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Green, brown, and now white certificates - are three one too many? A micromodel of market interaction

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Georg Meran • Nadine Wittmann

Green, Brown, and Now White Certificates: Are Three One Too Many?
A Micromodel of Market Interaction

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Green, Brown, and now White Certificates
Are three one too many?
A micromodel of market interaction

by

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June 2008

Abstract

Our paper deals with modeling the effects of introducing a market-based tool for improving end-users’ efficiency in an energy market which is already regulated through a cap-and-trade system for greenhouse gas emissions and a quota system meant to improve competitiveness of energy produced using renewable resources. Our results show that the regulation of energy demand achieves its underlying objects of energy savings and energy efficiency solely at the expense of other goals such as the environmental efficiency of energy production. In our model, the implementation of a market for White Certificates (WCTS) causes energy producers’ investment in abatement to decrease along with the price for Brown Certificates and the amount of renewable energy demanded. Once we turn to the currently more empirically relevant case of integrating end-users only partially into WCTS, the unregulated group compensates in parts for the decrease in demand of the regulated group, due to an indirect price effect. As both supply and demand side of the market are regulated, this special set of regulations applied can, therefore, be compared to the grip of pincers embracing the entire market, leaving some of it virtually scarred. Consequently, we intended to search for alternative policy measures, which are able to achieve an increase in end-users’ energy efficiency without the negative side-effects witnessed in case of a WCTS. In our model a subsidized reduction in the price for households’ investment in energy efficiency renders just slightly more favorable results than an implementation of WCTS. However, the most effective way to accomplish all goals of environmental policy alike is to reduce the cap on emissions.

Keywords: Energy Markets, Certificate Trading Scheme, White Certificates, Efficiency, Regulation, Market-based tool, pincers policy

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1. Introduction

There is nothing new about the idea of implementing emission trading systems (Montgomery, [6]). In order to achieve or surpass the standards of the Kioto-protocol, participating nations have established markets for permits (Sorrell, S., Sijm, J. [12]) controlling the emissions of various greenhouse gases (GHG), so called Brown Certificates. However, the climate change is not the only topic that keeps decision makers worried. The threat of an increase in scarcity of various non-renewable resources has resulted in fervent support for the implementation of further market-oriented regulation schemes in the energy market in order to improve overall efficiency and facilitate market acceptance of energy policy (Oikonomou, V., Jepma, C.J. [9]). Also, the recently issued 20/20/20 vision of the EU demands further measures to achieve its goals.

In addition to improving the competitiveness of energy produced from renewable resources (Green Energy) through the implementation of a Green Certificate Trading Scheme (Amundsen, E.S., Mortensen, J.B. [1]) the focus of attention has moved to the issue of increasing energy efficiency of consumers, since many are of the opinion that, in most cases, the present environmental policy mix does not yet render satisfying results (Quirion, P. [10]). Hence, many people propose the establishment of a market for tradable permits gained by consumers of energy through investing in measures that will increase their respective energy efficiency (RWI (2006) [11]). In a few European Countries, like Great Britain, France, and Italy such a market-based tool has already been implemented. The proposed system works as follows: if consumers fail to fulfill a minimum requirement of energy efficiency measures set by authorities they need to buy White Certificates, if they surpass their requirements they thereby generate White Certificates and are able to sell their surplus to others. In reality, in order to achieve a reduction in costs and complexity, distributors take the part of consumers. The latter are only indirectly affected by the new market-system through an incentive mechanism. This describes, in short, the workings of a White Certificate Scheme (WCTS).

However, despite all of the arguments in favor of WCTS stated in the present literature, our model presents quite contradictory results. It shows that, in combination with all the other environmental policy measures already in place, the implementation of WCTS results in several negative side-effects, if evaluated from an environmental point of view; for example, in case of a WCTS energy producers’ investment in abatement decreases along with the price for Brown Certificates, which is a clear loss of environmental efficiency regarding producers of energy. Similarly, if we turn to the case of a partial integration of end-users into WCTS, the unregulated group compensates in parts for the decrease in demand of the regulated group, due to an indirect price effect. As both supply and demand side of the market are regulated, this special set of regulations applied can, therefore, be compared to the grip of pincers embracing the entire market, leaving some of it virtually scarred.

Consequently, we decided to present an alternative policy measure, which, in contrast to pincers policy, achieves an increase in end-users’ energy efficiency without any of the negative side-effects witnessed as WCTS is implemented. Before dealing with the implementation of a White Certificate Trading Scheme (WCTS) we present
the outline of the model in section two. In section three, we model a total integration of end-users into WCTS. Section four deals with relaxing the assumption that all households need to take part in WCTS. This constitutes a model of partial integration of only one group of end-users into WCTS. Section five discusses the effects of assuming investment in abatement technology to be constant. Due to the fact, that the implementation of WCTS causes several side-effects, which can be considered contradictory to environmental policy objects, we propose alternative solutions to the problem of how to increase end-users’ investment in energy efficiency and energy savings in sections six and seven. The concluding analysis of section eight shows, that, in fact, the most effective way to accomplish all goals of environmental policy alike is to reduce the cap on emissions.

2. Basics of the Model

First of all, we assume a perfectly competitive market system. We are aware of the fact that the energy market, in reality, is characterized by imperfect competition. However, in order to clearly isolate the effects of WCTS we dispense with this fact. There are four groups that are involved in the market for energy: producers of energy produced using non-renewable resources (Brown Energy), producers of energy produced using renewable resources (Green Energy), distributors and end-users of energy. In the absence of WCTS markets for Green and Brown Certificates are already in place and assumed to be working properly. Sanctions for noncompliance with regulations, quotas or standards are assumed to be effective as well, so that there is no incentive to defect. Whereas the market for Brown Certificates is designed as a cap-and-trade market with maximum emissions set at level $\bar{e}$, the market for Green Certificates is based on a minimum quota system $\alpha \epsilon [0, 1]$ of green energy in relation to total energy consumed and produced as seen in the paper of Amundsen and Mortensen (Amundsen, E.S., Mortensen, J.B. [1]). Now, properties and equations for each set of actors need to be defined.

The Supply Side

This side of the market consists of producers of energy produced using non-renewable resources (Brown Energy) and producers of energy produced using renewable resources (Green Energy).

