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## The Debt-to-GDP Ratio as a Tool for Debt Management: Not Good for LICs

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# The Debt-to-GDP Ratio as a Tool for Debt Management: Not Good for LICs

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# The Debt-to-GDP Ratio as a Tool for Debt Management: Not Good for LICs

## Abstract

There have been criticisms of debt sustainability analysis in general, including the IMF's own evaluation of the usefulness of its debt sustainability methodology (e.g., IMF, 2017). This paper's focus is narrow. On the basis of theoretical arguments and empirical evidence, it argues that the debt-to-GDP ratio is a poor metric for debt management in low-income countries (LICs). It makes a case for explicit revenue-based metrics of debt management. In LICs or countries with weak institutions, the debt-to-GDP may be manipulated by understating the stock of debt, resorting to dubious accounting methods, and there is a weak correlation between GDP and revenue as result of inefficiencies in the tax administration and a large informal sector. It is also a relatively inefficient predictor of debt distress. Other reasons are given in the paper.

JEL-Codes: H630, E620.

Keywords: debt-to-GDP ratio, debt service-to-revenue ratio, debt sustainability, liquidity, solvency.

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## 1. Introduction

Twenty-three years after the Heavily Indebted Poor Country (HIPC) initiative, fears of debt distress or default by developing countries have resurfaced. As part of the African Monetary Co-operation Program (AMCP) for developing economies, the IMF suggested a maximum debt-to-GDP ratio of 60%. Ghana's debt-to-GDP ratio is now more than 100% and it has currently suspended servicing its external debt, will not pay interest on more than \$13 billion worth of domestic debt in 2023, and is negotiating with external and domestic creditors to restructure its debt. Sri Lanka defaulted on its debt in 2022 and Zambia defaulted on its external debt in 2020 and is also negotiating with creditors.

In the analysis of debt sustainability, various metrics or indicators are used. These include debt-to-GDP ratio, debt service-to-exports ratio, and debt service-to-revenues ratio, etc. All three ratios are used in the IMF-World Bank debt sustainability analysis and by international credit rating agencies like Moody's and Fitch. However, in practice, the debt-to-GDP ratio gets a disproportionate attention in public discussions and it is, by and large, the metric quoted by various governments, researchers, the IMF, etc. Work by Carmen Reinhart and Kenneth Rogoff of Harvard University has ignited a debate on whether there is a threshold debt-to-GDP ratio beyond which high levels of public debt stifle economic growth (Reinhardt and Rogoff, 2010; Ghosh et al., 2013; Chudik et al., 2017). An arbitrary threshold of 70% for debt-to-GDP ratio has been mentioned in budget statements by the government of Ghana.

If a researcher decides to use only one of the metrics of debt sustainability mentioned above, she will typically use the debt-to-GDP ratio. For example, debates about debt sustainability between the major political parties in Ghana, Nigeria, Kenya, and other countries invariably focus on the debt-to-GDP ratio. In spite of its prominence and popularity, I shall argue

in this paper that the debt-to-GDP ratio is a poor or misleading metric for low-income countries (LICs). My focus is narrow.<sup>1</sup>

## 2. The debt-to-GDP ratio as a metric for debt management: a brief intellectual history

According to Ley (2010), “It is useful to normalise the (debt)... by some measure of the government’s *ability to service and repay its debt ... The most common choice used for normalising government debt is GDP.*” Italics mine. Cassimon et al (2008) refer to normalizing government debt by GDP as the “conventional approach to fiscal sustainability” and in IMF (2022), it is stated that “Public debt is typically analyzed when expressed as a percent of GDP.”

It is obvious that governments do not directly service and repay public debts from GDP. They service and repay debts from sources of cash flow: royalties, tax revenue, export revenue, and bonds (loans). Why then do we use debt-to-GDP ratios? When was debt management first linked to national income (GDP)? A pioneering paper that was published in the *American Economic Review* in 1944 (during World War II) by Evsey Domar, then at the US Federal Reserve, provides an answer to this question. Its title was “The ‘Burden of the Debt and the National Income.’” In this paper, Evsey Domar observed that:

*“The phrase “burden of the debt,” ... evidently refers to the tax rate (or rates) which must be imposed to finance the (debt) service charges, and that the tax rate will rise is far from evident. ... It has been pointed out ... particularly by Professor Alvin Hansen, that **the debt problem should be studied in its relation to national income, and that with a growing national income the “debt burden” is likely to be confined within manageable limits.**”*

Evsey Domar concluded his paper by stating that “*It is hoped that this paper has shown that the problem of the debt burden is essentially a problem of achieving a growing national income .... The faster income grows, the lighter will be the burden of the debt.*”

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<sup>1</sup>Debrun (2020), for example, present a discussion of the general issues of debt sustainability.

