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## Distribution of Labour Productivity in Japan over the Period 1996--2006

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## Distribution of Labour Productivity in Japan over the Period 1996–2006

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### Abstract

The distribution of labour productivity is investigated by analyzing the longitudinal micro-level data set which contains detailed financial condition of large numbers of Japanese companies over the period 1996–2006. The generalized beta function of the second kind is applied to explain the distribution. We calculate marginal labour productivity by using the fitting parameters, and show that the economy in the labour market is not in equilibrium. By comparing parameters characterizing high productivity range and low productivity range, we show that inequality of low productivity range is larger than that of high productivity range. In addition, it is shown that the change of inequality in low productivity has strong correlation with GDP.

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## I. Introduction

The productivity is important quantity to discuss the economic condition and the national power. Therefore the study of the productivity has long history. A significant change happened in the 1990s. Many studies began to analyze longitudinal micro-level data of establishments and firms. The article by Bartelsman and Doms (2000) reviews the studies of productivity in the manufacturing industry until 2000.

Many studies focused on productivity in the manufacturing industry. For example, Bartelsman and Dhrymes (1998) investigated total factor of productivity in large manufacturing plants in high-tech industries from 1972 through 1986. Aw et al. (2001) measured total factor of productivity by using comprehensive firm-level panel data from the Taiwanese Census of Manufactures from 1981, 1986, and 1991. Haskel and Martin (2002) studied the productivity spread by analyzing the ARD micro-level panel of UK manufacturing establishments from 1998 through 2000.

However, the studies of the productivity in the non-manufacturing industry has emerged in the late 1990s. For example, Oulton (1998) studied a sample of 140,000 UK companies including both the manufacturing and non-manufacturing industry over the period 1989-93. Faggio et al. (2007) investigated the relation between productivity inequality and wage inequality by analyzing a UK longitudinal panel data covering the manufacturing and non-manufacturing sectors since the early 1980s. Morikawa (2008) empirically analyzed the relationship between union presence and firm performance in area such as productivity and profitability by using data on a large number of Japanese firms, covering both manufacturing and non-manufacturing industries.

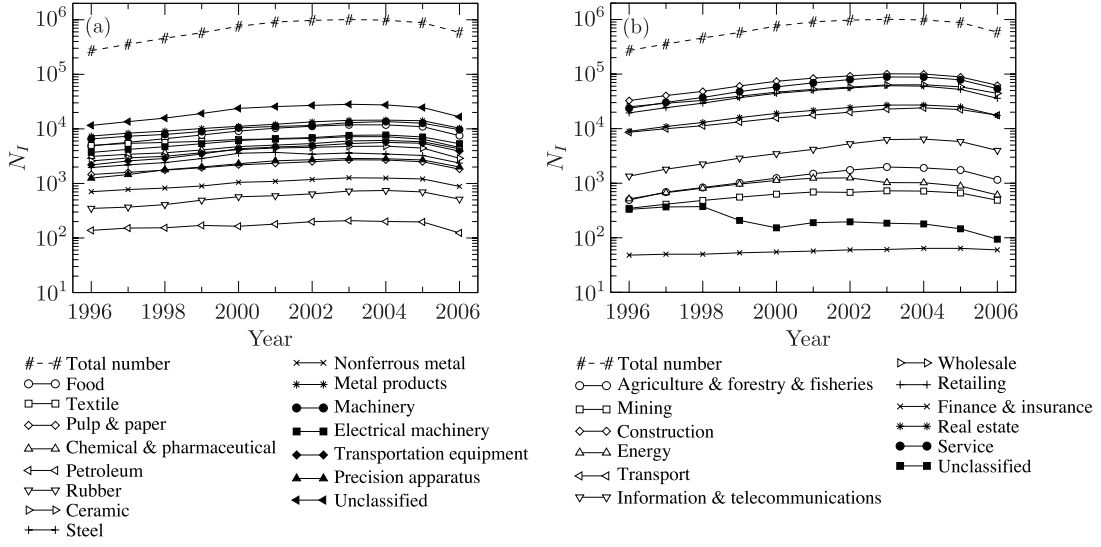
Although a dispersion of productivity across companies is investigated in many studies, the shape of the distribution function of productivity is not sufficiently discussed. Hence, one purpose of this article is to clarify the shape of the distribution function of productivity by analyzing massive data about the financial information of Japanese companies from 1996 through 2006. In addition, by analyzing parameters characterizing distributions, we investigate the dispersion of productivity across business sectors. Recently, Aoyama et al. (2008), Aoyama et al. (2009a), and Aoyama et al. (2009b) showed that the power law distribution is applicable to the distribution in high productivity region in both firm's productivity and worker's productivity. In addition, the relation between the exponent of firm's productivity and that of worker's productivity is explained by the theory of superstatistics. The power law distribution in terms of firms and sector productivity was also found in both US. and Japan (Ikeda and Souma, 2009).

