are not materials balance equations, but rather flow-regulators: the larger the quantity of productive factors employed, the more materials will flow through the process, the proportion between useful output and waste being determined by eco-efficiency. I consider three factors: physical capital, measured in tons as in the preceding section, labour, measured in time-units, and human capital, enhancing capital productivity. Clearly, tons of physical capital make sense in materials balance equations, but they make less sense in economic equations, since a ton of capital mass may provide different productive services, depending on the ‘shape’ or ‘form’ given to capital-matter by historically accumulated engineering knowledge. For this reason, economic equations must contain some measure of the quality of this ‘shape’. I shall measure the quality of capital mass by a productivity index  $\pi$ , depending on human capital.  $K_i$  is therefore mass capital in sector  $i$ , whereas  $\pi K_i$  is quality capital in sector  $i$ .  $\pi$  is assumed to be the same in all sectors.

From (7) we know that refined virgin materials are equal to  $\eta v$ . The production function for refined virgin materials can therefore be written as:

$$\eta v = (\pi K_V)^\lambda (\gamma l_V)^{1-\lambda} \quad 0 < \lambda < 1 \quad (12)$$

where  $K_V$  is physical capital (in tons), applied to the extraction and refinement of virgin materials,  $l_V$  is labour employed in the same sector, and  $\gamma$  is a productivity coefficient for labour, which is considered to be given and the same in all sectors. Assuming that  $\pi$  depends on human capital, whereas  $\gamma$  is constant, is tantamount to saying that technical progress is capital-mass-saving.

Recycled materials are equal to  $\eta r$ . The economic equation for the recycling sector can be written therefore as:

$$\psi \eta r = (\pi K_R)^\lambda (\gamma l_R)^{1-\lambda} \quad \psi > 1 \quad (13)$$

$K_R$  and  $l_R$  are capital and labour in the recycling sector.  $\psi$  measures the additional effort required for recycling as compared to virgin materials extraction. Recycling is assumed to be more costly than virgin materials extraction because it consists of two stages: sequestration and materials processing. Nevertheless, the additional costs of recycling may be worth incurring because of the positive impacts on the environment. If  $\psi$  were equal to one, there would be no virgin materials extraction.

Since output is equal to  $\eta^2(v+r)$  the production function for final output is:

$$\eta^2(v+r) = (\pi K_P)^\lambda (\gamma l_P)^{1-\lambda} \quad (14)$$

$K_P$  and  $l_P$  are capital and labour in final output.

With (12) to (14) it is implicitly assumed, that the production function is the same in all sectors.

### 2.3. Consumption

Like other material flows, consumption is also measured in tons. In the same way as one physical unit of capital can be more or less productive, depending on how skilfully it is shaped, so can a physical unit of consumption deliver a higher contribution to welfare,

if consumption goods are more conveniently ‘shaped’. In this way, the benefits of physical consumption are enhanced by higher quality. Physical per capita consumption ( $b$ ) is:

$$b = \frac{c}{N} \quad (15)$$

$N$  is the number of identical individuals of a given population. Qualified per capita consumption ( $z$ ) is:

$$z = \mathcal{G}b \quad (16)$$

where  $\mathcal{G}$  is a quality index, depending on human capital, and measuring the ‘shape’ given to consumption mass. Using (6), (10), (15) and (16) qualified per-capita consumption can be also expressed as a function of the quality index  $\mathcal{G}$ , eco-efficiency, the flow of materials and the physical capital stock:

$$z = \frac{\mathcal{G}}{N} [\eta^2 (v + r) - \delta_K K] \quad (17)$$

Since both quality indexes for physical capital ( $\pi$ ) and consumption goods ( $\mathcal{G}$ ) depend on human capital only, the quality of all goods does not depend on whether they are produced from virgin or recycled materials. Processed virgin materials and recycled materials can be indifferently used therefore as perfect substitutes in final output production.

The model discussed in this paper is the model of an affluent society. This means, that consumption is well above the level of survival and that its main role consists of enhancing life enjoyment. In this case, consumption mass can be substituted by quality, and  $z$  is a more adequate measure for the benefits of consumption than  $b$ . There is a lower bound to this kind of substitution however, since physical per capita consumption cannot decline below a minimum level of calories, necessary for survival.

#### 2.4. Human capital

Knowledge begins to shape materials already at extraction and recycling level, when materials to be extracted and recycled are selected and shaped in such a way, as they can best serve their purpose further down in the production process. The same is true of

eco-efficiency: already at the stage of extraction, materials can be selected in such a way, that losses at later stages of the process can be minimized. Knowledge thus permeates all stages of the economic process, although its results are only measurable when output reaches its final destination as capital or consumption good.

The productivity, eco-efficiency, and consumption quality coefficients  $\pi$ ,  $\eta$  and  $\vartheta$  are determined by human capital.

$$\pi = \pi(H) \qquad \pi' > 0 \qquad \pi'' < 0 \qquad (18)$$

$$\eta = \eta(H) \qquad \eta' > 0 \qquad \eta'' < 0 \qquad (19)$$

$$\vartheta = \vartheta(H) \qquad \vartheta' > 0 \qquad \vartheta'' < 0 \qquad (20)$$

where  $H$  is human capital.  $\eta$  has a logical upper bound of one, if perfect eco-efficiency is considered to be possible. Otherwise, the upper bound of eco-efficiency will be lower than one.  $\pi$  and  $\vartheta$  may, or may not, have empirical upper bounds.

Equations (18) to (20) imply that knowledge is treated as an externality in this paper: the *same* stock of knowledge increases productivity in all capital using sectors; at the same time it raises eco-efficiency and consumption quality.

Assuming that eco-efficiency and commodity quality are jointly enhanced by human capital accumulation is obviously a strong assumption, since negative cross effects may occur in the real world. A shaped piece of marble, a statue for example, is more likely to produce aesthetic pleasure on the beholder than an unshaped piece of the same matter. Sculptured marble will “loose” however more matter in transformation than unsculptured one, and eco-efficiency will be lower as a consequence of an improved aesthetic quality of marble, so that eco-efficiency is negatively affected by quality. In a baroque Pietà, for example, quality is high, and eco-efficiency is likely to be rather low. Assuming a joint progress of eco-efficiency and quality means that, in the aggregate, as product quality improves, also the proportion of materials finding any kind of useful employment (either as statue or as marble dust) will increase. The assumption of joint progress at aggregate level is therefore less unrealistic than it may seem for individual processes.

Any given stock of knowledge depreciates through time and requires a continual maintenance and renovation effort in scientific institutions. Knowledge generates therefore maintenance costs in terms of capital and labour. I shall model knowledge maintenance and renovation as:

$$\varphi H = (\pi K_H)^\lambda (\gamma l_H)^{1-\lambda} \quad \varphi > 0 \quad (21)$$

where  $K_H$  and  $l_H$  are capital and labour requirements in the maintenance of knowledge and  $\varphi$  is a coefficient defining human capital depreciation and maintenance costs.

