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Upfront Payments and Listing Decisions*

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Abstract

We analyze the listing decisions of a retailer who may ask her suppliers to make upfront payments in order to be listed. We consider a sequential game with upfront payments being negotiated before short-term delivery contracts. We show that the retailer is more likely to use upfront payments the higher her bargaining power and the higher the number of potential suppliers. Upfront payments tend to lower the number of products offered by the retailer when the products are rather close substitutes. However, upfront payments can increase social welfare if they ameliorate inefficient listing decisions implied by short-term contracts only.

JEL-Classification: L14, L42

Keywords: Buyer Power, Upfront Payments, Retailing
1 Introduction

During the last decades the retail industry has witnessed significant changes. Both the growing concentration among retailers as well as the ongoing consolidation process towards fewer but larger store outlets have significantly altered the vertical relations in the grocery channel (OECD 1998, EU 1999, FTC 2001). Large retailers have become the essential intermediaries between manufacturers and consumers. Unless manufacturers have not passed “the decision-making screen of a single dominant retailer” (FTC 2001), their products are not sold in final consumer markets. Retailers have therefore gained significant gatekeeper control to final consumer markets. Additionally, the high frequency of new product launches has intensified competition among suppliers for getting access to retail shelf space.\(^1\) As a result, bargaining power has shifted in favor of retailers which enables them to set up rather complex delivery contracts.\(^2\) This holds especially for new products where “…retailers and suppliers negotiate over the amount of upfront payments, introductory allowances per unit, marketing funds, and other special funds such as those used for in-store displays and demonstrations, couponing and customers’ saving cards” (FTC 2003). However, suppliers are also charged to keep already established goods on the shelf. All these different types of fees and allowances are lump-sum payments which are paid upfront.\(^3\) Considering the competitive and allocative effects of upfront payments, there is a contentious debate to what extent they may harm competition, consumers and suppliers.\(^4\) Despite the growing literature on the pro- and anti-competitive effects of upfront payments, however, no consensus concerning the pretended anti-competitive effects of upfront payments has been reached until now.

Our model focuses on the interdependence between the listing decision of a retailer and her incentives to use upfront payments in order to extract surplus from her suppliers. Assuming that contracts between the retailer and her suppliers have to be negotiated, we show that a retailer is more likely to use upfront payments the more buyer power she has vis-à-vis suppliers. The

\(^1\) For example, the German food industry launches about 150,000 products every year, while retail assortments consist merely of 6,200 to 30,000 products in average (see Lebensmittelzeitung 2005). Similar data for the U.S. is quoted by Shaffer (2005).

\(^2\) Both the trade press as well as the academic literature have documented a shift of relative bargaining power in the grocery channel in favor of retailers (see Lariviere and Padmanabhan 1997, Sullivan 1997).

\(^3\) A survey on the actual debate on slotting allowances is provided by Klein and Wright (2007).

retailer’s buyer power increases in the number of her potential suppliers, the substitutability of suppliers’ products, and the extent of her exogenously given bargaining power. Furthermore, upfront payments can increase social welfare if they ameliorate inefficient listing decisions implied by short-term contracts only.

We consider a monopolistic retailer and a potentially high number of upstream suppliers. Before the retailer negotiates delivery contracts with a subset of suppliers she may also ask her suppliers to make upfront payments in order to be listed. Whereas annual listing decisions serve to determine the suppliers whose products are to be offered, terms of trade are determined for shorter time periods and can be readjusted during the period products are listed. While listing decisions and the associated upfront payments refer to long-term contracts, delivery contracts are determined for shorter time periods and can be readjusted during the period products are listed. This two-stage setting fits the bargaining procedures typically observed in intermediate good markets.

Furthermore, we assume that the retailer is not in a position to make take-it or leave-it offers. Instead, long-term contracts and the associated upfront payments as well as short-term delivery contracts rely on negotiations between the retailer and her suppliers where gains from trade have to be shared. This approach is based on the observation that there are several reasons which restrict the bargaining power of a retailer. For example, after having built her sales outlet, the retailer is committed to a particular assortment structure. Sales counters for goods that need special treatments, such as frozen food, dairy products, fresh fish and meat, can not be built up or reduced in the short run. There also exist “focal goods” and well-known brands the retailer has to offer in order to attract consumers. Hence, although the retailer can use her gatekeeper control to final consumer markets in order to extract surplus form her suppliers, she may not be able to fully extract all surplus. We therefore suppose that both short-term and long-term contracts rely on negotiations between the retailer and her suppliers. However, the retailer can decide whether to use long-term contracts or not.

Our model shows that upfront payments gain in importance when the retailer’s buyer power increases. Furthermore, upfront payments significantly alter the retailer’s listing decision. Without upfront payments the retailer tends to choose an inefficiently high number of products if products are rather close substitutes. Upfront payments induce the retailer to decrease the num-
ber of products significantly. The same results hold vice versa if the substitutability between the suppliers’ products is rather low. In this case, the retailer will extend her assortment if she uses upfront payments. These observations are based on the fact that upfront payments allow the retailer to extract parts of suppliers’ rents. The retailer’s listing decisions thus tend to maximize overall profits when upfront payments are used.

Our paper contributes to the expanding literature on upfront payments and retailer’s listing policy. Aydin and Hausman (2007) consider a setting with a single retailer and a single multi-product manufacturer. They find that due to double marginalization the industry-optimal level of variety is higher than that the retailer would offer. The retailer increases her offered variety and thus resells the industry-optimal assortment if she demands upfront payments for each additional product to be listed. Slotting allowances can also be interpreted as a signaling (Kelly 1992, Chu 1992 and Larivière and Padmanabhan 2001) or screening device (DeVuyst 2005 and Sullivan 1997) which constitutes an efficient mechanism for allocating limited shelf space. Suppliers expecting their products to be successful on downstream markets are willing to pay higher slotting fees than those expecting their products to fail. Suppliers may also use upfront payments in order to raise rival’s costs (Shaffer 2005). Furthermore, there are several papers which explicitly focus on the competitive effects of upfront payments imposed by retailers. For instance, Shaffer (1991) considers a model with upfront payments leading to higher wholesale prices which in turn imply that downstream competition is softened. In Marx and Shaffer (2007) competing retailers offer a common supplier a three-part tariff which entails a slotting fee and a two-part delivery tariff. By offering the manufacturer its own monopoly profit as a compensation for the upfront payment, a retailer can induce the manufacturer to rely on exclusive dealing which in turn reduces downstream competition. The exclusionary effect of upfront payments is based on the assumption that the retailer can resign to buy positive quantities if the manufacturer has signed a delivery contract with other retailers. Rey et al. (2006) use a similar model but they assume that tariffs can be conditioned on actual trade. Their results show that conditional three-part tariffs allow firms to sustain monopoly profits in a common agency situation. While upfront payments do not imply downstream exclusion, they lead to a fully collusive outcome in downstream markets. Our paper differs from both Marx and Shaffer (2007) and Rey et al. (2006) since we assume an inverted market structure with one monopolistic retailer and a
potentially high number of suppliers. Furthermore, we assume that neither the retailer nor the suppliers have take-it or leave-it power and that contracts have to be negotiated.

The model closest to ours is Marx and Shaffer (2004). They show that upfront payments may induce a retailer to limit her shelf space in order to capture more of the suppliers’ profits. Considering two suppliers and sequential Nash bargaining between the retailer and the suppliers, the model of Marx and Shaffer implies that upfront payments can mirror the outcome of an auction for getting access to limited shelf space. However, upfront payments and the induced limitation of shelf space are unprofitable for the retailer when her bargaining power is sufficiently high. In contrast to this result, our framework implies that upfront payments are more likely to be used by the retailer the higher her bargaining power.