Green Energy:
The producers of Green Energy receive wholesale price $q$ for each unit of Green Energy $y$ produced as well as the price $s$ of Green Certificates, which can be perceived as a market based subsidy paid by distributors. The costs of production $K(y)$, $K_y > 0, K_{yy} > 0$, are assumed to be relatively high. Hence, without subsidies and quota $\alpha$ Green Energy would not be able to compete successfully with Brown Energy. Profits are maximized according to

$$\max_y [(q + s)y - K(y)]$$

which results in the following first order condition (FOC):

$$K_y(y) = (q + s) \tag{1}$$

1For details on calculations of the results presented in all following sections please refer to the denoted sections of the appendix.
As a result, the amount of Green Energy produced can be expressed\footnote{appendix I.1, equation (38)} by the following supply function:
\[
\hat{y} = \hat{y}(q + s), \quad \hat{y}_q > 0 \tag{2}
\]

**Brown Energy:**

The producers of Brown Energy face production costs $C(x), C_x > 0, C_{xx} > 0$ as a function of units of energy produced $x$, which are assumed to be relatively low, compared to the costs of producing Green Energy. However, the production of energy using non-renewable resources is assumed to result in emissions of Greenhouse Gases, such as $CO_2$. Therefore, producers of Brown Energy incur additional costs as they need to comply with emission cap $\bar{e}$ set by regulation authorities and need to buy a certificate at price $z$ for each unit ($e$) of $CO_2$ emitted. Additionally, they could also invest in abatement technologies $I$ at price $D$ which reduces the amount of emission per unit of energy produced (Montgomery [6]). Emission can, thereby, be expressed as a function of amount of energy produced and investment in abatement:
\[
e = e(x, I), \quad e_x > 0 \text{ and } e_I < 0 \tag{3}
\]

We assume that emissions rise as $x$ rises and fall as $I$ rises. In both cases, we propose diminishing marginal effects, i.e. $e_x > 0, e_{xx} > 0, e_I < 0, e_{II} > 0$. Moreover, it is assumed that the marginal impact on emissions through an increase in output diminishes as $I$ rises, i.e. $e_{xI} = e_{Ix} < 0$. Finally, $e(x, I)$ is convex and, hence, producers’ resulting profit function is concave. Profits are maximized according to
\[
\max_{x,I} [qx - C(x) - ze(x, I) - ID]
\]
which results in the following FOCs:
\[
q = C_x(x) + ze_x(x, I) \tag{4}
\]
\[
D = -ze_I(x, I) \tag{5}
\]

As a result, the amount of Brown Energy produced and the amount of investment made can be expressed as a function of prices $q, z$, i.e.
\[
\hat{x} = \hat{x}(q, z) \quad \text{and} \quad \hat{I} = \hat{I}(q, z) \tag{6}
\]

since we assume $D = \bar{D}$ to be constant. In order to analyze the effects on $\hat{x}$ and $\hat{I}$ with respect to changes in prices $q$ and $z$, equations (4) and (5) are used. Changes in $q$ render the following results: As it is shown in in the appendix\footnote{appendix I.1, equation (39)}, both $\hat{x}$ and $\hat{I}$ rise in $q$, i.e. $\hat{x}_q > 0$ and $\hat{I}_q > 0$. Changes in $z$ result in the subsequent effects: As it is shown in in the appendix\footnote{appendix I.1, equation (42)}, while $\hat{x}$ is negatively correlated to $z$, $\hat{I}$ increases if $z$ rises, i.e. $\hat{x}_z < 0$ and $\hat{I}_z > 0$. To summarize
\[
\hat{x}_q > 0, \quad \hat{I}_q > 0, \quad \hat{x}_z < 0, \quad \hat{I}_z > 0 \tag{7}
\]
Subsequently, we need to determine the reaction of the emission function, i.e.
\[ \hat{e}(q, z) = e(\hat{x}, \hat{I}) \], to possible changes in \( q \) and \( z \):
\[ \hat{e}(q, z) = e(\hat{x}(q, z), \hat{I}(q, z)) \] (8)

From equations (3) and (7) it follows that
\[ \hat{e}_z = e_x \hat{x}_z + e_I \hat{I}_z < 0 \] (9)

The reaction of \( \hat{e} \) to an increase in \( q \) is ambiguous. This follows from
\[ \hat{e}_q = e_x \hat{x}_q + e_I \hat{I}_q \] (10)

and equation (7) to (8). In the following, we will assume that \( \hat{e}_q > 0 \), since, as energy wholesale price increases, production rises and, therefore, emissions increase, as well. The countervailing effect of an increase in abatement investment (\( I \)) does not compensate for an increase in emissions due to an increase in the production of Brown Energy \( x \).

The Demand Side
This side of the market consists of distributors and end-users of energy.

Distributors:
The distributors buy energy from producers at wholesale price \( q \) and sell it at market price \( p \), i.e. \( p > q \). However, they also need to ensure that the amount of Green Energy satisfies quota \( \alpha \) and, hence, are also forced to buy the respective amount of Green Certificates at price \( s \). As Brown Energy is assumed to be relatively cheaper, distributors demand the minimum amount of Green Energy, \( y = \alpha(x + y) \) (minimum quota system). The total amount of energy demanded is denoted by \( \hat{v} = \hat{x} + \hat{y} \) (market equilibrium). Hence, distributors profits are
\[ (p - q - \alpha s)(\hat{x} + \hat{y}) \]

Utilizing the market equilibrium condition \( \hat{v} = \hat{x} + \hat{y} \) we arrive at
\[ (p - q - \alpha s)\hat{v} \] (11)

which yields
\[ p = q + \alpha s \quad \text{or} \quad q = p - \alpha s. \] (12)

End-users:
We assume a strictly concave utility function \( U \), which depends on the amount of energy consumed \( v \) at price \( p \) and the amount of investment \( i \) in energy efficiency at price \( d \). The utility function \( U \) can be perceived as a reduced form of a traditional utility function and a household production function (Wirl [14]). In the absence of White Certificates, utility is maximized according to
\[ \max_{v,i} [U(v, i) - pv - id] \]
which results in the following FOCs:

\[ p = U_v(v, i) \quad \text{and} \quad d = U_i(v, i) \] (13)

The amount of energy demanded \( \hat{v} \) and the amount of investment chosen \( \hat{i} \) can be expressed as a function of \( p \)

\[ \hat{v} = \hat{v}(p) \quad \hat{i} = \hat{i}(p) \]

as we assume \( d = \bar{d} \) to be constant.

