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Urban income inequality and social welfare

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Tinbergen Institute Discussion Paper

Urban income inequality and social welfare

*Paul Koster*¹

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URBAN INCOME INEQUALITY AND SOCIAL WELFARE

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SUMMARY — When income inequality increases when average income levels increase, rises in average income levels might result in inequality costs. This paper develops marginal social welfare measures that account for the possibility that income inequality changes when average income levels change. Applications are given for the city of Amsterdam, The Netherlands. For this city, the income elasticity of the Gini is estimated in the range 0.25-0.48. Estimates of marginal welfare changes vary greatly with model choice. For plausible cases, estimates can be negative, raising doubts whether average income increases are beneficial in rich urban areas with high valuations of equality where income inequality increases in income.

Keywords — Social welfare; income inequality; inequity aversion; post-growth.

JEL Codes — A13; D61; D63; E24

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I. Introduction

Urban and transport economists often have motivated their policies by the potential indirect impacts on average income levels. For example, in a recent meta-study on urban advantages resulting from agglomeration effects, Donovan et al. (2021, p.2) find that the density elasticity of wages is in the range of 0.015-0.039 with 95% credibility. A policy that doubles the density, therefore predicts an increase in average wages of about 1.045%-2.740%. Or consider the impact of accessibility improvements on wages. Knudsen et al. (2022) estimate the elasticity in the range 0.025-0.029 for Denmark, implying that average wages increase with 1.748%-2.030% when doubling the accessibility in Denmark. These estimates are useful for the estimation of orders of magnitude for the wider economic benefits of infrastructure investments.

As wages are a substantial part of disposable household income, average disposable household income after taxes will increase when wages increase. However, policies that impact average income levels might also impact income inequality. In order to evaluate the wider welfare impacts, one therefore needs welfare indicators that account for average levels of income and the inequality of income in the population. These indicators aim to account for two important societal values: *efficiency* and *equity*.

Suppose a policy leads to growth in average income levels. A 1% increase in income for a household with an income level of 10.000 euros per year is equal to 100 euros, whereas a household with an income level of 100.000 euros per year gains 1000 euros for the same percentage increase. This 1% increase for all households will not impact the share of households' income in the total income. But an equal percentage increase for all households will increase the absolute difference in income levels for these two households from 90.000 to 90.900 euros per year. The way inequality costs are modelled in social welfare is therefore likely to be relevant.

The contribution of this paper is fourfold. First, it derives tractable marginal average social welfare expressions for an average income increase. The result of this analytical investigation is relevant for any policy that changes average income levels. Second, the paper analyses how income inequality is empirically related to average income levels. When income inequality is decreasing (increasing) in average income levels, efficiency and equity are complements (substitutes). When income growth is unevenly distributed over households, -with more growth for higher incomes-, a positive relationship between income inequality and average income is expected. Third, the paper investigates with how much average social welfare will change when average income levels increase using the empirical estimates and the analytical marginal expressions. The fourth contribution is the integrative empirical and analytical analysis of the welfare effects of income inequality for the city of Amsterdam, The Netherlands. This will give new insights on urban income inequality and social welfare and shows how a combination of empirical and theoretical analysis can result in easy-to-use quantitative results.

The paper proceeds as follows. Section II develops and reviews some important analytical approaches to derive social welfare indicators that account for both *levels* and *inequality*. In the axiomatic literature, Sen (1976, p.31) showed how average income levels and income inequality can be accounted for in aggregated social welfare in a very tractable way. Section II also builds on recent developments in the behavioural economics literature by Schmidt and Wichardt (2019) who link the inequity model of Fehr and Schmidt (1999) with an aggregate social welfare function that has the Sen result as a special case. From these social welfare functions marginal expressions for average income changes are derived which require a minimum amount of information: (i) the income elasticity of the Gini; (ii) the Gini coefficient and; (iii) the (estimated) preferences for income equality.

Section III then discusses the empirical relationship between income inequality and average income levels. Inspired by the work of Kuznets (1955), it estimates the non-linear income

elasticity of the Gini for the city of Amsterdam in The Netherlands. The regressions use urban neighbourhoods as the relevant unit of analysis and the average disposable household income as the measure of income. Therefore, income inequality is measured *after* institutional redistribution via taxes has taken place. Informal redistribution between households is ignored in the analysis. Glaeser et al. (2009, p.617) state that the focus on neighbourhoods is justified because envy is more likely to be directed toward near neighbours. Luttmer (2005) provides empirical evidence for the presence of envy in neighbourhoods using happiness data with household income levels and average neighbourhood income levels as explanatory variables. Using neighbourhoods as units of comparison implicitly assumes that households compare their income to the income levels in their neighbourhood and not (additionally) to their network of family members, friends or colleagues. The panel data regressions in Section III improve on (some of the) earlier studies by using: (i) a quadratic logged specification of mean-scaled explanatory variables; (ii) controls for time *and* spatial unobserved heterogeneity and (iii) controls for neighbourhood characteristics.¹ Close to our regression methodology are the contributions of Partridge et al. (1996) and Levernier et al. (1998) who use state fixed effects and covariates to predict aggregate income inequality.

Section IV combines the insights of section II and III by applying the marginal expressions derived in Section II using the elasticity estimates of Section III as an input. This section gives insights on the quantitative impact of income inequality. Section V concludes with a discussion.

¹ Early cross-section contributions explaining (urban) income inequality are Aigner and Heins (1967), Al-Samarrie and Miller (1967), Conlisk (1967), Long et al. (1970) and Nord (1980). Glaeser et al. (2009, p. 624) report cross-sectional estimates with the log of *median* income as an explanatory variable. Castells-Quintana (2018) use the quadratic log specification for the analysis of a Kuznets curve on country level data. Houthakker (1959), Soltow (1960) and Morgan (1962) are early contribution discussing the impacts of covariates such as age, education, occupation and household composition.

II. Social welfare and income inequality

A. Introduction

Let μ_{Yr} the average disposable income level for the selected population in area r . Furthermore, let I_{Yr} be an indicator of income inequality, and let N_r be the number of households. For notational simplicity, the subscript r is omitted in the theoretical analysis. Average social welfare (SW) is defined as:

$$(1) \quad \frac{SW}{N} = h[\mu_Y, I_Y(\mu_Y)].$$

The marginal impact of average income on average social welfare is then given by:

$$(2) \quad \frac{\partial \frac{SW}{N}}{\partial \mu_Y} = \frac{\partial h}{\partial \mu_Y} + \frac{\partial h}{\partial I_Y} \frac{\partial I_Y(\mu_Y)}{\partial \mu_Y}.$$

The first term in Eq. (2) is the direct marginal welfare of average income. It is usually positive implying that average social welfare increases when average income levels increase. When $\frac{\partial h}{\partial I_Y} < 0$, the second part of Eq. (2) is negative when income inequality is increasing in average income. The fact that I_Y changes with changes in average income levels make ordinal comparisons more complicated without information on the sign of this change. Data analysis is therefore needed to investigate $\frac{\partial I_Y(\mu_Y)}{\partial \mu_Y}$ and preferences for income inequality $\frac{\partial h}{\partial I_Y}$ in Eq. (2).

An ordinal analytical welfare comparison of distributions using generalized Lorenz dominance in the spirit of Schorroek (1983) will give insights on whether social welfare increases or not, but will not deliver quantitative entities for policy analysis that provide insight on the size of the marginal welfare impacts.

The first approach pursued in the literature to develop Eq. (1) is to use axiomatic analysis. Some results are discussed in subsection II.B and marginal expressions as in Eq. (2) are

provided. The second approach is to start with a household welfare function and then view Eq. (1) as the sum of households' welfare functions.² This is discussed in subsection II.C.

B. Review of some important earlier axiomatic results

Let $0 < G_Y(\mu_Y) < 1$ be the Gini coefficient (Gini, 1912). Using axiomatic analysis, Sen (1976, p.31) derived a social welfare measure that accounts for income inequality: $SW = N\mu_Y(1 - G_Y(\mu_Y))$. Total disposable household income $N\mu_Y$ is scaled with a measure of equality $(1 - G_Y(\mu_Y))$. Complete income inequality results in social welfare equal to 0 for this measure.

Kakwani (1980) used different assumptions on transfer-sensitivity leading to a social welfare function that divides total spendable income by an index of inequality $1 + G_Y(\mu_Y)$: $SW = \frac{N\mu_Y}{1 + G_Y(\mu_Y)}$. Complete income inequality for this social welfare function means dividing the total income by 2.

Dagum (1990, Eq. 35) used more general axiomatic assumptions and arrived at a result $SW = N\mu_Y \left(1 - \frac{2G_Y(\mu_Y)}{1 + G_Y(\mu_Y)}\right)$ (see also Gruen and Klasen (2008, Eq. 4)). Like the Sen measure, complete income inequality leads to social welfare equal to 0 for the Dagum measure. Dagum (1990, p.93) also showed that each axiomatically derived social welfare function has an 'aggregated' dual counterpart based on households' utility functions.