With respect to the bargaining on upfront payments, our work is similar to de Fontenay and Gans (2003) who model the employment decision of a firm taking into account wage bargaining in labor markets. Assuming that already employed workers are immediately replaceable by outside workers, they show that underemployment constitutes a profit-maximizing strategy for the firm because the increased pool of potential workers outside can be used for squeezing inside wages. This result contrasts the insights gained by Stole and Zwiebel (1996), who show that firms - given that workers are not replaceable - tend to hire an inefficiently high number of workers in order to overcome their hold-up power. By considering different bargaining frameworks for upfront payments and short-term delivery contracts, our model combines the approaches of de Fontenay and Gans (2003) and Stole and Zwiebel (1996).

The remainder of the paper is organized as follows: In Section 2 we first introduce our model and explain the different bargaining stages. In Section 3 we consider optimal consumer prices and the different contracts between the retailer and her suppliers. Section 4 focuses on the listing decision and the impact which long-term contracts have on social welfare. To illustrate our results, we consider a numerical example in Section five. The final section summarizes the main findings.

2 The Model

We consider a model with homogeneous consumers, one retailer and a set $S_N = \{1, 2, \ldots, N\}$ of manufacturers $i = 1, 2, \ldots, N$ producing one product each. All products are supposed to be
substitutable and the retailer decides which and how many products \( n \leq N \) she distributes to final consumers. Let \( S_n \subseteq S_N \) with \( |S_n| = n \leq N \) denote the set of suppliers whose products are resold by the retailer. Employing the generalized Dixit utility function, consumers’ utility can be written as\(^5\)

\[
U(\cdot) = \alpha \sum_{i \in S_n} q_i - \frac{1}{2} \left( \sum_{i \in S_n} q_i^2 + 2\sigma \sum_{i \in S_n} \sum_{j \in S_n, j \neq i} q_i q_j \right) - \sum_{i \in S_n} p_i q_i, \tag{1}
\]

where \( q_i \) and \( p_i \) denote the quantity and the price of a specific good \( i \). While \( \alpha \) indicates the consumers’ reservation price, substitutability between goods is measured by \( \sigma \in [0, 1] \). The number of consumers is normalized to one.

We assume that the suppliers bear no fixed costs and have constant marginal costs which we normalize to zero. In contrast, the retailer incurs fixed costs \( c(n) \) for the maintenance of outlet space and investments for in-store facilities like shelves, freezer and sales counters. We assume that these costs are increasing and strictly convex in \( n \):\(^6\)

\[
c'(n), c''(n) > 0 \text{ and } c''(n)/c'(n) > (1 - 2n)/(n - n^2). \tag{2}
\]

We distinguish two different types of contracts between the retailer and her suppliers. First, there are short-term contracts which specify the conditions under which the retailer can buy the products from the respective supplier. These short-term delivery contracts entail two-part tariffs with a wholesale price \( w_i \) and a fixed fee \( F_i \). We assume bilateral negotiations taking place simultaneously, whereas we focus on efficient bargaining.

The second kind of contracts are long-term contracts which entail upfront payments to be paid by the suppliers. Long-term contracts are negotiated before short-term contracts and serve as a commitment device for the retailer. That is, given the retailer has agreed on long-term contracts with a set \( S_n \subseteq S_N \) of suppliers, she can enter into short-term contracts with suppliers \( i \in S_n \) only. Again, we assume bilateral bargaining and that negotiations take place simultaneously. In contrast to the short-term contracts, however, we assume that renegotiations

\(^5\)In order to simplify the notation, we omit the arguments of the functions where this does not lead to any confusion.

\(^6\)Strict convexity can be justified by the observation that opportunity costs for the use of real estate are increasing.
are possible.

Summarizing, we analyze the following four-stage game which we solve by backward induction: In the first stage, the retailer decides about the number of products \( n \) she offers and whether or not she uses long-term contracts. If long-term contracts are used, they are negotiated in the second stage. In the third stage, short-term delivery contracts are negotiated. Finally, the retailer sets prices \( p_i \) for all products she offers.

### 3 Consumer Prices and Contracts

Starting with the market stage, we get consumers’ demand by maximizing (1) with respect to all quantities \( q_i \) with \( i \in S_n \). Solving the respective first-order conditions and assuming interior solutions, optimal demand \( q_i(p_i, n, \cdot) \) is given by

\[
q_i(p_i, \cdot) = \frac{\alpha - p_i - \left[ \alpha + (n - 2)p_i - \sum_{j \in S_n, j \neq i} p_j \right] \sigma}{(1 - \sigma) \left[ 1 + (n - 1)\sigma \right]}.
\]

Using (3) and taking into account the payments induced by short-term delivery contracts, the (gross) profits \( \Pi^R_i \) and \( \Pi^S_i \) of the retailer and the supplier \( i \in S_n \) are given by

\[
\Pi^R_i(\cdot) = \sum_{i \in S_n} (p_i - w_i)q_i(p_i, \cdot) - \sum_{i \in S_n} F_i
\]

and

\[
\Pi^S_i(\cdot) = (w_i - c_i)q_i(p_i, \cdot) + F_i.
\]

Note that \( \Pi^R \) and \( \Pi^S_i \) do neither cover retailer’s cost \( c(n) \) nor possible upfront payments implied by long-term contracts. Maximizing (4) with respect to the prices \( p_i \) and using (3), it is easy to show that optimal prices \( p_i^*(w_i) \) are given by

\[
p_i^* = \frac{a + w_i}{2}.
\]
Substituting $p_i^*$ into the profit functions (4) and (5), let $\Pi^R(n, \cdot)$ and $\Pi^S_i(\cdot)$ denote the reduced profit functions of the retailer and the suppliers respectively:

$$\Pi^R(n, \cdot) = \sum_{i \in S_n} (p_i^* - w_i)q_i(p_i^*, \cdot) - \sum_{i \in S_n} F_i$$

(7)

and

$$\Pi^S_i(\cdot) = (w_i - c_i)q_i(p_i^*, \cdot) + F_i.$$  

(8)

### 3.1 Short-Term Contracts

Turning to the third stage of the game and thus to the negotiation on short-term delivery contracts, we assume that the retailer selects a set $S_n \subseteq S_N$ of suppliers with $|S_n| = n$ whose products she resells to final consumers. With each $i \in S_n$ the retailer negotiates a simple two-part tariff with a wholesale price $w_i$ and a fixed fee $F_i$. Negotiation takes place simultaneously. Using the generalized Nash bargaining solution, the wholesale price $w_i$ is determined in order to maximize the joint profit of the retailer and each supplier. Incremental gains from trade are shared by the fixed fee $F_i$. More precisely, the retailer and each supplier receive their disagreement payoff plus a share of the joint profit according to the weights $\delta \in (0, 1)$ and $1 - \delta$ respectively. These weights reflect possible asymmetries in the bargaining procedure, in retailer’s and suppliers’ time preferences or their beliefs about potential negotiation breakdowns.\textsuperscript{7}

For simplicity, we assume that suppliers’ disagreement payoffs are equal to zero. Furthermore, contracts are binding and not contingent on other contracts (see Horn and Wolinsky 1988a,b, McAfee and Schwartz 1994, O’Brien and Shaffer 1998). Accordingly, we do not allow for renegotiation in cases of negotiation breakdown with one supplier. The bargaining solution between each retailer-supplier pair $R, i \in S_n$ can then be characterized by the solution of

$$\max_{w_i, F_i} \left[ \Pi^R(n, \cdot) - \Pi^R_{-i}(n, \cdot) \right]^{\frac{\delta}{\delta}} \left[ \Pi^S_i(\cdot) \right]^{1-\delta},$$