Equation (21) implies that the only material requirement in the science sector is physical capital, and ignores therefore other types of material inputs (e.g.: paper, fuel, etc.). The material body of knowledge (e.g.: paperboard, paper and ink for books, hard disks for electronic storage, etc.) is also neglected. From this it follows, that depreciating capital is the only kind of waste arising from the research sector. The reason for these simplifying assumptions is that the use of physical capital in the knowledge sector is sufficient to generate a material constraint on knowledge, and the introduction of other material constraints would only complicate, but not substantially modify the picture. Equation (21) also implicitly defines a measure of knowledge in terms of opportunity costs: the maintenance of one unit of knowledge requires the same quantity of factors as  $\varphi$  units of output. In this way, human capital can be measured in terms of forsaken output, i.e. in terms of that quantity of output which could have been produced if capital and labour were employed in final output, instead of being employed in the human capital sector.

In the following discussion of the model I shall prefer to invert (19) and express human capital and quality coefficients in terms of eco-efficiency:

$$H = H(\eta) \quad (22)$$

$$\pi = \pi(\eta) \quad (23)$$

$$\vartheta = \vartheta(\eta) \quad (24)$$

## 2.5. Model reduction

There are in the above described model four types of physical capital and labour in extraction, recycling, final output and human capital maintenance. Total capital is therefore equal to:

$$K = K_V + K_R + K_P + K_H \quad (25)$$

One unit of labour is inelastically supplied by each of the identical individuals in the population, so that total labour supply is equal to  $N$ . Therefore:

$$N = l_V + l_R + l_P + l_H \quad (26)$$

Equations (12) to (14) and (21) can be reduced to a single equation, by noticing that with the same technologies everywhere, efficient capital and labour allocation requires:

$$\frac{K_V}{l_V} = \frac{K_R}{l_R} = \frac{K_P}{l_P} = \frac{K_H}{l_H} \quad (27)$$

Using (27) one obtains:

$$(\pi K)^\lambda (\gamma N)^{1-\lambda} = \eta(v + \psi r) + \eta^2(v + r) + \phi H \quad (28)$$

Equation (28) is the aggregate production function of the economy. It states that aggregate factors  $(\pi K)^\lambda (\gamma N)^{1-\lambda}$  can be applied to three types of productive effort.  $\eta(v + \psi r)$  represents extraction and refinement of materials (virgin and recycled), suitable to enter the final output process.  $\eta^2(v + r)$  represents final output.  $\phi H$  are services to the knowledge sector, measured in terms of forsaken output. The assumption that production functions are the same in all sectors implies that one ton of refined virgin materials requires the same effort as one ton of final output and as  $\frac{1}{\psi}$  tons of recycled materials. The factor allocation to final output production, relative to materials processing (virgin and recycled), i.e. the expression:  $\frac{\eta(v + r)}{v + \psi r}$  increases with  $\eta$ . This means, that with rising eco-efficiency factors are shifted from materials processing to final output. Low eco-efficiency implies heavy losses of materials during processing. As a consequence, a greater portion of aggregate factors will have to be invested in the

materials processing sectors, and less will be left for final output. For this reason, eco-efficiency plays an important role at aggregate level, since, given aggregate capital and labour, the system will be more productive if a greater portion of aggregate factors is employed in final output production.

Equations (5), (17) and (28), together with technology functions (22) to (24) describe a feasible stock-flow equilibrium, constrained by the mass conservation principle.

## 2.6. Preferences

I assume that welfare is determined for each individual by the stream of qualified per-capita consumption  $z$ , and by the state of the environment. The state of the environment is a public good and affects all of the identical  $N$  individuals in the same way.

Physical capital affects welfare in two ways: a direct, and an indirect one. The indirect effect has been extensively analysed in economic theory: capital enhances labour productivity and contributes in this way to output and consumption. On the other hand however, physical capital encroaches upon natural spaces, spoils landscapes, destroys biotopes, and for this reason also has direct negative effects on welfare. Physical capital is, in other words, a necessity, not a pleasure<sup>3</sup>. This suggests the idea of an optimal level of physical capital, which will have to be determined by a compromise between its positive contribution to production and its negative effects on the material scale of the economy. The stock of accumulated emissions  $D$  is another argument of the welfare function.

With these premises in mind, preferences can be modelled as:

$$U = U(z, K, D)$$

$$U_z > 0, \quad U_{zz} < 0, \quad U_K < 0, \quad U_{KK} < 0, \quad U_D < 0, \quad U_{DD} < 0 \quad (29)$$

At this point, the difference between physical and human capital may be summarized as follows. Both types of capital enhance productivity in all sectors, in which they are employed. Physical capital directly encroaches upon the environment by occupying natural spaces, whereas human capital only indirectly affects the environment, insofar as physical capital is needed for the maintenance of knowledge.



### 3. THE SOCIAL PLANNER'S PROBLEM

#### 3.1. The problem

In order to determine the optimal stationary material state of society, the social planner maximizes (29), subject to (5), (17), (28) and technology functions (22) to (24). It is intuitive, that optimal human capital is finite in the model presented in this paper. Maintenance and renewal of depreciated human capital require capital and labour. Human capital is limited by the same environmental constraints limiting the quantity of physical capital and is endogenously determined therefore along with the other stocks. In order to give an intuitive interpretation of the first-order conditions, I shall use a more general form for (17) and (28):

$$z = z(\vartheta, \eta, v, r, K) \quad (17')$$

$$F(\pi, \eta, v, r, K, H) = 0 \quad (28')$$

#### 3.2. First order conditions

First-order conditions for this problem are:

$$U_z z_v = \frac{F_v}{F_K} (U_K + U_z z_K) - \frac{U_D}{\tau} \quad (30)$$

$$U_z z_r = \frac{F_r}{F_K} (U_K + U_z z_K) \quad (31)$$

$$U_z (z_\eta + z_\vartheta \vartheta') = \left( \frac{F_\eta + F_H H' + F_\pi \pi'}{F_K} \right) (U_K + U_z z_K) \quad (32)$$

First-order condition (30) equates marginal benefits to consumption from an increase in  $v$  to marginal environmental degradation deriving from induced increases in stocks  $K$  and  $D$ .  $U_z z_K$  is the direct negative marginal effect on consumption, deriving from the burden of capital depreciation. Similarly, (31) compares benefits and costs of a marginal increase in recycling. The lack of a second member on the right hand side is due to the fact that recycling does not increase the pollution stock. Equation (32) compares the

consumptive benefits deriving from a marginal increase in eco-efficiency to the negative effects of an induced marginal increase in physical capital. Of course, this effect is mitigated by a raise in productivity  $\pi$ .

First order conditions, together with the problem's constraints, determine the optimal values for variables:  $K$ ,  $D$ ,  $H$ ,  $v$ ,  $r$ ,  $z$ ,  $\pi$ ,  $\eta$  and  $\mathcal{G}$ .

#### 4. REFERENCE CASE

Given the number of variables and the non-linearity of the system, finding a solution may become quite an intricate business. In the next section I shall present a graphic discussion of a simplified reference case, which allows to study the basic structure of the model.

