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**Working Paper**

## Optimal deficit-spending in a liquidity trap with long-term government debt

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# Working Paper Research

July 2022 N° 409

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with long-term government debt  
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## Abstract

When the government issues long-term bonds, the optimal time-consistent fiscal and monetary policy is to consolidate debt in a liquidity trap by increasing taxes and by taming public spending. This prescription is at odds with large deficit-spending undertaken in the US during previous liquidity trap episodes. In this article, I show that accumulating debt turns optimal with long-term bonds and flexible wages if labor taxes are kept constant or if monetary policy is conducted non optimally. Moreover, even when labor taxes fluctuate and policy is fully coordinated, optimal deficit-spending in a liquidity trap emerges in a medium-scale model with sticky wages and rule-of-thumb consumers. In this case, debt consolidation occurs only after the nominal interest rate has lifted-off the zero lower bound, in accordance with conventional wisdom that a government should fix the roof while the sun is shining

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The views expressed in this paper are those of the author and do not necessarily represent those of the National Bank of Belgium.

## Non-technical summary

Since the Great Recession, modern economies have experienced periods of low natural interest rates that may be explained (at least partially) by a structural demand shortfall. A central question for policy makers concerns the role of fiscal and monetary policy in dealing with this situation. A major issue for central banks consists in the presence of an operational zero lower bound (ZLB) on the nominal interest rate that prevents (conventional) monetary policy to be sufficiently accommodative in stimulating inflation and economic activity when the natural interest rate falls below zero. This unpleasant scenario has been labelled “liquidity trap” in the macroeconomic literature. At the same time, fiscal policy has been put more and more forward to address the underlying structural issues in the economy.

The macroeconomic literature has been interested in the optimal discretionary (i.e., when policy makers cannot commit to a future course of action) fiscal and monetary policy response to a large contractionary demand shock that pushes the economy in a liquidity trap. When the government does not have access to lump-sum taxes to raise revenues, an accumulation of government debt may influence expectations about distortionary taxes and inflation. In this context, the anticipated response of monetary policy upon the exit of the ZLB becomes a key driver of forward-looking households’ consumption decisions. So far, those studies have found that deficit-spending is optimal when government debt is short-term but that a longer debt maturity overturns this result in favour of an optimal debt consolidation financed with an increase in labor taxes. Given that the observed average maturity of government debt in the US is long-term, this conclusion raises questions about the merits of the large expansionary fiscal policies undertaken during previous liquidity trap episodes.

In this paper, I revisit those results to show that deficit-spending can turn optimal when government debt is long-term, in line with empirical evidence. To obtain this policy, I conduct several experiments in a standard New-Keynesian model subject to the ZLB constraint. First, I keep the tax rate constant so that fiscal adjustments are confined to government spending and the tax base. Second, I assume that monetary policy is conducted independently according to a Taylor rule. Third, I assume that fiscal and monetary policy are coordinated but that wages adjust slowly, and a share of the households are constrained to consume all their income every period. In all those cases, deficit-spending emerges as the optimal policy in the liquidity trap. Only in the third case, debt consolidation occurs but not before the liquidity trap is over, in accordance with conventional wisdom that a government should fix the roof while the sun is shining.

## **TABLE OF CONTENTS**

1.	Introduction.....	1
2.	The model .....	2
2.1	Households and firms.....	2
2.2	Public authorities.....	4
2.3	Log-linear approximation .....	4
3.	Flexible wages .....	5
3.1	Loss function.....	5
3.2	Time-consistency .....	5
3.3.	Coordinated policy .....	6
3.3.1	The policy problem.....	6
3.3.2	Numerical results .....	6
3.4.	Taylor rule.....	9
3.4.1.	The policy problem.....	9
3.4.2.	Numerical results .....	10
3.5.	Welfare comparison .....	11
4.	Sticky wages and rule-of-thumb consumers .....	13
4.1.	Model extension.....	13
4.1.1.	RoT consumers.....	13
4.1.2.	Labor unions and sticky wages.....	14
4.1.3.	Competitive equilibrium .....	15
4.2	Optimal fiscal-monetary mix .....	15
4.2.1.	The policy problem.....	15
4.2.2.	Numerical results .....	17
4.2.3.	Commitment .....	18
5.	Conclusion.....	19
	Appendices.....	19
	Literature cited .....	26
	National Bank of Belgium - Working Papers Series .....	28

# 1 INTRODUCTION

The recent literature on the optimal time-consistent management of a liquidity trap with government debt (Eggertsson, 2006; Burgert and Schmidt, 2014) demonstrated that a government with short-term liabilities should provide a large deficit-financed fiscal stimulus in the liquidity trap. The coordinated policy is for fiscal policy to support the economy in the recession with low taxes and stimulative government spending and for monetary policy to accommodate inflation upon the exit of the zero lower bound (ZLB). More recently, Matveev (2021) showed that this optimal coordinated policy hinges on government debt being short-term. If instead, the government issues bonds of longer maturity, the prescription becomes to consolidate government debt during the liquidity trap by increasing taxes and by taming spending. The reason is that the long maturity of debt reduces the fiscal benefits of monetary accommodation because the yield to maturity of the long bond reacts less to changes in the nominal interest rate. Consequently, monetary policy focuses more on inflation stabilisation and fiscal policy consolidates debt to avoid inflation stemming from the cost-push effect of labor taxes. Given empirically observed long maturity of government debt in the US, this result of optimal debt consolidation raises questions about the merits of large deficit-spending undertaken during previous liquidity trap episodes. In this article, I modify the baseline model along several dimensions and show that deficit-spending in the liquidity trap may be optimal even when debt is long-term. In particular, this holds in the following cases:

1. *Optimal policy is coordinated and labor taxes are kept constant* — Shutting-down the cost-push channel of labor taxes alleviates the inflationary effect of accumulating debt in the liquidity trap. This, in turn, mitigates the negative wealth effect of deficit-spending stemming from expected monetary tightening at positive interest rates.
2. *Monetary policy is non optimal and follows a standard Taylor rule* — The Taylor rule targets both inflation and output gap. Since a high debt level impacts output negatively (because of fiscal tightening), monetary policy keeps interest rates sufficiently low during the recovery to sustain the initial deficit-spending. This result may be overturned if a hawkish central bank is assumed instead (this is, if the weight on inflation stabilisation is sufficiently high).
3. *Optimal policy is coordinated, wages are sticky and a share of consumers are rule-of-thumb (RoT)* — Deficit-spending supports consumption of RoT households in the liquidity trap. Optimal fiscal-monetary policy trades-off this Keynesian demand effect against the wealth effect from expected wage inflation. The result is a debt consolidation that occurs only after the interest rate has lifted-off the ZLB, in accordance with conventional wisdom that a government should fix the roof while the sun is shining.

This paper is related to the voluminous literature on fiscal and monetary policy in a low-rate environment. An important strand of this literature abstracts from debt sustainability issues to focus on the stabilising role of fiscal policy. Those models typically predict that the fiscal multiplier is large (superior to one) at the lower bound (Woodford, 2011; Christiano et al., 2011; Erceg and Lindé, 2014) and optimal fiscal policy is therefore expansionary (Schmidt, 2013; Nakata, 2016).

Closer to this paper, another strand of the literature attributes a meaningful role to government debt by confining tax instruments to labor income. Eggertsson (2006); Burgert and Schmidt (2014) study the optimal time-consistent fiscal-monetary policy mix in a liquidity trap when government budget is a binding constraint. Eggertsson (2013) extends this analysis to a policy game between fiscal and monetary authorities. His result that an independent central bank reduces the deficit-spending multiplier is consistent with the low debt accumulation I find under the hawkish Taylor rule. Matveev (2021) explores the role



of debt maturity while keeping the assumption of flexible wages. His finding about optimal debt consolidation in the liquidity trap constitutes the main motivation of this paper (this is, generalising optimal deficit-spending to long-term debt).

The analysis on optimal fiscal policy and non optimal monetary policy is inspired by [Schmitt-Grohé and Uribe \(2005\)](#); [Faraglia et al. \(2013\)](#); [Gnocchi and Lambertini \(2016\)](#) that follow a similar methodology, however abstracting from the ZLB constraint. Finally, my model extension with sticky wages and rule-of-thumb consumers is grounded in the rapidly growing literature on heterogeneous agents. The wage setting by labor unions draws from [Hagedorn et al. \(2019\)](#). In a model with ad-hoc policy rules, [Drautzburg and Uhlig \(2015\)](#) show that delayed debt consolidation increases the fiscal multiplier at the ZLB when taxes are distortionary, wages are sticky and a share of consumers are rule-of-thumb while in a similar model, [Kaszab \(2016\)](#) finds that a tax-cut can increase output in a liquidity trap. The optimal deficit-spending in the extended model is analogous to their results. Moreover, numerous papers have revisited New-Keynesian models adding rule-of-thumb consumers. Related to this paper, [Bilbiie \(2008\)](#) characterises (optimal) monetary policy with limited asset participation while [Colciago \(2011\)](#) combines rule-of-thumb consumers with sticky wages as in the present model, however abstracting from fiscal policy and the ZLB constraint.

The remainder of the paper is organised as follows. Section 2 presents the main ingredients of the benchmark model with flexible wages. Section 3 lays out the policy problem and proposes two modifications (constant taxes and non optimal monetary policy) that produce an optimal deficit-spending in the liquidity trap. Section 4 extends the benchmark model to sticky wages and rule-of-thumb consumers and Section 5 concludes.

## 2 THE MODEL

The model is a cashless New Keynesian economy in which fiscal policy is non-Ricardian due to the presence of distortionary taxation. I provide an overview of the main ingredients here and leave the details to Appendix A.1. In the benchmark model, the private sector is composed of an infinitely lived representative household, a representative aggregate good producer and intermediate good producers which compete monopolistically and are subject to costly price adjustments. Section 4 extends this benchmark model to costly wage adjustments and rule-of-thumb consumers. The public sector is represented by two institutions, a central bank and a government, respectively in charge of monetary policy and fiscal policy.

### 2.1 Households and firms

The representative household derives utility from consuming the private good  $c_t$  and the public good  $G_t$  while it dislikes hours worked  $h_t$ . I assume a separable utility function leading to the following expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[ \frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right] \quad (1)$$

where  $\mathbb{E}_t$  is the rational expectations operator conditional on information in period  $t$ ,  $\beta$  is the time discount factor. Parameters  $\gamma_c$  and  $\gamma_g$  are respectively the intertemporal elasticity for private and government consumption and,  $\gamma_h$  is the inverse of the Frisch elasticity. I also attach utility weights  $\nu_g$  and  $\nu_h$  to characterise preference for government consumption and hours, relatively to private consumption.

The variable  $\xi_t$  is an exogenous process characterising the preference for time. Under this

specification, time preference between states of two consecutive periods evolves according to  $\xi_t/(\beta\xi_{t+1})$ . I assume the following autoregressive structure for this process:

$$\log(d_t) = \rho_\epsilon \log(d_{t-1}) + \epsilon_t \quad (2)$$

where  $d_t \equiv \xi_{t+1}/\xi_t$  and  $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$  is a normally distributed exogenous shock that will be the source of liquidity trap episodes.

The household sells hours to intermediate firms for a wage  $W_t$  net of labor taxes  $\tau_t$  and may save via two nominal and non state-contingent assets: a one-period bond  $B_t^s$  and a perpetual bond  $B_t$ . Following [Woodford \(2001\)](#), the perpetuity yields a coupon with payoff decaying at exponential rate  $\rho$ . Consequently, when  $\rho = 0$ , the short-term bond and the perpetuity have the same one-period maturity.<sup>1</sup> Firm profits yield a dividend  $\Pi_{i,t}$  and lump-sum transfers  $T_t$  are collected from the government. The household budget constraint (in real terms) reads

$$c_t + \frac{b_t^s}{i_t} + q_t b_t = (1 - \tau_t)w_t h_t + \frac{b_{t-1}^s}{\pi_t} + (1 + \rho q_t) \frac{b_{t-1}}{\pi_t} + \int_0^1 \frac{\Pi_{i,t}}{P_t} di + \frac{T_t}{P_t} \quad (3)$$

where  $i_t$  is the gross nominal interest rate,  $q_t$  is the price of the perpetual bond and  $\pi_t$  is the gross inflation rate. The household chooses  $\{c_t, h_t, b_t^s, b_t\}_{t=0}^\infty$  to maximise expected lifetime utility (1) subject to (A.1.1) and no-Ponzi scheme conditions on the two bonds.

Intermediate firms operate under monopolistic competition and seek to maximise profits subject to linear technology  $y_{i,t} = h_{i,t}$  and quadratic price adjustment costs à la [Rotemberg \(1982\)](#). From the profit maximisation problem of the final producer, the demand function of the generic firm producing  $i$  is given by

$$y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\theta_\pi} y_t \quad (4)$$

where  $\theta_\pi$  is the marginal rate of substitution between varieties. The program of the firm  $i$  is

$$\max_{P_{i,t}} \mathbb{E}_0 \sum_{t=0}^\infty \lambda_t \left[ \left( \frac{P_{i,t}}{P_t} \right)^{-\theta_\pi} y_t \left( \frac{P_{i,t}}{P_t} - (1-s)w_t \right) - \frac{\psi_\pi}{2} \left( \frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 y_t \right] \quad (5)$$

where  $\lambda_t$  is the multiplier of budget constraint from the household problem and  $\psi_\pi$  is the price adjustment cost factor.