(9)

where

$$\Pi^R_{-i}(n, \cdot) := \sum_{j \in S_n, j \neq i} (p_j^* - w_j)q_j(p_j^*, n - 1, \cdot) - \sum_{j \in S_n, j \neq i} F_j$$

\textsuperscript{7}For a detailed discussion see Binmore et al. (1986).
denotes retailer’s profit if the negotiation with one particular supplier \( i \in S_n \) fails. Differentiating (9) with respect to \( F_i \) and \( w_i \), we get

\[
(1 - \delta) \left[ \Pi^{R*}(n, \cdot) - \Pi^{R*}_{-i}(n, \cdot) \right] - \delta \Pi^{S*} = 0
\]

(10)

and

\[
(1 - \delta) \left[ \Pi^{R*}(n, \cdot) - \Pi^{R*}_{-i}(n, \cdot) \right] \frac{\partial \Pi^{S*} (\cdot)}{\partial w_i} + \delta \Pi^{S*} \frac{\partial \Pi^{R*} (n, \cdot)}{\partial w_i} = 0
\]

(11)

Following Chipty and Snyder (1999) we assume that agents believe that efficient trade will occur between the retailer and all other suppliers. Since these beliefs will be justified in equilibrium, we can solve the system of equations (10)—(11) simultaneously for all \( F_i \) and \( w_i \forall i \in S_n \). Using symmetry, we get

\[
w_i^* = c_i = 0 \quad \text{and} \quad F^*(n, \cdot) = \frac{a^2(1 - \delta)(1 - \sigma)}{4[1 + (n - 2)\sigma][1 + (n - 1)\sigma]}. \]

(12)

Employing (12), the reduced profit functions of the retailer and the suppliers without considering long-term contracts, i.e. \( \Pi^{R*}_i (n, \cdot) \) and \( \Pi^{S*}_i (\cdot) \), can be written as

\[
\Pi^{R*}(n, \cdot) = \sum_{i \in S_n} (p_i^* - w_i^*) q_i - nF^*(n, \cdot) = R(n, \cdot) (1 - \psi(n, \cdot))
\]

(13)

and

\[
\Pi^{S*}_i (\cdot) = F^*(n, \cdot) = \frac{1}{n} R(n, \cdot) \psi(n, \cdot) \quad \text{for all} \ i \in S_n,
\]

(14)

where \( R(n, \cdot) \) and \( \psi(n, \cdot) \) are given by

\[
R(n, \cdot) := \frac{\alpha^2 n}{4(1 + (n - 1)\sigma)} \quad \text{and} \quad \psi(n, \cdot) := \frac{(1 - \delta)(1 - \sigma)}{1 + (n - 2)\sigma}.
\]

(15)

Analyzing (13) and (14) simple comparative statics with respect to \( n \) leads to:

**Lemma 1** The reduced profit function \( \Pi^{R*}_i (n, \cdot) \) is strictly increasing in \( n \), while \( \Pi^{S*}_i (\cdot) \) and thus \( F^*(n, \cdot) \) are strictly decreasing in \( n \). Furthermore, considering the aggregate fixed-fee payments
by the retailer, we get

\[
\frac{\partial nF^*(n, \cdot)}{\partial n} > 0 \iff n \leq n^k(\sigma) := \begin{cases} 
\frac{1}{\sigma} \sqrt{(1 - \sigma)(1 - 2\sigma)} & \text{for } \sigma \in (0, 0.5) \\
0 & \text{else} \end{cases}
\]

**Proof.** These results can be proved by differentiating (13) and (14) with respect to \( n \) and taking into account \( \delta, \sigma \in (0, 1) \) as well as \( n \geq 1 \).

The intuition for these results relies on the fact that aggregate demand increases in the number of products, while substitutability of products implies that suppliers’ marginal contributions are decreasing in \( n \). Hence, an increase in \( n \) has two positive effects for the retailer: First, her revenues will increase; second, the fixed \( F^* \) payments will decrease.

### 3.2 Long-Term Contracts

Before the retailer starts to negotiate short-term delivery contracts with a subset of suppliers, she can also decide whether to employ long-term contracts in order to get upfront payments from her suppliers. In contrast to short-term delivery contracts, long-term contracts serve as a commitment device for the retailer. The agreement on long-term contracts enforces the retailer to negotiate delivery contracts with the respective suppliers. Correspondingly, upfront payments are tantamount to an assurance for suppliers to enter into negotiations on delivery contracts. At the same time, long-term contracts allow the retailer to exploit her gatekeeper position by reaping at least some of the suppliers’ profits. However, we assume that the retailer is not able to extract all surplus. Long-term contracts and the implied upfront payments are presumed to be based on negotiations between the retailer and her suppliers.

Considering the bargaining process on long-term contracts we follow the model of de Fontenay and Gans (2003). We assume that the retailer can immediately replace suppliers with whom negotiations on long-term contracts have failed. Let the initially selected suppliers \( i \in S_n \subseteq S_N \) be the insiders and the remaining \( N - n = N - |S_n| \) suppliers be the outsiders. If the retailer bargains over long-term contracts with the insiders and if negotiations with one of the \( i \in S_n \) outsiders fails, the retailer can start to negotiate with one of the remaining outsiders. Moreover, we assume that the retailer will never again enter into negotiations with those suppliers with whom negotiations have failed. Therefore, the number of outsiders is reduced by one, if negotiations...
with one of the insiders have failed. With \( |S_n| = N \) the retailer is not able to replace any of the initially selected suppliers. To illustrate the implied bargaining process suppose \( n = 1 \) and \( N = 2 \). If the retailer starts negotiations with one of the two suppliers and if this negotiation fails, she can immediately start to negotiate with the other supplier. However, with \( n = 2 \) the retailer cannot replace any supplier in the case of negotiation breakdown. Consequently, the higher \( N \) and the lower \( n \), the more credible the retailer can threaten to replace suppliers she bargains with. Thus, the retailer’s bargaining position is the weaker the lower the number of potential suppliers.

We assume that negotiations between the retailer and all inside suppliers \( i \in S_n \) are bilateral and take place simultaneously. Furthermore, we assume rational beliefs and focus on the Nash bargaining solution. The analysis is further simplified by the assumption of no renegotiations if \( n = N \) holds.\(^8\)

Starting with the case \( n = N \), where outside suppliers are lacking for immediate replacement in the case of negotiation breakdown, the upfront payment \( G_i \) of supplier \( i \) is determined by (see (13) and (14)):

\[
\max_{G_i} \left[ \Pi_i^{Rs}(N, \cdot) + G_i - \Pi_i^{Rs}(N - 1, \cdot) \right]^\delta \left[ \Pi_i^{Qs}(N, \cdot) - G_i \right]^{1-\delta}.
\]

Maximizing (16) with respect to \( G_i \), defining \( \Delta(n, \cdot) := \Pi_i^{Rs}(n, \cdot) - \Pi_i^{Rs}(n - 1, \cdot) \) and assuming symmetry leads to the following equilibrium payments \( G^*(N, n, \cdot) \)

\[
G^*(N, N, \cdot) = -(1 - \delta)\Delta(N, \cdot) + \delta F^*(N, \cdot).
\]

With \( n = N - 1 \), there is one outside supplier for immediate replacement. Thus, the retailer’s threat point in the initial negotiations is determined by \( \Pi_i^{Rs}(N - 1, \cdot) + (N - 1)G^*(N - 1, N - 1, \cdot) \). Since \( \Pi_i^{Rs}(N - 1, \cdot) \) does not change if an inside supplier is replaced by an outsider, the Nash bargaining solution can be determined by maximizing the following expression with respect to