For the simplified reference case I shall make following assumptions:

*a) Non depreciating physical capital*

Non-depreciating physical capital implies:

$$\delta_K = 0 \quad (33)$$

*b) Technology functions*

I shall assume that the upper bound of  $\eta$  is 1, and model eco-efficiency as:

$$\eta = \frac{H}{\alpha + H} \quad (34)$$

which implies:

$$H = \alpha \frac{\eta}{1-\eta} \quad H' = \alpha \frac{1}{(1-\eta)^2} \quad (35)$$

Productivity and consumption quality are assumed to be bounded and fixed multiples of eco-efficiency:

$$\pi = \bar{\pi} \eta \quad \pi' = \bar{\pi} \quad (36)$$

$$\mathcal{G} = \bar{\mathcal{G}} \eta \quad \mathcal{G}' = \bar{\mathcal{G}} \quad (37)$$

$\bar{\pi}$  and  $\bar{g}$  are arbitrarily chosen upper bounds of productivity and consumption quality.

*c) Preferences*

I shall model utility as a logarithmic function of qualified consumption, net of quadratic environmental damage from stocks:

$$U = q_z \log z - \frac{1}{2}(q_K K^2 + q_D D^2) \quad (29')$$

where  $q_i$  are weights of the arguments in the utility function. With these assumptions (17), (28) and (30) to (32) become:

$$z = \frac{\bar{g} \eta^3}{N} (v + r) \quad (38)$$

$$(\bar{\pi} \eta K)^\lambda (\gamma N)^{1-\lambda} = \eta(v + \psi r) + \eta^2(v + r) + \alpha \varphi \frac{\eta}{1-\eta} \quad (39)$$

$$\lambda (\bar{\pi} \eta K)^\lambda (\gamma N)^{1-\lambda} [q_z \tau^2 - q_D v(v + r)] = q_K \tau^2 \eta (1 + \eta) (v + r) K^2 \quad (40)$$

$$\lambda q_z (\bar{\pi} \eta K)^\lambda (\gamma N)^{1-\lambda} = q_K \eta (\psi + \eta) (v + r) K^2 \quad (41)$$

$$\begin{aligned} 3\lambda q_z (\bar{\pi} \eta K)^\lambda (\gamma N)^{1-\lambda} &= \\ &= q_K \left[ \alpha \varphi \frac{\eta}{(1-\eta)^2} + \eta(1 + 2\eta)v + \eta(\psi + 2\eta)r - \lambda (\bar{\pi} \eta K)^\lambda (\gamma N)^{1-\lambda} \right] K^2 \end{aligned} \quad (42)$$

For a given size of the population, (38) to (42) yield optimal values for  $z$ ,  $v$ ,  $r$ ,  $\eta$  and  $K$ .

For a graphic analysis of the solution, (39) to (42) can be more conveniently manipulated into:

$$q_D (\psi + \eta)v^2 + q_D (\psi + \eta)rv - q_z \tau^2 (\psi - 1) = 0 \quad (43)$$

$$v = \frac{\alpha \varphi (1 - \lambda + \lambda \eta) - (1 - \eta)^2 [(2 + \lambda)\psi + (1 + \lambda)\eta] r}{(1 - \eta)^2 [3\psi - (1 - \lambda) + (1 + \lambda)\eta]} \quad (44)$$

$$\begin{aligned} & \lambda^2 q_D^2 \bar{\pi}^{2\lambda} (\gamma N)^{2(1-\lambda)} (1-\eta)^{2-\lambda} v^2 = \\ & = [q_K \tau^2 (\psi - 1)]^2 \eta^{2(1-\lambda)} [q_z \tau^2 (\psi - 1)(1-\eta) + q_D \alpha \phi v - q_D (\psi - 1)(1-\eta)v^2]^{2-\lambda} \end{aligned} \quad (45)$$

$$\lambda q_D (\psi - 1)(1-\eta)v^2 - \lambda q_D \alpha \phi v + \tau^2 (\psi - 1)(1-\eta)(q_K K^2 - \lambda q_z) = 0 \quad (46)$$

Physical and qualified per-capita consumption are:

$$b = \frac{\eta^2 q_z \tau^2 (\psi - 1)}{N q_D (\psi + \eta) v} \quad (47)$$

$$z = \frac{\bar{g} \eta^3 q_z \tau^2 (\psi - 1)}{N q_D (\psi + \eta) v} \quad (48)$$

The logic of the solution is as follows. Equations (43) and (44) yield an efficient locus of points in  $v/\eta$  space. Each point on the locus is associated with a different level of recycling. Equation (45) identifies the optimal point on the efficient locus (and therefore optimal recycling) for an exogenously given size of the population. Equation (46) can then be solved for optimal physical capital. Equations (47) and (48) determine the values of physical and qualified per-capita consumption.

## 5. A STUDY OF EQUATIONS (43) TO (47)

### 5.1. Efficient locus of non-negative recycling in $v/\eta$ space

Consider first (43) and (44). Setting  $r = 0$  yields:

$$v = \tau \sqrt{\frac{q_z (\psi - 1)}{q_D (\psi + \eta)}} \quad (49)$$

$$v = \frac{\alpha \phi (1 - \lambda + \lambda \eta)}{(1 - \eta)^2 [3\psi - (1 - \lambda) + (1 + \lambda)\eta]} \quad (50)$$

Since the graph of (49) is monotonically declining, whereas the graph of (50) is monotonically rising, the two equations will only have a  $v > 0$ ,  $\eta > 0$  solution, if

$$\tau \sqrt{\frac{q_z}{q_D} \frac{\psi - 1}{\psi}} > \frac{\alpha \phi (1 - \lambda)}{3\psi - (1 - \lambda)} \quad (51)$$

It can be also shown, that if (51) is **not** satisfied, then a solution  $\eta = 0, v > 0, r \geq 0$  exists. Setting  $\eta = 0$  into (43) and (44) yields:

$$v = -\frac{r}{2} + \sqrt{\left(\frac{r}{2}\right)^2 + \tau^2 \frac{q_z(\psi - 1)}{q_D \psi}} \quad (52)$$

$$v = \frac{\alpha\phi(1-\lambda) - (2+\lambda)\psi r}{3\psi - (1-\lambda)} \quad (53)$$

The value of  $v$  in (52) tends to zero, as  $r$  tends to infinity, whereas in (53)  $v$  declines to zero for a finite value of  $r$ . Therefore (53) cuts (54) from above if the inequality in (51) is reversed. From this I conclude that (43) and (44) have **either** a  $r = 0, v > 0, \eta > 0$  **or** a  $\eta = 0, v > 0, r \geq 0$  solution, depending on the direction of the inequality in (51). I shall call this point the **origin** of the solution locus.

For increasing values of  $r$  the crossing points of (43) and (44) generate an efficient locus in  $v/\eta$  space (Figure 2).

Figure 2

Point  $Q$  represents the origin of the locus in the case where (51) holds. If (51) does **not** hold, point  $Q$  is on the vertical axis. Pairs of  $v$  and  $\eta$  on this locus satisfy:

$$v \leq \frac{\alpha\phi(1-\lambda + \lambda\eta)}{(1-\eta)^2 [3\psi - (1-\lambda) + (1+\lambda)\eta]} \quad (54)$$

and also:

$$v \leq \tau \sqrt{\frac{q_z(\psi - 1)}{q_D(\psi + \eta)}} \quad (55)$$

Inequalities (54) and (55) define an area of non-negative recycling in  $v/\eta$  space.