Parameter  $s$  is an employment subsidy which offsets steady state distortions stemming from monopolistic competition and distortionary taxation. For the rest of this paper, this subsidy is kept constant over time and lump-sum transfers are restricted to the sole purpose of financing it.<sup>2</sup> The reason for introducing this subsidy is to make a positive amount of debt sustainable at the steady state. Absent of the subsidy, a time-consistent policy maker wants to reduce any positive amount of liabilities that successive policy makers will inherit. This debt consolidation removes future incentives to inflate debt away and lowers inflation expectations to their efficient level.<sup>3</sup>

<sup>1</sup>I only introduce the short-term bond  $B_t^s$  because I want to be able to refer to the short-term yield even when the maturity of the perpetuity is superior to one period.

<sup>2</sup>This implies that distortions from labor taxation do occur outside the steady state. See [Leith and Wren-Lewis \(2013\)](#) for a similar use of a steady state subsidy.

<sup>3</sup>For a detailed analysis of those dynamics in a real economy and Markov-Perfect Equilibrium, see e.g. [Debtoli and Nunes \(2013\)](#).

The symmetric pricing of intermediate firms implies that they all produce the same amount  $y_{i,t} = y_t$  and so the resource constraint reads

$$y_t = c_t + G_t + \frac{\psi_\pi}{2}(\pi_t - 1)^2 y_t \quad (6)$$

## 2.2 Public authorities

There are two authorities exercising policy in this economy: a central bank and a government. Each authority uses its own instruments to pursue its objective:

**The central bank** chooses short-term nominal interest rates  $\{i_t\}$  while being constrained by a zero lower bound (ZLB).

**The government** chooses labor taxes and government spending  $\{\tau_t, G_t\}$  to finance its net debt position. Assuming that the short-bond is in zero net supply, the budget constraint of the government in real terms reads

$$\left(\frac{1 + \rho q_t}{\pi_t}\right) b_{t-1} = q_t b_t + \tau_t w_t h_t - G_t - s(w_t h_t - wh)$$

The last term implies that the subsidy is not rebated at the steady state.

## 2.3 Log-linear approximation

For the remaining of the paper, I work with a log-linear approximation of the system equations around the efficient steady state. Variables without time-subscript represent steady state values and hatted variables are log-deviations from the steady state.

$$y \hat{y}_t = c \hat{c}_t + G \hat{G}_t \quad (7)$$

$$\hat{\pi}_t = \kappa_\tau \hat{\tau}_t + \kappa_c \hat{c}_t + \kappa_y \hat{y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \quad (8)$$

$$\hat{c}_t = -\frac{1}{\gamma_c} (\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} + \hat{d}_t) + \mathbb{E}_t \hat{c}_{t+1} \quad (9)$$

$$\hat{i}_t = \rho \beta \mathbb{E}_t \hat{q}_{t+1} - \hat{q}_t \quad (10)$$

where  $\kappa_\tau = w\tau(\theta_\pi - 1)/\psi_\pi$ ,  $\kappa_c = \gamma_c(\theta_\pi - 1)/\psi_\pi$  and  $\kappa_y = \gamma_h(\theta_\pi - 1)/\psi_\pi$ .

The log-linear budget constraint reads

$$\Omega \left( \hat{b}_t + (1 - \rho) \hat{q}_t - \beta^{-1} (\hat{b}_{t-1} - \hat{\pi}_t) \right) - G \hat{G}_t + w\tau y \frac{\theta_\pi - 1}{\theta_\pi} \hat{\tau}_t - \frac{y}{\theta_\pi} (\gamma_c \hat{c}_t + (1 + \gamma_y) \hat{y}_t) = 0 \quad (11)$$

where  $\Omega \equiv qb$  is the steady state market value of debt.

A private-sector rational expectations equilibrium consists of a sequence  $x_t \equiv \{\hat{c}_t, \hat{\pi}_t, \hat{b}_t, \hat{y}_t, \hat{q}_t\}$  satisfying equations (7)–(11), given the policies  $p_t \equiv \{\hat{i}_t \geq -r^*, \hat{G}_t, \hat{\tau}_t\}$ , exogenous process  $\{\epsilon_t\}$ , and initial conditions  $\hat{b}_{-1}$ .

Optimal choice of policy instruments is described in the respective sections below. Nominal interest rate choice is constrained by the ZLB and  $r^* \equiv \log(1/\beta)$  is the net real interest rate in the steady state.

### 3 FLEXIBLE WAGES

Flexible wages constitute the main assumption in the optimal time-consistent literature (Eggertsson, 2006; Adam and Billi, 2007; Niemann et al., 2013; Burgert and Schmidt, 2014; Matveev, 2021). In this section, I show that two ingredients are necessary to obtain an optimal debt consolidation with flexible prices: control over labor taxes and coordinated policy. Section 4 considers the case of sticky wages. When a share of the consumers are rule-of-thumb, those two ingredients are no longer sufficient for optimal debt consolidation.

#### 3.1 Loss function

In my model, the presence of nominal rigidities and distortionary taxation generates inefficient economic dynamics. I employ a quadratic microfounded loss function to highlight the welfare costs associated with these distortions. Assuming that the time-invariant employment subsidy described in Section 2.1 holds, the welfare criterion is obtained by taking a second-order approximation of household's utility function (1) around the efficient steady state.<sup>4</sup> Leaving the details of the derivation to Appendix A.2, the loss function reads

$$\mathcal{L}_t = \frac{1}{2} \left[ \gamma_c c \hat{c}_t^2 + \gamma_g G \hat{G}_t^2 + \gamma_h y \hat{y}_t^2 + \psi_\pi y \hat{\pi}_t^2 \right] \quad (12)$$

The first term in equation (12) highlights the presence of taxes on labor income that distort the consumption intertemporal trade-off of the households. The second term captures the distortions stemming from government spending that influence the tax rate and inflation rate in the economy.<sup>5</sup> The third term and fourth term relate to the output-inflation trade-off that arises because of relative price frictions and distortionary taxation.

#### 3.2 Time-consistency

I focus on the discretionary fiscal and monetary policy. This is, in minimising its intertemporal loss function, the policymaker cannot commit to time-inconsistent actions in the future to influence private sector expectations and improve current policy trade-offs as they would under Ramsey Policy.<sup>6</sup> Instead, rational agents correctly anticipate the states of the economy that next policy makers will inherit and form expectations accordingly. To capture the dependence of private sector expectations to the equilibrium mapping between the state-space and endogenous variables, I write expectations using the following state-dependent auxiliary functions

$$\mathbb{E}_t \pi_{t+1} \equiv \mathbb{E}_t \Pi(s_{t+1}) \quad \mathbb{E}_t c_{t+1} \equiv \mathbb{E}_t \mathcal{C}(s_{t+1}) \quad \mathbb{E}_t q_{t+1} \equiv \mathbb{E}_t \mathcal{Q}(s_{t+1}) \quad (13)$$

where  $s_t \equiv \{\hat{b}_{t-1}, \hat{d}_t\}$  is the state vector. In the present model with debt as an endogenous state, the policymaker takes into account the effect of its debt choices on the next period mapping and related impact on private expectations.

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<sup>4</sup>Woodford (2003) shows that the linearised equilibrium conditions allow to evaluate such welfare criterion accurately up to second order.

<sup>5</sup>If one were to assume lump-sum taxes to finance government spending, those two first terms would vanish (see e.g. Schmidt, 2017)

<sup>6</sup>However, the government commits to repay its debt obligations fully. For an analysis of optimal time-consistent distortionary taxation when default can occur on government debt, see Karantounias (2017).

### 3.3 Coordinated policy

Under coordinated fiscal and monetary policy, the policy makers jointly minimise the utility loss stemming from exogenous demand shocks by choosing available policy instruments subject to a consolidated budget constraint. This view of the optimal policy problem is standard in the literature and has been studied in a context of time-consistent policy at the ZLB in [Burgert and Schmidt, 2014](#); [Matveev, 2021](#). Hence, this section constitutes a useful benchmark to study the optimal path of debt.

**3.3.1 The policy problem.** Given Markov-Perfection, the joint policymaker is represented by a sequence of authorities with identical preferences, each one leading its future selves. The Lagrangean for the policy problem reads

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \left[ \gamma_c c \hat{c}_t^2 + \gamma_g G \hat{G}_t^2 + \gamma_h y \hat{y}_t^2 + \psi_\pi y \hat{\pi}_t^2 \right] + \beta \mathbb{E}_t V(s_{t+1}) + \lambda_t^{zlb} \left[ \rho \beta \mathbb{E}_t \mathcal{Q}(s_{t+1}) - \hat{q}_t + r^* \right] \\ & + \lambda_t^r \left[ c \hat{c}_t + G \hat{G}_t - y \hat{y}_t \right] + \lambda_t^p \left[ \kappa_\tau \hat{\tau}_t + \kappa_c \hat{c}_t + \kappa_y \hat{y}_t + \beta \mathbb{E}_t \Pi(s_{t+1}) - \hat{\pi}_t \right] \\ & + \lambda_t^b \left[ \Omega \left( \hat{b}_t + (1 - \rho) \hat{q}_t - \beta^{-1} (\hat{b}_{t-1} - \hat{\pi}_t) \right) - G \hat{G}_t + w \tau y \frac{\theta_\pi - 1}{\theta_\pi} \hat{\tau}_t - \frac{y}{\theta_\pi} \left( \gamma_c \hat{c}_t + (1 + \gamma_y) \hat{y}_t \right) \right] \\ & + \lambda_t^q \left[ \gamma_c \hat{c}_t + \rho \beta \mathbb{E}_t \mathcal{Q}(s_{t+1}) - \hat{q}_t - \mathbb{E}_t \Pi(s_{t+1}) + \hat{d}_t - \gamma_c \mathbb{E}_t \mathcal{C}(s_{t+1}) \right] \end{aligned} \quad (14)$$

The coordinated policy maker optimises (14) by choosing  $\hat{c}_t$ ,  $\hat{G}_t$ ,  $\hat{y}_t$ ,  $\hat{\pi}_t$ ,  $\hat{\tau}_t$ ,  $\hat{q}_t$ ,  $\hat{b}_t$  and the multipliers  $\lambda_t^r$ ,  $\lambda_t^p$ ,  $\lambda_t^q$ ,  $\lambda_t^b$ ,  $\lambda_t^{zlb}$ . The first order necessary conditions (FONCs) of the program can be found in [Appendix A.3](#) and can be combined to give

$$y \hat{y}_t = - \frac{\gamma_g y \eta_y + (\gamma_c G + \gamma_g c) \eta_y}{D} \vartheta \hat{\pi}_t + \frac{\gamma_g \gamma_c y}{D} \lambda_t^{zlb} \quad (15)$$

$$c \hat{c}_t = - \frac{\gamma_g c \eta_y + (\gamma_h G + \gamma_g y) \eta_c}{D} \vartheta \hat{\pi}_t + \frac{(\gamma_h G + \gamma_g y) \gamma_c}{D} \lambda_t^{zlb} \quad (16)$$

$$(\Omega - \beta \frac{\psi_\pi y}{\theta_\pi} \Theta_{1,t} - \Omega(1 - \rho) \Theta_{2,t}) \vartheta \hat{\pi}_t = \Omega \vartheta \mathbb{E}_t \hat{\pi}_{t+1} + \Theta_{3,t} \lambda_t^{zlb} \quad (17)$$

$$\lambda_t^{zlb} (\hat{i}_t + r^*) = 0 \quad (18)$$

$$\hat{i}_t \geq -r^* \quad (19)$$

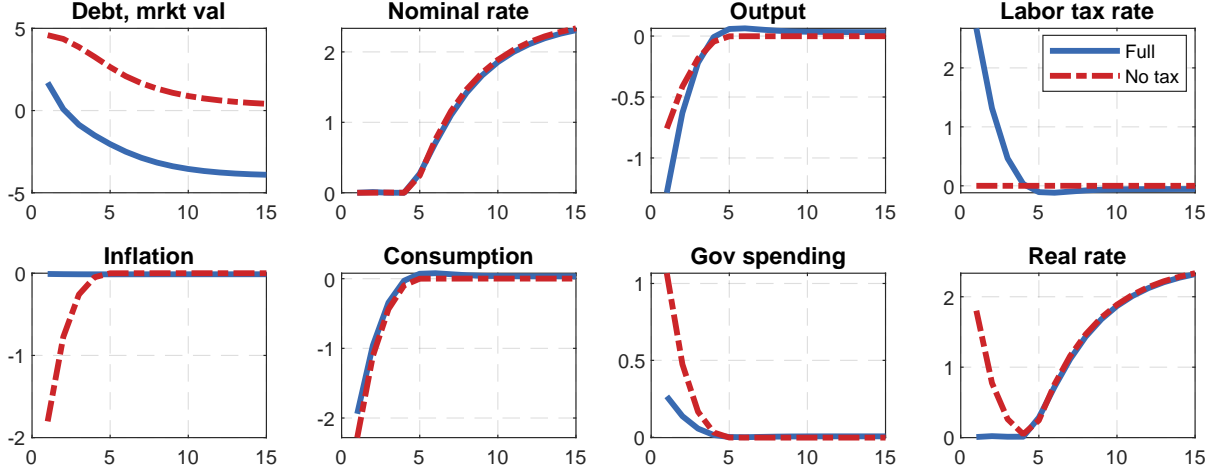
where  $\eta_y \equiv y + \frac{\psi_\pi y}{\theta_\pi} \kappa_y + \frac{y}{\theta_\pi} (1 + \gamma_h)$ ,  $\eta_c \equiv \frac{\psi_\pi y}{\theta_\pi} \kappa_c + \frac{y}{\theta_\pi} \gamma_c - c - \gamma_c \Omega (1 - \rho)$ ,  $D \equiv \gamma_h \gamma_c G + \gamma_h \gamma_g c + \gamma_c \gamma_g y$ ,  $\vartheta \equiv \frac{y \psi_\pi}{y \frac{\psi_\pi}{\theta_\pi} + \beta^{-1} \Omega}$  and  $\Theta_{1,t} \equiv \frac{\partial \mathbb{E}_t \Pi(s_{t+1})}{\partial \hat{b}_t}$ ,  $\Theta_{2,t} \equiv \Theta_{1,t} + \gamma_c \frac{\partial \mathbb{E}_t \mathcal{C}(s_{t+1})}{\partial \hat{b}_t}$ ,  $\Theta_{3,t} \equiv \Theta_{2,t} - \rho \beta \frac{\partial \mathbb{E}_t \mathcal{Q}(s_{t+1})}{\partial \hat{b}_t}$ .