Once we introduce WCTS consumers are assumed to be directly affected by the mechanism and need to buy certificates at price \( w \) if their consumption of energy exceeds the standard set at \( \bar{v} \) by regulation authorities. If they consume less, they are able to sell their certificates generated at price \( w \). Hence, utility is maximized according to

\[ \max_{v,i} \left[ U(v, i) - pv - id + w(\bar{v} - v) \right] \]

which results in the following FOCs:

\[ U_v(v, i) = p + w \quad \text{and} \quad U_i(v, i) = d \] (14)

Therefore, the amount of energy demanded \( \hat{v} \) and the amount of investment chosen \( \hat{i} \) can be expressed as a function of \( p \) and \( w \)

\[ \hat{v} = \hat{v}(p + w) \quad \hat{i} = \hat{i}(p + w) \] (15)

as we assume \( d = \bar{d} \) to be constant, again. 5 Before we move on, however, two issues need to be taken care of. Firstly, the characteristics of end-users’ utility function need to be determined. There are two cases we pay attention to 6:

\textit{Case 1:} energy consumption and investment in energy efficiency are substitutes, i.e. \( U_{iv} = U_{vi} < 0 \).

\textit{Case 2:} energy consumption and investment in energy efficiency are complements, i.e. \( U_{iv} = U_{vi} > 0 \).

Secondly, changes in energy demand and amount of investment in energy efficiency with respect to \( p \) and \( d \) are analyzed as shown in the appendix 7. Results regarding changes in \( p \) are as follows: While \( \hat{v} \) is always negatively correlated regarding changes in \( p \), \( \hat{i} \) is positively (negatively) related to \( p \), if \( \hat{i} \) and \( \hat{v} \) are substitutes (complements). Considering changes in \( d \) we obtain the following results as shown in the appendix 8: While \( \hat{i} \) is always negatively correlated regarding changes in \( d \), \( \hat{v} \) is positively (negatively) related to \( d \), if \( \hat{i} \) and \( \hat{v} \) are substitutes (complements).

Now, we are able to move on to the core of our model.

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5If we separate consumers into more than one group, we assume that the utility function and all other assumptions introduced in this section hold for all groups alike.

6Calculations are to be found in appendix I.2

7Using equation (13), and appendix I.2 equation (44)

8Using equation (13), and appendix I.2 equation (49)
3. **Total Integration of End-users into WCTS**

Taking all assumptions and previously stated settings into account, we can determine the equilibrium conditions for the energy markets and the market for Brown Certificates through equations (16) to (18):

\[
(1 - \alpha) \hat{v}(p) - \hat{x}(p - \alpha s, z) = 0 \quad (16)
\]
\[
\alpha \hat{v}(p) - \hat{y}(p + (1 - \alpha)s) = 0 \quad (17)
\]
\[
\bar{e} - \hat{e}(p - \alpha s, z) = 0 \quad (18)
\]

Solving these equations, we arrive at a set of equilibrium prices \(p^0, s^0, z^0 > 0\) and \(q^0 = p^0 - \alpha s^0 > 0\) in the absence of WCTS. After introducing an effective WCTS into the market system by establishing a maximum quantity demanded \(\bar{v}\), we are faced with the following equations, taking into account that \(\bar{v} < \hat{v}(p^0)\):

\[
(1 - \alpha) \bar{v} - \hat{x}(p - \alpha s, z) = 0 \quad (19)
\]
\[
\alpha \bar{v} - \hat{y}(p + (1 - \alpha)s) = 0 \quad (20)
\]
\[
\bar{e} - \hat{e}(p - \alpha s, z) = 0 \quad (21)
\]

The solution obtained by these equations leads to equilibrium prices defined as \(p^w, s^w, z^w > 0\) and \(q^w = p^w - \alpha s^w > 0\). Also, we are able to calculate \(w > 0\) through \(\bar{v} = \hat{v}(p^w + w)\).

**Proposition 1** The introduction of a WCTS leads to an increase (decrease) of \(\hat{i}\), if \(\hat{i}\) is a substitute (complement), whereas \(\hat{I}\) decreases in any case.

**Proof:** We begin with the latter statement. Since, by equation (8), \(\bar{e} = \hat{e}(p - \alpha s, z) = e((1 - \alpha)\bar{v}, \hat{I})\), it follows that \(\hat{I} > 0\):

\[
\hat{I} = \frac{-(1 - \alpha)e_x}{\epsilon_i} > 0 \quad (22)
\]

In order to prove the former assertion, we differentiate equation (13) with respect to \(\bar{v}\) which yields:

\[
\frac{\partial \hat{I}}{\partial \bar{v}} = -\frac{U_{iv}}{U_{ii}}
\]

Thereby, the results can be deducted from the two familiar cases \(U_{iv} > 0\) and \(U_{iv} < 0\).

Applying some comparative statics to equations (19) to (21)\(^9\) we arrive at

\[
p^w \bar{v} > 0 \quad s^w \bar{v} \geq 0 \quad z^w \bar{v} > 0
\]

\(^9\)appendix II.1 and II.2
Now, we are able to infer all of the effects of implementing WCTS. In order to increase end-users’ energy savings and energy efficiency regulation authorities set $\bar{v} < \hat{v}(p^o)$. As sanctions are supposed to be working properly, end-users will comply with $\bar{v}$ and either reduce their energy consumption or buy White Certificates at price $w$. But that is not the end of the story; the following effects result from this change in demand. Setting $\bar{v} < \hat{v}(p^o)$ leads to a reduction in demand. Due to this effect, the price for energy decreases, $\hat{p} \downarrow$, along with the price for Brown Certificates, $\hat{z} \downarrow$. As demand decreases, supply of $\hat{x}$ and $\hat{y}$ decreases likewise. If $\hat{v}$ and $\hat{i}$ are substitutes (complements) households’ investment increases (decreases), i.e. $\hat{i} \uparrow$ ($\hat{i} \downarrow$).

Therefore, according to our model, the intended goal of WCTS to increase end-users’ investment in energy efficiency measures cannot be ensured. The effect depends on whether the latter is a substitute for or a complement to energy consumption.

Moreover, the effects on energy producers’ behavior are quite unambiguous: The reduced amount of Brown Energy $\hat{x}$ is produced using less investment in abatement $\hat{I} \downarrow$ and at a lower price for Brown Certificates $\hat{z} \downarrow$, which results in an increase in emissions per unit of energy $\hat{\epsilon}(q,z) \uparrow$. From an environmental point of view, this is clearly a negative effect. Due to the minimum quota system, production of Green Energy has decreased as well, i.e. $\hat{y} \downarrow$. To sum it up, while the new goal of energy efficiency is not necessarily achieved, the remaining production of Brown Energy runs at a relatively higher emission level per unit of output. As a result, the environmental efficiency in energy production is clearly reduced.