Inserting  $r$  from (43) into (44) yields the equation of this locus:

$$q_D(1-\lambda)(\psi-1)(\psi+\eta)(1-\eta)^2 v^2 - q_D \alpha \phi (\psi+\eta)(1-\lambda+\lambda\eta)v + q_z \tau^2 (\psi-1)(1-\eta)^2 [(2+\lambda)\psi + (1+\lambda)\eta] = 0 \quad (56)$$

Derivation of the implicit function (56) yields:

$$\frac{dv}{d\eta} = \frac{g_1(v, \eta)}{g_2(v, \eta)} \quad (57)$$

where:

$$g_1(v, \eta) = q_D(1-\lambda)(\psi-1)(1-\eta)(2\psi-1+3\eta)v^2 + q_D\alpha\varphi[1+(\psi-1)\lambda+2\lambda\eta]v + q_z\tau^2(\psi-1)(1-\eta)[2(2+\lambda)\psi-(1+\lambda)+(3-\lambda)\eta] \quad (58)$$

$$g_2(v, \eta) = q_D(\psi+\eta)[2(1-\lambda)(\psi-1)(1-\eta)^2v - \alpha\varphi(1-\lambda+\lambda\eta)] \quad (59)$$

It can be seen, that:  $g_1(v, \eta) > 0$ , and that:  $g_2(v, \eta) < 0$  if :

$$v < \frac{\alpha\varphi(1-\lambda+\lambda\eta)}{2(1-\lambda)(\psi-1)(1-\eta)^2} \quad (60)$$

A quick check shows, that if (54) is satisfied, (60) is satisfied as well. This implies, that the locus is monotonically falling.

It can be seen that complete recycling ( $D = v = 0$ ) can only be efficient at  $\eta = 1$ . The reason for this is simple: at  $D = 0$  marginal damage from pollution is zero and there is no incentive therefore to prevent some materials to diffuse into the environment.

### 5.2 The solution point as a function of the size of the population

An easy graphic interpretation of (45) can be given after manipulating it into:

$$\begin{aligned} \lambda^\lambda q_D^2 \bar{\pi}^{2\lambda} (\gamma N)^{2(1-\lambda)} &= \\ &= [q_k \tau^2 (\psi-1)]^\lambda \frac{\eta^{2(1-\lambda)}}{v^\lambda} \left[ \frac{q_z \tau^2 (\psi-1)}{v} + q_D \frac{\alpha\varphi}{1-\eta} - q_D (\psi-1)v \right]^{2-\lambda} \end{aligned} \quad (61)$$

For any value of  $\eta$  such that  $0 < \eta < 1$ , a unique value of  $v$  exists, setting the right hand side of the equation equal to the left hand side, since the right hand side is monotonically declining from infinity to zero as  $v$  increases from zero to the positive value reducing the expression in square brackets to zero. Without the necessity of

calculating derivatives, one can also see, that  $\frac{dv}{d\eta} > 0$ , and that the graph shifts

downwards for rising values of  $N$ .

For the case where (51) holds, the graph of (61) passing through the origin of the locus of efficient points is represented in Figure 3:

Fig 3

If the population increases, the graph shifts downwards. Eco-efficiency increases and virgin material inputs into the economic system are substituted by recycled materials. It must be noted, that the size of the population in the origin of the efficient locus is strictly positive, and recycling is zero. Therefore, if (51) holds, there is for non-negative recycling a lower limit to the size of the population. In the case where (51) does **not** hold, the origin of the efficient locus is on the vertical axis and the size of the population in the origin is zero.

### 5.3. Physical capital

Solving equation (46) for  $v$  yields:

$$v = \frac{\lambda q_D \alpha \varphi \pm \sqrt{(\lambda q_D \alpha \varphi)^2 - 4 \lambda q_D \tau^2 (\psi - 1)^2 (1 - \eta)^2 (q_K K^2 - \lambda q_z)}}{2 \lambda q_D (\psi - 1)(1 - \eta)} \quad (62)$$

For values of  $K \geq \sqrt{\frac{\lambda q_z}{q_K}}$  this equation yields two branches, one ascending, the other descending. The locus of bifurcation points is given by:

$$v = \frac{\alpha \varphi}{2(\psi - 1)(1 - \eta)} \quad (63)$$

Figure 4 represents the graph of (62) together with the locus of bifurcation points (63) and the efficient locus (56).

Figure 4

The graph shows, that physical capital declines as the growth of the population shifts the solution point downwards along the efficient locus. This is because physical capital is substituted by labour and human capital as the population increases.

If physical capital and virgin material inputs both decline, this means, that the state of the environment improves. It may seem strange that in the stationary state the quality of the environment is better with a larger size of the population. This result follows

however from the assumption of good substitutability between consumption and environmental quality in the social welfare function. When individual consumption declines, an improvement in environmental conditions helps reduce the loss in welfare. There is an exception however to the decline of physical capital with an increase of the population. If the origin of the efficient locus is above the locus of bifurcation points, physical capital rises first and later declines as the population increases, as is shown in Figure 5.

Figure 5

#### 5.4. Physical per capita consumption.

Substituting  $N$  from (47) into (45) yields:

$$\begin{aligned} & \left( \lambda q_D^2 \bar{\pi}^2 \right)^\lambda (\gamma q_z)^{2(1-\lambda)} \left[ \tau^2 (\psi - 1) \right]^{2-3\lambda} \left( \frac{\eta}{\psi + \eta} \right)^{2(1-\lambda)} (1-\eta)^{2-\lambda} v^{2\lambda} = \\ & = q_K^\lambda b^{2(1-\lambda)} \left[ q_z \tau^2 (\psi - 1)(1-\eta) + q_D \alpha \varphi v - q_D (\psi - 1)(1-\eta)v^2 \right]^{2-\lambda} \end{aligned} \quad (64)$$

This equation identifies points in  $v/\eta$  space for given values of physical per capita consumption. Any point on the efficient locus is associated with one level of per capita consumption. The graphs of (56) and (64) must therefore cross. How this looks like, depends however on the value of the Cobb-Douglas parameter  $\lambda$ .

The first derivative of (64) is:

$$\frac{dv}{d\eta} = \frac{v}{\eta(1-\eta)(\psi + \eta)} \frac{g_3(v, \eta)}{g_4(v, \eta)} \quad (65)$$

where:

$$\begin{aligned} g_3(v, \eta) &= 2q_D(1-\lambda)\mu(\psi - 1)(1-\eta)^2 v^2 + \\ &+ q_D \alpha \varphi \left[ (4 - 3\lambda)\mu\eta + (2 - \lambda)\eta^2 - 2(1-\lambda)\mu \right] v - 2q_z \tau^2 (1-\lambda)\mu(\psi - 1)(1-\eta)^2 \end{aligned} \quad (66)$$

$$g_4(v, \eta) = 4q_D(1-\lambda)(\psi - 1)(1-\eta)v^2 + q_D \alpha \varphi (3\lambda - 2)v + 2\lambda q_z \tau^2 (\psi - 1)(1-\eta) \quad (67)$$

$v/\eta$  space can be divided in sections, depending on the signs of  $g_3(v, \eta)$  and  $g_4(v, \eta)$ .