The discretionary equilibrium is determined by the system given by the FONCs (15)–(19), the private-sector equilibrium conditions (7)–(11), the state-dependent auxiliary functions in (13) and the exogenous process for the markup shock, (2). The solution to this system is a set of piece-wise linear equilibrium Markov-perfect policy rules  $z_t = \mathcal{H}(s_{t-1})$  mapping the vector of states  $s_{t-1} = \{\hat{b}_{t-1}, \hat{d}_t\}$  to the optimal decisions for  $z_t = \{\hat{c}_t, \hat{G}_t, \hat{y}_t, \hat{\pi}_t, \hat{\tau}_t, \hat{q}_t, \hat{b}_t, \hat{i}_t, \lambda_t^{zlb}\}$ .

**3.3.2 Numerical results.** The numerical experiment consists in a large contractionary demand shock of four standard deviations that drives the economy in a liquidity trap. Throughout, I consider that the shock is fully unanticipated and occurs at a given point in time (period 1). No other shocks or news occur after that, so the environment becomes deterministic. As it is customary in previous studies, I assume that the economy starts

at the risky steady state before the shock occurs.<sup>7</sup> For each model extension considered, I solve the model using a collocation method on a finite domain for the states. This method allows to deal with the complexities related to non-linearity (due to the ZLB constraint) and time-consistency. I restrict my analysis to differentiable equilibria and numerically obtain convergence to a unique equilibrium. More details about the algorithm are provided in Appendix D.

Figure 1: **Impulse responses to a contractionary demand shock with coordinated fiscal and monetary policy**



**Notes:** Impulse responses to a four standard deviations contractionary shock on private demand. The solid-blue line corresponds to the benchmark model with full set of policy instruments while the dashed-red line to constant labor taxes. Variables are expressed in percentage deviations from steady state. Debt market value and public spending are in percentage of GDP. Interest rates and inflation are annualised. Labor tax rate deviations are in percentage points.

**Calibration** The model is calibrated on the US economy before the Great Recession. A time period represents one quarter of a year. The preference parameters,  $\nu_g$  and  $\nu_h$ , are chosen to be consistent with households working one quarter of their time endowment and with a share of government spending equating one fifth of output. The annual interest rate in the intended steady state is set to 2.5% and pins down the time discount factor,  $\beta$ . The parameters of the Phillips Curve are standard to the literature. The intertemporal elasticity of private and public consumption and, the inverse of the Frisch elasticity are equal to 1. The elasticity of substitution between intermediate goods,  $\theta_\pi$ , corresponds to a price markup over the marginal cost of 10%. Given the value of  $\theta_\pi$ , the parameter of the price adjustment cost,  $\psi_\pi$ , matches its Calvo (1983) equivalent (up to a first-order approximation around the deterministic steady-state) when the average duration for setting prices equals one year. The market value of debt in the efficient steady state corresponds to 40% of output. Throughout the paper, I assume that  $\rho = 0.9434$  which corresponds to an average maturity of debt equal

<sup>7</sup>In an economy with occasionally binding zero lower bound, the deterministic steady state is unstable because of a deflationary bias (Nakov, 2008) that leads a discretionary policy maker to accumulate more debt in the risky steady state (Matveev, 2021).

to 16 quarters in line with empirical evidence detailed in [Matveev \(2021\)](#). The parameter values are summarised in [Table 1](#).

Table 1: Baseline calibration

Symbol	Description	Value
$\beta$	Subjective discount factor	0.99
$\theta_\pi$	Elasticity of substitution among goods	11
$\psi_\pi$	Price adjustment cost	117.805
$\gamma_c$	Intertemporal elasticity of c	1
$\gamma_g$	Intertemporal elasticity of G	1
$\gamma_h$	Intertemporal elasticity of h	1
$\nu_h$	Utility weight on labor	20
$\nu_g$	Utility weight on labor	0.25
$\rho_\epsilon$	AR coefficient on demand shock	0.77
$\sigma$	S.D. of demand shock (%)	0.4
$\rho$	Average maturity of government debt	0.94

*Note:* See text for rationale behind the parameter values.

**Variable taxes** Consider first a government that can set optimally labor taxes and public spending. The central bank sets the nominal interest rate while being constrained by the ZLB. This environment corresponds to [Matveev \(2021\)](#). [Figure 1](#) displays the impulse responses of the government debt market value, the nominal interest rate, output, the labor tax rate, inflation, public spending and the real interest rate to a contractionary shock that drives the economy into a liquidity trap. Consistently with the findings of [Matveev \(2021\)](#), the government consolidates debt in the liquidity trap (following a small increase on impact) and returns it only progressively to steady state so that debt remains subdued long after the nominal interest rate has recovered. As explained by [Matveev \(2021\)](#), this optimal policy lowers expected real interest rates by reducing labor taxes outside the ZLB and by pushing inflation into negative territory so that, in turn, monetary policy turns expansionary.

**Constant taxes** Labor taxes play a central role in the optimal consolidation of government debt. Absent of the consolidation, the cost-push effect of labor taxes would cause a monetary tightening upon the exit of the ZLB and a negative wealth effect that reduces consumption of forward-looking consumers in the liquidity trap. A natural extension is thus to understand how optimal fiscal and monetary policy changes when tax rates are kept constant at their steady state value and stabilisation of real debt is achieved through changes in government spending and the tax base. The assumption of constant tax rates is also relevant in a context of high levels of public indebtedness because it can proxy a situation in which taxes are unresponsive to the level of government debt for example because the top of the Laffer curve has been reached.<sup>8</sup> I avoid repeating the FONCs of the modified problem here and only point out that they are equivalent to the ones with full set of policy instruments except for the derivative of the Lagrangian with respect to labor taxes that is removed.

[Figure 1](#) displays the impulse responses to the large contractionary demand shock when taxes are kept constant at their steady state level (dashed-red line). The optimal policy now

<sup>8</sup>For a paper tackling the constraint exerted by the Laffer curve on optimal time-consistent fiscal policy with distortionary taxation, see [Debortoli et al. \(2021\)](#).



becomes to finance a large increase in public spending with deficits and to return it to steady state before the liquidity trap ends. Despite the debt accumulation, the interest rate path remains very similar to the case of variable taxes and debt consolidation. Hence, shutting down the cost-push channel of taxes sustains a deficit-financed fiscal expansion without the negative wealth effect of expected monetary tightening upon the exit of the ZLB. This policy mitigates the initial drop in output at the cost of a welfare-reducing shortfall in inflation.

This result complements [Burgert and Schmidt \(2014\)](#) who show that removing labor taxes from the instrument set of the policy maker does not modify the conclusion of an optimal debt accumulation in the liquidity trap when debt is short-term.<sup>9</sup> Indeed, when labor taxes are kept constant, deficit-spending is optimal *no matter the maturity of government debt*.

### 3.4 Taylor rule

An important determinant of the optimal debt policy is the expected stance of monetary policy upon the exit of the ZLB. [Matveev \(2021\)](#) argues that increasing the debt maturity lowers the fiscal benefits from monetary accomodation because the yield to maturity of government bonds becomes less sensitive to changes in the short-term interest rate. As a result, monetary policy focuses more on inflation stabilization and less on the fiscal benefits of inflation. The future monetary tightening in response to the cost-push effect of high debt and labor taxes creates a negative wealth effect that fiscal policy avoids by consolidating debt. The specific conduct of monetary policy thus matters for the conclusion that governments should diminish the level of public debt during an economic downturn.

More specifically, discretion is an attractive representation of fiscal policy because the political decision process often lacks transparency and is subject to uncertainty due to changes in the composition of the government. In contrast, central banks have a very stable mandate that is revised only occasionally and communicate regularly about their strategy. Those characteristics of central banks call for a degree of commitment and independence that go beyond the coordination assumption. To account for this, I now assume monetary policy commits once and for all to the following standard Taylor rule

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \quad (20)$$

For example, [Schmitt-Grohé and Uribe, 2005](#); [Faraglia et al., 2013](#); [Gnocchi and Lambertini, 2016](#) adopt a similar approach to characterise optimal fiscal policy when monetary policy is conducted non optimally by an independent central bank.

**3.4.1 The policy problem.** The government is constrained by the Taylor rule so that the Lagrangean for the policy problem now reads

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \left[ \gamma_c c \hat{c}_t^2 + \gamma_g G \hat{G}_t^2 + \gamma_h y \hat{y}_t^2 + \psi_\pi y \hat{\pi}_t^2 \right] + \beta \mathbb{E}_t V(s_{t+1}) \\ & + \lambda_t^r \left[ c \hat{c}_t + G \hat{G}_t - y \hat{y}_t \right] + \lambda_t^p \left[ \kappa_\tau \hat{\tau}_t + \kappa_c \hat{c}_t + \kappa_y \hat{y}_t + \beta \mathbb{E}_t \Pi(s_{t+1}) - \hat{\pi}_t \right] \\ & + \lambda_t^q \left[ \gamma_c \hat{c}_t + \rho \beta \mathbb{E}_t \mathcal{Q}(s_{t+1}) - \hat{q}_t - \mathbb{E}_t \Pi(s_{t+1}) + \hat{d}_t - \gamma_c \mathbb{E}_t \mathcal{C}(s_{t+1}) \right] \\ & + \lambda_t^b \left[ \Omega \left( \hat{b}_t + (1 - \rho) \hat{q}_t - \beta^{-1} (\hat{b}_{t-1} - \hat{\pi}_t) \right) - G \hat{G}_t + w \tau y \frac{\theta_\pi - 1}{\theta_\pi} \hat{\tau}_t - \frac{y}{\theta_\pi} \left( \gamma_c \hat{c}_t + (1 + \gamma_y) \hat{y}_t \right) \right] \\ & + \lambda_t^m \left[ \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \hat{q}_t - \rho \beta \mathbb{E}_t \mathcal{Q}(s_{t+1}) \right] + \lambda_t^{zlb} \left[ \rho \beta \mathbb{E}_t \mathcal{Q}(s_{t+1}) - \hat{q}_t - r^* \right] \end{aligned} \quad (21)$$

---

<sup>9</sup>In their model, labor taxes are only used to engineer a desired level of debt but do not change the central prescription of deficit-spending.