4. Partial integration of End-users into WCTS

In this section we separate consumers into two groups and only one of them is forced to take part in WCTS. This results from the fact, that in some of the EU countries where WCTS has already been implemented only a representative group of consumers is (indirectly) subject to regulations (Bürger, V., Wiegmann, K. [3]). Therefore, we separate aggregate demand into $v$ and $v_f$, the latter denoting the unregulated part of demand. $^10$ We intend to find out whether the group whose demand is not regulated serves as a vent for excess supply of energy. We modify equations (16) to (18) as follows:

$$\begin{align*}
(1 - \alpha) \left( \hat{v}(p) + \hat{v}_f(p) \right) - \hat{x}(p - \alpha s, z) &= 0 \\
\alpha \left( \hat{v}(p) + \hat{v}_f(p) \right) - \hat{y}(p + (1 - \alpha)s) &= 0 \\
\bar{v} - \hat{e}(p - \alpha s, z) &= 0
\end{align*}$$

Solving these equations yields equilibrium prices $p^{o_f}, s^{o_f}, z^{o_f} > 0$, and $q^{o_f} = p^{o_f} - \alpha s^{o_f} > 0$.

After introducing an effective WCTS into the market system by establishing a maximum quantity demanded $\bar{v}$, we are faced with the following equations, taking into account that $\bar{v} < \hat{v}(p^{o_f})$:

$$\begin{align*}
(1 - \alpha) \left( \bar{v} + \hat{v}_f(p) \right) - \hat{x}(p - \alpha s, z) &= 0
\end{align*}$$

$^10$\(v_f\) is also determined by equation (13).
\[ \alpha (\bar{v} + \hat{v}_f(p)) - \hat{y}(p + (1 - \alpha)s) = 0 \]  
(27)

\[ \bar{v} - \hat{v}(p - \alpha s, z) = 0 \]  
(28)

Solving these equations renders equilibrium prices \( p^{wf}, s^{wf}, z^{wf} > 0 \), and \( q^{wf} = p^{wf} - \alpha s^{wf} > 0 \). Also, we are able to calculate \( w > 0 \) through \( \bar{v} = \hat{v}(p^{wf} + w) \). Before the main propositions characterizing the case of partial integration are stated, a lemma is introduced which is meant to prepare for the analysis lying ahead.

**Lemma 1** The increase in \( \hat{v}_f(p) \) does not completely offset the decrease in demand caused by implementing \( \bar{v} \), i.e. \( \frac{\partial \hat{v}_f}{\partial \bar{v}} = \hat{v}_f \frac{\partial p}{\partial \bar{v}} > -1 \).

**Proof:** Please refer to appendix III.1

**Proposition 2** The introduction of a WCTS leads to an increase (decrease) of \( \hat{i} \), if \( \hat{i} \) is a substitute (complement), whereas \( \hat{I} \) decreases in any case. The investment in energy efficiency, \( \hat{i}_f \), of the unregulated market increases (decreases) if \( \hat{i}_f \) is a substitute (complement).

**Proof:** Since \( \bar{v} = e((1 - \alpha)(\bar{v} + \hat{v}_f), \hat{I}) \), we are able to determine that

\[ \hat{I}_\bar{v} = \frac{-(1 - \alpha)e_x[1 + \frac{\partial \hat{i}_f}{\partial p^{wf}} \frac{\partial p^{wf}}{\partial \bar{v}}]}{\epsilon_I} > 0 \]  
(29)

The sign follows immediately from Lemma 1. To proof the other assertions of proposition 2 we first need to calculate the price change in \( p \), i.e. the difference between \( p^{e} \) and \( p^{wf} \).

Therefore, we apply some comparative static analysis to equations (26) to (28) as shown in the appendix. As a result, the following results can be obtained:

\[ p^{wf}_\bar{v} > 0 \quad s^{wf}_\bar{v} \leq 0 \quad z^{wf}_\bar{v} > 0 \]  
(30)

The first statement of proposition 2 has already been proven in proposition 1. The only missing part is the reaction of \( \hat{i}_f \). From equation (15) it follows that

\[ \frac{\partial \hat{i}_f}{\partial \bar{v}} = \hat{i}_f \frac{p^{wf}}{p^{e}} \]  
(31)

Taking into account equation (30) and the two possible relationships between energy consumption and investment in energy efficiency we arrive at \( \frac{\partial \hat{i}_f}{\partial \bar{v}} > 0 \) (\( \frac{\partial \hat{i}_f}{\partial \bar{v}} < 0 \)) in the case of them being substitutes (complements).

The effects are as follows: In order to increase end-users’ energy savings and energy efficiency regulation authorities set \( \bar{v} < \hat{v}(p^{wf}) \). Hence, consumers will comply with \( \bar{v} \) and reduce their energy consumption or buy White Certificates at price \( w \), while the unregulated group remains seemingly unaffected, at first glance. But that is not the end of the story. As demand of the regulated group decreases, \( v \downarrow \), price of energy decreases as well, \( p \downarrow \), along with the price for Brown Certificates \( z \downarrow \). Therefore, quantity demanded of the unregulated group increases \( \hat{v}_f \uparrow \), due to an indirect

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\textsuperscript{11} appendix III.1, equation (55) and appendix III.2
price effect. However, as overall quantity demanded decreases (Lemma 1), quantity supplied also decreases, \( \hat{x} \downarrow \) and \( \hat{y} \downarrow \). Moreover, investment in abatement technology decreases \( \hat{I} \downarrow \). Also, if \( \hat{i} \) and \( \hat{v} \) are substitutes (complements), investment in energy efficiency of the regulated group increases (decreases), i.e. \( \hat{i} \uparrow (\hat{i} \downarrow) \). However, the investment in energy efficiency of the unregulated group decreases (increases), as \( \hat{i}_f \) and \( \hat{v}_f \) are substitutes (complements). Therefore, the originally intended effect is clearly reduced.

On the whole, again, the intended goal of WCTS to increase overall end-users' investment energy efficiency measures cannot be ensured. The effect depends on whether investment and energy consumption are substitutes or complements. Even if the more likely case of them being substitutes, the effect on energy efficiency is still twofold, as the effects on energy producers' behavior are quite unambiguous: The reduced amount of Brown Energy \( \hat{x} \) is produced using less investment in abatement \( \hat{I} \downarrow \) and at a lower price for Brown Certificates \( z \downarrow \), which results in an increase in emissions per unit of energy \( \frac{e(q,z)}{\hat{x}(q,z)} \uparrow \). From an environmental point of view, this is clearly a negative effect. Due to the minimum quota system, production of Green Energy has decreased as well, i.e. \( \hat{y} \downarrow \). To sum it up, while the new goal of energy efficiency cannot be achieved successfully, the remaining production of Brown Energy runs at a relatively higher emission level per unit of output. As a result, the environmental efficiency in energy production is clearly reduced.