##### 5.4.1. The $g_3(v, \eta) = 0$ locus.



The first derivative of  $g_3(v, \eta) = 0$  is:

$$\left. \frac{dv}{d\eta} \right|_{g_3(v, \eta)=0} = -\frac{q_D \alpha \varphi v^2 [2(2-\lambda)\eta + (4-3\lambda)\psi\eta + \lambda\psi]}{2(1-\lambda)\psi(\psi-1)(1-\eta)^3 (q_D v^2 + q_z \tau^2)} \quad (68)$$

The graph of  $g_3(v, \eta) = 0$  is downwards sloping, as is shown in Figure 6.

Figure 6

5.4.2. The  $g_4(v, \eta) = 0$  locus.

If  $\lambda \geq \frac{2}{3}$ ,  $g_4(v, \eta)$  is always positive. If  $\lambda < \frac{2}{3}$  the locus:  $g_4(v, \eta) = 0$  is given by:

$$v = \frac{q_D \alpha \varphi (2-3\lambda) \pm \sqrt{[q_D \alpha \varphi (2-3\lambda)]^2 - 32\lambda(1-\lambda)q_z q_D \tau^2 (\psi-1)^2 (1-\eta)^2}}{8q_D (1-\lambda)(\psi-1)(1-\eta)} \quad (69)$$

Equation (69) consists of two branches. The bifurcation is located at the value of  $\eta$  which sets the expression under the square root equal to zero, i.e.:

$$\eta = 1 - \frac{\alpha \varphi}{4\tau(\psi-1)} \sqrt{\frac{q_D (2-3\lambda)^2}{2q_z \lambda (1-\lambda)}} \quad (70)$$

The expression under the square root rises monotonically from zero to infinity as  $\lambda$  declines from  $\lambda = \frac{2}{3}$  to zero. The bifurcation point shifts therefore from  $\eta = 1$  for

$\lambda = \frac{2}{3}$  to  $\eta = -\infty$  for  $\lambda = 0$ .

Figure 7 represents the graph of the  $g_4(v, \eta) = 0$  locus with  $\lambda < \frac{2}{3}$ .

Figure 7

5.4.3. The division of  $v/\eta$  space.

The graphs of  $g_3(v, \eta) = 0$  and of  $g_4(v, \eta) = 0$  can be combined and generate different patterns, depending on the value of  $\lambda$ . Three possible patterns can be distinguished. a)

If  $\lambda \geq \frac{2}{3}$ , the graph of (64) only depends on the  $g_3(v, \eta) = 0$  locus. I shall call this the case of a “large” value of  $\lambda$ . b) If  $\lambda$  is small enough for the two graphs to interfere, the pattern will be called that of a “small” value of  $\lambda$ . c) In the “intermediate” case,  $\lambda$  is smaller than  $\frac{2}{3}$ , but the two graphs do not interfere.

#### 5.4.4. A “large” value of $\lambda$ .

The graph of  $g_3(v, \eta) = 0$  (pink curve), and the graph of (64) (green curves for two different values of physical per capita consumption) are represented in Figure 8:

Figure 8

The direction of a shift as a consequence of a change of  $b$  can be determined by calculating  $\frac{dv}{db}$  for a given value of eco-efficiency:

$$\frac{dv}{db} = \frac{2(1-\lambda)v}{b} \frac{q_z \tau^2 (\psi - 1)(1 - \eta) + q_D \alpha \phi v - q_D (\psi - 1)(1 - \eta)v^2}{g_4(v, \eta)} \quad (71)$$

With  $g_4(v, \eta) > 0$ , this expression is positive. For this reason, the graph shifts downwards as per capita consumption declines, and this means that with increasing population physical per capita consumption declines.

#### 5.4.5. A “small” value of $\lambda$ .

The division of  $v/\eta$  space with respect to the derivatives and the graph of (64) for three different levels of physical per capita consumption are shown for a “small” value of  $\lambda$  in Figure 9.

Figure 9

The graph of (64) is a loop, and the relevant feature is that an increase in  $b$  will not shift the graph upwards, as in the “large”  $\lambda$  case, but rather contract the loop. This is because the upper part of the loop is in the  $g_4(v, \eta) < 0$  section, whereas the lower part is in the  $g_4(v, \eta) > 0$  section of  $v/\eta$  space. According to (71) therefore, the upper part shifts downwards, and the lower part shifts upwards (and the loop therefore contracts)

as  $b$  increases. This means, that physical per capita consumption increases with increasing population up to the tangency point (point A) between the locus of efficient points and the loop representing (64). For increases in the size of the population beyond the tangency point, obviously physical per capita consumption declines again. As long as per capita consumption and the population both grow, aggregate consumption increases. This can happen without disrupting the environment, because capital is substituted by labour and recycling is increased.

This result seems to be rather counter-intuitive, and requires therefore some comment. The supply of labour in this model is inelastic. For this reason, an increase in the size of the population results in an increase of the labour force. If  $\lambda$  has a “low” value, this means that  $1 - \lambda$ , that is the share of labour in the Cobb-Douglas production function, is “high”. This is true in all sectors where labour is employed, therefore also in research. For this reason, the positive contribution of eco-efficiency can temporarily offset the decline in marginal returns to labour. Finally however, declining marginal returns and a decline in per capita consumption must prevail.

This allows to characterize the logic structure of the present model. For a **given** level of the population the model is bounded by the environment, since physical capital cannot grow beyond a certain level without causing environmental disruption. With an increase in the labour force, there is no environmental disruption, since labour is considered in this model to be an environmentally “clean” factor. The economy is bounded however by declining marginal returns to labour.

#### 5.4.6. The “intermediate” case.

In the intermediate case  $\lambda < \frac{2}{3}$ , but  $g_3(v, \eta) = 0$  and  $g_4(v, \eta) = 0$  do not cross. The graph is represented in Figure 10.

Figure 10

Although the graph is somewhat different, the result is similar to the “large”  $\lambda$  case. On the efficient locus  $g_4(v, \eta)$  is positive, and therefore  $b$  declines as the population increases.

### 5.5. *Carrying capacity*

The present model is built on the assumption that consumption quantity can be substituted by quality. There are obvious lower limits to such a kind of substitution, since, once consumption per head is reduced to a minimum caloric level, a further substitution of quantity with quality is no longer possible. For this reason, there is an upper bound to the size of the population, given by **carrying capacity**, and this means, that there is a point on the efficient locus, below which a growth of the population is no longer sustainable.

## 6. CONCLUSIONS AND SOME WARNINGS

The model discussed in the previous sections investigates a material stock-flow equilibrium, supporting a sustainable economy. The constraints imposed by the environment upon the accumulation of physical and human capital are described in this paper by two basic assumptions. The first is that the mere existence of physical capital negatively affects the environment. Thus, physical capital directly enters the social welfare function with a negative first derivative. Increasing marginal environmental damage prevents physical capital from growing without bounds and requires that human activities should not encroach upon natural capital beyond a reasonable level (Ekins 2003; Ekins et al. 2003). Clearly, the quality of physical capital is enhanced by human capital, and in this way the benefits of capital accumulation can be expanded beyond its physical limits. The second basic assumption is that the aggregate production function is subject to declining marginal returns to human capital, whereas maintenance and depreciation costs of human capital linearly rise. For this reason, human capital cannot grow without limits, and an optimal level of human capital has to be endogenously determined.