The government optimises (21) by choosing  $\hat{c}_t$ ,  $\hat{G}_t$ ,  $\hat{y}_t$ ,  $\hat{\pi}_t$ ,  $\hat{\tau}_t$ ,  $\hat{q}_t$ ,  $\hat{b}_t$  and the multipliers  $\lambda_t^r$ ,  $\lambda_t^p$ ,  $\lambda_t^m$ ,  $\lambda_t^q$ ,  $\lambda_t^b$ ,  $\lambda_t^{zlb}$ . The FONCs can be combined to give

$$Dy\hat{y}_t = \Psi_{y,1}\lambda_t^b + \Psi_{y,2}\hat{\pi}_t + \Psi_{y,3}\lambda_t^{zlb} \quad (22)$$

$$Dc\hat{c}_t = \Psi_{c,1}\lambda_t^b + \Psi_{c,2}\hat{\pi}_t + \Psi_{c,3}\lambda_t^{zlb} \quad (23)$$

$$\Omega\mathbb{E}_t\lambda_{t+1}^b = \Psi_{b,1}\lambda_t^b + \Psi_{b,2}\hat{\pi}_t + \Psi_{b,3}\lambda_t^{zlb} \quad (24)$$

$$\lambda_t^{zlb}(\hat{i}_t + r^*) = 0 \quad (25)$$

$$\hat{i}_t + r^* \geq 0 \quad (26)$$

The definition of a discretionary equilibrium is analogous to the benchmark model in Section 3.3. The coefficients are defined as

$$\begin{aligned} \Psi_{y,1} &\equiv \gamma_g y \Lambda_c + (\gamma_c G + \gamma_g c) \Lambda_y, \quad \Psi_{y,2} \equiv \frac{\psi_{\pi} y}{\phi_{\pi}} \left[ \gamma_c \gamma_g y G + (\gamma_c G + \gamma_g c) \phi_y \right], \quad \Psi_{y,3} \equiv \gamma_g \gamma_c y \\ \Psi_{c,1} &\equiv \gamma_g c \Lambda_y + (\gamma_h G + \gamma_g y) \Lambda_c, \quad \Psi_{c,2} \equiv \frac{\psi_{\pi} y}{\phi_{\pi}} \left[ \gamma_c \gamma_g c G \phi_y + \gamma_h G + \gamma_g y \right], \quad \Psi_{c,3} \equiv (\gamma_h G + \gamma_g y) \gamma_c \\ \Psi_{b,1} &\equiv \Omega - \beta \frac{\psi_{\pi} y}{\theta_{\pi}} \Theta_{1,t} + \frac{\Theta_{2,t}}{\phi_{\pi}} \left( \frac{\psi_{\pi} y}{\theta_{\pi}} + \Omega \beta^{-1} \right) - \Omega(1 - \rho) \Theta_{3,t}, \quad \Psi_{b,2} \equiv \frac{\Theta_{2,t}}{\phi_{\pi}} \psi_{\pi} y, \quad \Psi_{b,3} \equiv \Theta_{2,t} \end{aligned}$$

where  $\Lambda_c \equiv \eta_c + \frac{\gamma_c}{\phi_{\pi}} \left( \frac{\psi_{\pi} y}{\theta_{\pi}} + \Omega \beta^{-1} \right)$  and  $\Lambda_y \equiv \eta_y + \frac{\phi_y}{\phi_{\pi}} \left( \frac{\psi_{\pi} y}{\theta_{\pi}} + \Omega \beta^{-1} \right)$ .

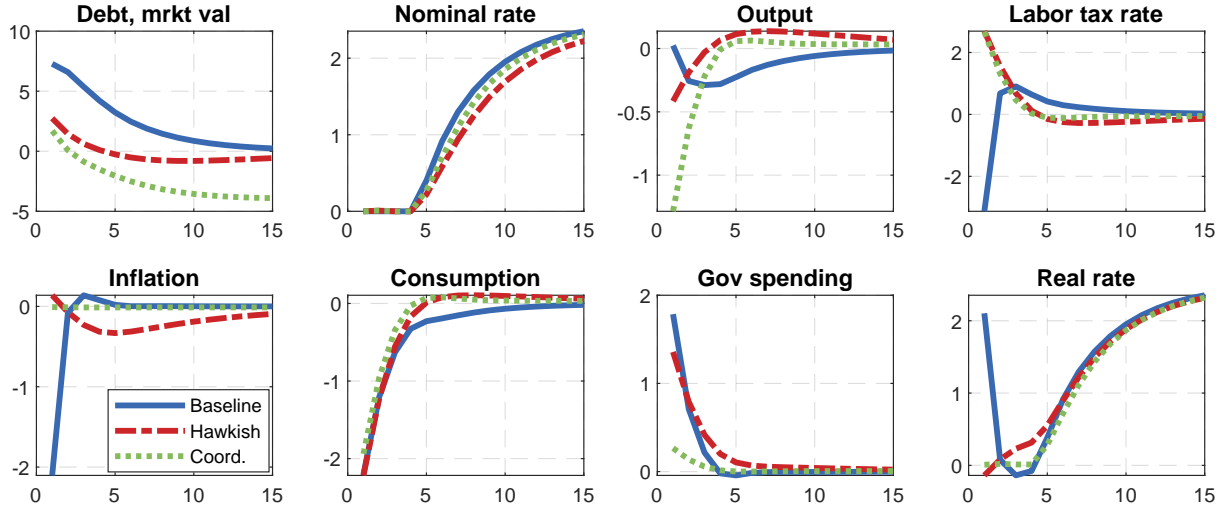
Assume that  $\eta_c > 0$ , this would hold under baseline calibration, then  $\Psi_{y,1}, \Psi_{y,2}, \Psi_{y,3} > 0$ . Outside the ZLB,  $\lambda_t^{zlb} = 0$  and thus target rule (22) implies that output responds positively to inflation and to a budget constraint tightening. This is intuitive because a recovery in private demand stimulates both inflation and output while an expansionary fiscal policy, in the form of a tax-cut or government spending, stimulates output and tightens the budget constraint. Given Taylor rule (20), the net response of output is key to understand the monetary policy stance upon the exit of the ZLB, as the following numerical exercise demonstrates.

**3.4.2 Numerical results.** To calibrate the Taylor parameters, I borrow from [Faraglia et al. \(2013\)](#) and set  $\phi_{\pi} = 1.3$  and  $\phi_y = 6h/LF = 2.34$  with  $LF$  representing the labor force participation.<sup>10</sup> I reiterate here the experiment of a large demand shock of four standard deviations that pushes the economy for several quarters into a liquidity trap. Figure 2 shows the impulse responses for the selected variables. Under the baseline calibration (solid blue line), government debt is increased initially and then returned progressively to steady state. The debt expansion outlives the liquidity trap and implies lower labor taxes and higher government spending. This optimal fiscal policy recalls [Burgert and Schmidt \(2014\)](#); [Matveev \(2021\)](#) when the government issues debt of short-term maturity. This similarity can be traced back to the reaction of monetary policy upon the exit of the liquidity trap.

Figure 3 shows the equilibrium response of selected variables to beginning-of-period government debt in the absence of demand shock (solid-blue line) and when a negative demand shock of two standard deviations occurs (dashed-red line). As can be seen from the first panel, output drops outside the liquidity trap when beginning-of-period debt increases because of the consolidation effort needed to return debt to the pre shock level. At the same time, inflation increases (top right panel) due to the cost-push effect of labor taxes. Mon-

<sup>10</sup>The calibration strategy draws on [Rudebusch et al. \(2009\)](#) who regress the interest rate on inflation and unemployment in the US. In my model, the non-linear Taylor rule takes the form  $i_t = (\pi_t)^{\phi_{\pi}} (y_t/y)^{\phi_y}$ . Since  $y_t = h_t$ , I follow [Faraglia et al. \(2013\)](#) to map hours worked into unemployment. I refer to their paper for more details.

Figure 2: Impulse responses to a contractionary demand shock with optimal fiscal policy and non optimal (Taylor rule) monetary policy



**Notes:** Impulse responses to a four standard deviations contractionary shock on private demand. The solid-blue line corresponds to baseline calibration  $\phi_\pi = 1.29$  and  $\phi_y = 2.33$  while the dashed-red line to a hawkish Taylor rule  $\phi_\pi = 5$  and  $\phi_y = 2.33$ . As a reference, the dotted-green line shows the response under policy coordination. Variables are expressed in percentage deviations from steady state. Debt market value and public spending are in percentage of GDP. Interest rates and inflation are annualised. Labor tax rate deviations are in percentage points.

etary policy trades-off those two effects in its Taylor rule. Under baseline calibration, the resulting stance is sufficiently accommodative to support a deficit-financed fiscal expansion in the liquidity trap while avoiding a strongly negative wealth effect from expected monetary tightening.

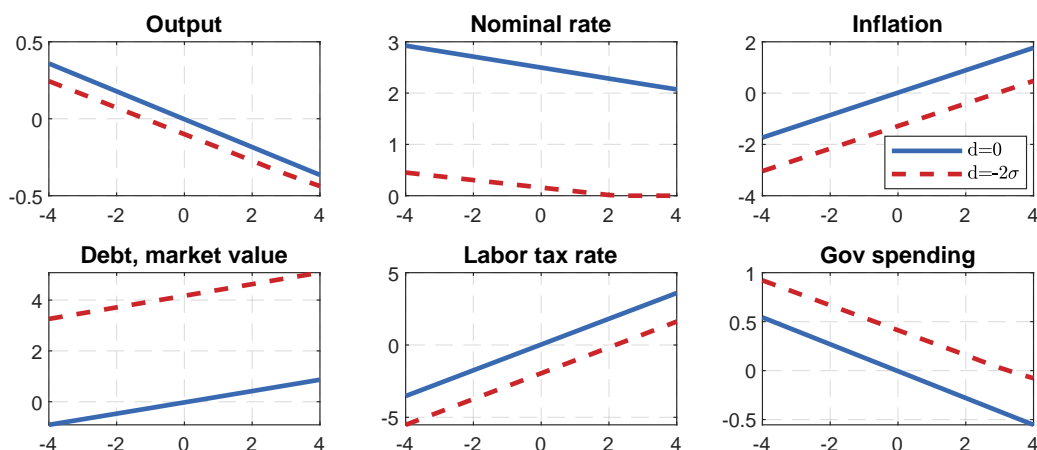
Evidently, this result is sensitive to the monetary response to inflation. For example, the dashed-red line in Figure 2 shows the impulse responses when the inflation parameter in the Taylor rule is increased to  $\phi_\pi = 5$ .<sup>11</sup> The hawkish central bank restrains the debt accumulation in the liquidity trap. Moreover, the government slightly consolidates debt upon the exit of the ZLB to lower the tax rate and to provoke an expansionary monetary policy during the recovery of private demand.

### 3.5 Welfare comparison

So far, I have demonstrated that deficit-spending may be desirable in a liquidity trap even if debt is long-term provided that available fiscal instruments or institutional set-up are modified. In this case, the fiscal stimulus alleviates the output loss at the onset of the liquidity trap. A natural follow-up exercise is thus to evaluate this policy from a welfare point of view and to compare it with the coordinated policy. The results of this exercise can be found in Table 2 that computes average welfare losses over 50 samples of 10,000 periods

<sup>11</sup>All else equal, this calibration implies that an increase of one percentage point in annual inflation causes the interest rate to rise by 5 percentage points on annual basis (instead of 1.3 percentage point under baseline calibration).

Figure 3: Equilibrium responses to beginning-of-period government debt



**Notes:** The x-axis represents the beginning-of-period government debt in percentage deviations from steady state. The solid-blue line corresponds to the response absent of demand shock while the dashed-red line to a demand shock of two standard deviations. Output is expressed in percentage deviations from steady state. Debt market value and public spending are in percentage of GDP. Interest rates and inflation are annualised. Labor tax rate deviations are in percentage points.

each for different institutional arrangements.<sup>12</sup> Unsurprisingly, the occurrence of occasional liquidity trap episodes causes the lowest welfare loss (0.0009) when fiscal and monetary policies coordinate with a full set of policy instruments. This loss goes up to 0.0045 and 0.0249 when considering the absence of tax instrument and the independent central bank, respectively. As explained earlier, the coordinated monetary policy focuses more on inflation stabilisation when debt is long-term. Knowing this, fiscal policy increases taxes to avoid piling up debt and causing a monetary policy tightening upon the exit of the liquidity trap. As a result, inflation is smoothed in and out the liquidity trap, as can be seen from its lower standard deviation (in the second column of the table).

In contrast, the fiscal policy with constant taxes or independent central bank accumulates a large amount of debt in the liquidity trap to benefit from the subsequent accommodative stance of monetary policy. Fiscal policy achieves this higher debt level by increasing government spending (and cutting labor taxes in the case of Taylor rule). While this policy curbs the recession, it does so at the cost of a welfare reducing increase in government spending and inflation volatility. In this sense, a more hawkish central bank reduces the welfare loss (to 0.005) by inciting fiscal policy to finance its spending stimulus with taxes instead of deficits. As with full coordination, the cost-push effect of labor taxes alleviates the inflation shortfall during the downturn and is reverted when the nominal rate lifts-off the lower bound.

<sup>12</sup>Welfare loss is expressed in perpetual consumption transfer that would make a household in the artificial economy without any fluctuations indifferent to living in the economy. Let the unconditional expected lifetime utility loss from fluctuations be  $\mathcal{S} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\gamma_c c \hat{c}_t^2 + \gamma_g G \hat{G}_t^2 + \gamma_h y \hat{y}_t^2 + \iota_y \hat{\pi}_t^2]$ . A permanent reduction in private consumption by the share  $\mathcal{W}$  lowers the utility of the household by an amount equivalent to  $\frac{U_c C \mathcal{W}}{1-\beta}$ . Hence, the welfare loss is  $\mathcal{W} = \frac{1}{2}(1-\beta)C^{-1}\mathcal{S}$ .