---

\(^8\)This assumption allows us to avoid a rather complicated recursion problem but does not affect the main qualitative results of our model.
\( G_i \)

\[
\max_{G_i} \left[ G_i + \sum_{j \in S_i, j \neq i} G_j - (N - 1)G^*(N - 1, N - 1, \cdot) \right]^{\delta} \left[ \Pi_i^{Ss}(N - 1, \cdot) - G_i \right]^{1-\delta}.
\] (18)

Differentiating (18) with respect to \( G_i \) and using symmetry, we get

\[
-(1 - \delta)(N - 1) \left[ G_i - G^*(N - 1, N - 1, \cdot) \right] + \delta(F^*(N - 1, \cdot) - G_i) = 0.
\] (19)

Solving (19) for the equilibrium payment \( G^*(N, N - 1, \cdot) \) leads to

\[
G^*(N, N - 1, \cdot) = \frac{(1 - \delta)(N - 1)G^*(N - 1, N - 1, \cdot) + \delta F^*(N - 1, \cdot)}{(1 - \delta)(N - 1) + \delta}.
\] (20)

Increasing the difference between \( N \) and \( n \) further and solving the implied recursion formula for \( G^*(N, n, \cdot) \) yields

\[
G^*(N, n, \cdot) = F^*(n, \cdot) - (1 - \delta) \left[ \frac{n(1 - \delta)}{n(1 - \delta) + \delta} \right]^{N-n} [\Delta(n, \cdot) + F^*(n, \cdot)].
\] (21)

Employing (21), retailer’s profit with long-term contracts can be written as

\[
\Pi^{RL}(N, n, \cdot) = R(n, \cdot) [1 - \psi(n, \cdot) \mu(N, n, \cdot)]
\] (22)

with

\[
\mu(N, n, \delta, \sigma) := \left[ 1 + \frac{\delta}{\delta(n - 1) - n} \right]^{N-n} \left[ 1 + \frac{2\sigma(n - 1)(1 - \delta)}{1 + (n - 3)\sigma} \right].
\] (23)

Using \( \delta \in (0, 1) \) and analyzing \( \mu(N, n, \cdot) \) shows \( \mu(N, N, \delta, \sigma) > 1 \) and \( \lim_{N \to \infty} \mu(n, N, \delta, \sigma) = 0 \). Comparing (13) and (22), we therefore get that the retailer will never use long-term contracts if \( N = n \). On the other hand, the retailer will always benefit from long-term contracts if \( N \) tends to infinity as this implies that upfront payments are equal to the suppliers’ profits.\(^9\) Considering the impact of \( n \) and \( N \) more carefully, yields:

\(^9\) Note that one would obtain the same result if the retailer could make take-it or leave-it offers or if she could auction off access to her shelf space (provided that \( n < N \)).
Lemma 2 With $N > n \geq 2$ there exist a unique critical $\delta^k(N, n, \sigma)$ such that

$$G^*(N, n, \delta, \sigma) > 0 \Leftrightarrow \delta > \delta^k(N, n, \sigma).$$

Furthermore, $\delta^k(N, n, \sigma)$ decreases in $N$ while it increases in $\sigma$.

Proof. See appendix.

The retailer benefits from long-term contracts whenever her bargaining power is high enough. Furthermore, Lemma 2 shows that the greater the number of potential suppliers and the less substitutable their products the more attractive are long-term contracts for the retailer. Whereas an increase in the number of potential suppliers enhances the bargaining position of the retailer, a higher level of product substitutability strengthens the bargaining position of the suppliers. This is due to the fact that the fixed payments negotiated under short-term contracts are the lower the higher $\sigma$. While this decreases the suppliers’ willingness-to-pay for being listed, it also increases the retailer’s valuation of additional suppliers. Therefore, upfront payments tend to decrease in $\sigma$. Summarizing these results, we get:

**Proposition 1** The retailer can benefit from long-term contracts if and only if her bargaining power is high enough and if the number of potential suppliers exceeds the number of products which can be listed. Furthermore, the retailer is more likely to use long-term contracts, the less substitutable the suppliers’ products are.

Finally, it turns out that the following reformulation of Lemma 2 is quite helpful for the analysis of the retailer’s listing decision:

**Corollary 1** With $N > n \geq 2$ there exists an critical value $N^k(\sigma, \delta, n)$ such that $G^*(N, n, \delta, \sigma) > 0 \Leftrightarrow N > N^k(\sigma, \delta, n)$. Furthermore, $N^k(\sigma, \delta, n)$ decreases in $\delta$ while it increases in $\sigma$.

Proof. See appendix.

4 Assortment, Contracts and Social Welfare

Turning to the first stage, the retailer decides about the number of products she offers. Besides fixing her assortment, she also determines whether or not she will negotiate with her suppliers
about an upfront payment. We first analyze the optimal number of products the retailer offers to final consumers if the interaction between the retailer and her suppliers is based on short-term contracts only. Subsequently, we turn to the case where suppliers have to agree on upfront payments before they enter into negotiations on short-term delivery contracts. The comparison of the optimal listing decisions under both regimes shows that long-term contracts tend to reduce the number of products listed by the retailer if products are rather close substitutes. Furthermore, upfront payments are more likely to be used if the number of potential suppliers is high or if their bargaining power is low. While these results are in line with Proposition 1, it turns out that the substitutability between the suppliers’ products has ambiguous effects on the retailer’s decision to use long-term contracts. In fact, in Section 5 we will analyze a numerical example which shows that the retailer may well use long-term contracts only if the products are rather close substitutes. Finally, considering social welfare, long-term contracts are more likely to lead to socially more efficient listing decisions the higher the substitutability between the suppliers’ products and the higher the retailers’ bargaining power.

4.1 Short-term contracts

Considering short-term contracts only and taking into account the costs for providing shelf space, the maximization problem of the retailer is given by

\[
\max_n \Pi^R_s(n, \cdot) := \Pi^R_s(n, \cdot) - c(n) = R(n, \cdot) (1 - \psi(n, \cdot)) - c(n) \quad \text{s.t. } n \leq N.
\]  

(24)

Differentiating \(\Pi^R_s(n, \cdot)\) with respect to \(n\), the first-order conditions for (24) can be written as\(^{10}\)

\[
\Pi^R_{n s}(\cdot) \geq 0 \quad \text{and} \quad \Pi^R_{n s}(n, \cdot)(N - n) = 0.
\]  

(25)

\(^{10}\)For analytical purposes we ignore the integer constraint with respect to \(n\) in the following. We take \(n \in \mathbb{N}\) explicitly into account, when we analyze a numerical example (see Section 5).
Let \( n^s(\delta, \sigma, N) \) denote the solution of (25) and let \( n^*(\sigma, N) \) define the number of suppliers that maximizes industry profit, i.e.

\[
n^*(\sigma, N) := \arg \max_{n \leq N} [R(n, \cdot) - c(n)]. \tag{26}
\]

Comparing \( n^s(\delta, \sigma, N) \) and \( n^*(\sigma, N) \), we get:

**Proposition 2** If only short-term contracts are negotiated and \( n^*(\sigma, N) \leq N \), the retailer overlists as long as \( n^s(\sigma, \cdot) > n^k(\sigma) \), i.e. \( n^s(\delta, \sigma, N) > n^*(\sigma, N) \). With \( n^s(\sigma, \cdot) < n^k(\sigma) \) and \( n^*(\sigma, N) \leq N \), the retailer underlists, i.e. \( n^s(\delta, \sigma, N) < n^*(\sigma, N) \).