As mentioned by Oulton (1998), the fact that labour productivity varies between plants and companies is well-known (Salter et al. (1969), Caves (1992), Green and Mayes (1991), Hart and Shipman (1992), Lansbury and Mayes (1996a) and Lansbury and Mayes (1996b)). This means that the labour market is not in equilibrium. To clarify whether the labour market in Japan is in equilibrium or not is the second purpose of this article. To accomplish this purpose, we consider marginal labour productivity of each business sector.

The organization of the paper is as follows. In Section 2, we explain the data sets investigated in this article. In Section 3, we define labour productivity as value added per employee, and derive the constraint that marginal labour productivity in equilibrium must satisfy. In Section 4, we investigate the distribution of labour productivity in the manufacturing industry, and apply the generalized beta function of second kind (GB2) to explain the distribution (Kleiber and Kotz, 2003). By using parameters contained in the GB2, we consider marginal productivity and the dispersion of both the high and low productivity region. In Section 5, we investigate the distribution of labour productivity in the non-manufacturing industry. Finally, Section 6 presents some conclusions and discussions.

## II. Data sets

In this article, we use the data constructed from two data sets. One is the Nikkei Economic Electronic Database System (NEEDS) sold by the Nikkei Media Marketing Inc (2007), and the other is the Credit Risk Database (CRD) compiled by the CRD Association (2007). We call these data sets as the NEEDS and the CRD. The NEEDS contains financial and non-financial data on companies listed on stock markets. The number of financial items for each company filed in the NEEDS exceeds eight hundred. The CRD is the collection of financial and non-financial data, including default information, on small and medium enterprises. The total number of companies recorded by the



**Figure 1:** Change of the number of companies  $N_I$ . (a) Manufacturing industry. (b) Non-manufacturing industry.

CRD is illustrated as the dashed lines in Figs. 1. It is reported by the Ministry of Internal Affairs and Communications in Japan that 1,529,619 Japanese companies existed in 2004, while the CRD contains 985,561 companies in that year. Hence, 65% of Japanese companies are covered by the CRD in 2004. This coverage ratio is almost same over the period 1996-2006. Although the CRD contains 93 financial items for each company, almost 60% of the companies fill completely these items. We investigate these companies in this article.

We identify companies included in both the NEEDS and the CRD, and merge these two data sets by avoiding overlaps. The number of companies,  $N_I$ , included by merged data is shown in Figs. 1. Here  $I$  identify each business sector. Figs. 1 (a) and (b) show the change of the number of companies for the manufacturing industry and the non-manufacturing industry, respectively. The manufacturing industry is constructed from food, textile, pulp & paper, chemical & pharmaceutical, petroleum, rubber, ceramic, steel, nonferrous metal, metal products, machinery, electrical machinery, transportation equipment, precision apparatus, and unclassified manufacturing. The non-manufacturing industry is constructed from agriculture & forestry & fisheries, mining, construction, energy, transport, information & telecommunications, wholesale, retailing, finance & insurance, real estate, service, and unclassified non-manufacturing.

### III. Marginal labor productivity in equilibrium

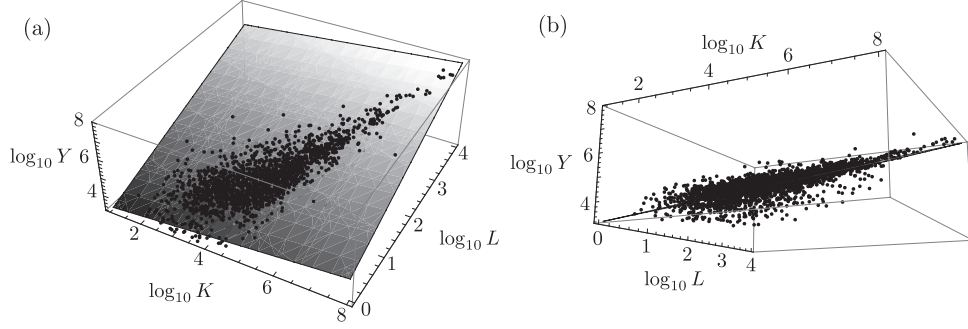
Though many definitions of productivity has been proposed by many researchers and organizations, we consider labor productivity in this article. We define labor productivity  $x$  as value added per employee:

$$x = \frac{Y}{L}, \quad (1)$$

where  $Y$  is value added and  $L$  is employee. Though there are many definitions of value added, we use the definition proposed by the Bank of Japan:

$$Y = O + C + I + R + T + D, \quad (2)$$

where  $O$ ,  $C$ ,  $I$ ,  $R$ ,  $T$ , and  $D$  are ordinary gain, labor and welfare expenses, interest expense and discount premium, rent, taxes and public charges, and depreciation expense during the year, respectively.



**Figure 2:** Distribution of  $\log_{10} K$ ,  $\log_{10} L$ , and  $\log_{10} Y$  of the companies belonging to the precision apparatus sector in 2003.

