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**Article**

## Optimal city zoning and Big Data

Journal of Urban Management

**Provided in Cooperation with:**

Chinese Association of Urban Management (CAUM), Taipei

*Suggested Citation:* Colombo, Stefano (2022) : Optimal city zoning and Big Data, Journal of Urban Management, ISSN 2226-5856, Elsevier, Amsterdam, Vol. 11, Iss. 2, pp. 269-277, <https://doi.org/10.1016/j.jum.2022.04.003>

This Version is available at:

<https://hdl.handle.net/10419/271465>

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## Journal of Urban Management

journal homepage: [www.elsevier.com/locate/jum](http://www.elsevier.com/locate/jum)

## Research Article

## Optimal city zoning and Big Data

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## ARTICLE INFO

## JEL classification:

L13

R38

## Keywords:

Zoning

Big data

Hotelling linear city

## ABSTRACT

The purpose of this paper is to characterize the optimal design of a residential zone in a linear town by a welfare-maximizing regulator when firms might know to some extent the position of the customers/citizens in the city. Information might have different degrees of imperfectness. A micro-founded theoretical approach is adopted throughout the paper. The main findings are the following: a peripheral zoning is more likely to arise when information is scarcely precise, whereas central zoning is more likely when information is highly precise. Moreover, peripheral zoning is more likely the greater is the bias of the regulator toward the consumer surplus. The main policy implication is the following: public authorities should implement city zoning by taking into account the amount of data at the disposal of the firms.

## 1. Introduction

In modern cities zoning is a ubiquitous practice (Gabbe, 2018; Bárcena-Ruiz & Casado-Izaga, 2020). Indeed, municipal authorities are used to impose zones where plants cannot be located, with the aim to prevent disturbances, pollution and traffic and/or to provide a high quality environment for the residents (Pogodzinski & Sass, 1990). In this sense, zoning ordinances by local authorities determine different uses for the town land (for example, residential, industrial, and commercial uses).

Optimal zoning has been usually characterized in the economic literature under the assumption of either minimal or maximal information at the disposal of firms. However, the recent years have been characterized by an impressive wave of Big Data at the disposal of firms when determining their strategies (Choe et al., 2018). It follows that Big Data, by crucially affecting the behaviour of firms, also affect planning strategies by urban authorities (Hao et al., 2015). As a consequence, it is worthy to assess the effect of Big Data on optimal city zoning policies.<sup>1</sup>

In particular, while Big Data have opened the way for price discrimination based on customers' location (Colombo & Pignataro, 2022), it is still rare that firms have *perfect* information, that is, it is uncommon that they are able to distinguish the location of a particular consumer from that of *any other* consumer. Therefore, it is important to discuss optimal zoning under the more realistic assumption of *imperfect* price discrimination, that is, under the circumstance that firms are able to distinguish between groups of consumers.

The aim of this paper is to explore how *better* information of firms about the location of consumers (or citizens) along the city affects optimal zoning regulation. In particular, the present paper includes Big Data into the Hotelling linear town by adopting the framework

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<sup>1</sup> In this paper, another important question which has been widely discussed in urban management literature, namely the use of Big Data by regulators to develop better planning practices (see for example Engin et al., 2020, and the references therein) is left aside. Instead, in this paper, it is assumed that Big Data is at the disposal of firms, and it is asked how this possibility, by altering the behaviour of firms, modifies optimal city zoning by regulators.

<https://doi.org/10.1016/j.jum.2022.04.003>

Received 15 December 2021; Received in revised form 22 March 2022; Accepted 5 April 2022

Available online 22 April 2022

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developed by Colombo (2011) and Liu and Serfes (2004). In these papers, the customers, which are distributed along the Hotelling town, are classified by the firms into several sub-segments, or urban districts, and firms might price discriminate across urban districts but not within each urban district. Therefore, the number of urban districts is a proxy of the precision of the information about the customers' location. It follows that when there is just one urban district, the model collapses into the uniform pricing case (i.e. the precision of the information is minimal) whereas when the number of urban districts tends to infinite, the model collapses into the perfect price discrimination case (i.e. the precision of the information is maximal): all the other intermediate cases represent imperfect price discrimination. This article explores how optimal zoning modifies when the precision of information moves from minimal to maximal.

Two firms compete within the same city, so that there is just one municipal authority setting the optimal zone in order to maximize welfare. The regulator might have a bias toward the profits or the consumer surplus when deciding about zoning regulation. It is shown that optimal zoning regulation by a regulator is crucially affected by the amount of information at the disposal of the firms. In particular, peripheral zoning is more likely to emerge when the information at the disposal of firms is low, whereas central zoning is more likely when information accuracy is high. Moreover, peripheral zoning is more likely the greater is the bias of the regulator toward the consumer surplus.