Gruen and Klasen (2008, Table 1) provide a comparison of countries social welfare *levels* by applying the insights of Sen (1976), Dagum (1990) and Atkinson (1970). Dollar et al. (2015, Table 1) analyse welfare changes in countries, using different types of social welfare functions and an inequality indicator which is independent of average income levels. Zheng (1997)

² Capraro and Perc (2021, Table 2) give an overview of 6 specifications of utility functions that account for inequality measures at the level of the individual or household. This paper re-interprets two of these functions as welfare functions of households over the division of the total income in a neighbourhood.

provides a further extensive overview of the axiomatic literature on poverty indices. Vaughan (1987) shows how to aggregate homogeneous iso-elastic surplus functions with non-linear income effects without relative income comparisons and homogeneous marginal utility of income in the spirit of Atkinson (1970). These earlier contributions do not investigate changes in average social welfare when the Gini depends on mean income levels (see Eq. (2)) and do not employ measures of disposable household income for their policy analysis.

Define $\epsilon_Y \equiv \frac{\partial G_Y(\mu_Y)}{\partial \mu_Y} \frac{\mu_Y}{G_Y(\mu_Y)}$ as the income elasticity of the Gini. Table 1 summarizes the results on marginal *average* social welfare that will be used in this paper. The results show that information on ϵ_Y and the level of the Gini is sufficient to determine the change in average social welfare for a change in average income. The informational requirements for applying the model are therefore low.

Table 1: summary of axiomatic results used in this paper and their marginal impacts.

| <i>Author</i> | <i>Average social welfare $\frac{SW}{N}$</i> | <i>Marginal impact of increase in average income $\frac{\partial \frac{SW}{N}}{\partial \mu_Y}$</i> |
|----------------------|---|--|
| Sen (1976) | $\mu_Y(1 - G_Y)$ | $1 - (1 + \epsilon_Y)G_Y$ |
| Kakwani (1980) | $\frac{\mu_Y}{1 + G_Y}$ | $\frac{1}{1 + G_Y} \left[1 - \epsilon_Y \frac{G_Y}{1 + G_Y} \right]$ |
| Dagum (1990, Eq. 35) | $\mu_Y \left(1 - \frac{2G_Y}{1 + G_Y} \right)$ | $1 - \frac{2G_Y}{1 + G_Y} \left[1 + \epsilon_Y \frac{1}{1 + G_Y} \right]$ |

Notes: marginal impacts derived by the author.

C. Fair shares

C.1 The Bolton and Ockenfels household welfare function

This section discusses the Bolton and Ockenfels (2000) (BO) approach to account for inequality. The BO-approach conceptualize inequality as obtaining a fair *share* of the total disposable income that is available in the population of interest. Households' welfare (HW) is defined as:

$$\begin{aligned}
(3) \quad HW_n &= Y_n - \frac{\beta_n}{2} \left[\frac{Y_n}{N\mu_Y} - \frac{\mu_Y}{N\mu_Y} \right]^2 = Y_n - \frac{\beta_n}{2} \left[\frac{1Y_n}{N\mu_Y} - \frac{1}{N} \right]^2 \\
&= Y_n - \frac{\beta_n}{2} \left(\frac{1}{N} \right)^2 \left[\frac{Y_n}{\mu_Y} - 1 \right]^2.
\end{aligned}$$

Other factors than income that impact HW , are assumed to be additive and can therefore be ignored. The first term in Eq. (3) gives the level of income for household n . It is assumed that it has unit marginal utility of income. Extensions that alleviate this assumption will be provided in Section IV. The second term in Eq. (3) takes the squared difference of the share of n in the total. For a household with an average income level ($Y_n = \mu_Y$), the inequality part between brackets will be equal to 0. The household with an average income level therefore does not contribute to the inequality related decrease in households' welfare. For all other cases the squared term will be positive, leading to a downward effect of inequality on households' welfare when $\beta_n > 0$. According to this model, households do not compare income shares with each other, but only compare their situation to the average share.

Eq. (3) assumes that inequality enters in a symmetric way: households' deviations from the average share count in the same way regardless whether their average share is higher or lower than $\frac{1}{N}$. Both situations of *envy in shares* (lower share than average) and *altruism in shares* (higher share than average) are considered to be undesirable when $\beta_n > 0$. High-income individuals *and* low-income individuals therefore consider it to be undesirable that they do not have an average share of the total income, which might not be that intuitive. An equal percentage increase in all income levels does not change the inequality costs and only raises the first term in Eq. (3).

The second line in Eq. (3) shows that the inequality term decreases quadratically in N . A social welfare measure is obtained by *aggregating* all the household welfare functions of in a population of interest into one social welfare measure for a population. This social welfare

measure can be used to describe a population of households and accounts for levels and inequality. Appendix A shows that:

$$(4) \quad SW = N\mu_Y - \frac{\mu_\beta}{2} \frac{1}{N} cv_Y^2.$$

Social welfare in Eq. (4) is linearly decreasing in the squared of the coefficient of variation $cv_Y = \frac{\sigma_Y}{\mu_Y}$. Therefore, cv_Y^2 is the relevant income inequality indicator. All else equal, a higher variance of income results in lower social welfare. Based on the data of Roth et al. (1991) and Ochs and Roth (1989), De Bruyn and Bolton (2008, Table 5) report a mean preference parameter in the range $\mu_\beta = [6.692; 12.081]$, implying that there is a preference for equality of income shares. Their estimated preference for income equality is increasing in the number of bargaining rounds.

As the variance of income potentially changes the mean income, one needs the empirical relationship between μ_Y and σ_Y^2 . This relationship will determine whether an increase in the mean income leads to an increase in (average) social welfare. When the variance is increasing in the mean, the sign of the impact of an increase in the mean income on social welfare is unknown without further information.

Eq. (4) shows that income inequality becomes ‘asymptotically irrelevant’ for social welfare when the number of households N in the population is high: the second term is very small compared to the first. This result of the fair share model is already captured in the specification of the social surplus (the second line in Eq. (3)) and remains true even when preference parameters for equality depend on income levels.

C.2 Comparison of shares model

This sub-section provides an extension of the BO-model by assuming that the households compare their income share with the income share of all other households in the population of interest. Households’ welfare is defined as:

$$(5) \quad HW_n = Y_n - \frac{\beta_n}{2} N \int \left[\frac{Y_n}{N\mu_Y} - \frac{Y}{N\mu_Y} \right]^2 f[Y] dY.$$

Social welfare is then given by (see Appendix A):

$$(6) \quad SW = N\mu_Y - \mu_\beta cv_Y^2.$$

This shows that social welfare is linearly decreasing in the squared of the coefficient of variation $cv_Y = \frac{\sigma_Y}{\mu_Y}$. Although the inequality term is much higher compared to Eq. (4), the size of the second term is again low compared to the total economic income (first term). These analytical results show that it matters how the households' welfare is specified and to how many households a household is comparing its own income level.

Eqs. (4) and (6) show that the marginal impact of changes in the average income level will be quantitatively small. For completeness the analytical results are given in Table 2. These marginal impacts are derived assuming that the coefficient of variation potentially changes when the average income changes.

Table 2: summary of fair share results and marginal impacts.

| <i>Author</i> | <i>Average social welfare $\frac{SW}{N}$</i> | <i>Marginal impact of increase in average income $\frac{\partial \frac{SW}{N}}{\partial \mu_Y}$.</i> |
|---|---|---|
| Bolton and Ockenfels (2000): comparison to average share. | $\mu_Y - \frac{1}{N^2} \frac{\mu_\beta}{2} cv^2$ | $1 - \frac{1}{N^2} \mu_\beta \frac{\partial cv}{\partial \mu_Y} cv$ |
| This paper: comparison to all other shares. | $\mu_Y - \frac{1}{N} \mu_\beta cv^2$ | $1 + \frac{2}{N} \mu_\beta \frac{\partial cv}{\partial \mu_Y} cv$ |

Notes: marginal impacts derived by the author.

D. Inequity aversion

A first disadvantage of the model in section II.B and II.C is that downward and upward inequality are counted in exactly the same way. This disadvantage was accommodated by the model of Bolton (1991) (see also De Bruyn and Bolton, 2008, p.1777).

A second disadvantage is that only comparisons of shares matter and not the absolute distance in income levels *between* households. The latter seems to be more relevant as it is the absolute income that governs the budget constraint and thereby the opportunity for savings and consumption of households. Surprisingly, De Bruyn and Bolton (2008) show that the fair share model often provides the best fit for their collection of datasets, but sometimes the inequity aversion model of Fehr and Schmidt (1999) (FS) model is preferred. Engelmann and Strobel (2000) find that the FS-model performs better for their sample.

Fehr and Schmidt (1999) define their model to include aversion to disadvantageous and advantageous inequality. Welfare of household n is given by:

$$(7) \quad HW_n = Y_n - \alpha_n \int \max(Y - Y_n, 0) f[Y] dY - \beta_n \int \max(Y_n - Y, 0) f[Y] dY.$$

The first term in Eq. (7) increases linearly in income because of the assumption of unit marginal utility of income. The second negative term captures disadvantageous inequality or envy in income levels, meaning that households compares the absolute distance in income of the household with all other households in the population that have a *higher* income. This is because the $\max(\cdot)$ function evaluates to 0 for all indicators that are lower than Y_n and to a positive value when $Y > Y_n$.