Operating profit  $\Pi$  is defined by

$$\Pi = pY - rK - wL, \quad (3)$$

where  $p$ ,  $r$ ,  $K$ , and  $w$  are price, interest rate, capital, and wage rate, respectively. Each company maximizes its profit  $\Pi$  by adjusting  $L$ :

$$\frac{\partial \Pi}{\partial L} = p \frac{\partial Y}{\partial L} - w = 0. \quad (4)$$

Thus, marginal labor productivity  $\partial Y / \partial L$  satisfies

$$\frac{\partial Y}{\partial L} = \frac{w}{p}. \quad (5)$$

In equilibrium, there is no arbitrage opportunity for wage rates, therefore the actual wage rate  $w/p$  is equal for each company. Hence, marginal labor productivity of  $i$ -th company and that of  $j$ -th company are equal, i.e.,

$$\left( \frac{\partial Y}{\partial L} \right)_i = \left( \frac{\partial Y}{\partial L} \right)_j. \quad (6)$$

If we assume the Cobb-Douglass production function given by

$$Y = AK^\alpha L^\beta, \quad (7)$$

Eq. (6) can be rewritten as

$$\beta_i x_i = \beta_j x_j. \quad (8)$$

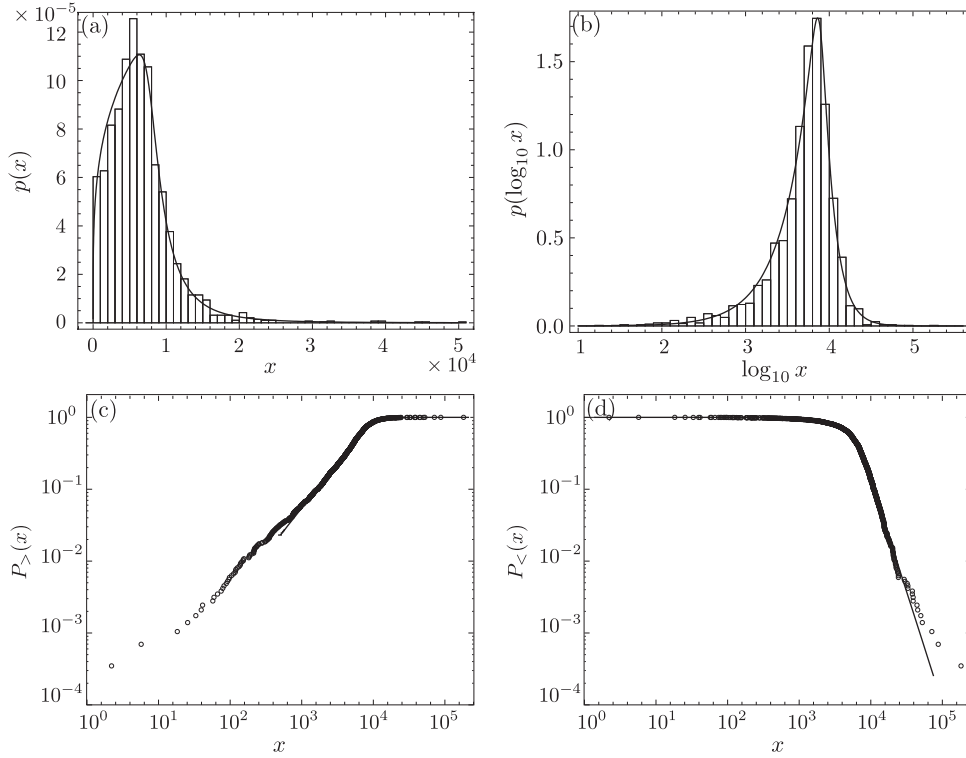
If we have sufficiently long time series data for  $Y_{i,t}$ ,  $K_{i,t}$ , and  $L_{i,t}$ , we can estimate  $\beta_i$  for each company. Here the subscript  $t$  means time. In addition, if we have sufficiently long time series data for  $x_{i,t}$ , we may regard the averaged value or the mode value of  $x_{i,t}$  as  $x_i$  in Eq. (8). Therefore, we can investigate the validity of Eq. (8). However, it is difficult to estimate  $\beta_i$  and  $x_i$  by using such manipulation, because we do not have such a long time series data. Hence, to overcome this problem, we abandon the investigation of the validity of Eq. (8) in the company level, and consider the validity of Eq. (8) in the business sector level. Thus we modify Eq. (8) to

$$\beta_I x_I = \beta_J x_J, \quad (9)$$

where  $I$  and  $J$  identify the business sectors.