Table 2: Welfare losses with flexible wages

Framework	$\mathcal{W}$	Std dev.			
		$\hat{\pi}_t$	$\hat{c}_t$	$\hat{y}_t$	$\hat{G}_t$
<i>Coordinated policy</i>					
Variable tax rate	0.0009	0.0231	0.3077	0.2078	0.2187
Constant tax rate	0.0045	0.2626	0.3666	0.1755	1.4676
<i>Taylor rule</i>					
Baseline	0.0249	0.6510	0.5191	0.2053	1.5285
Hawkish	0.0050	0.2013	0.3861	0.1113	1.2833

*Note:* Welfare loss ( $\mathcal{W}$ ) is expressed in percentage of perpetual consumption transfer that would make a household in the artificial economy without any fluctuations indifferent to living in the economy.

## 4 STICKY WAGES AND RULE-OF-THUMB CONSUMERS

To obtain an optimal deficit-spending policy under flexible wages, I have imposed additional constraints to the discretionary policy maker namely, the removal of the labor tax instrument and the independent central bank. Now, I want to answer the following question: is it possible to have an optimal time-consistent debt accumulation in the liquidity trap when the two authorities coordinate and government debt is long-term?

In the representative agent model with flexible wages assumed so far, labor taxes do not have direct demand effects because wage adjustments preserve the net disposable income following variations in labor taxes. In reality, increasing taxes in the liquidity trap may have adverse effects if it reduces the net disposable income of consumers that do not have the possibility to trade consumption intertemporally. For example, in a model with rules, [Drautzburg and Uhlig \(2015\)](#) show that the fiscal multiplier of tax-financed government spending increases when taxes are adjusted only slowly. On the one hand, deficit spending avoids a negative demand effect at the lower bound from lower net disposable income of the credit-constrained agents (see also [Kaszab, 2016](#)). On the other hand, imperfect wage adjustments contain the inflationary effect of debt (through reduced cost-push effect of labor taxes) and thus alleviate the negative wealth effect due to monetary tightening upon the exit of the ZLB.

### 4.1 Model extension

To see how those mechanisms may shape the optimal time-consistent fiscal and monetary policy in a liquidity trap, I now introduce two additional ingredients in the model: sticky wages and rule-of-thumb (RoT) consumers.

**4.1.1 RoT consumers.** The economy is populated by two types of consumers. Ricardian consumers whose maximisation problem corresponds to the one previously described and RoT consumers who cannot save and do not own shares of firms. Respective share of those consumers in the economy are given by  $(1 - \delta)$  and  $\delta$ . RoT consumers consume their full

income

$$c_{H,t} = (1 - \tau_t)w_t h_t - \frac{T_{H,t}}{P_t} - \frac{\psi_w}{2}(\pi_{w,t} - 1)^2 y_t \quad (27)$$

The last term represents the adjustment costs to wages that are passed on consumers by labor unions (see next subsection). RoT consumers delegate their wage and labor decisions to the labor unions. Hence, they do not optimise and there are no FONCs to consider.

**4.1.2 Labor unions and sticky wages.** I model sticky wages following [Hagedorn et al. \(2019\)](#). Worker  $i$  provides differentiated labor services,  $h_{i,t}$ , that are sold by a labor union to a representative, competitive labor recruiting firm. The labor recruiting firm combines labor services into an aggregate effective labor input according to technology

$$h_t = \left( \int_0^1 h_{i,t}^{\frac{\theta_w-1}{\theta_w}} di \right)^{\frac{\theta_w}{\theta_w-1}}$$

where  $\theta_w$  is the elasticity of substitution between labor services. Given aggregate demand  $h_t$  by the intermediate goods sector, the cost minimisation problem of the labor recruiting firm reads

$$\min_{h_{i,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi^t \left[ \int_0^1 W_{i,t} h_{i,t} di + W_t \left\{ h_t - \left( \int_0^1 h_{i,t}^{\frac{\theta_w-1}{\theta_w}} di \right)^{\frac{\theta_w}{\theta_w-1}} \right\} \right]$$

the FONC of this problem gives the demand for labor services of worker  $i$

$$h_{i,t} = \left( \frac{W_{i,t}}{W_t} \right)^{-\theta_w} h_t$$

where  $W_t$  is the wage index that can be obtained by substituting the solution  $h_{i,t}$  into the profit function to yield

$$W_t = \left( \int_0^1 W_{i,t}^{1-\theta_w} di \right)^{\frac{1}{1-\theta_w}}$$

In the presence of RoT consumers, I assume that the labor union maximises the utility of the Ricardian household. This assumption is motivated by a minority of RoT consumers in the economy ( $\delta < 1/2$ ). Define the marginal rate of substitution of the Ricardian household as

$$MRS_{S,t} = - \frac{u'(h_{S,t})}{(1 - \tau_t)u'(c_{S,t})} \quad (28)$$

The labor union maximises profits defined as the difference between the net income and the dis-utility of labor (expressed in marginal utility units) subject to wage adjustment costs in the spirit of [Rotemberg \(1982\)](#) and proportional to aggregate output. The maximisation program of the labor union reads

$$\max_{W_{i,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi^t \left[ \frac{(1 - \tau_t)W_{i,t}}{P_t} h_{i,t} - \frac{g(h(W_{i,t}, W_t, h_t))}{u'(c_{S,t})} - \frac{\psi_w}{2} \left( \frac{W_{i,t}}{W_{i,t-1}} - 1 \right)^2 y_t \right] \quad (29)$$

with  $g(h(W_{i,t}, W_t, h_t)) = \nu_h \frac{h_{i,t}^{1+\gamma_h}}{1+\gamma_h}$ .

Solving this problem gives the following wage Phillips curve (see Appendix B.1.1)

$$\frac{\psi_w}{\theta_w}(\pi_{w,t} - 1)\pi_{w,t} = \beta d_t \frac{\psi_w}{\theta_w} \mathbb{E}_t \left[ (\pi_{w,t+1} - 1)\pi_{w,t+1} \frac{y_{t+1}}{y_t} \right] - (1 - \tau_t) \left( \frac{\theta_w - 1}{\theta_w} w_t - MRSS_{S,t} \right) \quad (30)$$

**4.1.3 Competitive equilibrium.** Appendix B.1.2 shows that market clearing yields the non-linear resource constraint:  $y_t \left( 1 - \frac{\psi_\pi}{2} (\pi_t - 1)^2 - \frac{\psi_w}{2} (\pi_{w,t} - 1)^2 \right) = c_t + G_t$ . The system is log-linearised around the efficient steady state to give

$$y \hat{y}_t = \delta c \hat{c}_{H,t} + (1 - \delta) c \hat{c}_{S,t} + G \hat{G}_t \quad (31)$$

$$\psi_w \hat{\pi}_{w,t} = \beta \psi_w \mathbb{E}_t \hat{\pi}_{w,t+1} - (1 - \tau) (\theta_w - 1) w \hat{\mu}_t \quad (32)$$

$$\hat{\mu}_t = \hat{w}_t - \gamma_h \hat{y}_t - \gamma_c \hat{c}_{S,t} - \frac{\tau \hat{\tau}_t}{1 - \tau} \quad (33)$$

$$\hat{i}_t = \rho \beta \mathbb{E}_t \hat{q}_{t+1} - \hat{q}_t \quad (34)$$

$$\gamma_c \hat{c}_{S,t} = \hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} + \hat{d}_t + \gamma_c \mathbb{E}_t \hat{c}_{S,t+1} \quad (35)$$

$$\hat{c}_{H,t} = \hat{w}_t + \hat{y}_t - \frac{\tau \hat{\tau}_t}{1 - \tau} \quad (36)$$

$$\hat{w}_t - \hat{w}_{t-1} = \hat{\pi}_{w,t} - \hat{\pi}_t \quad (37)$$

$$\psi_\pi \hat{\pi}_t = \beta \psi_\pi \mathbb{E}_t \hat{\pi}_{t+1} + (\theta_\pi - 1) \hat{w}_t \quad (38)$$

$$\Omega \left( \hat{b}_t + (1 - \rho) \hat{q}_t - \beta^{-1} (\hat{b}_{t-1} - \hat{\pi}_t) \right) - G \hat{G}_t + (\tau - s) w y (\hat{w}_t + \hat{y}_t) + w y \tau \hat{\tau}_t = 0 \quad (39)$$

where  $\hat{\mu}_t$  is the household real wage markup. Equation (32) states that labor unions will reset nominal wages upwards (and thus increase wage inflation) whenever the real wage markup is below the natural level (equivalent to steady state level). With infinite wage adjustment costs ( $\psi_w \rightarrow \infty$ ), nominal wages are fixed. As before,  $s$  is a steady state subsidy eliminating the distortions in the model to recover an efficient steady state. This subsidy takes a different value than in the flexible wages model because it also cancels out distortions stemming from market power in the market for labor services. More details about the efficient steady state in the extended model can be found in Appendix B.1.3.

A private-sector rational expectations equilibrium in the extended model consists of a sequence  $x_t \equiv \{\hat{c}_t, \hat{\pi}_t, \hat{\pi}_{w,t}, \hat{\mu}_t, \hat{w}_t, \hat{b}_t, \hat{y}_t, \hat{q}_t\}$  satisfying equations (31)–(39), given the policies  $p_t \equiv \{\hat{i}_t \geq -r^*, \hat{G}_t, \hat{\tau}_t\}$ , exogenous process  $\{\epsilon_t\}$ , and initial conditions  $\hat{b}_{-1}$ .

## 4.2 Optimal fiscal-monetary mix

As before, I assume that fiscal and monetary policy are discretionary. Given the law of motion (37), real wages become a state variable of the extended model and thus influence the mapping between the state-space and endogenous variables. This implies that the policy maker now takes into account the effect of both debt and wages on the private expectations. In a linear model, this effect is additive so that cross-derivative terms are zero. Since we have four expectation terms in the extended model ( $\mathbb{E}_t \Pi(s_{t+1}), \mathbb{E}_t \mathcal{C}(s_{t+1}), \mathbb{E}_t \mathcal{Q}(s_{t+1}), \mathbb{E}_t \Pi_w(s_{t+1})$ ), this results in eight derivative terms (4 differential expectations terms  $\times$  2 states).

**4.2.1 The policy problem.** The coordinated policy maker minimises the weighted approximated loss of utility for the two types of consumers. Leaving the derivation of the

modified loss function to Appendix B.2, the Lagrangean for the policy problem reads

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2} \left[ \gamma_c c \delta \hat{c}_{H,t}^2 + \gamma_c c (1 - \delta) \hat{c}_{S,t}^2 + \gamma_g G \hat{G}_t^2 + \gamma_h y \hat{y}_t^2 + \psi_\pi y \hat{\pi}_t^2 + \psi_w y \hat{\pi}_{w,t}^2 \right] + \beta \mathbb{E}_t V(s_{t+1}) \\
& + \lambda_t^r \left[ \delta c \hat{c}_{H,t} + (1 - \delta) c \hat{c}_{S,t} + G \hat{G}_t - y \hat{y}_t \right] + \lambda_t^h \left[ \hat{c}_{H,t} - \hat{w}_t - \hat{y}_t + \frac{\tau \hat{\tau}_t}{1 - \tau} \right] \\
& + \lambda_t^\mu \left[ \hat{\mu}_t - \hat{w}_t + \gamma_h \hat{y}_t + \gamma_c \hat{c}_{S,t} + \frac{\tau \hat{\tau}_t}{1 - \tau} \right] + \lambda_t^{zlb} \left[ \rho \beta \mathbb{E}_t \mathcal{Q}(s_{t+1}) - \hat{q}_t - r^* \right] \\
& + \lambda_t^q \left[ \gamma_c \hat{c}_{S,t} + \rho \beta \mathbb{E}_t \mathcal{Q}(s_{t+1}) - \hat{q}_t - \mathbb{E}_t \Pi(s_{t+1}) + \hat{d}_t - \gamma_c \mathbb{E}_t \mathcal{C}_S(s_{t+1}) \right] \\
& + \lambda_t^p \left[ \psi_\pi \hat{\pi}_t - \beta \psi_\pi \mathbb{E}_t \Pi(s_{t+1}) - (\theta_\pi - 1) \hat{w}_t \right] + \lambda_t^i \left[ \hat{w}_t - \hat{w}_{t-1} - \hat{\pi}_{w,t} + \hat{\pi}_t \right] \\
& + \lambda_t^b \left[ \Omega \left( \hat{b}_t + (1 - \rho) \hat{q}_t - \beta^{-1} (\hat{b}_{t-1} - \hat{\pi}_t) \right) - G \hat{G}_t + (\tau - s) w y (\hat{w}_t + \hat{y}_t) + w y \tau \hat{\tau}_t \right] \\
& + \lambda_t^w \left[ \psi_w \hat{\pi}_{w,t} - \beta \psi_w \mathbb{E}_t \Pi_w(s_{t+1}) + (1 - \tau) (\theta_w - 1) w \hat{\mu}_t \right] \tag{40}
\end{aligned}$$

where  $s_t = \{\hat{b}_{t-1}, \hat{w}_{t-1}, \hat{d}_t\}$ .

