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## Avoiding Extinction: Equal Treatment of the Present and the Future

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## ***Avoiding Extinction: Equal Treatment of the Present and the Future***

*Graciela Chichilnisky*

*Columbia University, New York*

### **Abstract**

Equal treatment for the present and the future was required in two axioms for sustainable development introduced by the author. This article shows that the two axioms are equivalent to awareness of physical limits in the long run future. We prove that two optimization problems are equivalent: maximizing discounted utility with a long run survival constraint, and maximizing utilities that treat equally the present and the future. The equal treatment axioms are therefore the essence of sustainable development. The "weight"  $\lambda$  given to the long run future is identified with the marginal utility of the environmental asset along a path that narrowly avoids extinction. An existence theorem is provided for optimizing according to the welfare criterion that treats equally the present and the future. We show that no prior welfare criteria satisfy the axioms for sustainable development introduced in [12].

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### **Correspondence:**

*Graciela Chichilnisky, Columbia University, New York 10027, USA,*

*email: [chichilnisky1@gmail.com](mailto:chichilnisky1@gmail.com)*

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# 1 Introduction

Global warming and the plight of extinguishing species are attracting increasing public attention and leading to calls for new forms of economic development. But sustainable development is hardly a new issue. The need for development that satisfies the basic needs of present and future generations was introduced 30 years ago<sup>1</sup> (Chichilnisky [9] [10] [37]), it was developed by the ILO and the World Bank in country studies, and reaffirmed by international vote as a global development priority 15 years ago<sup>2</sup> (Brundtland<sup>3</sup> [7]). Yet the challenge to achieve sustainable development remains today as elusive as ever. A change in preferences is essential. What this article shows is that once we become aware of new long term physical constraint on resources, such as the possible extinction of a species, this invokes new behavioral axioms introduced in [12], that require equal treatment of the present and the future – and we behave according to the decision criterion or preferences they imply. This article shows that the "equal treatment" axioms are equivalent to preferences that reflect awareness of physical limits in the long run future. Therefore the new axioms are the essence of sustainable development.

In reality implementing sustainable development is a moving target that requires more than public attention and a change in values and preferences. It requires a solid analysis of sustainability with the level of clarity and substance of neoclassical theory, to support the practical scope and the current widespread use of markets and cost benefit analysis.<sup>4</sup> The crux of the matter is how, through markets, we can define economic values that go beyond immediate individual

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<sup>1</sup>The concept of *development based on the satisfaction of basic needs* was introduced in the mid 1970s in the Bariloche Model and several other publications (Chichilnisky [9] [10] [37]) and became the cornerstone of efforts to define sustainable development (see [28]) in the 1987 Brundland Report [7], which uses the concept explicitly, defining sustainable development as 'development that satisfies the needs of the present and the future'. This was introduced in the 1992 Earth Summit of Rio de Janeiro and voted by 150 nations as the main priority of economic development.

<sup>2</sup>At the Earth Summit of Rio de Janeiro 1992, where the concept was voted by 150 nations as the main concept of development.

<sup>3</sup>Brundland's Report in 1987 [7] defined 'sustainable development' on the basis of *basic needs*, as "a form of development that satisfies the needs of the present without preventing the future from satisfying its own needs". This was voted by 150 nations as a global priority in the Earth Summit of 1992 in Rio de Janeiro. Yet Basic Needs and Sustainable Development are only achieving mainstream status now.

<sup>4</sup>Solow [46] pointed out that discussion of sustainability has been mainly an occasion for the expression of emotions and attitudes, with very little formal analysis of sustainability or of sustainable paths for a modern industrial economy. One purpose of Chichilnisky [12] and [15] was to attempt to resolve this problem.

gain and encompass the needs of future generations.<sup>5</sup> This motivation led me<sup>6</sup> to propose in 1993 two axioms to define sustainable development requiring equal treatment for the present and for the future, and to derived the decision criteria that they imply, [12] [15]). The two axioms require that neither the present nor the future should be "dictatorial." This article takes the matter further by solving outstanding issues that were not covered in the original treatment: (i) what is the practical basis for the new axioms, where do they come from? Exactly why and how do individuals change their preferences and start taking into consideration the long term future?, (ii) how to identify in practice the weight given to the long run future, and (iii) how to ensure the existence of sustainable solutions. The three issues were raised in the subsequence literature on the topic [17] [18] [19] [20] [21] [36] [40] [41] and are resolved here. In a nutshell we show in Theorem 3 that the new axioms arise from a rational response to a new objective reality, a reaction to new long term physical constraints that we face today and did not exist before, and how the weight given to the long run future derives from the losses due to the extinction of the resource in the long run. I also show that the existence of solutions is guaranteed by the inability of physical systems to adapt beyond a certain speed of response. The implications of these results is clear. It is indisputable the world economy has changed, and that we face physical limits that did not exist before. In particular we are now aware of the possibility of the extinction of our species in the long run. This article says that this awareness leads us to behave according to the new axioms [12] and the decision criterion that they imply.

The change in preferences takes place now because the situation has changed. It is worth underscoring the enormous physical transition that took place since the middle of the 20th century, a period in which most economic analysis of preferences was developed. For the first time in history, humans dominate the planet and consume resources in a way that can alter the planet's climate, its water bodies and its biological mix. Fossil fuel energy used for production since the second world war emitted carbon that could alter irreversibly the earth's climate with catastrophic consequences. Biologists see the loss of biodiversity during the last sixty years as one of the four or five largest incidents of destruction of life on the planet, 1,000 times larger than fossil records<sup>7</sup>. Of the 5487

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<sup>5</sup>Standard cost-benefit analysis discounts and undervalues the future [12] [15] [35]. It is therefore biased against policies designed to provide benefits in the very long run. A sharp example is the evaluation of projects for the safe disposal of waste from a nuclear power plant. Another is policies designed for the prevention of global warming. The benefits of both may be at least fifty to a hundred years into the future. The costs, however, are here today. In these cases, the inherent asymmetry between the treatment of present and future makes it hard to justify investment decisions that large numbers of individuals and organizations clearly feel are well merited.

<sup>6</sup>Neoclassical theory of choice and preference theory was developed in the first part of the 20th century and is based in *axioms* from which 'present discounted value' and 'expected utility analysis' were derived. They were created by Koopmans for the case of choice over time and by Von Neumann for the case of choice under uncertainty and grew to achieve the status of common knowledge [1] [35] [15] [12] [36].

<sup>7</sup>According to the UN 2000 Millenium Report [39].

known species of mammals - our relatives - 25% have become extinct<sup>8</sup> sending a somber message to the rest. The voracious use of resources since World War II originated largely in the industrial countries, and was accompanied by increasing discrepancies in resource consumption and welfare between industrial and developing countries, the North and the South [22]. The problem has been high in the international agenda since the 1992 United Nations Earth Summit in Rio de Janeiro, where the issue of sustainable development<sup>9</sup> emerged as one of the most urgent topics of international policy and one hundred and fifty nations endorsed UN Agenda 21 requiring new patterns of sustainable development that can satisfy the basic needs of the present and the future.<sup>10</sup> Yet little progress has been achieved since then. The use of biodiversity and fossil fuels has increased rapidly and water is now the most scarce resource in the world.<sup>11</sup>

The "equal treatment" axioms for the present and the future have not been debated, partly because there is general agreement that economics has to take account of the long run [29] [30] [33] [35] [36] [38] [40] [42] [43] [46]. Yet three outstanding issues emerged in the ensuing literature<sup>12</sup> and are resolved here. The first is how to explain the origin of the new axioms - where they come from? Why do the issues of sustainable cost benefit analysis, sustainable preferences and sustainable markets, arise today and not before? It seems important to know this, because only a rapid change of preferences can help achieve sustainable development under current circumstances. Theorem 3 shows that if we are aware of long term constraints, then we must accept the two "equal treatment" axioms proposed in 1993 and the decision criteria that they imply, since both criteria lead to the same decisions. This article shows that the new axioms are equivalent to the awareness of physical limits in the long run future.