In this pioneering work, Domar (1944) derived what has become known as the “Domar debt sustainability condition”. That is, GDP should grow faster than public debt for the debt to be sustainable. Domar dispelled fears of debt accumulation and emphasized the need to increase economic growth at a rate that was faster than the rate of accumulation of public debt. He showed that for the public debt of a country to be sustainable, the nominal rate of growth of national income should be greater than the growth rate of nominal public debt. As long as this condition is met any level of public debt is sustainable and the country remains solvent. He showed that the debt-to-GDP ratio and the debt burden (i.e., the tax rate) approached a finite limit that is less than 100%. Evsey Domar’s paper in 1944 was the genesis of how the debt-to-GDP ratio became the central metric in the analysis of debt sustainability.

### **3. Why the debt-to-GDP ratio is a poor metric for debt sustainability in LICs**

#### *(i) Weak correlation between revenue and GDP in LICs*

Domar (1944) made a crucial assumption. He assumed a directly proportional and fixed relationship between a country’s tax revenue and its national income (GDP) and that any debt service payment can be financed via taxes on national income (GDP). On page 802 of his paper, he wrote “*It will be assumed that (debt) service charges are raised by means of a proportional income tax imposed on the total taxable income (without any exemptions).*” Based on this assumption, it follows that tax revenue and national income will grow at the same rate. That is, in this case, a 1% increase in national income results in a 1% increase in a tax revenue and thus an increase in debt-service capacity. And when a country’s currency is internationally convertible because it is, for example, a global reserve currency like the dollar, domestic tax revenue is what is required to service external debt. However, this argument does not hold for LICs whose

domestic currency is not internationally accepted and have a weak relationship between tax revenue and GDP.

In fact, Debrun et al (2020) made an explicit connection between revenues and GDP in debt sustainability analysis when they stated that:

*“Because the economy’s taxable income roughly grows with nominal GDP, it is common to scale the nominal amounts in identity (2)<sup>2</sup> in terms of ratios to nominal GDP. The idea is simply that if government’s revenues can grow indefinitely, so could expenditure and debt.”*

It is instructive to note that *tax buoyancy* measures the response of tax revenue to changes in GDP and is interpreted as the percentage change in tax revenue in response to a 1 percent change in GDP. A tax buoyancy value of 1 means that a 1 percentage increase in GDP leads to a 1 percentage increase in tax revenue (Dudine and Jalles, 2017). A tax buoyancy of 1 or a strong relationship between tax revenue and GDP is likely if there is/are (a) no corruption or inefficiencies in tax administration, (b) few tax exemptions, and (c) the informal sector is small, so there are almost no taxpayers outside the tax net. Otherwise, a country’s tax revenue may grow at a much slower rate than its nominal GDP. In short, we expect a tax buoyancy value of close to 1 if tax collection/administration is efficient or there is no waste in tax administration. There is no guarantee that these conditions (as implicitly assumed in, for example, Domar, 1994; Debrun, 2020) hold in low-income countries (LICs). Dudine and Jalles (2017) estimated a tax buoyancy of less than 1 for some low-income countries. Even if tax buoyancy is greater than 1 in low-income countries, what is the point of using a proxy (i.e., GDP) when there is data available for the main variable (i.e., revenue) of interest? I elaborate on this point below.

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<sup>2</sup>In Debrun et al. (2020), the identity in (2) refers to an expression for the evolution of debt. I discuss this identity below.