Enhancing eco-efficiency is an important social response to the growing encroachment of the economy upon the environment, but high levels of eco-efficiency require a well developed system of knowledge and technology, and knowledge and technology cannot be acquired at zero environmental costs. Productive factors are necessary in order to support human capital maintenance, and the natural environment cannot absorb

unlimited quantities of physical capital. On the other hand, also an increase of the factor labour, which has been optimistically assumed to be an environmentally “clean” factor in this paper, must finally result in declining per capita consumption and in an overshooting of the carrying capacity potential of the natural environment.

Recycling reduces flows of waste materials to the environmental sink, but achieves this by expanding other types of stocks, such as sequestration and recycling capital. For this reason, although recycling can be useful to a certain extent, it cannot fully overcome the basic environmental limitation of the economic system.

The model results, which most diverge from standard intuition may be listed as follows.

- aggregate physical capital may decline as the population increases
- physical per capita consumption may increase with an increase of the population, if the Cobb-Douglas exponent is biased towards labour
- the state of the environment may improve for a larger size of the population, if consumption and environmental quality are good substitutes in the social welfare function
- recycling may be useful, but complete recycling is unlikely to be a meaningful option

A stationary state, in order to be sustainable, must also satisfy some basic ecological conditions that have remained outside the scope of the present paper. A fundamental sustainability condition requires that the chemical elements entering the system must exit the system in the same proportions; otherwise the economic system would work as a filter, increasing over time the concentration of some chemical elements at the expense of others. For this reason, a stationary state, defined at aggregate level, will only be sustainable if dispersed waste is qualitatively recombined by natural processes in such a way as to preserve a balance between nature and the economy for a long time period. If this is not the case, an economic stationary state may turn out to be environmentally disruptive in the long run, and therefore ecologically unsustainable. An important condition also concerns the ecological quality of the economy, and in particular of resources and energy. It is not unrealistic to assume abundant resources in an aggregate model, although individual resources may face exhaustion in a shorter time

period. Obviously, a source of energy is necessary in order to set flows in motion. If some of the materials here described are fossil fuels, they may provide the necessary source of energy. Alternatively, some of the capital goods of this paper may be thought of as consisting of appliances able to capture solar energy. A stationary state based on fossil fuels is however completely different from a stationary state based on solar energy. In a fossil fuel system carbon is extracted from below the earth crust and is emitted into the atmosphere. In the terminology of this paper eco-efficiency is low in such a system, since it is based on carbon dispersion. A fossil system will be capable of producing a stationary state of some duration, if an adequate technology for carbon recovery from the atmosphere becomes available (Holloway 2001). Low eco-efficiency and storage of waste is therefore a characteristic feature of such a system. In a solar economy waste production is low, since it is basically reduced to the wear and tear of solar capital stock. At high levels of eco-efficiency the energy system in this paper may be best thought of therefore as being based on solar energy.

This paper only investigates some economic aspects of material sustainability, but it must be added, that the scientific knowledge on the quality of an economic-ecological equilibrium may change over time and for this reason, the sensitivity of society to environmental warnings and the flexibility to adapt to new environmental insights remains the basic social resource, needed to keep anthropic activities in balance with the natural environment.

## NOTES

<sup>1</sup> I do not consider consumer durables in this paper. I assume that all materials consumed are transformed into waste during the same time period.

<sup>2</sup> I neglect end-of-the-pipe waste treatment and storage, in order to reduce the complexity of the model. Waste treatment can improve environmental conditions however, both in the short and in the long run.

<sup>3</sup> I am only concerned with productive capital in this paper. Matters are obviously different for architectural capital, architected landscapes, gardens, etc. as expressions of aesthetic and cultural values.

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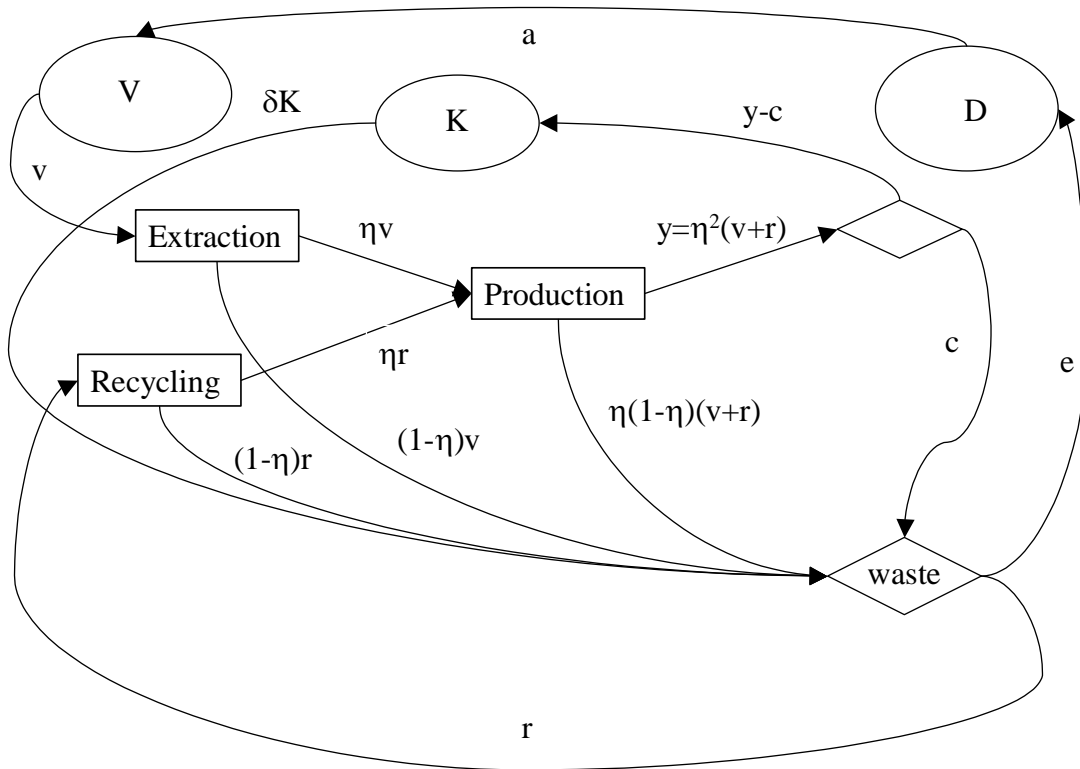
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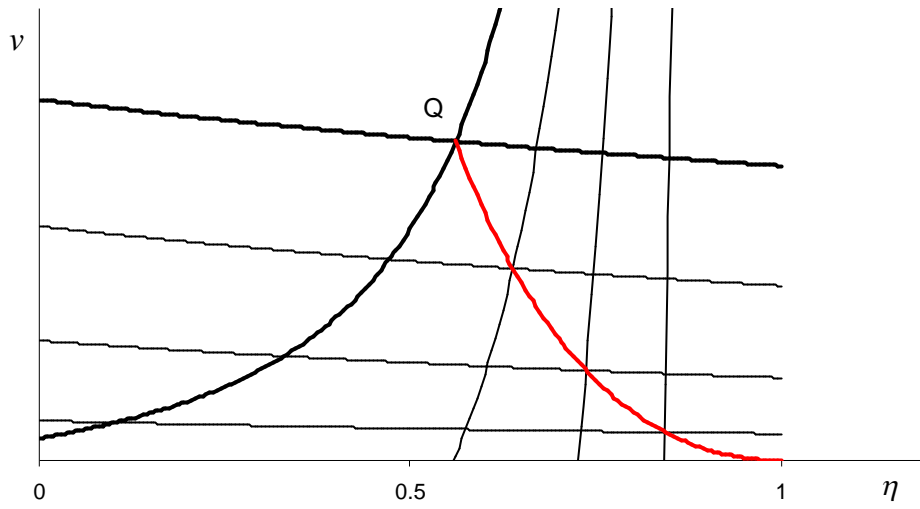
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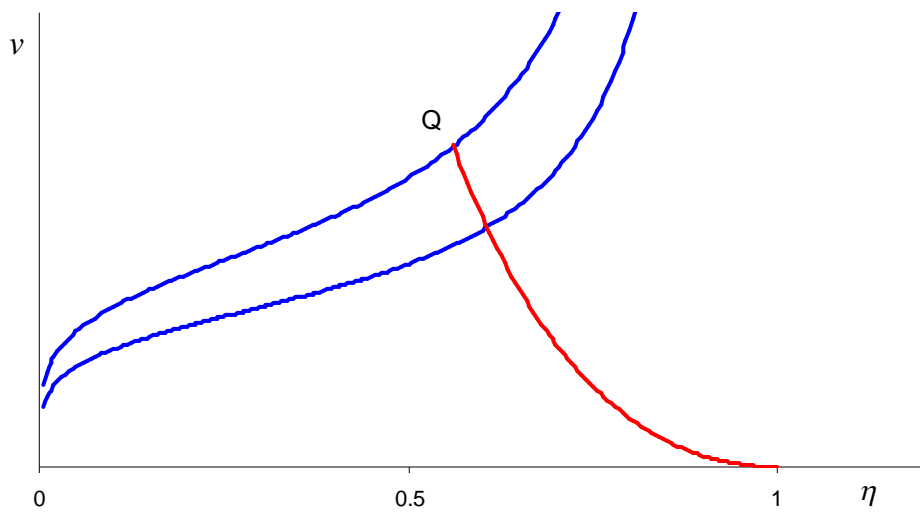
The stock-flow model