A second related issue is how to compute the "weight"  $\lambda$  that is assigned to the long run future by the sustainable preference.<sup>13</sup> The initial representation

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<sup>8</sup>See Jan Schipper et al, *Science* October 10 2008 [47]: "The Status of the World's Land and Marine Mammals: Diversity Threat and Knowledge" p. 225-229.

<sup>9</sup>At the 1992 United Nations Earth Summit in Rio de Janeiro, sustainable development emerged as one of the most urgent subjects for international policy. One hundred and fifty participating nations endorsed UN Agenda 21, proposing as part of its policy agenda sustainable development based on the satisfaction of basic needs in developing countries as defined in [9] [10] [37].

<sup>10</sup>The development criterion based on the satisfaction of basic needs, was introduced and developed empirically by us in 1976 in the 'Bariloche Model' and in several scientific publications in the mid 1970s, [9] [10] [37]. and was given further impetus in 1987 when the Brundtland Commission proposed that "sustainable development is development that satisfies the needs of the present without undermining the needs of the future" see Brundtland (1987, chap. 2, para. 1).

<sup>11</sup>The global use of resources and the difference of consumption between poor and rich nations has become more acute since 1992, and could threaten the future of humankind.

<sup>12</sup>For example, in a recent presentation I gave at a Seminar in Cargese, France in the Spring of 2007 Charles Perrings argued that my criterion of sustainable preferences (which arises from my two axioms) was less applicable because we did not know the exact value of the parameter  $\lambda$  that appears in front of the long run future utility. A similar point was made for several years by another Columbia University colleague, Peter Eisenberger.

<sup>13</sup>This was represented by a real number " $\lambda$ " that appears in the representation of a sustainable utility in Chichilnisky [12] [15].

theorem provided in [12] had a degree of freedom for this parameter, which remained undefined. This can be considered a useful feature of the theory, and parallels the two degrees of freedom that appear in traditional discounted utility analysis, namely the ‘instantaneous utility’ and the ‘discount factor’ that are used to define discounted utility - and are left undefined. Yet it can be useful to identify the value of the "weight"  $\lambda$  in specific cases, to show how it can be computed in practice. In this article we consider the case of renewable resources that can become extinct when overused, a timely issue since 25% of all known mammals are now extinct [47]. Theorem 3 establishes that the optimizing according to the new axioms and the attendant preferences, is equivalent to optimizing based on a new physical constraint on the resource in the long term, and that the factor  $\lambda$  is the marginal utility of the resource at the point of extinction. This is the point where the resource is presumably most valuable. A Proposition resolves the issue of existence of optimal solutions, which is assured with conditions of "bounded variation" on the consumption paths, see also [15] and [14]. The results provided here add a new angle to the initial axioms and the sustainable preferences they imply. These results help to compute solutions and explain further the emergence of sustainable preferences as awareness about the depth and scope of biodiversity destruction and the risk of potentially catastrophic climate change in the long run future. In other words, to change our preferences what is needed is to increase awareness of the new long term physical constraints. The last section examines a list of previously used preferences and shows that none satisfy the axioms of sustainability defined here, so this theory of sustainable development represents genuine change

## 2 Experimental Evidence on How We Value the Long Run

In the last twenty years many experiments have measured how people value the long run (see, e.g., Lowenstein and Thaler [42], Cropper, Aydede, and Portney [30], and the references in Lowenstein and Elster [43]). Their findings clash with the traditional discounted approach. People value the present and the future differently from the predictions of the standard analysis, the present and the future are treated more evenhandedly. Typically we discount the future, but the trade-off between today and tomorrow blurs as we move into the future. Tomorrow acquires increasing relative importance as time progresses. It is as if we viewed the future through a curved lens. The relative weight given to two subsequent periods in the future is inversely related to their distance from today. The period-to-period rate of discount is inversely related to the distance into the future. The experimental evidence shows that rate of discount between period  $t$  and period  $t + 1$  decreases with  $t$ . Interestingly, studies of human responses to sound summarized in the Weber-Fechner law [15] [36], indicate similar responses to changes in sound intensity. The human ear responds to sound stimuli in an inverse relation to the initial stimulus.

How to explain this experimental evidence, our sensitivity to time, and how to integrate it into a criterion of optimality? Several interesting alternatives to the discounted utility analysis have been proposed. So far none had reached the clarity and consistency of the discounted utilitarian criterion used in cost-benefit analysis, nor its analytical tractability. Prominent examples are the "overtaking criterion," [48] Ramsey's criterion [44] and Basic Needs [9] [10] . However, these criteria are incomplete, failing to rank many reasonable paths. The ordering induced by the overtaking criterion cannot be represented by a real valued function, making it impractical to use. As a result, they lack the corresponding "shadow" prices to evaluate costs and benefits in an impartial fashion. These criteria therefore fail on practical grounds. The last section of this article examines these and other criteria that were previously used.

In 1993 [12] proposed simple axioms that capture the concept of sustainability, and derived the welfare criterion which they imply, see also [15]. The criterion that emerges is complete, analytically tractable, and is represented by a real valued ad continuous function. In optimization it leads to well-defined shadow prices which can be used for a "sustainable cost-benefit analysis." The new axioms provided internal consistency and ethical clarity. They imply a more symmetric treatment of generations in the sense that neither the "present" nor the "future" should be favored over the other. They neither accept the romantic view which relishes the future without regards for the present, nor the consumerist view which ranks the present above all. The axioms lead to a complete characterization of sustainable preferences, which are sensitive to the welfare of all generations and offer an equal opportunity to the present and to the future. Trade-offs between present and future consumption are allowed. The existence and characterization of sustainable preferences appears in Chichilnisky [15], Theorems 1, 2 and 3, <sup>14</sup> and sustainable preferences were shown in [15] to be a natural extension of the "equal treatment criterion" for finitely many generations, in the sense that the optimal solutions for such preferences approach the "turnpike" of an 'equal-weight finite horizon optimization problem as the horizon increases [15]. Theorem 3 of Chichilnisky [15] showed that sustainable preferences match the experimental evidence in these cases, in the sense that they imply a rate of discount that decreases and approaches zero as time goes to infinity, and Theorem 4 investigated the relationship between the optimal paths according to sustainable preferences and discounted utilitarianism in an extension of the classical Hotelling problem of the optimal depletion of an exhaustible resource. Theorem 5 in [15] showed that sustainable optima can be quite different from discounted optima, no matter how small is the discount factor. Subsequent examples show the implications for shadow prices.

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<sup>14</sup>An alternative name suggested by Robert Solowin personal communication is "intertemporally equitable preferences." shows that sustainable preferences are different from all other criteria used so far in the analysis of optimal growth and of markets. Theorem 3 studies a standard dynamical system representing the growth of a renewable resource.