The intuition is the following. Due to the importance of the overall transportation costs for the overall welfare, the optimal locations of the firms are “close” to the first and third quartile of the segment. At the opposite, the equilibrium locations tend to be more dispersed from the first and third quartile. In particular, when the information at the disposal of the firms is scarce, each firm tends to locate close to the endpoints of the city in order to minimize the degree of competition with the rival. As the two firms separate too much in equilibrium, peripheral zoning emerges to induce the firms to locate closer. Instead, when the information accuracy is high enough, the firms enjoy the possibility to monopolize some urban districts. By moving toward the centre of the city, each firm tries to enlarge such monopolized space. Therefore, there is a strong incentive for each firm to locate close to the city centre. As the two firms are too close in equilibrium, central zoning emerges in order to induce the firms to locate far apart.

The rest of the paper proceeds as follows. After a review of the relevant literature, in Section 2 the model is introduced. In Section 3 the location equilibrium in the absence of zoning is characterized. Section 4 discusses optimal zoning as a function of information accuracy. Section 5 concludes.

### 1.1. Literature review

Economists have typically investigated optimal zoning in the context of spatial competition theory (Colombo, 2020). A number of papers have discussed the optimal shape of city zones under different assumptions. One common feature of these models is the adoption of the classic linear city model of Hotelling (1929), where consumers or citizens are uniformly distributed along a segment, which represents the city.<sup>2</sup> Lai and Tsai (2004) assume that there is an asymmetric zoning within the linear town. In particular, the firms are prohibited to locate near to the left extremity of the segment. In that paper, it is shown that maximal differentiation still arises, as the most-to-the left firm positions itself as near as possible to the left endpoint of the city. In this case, overall transportation costs are distorted from optimality. Tsai et al. (2006) consider symmetric peripheral zoning (that is, the firms are forced to position themselves in the inner part of the town) and they demonstrate that, when there is zoning, the area where the two firms might position themselves is narrower. Chen and Lai (2008) and Colombo (2012) study optimal zoning in a different set up, by assuming that firms deliver quantities in the different points of the city rather than setting prices. Chen and Lai (2008) assume spatial discrimination, and show that in this case central zoning might improve the welfare of the city. In contrast, Colombo (2012) assumes uniform delivered quantities, and shows that the optimal zone is nil, that is, zoning is welfare detrimental (both consumer surplus and profits decrease with the extent of the zone). Bárcena-Ruiz and Casado-Izaga (2014) study zoning when the firms might adopt spatial price discrimination. Following Lederer and Hurter (1986), they show that in the absence of zoning, the two firms position themselves at 1/4 and 3/4 of the town. When zoning is imposed, it is shown that when the regulator has a bias toward the profits, a symmetric peripheral zoning is imposed in order to induce the firms to position themselves in the inner part of the town. Sometimes, zoning are imposed by different cities. In this case, zoning can be adopted strategically by nearby cities. Bárcena-Ruiz and Casado-Izaga (2017) focus on two towns which are located close each other. The regulator in each city decides about the zone in its own town by taking into account the possible reaction of the other regulator. It is shown that when the two cities have different dimensions, the regulator of the bigger city forces the domestic firm to position itself near to the frontier with the adjacent city. It is also possible that only one regulator chooses to zone the city, while the other does not.

All the mentioned papers assume that either the information on the customers' location in the town is nil or it is perfect. In the former case, the firms are constrained to set the same price or to deliver the same amount of good to everyone. In the latter case, the firms can set a different price or deliver a different amount of good at any location in the town (spatial discrimination). However, the literature has not considered yet optimal zoning in the case of imperfect information, that is, when the firms can distinguish between groups of customers/citizens, but not within the same group of consumers/citizens (imperfect price discrimination).

A second group of papers investigates optimal zoning from an urban/geographical perspective. Ahlfeldt et al. (2015) build an empirical model to test the consequences on the internal city structure of agglomeration and dispersion forces by focusing on the event of the cold war division of Berlin, where the Berlin wall can be interpreted as an extreme type of urban zone. By adopting a similar quantitative perspective, Albouy and Ehrlich (2018) consider the US house market and estimate a housing cost function. They find that regulatory restrictions like urban zones tend to increase the price of the houses with respect to both the price of land and the price of the construction inputs. Moreover, the costs of regulatory restrictions tend to overcome the benefits in terms of quality of life, with a

<sup>2</sup> In such a literature, there is no distinction between consumers and citizens.

negative impact on the overall welfare. On a similar vein, [Herkenhoff et al. \(2018\)](#) study the impact of urban restrictions on US macroeconomic dynamic. In particular, by using a dataset of US urban restrictions between 1950 and 2014, the authors claim that, on one hand, land-use restrictions have increased over time, and, on the other hand, they have depressed economic activity. A similar conclusion has been obtained by [Hsieh and Moretti \(2019\)](#), which consider the impact of a particular type of zoning (namely, the existence of constraints on new house constructions) on labour mobility across US cities. Such urban constraints, by limiting the possibility for workers to move toward high-productivity cities (where zoning regulation tends to be stricter), have determined a spatial misallocation of labour, thus lowering the aggregate US growth by more than 50% in the period between 1964 and 2009. [Allen et al. \(2016\)](#) build a quantitative general equilibrium model and show that, in the case of externalities, zoning is efficient in increasing the welfare of the inhabitants of the city. The implications of the model are then tested against the real-world zoning regulation in Chicago.