5. **Abatement Investment is Constant**

Let us assume, that investment in abatement technology does not or cannot be varied, i.e. \( \hat{I} = \bar{I} \). As a result, emission levels can only be altered through a change in production \( x, e = e(\hat{x}, \bar{I}) \). Therefore, changes in \( \hat{v} \) and \( \hat{v}_f \) only affect emission levels through their effect on \( \hat{x} \):

**Corollary 1** In case of total integration the introduction of WCTS leads to a breakdown of the market for Brown Certificates (Montgomery [6]), i.e. \( z = 0 \). In case of partial integration of consumers into WCTS the increase in demand of the unregulated group enables the market for Brown Certificates to persist, i.e. \( z > 0 \).

**Proof:** Please refer to appendix IV.1 and IV.2

6. **Back to subsidies?**

As we have seen so far, implementing WCTS cannot exactly be considered the icing on the cake of environmental policy, considering the various negative side-effects. Therefore, we tried to come up with a policy measure which achieves an increase in end-users' energy efficiency without any of the unintended results witnessed as WCTS is implemented.

A subsidized reduction of the price of end-users' investment in energy efficiency measures \( d \) could be an alternative. As a result, demand for investment in energy

\[ \text{appendix IV.1} \]

\[ \text{appendix IV.2} \]
efficiency increases, i.e. $\hat{i} \uparrow$. To determine all of the relevant effects of a subsidy on $d$ we set up the subsequent set of equations, while, according to equations (13) $v$ now depends on both, $p$ and $d$:

$$
(1 - \alpha) \hat{v}(p,d) - \hat{x}(p - \alpha s, z) = 0
$$

(32)

$$
\alpha \hat{v}(p,d) - \hat{y}(p + (1 - \alpha)s) = 0
$$

(33)

$$
\hat{e} - \hat{e}(p - \alpha s, z) = 0
$$

(34)

Solving these equations equilibrium prices are defined as $p^t$, $s^t$, $z^t$ and $q^t = p^t - \alpha s^t$.

**Proposition 3** A reduction in $d$ through an increase in subsidies for households’ investment in energy efficiency does render the intended results of an increase in $\hat{i}$. Compared to the effects of implementing WCTS, subsidies do pose an relatively more effective alternative, as a reduction in $d$ can lead to an increase in abatement investment $I$.

**Proof:** Analysis has to be divided according to the relevant cases:

**Case 1:** energy consumption and investment in energy efficiency are substitutes ($v_d < 0$). This condition results in $p_d^t$, $z_d^t < 0$ and $q_d^t < 0$ while $s_d^t$ remains ambiguous, therefore it follows that $\frac{\partial \hat{I}}{\partial d} = \hat{i}_p p_d^t + \hat{i}_d < 0$.

**Case 2:** energy consumption and investment in energy efficiency are complements ($v_d > 0$). This condition results in $p_d^t$, $z_d^t > 0$ and $q_d^t > 0$ while $s_d^t$ remains ambiguous, therefore it follows that $\frac{\partial \hat{I}}{\partial d} = \hat{i}_p p_d^t + \hat{i}_d < 0$.

To ascertain the second part of proposition (3) the indirect impact of a change in $d$ on $I$ has to be defined:

**Case 1:** energy consumption and investment in energy efficiency are substitutes:

$$
\frac{\partial \hat{I}}{\partial d} = \frac{-(1-\alpha)e_v[v_p^d \frac{\partial s^t}{\partial d} + \hat{v}_d]}{e_I} > 0
$$

**Case 2:** energy consumption and investment in energy efficiency are complements:

$$
\frac{\partial \hat{I}}{\partial d} = \frac{-(1-\alpha)e_v[v_p^d \frac{\partial s^t}{\partial d} + \hat{v}_d]}{e_I} < 0
$$

Now, all necessary analysis has been conducted an the effect of a subsidy can be inferred: Investment in energy efficiency increases $i \uparrow$ in both cases.

If $\hat{v}$ and $\hat{i}$ are substitutes, demand decreases $\hat{v} \downarrow$. As a result the price of energy decreases. Moreover, as demand decreases supply decreases as well. The price of Brown Certificates increases as $z_d < 0$. Investment in abatement decreases, i.e. $I_d > 0$.

If $\hat{v}$ and $\hat{i}$ complements, demand increases $\hat{v} \uparrow$. As a result the price of energy increases. Moreover, as demand increases supply increases as well. Also, as the price of energy increases significantly, demand is likely to decrease again. The price of Brown Certificates increases as $z_d > 0$. Investment in abatement increases, i.e. $I_d < 0$.

---

14 for details see appendix V.1 and V.2
Overall, however, these results can be considered only slightly more favorable than the effects of implementing WCTS concerning incentives on investment in abatement and energy efficiency. This conclusion stems from the fact that, in both cases, $\hat{v}$ and $\hat{i}$ being substitutes or complements, negative effects can be witnessed. In the less likely case of $\hat{v}$ and $\hat{i}$ being complements, investment in abatement technology decreases, similar to the general effect displayed in the case of WCTS. In the likely case of $\hat{v}$ and $\hat{i}$ being substitutes, investment abatement increases, however, the price of Brown Certificates decreases, which results in emissions to become relatively cheaper. Therefore, in both cases, subsidies cannot be considered the most favorable measure, if analyzed from an environmental point of view.

Possible criticism of the alternative proposed could also spring from the fact that subsidies are considered a costly measure of environmental policy which might, therefore, face more opposition than a market-based tool. However, one has to keep in mind that, in many countries, systems for subsidizing energy savings and efficiency measures of households are already established (Bürger, V., Wiegmann, K. [3]). Therefore, for the most part, only the additional cost of an increase in subsidies would have to be incurred. However, in the case of a WCTS, although a market-based tool seems to be less costly at first glance, we argue that establishing an effective certificate system which includes supervising trade, compliance, or imposing sanctions cannot, at all, be considered cheap and easy to handle.

7. Less is more

Instead of further dwelling measures like WCTS or an increase in subsidies that render only unsatisfactory results, we make one more attempt to propose a successful way of increasing end-users’ investment in energy efficiency without the negative side-effects witnessed so far. As markets for Brown and Green Certificates are supposed to be implemented and working properly, we suggest a reduction in the cap of emissions $\bar{e}$ to achieve satisfactory results. Demand for energy remains unregulated and undivided, i.e. $\hat{v} = \hat{v}(p)$.  