The third term in Eq. (7) accounts for advantageous inequality or altruism. It compares Y_n with all lower income levels resulting in a negative impact on households' welfare when one has higher income levels than others. When $\alpha_n > 0$ and $\beta_n > 0$, households are averse to inequity meaning that they dislike the fact that other persons in society have different *levels* of income than their own (Fehr and Schmidt, 1999). Because α_n and β_n can have different numerical

values, there can be asymmetry in the value of inequity. Furthermore, these welfare parameters can be scaled with normative or ethical parameters. For example, when envy is considered to be a negative attitude that *should* not play a role in the calculation of value, one can set $\alpha_n = 0$ (Luttmer, 2005, p.963). Frank (2005, p.141) motivates this normative view as follows: “*we should continue to teach our children not to envy the good fortunes of others.*”

Social welfare for the FS-model was derived by Schmidt and Wichardt (2019). For the interested reader, this result is replicated for continuous distributions using integration techniques in Appendix A:

$$(8) \quad SW = N\mu_Y - N(\mu_\alpha + \mu_\beta)\mu_Y G_Y = N\mu_Y[1 - (\mu_\alpha + \mu_\beta)G_Y].$$

Eq. (8) is linear in the mean income, the Gini and the preference parameters. The term $N\mu_Y$ is the standard term capturing total spendable income. The term $N(\mu_\alpha + \mu_\beta)\mu_Y G_Y$ is equal to the number of people multiplied by the *absolute* Gini coefficient and then in turn multiplied by the sum of the average social preference parameters. When the absolute Gini coefficient $\mu_Y G_Y$ increases, inequality costs are higher and social welfare decreases.

The result of Schmidt and Wichardt (2019) provides a useful and beautiful micro-economic underpinning for the axiomatic result of Sen (1976) as it starts with the households’ welfare functions and performs aggregation to arrive at the social welfare measure. In principle, the mathematical model does not rule out a preference for inequity as $\mu_\alpha + \mu_\beta$ can be smaller than 0. It is also possible that (monetized) social welfare is negative when $(\mu_\alpha + \mu_\beta)G_Y > 1$. This implies inequality costs dominate the benefits of high average income levels.

Eckel and Gintis (2010, Table 1) provide an overview of estimated and calibrated preference parameters of the FS-model. They find that at the population level individuals are on average averse to inequity, but mainly report preferences of students (except for Bellemare et al. (2008) who use a population of Dutch adults). The estimated mean parameter for envy is larger than the estimated mean parameter for altruism ($\mu_\alpha > \mu_\beta$). The sum $\mu_\alpha + \mu_\beta$ is estimated in the

range 0.67-2.69 depending on the country and the sample. According to Eckel and Gintis (2010), Bellemare et al. (2008) estimate $\mu_\alpha + \mu_\beta = 1.892 + 0.801 = 2.693$ for a representative sample of adults in The Netherlands which is at the high end compared to the international literature. De Bruyn and Bolton (2008, p.1785) review many studies from the literature and find a sum equal to $\mu_\alpha + \mu_\beta = 1.056$ which is in the range reported by Eckel and Gintis (2010). This suggests that Sen's (implicit) assumption $\mu_\alpha + \mu_\beta = 1$, was not that far off. It is interesting to relate these empirical findings to the definition of the household welfare function in Eq. (7). In Appendix A we show that it can be written as:

$$(9) \quad HW_n = Y_n + \alpha_n(Y_n - \mu_Y) - (\alpha_n + \beta_n) \int_{-\infty}^{Y_n} F[Y] dY.$$

The last term is decreasing in Y_n and multiplies the area under the cumulative distribution function up to Y_n with the preference parameters. The marginal change in household welfare is given by:

$$(10) \quad \frac{\partial HW_n}{\partial Y_n} = 1 + \alpha_n - (\alpha_n + \beta_n)F[Y_n] = 1 + \alpha_n(1 - F[Y_n]) - \beta_n F[Y_n],$$

$$\frac{\partial HW_n}{\partial Y_n} > 0 \leftrightarrow \frac{\alpha_n + 1}{\alpha_n + \beta_n} > F[Y_n].$$

The first line shows that the marginal household welfare is decreasing in the rank $F[Y_n]$, showing that household welfare increases most for households with the lowest income *at the same level of the preference parameters and no changes in the distribution*. In order to have positive impacts of income on household welfare, the condition in the third line must be satisfied at the household level. When $\alpha_n \geq 0$, for the richest household this implies that $\beta_n > 1$, is sufficient to have positive marginal welfare. The sum $\mu_\alpha + \mu_\beta$ can be larger than 1 as long as condition Eq. (10) is satisfied at the household level.

The social welfare functions do not make a distinction between 'unchosen' income (related to luck and circumstances) and 'chosen' income related to individual efforts (see Lefranc et al.

2009). How and whether to correct the inequality indicator for merit, luck, circumstances and other normative considerations depends on the normative school that somebody belongs to. Because in the FS-model the inequality part is a homogenous function of income one can scale down the income levels in the inequality part with a factor $0 \leq k \leq 1$ which increases in the degree of luck. For tractability it is assumed that k does not depend on the income level and is unrelated to the other preference parameters. Admiraal (2021) shows that the resulting social welfare measure can then be adjusted, resulting in costs of inequality that are linear in k (see Appendix A):

$$(11) \quad SW = N\mu_Y[1 - (\mu_\alpha + \mu_\beta)kG_Y].$$

When on average 60% of the income is related to luck and circumstances and 40% to effort one can assume $k = 0.6$. Table 3 summarizes the analytical results and provides the marginal expressions in the third column. These expressions only depend on the Gini, the elasticity and the preference parameters. McDonald (1984, Table 1) derives closed-form expressions for the Gini coefficient for particular distributions that can be used when data on G_Y is lacking but distributional parameters are known.

Table 3: average social welfare for (extensions) of the Fehr and Schmidt (1999) model.

| <i>Author</i> | <i>Average social welfare $\frac{SW}{N}$</i> | <i>Marginal impact of an increase in average income $\frac{\partial \frac{SW}{N}}{\partial \mu_Y}$.</i> |
|-----------------------------|---|--|
| Schmidt and Wichardt (2019) | $\mu_Y[1 - (\mu_\alpha + \mu_\beta)G_Y]$ | $1 - (1 + \epsilon_Y)(\mu_\alpha + \mu_\beta)G_Y$ |
| Admiraal (2021) | $\mu_Y[1 - (\mu_\alpha + \mu_\beta)kG_Y]$ | $1 - (1 + \epsilon_Y)(\mu_\alpha + \mu_\beta)kG_Y$ |

Note: marginal expressions derived by the author.

III. How is income inequality related to average income levels?

A. Model specification

The next step is to estimate the income elasticity of the Gini. The relationship between income inequality and average disposable income has been hypothesized by Kuznets (1955) to be inverse U-shaped. According to one of his hypotheses at rising income levels economies can transform from agricultural (lower wages) into service/manufacturing economies (higher wages).³ Income inequality first increases when the agricultural sector decreases in size, but eventually decreases when the service sector becomes large. Whether the pattern holds for urban areas in a service economy remains to be seen and according to Ravallion (2018, p.629), the evidence on the Kuznets curve for countries is limited.

For the specification of an ‘urban Kuznets curve’ it is important to transform the dependent and independent variables to logs. Otherwise there is the risk to fit the right quadratic part of the curve on the basis of the left increasing part of the Kuznets curve. By adding a squared quadratic ln-term one can add asymmetry: a strong increase at the left side of the curve in combination with a mildly decreasing right part of the curve is possible.

Define $GI_{Y_{tr}}$ as the income Gini *index* for neighbourhood r at time t , and $\mu_{Y_{tr}}$ the average disposable income in the neighbourhood. The Gini index multiplies the Gini coefficient with 100. A basic OLS specification is given by:

$$(12) \quad \ln[GI_{Y_{tr}}] = c + \beta_1 \ln[\mu_{Y_{tr}}] + \beta_2 (\ln[\mu_{Y_{tr}}])^2 + \varepsilon_{tr}.$$

It is useful to scale the independent variables with the mean income level in the sample to enhance interpretation and to limit multicollinearity. Model (1) is specified as:

$$(13) \quad \ln[GI_{Y_{tr}}] = c + \beta_1 \ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right] + \beta_2 \left(\ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right]\right)^2 + \varepsilon_{tr}.$$

³ Note that this was not the only hypothesis of Kuznets (1955) who also stated that: “the paper is perhaps 5 percent empirical information and 95 percent speculation” (p.26).