**Proof.** See appendix. ■

Proposition 2 shows that the retailer has a strong incentive to offer an inefficiently high number of products as long as the fixed costs to extend her outlet are sufficiently low and \( \sigma \) is high enough. Note that low (high) fixed costs imply \( n^s(\sigma, \cdot) > n^k(\sigma) \) \((n^s(\sigma, \cdot) < n^k(\sigma))\). Given low fixed costs and a high level of substitutability, the retailer benefits from the fact that the marginal contribution of each product and thus total payments to the suppliers decrease with each additional product. With high investment costs or highly differentiated products, i.e. sufficiently small \( \sigma \), the retailer underinvests. That is, the retailer has an incentive to reduce his assortment inefficiently since \( n < n^k(\sigma) \) implies that total payments to the suppliers are the higher the more products are listed.

### 4.2 Long-term contracts

Let \( \Pi^{RL}(N, n, \cdot) \) denote the retailer’s profit, when long-term contracts are used. Then, the maximization problem with respect to \( n \) can be written as

\[
\max_n \Pi^{RL}(N, n, \cdot) = \max_n \left[ \Pi^{RL}(N, n, \cdot) - c(n) \right] \tag{27}
\]

\[
= \max_n \left[ R(n, \cdot) (1 - \psi(n, \cdot)\mu(N, n, \cdot)) - c(n) \right] \text{ s.t. } n \leq N.
\]

Analyzing the first-order conditions for (27), i.e.

\[
\Pi^{RL}_n(\cdot) \geq 0 \text{ and } \Pi^{RL}_n(N, n, \cdot)(N - n) = 0 \tag{28}
\]
and letting $n^l(\sigma, \delta, N)$ denote the solution of (28), we obtain:

**Proposition 3** If long-term contracts are used, the retailer chooses

$$n^l(\sigma, \delta, N) \leq n^s(\sigma, \delta, N)$$

as long as $n^s(\sigma, \delta, N) \geq \max\{3, n^k(\sigma)\}$. With $n^s(\sigma, \delta, N) < n^k(\sigma)$ the retailer chooses $n^l(\delta, \sigma, N) > n^s(\sigma, \delta, N)$, as long as $N$ is high enough. Compared to efficient listing decisions, the retailer underlists, i.e. she chooses

$$n^l(\delta, \sigma, N) \leq n^*(\sigma, N),$$

as long as $n^*(\sigma, N) > n^k(\sigma)$ and $N$ large enough.

**Proof.** See appendix. ■

The implementation of upfront payments in intermediate goods markets can avoid potential overlisting that may occur if only short-term contracts are negotiated in intermediate goods markets. In fact, with $N$ high enough the retailer has an incentive to underlist in order to strengthen competition between suppliers for getting access to the retailer’s shelf space. While underlisting increases the fixed fee negotiated in the two-part tariff, it also increases the value of being listed and hence the upfront payment. This positive effect always dominates if $N$ is large enough. Note further, that the retailer has an incentive to expand her assortment under short-term contracts with $n^l(\cdot) \leq n^s(\cdot)$. Therefore, there is no commitment problem, when bargaining over long-term contracts is considered. With $n^l(\cdot) > n^s(\cdot)$, however, long-term contracts serve as a commitment device, which forces the retailer to bargain with all accepted suppliers.

### 4.3 Choice of contracts

Using $n^s(\sigma, \delta, N)$ and $n^l(\sigma, \delta, N)$ we now turn to the retailer’s decision of whether or not she will use long-term contracts. While the use of long-term contracts is more likely the higher $N$ and $\delta$ (see Proposition 1), the effect of $\sigma$ is less clear cut. Although upfront payments decrease in $\sigma$, the retailer can balance this negative effect by reducing $n$. Therefore, with an endogenously chosen number of products long-term contracts may be more beneficial for the retailer the higher the substitutability between the suppliers’ products.
Corollary 1 and the fact that $\Pi^{RI}(N, n^l(\cdot), \cdot) - c(n^l(\cdot))$ is monotonically increasing in $N$ imply that there must exist a critical value $N^*(\delta, \sigma)$ such that\footnote{Differentiating $\Pi^{RI}(N, n^l(\cdot), \cdot) - c(n^l(\cdot))$ with respect to $N$ and using the envelope theorem, it follows immediately that the retailer’s profit is monotonically increasing in $N$.}

$$\Pi^{RI}(N^*(\cdot), n^l(\cdot, N^*(\cdot)), \cdot) - c(n^l(\cdot, N^*(\cdot))) = \Pi^{Rs}(n^s(\cdot), \cdot) - c(n^s(\cdot))$$

(29)

and

$$\Pi^{RI}(N, n^l(\cdot, N), \cdot) - c(n^l(\cdot, N)) > \Pi^{Rs}(n^s(\cdot), \cdot) - c(n^s(\cdot))$$

(30)

for all $N > N^*$. Using (29)—(30) and analyzing the impact of $\delta$ on $N^*(\delta, \sigma)$ and thus on the retailer’s choice of contracts, we get:

**Proposition 4** The retailer is more likely to use long-term contracts, the higher the number of potential suppliers. Moreover, with $n^s(\sigma, \cdot) > n^k(\sigma)$ the critical number $N^*(\cdot)$ decreases in $\delta$, as long as $n^s(\cdot)$ is large enough.

**Proof.** See appendix. ■

While proposition 4 focuses on the impact of $N$ and $\delta$, the degree of substitutability between suppliers’ products affects the retailer’s contract decision ambiguously. This is due to the observation that

$$\frac{\partial}{\partial \sigma} R(n, \cdot) < 0, \quad \frac{\partial^2}{\partial \sigma \partial n} R(n, \cdot) < 0 \quad \text{and} \quad \frac{\partial}{\partial \sigma} [R(n, \cdot) \psi(n, \cdot)] < 0$$

(31)

as well as (see (44) in the appendix)

$$\frac{\partial}{\partial \sigma} \mu(N, n, \cdot) > 0 \quad \text{for} \quad N > n \geq 2.$$  

(32)

hold. Thus, while an increase of $\sigma$ reduces the retailer’s revenues, it has an additional negative effect on her profit when long-term contracts are used. Hence, $N^*(\delta, \sigma)$ increases in $\sigma$ if all other effects are ignored. However, assuming $n^s(\cdot) > n^l(\cdot)$, (31) points to a negative correlation between $N^*(\delta, \sigma)$ and $\sigma$. In Section 5, we analyze an example where $\partial N^*(\delta, \sigma)/\partial \sigma < 0$ holds which also implies that long-term contracts are more likely to be beneficial for the retailer the higher $\sigma$.  

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4.4 Social welfare

In order to analyze the implications of long-term for social welfare, we define social welfare as the sum of consumers’ and firms’ surplus. Denoting $U^*(\sigma, n, N)$ consumers’ indirect utility function and using (3) and (15), social welfare $W(\sigma, n, N)$ can be written as

$$W(\sigma, n, N) = U^*(\cdot) + R(n, \cdot) - c(n) = \frac{3}{2} R(n, \cdot) - c(n).$$  \hspace{1cm} (33)

Maximizing $W(\cdot)$ with respect to $n$ and defining the number of suppliers that maximizes social welfare, i.e.

$$n^w(\sigma, N) := \arg \max_{n \leq N} \left[ \frac{3}{2} R(n, \cdot) - c(n) \right],$$

it follows immediately that social welfare is maximized by a higher number of suppliers than industry profit, i.e. $n^w(\sigma, N) \geq n^*(\sigma, N)$. Furthermore, Proposition 2 implies $n^w(\sigma, N) \geq n^*(\sigma, \delta, N)$, whenever $n^*(\cdot) < n^k(\sigma)$. That is, if costs are sufficiently high, i.e. $n^*(\cdot) < n^k(\sigma)$, and only short-term contracts are negotiated, the number of products listed by the retailer undercuts the socially optimal number of products.