In its credit rating methodology, Moody's (2020) has a "fiscal strength" category that includes the key subcategories "debt burden" and "debt affordability". Debt burden includes debt-to-GDP ratio and debt-to-revenue ratio, and debt affordability includes interest payments-to-GDP ratio and interest payments-to-revenue ratio. Explaining the importance of GDP and the growth of GDP in the determination of a country credit rating, Moody's observed that:

*"A sovereign's ability to generate **sufficient revenue to service debt** over the medium term relies on sustained economic growth and prosperity. National Income ... is a **proxy for the revenue-generating** potential of a sovereign. ... Low or volatile levels of economic growth can, if sustained over a number of years, amplify debt serviceability challenges and can render a heavy debt burden unsustainable. ... Scale is an important indicator of an economy's diversity and complexity, which greatly influences **its ability to withstand shocks** and hence a **sovereign's capacity to generate stable revenue streams to service its debt**. For example, a very small country with a competitive economy but concentrated exposure to a few sectors can be subject to abrupt economic shifts, which can undermine a **sovereign's ability to raise revenue** from within the economy." (bold font mine).*

Clearly, the focus is on revenue because revenue because revenue is ultimately required for debt servicing. But a country's revenue is from its GDP and therefore the growth and volatility of GDP must be considered. One can focus on GDP (*without focusing on the debt-to-GDP ratio*) and factors like the strength of economic, legal, and political institutions to forecast economic growth, volatility of revenues, capacity to absorb negative shocks, the primary fiscal balance, the evolution of the stock of debt, the risk of debt default, etc.

Knowledge of the growth of the stock of debt and GDP is necessary to estimate future debt service costs and revenues. A higher stock of debt implies a higher debt service cost, even if interest rates are constant, and an increasing stock of debt can lead to rising interest rates. A higher GDP, all things being equal, implies a higher government revenue. But these considerations do not imply the use of the debt-to-GDP ratio. The outcome of these calculations (i.e., debt service costs and revenue), not the debt-to-GDP ratio, should be the ultimate focus of debt management.

(ii) *Using debt-to-GDP ratio may not be incentive compatible in LICs*

In countries with weak institutions, using the debt-to-GDP ratio leads to irresponsible fiscal management and perverse incentives. The debt-to-GDP ratio can be manipulated by understating the debt stock (e.g., the off-budget operations pointed out by the IMF in its 2019 DSA of Ghana) and inflating GDP (e.g., when GDP is rebased).

In 2009, Greek debt was downgraded after many statistical discrepancies, including underreporting of public debt, were exposed. This resulted in immediate downgrades of Greek debt.<sup>3</sup> In Ghana, the ministry of finance reported a debt-to-GDP ratio of 76.6% by the end of the December 2021<sup>4</sup> and, in his budget speech on November 24, 2022, the minister of finance said that "... the Public Debt-to-GDP ratio stood at 75.9 percent at the end of September 2022." But, on December 5, 2022 (less than two weeks later after the budget was read), the government -- as part of a debt sustainability analysis to get a three-year IMF facility -- admitted that the debt-to-GDP ratio was more than 100%. The opposition in Ghana accused the government of fudging the numbers.

In Ghana, ESLA bonds (to pay for energy debt), Daakye bonds (to pay for education), and the debt of state-owned companies, etc were not included in public debt on the dubious grounds that Daakye bonds, ESLA bonds, etc were separate because they were issued by Special Purpose Vehicles (SPVs): E.S.L.A. Plc and Daakye Trust Plc. Yet, in Ghana's debt restructuring program announced on December 5, 2022, the bonds issued by Daakye Trust Plc and ESLA Plc (more than \$2 billion) will be exchanged for bonds issued by the Government of Ghana.<sup>5</sup>

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<sup>3</sup>Greece condemned for falsifying data, The Financial Times, January 12, 2010: <https://www.ft.com/content/33b0a48c-ff7e-11de-8f53-00144feabdc0>

<sup>4</sup><https://mofep.gov.gh/public-debt/debt-newsletters>

<sup>5</sup> <https://mofep.gov.gh/press-release/2021-12-06/ghanas-domestic-debt-exchange-2022-exchange-memorandum>

In their debt sustainability analysis (DSA) for Ghana, the IMF-World Bank use a more holistic and reasonable definition of public debt, which is public and publicly-guaranteed (PPG) debt. For example, in their 2019 DSA for Ghana, the IMF and World Bank stated that "The DSA covers public and publicly guaranteed debt of the central government. It includes several state-owned enterprise (SOE) loans not explicitly guaranteed by the state for infrastructure and power projects." And in the same DSA report in 2019, it stated that "Off-budget operations, including ESLA, Sinohydro, and GETFund, contribute to public debt but their decentralized and not always transparent nature complicates oversight and management of public financing, raising the scope for corruption."