### 3 Two Axioms for Sustainable Development

The two axioms introduced in Chichilnisky [12] can be seen as non-dictatorship properties, see also [1] [8]. Axiom 1 requires that the present should not dictate the outcome in disregard for the future: it requires sensitivity to the welfare of generations in the distant future. Axiom 2 requires that the welfare criterion should not be dictated by the long-run future, and thus requires sensitivity to the present. To offer a formal perspective a few definitions are required. Each generation is represented by an integer  $g, g = 1, 2, \dots$ . An infinitely lived world obviates the need to make decisions contingent on an unknown terminal date. Generations could overlap or not. Indeed agents could be infinitely long-lived and evaluate development paths for their own futures. For ease of comparison, I adopt a formulation which is as close as possible to that of [12] and to the standard neoclassical model. Each generation  $g$  has a preference that can be represented by a utility function  $u_g$  for consumption of  $n$  goods, some of which could be environmental goods such as water, or soil, so that consumption vectors are in  $R^n$ , and  $u_g : R^n \rightarrow R$ . The availability of goods in the economy is constrained in a number of ways, for example by a differential equation which represents the growth of the stock of a renewable resource and/or the accumulation and depreciation of capital. Ignore for the moment population growth, although this issue can be incorporated with little change in the results, at the cost of more notation.<sup>15</sup>

The space of all feasible consumption paths is indicated by  $F$ .  $F = \{x : x = \{x_g\}, g = 1, 2, \dots, x_g \in R^n\}$ . We chose a utility representation so that each generation's utility function is bounded below and above:  $u : R^n \rightarrow R^+$ , and  $ess\ sup_{x \in R^n} (u_g(x)) < K$ . This choice is not restrictive: it was shown by Arrow [2] that when ranking infinite streams of utilities as done here one should work with bounded utility representations since doing otherwise could lead to paradoxes<sup>16</sup>. Utility across generations is assumed to be comparable. In order to eliminate some of the most obvious problems of comparability I normalize the supremum of utilities to be 1, see also Arrow [1] and [8].<sup>17</sup> In this case we are concerned with fairness across generations, see also Solow [46], Lauwers [40] [41] Chichilnisky [9] [8], and Beltratti, Chichilnisky, and Heal [5] [17] [18] [19]. The space of feasible utility streams is therefore  $Q = \{\alpha : \alpha = (\alpha_g), g = 1, 2, \dots, \alpha_g = u_g(x_g), g = 1, 2, \dots\}$ .

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<sup>15</sup>Population growth and utilitarian analysis are well known to make an explosive mix, which is however outside the scope of this paper.

<sup>16</sup>Daniel Bernouilli showed that without bounded utilities, one easily has the St. Petersburg's paradox, see Arrow [2], and bounded utilities avoid this problem. The need to work with bounded utility representation in models with infinitely many parameters was pointed out by Arrow [2], who required boundedness to solve the problem that originally gave rise to Daniel Bernouilli's famous paper on the "St. Petersburg paradox" Utility Boundedness Theorem (Arrow 1964, 27). If utilities are not bounded, one can find a utility stream for all generations with as large a welfare value as we wish, and this violates standard continuity axioms.

<sup>17</sup>A preference admits more than one utility representation ; among these one chooses a bounded representation. Utility functions  $u$  that are nonnegative and all share a common bound, which I assume without loss of generality to be 1.



## 4 The Present and the Future

Each utility stream is a sequence of positive real numbers bounded by the number 1. The space of all utility streams is therefore contained as a bounded set in the space of all infinite bounded sequences of real numbers, denoted  $F = l_\infty$ .<sup>18</sup> A welfare criterion  $W$  should rank elements of  $F$ , for all possible  $f \in F$ . The welfare criterion  $W$  is *complete* if it is represented by an increasing real valued function defined on all bounded utility streams  $W : l_\infty \rightarrow R$ . It is called *sensitive* or *increasing*, if whenever a utility stream  $\alpha$  is obtained from another  $\beta$  by increasing the welfare of some generation, then  $W$  ranks  $\alpha$  strictly higher than  $\beta$ .

### 4.1 The Present

Intuitively, the present is represented by all the utility streams which have no future: for any given utility stream  $\alpha$ , its "present" is represented by all finite utility streams which are obtained by cutting a off after any number of generations. Formally:

*DEFINITION 1* For any utility stream  $\alpha$ , and any integer  $K$  let  $\alpha_K$  be the " $K$ -cutoff" of the sequence  $\alpha$ , the sequence whose coordinates up to and including the  $K$ -th are equal to those of  $\alpha$ , and zero after the  $K$ -th.

*DEFINITION 2.* The present consists of all feasible utility streams which have no future: it consists of the cutoffs of all utility streams, as defined above in Definition 1.

### 4.2 The Future

By analogy, for any given utility stream  $\alpha$ , its "future" is represented by all infinite utility streams which are obtained as the "tail" resulting from cutting a off for any finite number of generations.

*DEFINITION 3.* The  $K$ -th tail of  $\beta$  is the sequence whose coordinates up to and including the  $K$ -th are zero and equal to those of  $\beta$  after the  $K$ -th generation. For any two  $\alpha, \beta$  let  $(\alpha K, \beta K)$  be the sequence defined by summing up or "pasting together" the  $K$ -th cutoff of  $\alpha$  with the  $K$ -th tail of  $\beta$ .

### 4.3 No Dictatorship of the Present

*DEFINITION 4.* A welfare function  $W : l_\infty \rightarrow R$  gives a dictatorial role to the present,<sup>19</sup> and is called a *dictatorship of the present*, if  $W$  is insensitive to the utility levels of all but a finite number of generations, i.e., if  $W$  is only sensitive

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<sup>18</sup>Formally :  $l_\infty = \{y : y = \{y_g\}, g = 1, \dots : y_g \in R^+ \text{ and } \sup_g \|y_g\| < K\}$ . Here  $\| \cdot \|$  denotes the absolute value that is used to endow  $l_\infty$  with a standard Banach space structure, defined by the norm  $\| \cdot \|$ . The space of sequences  $l_\infty$  was first used in economics by G. Debreu [32].

<sup>19</sup>The representability of the order  $W$  by a real valued function can be obtained from more primitive assumptions, such as, for example, transitivity, completeness, and continuity conditions.

to the "cutoffs" of utility streams, and it disregards the utility levels of all generations from some generation on. Formally, for every  $\alpha, \beta$ ,  $W(\alpha) > W(\beta)$  if and only if there exists an  $N > 0$ ,  $N = N(\alpha, \beta)$  such that  $W(\alpha') > W(\beta')$  for any  $\alpha', \beta'$  which differ from  $\alpha$  and  $\beta$  only in their elements after  $N$ , namely  $\forall g < N, \alpha_g = \alpha'_g$  and  $\beta_g = \beta'_g$ . The following axiom eliminates dictatorships of the present:

*Axiom 1: No dictatorship of the present.*

#### 4.4 No Dictatorship of the Future

*DEFINITION 5.* A welfare function  $W$  gives a dictatorial role to the future, and is called a *dictatorship of the future*, if  $W$  is insensitive to the utility levels of any finite number of generations, or equivalently it is only sensitive to the utility levels of the "tails" of utility streams. Formally, for every  $\alpha, \beta$ ,  $W(\alpha) > W(\beta)$  if and only if there exists an  $N > 0$ ,  $N = N(\alpha, \beta)$  such that  $W(\alpha') > W(\beta')$  for any  $\alpha', \beta'$  which differ from  $\alpha$  and  $\beta$  only in their elements prior to  $N$ , namely  $\forall g > N, \alpha_g = \alpha'_g$  and  $\beta_g = \beta'_g$ . The welfare criterion  $W$  is therefore only sensitive to the utilities of "tails" of streams, and in this sense the future always dictates the outcome independently of the present. The following axiom eliminates dictatorships of the future:

*Axiom 2: No dictatorship of the future.*

#### 4.5 Sustainable Preferences

*DEFINITION 6.* A sustainable preference is a complete sensitive preference satisfying Axioms 1 and 2: it is therefore neither a dictatorship of the present nor a dictatorship of the future.

### 5 Existence and Characterization of Sustainable Preferences

Why is it difficult to rank infinite utility streams? Ideally one would give equal weight to every generation. For example, with a finite number  $N$  of generations, each generation can be assigned weight  $1/N$ . But when trying to extend this criterion to infinitely many generations one encounters the problem that, in the limit, every generation is given zero weight. What is done usually to solve this problem is to attach more weight to the utility of near generations, and less weight to future ones. An example is of course the sum of discounted utilities. Discounted utilities give a bounded welfare level to every utility stream which assigns each generation the same utility. Two numbers can always be compared, so that the criterion so defined is clearly complete. However, the sum of discounted utilities is not even-handed: it disregards the long-run future. It was shown in Chichilnisky [12] [15] that it is a dictatorship of the present. Another solution is the criterion defined by the long-run average of a utility

stream, a criterion used frequently in repeated games. However, this criterion is not even-handed either: it is biased in favor of the future and against the present. It is insensitive to the welfare of any finite number of generations.<sup>20</sup> Here matters stood for some time. Asking for the two axioms together, the non dictatorship of the present and the non dictatorship of the future, as I do there, appears almost as if it would lead to an impossibility theorem. Not quite.