From a more theoretical perspective, [Joshi and Kono \(2009\)](#) consider optimal density restrictions (zoning) in a two-zones model with growing population. They show that the optimal regulation in a particular time depends on the expected population growth. More generally, optimal zoning is expected to change over time, depending on the population and buildings dynamics. [Rossi-Hansberg \(2004\)](#) considers a circular city and studies the optimal distribution of residential and business areas. In particular, he investigates how zoning regulation might close the gap between the equilibrium and the optimal distribution. It is found that zoning regulation alone is not enough to induce the optimal allocation, but it significantly reduces the distance between equilibrium and optimal allocations, especially when the commuting costs of citizens are high. This model is then extended by [Kantor et al. \(2014\)](#) to include congestion effects. Finally, [Zhang and Kockelman \(2016\)](#) consider a spatial general equilibrium model to explore the efficiency of first-best (as Pigouvian congestion tolling and/or subsidies) and second-best (as zoning and urban growth boundaries) urban policies. They find that, among the second-best policies, zoning is in general more efficient than an urban growth boundary.<sup>3</sup>

At the best of our knowledge, the literature on zoning has not considered yet the implications of the increasing amount of information at the disposal of firms. The present paper represents a first step toward the inclusion of Bid Data into the characterization of the optimal zoning regulation.

## 2. Model

The model, originally introduced by [Liu and Serfes \(2004\)](#) and [Colombo \(2011\)](#), runs as follows. Following [Hotelling \(1929\)](#), the customers are assumed to be distributed uniformly along a linear city whose length is equal to 1, ranging from 0 to 1. The position of a customer in the town is indicated by  $x \in [0, 1]$ . Two firms, say Firm *A* and Firm *B*, are positioned at  $x_J$ ,  $J = A, B$ , with  $0 \leq x_A \leq x_B \leq 1$ .<sup>4</sup> Zoning will be introduced later in Section 4.

Suppose that the linear town can be partitioned into  $n$  sub-segments, or urban districts. Each of these sub-segments is indexed by  $m = 1, \dots, n$ . The length of each sub-segment is  $1/n$ . It follows that the left (right) extremity of sub-segment  $m$  is given by  $\frac{m-1}{n}$  ( $\frac{m}{n}$ ), as illustrated in [Fig. 1](#).

A firm can set a different price for any different urban district, but it cannot set different prices on those customers that are located in the same urban district. Note that this practice coincides with imperfect (or third-degree) price discrimination.

Let  $p_{J,m}$  indicate the price charged by Firm  $J = A, B$  on those customers that are positioned in urban district  $m$ . We assume  $n = 2z + 6k$ , with  $k = 0, 1, 2, 3, 4, \dots$  and  $z = \begin{cases} \{1, 2\} & \text{if } k = 0 \\ 2 & \text{if } k \neq 0 \end{cases}$ . In other words, the number of urban districts is: 2, 4, 10, 16, 22, 28... As shown by [Colombo \(2011\)](#), this segmentation allows keeping the analysis tractable. The case where there is one sub-segment (uniform pricing, or  $n = 1$ ) will be treated separately. Therefore, this set-up encompasses the perfect spatial price discrimination model as a limit case: when  $n$  tends to infinite, perfect spatial price discrimination emerges because the firms can charge a different price for any different customer; when there is only one sub-segment, the firms must set the same price to all customers (that is, uniform pricing emerges). It follows that  $n$  is measure of information accuracy: when  $n$  increases, the precision of information augments. In other words, growing Big Data are represented by an increasing  $n$ . The aim of the model is to analyse the impact of  $n$  on an optimal city zoning.

Next, the utility function of the consumers is introduced. There are unit demand functions: each customer purchases no more than one unit of the item. The utility of a customer located at  $x \in [\frac{m-1}{n}, \frac{m}{n}]$  that purchases from Firm  $J$  is the following:

$$u_{J,m}(x) = v - p_{J,m} - t(x - x_J)^2 \quad (1)$$

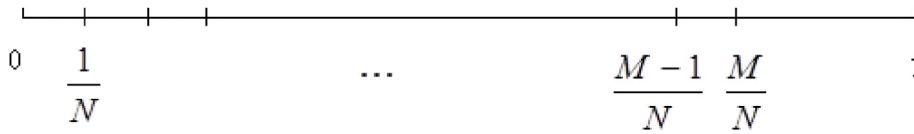
where  $t > 0$  represents the unit transportation cost and  $v$  represents the reservation price; suppose that  $v$  is so high that the market is always covered in equilibrium (that is, no consumer prefers not to purchase at the equilibrium prices).<sup>5</sup>

The game is the following. At stage 1, the two firms choose their position (that is,  $x_A$  and  $x_B$ ) in the admissible area (see later) of the town in a simultaneous and non-cooperative manner; at stage 2, the two firms choose the price in each urban district (that is,  $p_{A,m}$  and  $p_{B,m}$ ) of the town in a simultaneous and non-cooperative manner.