According to the IMF (2020):

*"Efforts to expand the coverage of public debt data in LICs to include SOEs have in several cases identified sizable amounts of public and publicly-guaranteed debt not previously captured (e.g., around 10 percent of GDP in the Republic of Congo)."*

In a press release by the World Bank on December 6, 2022, a senior Vice President and Chief Economist was quoted as saying that:

*"Poor debt transparency is the reason so many countries sleepwalk into a debt crisis ... Complete, transparent debt data improves debt management. It makes debt sustainability analyses more reliable. And it makes debt restructurings easier to implement."*<sup>6</sup>

In contrast, using a measure like the debt service-revenue ratio is more compatible with incentives. It is much more difficult to manipulate the debt service-to-revenue ratio because a government cannot understate its debt service payments (debt service payments can easily be verified) and it has no incentive to inflate its revenues.

A counter argument is that interest payments depend on the total stock of debt, so the total stock of debt can be inferred from interest payments. But that also requires knowledge of

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<sup>6</sup> "Debt-Service Payments Put Biggest Squeeze on Poor Countries Since 2000", The World Bank, DECEMBER 6, 2022): <https://www.worldbank.org/en/news/press-release/2022/12/06/debt-service-payments-put-biggest-squeeze-on-poor-countries-since-2000>

various interest rates on the debt. Besides, if data on debt service costs is available was is the point of focusing on a proxy (i.e., the debt-to-GDP ratio).

(iii) *The debt-to-GDP ratio is an inefficient predictor of debt stress*

The IMF-World Bank debt sustainability Framework (DSF) was introduced in April 2005 and is periodically reviewed. The current framework was approved by the IMF and World Bank Executive Boards in September 2017 and has been implemented since July 2018.<sup>7</sup>

The DSF is based on debt burden thresholds of five debt indicators: (i) Present Value (PV) of debt-to-GDP ratio; (ii) PV of debt-to-exports ratio; (iii) PV of debt-to-revenue ratio; (iv) debt service-to-exports ratio; and (v) debt service-to-revenue ratio.<sup>8</sup> The thresholds are country-specific and are derived from the World Bank's Country Policy and Institutions Assessment (CPIA) in terms of the quality of policy. Debt burden thresholds are set relatively higher for countries with CPIA scores. CPIA-determined debt burden thresholds are not the focus of this paper. The CPIA approach has been the subject of critique (e.g., Ferrarini, 2009, IMF, 2017).

Note that four of the five metrics are based on revenue (export revenue, tax revenue, etc) and the stock of debt is not used. The PV (present value) of debt is future projected debt-service payments, not the total stock of debt, discounted by market-based interest rates.

An examination of the IMF-World Bank debt sustainability analyses (DSAs) of Ghana for March 2015, August 2015, April 2018, November 2019 and other DSAs reveals that the debt

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<sup>7</sup><https://www.imf.org/en/About/Factsheets/Sheets/2016/08/01/16/39/Debt-Sustainability-Framework-for-Low-Income-Countries>

<sup>8</sup> In the 2017 review, the PV of external debt-to-revenue was dropped. It is no longer used.

service-to-revenue ratio was the most predictive indicator of debt distress in the sense that Ghana consistently breached this threshold when others like the debt-to-GDP ratio were not breached (or not breached in all years).<sup>9</sup>

Strictly speaking, breaching a threshold alone is not an indicator of the predictiveness of a metric. The breach must be juxtaposed against observed debt distress. Thus, a metric is poor predictor if debt distress was observed but its debt threshold was not breached or when debt distress was not observed but the debt threshold was breached (IMF, 2017).

In 2019, the IMF's DSA led to the conclusion that Ghana was at a **high risk** of external public debt distress with thresholds breached on the external debt to GDP ratio, the debt service-to-exports ratio, and the external debt service-to-revenues ratio. However, it sounded a conditional positive note by indicating that *if* the government was committed to fiscal/macroeconomic discipline and had favorable access to the Eurobond market, then Ghana's unsustainable debt could be on a sustainable path. In late 2021, Ghana's access to the Eurobond market was, amid ratings downgrades, blocked because the market saw Ghana as a very high-risk borrower. In 2022, Ghana announced suspensions of interest payments on external and domestic debt.