Figure 1



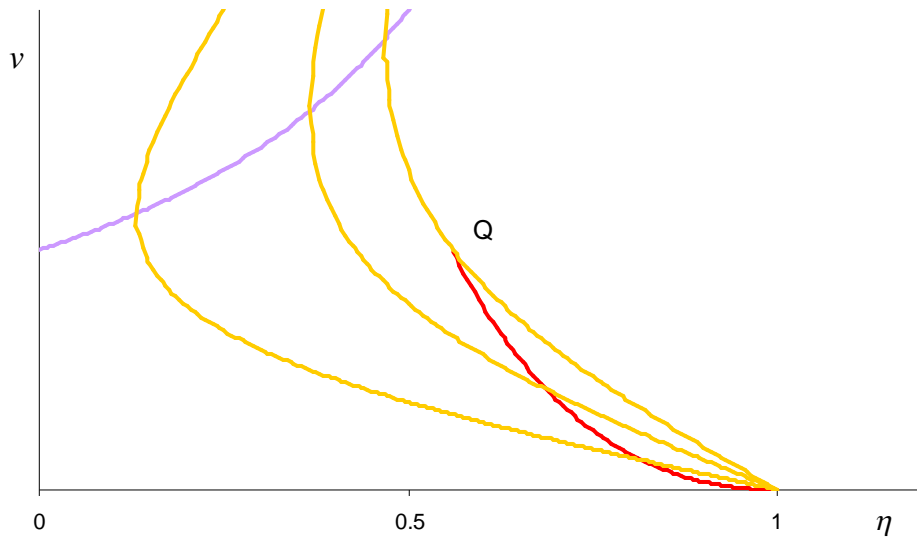
The locus of efficient points (red) if inequality (51) holds

Figure 2



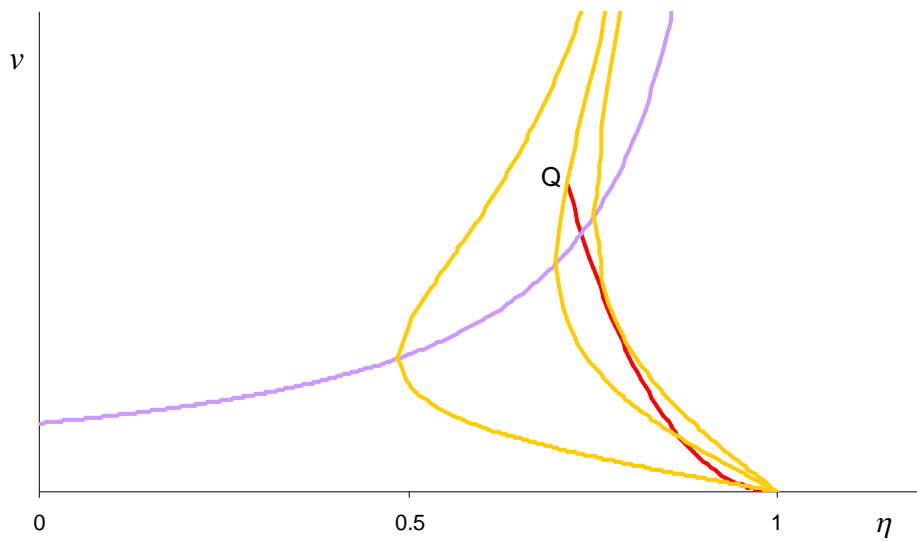
Equation (61) (blue) with increasing  $N$

Figure 3



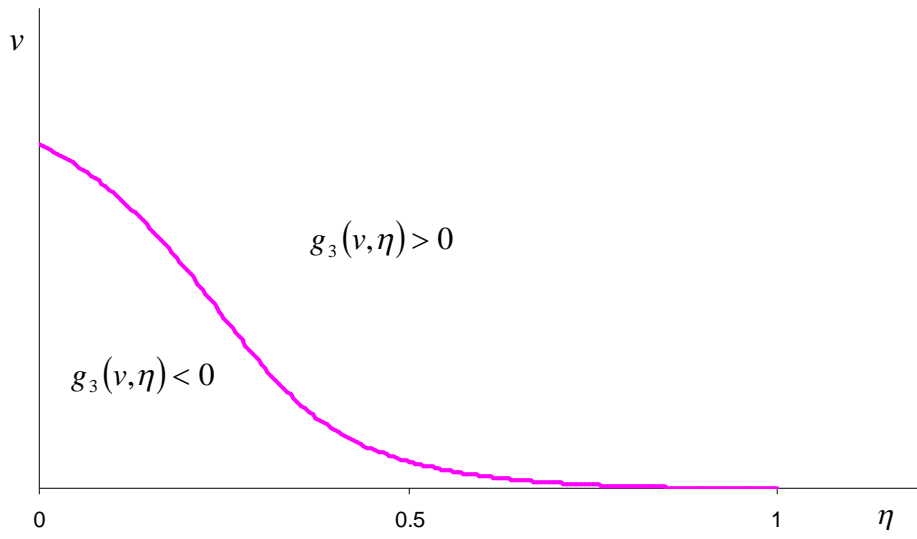
Bifurcation locus (violet) and different levels of physical capital (gold)