Here, the nominal interest rate has been eliminated by using the arbitrage condition (34). The coordinated policy maker optimises (40) by choosing  $\hat{c}_{H,t}$ ,  $\hat{c}_{S,t}$ ,  $\hat{G}_t$ ,  $\hat{y}_t$ ,  $\hat{\pi}_t$ ,  $\hat{\pi}_{w,t}$ ,  $\hat{w}_t$ ,  $\hat{\mu}_t$ ,  $\hat{\tau}_t$ ,  $\hat{q}_t$ ,  $\hat{b}_t$  and the multipliers  $\lambda_t^r$ ,  $\lambda_t^p$ ,  $\lambda_t^q$ ,  $\lambda_t^w$ ,  $\lambda_t^h$ ,  $\lambda_t^i$ ,  $\lambda_t^\mu$ ,  $\lambda_t^b$ ,  $\lambda_t^{zlb}$ . The FONCs read

$$\begin{aligned}
\partial \mathcal{L} / \partial \hat{c}_{H,t} & 0 = \gamma_c c \delta \hat{c}_{H,t} - \lambda_t^r c \delta + \lambda_t^h \\
\partial \mathcal{L} / \partial \hat{c}_{S,t} & 0 = \gamma_c c (1 - \delta) \hat{c}_{S,t} - \lambda_t^r c (1 - \delta) + \lambda_t^\mu \gamma_c + \lambda_t^q \\
\partial \mathcal{L} / \partial \hat{G}_t & 0 = \gamma_g G \hat{G}_t + \lambda_t^r G - \lambda_t^b G \\
\partial \mathcal{L} / \partial \hat{y}_t & 0 = \gamma_h y \hat{y}_t + \lambda_t^r y + \lambda_t^\mu \gamma_h - \lambda_t^h + \lambda_t^b (\tau - s) w y \\
\partial \mathcal{L} / \partial \hat{\pi}_t & 0 = \psi_\pi y \hat{\pi}_t + \lambda_t^p \psi_\pi + \lambda_t^i + \lambda_t^b \Omega \beta^{-1} \\
\partial \mathcal{L} / \partial \hat{\pi}_{w,t} & 0 = \psi_w y \hat{\pi}_{w,t} + \lambda_t^w \psi_w - \lambda_t^i \\
\partial \mathcal{L} / \partial \hat{\tau}_t & 0 = \tau (1 - \tau)^{-1} \lambda_t^\mu + \tau (1 - \tau)^{-1} \lambda_t^h + \lambda_t^b w y \tau \\
\partial \mathcal{L} / \partial \hat{\mu}_t & 0 = \lambda_t^w (1 - \tau) (\theta_w - 1) w + \lambda_t^\mu \\
\partial \mathcal{L} / \partial \hat{w}_t & 0 = -\lambda_t^\mu - \lambda_t^h - \lambda_t^p [(\theta_w - 1) + \beta \psi_\pi \Theta_{5,t}] + \lambda_t^i - \beta \mathbb{E}_t \lambda_{t+1}^i + \lambda_t^b (\tau - s) w h \\
& \quad - \lambda_t^w \beta \psi_w \Theta_{8,t} - \lambda_t^q \Theta_{7,t} + \lambda_t^{zlb} (\Theta_{6,t} - \Theta_{7,t}) \\
\partial \mathcal{L} / \partial \hat{q}_t & 0 = \lambda_t^b \Omega (1 - \rho) - \lambda_t^q - \lambda_t^{zlb} \\
\partial \mathcal{L} / \partial \hat{b}_t & 0 = \Omega (\lambda_t^b - \mathbb{E}_t \lambda_{t+1}^b) - \lambda_t^p \beta \psi_\pi \Theta_{1,t} - \lambda_t^w \beta \psi_w \Theta_{4,t} - \lambda_t^q \Theta_{3,t} + \lambda_t^{zlb} (\Theta_{2,t} - \Theta_{3,t}) \\
\text{CSC} & 0 = \lambda_t^{zlb} (\hat{i}_t + r^*) \\
\text{ZLB} & 0 \leq \hat{i}_t + r^*
\end{aligned}$$

where  $\Theta_{1,t}, \Theta_{2,t}$  and  $\Theta_{3,t}$  are defined as in Section 3.4.1 and  $\Theta_{4,t} \equiv \frac{\partial \mathbb{E}_t \Pi_w(s_{t+1})}{\partial \hat{b}_t}$ ,  $\Theta_{5,t} \equiv \frac{\partial \mathbb{E}_t \Pi(s_{t+1})}{\partial \hat{w}_t}$ ,  $\Theta_{6,t} \equiv \Theta_{5,t} + \gamma_c \frac{\partial \mathbb{E}_t \mathcal{C}(s_{t+1})}{\partial \hat{w}_t}$ ,  $\Theta_{7,t} \equiv \Theta_{6,t} - \rho \beta \frac{\partial \mathbb{E}_t \mathcal{Q}(s_{t+1})}{\partial \hat{w}_t}$  and  $\Theta_{8,t} \equiv \frac{\partial \mathbb{E}_t \Pi_w(s_{t+1})}{\partial \hat{w}_t}$ .

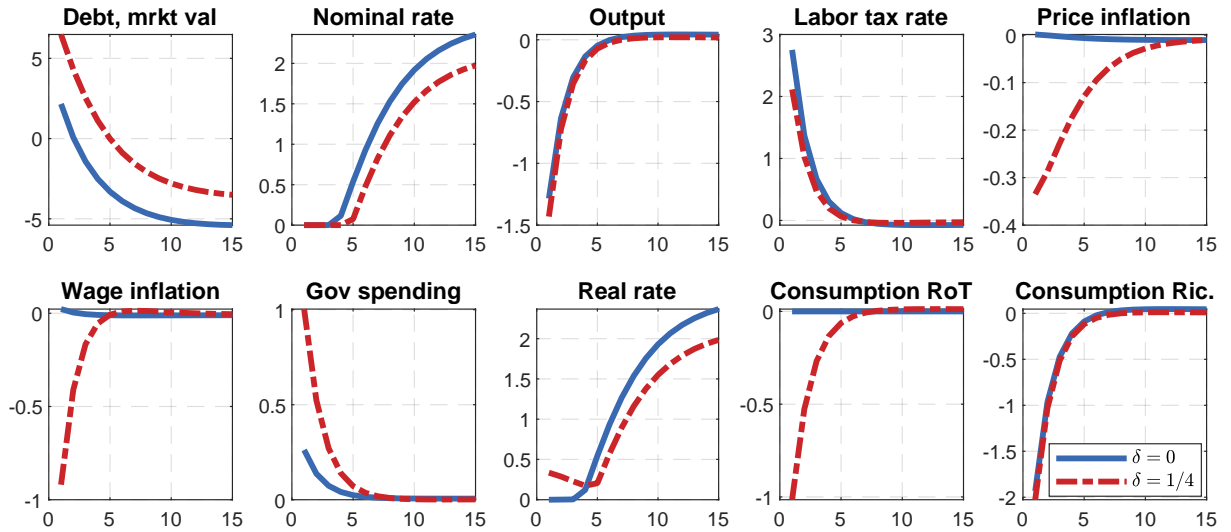
Together with the competitive equilibrium conditions, those FONCs form a balanced system of 20 equations for 20 unknowns that I solve using the projection method described in Section 3. The difference now is that there are two endogenous state variables,  $\hat{b}_{t-1}$  and  $\hat{w}_{t-1}$ , that result in 8 differential expectations terms.<sup>13</sup> The definition of a discretionary equilibrium is analogous to the benchmark model in Section 3.3.

<sup>13</sup>This is a much more involved problem to solve with a collocation method because of the enlargement of the state-space. Indeed, the curse of dimensionality constitutes an important reason for the perfect foresight approach adopted here.



**4.2.2 Numerical results.** The elasticity of substitution between labor services is set to  $\theta_w = 11$ , the same value used for the price Phillips curve. The adjustment cost parameter is then given by  $\psi_w = \frac{(\theta_w - 1)(1 - \tau)w\zeta}{(1 - \zeta)(1 - \beta\theta_w)}$  where  $\zeta = 3/4$  corresponds to a Calvo (1983) equivalent (up to first order) of annual frequency for resetting wages. Figure 4 displays the impulse responses of selected variables to a large contractionary demand shock when wages are sticky. Two models are considered: a representative household and a share of RoT consumers equal to one fourth (in line e.g. with Drautzburg and Uhlig, 2015).

Figure 4: Impulse responses to a contractionary demand shock with coordinated fiscal and monetary policy, sticky wages and RoT consumers



**Notes:** Impulse responses to a four standard deviations contractionary shock on private demand. The solid-blue line corresponds to representative household  $\delta = 0$  while the dashed-red line to a share of RoT consumers  $\delta = 1/4$ . Variables are expressed in percentage deviations from steady state. Debt market value and public spending are in percentage of GDP. Interest rates, price inflation and wage inflation are annualised. Labor tax rate deviations are in percentage points.

In a representative agent framework, the presence of sticky wages does not change the optimal consolidation of government debt in the liquidity trap. The contractionary demand shock puts downward pressure on wages. Since wage adjustments are costly, the policy maker wants to compensate this effect by increasing labor taxes. This policy stabilises wages and avoids a negative wealth effect from monetary tightening in response to expected wage inflation upon the exit of the ZLB.<sup>14</sup> As with flexible wages, public spending increases only weakly and debt drops persistently below its steady state level.

The dashed-red line in Figure 4 considers the model implied equilibrium response when the share of RoT consumers is increased from  $\delta = 0$  to  $\delta = 1/4$ . This modification shifts the path of government debt upwards with respect to the representative agent framework. The increase in labor taxes falls and the spending stimulus becomes much larger. Those policy measures aim at supporting disposable income of RoT consumers through the recession. This positive Keynesian demand effect is visible from the log-linear consumption equation of RoT households:

$$\hat{c}_{H,t} = \hat{w}_t + \hat{y}_t - \tau(1 - \tau)^{-1}\hat{\tau}_t$$

<sup>14</sup>A monetary tightening would occur because wage inflation is targeted by the central bank.



The spending expansion boosts public demand and hours worked by both RoT and Ricardian households. At the same time, the lower increase in labor taxes alleviates the loss in net income. This policy mix results in a debt increase in the liquidity trap which is overturned in favor of a moderate consolidation upon the exit of the ZLB. This late debt consolidation lowers expected labor taxes and wage inflation to loosen the stance of monetary policy. Hence, the sign-switching in the path of government debt reflects the trade-off between the Keynesian demand effect (attributed to RoT consumers) and the wealth effect (attributed to forward-looking Ricardian consumers). This trade-off resolves in an optimal deficit-spending in the liquidity trap. Thereafter, the government should fix the roof while the sun is shining — in accordance with conventional wisdom.

Table 3: Welfare losses with sticky wages

Framework	$\mathcal{W}$	Std dev.				
		$\hat{\pi}_t$	$\hat{\pi}_{w,t}$	$\hat{c}_t$	$\hat{y}_t$	$\hat{G}_t$
Representative ( $\delta = 0$ )	0.0006	0.0146	0.0152	0.3166	0.2136	0.2171
RoT ( $\delta = 1/4$ )	0.0038	0.0793	0.1330	0.4729	0.2239	0.7801

*Note:* Welfare loss ( $\mathcal{W}$ ) is expressed in percentage of perpetual consumption transfer that would make an average household in the artificial economy without any fluctuations indifferent to living in the economy.

Table 3 shows the welfare losses (following the methodology provided in Section 3.5) for the representative household economy and for the economy with RoT consumers. Introducing RoT consumers (with  $\delta = 1/4$ ) increases welfare loss from 0.0006 to 0.0038. The standard deviations in the table indicate that excess volatility comes mainly from inflation (wages and prices) and from government spending. On the one hand, the lower reaction of taxes following a negative demand shock increases the wage and price markups. On the other hand, the government relies more extensively on government spending to protect gross income of RoT consumers. The presence of heterogeneous households thus implies that the government has to address more policy trade-offs with as many instruments, resulting in additional welfare losses.

**4.2.3 Commitment.** To close the discussion, Appendix C investigates the implications of commitment for the optimal policy mix. As with discretion, the path of government debt switches sign once the ZLB stops binding. However now, the policy maker commits to keep the interest rate at the ZLB for an extended period of time to lower real rate expectations and to stimulate consumption in the liquidity trap. Through this commitment, monetary policy alone is able to offset most of the adverse output effects arising from the zero lower bound and thus government spending plays only a modest role in stabilising the economy in line with Schmidt (2013). In the presence of heterogeneous households, fiscal policy cuts taxes during the downturn to support net income of RoT consumers while monetary policy mitigates the inflation shortfall with forward guidance. This coordinated policy thus overturns the rise in taxes observed under discretion. All in all, macroeconomic outcomes depend less on the model specifics because of the stabilising power of credible commitments to state-contingent future policy actions.

## 5 CONCLUSION

I have revisited the optimal time-consistent fiscal-monetary policy mix in a liquidity trap with long-term government debt. The result previously found of an optimal debt consolidation in response to a large contractionary demand shock hinges on the fiscal instruments available, the expected reaction of monetary policy at positive interest rates and, the prevalence of flexible wages and representative household in the economy. I have shown that modifying the benchmark model along those dimensions overturns this result in favour of deficit-spending in the liquidity trap. For a realistic share of rule-of-thumb households, sticky wages imply a sign-switching in the path of government debt upon the exit of the ZLB. The government provides a deficit-financed fiscal stimulus in the liquidity trap to support disposable income of rule-of-thumb consumers and consolidates debt thereafter to lower wage inflation and to benefit from a positive wealth effect by loosening the expected stance of monetary policy. This policy highlights the importance of fixing the roof while the sun is shining.