It is important to note that the weakness of a metric as a predictor of debt distress does not necessarily imply that the metric is not good. Rather, it could be that the threshold for that

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<sup>9</sup> In its 2019 debt sustainability analysis (DSA) for Ghana, the IMF concluded that “Ghana is at high risk of external public debt distress with thresholds breached on the PV (present value) of external debt to GDP ratio, the debt service-to-exports ratio, and the external debt service-to-revenues ratio, **with the latter exceeding the threshold throughout the forecast horizon.**” And in its March 2015 DSA, it concluded that “all indicators **but debt service-to-revenue ratio** would remain below the policy dependent thresholds by comfortable margins under the baseline ...” In November 2019, it warned that “The risk of debt distress rating in the Debt Sustainability Analysis (DSA) remains high ... The DSA rating is **mainly driven by debt service to revenue exceeding the threshold throughout the forecast horizon.**” (all bold font mine).

metric was poorly chosen or estimated (IMF, 2017)<sup>10</sup> or stem from the inherent uncertainty of forecasting the variables in debt sustainability analysis (Debrun et al., 2020). In the case of LICs, one should be guided by the economic arguments in the paper to explain why the debt-to-GDP ratio is a poor metric. While the technical analysis of debt sustainability may give warning signs of a debt distress, this may not result in actual debt distress because the capital market may still give access to a country for some time and thereby allow the country to roll over the debt.

(iv) *The debt-to-GDP ratio compares a flow variable with a stock variable*

Export revenue or tax revenue or the size of the agricultural sector as a percentage of GDP compares a flow variable with another flow variable and, as expected, gives a percentage that does not exceed 100%. While the debt service-revenue ratio compares a flow variable (debt service) with a flow variable (revenue), the debt-to-GDP ratio compares GDP, a flow variable (output per period of time), to debt, a stock variable (debt accumulated at a point in time). It is not surprising that a country like Japan has a debt-to-GDP ratio of more 250%. Yet, the Japanese government can borrow at very low interest rates and has relatively low debt service costs. There is something much more important that determines debt sustainability.<sup>11</sup>

When Moody's refers to the "debt burden", it uses the total stock of debt but when it refers to "debt affordability", it uses interest payments. The total stock of debt as a percentage of GDP or revenue is used as indicator of the weight of a country's debt. But the total stock of debt does not have to be repaid immediately. The debt-to-GDP ratio as a measure of the debt burden is hypothetical. It is as though it looks at the percentage of GDP required to hypothetically pay

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<sup>10</sup>In the 2017 review, the IMF found that "while debt projections have been accurate in the near-term, they have tended to underestimate outcomes in the medium-term." This affects all debt indicators. As expected, underestimation errors in debt forecasts increase as the projection horizon is extended (IMF, 2017).

<sup>11</sup>It is not surprising that Debrun et al. (2020) observed that "... countries like Japan defy gravity with gross public debt levels above 200 percent of GDP, while others default on a considerably smaller stock of obligations (30 percent of GDP in Ukraine)."

off the debt. But the true burden of the debt stems from the taxes imposed on residents to service the debt. This was what Domar (1944) meant by “The phrase “burden of the debt,” ... evidently refers to the tax rate (or rates) which must be imposed to finance the (debt) service charges.”

The trajectory of debt affordability should be the metric for debt sustainability because the debt will be serviced over time, not at one point in time. This is consistent with the *intertemporal solvency condition* for debt sustainability. This condition requires that the initial debt plus the future stream of primary expenditure should equal the future stream of revenue (e.g., Debrun et al, 2020; Hamilton and Flavin, 1986). Future streams need to be discounted by calculating their present discounted value, using the interest rate paid on debt. The debt is sustainable if the intertemporal solvency condition is satisfied. That is, if the expected present value of future primary fiscal balances (revenue minus non-interest expenditure) covers the existing stock of debt. The long-run ability to service the debt includes the capacity to service the debt without perpetually rolling it over, a sort of “no Ponzi” condition.<sup>12</sup> Strictly speaking, long-run debt sustainability requires the ability to service the debt (including paying off maturing debt) *in each period*. That is, the government must be liquid in each period. This is consistent with Debrun et al. (2020) who observed that “liquidity crises triggered by senseless panic can lead otherwise solvent governments to default” and “By referring to sustainability as the ability to service debt, IMF (2014) effectively lumps together solvency and liquidity.”