The example of the estimation of  $\beta_I$  is shown in Figs. 2. These figures are for the precision apparatus sector in 2003. This sector is constructed from  $N_I = 2,872$  companies in 2003. In these figures, each dot corresponds to each company, and the surface is given by

$$\log_{10} Y = \log_{10} A_I + \alpha_I \log_{10} K + \beta_I \log_{10} L \quad (10)$$



**Figure 3:** Distribution of labour productivity of the precision apparatus sector in 2003. (a) Histogram of  $x$  in the range  $0 < x \leq 50000$ . (b) Histogram of  $\log_{10} x$ . (c) Cumulative distribution  $P_{>}(x)$ . (d) Cumulative distribution  $P_{<}(x)$ .

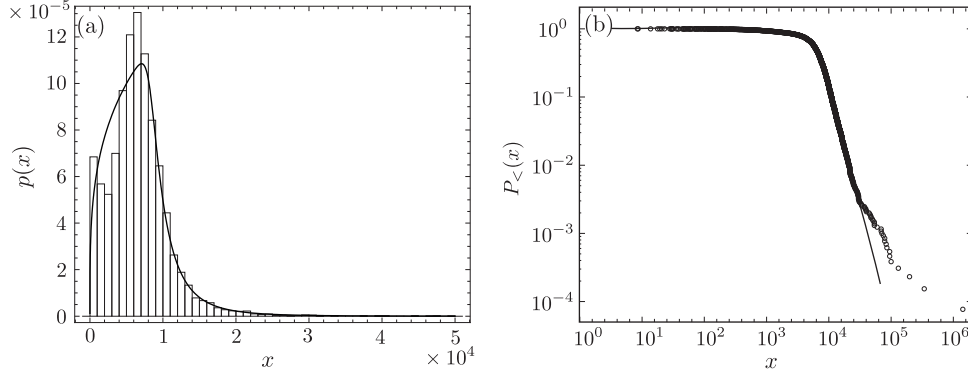
with  $A_I = 10^{3.09}$ ,  $\alpha_I = 0.159$ , and  $\beta_I = 0.890$ , respectively. Fig. 2 (a) shows the distribution over the surface. This figure shows the positive correlation between  $\log_{10} K$  and  $\log_{10} L$ . Fig. 2 (b) is the perspective from the parallel direction to the surface described by Eq. (10). This figure shows that each dot exists near the surface, and suggests that the Cobb-Douglass production function is applicable to the case.

To investigate the validity of Eq. (8), we must define  $x_I$ . In below, we use the model value of  $x_i$  as  $x_I$ . In next section, we explain the method to obtain  $x_I$ .

#### IV. Labour productivity of manufacturing industry

To obtain the value of  $x_I$ , we must study the distribution of  $x_i$ . In addition, by studying the distribution of  $x_i$ , we obtain the characteristics of labour productivity. The example of the distribution of  $x_i$  is shown in Figs 3. These figures are for the precision apparatus sector in 2003. This sector is constructed from  $N_I = 2,872$  companies in 2003. Figure. 3 (a) shows the histogram of  $x$  in the range  $0 < x \leq 50000$ . This figure shows that the distribution has the peak around  $x \approx 5000$ , and has the fat tail in high productivity region. Fat tail behavior is more clarified by considering the distribution of  $\log_{10} x$ . Fig. 3 (b) illustrates the histogram of the distribution of  $\log_{10} x$ . This figure shows that the fat tail is applicable to both side of the distribution, and that the distribution is asymmetry. Oulton (1998) pointed out that the distributions of labour productivity in UK are approximately lognormal. However, Fig. 3 (b) does not support the lognormal distribution.

The behavior of tail part is more clarified by studying a cumulative distribution. Fig. 3 (c) shows



**Figure 4:** Distribution of labour productivity of the machinery sector in 2003. (a) Histogram of  $x$  in the range  $0 < x \leq 50000$ . (b) Cumulative distribution  $P_{<}(x)$ .

the log-log plot of the cumulative distribution  $P_{>}(x)$  defined by

$$P_{>}(x) = \int_0^x p(y)dy, \quad (11)$$

in the continuous representation. Here  $p(y)$  is a probability density function. This figure shows that  $\log_{10} P_{>}(x)$  in low productivity region is represented by a linear function of  $\log_{10} x$ :

$$\log_{10} P_{>}(x) \propto \nu \log_{10} x, \quad (12)$$

where  $\nu$  is the value of the slope, therefore, the distribution in low productivity region is represented by the power law function given by

$$P_{>}(x) = Ax^\nu, \quad (13)$$

where  $\log_{10} A$  is an intercept.