Let us reason again by analogy with the case of finite generations. The number of generations one can assign weights which decline into the future, and then assign some extra weight to the last generation. This procedure, when extended naturally to infinitely many generations, is neither dictatorial for the present nor for the future. It is similar to adding to a sum of discounted utilities, the long-run average of the whole utility stream. Neither part of the sum is acceptable on its own, but together they are. This is Theorem 1 in Chichilnisky [12]. Theorem 2 in [12] proved that a similar procedure gives a complete characterization of all continuous sustainable preferences. The first part of Theorem 1 establishes that the sum of a dictatorship of the present plus a dictatorship of the future is in fact neither. The first part is sensitive to the present, and the second is sensitive to the future. Furthermore such a sum admits trade-offs between the welfare of the present and of the future. It is represented diagrammatically in Figure 1 of [12], which shows the trade-offs between the present's and the future's utilities. The three axes represent the utility levels of generations 1, 2, and, figuratively the generation  $\infty$ . The two triangular planes represent two indifference surfaces. One gives more utility to generations 1 and 2, and under a dictatorship of the present these choices would prevail; however the second surface gives more utility to the long run, so that under certain conditions the second surface is chosen over the first. Theorem 1 in [12] made this reasoning rigorous, see also Theorem 1 above and the Appendix. The second part of Theorem 1 above, shows that all known criteria of optimality used until now fail to satisfy the axioms postulated here, and is proved in the Appendix. Therefore the sustainable preferences defined here perform a role satisfied by no previously used criterion. What is perhaps more surprising is that the sustainable welfare criteria constructed here, namely the sum of a dictatorship of the present and one of the future, exhaust all the continuous utilities that satisfy my two axioms. This means that any continuous sustainable preference must be of the form just indicated. This is Theorem 2 above, see also [12]. The results of Theorem 3 below add another aspect to the characterization of sustainable preferences - it shows that a sum of a dictatorship of the present and one of the future represents the essence of sustainable development, since maximizing such preferences is equivalent to the awareness or consideration of a new resource constraint in the long run into the standard discounted utility framework we have used until now.

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<sup>20</sup>Other interesting incomplete intergenerational criteria which have otherwise points in common with sustainable preferences are found in Asheim [3] [4]

## 5.1 Existence of Sustainable Preferences

### Theorem 1

There exists a sustainable preference  $W : l_\infty \rightarrow R$ , i.e., a preference which is sensitive and does not assign a dictatorial role to either the present or the future

$$W(\alpha) = \sum_{g=1}^{\infty} \mu_g \alpha_g + \Phi(\alpha)$$

where  $\forall g, \mu_g > 0$ ,  $\sum_{g=1}^{\infty} \mu_g \alpha_g < \infty$  and where  $\Phi(\alpha)$  is the function  $\Phi(\alpha) = \lim_{g \rightarrow \infty} \alpha_g$  when this limit exists, and extended to all  $l_\infty$  otherwise using Hahn Banach's theorem.<sup>21</sup> None of the welfare criteria used until now in economics satisfies the axioms: (a) the sum of discounted utilities, for any fixed discount factor no matter how small, (b) Ramsey's criterion, (c) the overtaking criterion, (d) lim inf, (e) long-run averages<sup>22</sup>, (f) Rawlsian rules, (g) Basic Needs and (h) The Green Golden Rule  $g^*$ .

#### Proof:

For a proof see the Appendix and also Chichilnisky [12] [15]. The Appendix includes a definition of the prior criteria and a proof of how they fail the equal treatment axioms. ■

## 5.2 A Complete Characterization of Sustainable Preferences

The existence theorem presented above has been extended to provide a complete characterization of sustainable preferences in Chichilnisky [12]:

### Theorem 2

Let  $W : l_\infty \rightarrow R$  be a continuous sustainable preference. Then  $W$  is of the form

$$W(\alpha) = \sum_{g=1}^{\infty} \mu_g \alpha_g + \Phi(\alpha)$$

<sup>21</sup>The linear map "lim<sub>g</sub> α<sub>g</sub>" is defined by using the Hahn-Banach theorem, as follows: define first the function on the closed subset of / consisting of those sequences α<sub>g</sub> which have a limit, as that limit; the function is then extended continuously to all sequences in the space  $l_\infty$  by using the Hahn-Banach theorem, which ensures that such an extension exists and can be constructed while preserving the norm of the function on the closed subspace of convergent subsequences, see [12] [49].

<sup>22</sup>For example the two sequences (1, 0, 0,111, 0, 0, 0, 0,1,1,1,1,1, 0, 0, 0, 0, 0,1 . . . .) and (0,1,1, 0, 0, 0, 1,1,1,1, 0, 0, 0, 0, 0,1,1,1,1,1, 0, . . . .) are not comparable according to the long-run averages criterion.

where  $\forall g, \mu_g > 0, \sum_{g=1}^{\infty} \mu_g \alpha_g < \infty$  and where  $\Phi(\alpha)$  is a purely finitely additive measure on  $l_{\infty}$ .

**Proof:**

For a proof see the Appendix and also Chichilnisky [12],[15].■

## 6 How We Change Preferences, and Where the new Axioms Come from?

This section presents a new Theorem that explains exactly how we change preferences, and provides a practical criterion to explain why we should accept the new axioms defined in [12]. Consider ‘sustainable preferences’ that take into consideration environmental assets in the long term future. The theorem below proves that maximizing a sustainable preference over a set  $\Omega \subset l_{\infty}$  is mathematically equivalent to maximizing a standard preference with a new additional constraint in the long run, at infinity, a constraint that did not exist before.<sup>23</sup> An immediate corollary of the Theorem below is that if we are aware of a new physical constraint in the long run, and we consider these new constraints in making decisions, then we must accept the axioms proposed in [12] and the attendant decision criteria that they define [12] [15], as above.

The Theorem presented below resolves an outstanding issue that was pointed out about sustainable preferences: it identifies in practical terms a parameter, the "weight"  $\lambda$  that sustainable preferences assign to the long term future, appearing in the characterization Theorem 2 above and in [12]. This parameter  $\lambda$  can be identified with the marginal value of an environmental asset that could disappear in the long run, computed at the point of its extinction.

We need a few definitions to link this results with those presented above. Consider a standard optimization problem where one maximizes a continuous linear increasing function on the space of all sequences of utilities  $W : l_{\infty} \rightarrow R$  such as  $W(\alpha) = \sum_{g=1}^{\infty} \mu_g \alpha_g$ , where  $\mu_g \in l_1$  satisfy  $\forall g, \mu_g > 0$  and  $\sum_{g=1}^{\infty} \mu_g \alpha_g < \infty$ . This  $W$  is called standard discounted utility preference over infinite streams.

The following theorem establishes the equivalence between two optimization problems: (1) the problem of optimizing a sustainable preference  $W^*$  and (2) that of optimizing the standard preference  $W$  with an added (new) constraint in the long run, namely at infinity. The result suggests how sustainable preferences emerge from standard preferences: upon the consideration of new constraints at infinity – for example, from new survival constraints in the long run on a renewable resource such as an animal species, a constraint that was not considered or did not exist before.

The result provided below also identifies the value of the "weight"  $\lambda$  that sustainable preferences assign to the long run future in Theorem 2 above: this

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<sup>23</sup>For simplicity in the following we consider a simple preferences that are the sum of discounted utilities, and where  $\Omega = l_{\infty}$ . More general results are possible at the cost of more notation.

is the ‘shadow’ price of the new added constraint at infinity. For example, when the constraint represents a requirement to avoid extinction of the species, then the parameter  $\lambda$  that appears in the second term of the sustainable preference, the term that represents the value of the long run future, is identified with the marginal utility of this renewable resource at the point of its extinction.