Again, to determine all of the relevant effects we need to take a closer look at the conditions of a market equilibrium:

\begin{align}
(1 - \alpha) \hat{v}(p) - \hat{x}(p - \alpha s, z) &= 0 \quad (35) \\
\alpha \hat{v}(p) - \hat{y}(p + (1 - \alpha)s) &= 0 \quad (36) \\
\bar{e} - \hat{e}(p - \alpha s, z) &= 0 \quad (37)
\end{align}

Solving these equations equilibrium prices are defined as $p'_l, s'_l, z'_l$ and $q'_l = p'_l - \alpha s'_l$. Effects of a change in $\bar{e}$ need to be analyzed as well:

Comparative static analysis of equations (35) to (37) with respect to $\bar{e}$ yields $p'_l < 0, s'_l > 0, z'_l < 0$ and $q'_l = p'_l - \alpha s'_l < 0$.

**Proposition 4** As $\bar{e}$ is reduced, emissions become more costly due to an increase in scarcity. Therefore, investment in abatement ($I$) will increase. Households’ investment in energy efficiency ($i$) increases as well.

---

15 We assume $d = \bar{d}$ to be constant, again.

16 appendix VI.1, equation (63)
Proof: Firstly, households’ reaction ($i$) to a reduction in the cap of emission needs to defined, as well as the effect of the latter on investment in abatement ($I$). Comparative statics \(^{17}\) with respect to $\bar{e}$ yields, that investment in energy efficiency increases, i.e. $\hat{i}_e < 0$. Secondly, Brown Energy producers’ reaction to changes in $\bar{e}$ is analyzed \(^{18}\). As shown in the appendix, investment in abatement ($I$) increases as $\bar{e}$ is reduced, $\hat{I}_e < 0$.

Results can be summarized as follows: In order to increase end-users’ investment in energy efficiency, a reduction in the emission cap will render the intended results effectively. It also triggers additional investment in abatement technology.

8. Conclusions

To sum it up, the only sensible and effective way to achieve all of the environmental goals mentioned above is to keep things simple. Instead of introducing another certificate system or a subsidy the policy measures and the markets already in place need to be used to accomplish additional goals. Through a reduction in the cap of emissions all goals can successfully, efficiently, and effectively be achieved without having to incur any negative side-effects or additional costs on behalf of regulation authorities. Therefore, even in the case of trying to accomplish one more object through environmental policy, regarding the given circumstances, implementing less measures really seems to be the better deal.

Some experts assume energy demand to be relatively inelastic to price changes (Bertoldi et. al. [2]) and propose WCTS as a possible solution to successfully deal with this problem. However, in our opinion, WCTS is just another way to increase the relative price of energy. As a result, the effect of a possible increase in energy prices due to the implementation of an Emission Trading System (ETS) will have no effect regarding end-users’ energy efficiency or energy consumption, if the latter really reacted inelastic to price. It has also been argued that an increase in energy efficiency has been observed where WCTS has already been established (Mundaca, L., Leij, L. [7]). Additional positive effects such as reduction in emission have been identified as well. However, even though we arrive, in part, at similar findings, this cannot be perceived as being the bottom line of the story. According to our model, we agree on the statement that there are just too many other "surprising impacts" of this environmental policy instrument, due to the "multi-aspect nature" of the problems dealt with (OECD 2007 [8]). A critical remark, however, could be, that we have not addressed the critical issue that even in the European Union, in reality, only about half of the industry, whose production results in the emission of GHG, is integrated in the cap-and-trade system of Brown Certificates (Langniss, O., Praetorius, B. [5]). Furthermore, the effects of a recently proposed integration of White Certificates into the market for Brown Certificates seems to be of great interest, considering the results of our model. Moreover, in countries where WCTS has already been implemented, distributors are held responsible for compliance with $\bar{v}$ and have to buy or sell White Certificates on behalf of households. The latter have to be addressed through an incentive mechanism which has to be set up by distributors. We intend to pay more attention to these fact as our research proceeds.

\(^{17}\) appendix VI.3, equation (69)

\(^{18}\) appendix VI.3, equation (68)
Appendix

I. Basics of the Model

I.1 The Supply Side

This side of the market consists of producers of energy produced using non-renewable resources (Brown Energy) and producers of energy produced using renewable resources (Green Energy).

**Green Energy:**

\[
K_y(y) = q + s, \quad K_{yy} \hat{y}_q = 1, \quad \hat{y}_q = \frac{1}{K_{yy}} > 0
\]  

\[(38)\]

**Brown Energy:**

Changes in \(q\):

\[
\Delta \begin{pmatrix} \hat{x}_q \\ I_q \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}
\]  

\[(39)\]

where \(\Delta = \begin{pmatrix} -(C_{xx} + e_{xx}) & -ze_{xI} \\ -ze_{Ix} & -ze_{II} \end{pmatrix}\). Assuming concavity it follows that

\[
|\Delta| = z(e_{II}(ze_{xx} + K_{xx}) - ze_{xI}^2) > 0
\]  

\[(40)\]

From (39) it follows

\[
\hat{x}_q = \frac{ze_{II}}{|\Delta|} > 0 \quad \hat{I}_q = \frac{-ze_{xI}}{|\Delta|} > 0
\]  

\[(41)\]

Changes in \(z\):

\[
\Delta \begin{pmatrix} \hat{x}_z \\ I_z \end{pmatrix} = \begin{pmatrix} -e_x \\ e_I \end{pmatrix}
\]  

\[(42)\]

From (40) and (42) it follows

\[
\hat{x}_z = \frac{z(-e_Ie_{xI} - e_xe_{II})}{|\Delta|} < 0 \quad \hat{I}_z = \frac{z(-e_xe_{Ix} - e_I(C_{xx} + e_{xx}))}{|\Delta|} > 0
\]  

\[(43)\]

I.2 The Demand Side

This side of the market consists of distributors and end-users of energy.

**End-users:**

Changes in \(p\):

\[
\Omega \begin{pmatrix} \hat{v}_p \\ i_p \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]  

\[(44)\]

where \(\Omega = \begin{pmatrix} U_{vv} & U_{vi} \\ U_{iv} & U_{ii} \end{pmatrix}\).