The cumulative distribution is also defined by

$$P_{<}(x) = \int_x^\infty p(y)dy = 1 - P_{>}(x), \quad (14)$$

in the continuous representation. Fig. 3 (d) is the log-log plot of the cumulative distribution defined by Eq. (14). This figure shows that the cumulative distribution in high productivity region is also represented by the power law distribution given by

$$P_{<}(x) = A'x^{-\mu}. \quad (15)$$

Now, we fit the distribution of labour productivity by the generalized beta function of the second kind (GB2) given by

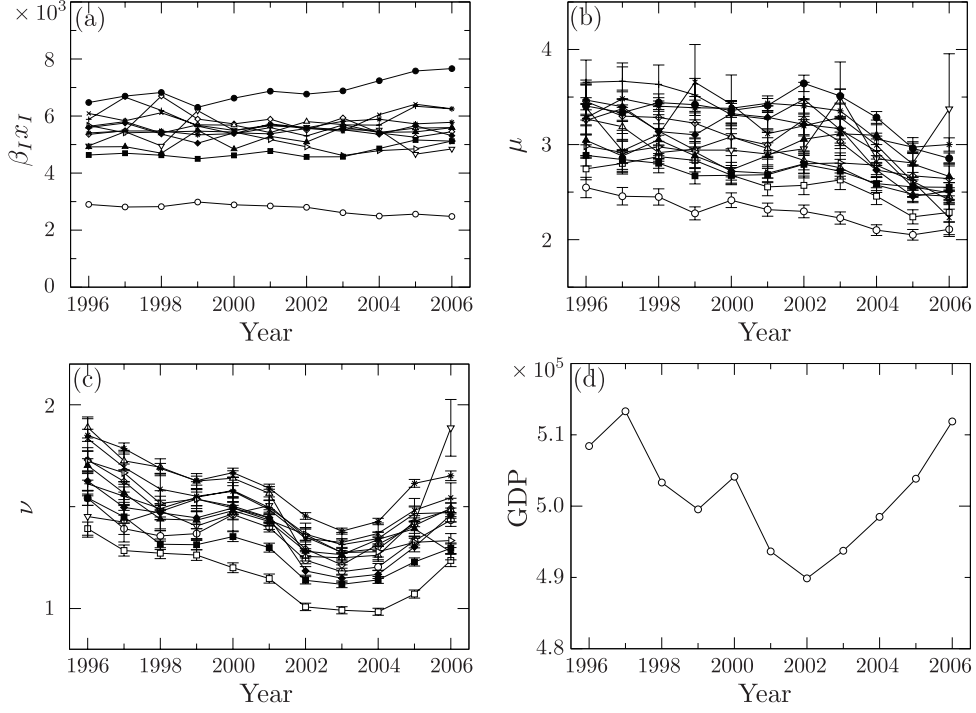
$$p(x; \mu, \nu, q, x_0) = N_B \frac{1}{x} \left( \frac{x}{x_0} \right)^\nu \left[ 1 + \left( \frac{x}{x_0} \right)^q \right]^{-(\mu+\nu)/q}, \quad (16)$$

where  $\mu$ ,  $\nu$ , and  $q$  determine the shape of the distribution, and  $x_0$  determines the scale (Kleiber and Kotz, 2003). Here  $N_B$  is the normalization factor given by

$$N_B = \frac{q}{B(\mu/q, \nu/q)}, \quad (17)$$

where the incomplete beta function  $B(r, s) = B(1, r, s)$  is defined by

$$B(z, r, s) = \int_0^z t^{r-1} (1-t)^{s-1} dt. \quad (18)$$



**Figure 5:** (a) Change of marginal labour productivity  $\beta_I x_I$ . (b) Change of  $\mu$ . (c) Change of  $\nu$ . (d) Change of GDP

The cumulative distribution of  $p(x; \mu, \nu, q, x_0)$  is given by

$$P_{<}(x; \mu, \nu, q, x_0) = \frac{B(z, \mu/q, \nu/q)}{B(\mu/q, \nu/q)}, \quad (19)$$

$$P_{>}(x; \mu, \nu, q, x_0) = 1 - P_{<}(x; \mu, \nu, q, x_0), \quad (20)$$

where

$$z = \left[ 1 + \left( \frac{x}{x_0} \right)^q \right]^{-1}. \quad (21)$$

In the limit  $x \rightarrow 0$  and  $x \rightarrow \infty$ , Eq. (16) behaves

$$p(x; \mu, \nu, q, x_0) = \begin{cases} \frac{N_B}{x} \left( \frac{x}{x_0} \right)^\nu & \text{for } x \rightarrow 0 \\ \frac{N_B}{x} \left( \frac{x}{x_0} \right)^{-\mu} & \text{for } x \rightarrow \infty \end{cases}. \quad (22)$$

Equation (16) has the maximum at

$$x = \left( \frac{\nu - 1}{\mu + 1} \right)^{1/q} x_0. \quad (23)$$

To fit the distribution of  $x$  by the GB2, we calculate the parameter set  $(\mu, \nu, q, x_0)$  that maximize