Figure 4



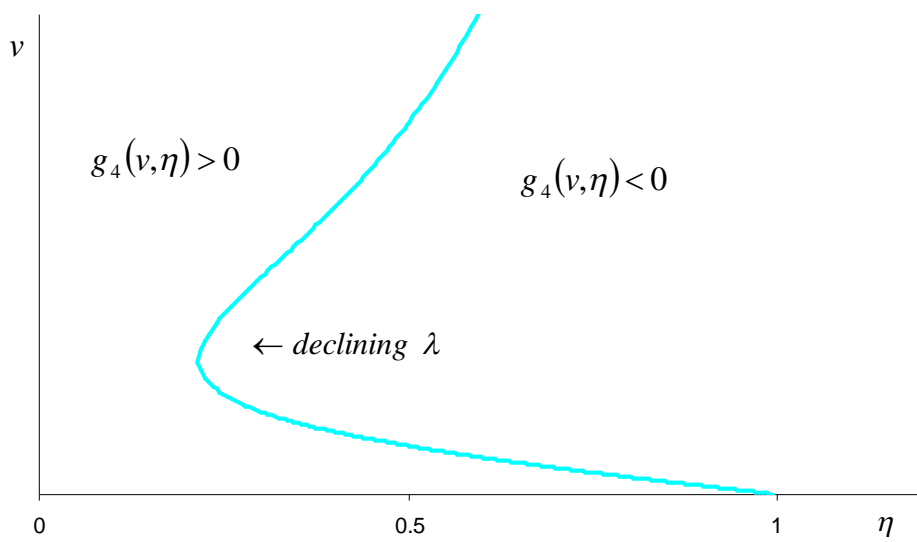
Origin of the efficient locus  $Q$  above the bifurcation locus

Figure 5



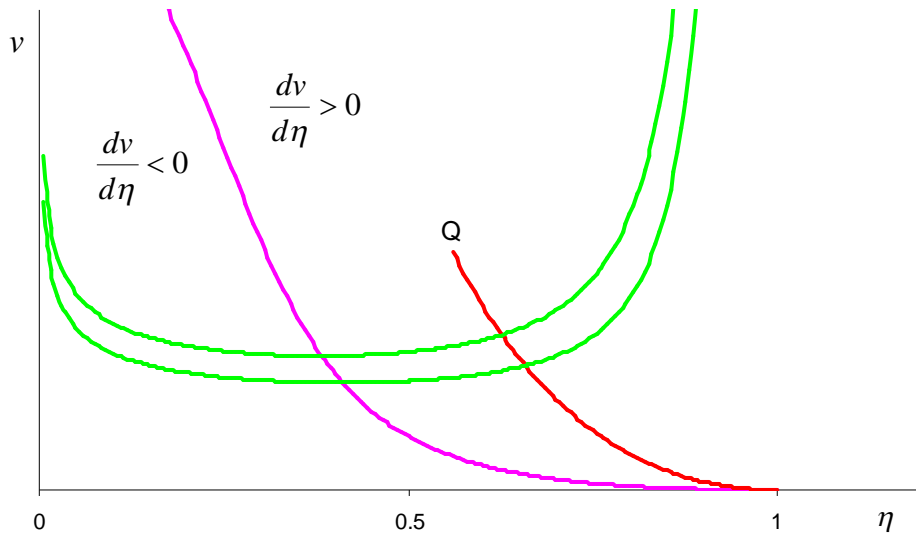
The  $g_3(v, \eta) = 0$  locus (pink)

Figure 6



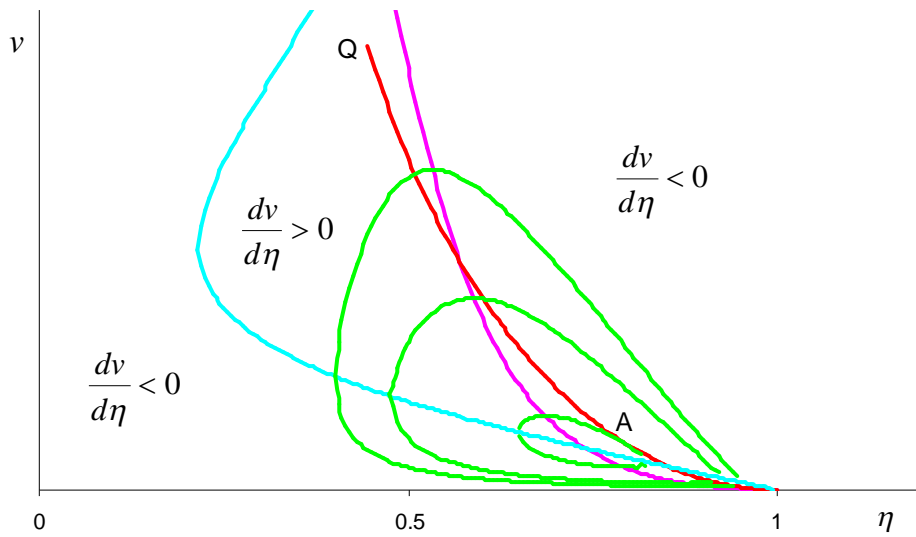
The  $g_4(v, \eta) = 0$  locus (cyan)

Figure 7



Physical per capita consumption with a “large”  $\lambda$

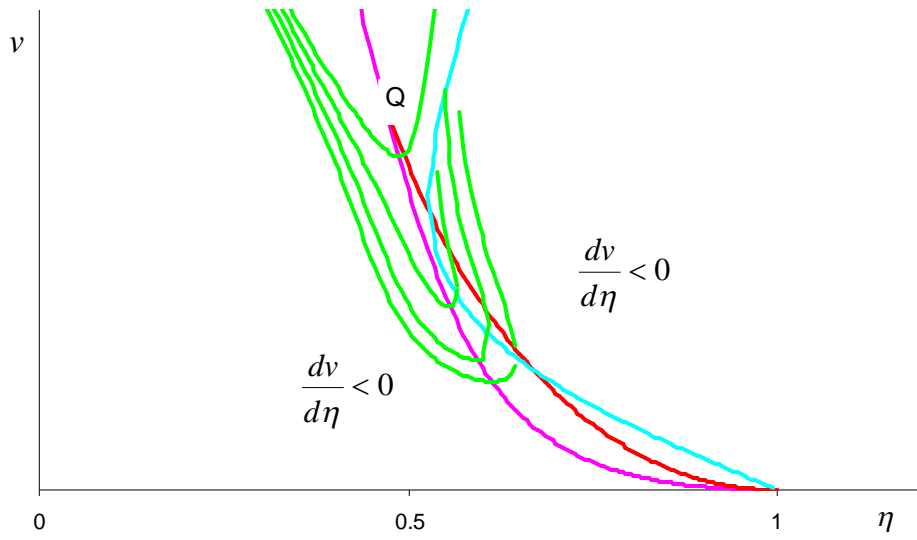
Figure 8



Physical per capita consumption with a “small”  $\lambda$

Figure 9





Physical per capita consumption with an “intermediate” value of  $\lambda$

Figure 10

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The Editor