By the assumption of concavity it follows that

\[
|\Omega| = U_{vv}U_{ii} - U_{vi}^2 > 0
\]  

\[(45)\]
From (44) and (45) it follows that
\[ \hat{v}_p = \frac{U_{ii}}{|\Omega|} < 0 \] (46)
\[ \hat{i}_p = -\frac{U_{iv}}{|\Omega|} > 0 \quad \text{if } i \text{ and } v \text{ are substitutes and} \] (47)
\[ \hat{i}_p = -\frac{U_{iv}}{|\Omega|} < 0 \quad \text{if } i \text{ and } v \text{ are complements} \] (48)

Changes in \( d \):
\[ \Theta \left( \begin{array}{c}
\hat{v}_d \\
\hat{i}_d
\end{array} \right) = \left( \begin{array}{c}
0 \\
1
\end{array} \right) \] (49)
\[ \hat{i}_d = \frac{U_{vy}}{|\Omega|} < 0 \] (50)
\[ \hat{v}_d = -\frac{U_{iv}}{|\Omega|} < 0 \quad \text{if } i \text{ and } v \text{ are complements} \] (51)
\[ \hat{v}_d = -\frac{U_{iv}}{|\Omega|} > 0 \quad \text{if } i \text{ and } v \text{ are substitutes} \] (52)

II. Total Integration of End-users into WCTS

II.1 From (16) to (18) we can derive the effects of changes in \( \bar{v} \):
\[ \Theta \left( \begin{array}{c}
p_{w}^v \\
s_{w}^v \\
z_{w}^v
\end{array} \right) = \left( \begin{array}{c}
(1 - \alpha) \\
\alpha \\
0
\end{array} \right) \] (53)
where \( \Theta = \left( \begin{array}{cc}
\hat{x}_q & -\alpha \hat{x}_q \\
\hat{y}_q & (1 - \alpha) \hat{y}_q \\
\hat{e}_q & -\alpha \hat{e}_q
\end{array} \right) \).

Due to assumption of concavity and due to equations (3), (7), and (38), we are able to infer that
\[ |\Theta| = \hat{y}_q (\hat{x}_q \hat{e}_z - \hat{x}_z \hat{e}_q) < 0 \] (54)

II.2 From (53) and (54) it follows that
\[ p_{w}^v = \frac{(1-\alpha)^2\hat{y}_q \hat{e}_z + \alpha^2(\hat{x}_q \hat{e}_z + \hat{x}_z \hat{e}_q)}{|\Theta|} > 0 \]
\[ s_{v}^w = \frac{\alpha(\hat{x}_q \hat{e}_z - \hat{x}_z \hat{e}_q) - (1-\alpha)\hat{y}_q \hat{e}_z}{|\Theta|} \geq 0 \]
\[ z_{w}^v = \frac{-2\alpha \hat{y}_q \hat{e}_q}{|\Theta|} > 0 \]

III. Partial Integration of End-users into WCTS

III.1 From (26) to (28) we can derive the effects of changes in \( \bar{v} \):
\[ \Lambda \left( \begin{array}{c}
p_{w}^f \\
s_{w}^f \\
z_{w}^f
\end{array} \right) = \left( \begin{array}{c}
(1 - \alpha) \\
\alpha \\
0
\end{array} \right) \] (55)
where \( \Lambda = \begin{pmatrix}
\dot{x}_q - (1 - \alpha)\dot{v}_p & -\alpha \dot{x}_q & \dot{x}_z \\
\dot{y}_q - \alpha \dot{v}_p & (1 - \alpha)\dot{y}_q & 0 \\
\dot{\bar{e}}_q & -\alpha \dot{\bar{e}}_q & \dot{\bar{e}}_z
\end{pmatrix} \)

\[
|\Lambda| = (\dot{x}_q \dot{\bar{e}}_z (\dot{y}_q - \alpha \dot{v}_p) + (1 - \alpha)\dot{y}_q) - \dot{x}_z \dot{\bar{e}}_q (\alpha \dot{y}_q - \alpha^2 \dot{v}_p + (1 - \alpha)\dot{y}_q - \frac{(1 - \alpha)^2 \dot{v}_p \dot{y}_q}{\dot{x}_q}) < 0 \tag{56}
\]

The sign of the determinant follows from equation (54) and the fact that

\[(\dot{y}_q - \alpha \dot{v}_p) + (1 - \alpha)\dot{y}_q > 0 \quad \text{and} \quad (\alpha \dot{y}_q - \alpha^2 \dot{v}_p + (1 - \alpha)\dot{y}_q - \frac{(1 - \alpha)^2 \dot{v}_p \dot{y}_q}{\dot{x}_q}) > 0 \]

due to equations (7), (38), and (46).

### III.2 Equations (26) to (28) and (56) render

\[
p_{wf}^v = \frac{(1 - \alpha)^2 \dot{y}_q \dot{\bar{e}}_z + \alpha^2 (\dot{x}_q \dot{\bar{e}}_z - \dot{x}_z \dot{\bar{e}}_q)}{|\Lambda|} > 0
\]

\[
s_{wf}^v = \frac{\dot{\bar{e}}_z (\alpha \dot{x}_q - (1 - \alpha)\dot{v}_p) - (1 - \alpha)\dot{y}_q - \alpha \dot{\bar{e}}_q - \alpha \dot{\bar{e}}_q}{|\Lambda|} \geq 0
\]

\[
z_{wf}^v = \frac{(1 - \alpha)\alpha^2 \dot{x}_q \dot{v}_p - \dot{x}_q (\alpha + (1 - \alpha)^2 \dot{y}_q)}{|-\alpha|} > 0
\]

### III.3 Inserting \( p_{wf}^v > 0 \) into \( \frac{\partial \dot{v}_f}{\partial v} \) yields:

\[
\frac{\partial \dot{v}_f}{\partial v} = \frac{\partial p_f}{\partial v} \frac{\partial \dot{v}_f}{\partial p_f} \leq -1
\]

\[
\frac{\partial p_f}{\partial v} = \frac{\dot{v}_f (1 - \alpha)^2 \dot{y}_q \dot{\bar{e}}_z + \alpha^2 (\dot{x}_q \dot{\bar{e}}_z - \dot{x}_z \dot{\bar{e}}_q)}{|\Lambda|} \geq -1
\]

\[
\dot{v}_f [(1 - \alpha)^2 \dot{y}_q \dot{\bar{e}}_z + \alpha^2 (\dot{x}_q \dot{\bar{e}}_z - \dot{x}_z \dot{\bar{e}}_q)] \leq -1 |\Lambda|
\]

\[
0 \leq \dot{y}_q (\dot{x}_q \dot{\bar{e}}_z - \dot{x}_z \dot{\bar{e}}_q)
\]

Due to equation (54) it follows determine that

\[
\frac{\partial \dot{v}_f}{\partial v} > -1 \tag{57}
\]

### IV. Abatement Investments are Constant

The effects can be analyzed utilizing equations (21) and (25) which represent the equilibrium conditions of the market for emission permits in case of total and partial integration into WCTS.