**Table 1:** Correlations between  $\mu$  and GDP, and  $\nu$  and GDP in the manufacturing industry.

| Sector                      | Cor( $\mu$ , GDP) | Cor( $\nu$ , GDP) |
|-----------------------------|-------------------|-------------------|
| Food                        | 0.3               | 0.62              |
| Textile                     | 0.15              | 0.69              |
| Pulp and paper              | 0.15              | 0.75              |
| Chemical and pharmaceutical | 0.23              | 0.66              |
| Rubber                      | 0.32              | 0.45              |
| Ceramic                     | 0.05              | 0.46              |
| Steel                       | 0.2               | 0.58              |
| Nonferrous metal            | -0.31             | 0.74              |
| Metal products              | -0.23             | 0.8               |
| Machinery                   | -0.66             | 0.83              |
| Electrical machinery        | 0.04              | 0.68              |
| Transportation equipment    | 0.14              | 0.77              |
| Precision apparatus         | -0.68             | 0.52              |

the log likelihood given by

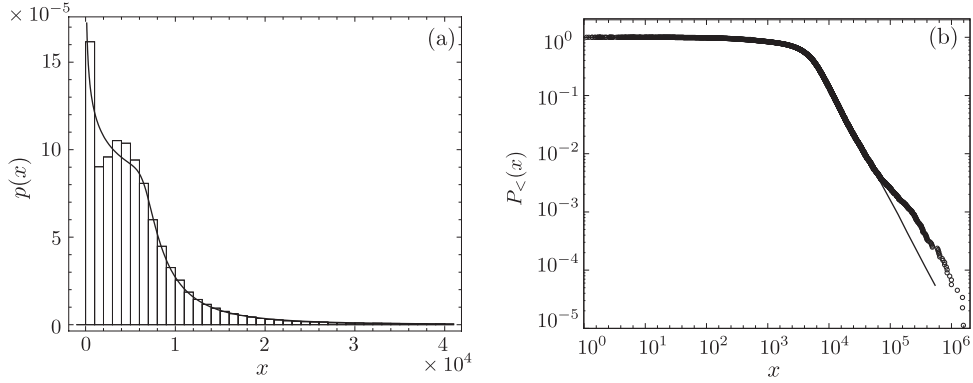
$$L(\mu, \nu, q, x_0) = \sum_{i=1}^{N_I} \log p(x_i; \mu, \nu, q, x_0) \quad (24)$$

$$= N_I (\log N_B + \nu \log x_0) + (\nu - 1) \sum_{i=1}^{N_I} \log x_i - \frac{\mu + \nu}{q} \sum_{i=1}^{N_I} \log \left[ 1 + \left( \frac{x_i}{x_0} \right)^q \right]. \quad (25)$$

In the case of the distribution shown in Figs. 3, Eq. (25) is maximized by the parameter set  $(\mu, \nu, q, x_0) = (3.11, 1.26, 13.17, 7762)$ . The solid lines in Figs. 3 correspond to Eqs. (16), (19), and (20) with these parameter values. These figures show that the distribution of  $x$  is well fitted by the GB2.

The GB2 is applicable to the distributions of  $x$  for many sectors belonging to the manufacturing industry. However the machinery sector is an exception. For example, the distribution of  $x$  of the machinery sector in 2003 is shown in Figs. 4. In these figures, the solid lines are GB2 with  $(\mu, \nu, q, x_0) = (3.51, 1.29, 17.13, 8299)$ . Figure 4 (a) shows that the distribution has tow peaks which exist in low productivity region and  $x \approx 8000$ . We call these peaks as the small peak and the large peak, respectively. From this figure, we recognize that the GB2 deviates from the distribution in the left side region of the large peak, i.e., the small  $x$  region. However, the distribution in the right side region of the large peak, i.e., the large  $x$  region is well fitted by the GB2. This is more clarified by considering the cumulative distribution as shown in Fig 4 (b), which is the log-log plot of the distribution. Therefore, the value of  $\mu$  is reliable even if the GB2 can not explain the distribution of small  $x$  region in the case of the machinery sector.

As mentioned in previous section, we use the modal value of  $x_i$  as  $x_I$ . It is reasonable to regard the value given by Eq. (23) as the modal value of  $x_i$ . Hence we can consider marginal labour productivity by multiplying  $\beta_I$  and  $x_I$  calculated from Eq. (23). The change of  $\beta_I x_I$  is shown in Fig. 5 (a). The symbols in this figure are same as that in Fig. 1 (a). Here, we ignore the textile sector, the petroleum sector, the machinery sector, and the unclassified sector. We obtain the value  $\nu < 1$  for the textile sector in some years, therefore Eq. (23) is not applicable. This is the reason why we ignore the textile sector. The petroleum sector contains small number of companies as shown in Fig. 1 (a), therefore we exclude this sector from the investigation. As mentioned above, the GB2 is not applicable to low



**Figure 6:** Distribution of labour productivity of the service sector in 2003. (a) Histogram of  $x$  in the range  $0 < x \leq 40000$ . (d) Cumulative distribution  $P_{<}(x)$ .

productivity region in the case of the machinery sector, therefore the value of  $\nu$  is not reliable. Thus we ignore the machinery sector in this study. The unclassified sector is mixture of many kinds of business, so it is meaningless to analyze this sector as one industry. Fig. 5 (a) shows that marginal labour productivity  $\beta_I x_I$  depends on the business sectors and the years. This means that equilibrium is not maintained in the labour market. The value of marginal labour productivity distributes in the range  $4 \times 10^3 < \beta_I x_I < 8 \times 10^3$  for many sectors except for the food sector.