The relation between $n^w(\cdot)$ and $n^s(\cdot)$ is ambiguous for low cost, i.e. $n^*(\cdot) > n^k(\sigma)$. Comparing the respective first-order conditions for $n^w(\cdot)$ and $n^s(\cdot)$ yields

$$n^w(\sigma, N) \geq n^s(\sigma, \delta, N) \iff \delta \geq \delta^w(n^s(\cdot), \sigma) \text{ if } n^s(\cdot) > n^k(\sigma)$$

with \( \delta^w(n^s(\cdot), \sigma) := 1 + \frac{(1 + (n^s(\cdot) - 2)\sigma)^2}{2(1 - \sigma(3 + (n^s(\cdot)^2) - 2)\sigma)}. \) \hspace{1cm} (34)

While (34) indicates that short-term contracts may induce the retailer to choose a socially inefficient high number of suppliers, it also shows that socially inefficient overinvestment only occurs, if the retailer’s bargaining power is rather low. More precisely, it is easy to show that

$$\delta^w(n^s(\cdot), \sigma) \in \left[ \frac{1}{2}, 1 \right] \text{ for } \sigma \geq \frac{1}{2} \text{ and }$$

$$\delta^w(n^s(\cdot), 1) = 1 - \frac{(n^s(\cdot) - 1)^2}{2n^s(\cdot)^2}. \hspace{1cm} (35)$$

Considering $\sigma < 1/2$, (34) implies that $\delta^w(n^s(\cdot), \sigma)$ tends to $-\infty$ as $n^s(\cdot) > n^k(\sigma)$ approaches
$n^k(\sigma)$. Furthermore, we get

$$\delta^w(n^s(\cdot), \sigma) > 0 \Leftrightarrow n^s(\cdot) > n^c(\sigma) \text{ and}$$

$$\delta^w(n^s(\cdot), \sigma) < \frac{1}{2} \forall \ n^s(\cdot) > n^c(\sigma) \tag{37}$$

with $n^c(\sigma) := \frac{1}{\sigma} \left[ 1 + \sqrt{4(2 + \sigma(6\sigma - 7))} \right] - 2 > n^k(\sigma) \tag{38}$

Therefore, although socially inefficient overlisting is possible for $\sigma < 1/2$, it never occurs if $\delta \geq 1/2$, if $\sigma$ is small enough or if retailer’s costs for offering additional products are such that $n^s(\cdot)$ is lower than the critical number $n^c(\sigma)$.

Combining these results with Proposition 3 reveals that long-term contracts and the implied listing decisions are detrimental for social welfare, whenever $n^w(\sigma, N) \geq n^s(\delta, \sigma, N)$ and $n^s(\delta, \sigma, N) \geq n^l(\delta, \sigma, N)$. On the other hand, long-term contracts can enhance social welfare if either socially inefficient overlisting is avoided, i.e. $n^w(\sigma, N) < n^s(\delta, \sigma, N)$ and $n^s(\delta, \sigma, N) > n^l(\delta, \sigma, N)$, or if we have $n^s(\delta, \sigma, N) < n^s(\sigma, N)$ and $n^s(\delta, \sigma, N) < n^l(\delta, \sigma, N)$.

Analyzing the first case more carefully, note that

$$\lim_{\sigma \to 1} n^w(\cdot) = \lim_{\sigma \to 1} n^s(\cdot) = 1. \tag{39}$$

Taking into account the integer constrained $n \in \mathbb{N}$ and comparing total payments to the suppliers, when the number of suppliers is increased from 1 to 2, we get

$$\lim_{\sigma \to 1} [F^*(1, \cdot) - 2F^*(2, \cdot)] <$$

$$\lim_{\sigma \to 1} [[F^*(1, \cdot) - G^*(N, 1, \cdot)] - 2 [F^*(2, \cdot) - G^*(N, 2, \cdot)]] \tag{40}$$

for all $N \geq 2$. While an increase of the number of suppliers from 1 to 2 decreases the total payments under short-term contracts, the use of upfront payments implies a lower reduction of overall payments. In view of (39), we thus get that upfront payments can avoid socially inefficient overlisting if the suppliers’ products are rather close substitutes and if the marginal costs of increasing shelf space are not too high.

Finally, with $n^s(\delta, \sigma, N) < n^s(\sigma, N)$ short-term contracts lead to socially inefficient underlisting which can be ameliorated by the use of upfront payments as long as $N$ is high enough.
(see Proposition 3).

5 Numerical Example

In order to illustrate the above results more explicitly, we examine a numerical example. Let consumers’ willingness to pay be $\alpha = 10$ and assume that the retailer’s costs $c(n)$ are given by $c(n) = n^2/10$. Considering the cases $N = 20$ and $N = 40$ allows us to point out the impact of $N$. Furthermore, we take the integer constraint $n \in \mathbb{N}$ explicitly into account.

Assuming $\delta = 0.5$, Figure 1 shows the optimal number of products listed, i.e. $n^s(\delta, \sigma)$ and $n^l(N, \delta, \sigma)$ with $N = 20$ and $N = 40$. Obviously, the use of long-term contracts results in a reduced number of products listed by the retailer, whereas the difference to the number of products accepted under short-term contracts only decreases the more substitutable the products are. Likewise, $n^l$ approaches $n^s$, the more potential suppliers are available for being listed.

![Figure 1: Assortment Decisions With and Without Long-Term Contracts](image)

Considering the use of long-term contracts and comparing the retailer’s profits with and without long-term contracts, it turns out that the critical value $N^*(\delta, \sigma)$ (see (29)) is decreasing in $\delta$ for all $\sigma$. Furthermore, for given $N$ we can define a threshold $\delta^{ls}(\sigma, N)$ such that

$$
\Pi^{Rl}(N, n^l(\cdot, N), \cdot) - c(n^l(\cdot, N)) \geq \Pi^{Rs}(n^s(\cdot), \cdot) - c(n^s(\cdot)) \text{ for } \delta \geq \delta^{ls}(\sigma, N). 
$$

(41)

Figure 2 shows $\delta^{ls}(\sigma, N)$ for $N = 20$ and $N = 40$ as well as the impact of long-term contracts on social welfare.
Note first that
\[ \text{sign} \frac{\partial}{\partial \sigma} \delta_{ls}(\sigma, N) = \text{sign} \frac{\partial}{\partial \sigma} N^* \left( \delta_{ls}(\cdot), \sigma \right) < 0, \]
since \( \delta_{ls}(\sigma, N) \) is decreasing in \( \sigma \). Hence, the retailer is more likely to choose long-term contracts the higher her bargaining power and the higher the degree of substitutability between the suppliers’ products. While these results are based on endogenously chosen \( n^l \) and \( n^s \), the results summarized in Proposition 1 are obtained for given \( n \).