The constant c is now equal to the estimated $\ln[GI_{Y_{tr}}]$ at the mean income level \bar{Y} in the sample. The estimated Gini index at \bar{Y} is then equal to $\exp(c)$ when it is assumed that regression error is measurement error.⁴ When regression error represents a stochastic unobserved explanatory variable for the Gini index, $\exp(c)$ represents the median estimate of the Gini. Model (2) includes year dummies γ_t and is specified as:

$$(14) \quad \ln[GI_{Y_{tr}}] = c + \gamma_t + \beta_1 \ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right] + \beta_2 \left(\ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right]\right)^2 + \varepsilon_{tr}.$$

These year dummies can account for general citywide trends in income inequality. For each year, the estimated Gini index is equal to $\exp(c + \gamma_t)$, where one year is normalized as the base year. Model (3) includes dummy variables κ_r for neighbourhoods:

$$(15) \quad \ln[GI_{Y_{tr}}] = c + \kappa_r + \beta_1 \ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right] + \beta_2 \left(\ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right]\right)^2 + \varepsilon_{tr}.$$

Model (4) includes dummy variables for years and neighbourhoods:

$$(16) \quad \ln[GI_{Y_{tr}}] = c + \gamma_t + \kappa_r + \beta_1 \ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right] + \beta_2 \left(\ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right]\right)^2 + \varepsilon_{tr}.$$

This specification therefore controls for unobserved variables related to general time trends and unobserved variables related to space. Note however, that there can also be observed characteristics that vary over time and space that are omitted. Some of these observed variables can be explanatory variables for the income levels Y_n in household welfare functions Eq. (3), Eq. (5) and Eq. (7) and thereby also for the Gini index. When households move from one neighbourhood to another, spatial sorting partly might explain changes in income levels and income inequality. Models (5) and (6) control for (some of) these variables by including a vector of explanatory variables Z_{tr} :

$$(17) \quad \ln[GI_{Y_{tr}}] = c + \gamma_t + \kappa_r + \alpha_Z \ln\left[\frac{Z_{tr}}{\bar{Z}}\right] + \beta_1 \ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right] + \beta_2 \left(\ln\left[\frac{\mu_{Y_{tr}}}{\bar{Y}}\right]\right)^2 + \varepsilon_{tr}.$$

⁴ When the error is not measurement error, the variance of the error impacts the predicted Gini at \bar{Y} . See also Goldberger (1968) for a discussion.

The control variables are scaled and logged in a similar way as the income variable, where \bar{Z} is the sample average. Therefore $\exp(c + \gamma_t + \kappa_r)$ is the estimated Gini for the reference year in the reference neighbourhood at average income levels and average levels of the neighbourhood characteristics. For this reason, a reference neighbourhood is chosen with income levels (almost) equal to the average income level. Then $\exp(c + \gamma_t)$ is the estimated Gini index for year t at the reference neighbourhood (with an average income level) assuming average levels for the neighbourhood characteristics. Model (5) is estimated without a squared term in (15) and model (6) includes the squared term.

The ultimate aim of the regressions is to estimate the income elasticity of the Gini. From the regression equations the income elasticity of the Gini is given by:⁵

$$\begin{aligned}
 \epsilon_Y &\equiv \frac{\partial \ln[G I_{Y_{tr}}]}{\partial \ln[\mu_{Y_{tr}}]} = \beta_1 + \beta_2 \frac{\partial}{\partial \ln[\mu_{Y_{tr}}]} [\ln[\mu_{Y_{tr}}] - \ln[\bar{Y}]]^2, \\
 &= \beta_1 + \beta_2 \frac{\partial}{\partial \ln[\mu_{Y_{tr}}]} [\ln[\mu_{Y_{tr}}] - \ln[\bar{Y}]] [\ln[\mu_{Y_{tr}}] - \ln[\bar{Y}]], \\
 (18) \quad &= \beta_1 + \beta_2 \frac{\partial}{\partial \ln[\mu_{Y_{tr}}]} [\ln[\mu_{Y_{tr}}]^2 - 2 \ln[\mu_{Y_{tr}}] \ln[\bar{Y}] + \ln[\bar{Y}]^2], \\
 &= \beta_1 + \beta_2 [2 \ln[\mu_{Y_{tr}}] - 2 \ln[\bar{Y}]], \\
 &= \beta_1 + 2\beta_2 \ln \left[\frac{\mu_{Y_{tr}}}{\bar{Y}} \right].
 \end{aligned}$$

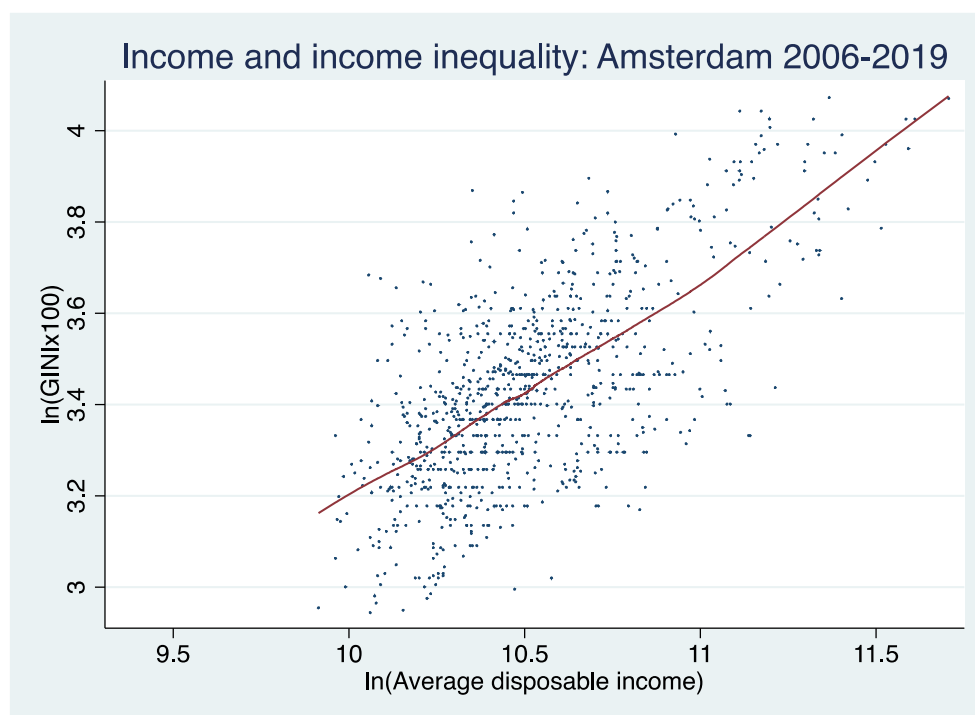
This shows that β_1 is the income elasticity of the Gini at the average income level in the sample. The sign of the second term can be positive or negative depending on the sign of β_2 and the log-ratio of the mean income and the sample average. For the purpose of this paper interaction effects of particular groups with the elasticity are ignored in the specification.

⁵ The mathematical impact of $\ln[\mu_{Y_{tr}}]$ on $\ln[\bar{Y}]$ is numerically negligible.

B. Data

Compared to Germany, France, United Kingdom, China, Russia, India and the United States, income inequality in The Netherlands is low with an average Gini index of 29.2 in 2019 and a small increase of 2-4 points in the last three decades (Nolan et al. 2019).⁶ This value is comparable to the level of the Gini index in Sweden in 2019. However, this country level measure does not imply that income inequality is low for particular urban areas. The descriptive statistics on the urban area of Amsterdam provide more insights on this.

The dataset that is used is from the Statistics Department of the city of Amsterdam. The neighbourhood data covers data on average disposable income and the Gini coefficient for the years 2006, 2009 and 2011-2019 (t) for neighbourhoods (r) in Amsterdam. Disposable income includes income generated by labour, business and wealth, but not wealth itself (Caminada et al., 2010, p.25). The sample mean \bar{Y} that is used for scaling the average income levels has a value of 38384 euros per year.



⁶ <https://data.worldbank.org/indicator/SI.POV.GINI?locations=GB-US-FR-DE-CN-RU-IN-NL-SE> (accessed on 17-01-2023). For more information on the Dutch development of income inequality we refer to Caminada et al. (2010).

*Figure 1: Household spendable income and income inequality for the city of Amsterdam
(years: 2006, 2009, 2011-2019).*

The log-log relationship in Figure 1 suggests increasing income inequality in average disposable household income. An inverse U-shaped pattern is not observed. Tables 4a and 4b provide the summary statistics of the Gini index and the average disposable household income by year at the neighbourhood level. Figure 1 suggests that the percentage income growth is not equal for all income levels: high incomes grow more proportionally speaking than low incomes resulting in an increasing Gini coefficient in income levels.

The mean and median Gini index in 2019 are higher than the Dutch average in 2019. The mean Gini index has increased with about 20% from 29.03 in 2006 to 35.00 in 2019 showing rising urban income inequality during this period. Average disposable incomes in neighbourhoods have increased with about 61% from 29279 euros to 46155 euros in the same time period. Minimum average incomes (+25%) and median average incomes (+58%) have increased at a slower pace than maximum average incomes (+116%) in the same time period.