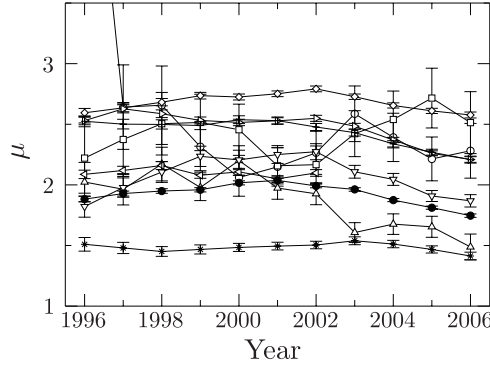
The inequality in high productivity region is quantified by the value of  $\mu$ . The small value of  $\mu$  means unequal distribution of labour productivity. The change of  $\mu$  is shown in Fig. 5 (b). The symbols in this figure are same as that in Fig. 1 (a). Here, we ignore the petroleum sector and the unclassified sector. The reason why these sectors are ignored is same as the case of Fig. 5 (a). This figure shows that the value of  $\mu$  distributes in the range  $2 < \mu < 4$ . The food sector has the smallest value over the all period.

The inequality in low productivity region is quantified by the value of  $\nu$ . As same as the case of  $\mu$ , the small value of  $\nu$  means unequal distribution of labour productivity. The change of  $\nu$  is shown in Fig. 5 (c). The symbols in this figure are same as that in Fig. 1 (a). Here, we ignore the petroleum sector and the unclassified sector for the same reason in the case of Fig. 5 (a). This figure shows that the value of  $\nu$  distributes in the range  $1 < \nu < 2$ . The distribution range of  $\nu$  is smaller than that of  $\mu$ . This means that the distribution of labour productivity in high productivity region is more equal than that in low productivity region. The typical behavior is observed in this figure, i.e.,  $\nu$  increases around the year 2000 and decreases around the year 2003.

These behaviors of  $\nu$  shown in Fig. 5 (c) reminds us the change of GDP. The change of GDP over the period 1996-2006 is shown in Fig. 5 (d). By comparing Fig. 5 (c) and (d), we find that there is strong and positive correlation between  $\nu$  and GDP. The correlations between  $\mu$  and GDP, and  $\nu$  and GDP are summarized in Table 1. This result shows that the correlation between  $\mu$  and GDP is weak or negative. On the other hand, the correlation between  $\nu$  and GDP is strong. This means that the inequality of labour productivity in low productivity region has positive and strong correlation with GDP.

## V. Labour productivity of non-manufacturing industry

As same as the manufacturing industry, we investigate the distribution of labour productivity in the non-manufacturing industry by applying the GB2. However, same as the case of machinery sector, the GB2 is not applicable to low productivity region in the non-manufacturing industry except for the real estate sector. The example of the distribution of labor productivity in the non-manufacturing industry is shown in Figs. 6. These figures show the distributions of labour productivity in the service sector in 2003. Figure 6 (a) shows the histogram of  $x$  in the range  $0 < x \leq 40000$ . This figure shows that there are two peaks in the distribution. This pattern is same as the case of



**Figure 7:** Change of  $\mu$  of each business sector constructing the non-manufacturing industry.

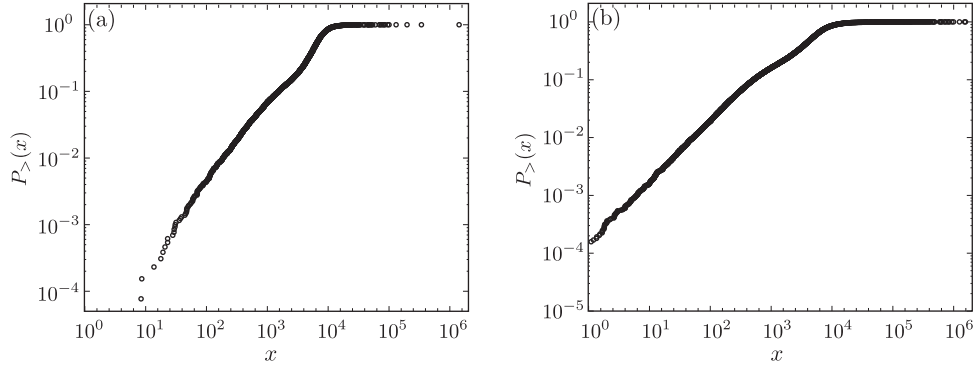
machinery sector shown in Fig. 4 (a). In this figure, the solid line is the GB2,  $p(x; \mu, \nu, q, x_0)$ , with  $(\mu, \nu, q, x_0) = (1.96, 0.83, 14.35, 6737)$ . This figure shows that distribution of the right side region of the large peak is well fitting by the GB2. However the distribution of the left side region of the large peak is not fitted by the GB2. Figure 4 (b) show the cumulative distribution  $P_<(x)$ . In this figure, the solid line is  $P_<(x; \mu, \nu, q, x_0)$  with  $(\mu, \nu, q, x_0) = (1.96, 0.83, 14.35, 6737)$ . This figure shows that the high productivity region is well explained by the GB2.