Turning to social welfare, the shaded areas in Figure 2 indicate the parameter constellations under which social welfare is higher if the retailer’s listing decisions are based on long-term contracts. Social welfare is always higher with long-term contracts if \( \sigma \) is large enough, i.e. if \( \sigma > 0.83 \). For lower values of the \( \sigma \), the bargaining power of the retailer has to be high enough in order to ensure that her listing decisions are more efficient with long-term contracts as compared to her decisions with short-term contracts only. These results are in accordance with our previous discussion. That is both a high bargaining power of the retailer or a high degree of product substitutability imply that socially inefficient overlisting decisions under short-term contracts are avoided by the use of long-term contracts.
6 Conclusion

In this paper we analyzed the use of upfront payments by a downstream retailer and considered their impact on the retailer’s listing decision. In the context of a non-cooperative bargaining framework, we analyzed a two-stage negotiation process where the retailer first decides how many products she lists and whether or not she employs long-term contracts associated with upfront payment to be made by suppliers. In the second stage the retailer and the suppliers negotiate on short-term delivery contracts which are based on non-linear tariffs. Within this bargaining process we identified two countervailing effects that affect the retailer’s listing decision. If only short-term delivery contracts are negotiated and if the suppliers’ products are close substitutes, the retailer has an incentive to expand her assortment in order to reduce the marginal contribution of each supplier. This strategic effect is reversed if the retailer’s costs for listing additional products are such that the number of products is rather small and if the substitutability between the products is relatively low. Upfront payments lead to opposite strategic incentives for the retailer. With close substitutes, the retailer tends to decrease the number of products she can list in order to increase the upfront payment she gets. The lower the number of suppliers, the higher their profits and, thus, the higher is their willingness to pay to get access to the retailers’ shelf space. On the other hand, if products are rather imperfect substitutes and if costs for providing shelf space are relatively high, upfront payments can increase the number of products listed by the retailer. These results combine the insights of de Fontenay and Gans (2003) and Stole and Zwiebel (1996). Our model extends the approach of de Fontenay and Gans (2003) as we consider a two-stage bargaining procedure where terms of trade are negotiated after potential suppliers have paid for the right to enter negotiations on delivery contracts.

Upfront payments are more likely to be used by the retailer the higher the buyer power the retailer has vis-à-vis her suppliers. That is, upfront payments are more likely to be beneficial for the retailer the higher her bargaining power, the higher the number of potential suppliers and the lower the degree of differentiation between the suppliers’ products. With respect to social welfare, we show that the use of upfront payments is socially beneficial if suppliers’ products are either highly substitutable or if products are rather imperfect substitutes. While long-term contracts can avoid socially inefficient overlisting induced by short-term contracts in the first case, they ameliorate socially inefficient underlisting in the second case. Apart from these cases,
long-term contracts are socially detrimental as they induce the retailer to inefficiently reduce
the extent of her retail assortment.

Considering the debate on the assessment of upfront payments, our results support a rule-of-
reason approach. While upfront payments can reduce social welfare, they are socially beneficial
if highly substitutable products, like dairy products, are concerned or if the costs for providing
shelf space are rather high. Furthermore, upfront payments are more likely to increase social
welfare, the higher the retailer’s buyer power. In view of the ongoing concentration process in
the retail industry and the implied shift of bargaining power toward retailers, upfront payments
thus tend to lead to socially more efficient listing decisions.

Appendix

Proof of Lemma 2

First note that we have $G^*(N, n, \cdot) \geq 0 \Leftrightarrow \Pi(N, n, \cdot) \geq \Pi^R(N, n, \cdot) \Leftrightarrow \mu(N, n, \cdot) \leq 1$.
Assuming $n \geq 2$, using (23) and taking limits show
\[
\lim_{\delta \searrow 0} \mu(N, n, \delta, \sigma) > 1 \text{ and } \lim_{\delta \nearrow 1} \mu(N, n, \delta, \sigma) = 0. \tag{43}
\]
Since $\mu(N, n, \delta, \sigma)$ is continuous in $\delta$, there must exists a $\delta^k(N, n, \sigma)$ such that $\mu(N, n, \delta^k(\cdot), \sigma) = 1$. Taking logs and differentiating $\mu(N, n, \delta, \sigma)$ with respect to $\delta$, $\sigma$ and $N$ leads with $N > n \geq 2$
and $\delta \in (0, 1)$ to:
\[
\frac{\partial}{\partial \delta} \log \mu(N, n, \cdot) = \frac{N - n}{(1 - \delta)(n - 1) + \sigma} - \frac{2(n - 1)\sigma}{1 - [5 + 2\delta(n - 1) - 3\sigma] \sigma} < 0 \tag{44}
\]
\[
\frac{\partial}{\partial \sigma} \log \mu(N, n, \cdot) = -\frac{2(1 - \delta)(n - 1)}{[1 + (n - 3)\sigma][-1 + [5 + 2\delta(n - 1) - 3\sigma] \sigma]} > 0 \tag{45}
\]
\[
\frac{\partial}{\partial N} \log \mu(N, n, \cdot) = \log \left[1 + \frac{\delta}{\delta(n - 1) - n}\right] < 0 \tag{46}
\]
While (43) implies that $\delta^k(\cdot)$ is unique, (44)—(46) and the implicit function theorem lead to the
comparative static properties of $\delta^k(\cdot)$, i.e. $\partial \delta^k(\cdot)/\partial N < 0 < \partial \delta^k(\cdot)/\partial \sigma$.

Proof of Corollary 1
Assuming \( \delta \in (0,1) \) and \( N \geq n \geq 2 \), note first that we have

\[
\lim_{N \to \infty} \mu(N, n, \cdot) = 0. \tag{47}
\]

Thus, while \( N = n \) leads to \( G(N, n, \cdot) < 0 \), we also have \( \lim_{N \to \infty} G^*(N, n, \cdot) = \lim_{n \to \infty} F^*(n, \cdot) \).

Furthermore, (46) indicates that there must exist a unique \( N^k(\sigma, \delta, n) \) such that \( \mu(N, n, \cdot) < 1 \Leftrightarrow N > N^k(\sigma, \delta, n) \).

**Proof of Proposition 2**

We first show that (25) has an unique maximum in \( n \). To this end it is sufficient to show that

\[
\frac{\partial}{\partial n} \Pi^{Rs}(N, n, \cdot) = 0 \quad \text{and} \quad n < N \Rightarrow \frac{\partial^2}{\partial n^2} \Pi^{Rs}(n, \cdot) < 0. \tag{48}
\]

Starting with the properties of \( \Pi^{Rs}(n, \cdot) \), it turns out that \( \Pi^{Rs}(n, \cdot) \) is log-concave, i.e.

\[
\frac{\partial^2}{\partial n^2} \log[\Pi^{Rs}(n, \cdot)] < 0 \Leftrightarrow \frac{\partial^2}{\partial n^2} \log[\Pi^{Rs}(N, n, \cdot)] < \left[ \frac{\partial \Pi^{Rs}(n, \cdot)}{\partial n} \right]^2 \frac{\Pi^{Rs}(N, n, \cdot)}{\Pi^{Rs}(n, \cdot)}. \tag{49}
\]

Using \( \partial \Pi^{Rs}(n, \cdot)/\partial n = c'(n) \) and (49), \( \partial \Pi^{Rs}(N, n, \cdot)/\partial n = 0 \) implies \( \partial^2 \Pi^{Rs}(N, n, \cdot)/\partial n^2 < 0 \) if

\[
\frac{c''(n)}{c'(n)} > \frac{\partial \Pi^{Rs}(n, \cdot)/\partial n}{\Pi^{Rs}(n, \cdot)}. \tag{50}
\]

Employing (13), simple calculations show that the right-hand side of (50) is decreasing in \( \delta \) and \( \sigma \). Using \( \sigma = \delta = 0 \), (50) can be written as

\[
\frac{c''(n)}{c'(n)} > \frac{1 - 2n}{n - n^2},
\]

which corresponds to (2). Turning to the comparison between \( n^*(\delta, \sigma, N) \) and \( n^*(\sigma, N) \) and using lemma 1, it follows that

\[
n^*(\delta, \sigma, N) \geq n^*(\sigma, N) \Leftrightarrow n^*(\sigma, \cdot) \geq n^k(\sigma)
\]

holds.