Table 4a: Summary statistics of the Gini index by year

| Year | National price index (2015=100) | N | Min | Median | Mean | Max |
|------|---------------------------------------|----|------|--------|-------|------|
| 2006 | 88.60 | 76 | 19.0 | 28.55 | 29.03 | 54.2 |
| 2009 | 92.71 | 77 | 19.1 | 29.40 | 29.76 | 46.9 |
| 2011 | 95.74 | 80 | 20.1 | 29.35 | 30.19 | 46.5 |
| 2012 | 97.71 | 80 | 19.6 | 30.35 | 31.02 | 49.2 |
| 2013 | 98.97 | 80 | 19.8 | 30.40 | 31.29 | 49.2 |
| 2014 | 99.60 | 86 | 20.6 | 30.65 | 31.74 | 49.6 |
| 2015 | 100.00 | 88 | 22.0 | 30.50 | 32.42 | 57.0 |
| 2016 | 100.25 | 88 | 23.0 | 31.00 | 32.10 | 57.0 |
| 2017 | 101.62 | 89 | 22.0 | 31.00 | 32.87 | 56.0 |
| 2018 | 103.01 | 90 | 23.0 | 32.00 | 32.48 | 56.0 |
| 2019 | 104.64 | 90 | 22.9 | 33.80 | 35.00 | 58.7 |

Table 4b: Summary statistics of average disposable household income (euros per year).

| Year | National price index (2015=100) | N | Min | Median | Mean | Max |
|------|---------------------------------------|----|-------|--------|-------|--------|
| 2006 | 88.60 | 82 | 20200 | 26100 | 29279 | 56200 |
| 2009 | 92.71 | 83 | 22600 | 28900 | 32466 | 58800 |
| 2011 | 95.74 | 87 | 24000 | 31400 | 35577 | 80200 |
| 2012 | 97.71 | 91 | 17077 | 31200 | 35854 | 83300 |
| 2013 | 98.97 | 88 | 23000 | 32200 | 36625 | 84100 |
| 2014 | 99.60 | 89 | 23500 | 34400 | 39189 | 100200 |
| 2015 | 100.00 | 89 | 23900 | 35300 | 39736 | 87900 |
| 2016 | 100.25 | 89 | 24600 | 36500 | 41310 | 101500 |
| 2017 | 101.62 | 90 | 27400 | 37200 | 42722 | 107500 |
| 2018 | 103.01 | 91 | 21200 | 38100 | 42934 | 110300 |
| 2019 | 104.64 | 91 | 25200 | 41000 | 46155 | 121400 |

The income levels are not corrected for prices. The national price index gives an indication about the development of prices. The second column in Table 4a shows this index has increased with 18% for the period 2006-2019. At the neighbourhood level, the least advantaged neighbourhood have a net increase in income corrected for prices. Nevertheless, the percentage *and* absolute increases in income of the least advantaged neighbourhoods are substantially lower than for the median and average neighbourhood. Disposable income growth is not equally distributed in Amsterdam.

Some neighbourhoods miss observations for the Gini coefficient and therefore the sample size reduces to 924. For some years (2006 and 2009) and neighbourhoods, covariates were not available at the time of analysis and therefore Models (5) and (6) have fewer observations.

C. Regression results

Table 5 reports the regression results for the six models. To account for spatial autocorrelation or dependence, all standard errors are clustered at the neighbourhood level. Model (1) is the basic model without fixed effects and control variables. The income elasticity of the Gini at sample average income levels is estimated at 0.474: for a 1% increase in average income levels, the Gini increases approximately with 0.474% according to this model. For larger changes one has to account for the non-linear relationship due to the log-log specification. When average income levels increase with a factor t , the Gini index will change with $100 \times (t^{\beta_1} - 1)\%$.

Doubling income levels lead to a predicted increase of the Gini index equal to 38.90% and raises the inequality costs in social welfare with the same percentage. The coefficient for the quadratic term is insignificant and small. This is in line with what is observed in Figure 1.

Model (2) controls for citywide time trends and estimates the elasticity at 0.497. The coefficient for the squared term is small and insignificant. Model (3) controls for neighbourhood unobserved heterogeneity and estimates the elasticity at 0.347. The squared term is positive, small and significant at the 5% level.

Table 5: Regression results

| | Dependent variable: $\ln[100 \times G_{Ytr}]$ | | | | | |
|--|---|----------------------|-----------------------|----------------------|---------------------|---------------------|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\ln\left[\frac{\mu_{Ytr}}{\bar{Y}}\right]$ | 0.474*** (9.16) | 0.497*** (7.40) | 0.347*** (10.40) | 0.145 (1.16) | 0.412*** (3.58) | 0.346** (2.86) |
| $\left(\ln\left[\frac{\mu_{Ytr}}{\bar{Y}}\right]\right)^2$ | 0.0335 (0.48) | 0.000614 (0.01) | 0.0602* (2.36) | 0.104** (2.94) | | 0.119* (2.03) |
| c | 3.451*** (182.90) | 3.466*** (205.44) | 3.443*** (1311.96) | 3.502*** (230.51) | 3.599*** (35.39) | 3.628*** (35.75) |
| Mean of $\ln[G_{Ytr}]$ | 3.433 | 3.433 | 3.433 | 3.433 | 3.447 | 3.447 |
| R^2 | 0.462 | 0.469 | 0.943 | 0.950 | 0.971 | 0.972 |
| Adjusted R^2 | 0.461 | 0.462 | 0.937 | 0.944 | 0.966 | 0.967 |
| AIC | -803.2 | -794.4 | -2879.3 | -2976.1 | -2843.4 | -2865.9 |
| BIC | -788.8 | -731.7 | -2869.6 | -2918.1 | -2750.9 | -2768.8 |
| N | 924 | 924 | 924 | 924 | 753 | 753 |
| Year fixed effects | No | Yes | No | Yes | Yes | Yes |
| Neighbourhood fixed effects | No | No | Yes | Yes | Yes | Yes |
| Control variables | No | No | No | No | Yes | Yes |

Notes: t statistics in parentheses; * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Standard errors are clustered at the neighborhood level.

Model (4) includes time and neighbourhood fixed effects. The linear term becomes insignificant, but the squared term becomes more positive and significant. Model (5) includes time and neighbourhood fixed effects, but excludes the squared term and estimates the elasticity at 0.412. Models (5) and (6) control for time varying observed heterogeneity in neighbourhoods by adding control variables related to aggregated household characteristics. The estimated elasticity at \bar{Y} is now close to the estimate of model (3) and equal to 0.346. The

squared term is close to the estimate of model (4) and equal to 0.119. Appendix C provides more details on the level and significance of the elasticities for the covariates.

Model (5) shows an increase in significance of the linear term and the elasticity at the average income level increases compared to model (6). In terms of model fit (AIC , BIC , but not the adjusted R^2), model (4) is a good candidate. But probably, model (4) produces different results due to time and spatial varying omitted variables. The estimates of model 5 and 6 are based on more recent years that have neighbourhood covariates. The estimated elasticities for the sample of model (6) are in the range 0.25 (minimum level of income) and 0.48 (maximum level of income).

Model (1) shows that the linear term explains a substantial part of the variation. Model (2) shows that the year fixed effects do not add much in terms of explaining variation. The control variables explain a bit more. Model (3) shows that mainly the neighbourhood dummies add much to explaining variation. Spatial unobserved heterogeneity is therefore relevant in terms of explanatory power.

IV. How does average social welfare changes with mean income levels?

A. Results at average levels

This sub-section investigates how an increase in mean household income in a particular neighbourhood will impact social welfare. In line with Table 5 an income elasticity of the Gini equal to 0.40 is chosen. For the Gini a value of 0.35 is chosen in line with the average of the year 2019 in Amsterdam. Table 6 reports the marginal impacts of an increase in average income levels. For the fair shares models the inequality costs are negligible as N is large in all neighbourhoods. The numbers in this table can be interpreted loosely as: “what remains of a 1-euro household income growth in terms of average social welfare when we correct for income

inequality?”. Table 6 shows that depending on the specification of the social welfare function 27%-100% of the income increase remains as social welfare.

Table 6: What remains of a 1-euro average income growth in terms of average social welfare?

| <i>Author</i> | <i>Marginal impact of increase in average income $\frac{\partial \frac{SW}{N}}{\partial \mu_Y}$.</i> | <i>Marginal change in social welfare.</i> |
|--|---|---|
| Sen (1976) | $1 - (1 + \epsilon_Y)G_Y$ | 0.51 euro |
| Schmidt and Wichardt (2019) | $1 - (1 + \epsilon_Y)(\mu_\alpha + \mu_\beta)G_Y$ | 0.27 euro |
| Kakwani (1980) | $\frac{1}{1 + G_Y} \left[1 - \epsilon_Y \frac{G_Y}{1 + G_Y} \right]$ | 0.66 euro |
| Dagum (1990, Eq. 35) | $1 - \frac{2G_Y}{1 + G_Y} \left[1 + \epsilon_Y \frac{1}{1 + G_Y} \right]$ | 0.33 euro |
| Fair shares: Bolton and Ockenfels (2001) | ≈ 1 | 1 euro |
| Comparison of shares | ≈ 1 | 1 euro |

Note: $\mu_\alpha + \mu_\beta = 1.5$ is assumed for the Schmidt and Wichardt (2019) model. $\epsilon_Y = 0.40$ and $G_Y = 0.35$.

B. Sensitivity analysis: preferences

This subsection illustrates the outcomes for values of $\mu_\alpha + \mu_\beta = 0.5, 1$ (Sens’ model), 1.5 and 2. The upper bound we choose is lower than the estimate for The Netherlands (Bellemare et al. 2008 as reported by Eckel and Gintis, 2010). For the Gini coefficient base levels between 0.20 and 0.50 are chosen. For the income elasticity of the Gini estimates between 0.25 and 0.45 are chosen based on the result of the previous section. Table 7 presents the results.

When $\mu_\alpha + \mu_\beta = 0.5$ or $\mu_\alpha + \mu_\beta = 1$, an increase in the average disposable income will result in positive effects on social welfare. Nevertheless, only about 50-60% of the income remains in terms of average social welfare. For high values of the Gini even less than 50% of the income increase remains for Sens’ model. The elasticity has a minor impact for these values of the preference parameters. For higher values of the preference parameters it is possible that average

social welfare decreases for an increase in average income. The negative impact of income inequality then dominates the benefits of the income increase.