## APPENDICES

### A MODEL WITH FLEXIBLE PRICES

#### A.1 The non-linear economy

**A.1.1 Equilibrium conditions.** The representative household maximises its expected lifetime utility  $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[ \frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right]$  subject to its budget constraint:

$$c_t + \frac{b_t^s}{i_t} + q_t b_t = (1 - \tau_t) w_t h_t + \frac{b_{t-1}^s}{\pi_t} + (1 + \rho q_t) \frac{b_{t-1}}{\pi_t} + \int_0^1 \frac{\Pi_{i,t}}{P_t} di + \frac{T_t}{P_t}$$

Attaching multiplier  $\lambda_t$  to the constraint, the FONCs of this problem are standard

$$\begin{aligned} \nu_h h_t^{\gamma_h} c_t^{\gamma_c} &= (1 - \tau_t) w_t \\ i_t^{-1} &= \beta d_t \mathbb{E}_t \left[ \frac{c_t^{\gamma_c}}{\pi_{t+1} c_{t+1}^{\gamma_c}} \right] \\ q_t &= \beta d_t \mathbb{E}_t \left[ \frac{c_t^{\gamma_c} (1 + \rho q_{t+1})}{\pi_{t+1} c_{t+1}^{\gamma_c}} \right] \end{aligned}$$

The intermediate firm faces demand function  $y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\theta_\pi} y_t$  from final good producer and maximises

$$\max_{P_{i,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t \left[ \left( \frac{P_{i,t}}{P_t} \right)^{-\theta_\pi} y_t \left( \frac{P_{i,t}}{P_t} - (1-s)w_t \right) - \frac{\psi_\pi}{2} \left( \frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 y_t \right]$$

giving rise to the following FONC

$$w_t = \frac{\theta_\pi - 1}{\theta_\pi(1-s)} - \frac{\psi_\pi}{\theta_\pi(1-s)} \left( \pi_t(\pi_t - 1) - \beta d_t \mathbb{E}_t \left[ \frac{c_t^{\gamma_c}}{c_{t+1}^{\gamma_c}} \frac{y_{t+1}}{y_t} \pi_{t+1} (\pi_{t+1} - 1) \right] \right)$$

Moreover, the budget constraint of the government in real terms reads

$$q_t b_t - (1 + \rho q_t) \frac{b_{t-1}}{\pi_t} - G_t + \tau_t w_t y_t - s(w_t y_t - w y) = 0$$

Together with the resource constraint,  $y_t = c_t + G_t + \frac{\psi_\pi}{2}(\pi_t - 1)^2 y_t$ , those equations can be log-linearised to obtain the system of equations in the text.

**A.1.2 The first-best allocation.** The first-best allocation solves the problem

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[ \frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g \frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h \frac{h_t^{1+\gamma_h}}{1+\gamma_h} + \omega_t (y_t - c_t - G_t) \right]$$

that gives the following FONCs

$$c_t^{-\gamma_c} = \omega_t, \quad \nu_h y_t^{\gamma_h} = \omega_t, \quad \nu_g G_t^{-\gamma_g} = \omega_t$$

Hence, at the first best, it holds that  $(1 - \tau_t)w_t = 1$ .

**A.1.3 The efficient steady state.** The steady state is characterised by the following set of equations

$$\begin{aligned} y &= c + G \\ R &= \beta^{-1} \\ q &= \beta(1 + \rho q) \end{aligned}$$

In the *efficient* steady state, the subsidy is set to attain the first-best,  $(1 - \tau)w = 1$  and is thus given by

$$s = 1 - \frac{\theta_\pi - 1}{\theta_\pi} (1 + G + (1 - \beta)by^{-1})^{-1}$$

Moreover, the equations for the steady state tax rate and wage become

$$\tau = \frac{\theta_\pi s - 1}{\theta_\pi - 1} \qquad w = \frac{\theta_\pi - 1}{(1 - s)\theta_\pi}$$

Hence, the preference parameters read

$$\nu_h = y^{-\gamma_h} c^{-\gamma_c} \qquad \nu_g = G^{\gamma_g} c^{-\gamma_c}$$

## A.2 Derivation of the loss function

A second-order approximation of the representative households' utility around the efficient steady state yields

$$\begin{aligned} U(c_t, y_t, G_t) &\approx c^{1-\gamma_c} \hat{c}_t + \frac{1}{2}(1 - \gamma_c)c^{1-\gamma_c} \hat{c}_t^2 - \nu_h y^{1+\gamma_h} \hat{y}_t - \frac{1}{2}\nu_h(1 + \gamma_h)y^{1+\gamma_h} \hat{y}_t^2 \\ &\quad + \nu_g G^{1-\gamma_g} \hat{G}_t + \frac{1}{2}\nu_g(1 - \gamma_g)G^{1-\gamma_g} \hat{G}_t^2 + tip \end{aligned}$$

At the efficient steady state, we have  $\nu_g = G^{\gamma_g} c^{-\gamma_c}$  and  $\nu_h = y^{-\gamma_h} c^{-\gamma_c}$ , thus we can write

$$U(c_t, y_t, G_t) \approx c^{-\gamma_c} (c\hat{c}_t + \frac{1}{2}(1-\gamma_c)c\hat{c}_t^2 - y\hat{y}_t - \frac{1}{2}(1+\gamma_h)y\hat{y}_t^2 + G\hat{G}_t + \frac{1}{2}(1-\gamma_g)G\hat{G}_t^2) + tip$$

Next, a second-order approximation of the resource constraint  $y_t \left(1 - \frac{\psi_\pi}{2}(\pi_t - 1)^2\right) = c_t + G_t$  gives

$$RC(c_t, G_t, \pi_t, \pi_{w,t}, y_t) \approx c\hat{c}_t + \frac{1}{2}c\hat{c}_t^2 + G\hat{G}_t + \frac{1}{2}G\hat{G}_t^2 + \frac{1}{2}\psi_\pi y \pi^2 \hat{\pi}_t^2 = y\hat{y}_t + \frac{1}{2}y\hat{y}_t^2$$

Solving for  $c\hat{c}_t + \frac{1}{2}c\hat{c}_t^2$ , substituting in the approximated utility and using  $\pi = 1$ , we arrive at the loss function in the text.

### A.3 FONCs of the coordinated problem

The Lagrangean of the coordinated fiscal and monetary policy problem is given by (14). The FONCs for this problem read

$$\begin{aligned} \partial \mathcal{L} / \partial \hat{c}_t & 0 = \gamma_c c \hat{c}_t + \lambda_t^r c + \lambda_t^p \kappa_c - \lambda_t^q \gamma_c - \lambda_t^b \frac{y}{\theta_\pi} \gamma_c \\ \partial \mathcal{L} / \partial \hat{G}_t & 0 = \gamma_g G \hat{G}_t + \lambda_t^r G - \lambda_t^b G \\ \partial \mathcal{L} / \partial \hat{y}_t & 0 = \gamma_h y \hat{y}_t - \lambda_t^r y + \lambda_t^p \kappa_y - \lambda_t^b \frac{y}{\theta_\pi} (1 + \gamma_h) \\ \partial \mathcal{L} / \partial \hat{\pi}_t & 0 = \psi_\pi y \hat{\pi}_t - \lambda_t^p + \lambda_t^b \Omega \beta^{-1} \\ \partial \mathcal{L} / \partial \hat{\tau}_t & 0 = \lambda_t^p \kappa_\tau + \lambda_t^b w \tau y \frac{\theta_\pi - 1}{\theta_\pi} \\ \partial \mathcal{L} / \partial \hat{q}_t & 0 = \lambda_t^q + \lambda_t^b \Omega (1 - \rho) - \lambda_t^{zlb} \\ \partial \mathcal{L} / \partial \hat{b}_t & 0 = \lambda_t^p \beta \Theta_{1,t} + \Omega (\lambda_t^b - \mathbb{E}_t \lambda_{t+1}^b) + \lambda_t^q \Theta_{2,t} + \lambda_t^{zlb} (\Theta_{3,t} - \Theta_{2,t}) \\ \text{CSC} & 0 = \lambda_t^{zlb} (\hat{i}_t + r^*) \\ \text{ZLB} & 0 \leq \hat{i}_t + r^* \end{aligned}$$

where the  $\Theta$ 's are defined as in the text. Applying straightforward algebra on those conditions, it is possible to eliminate  $\lambda_t^p$ ,  $\lambda_t^b$ ,  $\lambda_t^r$ ,  $\lambda_t^q$  and  $\hat{G}_t$  to arrive at the reduced system in the text.

## B MODEL WITH STICKY WAGES AND ROT CONSUMERS

### B.1 Model extensions

**B.1.1 Labor unions and wage maximization.** The maximisation program of the labor union reads

$$\max_{W_{i,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi^t \left[ \frac{(1-\tau_t)W_{i,t}}{P_t} h_{i,t} - \frac{g(h(W_{i,t}, W_t, h_t))}{u'(c_{S,t})} - \frac{\psi_w}{2} \left( \frac{W_{i,t}}{W_{i,t-1}} - 1 \right)^2 y_t \right]$$

with  $g(h(W_{i,t}, W_t, h_t)) = \nu_h \frac{h_{i,t}^{1+\gamma_h}}{1+\gamma_h}$  The FONC reads

$$0 = \beta^t \frac{(1-\tau_t)}{P_t} \left[ \left( \frac{W_{i,t}}{W_t} \right)^{-\theta_w} h_t - \theta_w W_{i,t} \left( \frac{W_{i,t}}{W_t} \right)^{-\theta_w-1} \frac{h_t}{W_t} \right]$$

$$\begin{aligned}
& + \frac{\beta^t}{u'(c_{S,t})} \left[ \nu_h \left( \left( \frac{W_{i,t}}{W_t} \right)^{-\theta_w} h_t \right)^{\gamma_h} \theta_w \left( \frac{W_{i,t}}{W_t} \right)^{-\theta_w - 1} \frac{h_t}{W_t} \right] - \psi_w \left( \frac{W_{i,t}}{W_{i,t-1}} - 1 \right) \frac{y_t}{W_{i,t-1}} \\
& \quad + \beta^{t+1} \psi_w \mathbb{E}_t \left[ \frac{\xi_{t+1}}{\xi_t} \left( \frac{W_{i,t+1}}{W_{i,t}} - 1 \right) \frac{y_{t+1} W_{i,t+1}}{W_{i,t} W_{i,t}} \right]
\end{aligned}$$

using  $W_{i,t} = W_t$  under symmetric equilibrium and  $h_t = y_t$ , we arrive to (30).

**B.1.2 Market clearing.** Assuming the short bond is in net zero supply and noticing that market clearing in the bond market requires  $b_{S,t} = \frac{b_t}{1-\delta}$ , the budget constraint of the Ricardian consumer reads

$$c_{S,t} + \frac{q_t b_t}{1-\delta} = (1-\tau_t)w_t h_t + \frac{(1+\rho q_t)b_{t-1}}{1-\delta} + \frac{\int_0^1 \frac{\Pi_{i,t}}{P_t} di}{1-\delta} - \frac{\psi_w}{2}(\pi_{w,t}-1)^2 y_t - \frac{T_{S,t}}{P_t} = 0$$

where the profit of the firm is given by  $\int_0^1 \frac{\Pi_{i,t}}{P_t} di = y_t(1 - (1-s)w_t) - \frac{\psi_\pi}{2}(\pi_t - 1)^2 y_t$ .

Moreover, the consumption of the RoT household is given by

$$(1-\tau_t)w_t y_t - c_{H,t} - \frac{\psi_w}{2}(\pi_{w,t}-1)^2 y_t - \frac{T_{H,t}}{P_t} = 0$$

The government budget constraint reads

$$(1+\rho q_t)\frac{b_{t-1}}{\pi_t} + G_t - (\tau_t - s)w_t y_t - q_t b_t - \delta \frac{T_{H,t}}{P_t} - (1-\delta)\frac{T_{S,t}}{P_t} = 0$$

where  $\delta \frac{T_{H,t}}{P_t} + (1-\delta)\frac{T_{S,t}}{P_t} = \delta T_H - (1-\delta)T_S = swh$  i.e. transfers are used for the sole purpose of financing the steady state subsidy.