<sup>3</sup> Another strand of literature focuses on the political economy of urban policies, including zoning. See for instance [Helsley and Strange \(1995\)](#), [Calabrese et al. \(2007\)](#), [Sole-Olle and Viladecans-Marsal \(2012\)](#), and [Hilber and Robert-Nicoud \(2013\)](#).

<sup>4</sup> The firms must be positioned within the town.

<sup>5</sup> Following [Colombo \(2011\)](#) and [Liu and Serfes \(2004\)](#), the customers are assumed to sustain the transport costs. The results are the same when the firms sustain the transport costs (see for example [Lederer & Hurter, 1986](#); [Thisse & Vives, 1988](#)).

Fig. 1. The linear town with  $n$  districts.

### 3. Equilibrium outcome in the absence of zoning

In this section, the location and price schedule equilibrium in the absence of any zoning regulation are characterized. By adopting backward induction, stage 2 is solved first, and then stage 1 is solved.

Indicate by  $d_{J,m}$  the Firm  $J$ 's demand in sub-segment  $m$ , and denote  $m_A \equiv \frac{n(a+b)}{2} - 1$  and  $m_B \equiv \frac{n(a+b)}{2} + 2$ . The consumer located in sub-segment  $m$  indifferent between the two firms is obtained by equating  $u_{A,m}$  and  $u_{B,m}$ , yielding  $k_m = \frac{p_{B,m} - p_{A,m}}{2t(x_B - x_A)} + \frac{x_A + x_B}{2}$ .

**Proposition 1.** Suppose that the number of sub-segments is  $n = 2z + 6k$ , with  $k = 0, 1, 2, 3, 4, \dots$  and  $z = \begin{cases} \{1, 2\} & \text{if } k = 0 \\ 2 & \text{if } k \neq 0 \end{cases}$ . In stage 2, the equilibrium price schedules and demand are:

- If  $m_A < m < m_B$ :

$$p_{A,m}(x_A, x_B) = \frac{t(x_B - x_A)}{3} \left( \frac{4 - 2m}{n} + x_A + x_B \right) d_{A,m}(x_A, x_B) = k_m - \frac{m - 1}{n} = \frac{2 - m}{3n} + \frac{x_A + x_B}{6}$$

$$p_{B,m}(x_A, x_B) = \frac{t(x_B - x_A)}{3} \left( \frac{2 + 2m}{n} - x_A - x_B \right) d_{B,m}(x_A, x_B) = \frac{m}{n} - k_m = \frac{m + 1}{3n} - \frac{x_A + x_B}{6}$$

- If  $m \leq m_A$ :

$$p_{A,m}(x_A, x_B) = t(x_B - x_A) \left( x_A + x_B - \frac{2m}{n} \right) d_{A,m} = \frac{1}{n}$$

$$p_{B,m} = 0 \quad d_{B,m} = 0$$

- If  $m \geq m_B$ :

$$p_{A,m} = 0 \quad d_{A,m} = 0$$

$$p_{B,m}(x_A, x_B) = t(x_B - x_A) \left( \frac{2m - 2}{n} - x_A - x_B \right) d_{B,m} = \frac{1}{n}$$

*Proof.* Refer directly to Colombo (2011, Proposition 1), for the case  $n \geq 4$ . The proof for case  $n = 2$  is analogous.<sup>6</sup> ■

It can be observed from Proposition 1 that the two most interior sub-segments (namely, sub-segment  $m_A + 1$  and sub-segment  $m_B - 1$ ) are shared by the firms, as no firm sells zero units in sub-segment  $m_A + 1$  and sub-segment  $m_B - 1$ . At the opposite, in all the other sub-segments there is a monopoly: Firm A is a monopolist in sub-segments  $m \leq m_A$  and Firm B is a monopolist in sub-segments  $m \geq m_B$ .

Given the equilibrium prices in stage 2, the profits of the firms in stage 1 are:

$$\begin{aligned} \pi_A(x_A, x_B) &= \sum_{m=1}^{m_A} \frac{p_{A,m}(x_A, x_B)}{n} + \sum_{m=m_A+1}^{m_B-1} p_{A,m}(x_A, x_B) d_{A,m}(x_A, x_B) = \\ &= \frac{t(x_B - x_A) [9n^2(x_A + x_B)^2 - 18n(x_A + x_B) + 40]}{36n^2} \end{aligned} \quad (2)$$

$$\pi_B(x_A, x_B) = \sum_{m=m_A+1}^{m_B-1} p_{B,m}(x_A, x_B) d_{B,m}(x_A, x_B) + \sum_{m=m_B}^n \frac{p_{B,m}(x_A, x_B)}{n} =$$

<sup>6</sup> The assumption about the possible number of sub-segments guarantees that, at the equilibrium locations (see later),  $m_A$  and  $m_B$  are integer numbers.