At a Gini coefficient equal to 0.4 and $\mu_\alpha + \mu_\beta = 2$, the social welfare effect of the income increase is equal to 0. Given the discussion on preferences and the levels of the Gini this is a real *possibility* in the Dutch context. For Amsterdam, the average Gini index in 2019 is about 35, and the average income elasticity of the Gini is about 0.35. At these average levels the remaining social welfare per euro income increase is in between 0.76 euro ($\mu_\alpha + \mu_\beta = 0.5$) and 0.06 euro ($\mu_\alpha + \mu_\beta = 2$). This shows that for a substantial number of neighbourhoods in Amsterdam average social welfare increases are not guaranteed when average disposable household incomes increase. The results of Table 7 also show that it is important to estimate the inequity preference parameters for urban regions precisely in order to determine the value of urban income inequality.

Table 7: What remains of a 1-euro average income growth in terms of average social welfare? Sensitivity analysis.

| Gini index (G_Y) | Income elasticity of the Gini coefficient (ϵ_G) | Change in average social welfare per household: $\frac{\partial \frac{SW}{N}}{\partial \mu_Y} = 1 - (1 + \epsilon_Y)(\mu_\alpha + \mu_\beta)G_Y$ (in euro 's). | | | |
|----------------------|--|---|------------------------------|--------------------------------|------------------------------|
| | | $\mu_\alpha + \mu_\beta = 0.5$ | $\mu_\alpha + \mu_\beta = 1$ | $\mu_\alpha + \mu_\beta = 1.5$ | $\mu_\alpha + \mu_\beta = 2$ |
| 20 | 0.25 | 0.88 | 0.75 | 0.63 | 0.50 |
| 20 | 0.30 | 0.87 | 0.74 | 0.61 | 0.48 |
| 20 | 0.35 | 0.87 | 0.73 | 0.60 | 0.46 |
| 20 | 0.40 | 0.86 | 0.72 | 0.58 | 0.44 |
| 20 | 0.45 | 0.86 | 0.71 | 0.57 | 0.42 |
| 30 | 0.25 | 0.81 | 0.63 | 0.44 | 0.25 |
| 30 | 0.30 | 0.81 | 0.61 | 0.42 | 0.22 |
| 30 | 0.35 | 0.80 | 0.60 | 0.39 | 0.19 |
| 30 | 0.40 | 0.79 | 0.58 | 0.37 | 0.16 |
| 30 | 0.45 | 0.78 | 0.57 | 0.35 | 0.13 |

| | | | | | |
|----|------|------|------|-------|-------|
| 40 | 0.25 | 0.75 | 0.50 | 0.25 | 0.00 |
| 40 | 0.30 | 0.74 | 0.48 | 0.22 | -0.04 |
| 40 | 0.35 | 0.73 | 0.46 | 0.19 | -0.08 |
| 40 | 0.40 | 0.72 | 0.44 | 0.16 | -0.12 |
| 40 | 0.45 | 0.71 | 0.42 | 0.13 | -0.16 |
| 50 | 0.25 | 0.69 | 0.38 | 0.06 | -0.25 |
| 50 | 0.30 | 0.68 | 0.35 | 0.02 | -0.30 |
| 50 | 0.35 | 0.66 | 0.33 | -0.01 | -0.35 |
| 50 | 0.40 | 0.65 | 0.30 | -0.05 | -0.40 |
| 50 | 0.45 | 0.64 | 0.28 | -0.09 | -0.45 |

Note: Authors own calculations based on Table 3. Negative impacts of income growth on social welfare are given in italics. G_Y is chosen in line with the range for Amsterdam (see Table 4). Elasticities are chosen in line with the estimates (see discussion Table 5).

C. Accounting for decreasing marginal utility of income

One could argue that the first term in the social welfare should be non-linear as the marginal (indirect) utility of income is expected to decrease for higher income levels. When this is the case, the welfare benefits of an income increase are overestimated. The implication of this assumption can be analysed by assuming that the first term in the household welfare (Eqs. (3), (5) and (7)) is equal to $\underline{Y}^\rho \frac{Y_n^{1-\rho}}{1-\rho}$, where $\mu_Y > \underline{Y} > 0$ is the minimum average income level in the population of interest. For $\rho = 0$, this reduces to the model in the previous sub-sections. It is expected that ρ is above 0 as households' welfare is concave in income. The marginal welfare of an income increase is then given by $\left(\frac{Y}{Y_n}\right)^\rho \leq 1$. This marginal income for the lowest income group is normalised to 1 in order to allow for welfare comparisons between neighbourhoods. Such a correction allows for the intuitive notion that a one euro increase in income in a rich neighbourhood is less beneficial than a one euro increase in a poor neighbourhood, independent of concerns related to income inequality.

Some analytical progress can be made in order to correct the social welfare measures for decreasing marginal utility of income. The sample expectation of welfare related to income, $E \left[Y^\rho \frac{Y_n^{1-\rho}}{1-\rho} \right]$, can be approximated with a Taylor expansion and therefore the average social welfare per household is approximately equal to:

$$(19) \quad \frac{SW}{N} \approx \frac{Y^\rho}{1-\rho} \left[\frac{\mu_Y^{1-\rho}}{1-\rho} - \frac{1}{2} \rho \frac{\sigma_Y^2}{\mu_Y^{\rho+1}} \right] - \mu_Y [1 - (\mu_\alpha + \mu_\beta) G_Y].$$

The marginal change in average social welfare is then given by:

$$(20) \quad \frac{\partial \frac{SW}{N}}{\partial \mu_Y} \approx \left(\frac{Y}{\mu_Y} \right)^\rho + \frac{1}{2} \rho (\rho + 1) \left(\frac{Y}{\mu_Y^2} \right)^\rho \frac{\sigma_Y^2}{\mu_Y^2} - [1 - (1 + \epsilon_Y)(\mu_\alpha + \mu_\beta) G_Y].$$

$$\approx \left(\frac{Y}{\mu_Y} \right)^\rho - [1 - (1 + \epsilon_Y)(\mu_\alpha + \mu_\beta) G_Y].$$

The second line results because the second term in the first line of Eq. (20) is numerically very small. This gives a quick way to estimate the marginal utility of income for different levels of ρ . In line with Table 4b (last line) it is assumed that $\frac{Y}{\mu_Y} = 0.55$ implying that for $\rho = 1$, the marginal utility of income is 45% lower compared to the estimate of 1 used in the previous sections.

Table 8 shows the results for Sen's model, where the third column is equal to the fourth column of Table 7 ($\rho = 0$). For quite some plausible cases, average social welfare is decreasing in the mean income level. These results suggest that for these cases priority should be given to the reduction of the Gini in order to make income increases beneficial. When this is not feasible, reducing average income levels would be a rational policy given the assumptions that are made although this might not be politically feasible. Correcting for decreasing marginal utility of income can therefore have substantial impacts on qualitative policy recommendations. For rich cities with high preferences for equality, social welfare impacts of average income increases can be low.

Table 8: What remains of a 1-euro average income growth in terms of average social welfare?

| Sensitivity analysis for Sen's model ($\mu_\alpha + \mu_\beta = 1$) | | | | | | |
|---|---|---|--------------|------------|--------------|--------------|
| Gini index ($100 \times G_Y$) | Income elasticity of the Gini (ϵ_G) | Change in average social welfare per household: | | | | |
| | | $\frac{\partial \frac{SW}{N}}{\partial \mu_Y} = \left(\frac{Y}{\mu_Y}\right)^\rho - (1 + \epsilon_Y)G_Y$ (in euro's). | | | | |
| | | $\rho = 0$ | $\rho = 0.5$ | $\rho = 1$ | $\rho = 1.5$ | $\rho = 2.0$ |
| 0.2 | 0.25 | 0.75 | 0.49 | 0.30 | 0.16 | 0.05 |
| 0.2 | 0.30 | 0.74 | 0.48 | 0.29 | 0.15 | 0.04 |
| 0.2 | 0.35 | 0.73 | 0.47 | 0.28 | 0.14 | 0.03 |
| 0.2 | 0.40 | 0.72 | 0.46 | 0.27 | 0.13 | 0.02 |
| 0.2 | 0.45 | 0.71 | 0.45 | 0.26 | 0.12 | 0.01 |
| 0.3 | 0.25 | 0.63 | 0.37 | 0.18 | 0.03 | -0.07 |
| 0.3 | 0.30 | 0.61 | 0.35 | 0.16 | 0.02 | -0.09 |
| 0.3 | 0.35 | 0.60 | 0.34 | 0.15 | 0.00 | -0.10 |
| 0.3 | 0.40 | 0.58 | 0.32 | 0.13 | -0.01 | -0.12 |
| 0.3 | 0.45 | 0.57 | 0.31 | 0.12 | -0.03 | -0.13 |
| 0.4 | 0.25 | 0.50 | 0.24 | 0.05 | -0.09 | -0.20 |
| 0.4 | 0.30 | 0.48 | 0.22 | 0.03 | -0.11 | -0.22 |
| 0.4 | 0.35 | 0.46 | 0.20 | 0.01 | -0.13 | -0.24 |
| 0.4 | 0.40 | 0.44 | 0.18 | -0.01 | -0.15 | -0.26 |
| 0.4 | 0.45 | 0.42 | 0.16 | -0.03 | -0.17 | -0.28 |
| 0.5 | 0.25 | 0.38 | 0.12 | -0.08 | -0.22 | -0.32 |
| 0.5 | 0.30 | 0.35 | 0.09 | -0.10 | -0.24 | -0.35 |
| 0.5 | 0.35 | 0.33 | 0.07 | -0.13 | -0.27 | -0.37 |
| 0.5 | 0.40 | 0.30 | 0.04 | -0.15 | -0.29 | -0.40 |
| 0.5 | 0.45 | 0.28 | 0.02 | -0.18 | -0.32 | -0.42 |

Note: Authors own calculations. Negative impacts of income growth on social welfare are given in italics. G_Y is chosen in line with the range for Amsterdam (see Table 4). Elasticities are chosen in line with the estimates (see Table 5). $\frac{Y}{\mu_Y} = 0.55$ is assumed (see Table 4b).