Solving the two first equations for  $\frac{T_{S,t}}{P_t}$  and  $\frac{T_{H,t}}{P_t}$ , respectively and substituting in the last one yields the following resource constraint

$$y_t \left( 1 - \frac{\psi_\pi}{2}(\pi_t - 1)^2 - \frac{\psi_w}{2}(\pi_{w,t} - 1)^2 \right) = c_t + G_t$$

**B.1.3 Steady state subsidy.** To obtain an efficient steady state, the subsidy must be set such that

$$MRS_S(1-\tau) = 1$$

At steady state, it holds that  $MRS_S = \frac{\theta_w - 1}{\theta_w} w$  and  $w = \frac{\theta_\pi - 1}{\theta_\pi(1-s)}$ . This gives

$$\tau = 1 - \frac{\theta_\pi \theta_w (1-s)}{(\theta_\pi - 1)(\theta_w - 1)}$$

The efficient subsidy can then be recovered from the steady state budget constraint

$$s = 1 - \frac{(\theta_\pi - 1)y}{\theta_\pi((1-q(1-\rho))b + G + \theta_w y(\theta_w - 1)^{-1})}$$

Moreover, I assume that the burden of the steady state subsidy is born unequally by the two types of households and guarantees that their respective consumption is equalised at

steady state. For the RoT consumer, this implies the following transfer

$$T_H = (1 - \tau)wy - c$$

where  $c = y - G = \delta c_H + (1 - \delta)c_S$ . As a result, the transfer of the Ricardian consumer reads

$$T_S = \frac{swy - \delta T_H}{1 - \delta}$$

This efficient steady state gives respectively the preferences for labor and government spending  $\nu_h = y^{-\gamma_h} c^{-\gamma_c}$  and  $\nu_g = G^{\gamma_g} c^{-\gamma_c}$

## B.2 Derivation of the loss function

A second-order approximation of each consumer type's utility around the efficient steady state yields

$$\begin{aligned} U_j(c_{j,t}, y_t, G_t) \approx & c^{1-\gamma_c} \hat{c}_{j,t} + \frac{1}{2}(1 - \gamma_c)c^{1-\gamma_c} \hat{c}_{j,t}^2 - \nu_h y^{1+\gamma_h} \hat{y}_t - \frac{1}{2}\nu_h(1 + \gamma_h)y^{1+\gamma_h} \hat{y}_t^2 \\ & + \nu_g G^{1-\gamma_g} \hat{G}_t + \frac{1}{2}\nu_g(1 - \gamma_g)G^{1-\gamma_g} \hat{G}_t^2 + tip \end{aligned}$$

for  $j \in \{H, S\}$

At the efficient steady state, we have  $\nu_g = G^{\gamma_g} c^{-\gamma_c}$  and  $\nu_h = y^{-\gamma_h} c^{-\gamma_c}$ , thus we can write

$$U(c_{j,t}, y_t, G_t) \approx c^{-\gamma_c} (c \hat{c}_{j,t} + \frac{1}{2}(1 - \gamma_c)c \hat{c}_{j,t}^2 - y \hat{y}_t - \frac{1}{2}(1 + \gamma_h)y \hat{y}_t^2 + G \hat{G}_t + \frac{1}{2}(1 - \gamma_g)G \hat{G}_t^2) + tip$$

Next, a second-order approximation of the resource constraint  $y_t \left(1 - \frac{\psi_\pi}{2}(\pi_t - 1)^2 - \frac{\psi_w}{2}(\pi_{w,t} - 1)^2\right) = c_t + G_t$  gives

$$\begin{aligned} RC(c_{H,t}, c_{S,t}, G_t, \pi_t, \pi_{w,t}, y_t) \approx & \delta(c \hat{c}_{H,t} + \frac{1}{2}c \hat{c}_{H,t}^2) + (1 - \delta)(c \hat{c}_{S,t} + \frac{1}{2}c \hat{c}_{S,t}^2) \\ = & y \hat{y}_t + \frac{1}{2}y \hat{y}_t^2 - G \hat{G}_t - \frac{1}{2}G \hat{G}_t^2 - \frac{1}{2}\psi_\pi y \pi_t^2 \hat{\pi}_t^2 - \frac{1}{2}\psi_w y \pi_w^2 \hat{\pi}_{w,t}^2 \end{aligned}$$

Substituting in the weighted approximated utility  $U = \delta U_H + (1 - \delta)U_S$  and using  $\pi = \pi_w = 1$ , we arrive at the loss function in the text.

## C COMMITMENT

Analysing the case of commitment is useful to understand the key role played by government debt in influencing policy outcomes when time-consistency is imposed and the first-best is out of reach. Under commitment, the government chooses the complete time path of variables in the initial period subject to the equilibrium conditions. In particular, the budget constraint only has to be satisfied intertemporally while the period-by-period level of debt is obtained residually given the optimal plan for other variables. Following the notation in the text for the multipliers, the FONCs of the extended model with commitment read

$$\begin{aligned} \partial \mathcal{L} / \partial \hat{c}_{H,t} & 0 = \gamma_c c \delta \hat{c}_{H,t} - \lambda_t^r c \delta + \lambda_t^h \\ \partial \mathcal{L} / \partial \hat{c}_{S,t} & 0 = \gamma_c c (1 - \delta) \hat{c}_{S,t} - \lambda_t^r c (1 - \delta) + \lambda_t^\mu \gamma_c + \lambda_t^q - \lambda_{t-1}^q \end{aligned}$$

$$\begin{aligned}
\partial\mathcal{L}/\partial\hat{G}_t & 0 = \gamma_g G\hat{G}_t + \lambda_t^r G - \lambda_t^b G \\
\partial\mathcal{L}/\partial\hat{y}_t & 0 = \gamma_h y\hat{y}_t + \lambda_t^r y + \lambda_t^\mu \gamma_h - \lambda_t^h + \lambda_t^b (\tau - s)wy \\
\partial\mathcal{L}/\partial\hat{\pi}_t & 0 = \psi_\pi y\hat{\pi}_t + \lambda_{t-1}^q \beta^{-1} + \lambda_t^p \psi_\pi - \lambda_{t-1}^p \psi_\pi + \lambda_t^i + \lambda_t^b \Omega \beta^{-1} \\
\partial\mathcal{L}/\partial\hat{\pi}_{w,t} & 0 = \psi_w y\hat{\pi}_{w,t} + \lambda_t^w \psi_w - \lambda_{t-1}^w \psi_w - \lambda_t^i \\
\partial\mathcal{L}/\partial\hat{\tau}_t & 0 = \tau(1 - \tau)^{-1} \lambda_t^\mu + \tau(1 - \tau)^{-1} \lambda_t^h + \lambda_t^b wy\tau \\
\partial\mathcal{L}/\partial\hat{\mu}_t & 0 = \lambda_t^w (1 - \tau)(\theta_w - 1)w + \lambda_t^\mu \\
\partial\mathcal{L}/\partial\hat{w}_t & 0 = -\lambda_t^\mu - \lambda_t^h - \lambda_t^p (\theta_w - 1) + \lambda_t^i - \beta \lambda_{t+1}^i + \lambda_t^b (\tau - s)wh \\
\partial\mathcal{L}/\partial\hat{q}_t & 0 = \lambda_t^q - \lambda_{t-1}^q \rho + \lambda_t^b \Omega (1 - \rho) - \lambda_t^{zlb} + \lambda_{t-1}^{zlb} \\
\partial\mathcal{L}/\partial\hat{b}_t & 0 = \lambda_t^b - \mathbb{E}_t \lambda_{t+1}^b \\
\text{CSC} & 0 = \lambda_t^{zlb} (\hat{i}_t + r^*) \\
\text{ZLB} & 0 \leq \hat{i}_t + r^*
\end{aligned}$$

The FONCs for the benchmark model can be easily recovered by setting  $\psi_w = 0$  and  $\delta = 0$  (in this case,  $\lambda_t^i = 0$  and  $\lambda_t^w$  becomes slack).

As can be seen from the last FONC on government debt, the multiplier attached to the government budget constraint is a near random walk. This characteristic reflects the consumption smoothing motive of the planner in the presence of distortionary taxes on labor income (see e.g. [Schmitt-Grohé and Uribe, 2004](#)). Figure 5 compares the IRFs to a large negative demand shock (four standard deviations) for two models: a model with flexible wages and representative agent (solid-blue line) and a model with sticky wages and RoT consumers ( $\delta = 1/4$ , dashed-red line).<sup>15</sup> The drop in consumption is curbed under commitment ( $\hat{c}_1 \approx -1\%$  instead of  $\hat{c}_1 \approx -2\%$  under discretion) because of the positive effect of forward guidance that lowers expectations about real interest rates (e.g. [Eggertsson and Woodford, 2003](#)). Through this commitment, monetary policy alone is able to offset most of the adverse effects arising from the zero lower bound and thus government spending plays only a modest role in stabilising the economy in line with [Schmidt \(2013\)](#). Interestingly, the path of government debt and output is very close between the two models reflecting the lower dependence of stabilisation outcomes to the distortions in the model when the policy maker can commit. Nevertheless, the path of labor taxes is now qualitatively different. The presence of sticky wages and RoT consumers overturns the rise in taxes observed in the benchmark model. This result confirms (in an optimal model) the multiplicative effect of a tax-cut found by [Kaszab \(2016\)](#) in a similar model with rules.

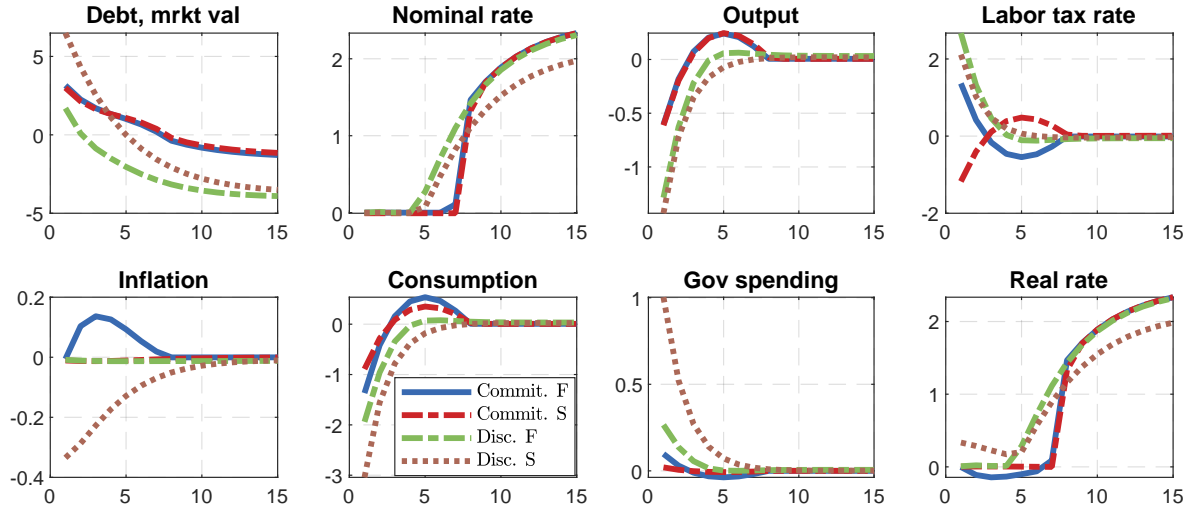
## D SOLUTION METHOD

The non-linearity of the time-consistent solution stemming from the occasionally binding ZLB constraint precludes traditional local methods. Instead, I approximate the linear policy functions using a projection method (collocation) on a finite number of points in the domain. Since the policy functions are linear with a kink, I rely on cubic splines for the basis function. The algorithm proceeds by checking convergence on two nested loops as follows:

1. Construct the grid for the state variables. Use a Gaussian quadrature scheme to discretise the normally distributed innovations to private demand.

<sup>15</sup>Under commitment the number of state variables increases to seven. This renders the projection method on a grid unhandy. Hence, I solve the model using the piece-wise solution method described in [Guerrieri and Iacoviello \(2015\)](#). The perfect foresight assumption of this method is consistent with my collocation method when I turn off uncertainty.

Figure 5: Impulse responses to a contractionary demand shock when fiscal and monetary policy can commit to future plans



**Notes:** See notes under Figure 4. The solid-blue line and the dashed-red line correspond to the case of commitment with flexible wages and sticky wages, respectively. The dashed-dotted-green line and the dotted-brown line correspond to the case of discretion with flexible wages and sticky wages, respectively. With sticky wages, the share of RoT consumers is set to  $\delta = 1/4$ .

2. Use the linear policy functions of the commitment solution from Dynare (outside the ZLB) to obtain an initial guess for the basis coefficients.
3. *Outer loop:* With the current guess of the basis coefficients, approximate the partial derivative of the expectation terms with respect to the endogenous states.
4. *Inner loop:* To solve the system of equilibrium equations, I proceed as follows.
  - (i) Use the guess of the basis coefficients to recover government debt from the budget constraint at the collocation nodes.
  - (ii) Build the new grid and approximate the expectation terms associated with next period's decisions.
  - (iii) Solve the linear system outside the ZLB with matrix inversion.
  - (iv) Verify whether ZLB constraint binds. In the affirmative, solve the alternative system at the ZLB and check KKT conditions.
  - (v) Update the guess for the basis coefficients based on the decision rules for the current period. If the difference between the old and the new guess is smaller than  $1.49e^{-8}$ , the inner loop has converged. Otherwise, go back to step (i).
5. Update the guess for the the partial derivative of the expectation terms with respect to government debt based on basis coefficients obtained from (4). If the difference between the old and the new guess is smaller than  $1.49e^{-8}$ , the outer loop has converged. Otherwise, go back to step (3).

I implement this procedure in Matlab by relying on the CompEcon toolbox of Miranda and Fackler (2002) for the function evaluation at the collocation nodes.



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