$$= \frac{t(x_B - x_A)[9n^2(2 - x_A - x_B)^2 - 18n(2 - x_A - x_B) + 40]}{36n^2} \quad (3)$$

The first-stage equilibrium locations in the absence of zoning regulations are obtained from the following system: 
$$\begin{cases} \frac{\partial \pi_A(x_A, x_B)}{\partial x_A} = 0 \\ \frac{\partial \pi_B(x_A, x_B)}{\partial x_B} = 0 \end{cases},$$

that is:

$$x_A^*(n \geq 2) = 1 - x_B^*(n \geq 2) = \frac{9n^2 - 40}{36n^2 - 36n} \quad (4)$$

By inspecting (4), it can be observed that when  $n = 2$  and  $n = 4$ , the equilibrium location is  $x_A^* = 0.08$  and  $x_A^* = 0.24$ , respectively. Therefore, when there are only two or four sub-segments and there is no zoning regulation, the two firms locate outside 1/4 and 3/4 of the city. When  $n = 10$ , it is  $x_A^* = 0.266$ : that is, the two firms position themselves within 1/4 and 3/4. When  $n$  increases further,  $x_A^*$  decreases monotonically and converges to 1/4 when  $n \rightarrow \infty$ . In other words, the two firms still locate within the first and the third quartile, but their distance in the city increases when information accuracy goes up.

Now the intuition behind the equilibrium locations is discussed. Given the number of sub-segments or urban districts,  $n$ , there are three different forces when Firm A moves toward the centre of the city (a symmetric discussion holds for Firm B moving to the left). First, Firm A is induced to move to the centre of the town so that some consumers become closer to it rather than to the rival (this force is similar to the *demand effect* postulated by Tirole, 1988). However, when Firm A moves to the right, there is also an impact on the prices which are charged on those customers that patronize Firm A. Looking to Proposition 1 for sub-segments  $m \leq m_A$ , it can be observed that such an impact is affected by i) the distance between Firm A and Firm B, and ii) the distance between sub-segment  $m$  and Firm A. When the distance between Firm A and Firm B goes up, the competition between the firms is less fierce and the price increases (this is the *strategic effect* postulated by Tirole, 1988), whereas the location that minimizes the overall distance between the sub-segments and the firm is 1/4. This last effect is the *minimizing transportation costs effect* (Colombo, 2011). The equilibrium position of the firms depends on the demand effect, the strategic effect, and the minimizing transportation costs effect. When  $n$  is small, there are few monopolized sub-segments (when  $n = 2$ , no urban district is monopolized, whereas when  $n = 4$  only two urban districts are monopolized). Therefore, the incentive to separate from the rival (i.e. the strategic effect) is strong. It follows that the two firms position themselves outside 1/4 and 3/4. When the number of sub-segments moderately increases, the demand effect becomes stronger, because each firm wants to increase the number of monopolized urban districts by moving toward the centre of the town. This explains why when passing from  $n = 4$  to  $n = 10$ , the firms locate closer in the space and within the first and the third quartile. However when  $n$  increases further, the competition in the two shared urban districts becomes fiercer, as these urban districts are now closer in the space. Consequently, the strategic effect pushes the two firms to locate more distantly. At the same time, the minimizing transportation costs effect prevents them to locate too close to the endpoints, as this would increase the cost of serving the consumers. It follows that the equilibrium locations converge to the first and the third quartile.

Using the equilibrium locations in (4), the equilibrium profits follow:

$$\pi_A^*(n \geq 2) = \pi_B^*(n \geq 2) = \frac{t(9n^2 - 18n + 40)^2}{648n^3(n - 1)} \quad (5)$$

In what follows, the case of uniform pricing (where there is no information) is discussed. This case is modelled by D'Aspremont et al. (1979), where it is shown that the firms position themselves at the extremities of the town, that is:

$$x_A^*(n = 1) = 1 - x_B^*(n = 1) = 0 \quad (6)$$

Indeed, since there is no partitioning of the town, the minimizing transportation costs effect is absent. As the demand effect is outweighed by the strategic effect, the two firms maximally separate in the city. The equilibrium profits are:

$$\pi_A^*(n = 1) = \pi_B^*(n = 1) = \frac{t}{2} \quad (7)$$

Summarizing, when there is no zoning regulation, the effect of greater information on the firms' position in the town is the following:

**Remark 1.** when information accuracy goes up, for low levels of information accuracy the two firms locate closer in the city, whereas for high levels of information accuracy they locate more distantly.

In what follows, the effect of information on the equilibrium profits is discussed. By considering (5) and (7), it can be observed that when  $n$  increases, the profits initially decrease, but when the number of sub-segments passes from four to ten, the profits go up and continue to monotonically increase with  $n$ , by converging asymptotically to 0.25 when  $n$  tends to infinite (recall that 0.25 is also the level of profits when there is no information about consumers' location, that is  $n = 1$  – see equation (7)). The intuition is the following (see Liu & Serfes, 2004). When  $n$  increases, there are two effects at work: on one hand, the number of monopolized urban districts increases, but the competition in the two shared urban districts is intensified. When  $n$  is low, the second effect dominates: it follows that when the number of urban districts increases, the profits go down. In contrast, when  $n$  is large, the first effect outweighs the second: therefore, the profits increase with  $n$ . The impact of more precise information on profits can be summarized in the following remark:

**Remark 2.** when the precision of information is low (high), the equilibrium profits decrease (increase) with information accuracy. However, profits are maximized when there is uniform pricing.