The IMF and World Bank use present-value metrics like (PV of debt)/GDP and (PV of debt)/exports as indicators of solvency. Solvency refers to an entity's capacity to meet its long-term financial commitments while liquidity refers to an entity's ability to pay short-term

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<sup>12</sup>But whether a government can roll over the debt in perpetuity is a subject of debate (Cuddington, 1997; Debrun et al., 2020; Hamilton and Flavin, 1986).

obligations. But even if a country is solvent, it has to meet its debt service obligations when they are due (i.e., it has to be liquid). Being solvent may help a country to borrow to meet current debt service obligations just as a central bank is expected to act as a lender of a last resort (provide liquidity) to a commercial bank only if the bank is solvent.

*(v) The debt-to-GDP ratio and revenue-based indicators of debt are not independent*

It makes sense to use different ratios if the ratios are independent just as it makes sense to diversify a portfolio if the assets in the diversified portfolio are independent or are negatively correlated but independently determined.

In the case of debt management, we know that the stock of debt is primarily driven by the fiscal primary balance (**revenue** minus expenditure, excluding interest costs). The debt service-to-GDP ratio and the debt-to-GDP ratio are not independent of each other. Once a path is determined for the debt service ratio-revenue ratio, that will automatically determine a path for the debt-to-GDP ratio.

The dynamics of debt is based on a simple equation that it is driven by the primary fiscal balance. Let  $D_t$  denote the stock of government debt at the end of year  $t$  (the current year), let  $i$  be the (average) interest rate, and  $B_t$  the primary fiscal balance. The primary balance is government revenue minus government expenditure (excluding interest payments on the debt). Therefore,  $B_t > 0$  means that the government runs a primary surplus and  $B_t < 0$  means a primary deficit. Then

$$D_t = D_{t-1} + iD_{t-1} - B_t, \tag{1}$$



where  $D_{t-1}$  is the stock of debt in the previous year ( $t - 1$ ) and  $D_t$  is the stock of the debt in the current year. Equation (1) is the basic/fundamental equation in debt sustainability analysis.<sup>13</sup>

Let the growth rate of revenue,  $R$ , be  $g$ . We may write  $R_t = (1 + g)R_{t-1}$ . Let  $d_t$  = debt-to-GDP ratio;  $r_t$  = debt service-to-revenue ratio;  $b_t$  = primary fiscal balance as percentage of revenue;  $\alpha$  = revenue-to-GDP ratio;  $i$  = (average) interest rate on the debt. Then, dividing equation (1) by  $R_t$ , it can be shown that the debt-to-GDP ratio,  $d_t$ , is related to the debt service-to-revenue ratio,  $r_{t-1}$ , according to the equation:

$$d_t = \alpha \left( \left( \frac{1+i}{i} \right) \frac{r_{t-1}}{1+g} - b_t \right). \quad (2)$$

Equation (2) shows that there is a relationship between the debt-to-GDP ratio and the debt service-to-revenue ratio and the primary fiscal balance.<sup>14</sup> If we choose fixed values (what one might call steady-state values) of these variables (i.e.,  $r_{t-1}$  and  $b_t$ ), then we get a steady-state value for  $d_t$  according to equation (2).<sup>15</sup>

One may argue that if there is a deterministic relationship between the debt-to-GDP ratio and the debt service-to-revenue, then the metric used should not matter. This argument is not correct because, as argued above, in countries with weak institutions, it is easier to manipulate the debt-to-GDP ratio than it is to manipulate the debt service-to-revenue ratio. Furthermore, if there is a stochastic or weak relationship between the two variables because, for example, shocks

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<sup>13</sup>This equation, and its variants (some of which include seignorage revenue), is well known in debt sustainability analysis (e.g., Cuddington, 1997; Ley, 2010; Ghosh et al., 2013; Hamilton and Flavin, 1986; Wilcox, 1989).