Assume the framework presented above. Formally:

**Theorem 3**

(i) *The following two optimization problems are equivalent:*

*Problem 1: Optimize a standard preference  $W$  with an additional constraint ‘at infinity’ namely:*

$$Max_{\alpha \in \Omega \subset l_\infty} (W(\alpha) = \sum_{g=1}^{\infty} \mu_g \alpha_g)$$

where  $W(\alpha) = \sum_{g=1}^{\infty} \mu_g \alpha_g$ ,  $\mu_g \in l_1 : \forall g, \mu_g > 0$  and  $\sum_{g=1}^{\infty} \mu_g \alpha_g < \infty$ , and the constraint at infinity

$$\lim_{g \rightarrow \infty} \alpha_g \geq K,$$

and

*Problem 2: Optimize a corresponding sustainable preference  $W^*(\alpha)$  as characterized in Theorem 2, with no constraint ‘at infinity,’ namely:*

$$Max_{\alpha \in \Omega \subset l_\infty} W^*(\alpha) = Max_{\alpha \in \Omega \subset l_\infty} \left\{ \sum_{g=1}^{\infty} \mu_g \alpha_g + \lambda \Phi(\alpha) \right\}$$

(ii) *We can furthermore identify in this case the parameter  $\lambda$  that appears in the sustainable preference  $W^*$  as the ‘marginal utility at the point of extinction’ of the resource, namely the Frechet derivative:*

$$\lambda = \frac{\partial}{\partial \alpha} W(\alpha) |_{\alpha^*}$$

computed at a path  $\alpha^*$  that satisfies  $\lim_{g \rightarrow \infty} \alpha_g = K$ , for the appropriate  $K > 0$ . In Problem 2,  $\frac{\partial}{\partial \alpha} W(\alpha)$  represents the Frechet derivative of  $W$  with respect to the variable  $\alpha \in l_\infty$ , and  $\Phi(\alpha)$  is a purely finitely additive measure on  $l_\infty$ ,  $0 < \lambda < 1$ .

**Proof:**

The space of all sequences  $l_\infty$  with the sup norm is a Banach space. Since  $W$  is a continuous linear function on  $l_\infty$  it is therefore differentiable. Using calculus in Banach spaces one can define the Frechet derivative of  $W$ , which is the analogue of standard derivatives in euclidean space. To establish the theorem we compactify time (compactify  $R$ ) using a Stone - Cech compactification and obtain the result in the compactified space, and then derive the implications for the original infinite horizon problem prior to the compactification.

The Banach space of sequences  $l_\infty$  can be defined as the space of all bounded real valued functions from the integers to the real numbers, namely all bounded functions of the form  $\alpha : Z \rightarrow R$ . Consider the Stone - Cech compactification of the integers  $Z$ , denoted  $\tilde{Z}$ , which is by definition the set of all ultrafilters of the integers.  $\tilde{Z}$  is formally equivalent to a finite interval  $\{1, \dots, N\}$  where the "point of infinity" is identified with the integer  $N$  for some  $N > 1$ . After the compactification, the sequences in the space  $l_\infty$  are defined on  $\tilde{Z}$  and the sequence space becomes the space of real valued (finite) sequences on  $\tilde{Z}$ , namely the functions of the form  $\alpha : \tilde{Z} \rightarrow R$ . By construction, therefore,  $l_\infty$  is formally identical to the space of all real valued sequences  $\alpha = (\alpha_1, \dots, \alpha_N)$  of finite length  $N$ , namely it is equivalent to euclidean space  $R^N$ .

$$l_\infty \approx R^N$$

Problem 1 above is formally equivalent to that of optimizing a finite dimensional problem having a finite number of time periods and with a constraint on the value in the last period

$$Max_{\alpha \in R^N} (\tilde{W}(\tilde{\alpha}) = \sum_{g=1}^{\infty} \mu_g \tilde{\alpha}_g)$$

with the constraint

$$\tilde{\alpha}_N \geq K > 0,$$

where by assumptions on  $W$ ,  $\tilde{W} : R^N \rightarrow R$  is a continuous and increasing function,  $\tilde{\alpha} \in R^N$ . By standard results of constrained maximization the maximum of this problem will be achieved at a boundary of the constrained set, so that when achieved the maximum coincides with

$$Max_{\alpha \in R^N} (\tilde{W}(\tilde{\alpha}) + \lambda(\tilde{\alpha} - \tilde{\alpha}_N))$$

with the (additional) constraint

$$\tilde{\alpha}_N = K > 0$$

Resolving the above (finite dimensional) problem implies that

$$\frac{\partial}{\partial \tilde{\alpha}} (\tilde{W} + \lambda(\tilde{\alpha}_N - K))$$

and

$$\tilde{\lambda} = \frac{\partial}{\partial \alpha} \tilde{W}(\alpha) |_{\alpha_N=K}$$

Reinterpreting the variables with " $\sim$ " and without, this implies that, in the original problem the optimum  $\alpha^*$  maximizes

$$Max_{\alpha \in l_\infty} (W(\alpha) + \lambda(\alpha - \lim_{g \rightarrow \infty} \alpha_g))$$

with the (additional) constraint

$$\lim_{g \rightarrow \infty} \alpha_g = K$$

and where

$$\lambda = \frac{\partial}{\partial \alpha} W(\alpha) \Big|_{\lim_{g \rightarrow \infty} \alpha_g = K}$$

as we wished to prove. ■

**Proposition 1**

*Consider a sustainable preference  $W$  satisfying Axioms 1 and 2 [12]. An optimal solution to Problems 1 and 2 above always exists on a constraint set of paths  $\Omega \subset l_\infty$  when there is a uniform bound for the variation of the paths in  $\Omega$ . For example, in problems involving consumption of a resource through time, existence is ensured when there is a uniform bound on the instantaneous change in consumption or the corresponding instantaneous change in the variation of stock of the resource within the constraint set  $\Omega$ .*

**Proof:**

By construction  $W$  is continuous on  $l_\infty$  <sup>24</sup>[12]. Therefore if the set  $\Omega$  is compact there is always an optimum for  $W$  on  $\Omega$ . With respect of compactness of  $\Omega$ , bounded variation is sufficient to establish compactness in function spaces; a similar proof of existence based on the fact that bounded variation of paths in  $\Omega$  implies compactness of  $\Omega$  is in [14], which establishes the existence of solutions to an optimal growth problem with or without convexity assumptions, in a Hilbert space of functions. See also the applications in [15]. ■

**Proposition 2**

*The equivalence between Problem 1 and Problem 2 established in Theorem 3 depends on Hahn Banach's Theorem and the Axiom of Choice.*

**Proof:**

The Frechet derivative used in Theorem 3 always exists because  $W$  is linear by construction [12]. The Frechet derivative at  $\lim_{g \rightarrow \infty} \alpha_g = K$  appearing in the proof of Theorem 3 can be constructed in many cases, yet this construction requires in general the use of a Hahn - Banach separating hyperplane. The proof of Hahn Banach's theorem in its most general form depends on the Axiom of Choice, and Kurt Godel [34] established that the most general solutions to the construction problems just mentioned (e.g. the hyperplanes in Hahn Banach Theorem) are independent of the rest of the axioms of mathematics: they can neither be established nor negated, as are the Continuum Hypothesis and the Axiom of Choice. ■

## 7 No Prior Welfare Criteria are Sustainable Preferences

Although the axioms presented here are reasonable, so far all preferences considered in the literature fail to satisfy them.

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<sup>24</sup>Any discounted utility criterion satisfies continuity in  $l_\infty$  with the sup norm.