**Proof of Proposition 3**
To prove part one, consider first \(N = N^k(\sigma, \delta, n^s(\cdot))\). Comparing the first-order conditions (25) and (28), we get

\[
\Pi^R_l(n^k(\cdot, n^s(\cdot)), n^s(\cdot)) - \Pi^R_l(n^s(\cdot), \cdot) = -R(n, \cdot)\psi(n, \cdot) \frac{\partial}{\partial n} \mu(N^k(\cdot, n^s(\cdot)), n^s(\cdot)).
\]  

(51)

Evaluating \(\partial \mu(N^k(n^s(\cdot), n^s(\cdot)), n^s(\cdot))/\partial n\) for all \(\delta, \sigma \in (0, 1)\) and \(n \geq 3\) reveals \(\partial \mu(N^k(n^s(\cdot), n^s(\cdot)), n^s(\cdot))/\partial n > 0\). Furthermore, using \(\partial \mu(N, n^s(\cdot))/\partial N < 0\) (see (46)) implies \(\partial N^k(\sigma, \delta, n)/\partial n > 0\) for all \(\delta, \sigma \in (0, 1)\) and \(n \geq 3\). Therefore, we must have \(n^l(\cdot) < n^s(\cdot)\) for all \(N \leq N^k(\sigma, \delta, n^s(\cdot))\). Considering \(N > N^k(\sigma, \delta, n^s(\cdot))\) note first that \(\lim_{N \to \infty} \mu(N, n^1, \cdot) = \lim_{N \to \infty} \mu_n(N, n^1) = 0\) and thus \(\lim_{N \to \infty} \Pi^R_l(\cdot) = R_n(n, \cdot) - c'(n)\). Hence, we get \(\lim_{N \to \infty} n^l(\cdot, N) = n^s(\cdot) < n^s(\sigma, \delta)\). Now, assuming to the contrary that there exists a \(\tilde{N} > N^k(\sigma, \delta, n^s(\cdot))\) such that \(n^l(\sigma, \delta, N_1) > n^s(\delta, \sigma)\), there must also exist \(N_1 < \tilde{N} < N_2\) such that

\[
n^l(\delta, \sigma, N_1) = n^l(\delta, \sigma, N_2) = n^s(\delta, \sigma) \text{ and } \quad n^l_N(\delta, \sigma, N)|_{N=N_1} > 0 > n^l_N(\delta, \sigma, N)|_{N=N_2}.
\]  

(52)  

(53)

Simple comparative statics for \(n^l(\delta, \sigma, N)\) leads to

\[
n^l_N(\cdot) \gtrless 0 \Leftrightarrow \varphi(n, \delta) \gtrless -\varphi(n, \delta) \frac{\partial}{\partial n} \log[R(n, \cdot)\psi(n, \cdot)\mu(N, n, \cdot)]
\]  

(54)

with

\[
\varphi(n, \delta) = \log\left[1 + \frac{\delta}{\delta(n-1) - n}\right] < 0; \quad \varphi(n, \delta) > 0.
\]  

(55)

However, since \(n^l(\delta, \sigma, N_1) = n^l(\delta, \sigma, N_2) = n^s(\delta, \sigma) > n^s(\delta, \sigma)\) requires

\[
\frac{\partial}{\partial n} \left[R(n, \cdot)\psi(n, \cdot)\mu(N_i, n, \cdot)\right]|_{n=n^l(\delta, \sigma, N_i)} < 0 \text{ with } i = 2, 3,
\]  

(56)

(52)–(56) lead to a contradiction. Turning to \(n^s(\sigma, \delta, N) < n^k(\sigma)\) and again using \(\lim_{N \to \infty} \mu(N, n, \cdot) = \lim_{N \to \infty} \mu_n(N, n, \cdot) = 0\) shows that we must have \(n^l(\delta, \sigma, N) > n^s(\delta, \sigma, N)\) for \(N\) large enough.

Finally, inspection of (54) shows that \(-\varphi(n, \delta) \frac{\partial}{\partial n} \log[R(n, \cdot)\psi(n, \cdot)\mu(N, n, \cdot)]\) is linearly increasing in \(N\). This and the fact that \(n^l(\delta, \sigma, N)\) is bounded from above due to convex costs implies that we have \(n^l_N(\cdot) > 0\) as \(N\) goes to infinity. Therefore, \(n^l(\delta, \sigma, N)\) approaches \(n^s(\delta, \sigma)\) from below.
Proof of Proposition 4

While the first part of the proposition simply reflects (29) and (30), the proof of the second part is more involved. Employing the envelope theorem, comparative statics with respect to $\delta$ leads to

$$\text{sign} \frac{\partial N^*(\cdot)}{\partial \delta} = \text{sign} \left[ \frac{\partial}{\partial \delta} \Omega^l(n^l(\cdot), \cdot) - \frac{\partial}{\partial \delta} \Omega^s(n^s(\cdot), \cdot) \right]$$

(57)

with $\Omega^l(n^l(\cdot), \cdot) := R(n^l(\cdot), \cdot)\psi(n^l(\cdot), \cdot)\mu(N^s(\cdot), n^l(\cdot), \cdot)$

and $\Omega^s(n^s(\cdot), \cdot) := R(n^s(\cdot), \cdot)\psi(n^s(\cdot), \cdot)$.

Furthermore, differentiating $\partial \Omega^s(n^s(\cdot), \cdot) / \partial n^s$ partially with respect to $\delta$ and using $n^s(\sigma, \cdot) > n^k(\sigma)$ yields

$$\text{sign} \left[ \frac{\partial^2 \Omega^s(n^s(\cdot), \cdot)}{\partial n^s \partial \delta} \right] = \text{sign} \left[ -1 + \sigma(3 + (n^s(\sigma, \cdot) - 2)\sigma) \right] > 0.$$  

(58)

Now, defining $\bar{n}(n^l(\cdot), \cdot) := \max \left\{ n | \Omega^l(n^l(\cdot), \cdot) = R(n, \cdot)\psi(n, \cdot) \right\}$

and evaluating $\frac{\partial}{\partial \delta} \Omega^l(n^l(\cdot), \cdot) - \frac{\partial}{\partial \delta} R(\bar{n}(\cdot), \cdot)\psi(\bar{n}(\cdot), \cdot)$ shows that

$$\text{sign} \left[ \frac{\partial}{\partial \delta} \Omega^l(n^l(\cdot), \cdot) - \frac{\partial}{\partial \delta} R(\bar{n}(\cdot), \cdot)\psi(\bar{n}(\cdot), \cdot) \right]$$

(59)  

$$= -\text{sign} \left[ N - n^l + \sigma(\delta^2(n^l - 1)^2 - 5N - 2\delta(n^l - 1)(N + n^l - 1) + n^l(3N - n^l + 3) \right] < 0.$$

Combining these results with

$$\frac{\partial}{\partial N} \left[ \Pi^m(N, n^l(\cdot), \cdot) - c(n^l(\cdot)) \right] > 0,$$

we must have $\partial N^*(\cdot) / \partial \delta < 0$ whenever $n^s(\cdot) > \bar{n}(n^l(\cdot), \cdot) \Leftrightarrow \Omega^l(n^l(\cdot), \cdot) > \Omega^s(n^s(\cdot), \cdot)$. Considering the case $\Omega^l(n^l(\cdot), \cdot) < \Omega^s(n^s(\cdot), \cdot)$, (58) and (59) indicate that $\partial N^*(\cdot) / \partial \delta < 0$ holds as long as $n^s(\cdot)$ is high enough.
References