#### 4. Optimal city zoning with Big Data

In this section, zoning regulation is introduced. In particular, the zone is the area of the city where no firm can locate (i.e. a residential area). There are two possible types of zones: *central zones* or *peripheral zones*. In the case of central zoning, the zone is identified by the area of the town between  $z_1$  and  $z_2$ , with  $0 \leq z_1 \leq z_2 \leq 1$ . Therefore, the complete set of admissible locations for the firms is  $x_A \in [0, z_1]$  and  $x_B \in [z_2, 1]$ . Under peripheral zoning, the zone is identified by the area outside  $z_1$  and  $z_2$ . Therefore, the complete set of admissible locations of the firms is  $x_A \in [z_1, x_B]$  and  $x_B \in [x_A, z_2]$ . Since the model is symmetric, only symmetric zoning, that is  $z_1 = z$  and  $z_2 = 1 - z$ , is considered. Furthermore, as in [Bárcena-Ruiz and Casado-Izaga \(2014\)](#), the regulator can be distorted toward the firms or the consumers. In other words, when setting the zone, the regulator maximizes a weighted average of the profits and the consumer surplus, that is:

$$W(z) = \alpha CS(x_A, x_B) + (1 - \alpha)[\pi_A(x_A, x_B) + \pi_B(x_A, x_B)] \quad (8)$$

where  $\alpha \in [0, 1]$  is the weight attached to consumer surplus: when  $\alpha > (<) 1/2$  there is a bias toward the consumer surplus (the profits).

The regulator chooses the optimal zone before the firms choose locations. The game is composed by three stages: in stage 1, the regulator chooses  $z$ ; in stage 2, the firms choose the locations in the admissible area; in stage 3, the firms set the prices.

The third-stage equilibrium prices are described in Proposition 1. Consider the second-stage equilibrium locations given  $z$  (that is, given the first-stage zoning decision of the regulator). In order to find the location decision by the firms in stage 2 when firms' locations are constrained by the zoning area, the first-stage choice of the regulator should be considered first. That is, the optimal locations of the firms from the regulator's perspective are characterized first.

The profits of the firms as a function of the firms' locations are given by (2) and (3), whereas the consumer surplus is given by<sup>7</sup>

$$\begin{aligned} CS(x_A, x_B) = & \sum_{m=1}^{m_A} \left[ \int_{\frac{m-1}{n}}^{\frac{m}{n}} [v - p_{A,m}(x_A, x_B) - t(x - x_A)^2] dx \right] + \int_{\frac{m_A}{n}}^{\frac{m_A+1}{n}} [v - p_{A,m}(m = m_A + 1; x_A, x_B) - t(x - x_A)^2] dx + \\ & + \int_{\frac{k_m(m=m_A+1)}{n}}^{\frac{m_A+1}{n}} [v - p_{B,m}(m = m_A + 1; x_A, x_B) - t(x - x_B)^2] dx + \int_{\frac{k_m(m=m_B-1)}{n}}^{\frac{m_B-1}{n}} [v - p_{A,m}(m = m_B - 1; x_A, x_B) - t(x - x_A)^2] dx + \\ & + \int_{\frac{k_m(m=m_B-1)}{n}}^{\frac{m_B-1}{n}} [v - p_{B,m}(m = m_B - 1; x_A, x_B) - t(x - x_B)^2] dx + \sum_{m=m_B}^n \left[ \int_{\frac{m-1}{n}}^{\frac{m}{n}} [v - p_{B,m}(x_A, x_B) - t(x - x_B)^2] dx \right] \end{aligned} \quad (9)$$

Basically, the consumer surplus is composed by three elements: the reservation price of the consumers, the price paid to the firms, and the overall transportation costs. Since the model is symmetric, the welfare maximizing locations of the firms must be symmetric as well. Therefore, only symmetric locations (i.e.  $x_B = 1 - x_A$ ) are considered. Under symmetry, welfare becomes:

$$W(z; x_B = 1 - x_A) = v\alpha + \frac{t[80 + n^2(18 - 39\alpha) - 168\alpha - 36n^2\alpha x_A^2 - 36n(1 - 2\alpha) - 2x_A[80 - 36n(1 - 2\alpha) - 168\alpha + 9n^2(2 - 5\alpha)]]}{36n^2} \quad (10)$$

By maximizing (10) with respect to  $x_A$ , the optimal firms' positions (from the point of view of the regulator) follow<sup>8</sup>

$$\tilde{x}_A(\alpha, n) = 1 - \tilde{x}_B(\alpha, n) = \frac{-80 + 36n(1 - 2\alpha) + 168\alpha - 9n^2(2 - 5\alpha)}{36n^2\alpha} \quad (11)$$

Consider first the case where consumer surplus and profits have the same weight for the regulator, that is  $\alpha = \frac{1}{2}$ . In this case the prices are irrelevant for welfare, because they are a transfer from the customers to the firms. Therefore, the overall transportation costs entirely determine the welfare. Note that  $\tilde{x}_A$  is strictly decreasing in  $n$  and converges to  $1/4$  when the number of urban districts tends to infinite. In other words, when the information at the disposal of firms increases, welfare is maximized when the two firms are more dispersed in the space but still remain within the first and the third quartiles of the town.