V. Conclusions

This paper has reviewed social welfare functions that account for income inequality and developed marginal expressions that can be used for policies that change average income levels. The marginal expressions account for the dependence of income inequality on average levels of income. This is relevant because income growth might not impact income levels in equal proportion. The quantitative impact of this dependence on marginal social welfare is larger for areas with high valuations of equality.

First, the analytical results of section II show that modelling matters as the fair share model of Bolton and Ockenfels (2000) leads to quantitatively low impacts of income inequality for social welfare compared to the inequity aversion model of Fehr and Schmidt (2019). For the fair share model inequality costs do not impact social welfare when all incomes increase with the same percentage. For the inequity aversion model percentage increases in all income levels impact the absolute Gini coefficient and thereby the social welfare.

Second, the relationship between average disposable income levels and income inequality for neighbourhoods in the city of Amsterdam was investigated. A positive relationship was estimated with an income elasticity of the Gini estimated between 0.25 and 0.48 implying that for a 50% increase in average income levels, the inequality costs in social welfare increase with about 10.7%-21.5%. Other metropolitan areas might find estimates with a different sign and/or size.

Third, the regression results and the social welfare measures were applied for the city of Amsterdam, The Netherlands. When correcting for a decreasing marginal utility of income, it was found that increases in average disposable household income might not always be beneficial. This has implications for local and urban policies that aim to increase average income levels: for some neighbourhoods it is better to focus on a reduction of income inequality first in order not to reduce social welfare.

Fourth, because of the focus on the trade-off between equity and efficiency, other external costs and benefits related to average income and income inequality are ignored (health, environment, social life, crime, resources etc.). The average impacts and inequality impacts can be added to the household welfare functions resulting in extended marginal expressions. Some of these aspects can be quantified using hedonic wage regressions and are therefore partly captured in the wage distribution in this paper.⁷ When this is the case, and one accepts the hedonic view on these matters, the results in this paper can also be used to estimate the marginal social welfare impacts of these aspects accounting for inequality costs.

Fifth, the focus of this paper was on the concept income inequality. Therefore, marginal costs of absolute poverty were ignored or only partly incorporated in the marginal utility of income in section IV.C. The household and social welfare functions can be adjusted to account for an absolute poverty line.⁸ When absolute poverty is reduced when average income increases, a welfare benefit can occur that is not part of the proposed social welfare functions.

Sixth, the household welfare functions are assumed to be meta-preferences for social outcomes. Although it could be possible, this author will not be surprised when these meta-preferences for social outcomes cannot be estimated using location choices of households. The reliance on survey estimates seems therefore justified for these kinds of preferences.

Returning to the examples of the introduction, this paper at least has showed that advantages resulting from density policies and transport investments might be lower than expected because of the negative side effects on income inequality. These negative side effects result for cities

⁷ For health and wage examples: see Baum and Ford (2004) and Bhattacharaya and Sood (2011) on obesity impacts on wages. For environment and wage examples see Sinha et al. (2021) for a recent example on climate amenities and hedonic wages.

⁸ Define the neighbourhood poverty line as \underline{Y} . The number of households in poverty is then given by: $N \int_{-\infty}^{\underline{Y}} f[Y] dY = NF[\underline{Y}]$. Let z_n be the costs in the household welfare function per poor household. Then the additional costs of poverty are given by: $\Delta HW_n = -z_n NF[\underline{Y}]$. The additional impact on average social welfare is then given by: $\Delta \frac{SW}{N} = -\mu_z NF[\underline{Y}]$, which is the average costs per poor household multiplied by the number of poor households in a neighbourhood. When $NF[\underline{Y}]$ decreases in the average income level, additional benefits of rising income levels can occur: an interesting topic for further study.

with a positive income elasticity of the Gini and might temper the enthusiasm about urban wage premiums. In rich cities with a low marginal utility of income and a high valuation of equality, negative (wider) social welfare impacts of average income increases are possible.

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Appendix A. Mathematical derivations

A.1 Fair shares: Bolton and Ockenfels model

Because the preference parameter β_n is entering linearly, the assumption of zero covariance with income is sufficient for a social welfare function that is linear in the mean value μ_β of this preference parameter. Social welfare can be written as:

$$\begin{aligned} SW &= E \left[N \int HW_n f[Y_n] dY_n \right] = N\mu_Y - \frac{\mu_\beta}{2} \frac{1}{N} E \left[\left(\frac{Y_n}{\mu_Y} \right)^2 - 2 \frac{Y_n}{\mu_Y} + 1 \right], \\ &= N\mu_Y - \frac{\mu_\beta}{2} \frac{1}{N} \left[\frac{E[Y_n^2]}{\mu_Y^2} - 2 + 1 \right], \end{aligned}$$

Because the second moment $E[Y_n^2] = \sigma_Y^2 + \mu_Y^2$, this results in:

$$SW = N\mu_Y - \frac{\mu_\beta}{2} \frac{1}{N} \left[\frac{\sigma_Y^2 + \mu_Y^2}{\mu_Y^2} - 1 \right],$$

Simplifying gives:

$$SW = N\mu_Y - \frac{\mu_\beta}{2} \frac{1}{N} \frac{\sigma_Y^2}{\mu_Y^2}.$$

Average social welfare is given by:

$$\frac{SW}{N} = \mu_Y - \frac{\mu_\beta}{2} \frac{1}{N^2} \frac{\sigma_Y^2}{\mu_Y^2}.$$

Now suppose we start with the household welfare function Eq. (5). Social welfare is given by:

$$\begin{aligned} SW &= E \left[N \int \left(Y_n - \frac{\beta_n}{2} N \int \left[\frac{Y_n}{N\mu_Y} - \frac{Y}{N\mu_Y} \right]^2 f[Y] dY \right) f[Y_n] dY_n \right], \\ &= N\mu_Y - \frac{\mu_\beta}{2} NE \left[\frac{Y_n^2 - 2Y_n\mu_Y + E(Y^2)}{(N\mu_Y)^2} \right], \\ &= N\mu_Y - \frac{\mu_\beta}{2} \frac{1}{\mu_Y^2} E[Y_n^2 - 2Y_n\mu_Y + E(Y^2)], \\ &= N\mu_Y - \frac{\mu_\beta}{2} \frac{1}{\mu_Y^2} [\mu_Y^2 + \sigma_Y^2 - 2\mu_Y^2 + \mu_Y^2 + \sigma_Y^2], \end{aligned}$$

$$= N\mu_Y - \frac{\mu_\beta}{2} \frac{2\sigma_Y^2}{N\mu_Y^2},$$

$$= N\mu_Y - \mu_\beta \frac{\sigma_Y^2}{\mu_Y^2}.$$

Average social welfare is therefore given by:

$$\frac{SW}{N} = \mu_Y - \frac{\mu_\beta}{2} \frac{1}{N} \frac{\sigma_Y^2}{\mu_Y^2}.$$

A.2 Inequity aversion: Fehr and Schmidt model

Rewriting households' welfare function gives (Fehr and Schmidt, 1999):

$$\begin{aligned} HW_n &= Y_n - \alpha_n \int \max(Y - Y_n, 0) f[Y] dY - \beta_n \int \max(Y_n - Y, 0) f[Y] dY. \\ &= Y_n - \alpha_n \int_{Y_n}^{\infty} (Y - Y_n) f[Y] dY - \beta_n \int_{-\infty}^{Y_n} (Y_n - Y) f[Y] dY. \\ &= Y_n - \alpha_n \int_{Y_n}^{\infty} Y f[Y] dY + \alpha_n \int_{Y_n}^{\infty} Y_n f[Y] dY - \beta_n \int_{-\infty}^{Y_n} Y_n f[Y] dY + \beta_n \int_{-\infty}^{Y_n} Y f[Y] dY. \\ &= Y_n - \alpha_n \mu_Y + \alpha_n Y_n \int_{Y_n}^{\infty} f[Y] dY - \beta_n Y_n \int_{-\infty}^{Y_n} f[Y] dY + (\alpha_n + \beta_n) \int_{-\infty}^{Y_n} Y f[Y] dY. \\ &= Y_n - \alpha_n \mu_Y + \alpha_n Y_n (1 - F[Y_n]) - \beta_n Y_n F[Y_n] + (\alpha_n + \beta_n) \int_{-\infty}^{Y_n} Y f[Y] dY. \\ &= Y_n - \alpha_n \mu_Y + \alpha_n Y_n (1 - F[Y_n]) - \beta_n Y_n F[Y_n] + (\alpha_n + \beta_n) \int_{-\infty}^{Y_n} Y f[Y] dY. \\ &= Y_n - \alpha_n \mu_Y + \alpha_n Y_n - (\alpha_n + \beta_n) Y_n F[Y_n] + (\alpha_n + \beta_n) \int_{-\infty}^{Y_n} Y f[Y] dY. \\ &= Y_n - \alpha_n \mu_Y + \alpha_n Y_n + (\alpha_n + \beta_n) \left(\int_{-\infty}^{Y_n} Y f[Y] dY - Y_n F[Y_n] \right). \end{aligned}$$