### Proposition 3

The following criteria for evaluating time paths fail our sustainability axioms:

(a) the sum of discounted utilities, for any fixed discount factor no matter how small, because it is always a dictatorship of the present, as established in Chichilnisky [12],

(b) Ramsey's criterion, which is seriously incomplete and therefore does not satisfy the definition of a sustainable preference see Chichilnisky [12] and [15],

(c) the overtaking criterion, because it is also incomplete, see Chichilnisky [15]

(d) *lim inf*, which is a dictatorship of the future [15],

(e) long-run averages, because it is a dictatorship of the future and also incomplete<sup>25</sup>

(f) Rawlsian rules

(g) Basic Needs because they are insensitive as defined here, since they rank as equal two paths that have the same infimum - even though one may assign a much higher utility to many (even to infinitely many) generations.

(h) The Green Golden Rule  $g^*$ , a stationary path  $g^* = \{c^*, s^*\}$  that achieves the maximum utility level which is sustainable forever, that is,  $g^* = \max u(c, s)$  subject to  $c < R(s)$ .

#### Proof:

For definitions and proofs see the Appendix and [15]. ■

## 8 APPENDIX

### 8.1 Continuity

In practical terms the continuity of  $W$  is the requirement that there should exist a sufficient statistic for inferring the welfare criterion from actual data. This is an expression of the condition that it should be possible to approximate as closely as desired the welfare criterion  $W$  by sampling over large enough finite samples of utility streams. Continuity of a sustainable criterion function  $W$  is not needed in Theorem 1; it is used solely for the characterization in Theorem 2. *Continuity* is defined here in terms of the standard topology of 'sup norm' defined by  $\|f\| = \sup_{x \in R} |f(x)|$ , a topology first used in economics by Debreu [32].

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<sup>25</sup>For example the two sequences  $(1, 0, 0, 111, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, \dots)$  and  $(0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, \dots)$  are not comparable according to the long-run averages criterion.

## 8.2 Independence

A welfare criterion  $W : l_\infty \rightarrow R$  is said to give independent trade-offs between generations, and is called *independent*, when the marginal rate of substitution between the utilities of two generations depends only on the identities of the generations, and not on the utility levels of the two generations. Independence of the welfare criterion is not needed in Theorem 1. It is used solely in the characterization of Theorem 2, to allow us to obtain a simple representation of all sustainable preferences. Formally : let  $l_\infty$  be the space of all continuous real valued linear functions on the integers  $Z$ .

*DEFINITION 7.* The welfare criterion  $W : l_\infty \rightarrow R$  is *independent* if  $\forall \alpha, \beta \in l_\infty$  :

$$W(\alpha) = W(\beta) \Leftrightarrow \exists \lambda > 0, \lambda \in l_\infty, \lambda = \lambda(W) : \lambda(\alpha) > \lambda(\beta)$$

This property has a simple geometric interpretation, which is perhaps easier to visualize in finite dimensions. For example: consider an economy with  $n$  goods and two (2) periods. Let  $\alpha = \{\alpha_1, \alpha_2\}$ ,  $\beta = \{\beta_1, \beta_2\}$  denote two feasible utility streams. Then  $\alpha$  and  $\beta$  are equivalent according to the welfare criterion  $W$ , that is,  $W(\alpha) = W(\beta)$  if and only if there exists a number  $\lambda = \lambda(W)$ ,  $\lambda > 0$ , such that

$$\frac{\alpha_2 - \beta_2}{\alpha_1 - \beta_1} = \lambda$$

The geometric interpretation is that the indifference surfaces of  $W$  are affine linear subspaces of  $R^2$ .

*Level independence* implies that the indifference surfaces of the welfare function  $W$  are affine hyperplanes in  $l_\infty$ . In particular,  $W$  can be represented by a linear function on utility streams, that is,  $W(\alpha + \beta) = W(\alpha) + W(\beta)$ . Examples of welfare criteria which satisfy this axiom are all 'time-separable' discounted utility functions, any linear real valued non-negative function on  $l_\infty$ , and the welfare criteria in Theorem 2. As already mentioned, this axiom is used to provide a tight representation of sustainable preferences, but is not strictly necessary for the main results.

*DEFINITION 8*

A *continuous independent sustainable* preference is a complete sensitive preference satisfying Axioms 1 and 2 and which is continuous and independent.

## 8.3 Previous Welfare Criteria

To facilitate comparison, this subsection defines some of the more widely used prior welfare criteria. A function  $W : l_\infty \rightarrow R$  is called a discounted sum of utilities if it is of the form:  $W(\alpha) = \sum_{g=1}^{\infty} \mu_g \alpha_g$  where  $\forall g, \alpha_g \geq 0$  and  $\sum \mu_g < \infty$ ,  $\mu_g$  is called a 'discount factor.'

*Ramsey's welfare criterion* (Ramsey [44]) ranks a utility stream  $\alpha = (\alpha_g)$   $g = 1, 2, \dots$  above another  $\beta = (\beta_g)$   $g = 1, 2, \dots$  if the utility stream  $\alpha$  is "closer" to the *bliss path*, namely to the sequence  $t = (1, 1, \dots, 1, \dots)$ , than is the sequence  $\beta$ . Formally:  $\sum_{g=1}^{\infty} 1 - \alpha_g < \sum_{g=1}^{\infty} 1 - \beta_g$ .

A *Rawlsian rule* (Rawls [45]) ranks two utility streams according to which has a higher infimum value of utility for all generations. This is a natural extension of the criterion proposed initially by Rawls [45]. Formally: a utility stream  $\alpha$  is preferred to another if  $\inf(\alpha_g) > \inf(\beta_g)$ . The criterion of *satisfaction of basic needs* introduced in Chichilnisky [9], [10] ranks a utility stream  $\alpha$  over another  $\beta$  if the time required to meet basic needs is shorter in  $\alpha$  than in  $\beta$ . Formally :  $T(\alpha) \leq T(\beta)$  where  $T(\alpha) = \min\{t : \alpha_g > b\}$ , for a given  $b$  which represents basic needs. The *overtaking criterion* (von Weizacker [48]) ranks a utility stream  $\alpha$  over another  $\beta$  if  $\alpha$  eventually leads to a permanently higher level of aggregate utility than does  $\beta$ . Formally:  $\alpha$  is preferred to  $\beta$  if  $\exists n$  such that

$$\forall M > N, \sum_{g=1}^M (\alpha_g) > \sum_{g=1}^M (\beta_g)$$

The *long-run average criterion* can be defined in our context as follows: a utility stream  $\alpha$  is preferred to another  $\beta$  if in average terms, the long-run aggregate utility achieved by  $\alpha$  is larger than achieved by  $\beta$ . Formally:  $\exists N, K > 0$ :  $\frac{1}{T}(\sum_{g=M}^{T+M} \alpha_g) \geq \frac{1}{T}(\sum_{g=M}^{T+M} \beta_g) \forall T > N$  and  $M > K$ .

## 8.4 Countable and Finitely Additive Measures

*DEFINITION 9.*

Let  $(S, \Sigma)$  denote the field of all subsets of a set  $S$  with the operations of unions and intersections of sets. A real valued, bounded additive set function on  $(S, \Sigma)$  is one which assigns a real value to each element of  $(S, \Sigma)$ , and assigns the sum of the values to the union of two disjoint sets.

*DEFINITION 10.*

A real valued bounded additive set function is called *countably additive* if it assigns the countable sum of the values to a countable union of disjoint sets.

*EXAMPLE 1.*

Standard probability measures on the real numbers,  $R$ , or on the integers  $Z$ , are examples of countably additive functions. Any sequence of positive real numbers  $\{\lambda_g\} g = 1, \dots$ , such that  $\sum \lambda_g < \infty$  defines a countably additive measure  $\mu$  on the integers  $Z$ , by the rule  $\mu(A) = \sum_{g \in A} \lambda_g, \forall A \subset Z$ .

*DEFINITION 11.*

A real valued bounded additive set function  $\varphi$  on  $(S, \Sigma)$  is called *purely finitely additive* (see Yosida and Hewitt [49] [50]) if whenever a countably additive function  $\nu$  satisfies:  $\forall A \in (S, \Sigma), \nu(A) < \varphi(A)$  then  $\nu(A) = 0 \forall A \in (S, \Sigma)$ . This means that the only countably additive measure which is absolutely continuous with respect to a purely finitely additive measure is the measure that is identically zero.