The intuition is the following. The transportation costs are minimized when each consumer purchases from the closer firm. When  $n \rightarrow \infty$ , all the consumers at the left (right) of  $1/2$  buy from Firm A (B) in equilibrium. Therefore, the optimal location is  $1/4$  for Firm A and  $3/4$  for Firm B. For lower values of  $n$ , some of the customers which are positioned at the left (right) of  $1/2$  buy from Firm B (A) (namely, these are some of the consumers which belong to the two shared urban districts,  $m_A + 1$  and  $m_B - 1$ ). Therefore, minimizing the overall transportation costs requires a lower distance between the two firms.

[Fig. 2](#) illustrates both the equilibrium locations and the optimal locations.<sup>9</sup>

<sup>7</sup> The explicit expression is quite long, and therefore it is omitted.

<sup>8</sup> If the locations in (11) do not guarantee that  $m_A$  and  $m_B$  are integer numbers, the optimal locations are the closest ones to  $\tilde{x}_A(\alpha, n)$  and  $\tilde{x}_B(\alpha, n)$  among those that guarantee that  $m_A$  and  $m_B$  are integer numbers.

<sup>9</sup> Even if  $n$  does not take all values,  $x_A^*$  and  $\tilde{x}_A$  are depicted as continuous lines in [Fig. 2](#). The case  $n = 1$  is considered separately.



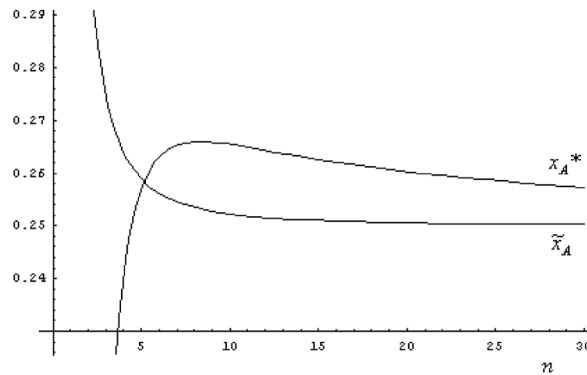


Fig. 2. Equilibrium locations and optimal locations.

By comparing  $x_A^*$  and  $\tilde{x}_A$  it can be observed that when  $n \leq 4$ , the firms are too separated in equilibrium (that is,  $x_A^* \leq \tilde{x}_A$ ), whereas when  $n \geq 10$  the two firms are too close in equilibrium (that is,  $x_A^* \geq \tilde{x}_A$ ).<sup>10</sup> Indeed, when information is low, the firms would like to locate quite far apart (see the discussion in Section 3) in order to reduce the fierceness of competition with the rival (strategic effect). At the opposite, there is a strong incentive in this case for the regulator to locate the two firms closer in the space in order to minimize the transportation costs. Therefore, peripheral zoning emerges to induce the firms to locate closer. Instead, when information accuracy is high, the two firms tend to locate closer in the space. Since now each firm monopolizes some urban districts, it tries to enlarge the monopolized space by moving toward the centre of the city. Moreover, as shown in Section 3, moving from  $n = 4$  to  $n = 10$  implies a “jump” in the equilibrium locations, as the firms now locate within the first and the third quartile. After that, the equilibrium locations converge monotonically to the first and the third quartile. In this case, the firms are too close each other from a welfare maximizing regulator's perspective. Therefore, central zoning emerges to induce the firms to locate far apart.

It remains to illustrate the case of no information ( $n = 1$ ). In this case, the equilibrium locations are at the endpoints of the town (d'Aspremont et al., 1979). Since all the consumers at the left (right) side of the city buy from Firm A (B), the welfare maximizing locations are  $\tilde{x}_A = \frac{1}{4}$  and  $\tilde{x}_B = \frac{3}{4}$ . Therefore, in equilibrium the two firms locate too distantly, similarly to cases  $n = 2$  and  $n = 4$ .

Given the relation between the equilibrium and the optimal locations, it is possible to describe the optimal city zoning for a regulator aiming to maximize the overall welfare when the consumer surplus and the profits have the same weight. The second-stage equilibrium positions of the firms given the zoning decision in stage 1 are also characterized.

**Proposition 2.** When  $n \leq 4$ , the regulator imposes peripheral zoning, otherwise it sets central zoning, with  $z = \tilde{x}_A$ . In equilibrium, Firm A positions itself at  $z$  and Firm B at  $1 - z$ .