<sup>14</sup>If we had divided equation (1) through by GDP, we would have obtained a difference equation involving  $d_t$  and  $d_{t-1}$ . Equation (2) is not a difference equation.

<sup>15</sup>To give another example, Moody's assigns its lowest credit rating of "Ca" (i.e., likelihood of being near or in default) if the interest payments-to-GDP ratio exceeds 7.5% or if the interest payments-to-revenue ratio exceeds 30%. But it can easily be shown that a revenue-to-GDP ratio of 25% implies that if the interest payments-to-revenue ratio exceeds 30%, then the interest payments-to-GDP ratio exceeds 7.5%.

to the revenue-to-GDP ratio or the primary fiscal balance, then it is important to focus on the revenue-based metric for reasons given in this paper. There is no need to use an imperfect proxy.

In a (probit) regression to determine the probability of a default, both metrics may be used because this may improve the econometric estimates. A very high debt-to-GDP can have a negative effect on growth and revenues and increase the interest rate at which the government borrows. But this econometric approach is different from the debt sustainability exercises that *separately* check whether debt-to-GDP ratio, debt service-to-revenue ratio, and other metrics breach country-specific thresholds.<sup>16</sup> This is what dominates public discussions of debt sustainability with the debt-to-GDP ratio being the disproportionately prominent indicator.

(vi) *Why use a proxy when it is not necessary?*

If data for revenues (e.g., tax revenue and export revenues) exist, it is not necessary to use a *proxy* (i.e., GDP), which has a weak relationship with revenue. In this case, it may be counterproductive to use an imperfect proxy.

A proxy is used when the ultimate variable of interest is not available. However, to the extent that international credit rating agencies (e.g., Standard & Poor's, Moody's, and Fitch) use, in conjunction with other metrics, debt-to-GDP ratios and investors use these credit ratings to guide their decisions, reporting debt-to-GDP ratios may be helpful. But an inordinate focus on an imperfect proxy metric instead of the fundamental metric implies that one may go way above the threshold value of the fundamental metric (e.g., debt service-to-revenue ratio) before realizing that the debt is unsustainable. It smacks of poor fiscal planning that does not allow timely corrective measures to be put in place.

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<sup>16</sup>In fact, the IMF makes a distinction between the probability approach and the debt threshold approach (e.g., IMF, 2017).

#### 4. Conclusion

There have been criticisms of debt sustainability analysis in general, including the IMF's own evaluation of the usefulness of its debt sustainability methodology (e.g., IMF, 2017). The focus of this paper is narrow. It has pointed out shortcomings in the use of the debt-to-GDP ratio as a metric for determining debt sustainability in low-income or developing countries.

Commenting on Ghana's debt, the IMF's Resident Representative in Ghana, Albert Touna-Mama stated in February 2020 that<sup>17</sup>:

*“When we think about debt and borrowing, ... we don't only measure it with respect to GDP. **An important metric that we look at and in the case of Ghana** is a metric that is of concern is **debt service to revenue**. ... We use debt service to revenue as a proxy of how sustainable the debt of Ghana is. At the moment, that ratio is close to 30 percent. ...we are concerned about the borrowing of Ghana.”*

Some of the reasons given in this paper to explain why the debt-to-GDP ratio is a poor metric for debt sustainability analysis are not peculiar to LICs. However, LICs are the focus of this paper because of their weak institutions, much greater vulnerability to debt crises, and limited capacity to deal with unfavorable economic shocks.

No country pays and services its debt from GDP. Debt is paid and serviced from revenues. LICs must focus on what matters for their capacity to service their debts: revenues. Ultimately, it is a country's revenue, fiscal and overall economic management that matter for debt sustainability. It need not be a puzzle that “... in Africa high economic growth has not translated into better sovereign ratings.”<sup>18</sup> Revenue-based measures should be given much more prominence in the debt management of LICs.

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<sup>17</sup> <https://www.ghanaweb.com/GhanaHomePage/business/We-are-concerned-about-the-borrowing-of-Ghana-IMF-Country-Rep-871075>

<sup>18</sup> <https://theconversion.com/african-countries-arent-borrowing-too-much-theyre-paying-too-much-for-debt-131053>

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