The distributions of labour productivity of each sector and each year in the non-manufacturing industry are almost same as Figs. 6. Hence the GB2 is not applicable to low productivity region, and the obtained value of  $\nu$  is not reliable. Therefore, marginal labour productivity derived from Eq. (23) is not trusted. On the other hand, the distribution in high productivity region is well fitted by the GB2. Thus we can investigate the value of  $\mu$ . The change of  $\mu$  is shown in Fig. 7. The symbols in this figure are same as that in Fig. 1 (b). Here, we ignore the finance & insurance sector and the unclassified sector. The finance & insurance sector contains small number of companies as shown in Fig. 1 (b), therefore we exclude this sector from the investigation. The unclassified sector is mixture of many kinds of business, so it is meaningless to regard this sector as one industry. This figure shows that the value of  $\mu$  distributes in the range  $1 < \mu < 3$ . This range is smaller than the range in the case of the manufacturing industry shown in Fig. 5 (b). This means that, in high productivity region, the inequality of labour productivity of the non-manufacturing industry is higher than that of the manufacturing industry.

## VI. Conclusion

In this article, the distributions of labour productivity in Japan over the period 1996-2006 is investigated by analyzing the micro-level data sets. We applied the GB2 to explain the distributions. It was clarified that the distributions of labour productivity in the manufacturing industry except from the machine sector are well explained by the GB2. We defined marginal labour productivity as  $\beta_I x_I$  and obtained it for each sector and each year. By comparing each value of marginal labour productivity, we clarified that marginal labour productivity depends on both the sectors and the years. This means that the economy in the labour market is not in equilibrium. In addition, we obtained  $\mu$  and  $\nu$  for each sector and each year. Here  $\mu$  represents the inequality in high productivity region, and  $\nu$  represents that in low productivity region. By comparing  $\mu$  and  $\nu$ , we clarified that the inequality in low productivity region is larger than that in high productivity region.

The GB2 is also applied to the distribution of labour productivity in the non-manufacturing industry. However, except for the real estate sector, the GB2 is not applicable to each sector in the non-manufacturing industry. The existence of tow peaks in the distribution, especially the peak in low productivity region, prevents the application of GB2 to the distribution. However, the GB2 is applicable to high productivity region of all sectors in the non-manufacturing industry. The distribution in high productivity region is characterized by the power law exponent  $\mu$ . By comparing



**Figure 8:** Cumulative distribution  $P_{>}(x)$  of labour productivity in 2003. (a) The machinery sector. (b) The service sector.

$\mu$  obtained for the manufacturing industry with that obtained for the non-manufacturing industry, we clarified that  $\mu$  of the non-manufacturing industry is smaller than that of the manufacturing industry. This means that the distribution in high productivity region of the manufacturing industry is more equal than that of the non-manufacturing industry.

We abandoned to apply the GB2 to the distribution in low productivity region. However, as shown in Figs. 8, the distributions in low productivity region show the power law distribution. Fig. 8 (a) and (b) are the log-log plot of the cumulative distribution  $P_{>}(x)$  for the machinery sector in 2003 and that for the service sector in 2003, respectively. Hence, asymptotic behavior of the fitting function  $p(x)$  is

$$p(x) \approx \begin{cases} x^\nu & \text{for } x \rightarrow 0 \\ x^{-\mu} & \text{for } x \rightarrow \infty \end{cases} . \quad (26)$$

One of the simple way to replicate asymptotic behavior given by Eq. (26) is the superposition of GB2:

$$p(x; \mu, \nu, q, x_0) = c_1 p(x; \mu_1, \nu_1, q_1, x_1) + c_2 p(x; \mu_2, \nu_2, q_2, x_2), \quad (27)$$

where  $c_1$  and  $c_2$  are superposition constants. To verify the validity of the applicability of Eq. (27) is future problem.

In this article, we did not consider the reason why two peaks emerge in the distribution of labor productivity for the machinery sector and almost all of the sectors in the non-manufacturing industry. However, it is natural to expect that the existence of peak in low productivity region has relation with the size of companies. It is also natural to expect that the peak in low productivity region has relation with the entry barrier of the sector, because many new companies enter the sector if the entry barrier is low. To verify these assumptions is also future problems.

The distribution of labour productivity must be explained by the microscopic model that describes the behavior of each company. It is natural to expect that the model is described by the stochastic process with entry and exit of companies. Therefore, it is important to estimate the entry and exit ratio by empirical study. In addition, estimation of the growth rate of labour productivity is also important to construct realistic model.

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