Using integration by parts results in:

$$\int_{-\infty}^{Y_n} Y f[Y] dY = [Y F[Y]]_{-\infty}^{Y_n} - \int_{-\infty}^{Y_n} F[Y] dY = Y_n F[Y_n] - \int_{-\infty}^{Y_n} F[Y] dY.$$

Substituting gives:

$$HW_n = Y_n + \alpha_n(Y_n - \mu_Y) - (\alpha_n + \beta_n) \int_{-\infty}^{Y_n} F[Y]dY.$$

This result provides a ‘micro-equation’ that describes how households compare their income to the distribution in a relevant population, assuming unit marginal utility of income (the first term). It can be used for the analysis of well-being, income and relative income using extensions in section IV.C (Luttmer, 2005, p.968). The social welfare is given by:

$$SW = E \left[N \int_{-\infty}^{\infty} HW_n f[Y_n]dY_n \right] = N\mu_Y - N(\mu_\alpha + \mu_\beta) \int_{-\infty}^{\infty} \int_{-\infty}^Y F[Y_n]dY_n f[Y]dY.$$

Using integration by parts results in:

$$\begin{aligned} SW &= N\mu_Y - N(\mu_\alpha + \mu_\beta) \left[\int_{-\infty}^{\infty} F[Y]dY - \int_{-\infty}^{\infty} F[Y]^2 dY \right] \\ &= N\mu_Y - N(\mu_\alpha + \mu_\beta) \left[\int_{-\infty}^{\infty} F[Y](1 - F[Y])dY \right]. \end{aligned}$$

The last step results in an integral which is equal to the mean level of the indicator multiplied by the absolute Gini coefficient:

$$\int_{-\infty}^{\infty} F[Y](1 - F[Y])dY = \mu_Y G_Y.$$

Substitution results in:

$$SW = N\mu_Y - N(\mu_\alpha + \mu_\beta)\mu_Y G_Y.$$

Average social welfare is given by:

$$\frac{SW}{N} = \mu_Y - (\mu_\alpha + \mu_\beta)\mu_Y G_Y = \mu_Y(1 - (\mu_\alpha + \mu_\beta)G_Y).$$

This resembles the result of Schmidt and Wichardt (2019). When only income related to effort is accounted for, the household welfare function can be written as (Admiraal, 2021):

$$\begin{aligned} HW_n &= Y_n - \alpha_n \int_{-\infty}^{\infty} \max(k_n Y - k_n Y_n, 0) f[Y]dY - \beta_n \int_{-\infty}^{\infty} \max(k_n Y_n - k_n Y, 0) f[Y]dY, \\ &= Y_n - \alpha_n \int_{-\infty}^{\infty} k_n \max(Y - Y_n, 0) f[Y]dY - \beta_n \int_{-\infty}^{\infty} k_n \max(Y_n - Y, 0) f[Y]dY. \end{aligned}$$

$$= Y_n - k_n \alpha_n \int_{-\infty}^{\infty} \max(Y - Y_n, 0) f[Y] dY - k_n \beta_n \int_{-\infty}^{\infty} \max(Y_n - Y, 0) f[Y] dY.$$

Therefore, the same analytical result arises for the social welfare function with the preference parameters multiplied by $k = E(k_n)$ as this parameter scales the absolute Gini coefficient.

This requires zero covariance between k_n and the other parameters.

Appendix B. Summary statistics of the regression sample

Summary statistics non-scaled variables

| Variable | N | Min | Median | Mean | Max | SD |
|--------------------|-----|-------|--------|--------|--------|------|
| $\ln(100xG_Y)$ | 924 | 2.94 | 3.41 | 3.43 | 4.07 | .21 |
| $\ln(\mu_{Ytr})$ | 970 | 9.75 | 10.45 | 10.51 | 11.71 | .3 |
| $\ln(\mu_{Ytr})^2$ | 970 | 94.97 | 109.30 | 110.51 | 137.05 | 6.42 |

Summary statistic sample mean

| Variable | Mean |
|-----------|-------|
| \bar{Y} | 38484 |

Summary statistics scaled variables

| Variable | N | Min | Median | Mean | Max | SD |
|------------------------------------|-----|------|--------|------|------|-----|
| $\ln(\frac{\mu_{Ytr}}{\bar{Y}})$ | 970 | -.81 | -0.10 | -.05 | 1.15 | .3 |
| $\ln(\frac{\mu_{Ytr}}{\bar{Y}})^2$ | 970 | 0 | 0.04 | .09 | 1.32 | .14 |

Summary statistics covariates

| Variable name | N | Min | Median | Mean | Max | SD |
|--------------------------------------|------|-----|--------|--------|--------|---------|
| WORKING PERSONS (>12h/week) per 1000 | 910 | 62 | 333.0 | 5242.8 | 270219 | 27340.2 |
| FEMALE POPULATION | 1001 | 0 | 3968.0 | 3937.9 | 9624 | 2398.6 |
| MAROK | 1000 | 0 | 279.0 | 685.5 | 4587 | 898.4 |
| ANTILLIAN | 1000 | 0 | 74.0 | 86.6 | 477 | 69.8 |
| SURINAM | 1000 | 0 | 322.0 | 489.4 | 3359 | 525.8 |
| TURKISH | 1000 | 0 | 143.0 | 377.9 | 2634 | 515.4 |
| AGE >65 | 1000 | 1 | 816.5 | 926.1 | 3705 | 710.8 |
| MARRIED | 819 | 8 | 762.0 | 882.3 | 2715 | 627.2 |
| LIVING DURATION | 1000 | .2 | 8.5 | 8.3 | 15.9 | 2.7 |
| BIRTH | 808 | 1 | 105.0 | 105.2 | 333 | 68.3 |
| DEATH | 795 | 1 | 45.0 | 53.4 | 232 | 44.4 |

Notes: MOROCCAN/ANTILLIAN/SURINAM/TURKISH means the number of persons that is born in that country, or for which one of the parents is born in that country.

Summary statistics of the log of the scaled covariates

| Variable name | N | Min | Median | Mean | Max | SD |
|--------------------------------------|------|-------|--------|-------|------|------|
| WORKING PERSONS (>12h/week) per 1000 | 910 | -4.44 | -2.76 | -2.39 | 3.94 | 1.45 |
| FEMALE POPULATION | 1000 | -5.14 | 0.01 | -.35 | .89 | 1.1 |
| MAROCCAN | 991 | -6.53 | -0.89 | -1.15 | 1.9 | 1.91 |
| ANTILLIAN | 991 | -6.53 | -0.89 | -1.15 | 1.9 | 1.91 |
| SURINAM | 984 | -4.46 | -0.15 | -.38 | 1.71 | 1.07 |
| TURKISH | 998 | -6.19 | -0.42 | -.67 | 1.93 | 1.41 |
| AGE >65 | 993 | -5.93 | -0.94 | -1.05 | 1.94 | 1.7 |
| MARRIED | 1000 | -6.83 | -0.13 | -.47 | 1.39 | 1.27 |
| LIVING DURATION | 819 | -4.7 | -0.15 | -.39 | 1.12 | 1.12 |

| | | | | | | |
|---------------------------------|------|-------|-------|------|------|------|
| BIRTH | 1000 | -3.73 | 0.02 | -.08 | .65 | .46 |
| DEATH | 808 | -4.66 | -0.00 | -.36 | 1.15 | 1.06 |
| WORKING PERSONS (>12h) per 1000 | 795 | -3.98 | -0.17 | -.45 | 1.47 | 1.13 |

Notes: MOROCCAN/ANTILLIAN/SURINAM/TURKISH means the number of persons that is born in that country, or for which one of the parents is born in that country.

Appendix C. Estimated elasticities of the covariates

| <i>Variable name</i> | <i>Model 5</i> | <i>Model 6</i> |
|--------------------------------------|----------------|----------------|
| WORKING PERSONS (>12h/week) per 1000 | 0.0329 | 0.0488 |
| FEMALE | 0.236*** | 0.209*** |
| MAROCCAN | -0.00226 | -0.000701 |
| ANTILLIAN | -0.0168 | -0.0156 |
| SURINAM | 0.0992*** | 0.0904*** |
| TURKISH | 0.0583*** | 0.0435*** |
| AGE >65 | -0.0994** | -0.102** |
| MARRIED | -0.279*** | -0.273*** |
| LIVING DURATION | 0.0534 | 0.0474 |
| BIRTH | -0.0379*** | -0.0304** |
| DEATH | -0.00195 | -0.00201 |

*Notes: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Standard errors are clustered at the neighborhood level.*

MOROCCAN/ANTILLIAN/SURINAM/TURKISH means the number of persons that is born in that country, or for which one of the parents is born in that country. The author does not consider all these variables to be of normative significance for policies aiming at reductions in the Gini coefficient.