#### IV.1

In case of total integration, where \( \bar{v} = \dot{v}(p^w + w) \), the implementation of WCTS will result in a reduction of total energy demanded and, hence, lead to a reduction in energy produced, \( \dot{x} \) and \( \dot{y} \). As emissions correlate positively with the respective level of production, \( e(\dot{x}, \dot{y}) \) will also decline. Hence, \( \dot{e} \) is no longer binding and, taking into account equation(18), the price of for Brown Certificates is zero, i.e. \( z = 0 \). This case displays most significantly the negative effects triggered by pincers policy.

#### IV.2

In case of partial integration into WCTS demand of the regulated group decreases, i.e. \( \bar{v} < \dot{v}(p^{wf}) \). However, the unregulated market compensates for all of the otherwise excess supply, i.e.
\((\hat{v}(p^o) + \hat{v}_f(p^o)) = (\bar{v} + \hat{v}_f(p^o))\). Therefore, the market for brown certificates persists. However, WCTS fails to achieve its goal of a reduction in aggregate energy consumption, completely.

**V. Back to subsidies?**

**V.1** From (32) to (34) we can derive the effects of changes in \(\bar{v}\):

\[
\Sigma \begin{pmatrix}
p^t_d \\
s^t_d \\
z^t_d
\end{pmatrix} = \begin{pmatrix}
(1 - \alpha)\hat{v}_d \\
\alpha\hat{v}_d \\
0
\end{pmatrix}
\]  

(58)

where \(\Sigma = \begin{pmatrix}
\hat{x}_q - (1 - \alpha)\hat{v}_p & -\alpha\hat{x}_q & \hat{x}_z \\
\hat{y}_q - \alpha\hat{v}_p & (1 - \alpha)\hat{y}_q & 0 \\
\hat{e}_q & -\alpha\hat{e}_q & \hat{e}_z
\end{pmatrix}\).

and

\[
|\Sigma| = (\hat{x}_q \hat{e}_z ((\hat{y}_q - \alpha\hat{v}_p) + (1 - \alpha)\hat{y}_q) - \hat{x}_z \hat{e}_q (\alpha\hat{y}_q - \alpha^2 \hat{v}_p + (1 - \alpha)\hat{y}_q - (1 - \alpha)^2 \hat{v}_p \hat{y}_q)) < 0
\]  

(59)

Recalling the assumption that \(\hat{v}_p = \hat{v}_f < 0\), the sign follows from equation (56).

**V.2** From (32) to (34) and (59) it follows that

\[
p^t_d = \frac{\hat{v}_d |(1 - \alpha)^2 \hat{y}_q \hat{e}_z + \alpha^2 \hat{x}_z \hat{e}_q|}{|\Sigma|} \\
s^t_d = \frac{\hat{v}_d |\alpha \hat{x}_q (\hat{y}_q - (1 - \alpha)\hat{v}_p) - (1 - \alpha) \hat{x}_z |}{|\Sigma|} \\
z^t_d = \frac{\hat{v}_d |-(1 - \alpha)\hat{v}_p + (1 - \alpha) \alpha \hat{x}_q - (1 - \alpha)^2 \hat{e}_q \hat{y}_q|}{|\Sigma|}
\]

If \(\hat{v}_d < 0\) (i.e. \(v\) and \(i\) are substitutes):

\[
p^t_d < 0, \quad s^t_d \leq 0, \quad z^t_d < 0
\]  

(60)

If \(\hat{v}_d > 0\) (i.e. \(v\) and \(i\) are complements):

\[
p^t_d > 0, \quad s^t_d \leq 0, \quad z^t_d > 0
\]  

(61)

**V.3** Changes in \(d\):

\[
\frac{\partial \hat{I}}{\partial d} = -\frac{(1 - \alpha)\epsilon_x [\hat{v}_p \frac{\partial p^t_d}{\partial d} + \hat{v}_d]}{e_t} \leq 0
\]  

(62)

It follows from equations (51), (52), (60) and (61) that in the case of \(\hat{v}_d < 0\):
\[
\frac{\partial I}{\partial d} = -\frac{(1-\alpha)e_x[v_p \frac{\partial v}{\partial x} + \hat{v}_d]}{e_I} > 0
\]

in the case of \( \hat{v}_d > 0 \):

\[
\frac{\partial I}{\partial d} = -\frac{(1-\alpha)e_x[v_p \frac{\partial v}{\partial x} + \hat{v}_d]}{e_I} < 0
\]

VI. Less is more

VI.1 From (35) to (37) we can derive the effects of changes in \( e \):

\[
\Sigma \begin{pmatrix} p_l^e \\ s_l^e \\ z_l^e \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\]

(63)

where \( |\Sigma| < 0 \) as shown in equation (59).

VI.2 From (35) to (37) and (59) it follows that

\[
p_l^e = -\frac{\hat{x}_z(1-\alpha)\hat{y}_q}{|\Sigma|} < 0
\]

(64)

\[
s_l^e = \frac{\hat{x}_z(\hat{y}_q - \alpha\hat{v}_p)}{|\Sigma|} > 0
\]

(65)

\[
z_l^e = \frac{(\hat{x}_q - (1-\alpha)\hat{v}_p)(1-\alpha)\hat{y}_q + \alpha\hat{x}_q(\hat{y}_q - \alpha\hat{v}_p)}{|\Sigma|} < 0
\]

(66)

\[
q_l^e = p_l^e - \alpha s_l^e < 0
\]

(67)

VI.3 Regarding \( I \): It follows from equations (6), (7), and (64) to (67) that

\[
\hat{I}_e = \hat{I}_q \frac{\partial q^l}{\partial \hat{e}} + \hat{I}_z \frac{\partial z^l}{\partial \hat{e}} < 0
\]

(68)

Regarding \( i \):

\[
\Omega \begin{pmatrix} \hat{v}_e \\ \hat{e}_e \end{pmatrix} = \begin{pmatrix} \hat{p}_e^l \\ 0 \end{pmatrix}
\]

(69)

It follows from (59) and (69) that

\[
\hat{v}_e = -\frac{U_{iv} p_l^e}{|\Sigma|} \leq 0
\]

\[
\hat{e}_e = \frac{U_{iv} p_l^e}{|\Sigma|} < 0
\]

\[
\hat{v}_e = -\frac{U_{iv} p_l^e}{|\Sigma|} < 0, \text{ if } i \text{ and } v \text{ are substitutes}
\]

\[
\hat{v}_e = -\frac{U_{iv} p_l^e}{|\Sigma|} > 0, \text{ if } i \text{ and } v \text{ are complements}
\]
References