*EXAMPLE 2.*

Any real valued linear function  $V : l_\infty \rightarrow R$  defines a bounded additive function  $V$  on the field  $(Z, \Sigma)$  of subsets of the integers  $Z$  as follows:  $\forall A \subset Z, \hat{V}(A) = V(\alpha^A)$  where  $\alpha^A$  is the "characteristic function" of the set  $A$ , namely

the sequence defined by  $\alpha^A = \{\alpha_g^A\}, g = 1, 2, \dots$  such that  $\alpha_g^A = 1$  if  $g \in A$  and  $\alpha_g^A = 0$  otherwise .

*EXAMPLE 3.* Typical purely finitely additive set functions on the field of all subsets of the integers,  $(Z, \Sigma)$ , are the *lim inf* function on  $l_\infty$ , defined for each by  $\lim inf (\alpha) = \lim inf \{\alpha_g\} g = 1, 2, \dots$

Recall that the *lim inf* of a sequence is the *infimum* of the set of points of accumulation of the sequence. The "*long - run averages*" function is another example of a purely finitely additive measure: it is defined for each a  $\alpha \in l_\infty$  by

$$\lim_{K, N \rightarrow \infty} \left( \frac{1}{K} \sum_{g=N}^{K+N} \alpha_g \right).$$

It is worth noting that a purely finitely additive set function  $\phi$  on the field of subsets of the integers  $(Z, \Sigma)$  cannot be represented by a sequence of real numbers in the sense that there exists no sequence of positive real numbers,  $\lambda = \{\lambda_g\}$  which defines  $\phi$ , that is, there is for no  $\lambda$  such that  $\forall A \subset Z, \phi(A) = \sum_{n \in A} \lambda_n$ . For example the *lim inf* :  $l_\infty \rightarrow R$ , defines a purely finitely additive set function on the integers which is not representable by a sequence of real numbers.

## 8.5 Proof of Theorem 1

See also [12]. To establish the existence of a sustainable preference it suffices to exhibit a well defined continuous linear function  $W : l_\infty \rightarrow R$  satisfying the two axioms (non dictatorship of the present or the future). For any sequence  $\alpha \in l_\infty$  define  $W(\alpha) = \sum \lambda_g \alpha_g + \lim(\alpha_g) g = 1, 2, \dots . \forall \alpha = (\alpha_g), \forall g, \lambda_g > 0, \sum \lambda_g < \infty$ , if the sequence  $\alpha$  has a well defined limit  $\lim_{g \rightarrow \infty} \alpha_g$ , and otherwise extend  $W(\alpha)$  to be defined on all of  $l_\infty$  using Hahn Banach's theorem.  $W$  satisfies the axioms because it is a well-defined, non-negative, increasing function on  $l_\infty$ ; it is not a dictatorship of the present (Axiom 1) because its second term makes it sensitive to changes in the "tails" of sequences; it is not a dictatorship of the future (Axiom 2) and because its first term makes it sensitive to changes in "cutoffs" of sequences. The next task is to show that the following welfare criteria do not define sustainable preferences: (a) Ramsey's criterion, (b) the overtaking criterion, (c) the sum of discounted utilities, (d) lim inf, (e) long-run averages (f) Rawlsian criteria, and (g) basic needs. The Ramsey's criterion defined above fails because it is not a well-defined real valued function on all of  $l_\infty$  and cannot therefore define a complete order on  $l_\infty$ . To see this it suffices to consider any sequence for which the sum does not converge. For example, let  $\alpha = (\alpha_g, g = 1, 2, \dots$  where  $\forall g, \alpha_g = (g - 1)/g$ . Then  $\alpha_g \rightarrow 1$  so that the sequence approaches the "bliss" consumption path  $\beta = (1, 1, \dots, 1, \dots)$ . The ranking of  $\alpha$  is obtained by the sum of the distance between  $\alpha$  and the bliss path. Since  $\lim_{N \rightarrow \infty} \sum_{g=1}^N (1 - \alpha_g)$  does not converge, Ramsey's welfare criterion does not define a sustainable preference as defined. The overtaking criterion defined above is not a well-defined function of  $l_\infty$ , since it cannot rank those pairs of

utility streams in which neither  $\alpha$  overtakes  $\beta$  nor  $\beta$  overtakes  $\alpha$ . Figure 2 in Chichilnisky [15] exhibits a typical pair of utility streams which the overtaking criterion fails to rank. The long-run averages criterion defined above and the lim inf criterion defined above fail on the grounds that neither satisfies Axiom 2; both are dictatorships of the future. Finally any discounted utility criterion of the form  $W(\alpha) = \sum \alpha_g \lambda_g$  for  $\forall g, \lambda_g > 0$  and  $\sum \lambda_g < \infty$  is a dictatorship of the present, as shown in Chichilnisky [12] and therefore fails to satisfy Axiom 1. Finally the Rawlsian welfare criterion and the criterion of satisfaction of basic needs do not define independent sustainable preferences: the Rawlsian criterion defined above fails because it is not sensitive to the welfare of many generations: only to that of the less favored generation. Basic Needs has the same drawback. ■

## 8.6 Proof of Theorem 2

Consider a continuous independent sustainable preference that satisfies Axioms 1 and 2, so there exists a utility representation for  $W$ . The welfare criterion  $W : l_\infty \rightarrow R$ , defines a non-negative, continuous linear functional on  $l_\infty$ . As seen above in Example 3, such a function defines a non-negative, bounded, additive set function denoted  $W$  on the field of subsets of the integers  $Z, (Z, \Sigma)$ . The representation theorem of Yosida and Hewitt (Yosida [49]; Yosida and Hewitt [50]) establishes that every non-negative, bounded, additive set function on  $(S, \Sigma)$ , the field of subsets  $\Sigma$  of a set  $S$  can be decomposed into the sum of a non-negative measure  $\mu$  and a purely finitely additive, which is a non-negative set function  $\phi$  on  $(S, \Sigma)$ . It follows from this theorem that  $W$  can be represented as the sum of a countably additive measure  $\mu$  and a purely finitely additive measure  $\phi$  on the integers  $Z$ . It is immediate to verify that this is the representation provided above. To complete the characterization proof that this is an independent sustainable preference it suffices to show that neither  $\mu$  nor  $\phi$  are identically zero. This follows from Axioms 1 and 2: we saw above that discounted utility is a dictatorship of the present, so that if  $\phi \cong 0$ , then  $W$  would be a dictatorship of the present, contradicting Axiom 1. If on the other hand  $\mu \cong 0$ , then  $W$  would be a dictatorship of the future because all purely finitely additive measures are, by definition, dictatorships of the future, contradicting Axiom 2. Therefore neither  $\phi$  nor  $\mu$  can be identically zero. This completes the proof of the theorem. ■

## References

- [1] Arrow, K. J. 1953. *Social Choice and Individual Values*. Cowles Foundation Monographs. New York: Wiley 1964.
- [2] *Aspects of the Theory of Risk-Bearing*. Yrjo Jahnsson Lectures, Yrjo Jahnssonin Saatio, Helsinki, 1971.

- [3] Asheim, G. 1988. "Rawlsian Intergenerational Justice as a Markov-Perfect Equilibrium in a Resource Economy." *Review of Economic Studies* 55 (3): 469-84.
- [4] "Unjust Intergenerational Allocations." *Journal of Economic Theory* 54 (2): 350-71 .
- [5] Beltratti, A., G. Chichilnisky, and G. M. Heal. 1995. "The Green Golden Rule: Valuing the Long Run." *Economics Letters* 49 (2):175-79.
- [6] Broome, J. 1992. *Counting the Cost of Global Warming*. London: White Horse Press.
- [7] Brundtland, G. H. 1987. The U.N. World Commission on Environment and Development: *Our Common Future*. Oxford : Oxford University Press.
- [8] Chichilnisky, G. "Social Aggregation Rules and Continuity" *Quarterly Journal of Economics* 97 (2) 337-52, 1982.
- [9] Chichilnisky, G. "Economic Development and Efficiency Criteria in the Satisfaction of Basic Needs." *Applied Mathematical Modeling* 1 (6):290-97. 1977
- [10] Chichilnisky, G: "Development Patterns and the International Order." *Journal of International Affairs*. 1977, 31 (2):275-304.
- [11] Chichilnisky, G: "Social Diversity, Arbitrage and Gains from Trade: A Unified Perspective on Resource Allocation" *American Economic Review* 84 (2):427-34. .1996.
- [12] Chichilnisky, G: "An Axiomatic Approach to Sustainable Development." *Social Choice and Welfare* 13 (2):231-57. 73(4), 1993.
- [13] Chichilnisky, G., and P. J. Kalman. 1980. "Application of Functional Analysis to Models of Efficient Allocation of Economic Resources." *Journal of Optimization Theory and Applications* 30:19-32.
- [14] Chichilnisky, G. "Nonlinear Functional Analysis and Optimal Economic Growth," *Journal of Mathematical Analysis and Applications*, Vol. 61, No. 2, November 15, 1977, p. 504-520.
- [15] Chichilnisky, G. 1997, "What is Sustainable Development" *Land Economics*, 73 (4) 467-491.
- [16] Chichilnisky, G. "The Topology of Fear" Working Paper Columbia University 2006, presented at NBER GE Conference in Honor of G. Debreu UC Berkeley 2006, submitted for publication.
- [17] Chichilnisky, G. "The Environment and the Long Run: A Comparison of Different Criteria" (with A. Beltratti and G. M. Heal), *Ricerca Economica*, Vol. 48, 1994, p. 319-340.

- [18] Chichilnisky, G. "The Green Golden Rule" (with G. Heal and A. Beltratti), *Economic Letters*, August 1995, Vol. 49, No. 2, p. 175-180.
- [19] Chichilnisky, G. "Sustainable Growth and the Green Golden Rule" (with A. Beltratti and G. M. Heal), presented at conference on Sustainable Growth, Paris, May 1993, (I. Goldin and A. Winters eds) Conference *Economic Approaches to Sustainable Development*, OECD, Paris, 1995, p. 147-165.
- [20] Chichilnisky, G. "Sustainable Use of Renewable Resources," (with A. Beltratti and G.M. Heal), Chapter 2.1, *Sustainability: Dynamics and Uncertainty*, (eds. G. Chichilnisky, G.M. Heal, and A.Vercelli), Kluwer Academic Publishers, The Netherlands, 1998, p. 49-76.
- [21] Chichilnisky, G. "Uncertain Future Preferences and Conservation," (with A. Beltratti and G.M. Heal), Chapter 3.4 in *Sustainability: Dynamics and Uncertainty*, (eds. G. Chichilnisky, G. M. Heal, and A. Vercelli), Kluwer Academic Publishers, The Netherlands, 1998, p. 257-276.
- [22] Chichilnisky, G, "Sustainable Development and North-South Trade", *Protection of Global Biodiversity: Converging Strategies*, (eds. Lakshman D. Guruswamy and Jeffrey A. McNeely), Duke University Press, Durham and London, 1998, p. 101-117.
- [23] Chichilnisky, G. "An Axiomatic Approach to Choice Under Uncertainty with Catastrophic Risks", *Resource & Energy Economics*, Volume 22, Issue 3, July 2000, pp. 221 -231.
- [24] Chichilnisky, G. "Catastrophic Risk" *Encyclopedia of Environmetrics*, edited by Abdel H. El-Shaarawi and Walter W. Piegorsch, published by John Wiley & Sons, Ltd, Chichester, UK 2002, Volume 1, pp 274-279.
- [25] Chichilnisky, G. and Y. Baryshnikov: "Intergenerational Choice: A Paradox and A Solution" in *Topology and Markets* (G. Chichilnisky ed.) American Mathematical Society and Field Institute for Mathematical Sciences Vol 22, 1999. P. 99-110.
- [26] Chichilnisky, G. "Catastrophic Risks: The Need for New Tools, Financial Instruments and Institutions," Symposium on the Privatization of Risk - Social Science Research Council, New York June 2006.
- [27] Chichilnisky, G. "NP Estimation in Hilbert Spaces: The Limits of Econometrics" to appear *Econometric Theory*, 2008 - 9.
- [28] Chichilnisky, G., G. Heal and S. Vercelli *Sustainability, Dynamics and Uncertainty*, Kluwer Academic Publishers, 1998.
- [29] Cline, W. R. 1992. *The Economics of Global Warming*. Washington, DC: Institute for International Economics.

- [30] Cropper, M. L., S. K. Aydede, and P. Portney. 1994. "Preferences for Life Saving Programs : How the Public Discounts Time and Age." *Journal of Risk and Uncertainty* 8 :243-65.
- [31] Dasgupta, P., and G. Heal. 1979. *Economic Theory and Exhaustible Resources*. Cambridge: Cambridge University Press.
- [32] Debreu, G. 1954. "Valuation Equilibrium and Pareto Optimum." *Proceedings of the National Academy of Sciences* 40 (July) 588-92.
- [33] Dutta, P. 1991 . "What Do Discounted Optima Converge To?" *Journal of Economic Theory* 55, 64-94 .
- [34] Godel, K. *The Consistency of the Continuum Hypothesis and the Independence of the Axiom of Choice*, Princeton University Press, 1970.
- [35] Hammond, P. J. 1993. "Is There Anything New in the Concept of Sustainable Development?" Presented at the conference The Environment after Rio, Courmayeur, Italy, 10-12 February .
- [36] Heal, G. 1995 *Valuing the Future: Economic Theory and Sustainability*, Columbia University Press, 2000.
- [37] Herrera, A., H. Scolnik, G. Chichilnisky et al . 1976. *Catastrophe or New Society: A Latin American World Model* (The Bariloche Model).Ottawa, Canada: International Development Research Center.
- [38] Krautkramer, Jeffrey, 1985. "Optimal Growth, Resource Amenities and the Preservation of Natural Environments." *Review of Economic Studies* 52:153-70.
- [39] United Nations: *Millenium Report 2000*.
- [40] Lauwers, L. 1993. "Infinite Chichilnisky Rules." Discussion Paper, Katholik Universitaet, Leuven, March; *Economics Letters*, April 1993, 42 (4):349-52.
- [41] 1997. "A Note on Weak - Chichilnisky Rules." *Social Choice and Welfare* 14 (2):357-59.
- [42] Lowenstein, G., and J. Elster, eds. 1992. *Choice Over Time*. New York: Russell Sage Foundation.
- [43] Lowenstein, G., and R. Thaler. 1989 "Intertemporal Choice." *Journal of Economic Perspectives* 3 (4):181-93.
- [44] Ramsey, F. 1928. "A Mathematical Theory of Saving ." *Economic Journal* 38:543-59.
- [45] Rawls, J. 1971 . *A Theory of Justice*. Cambridge, MA: Bellknap Press.



- [46] Solow, R. 1992. "An Almost Practical Step Toward Sustainability ." Lecture on the Occasion of the Fortieth Anniversary of Resources for the Future, Washington, DC, October.
- [47] Schipper Jan et al. "The Status of the World's Land and Marine Mammals: Diversity Threat and Knowledge" *Science* October 10, 2008, p. 225-229.
- [48] Weizacker, C. C. von. 1967. "Lemmas for a Theory of Approximate Optimal Growth." *Review of Economic Studies* 34 (Jan.):143-51.
- [49] Yosida, K. 1974. *Functional Analysis*. 4th ed. Berlin, Heidelberg, New York: Springer Verlag.
- [50] Yosida, K., and E. Hewitt. 1952. "Finitely Level Independent Measures ." *Transactions of the American Mathematical Society* 72:46-66.

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