*Proof.* The first part of the proposition comes directly from the observation of Fig. 2. When  $n \leq 4$ , the two firms tend to locate too far apart in equilibrium. Therefore, the regulator impedes the firms to locate close to the extremities of the city (peripheral zoning). In this case, the admissible area for Firm A (B) is  $x_A \in [z, x_B]$  ( $x_B \in [x_A, 1 - z]$ ), where  $z = \tilde{x}_A$ . When  $n \geq 10$ , the two firms tend to locate too close in equilibrium. Therefore, the regulator impedes the firms to locate too close to centre of the town (central zoning). In this case, the admissible area for Firm A (B) is  $x_A \in [0, z]$  ( $x_B \in [1 - z, 1]$ ), where  $z = \tilde{x}_A$ . Finally, it can be observed that at  $x_A = z$  and  $x_B = 1 - z$ , the derivative of the profits is  $\frac{\partial \pi_A(x_A=z, x_B=1-z)}{\partial x_A} = \frac{t(64-144n^2(43-z)+5184n^3z+243n^4(3-8z+16z^2))}{5184n^4}$ , which is negative if  $n \leq 4$  and positive if  $n \geq 10$ . Therefore, Firm A locates at  $z = \tilde{x}_A$ , and, by symmetry, Firm B locates at  $1 - z = \tilde{x}_B$ . ■

Suppose now that zoning implies a cost (for example, an administrative cost to impose zones and check that zoning regulation is respected). It is reasonable to assume that such a cost increases in the distance between the optimal firms' position,  $\tilde{x}_A$ , and the equilibrium position,  $x_A^*$ . Therefore,  $|x_A^* - \tilde{x}_A|$  is a measure of the inefficiency due to the administrative costs of zoning. Note that when information increases from  $n = 1$  to  $n = 4$ , the distance between the optimal location and the equilibrium location reduces, and thus the costs due to zoning reduce as well. Then, moving from  $n = 4$  to  $n = 10$ , there is a “jump” in the distance between the optimal and the equilibrium location, yielding greater zoning costs. Finally, when  $n$  increases further, the distance between  $\tilde{x}_A$  and  $x_A^*$  (and so the zoning costs) smoothly reduces and becomes zero when  $n \rightarrow \infty$ .

Next, the more general case where the regulator has a bias toward either the firms' profits ( $\alpha < \frac{1}{2}$ ) or the consumer surplus ( $\alpha > \frac{1}{2}$ ) is briefly considered. First, by comparing (4) and (11), it can be observed that the firms are too distant in equilibrium ( $x_A^* \leq \tilde{x}_A$ ) when  $\alpha \geq \bar{\alpha} \equiv \frac{80-116+54n^2-18n^3}{168-280+117n^2-36n^3}$ . At the opposite, the firms are too close in equilibrium ( $x_A^* \geq \tilde{x}_A$ ) when  $\alpha \leq \bar{\alpha}$ . Therefore, following the same argument as above, the regulator imposes peripheral zoning when  $\alpha \geq \bar{\alpha}$ , whereas it imposes central zoning when  $\alpha \leq \bar{\alpha}$ .<sup>11</sup> The following remark summarizes:

**Remark 3.** peripheral zoning is more likely the greater is the bias of the regulator toward the consumer surplus.

<sup>10</sup> When  $n = 2$ ,  $\tilde{x}_A = 0.3$ .

<sup>11</sup> It can be easily shown that, under zoning, the firms locate at  $z = \tilde{x}_A$  and at  $1 - z = \tilde{x}_B$ . The proof is analogous to the proof of Proposition 2 and therefore it is omitted.



Indeed, when  $\alpha$  is high, the regulator is biased toward the consumer surplus. Therefore, it chooses optimal locations in such a way to lower both the transportation costs of the consumers and the prices set by the firms. This is achieved when the firms are located within  $1/4$  and  $3/4$  and close to the centre of the town: indeed, when the firms are close each other, the equilibrium prices are low.<sup>12</sup> Given that the equilibrium positions of the firms in the absence of zoning are close to  $1/4$  and  $3/4$ , the regulator is more likely to impose peripheral zoning. The opposite holds when  $\alpha$  is low, that is the regulator is biased toward the profits. In this case, in order to maximize the profits by means of high equilibrium prices, the regulator imposes greater distance between the firms, that is, it sets central zoning.<sup>13</sup>

## 5. Conclusions

This paper considers the effect of growing information accuracy (i.e. expansion of Big Data) about the consumers' location in the town on the optimal design of a residential zone. In particular, building on Liu and Serfes (2004) and Colombo (2011), it is assumed that the firms are able to partition the linear town into a number of sub-segments or urban districts, and that they are able to distinguish between customers that are located in different urban districts, but not between customers that are located in the same urban district. Therefore, the number of the urban districts is a proxy of the precision of information.

Two different types of zones, namely central zoning and peripheral zoning, are considered. The former refers to the case where the firms cannot position themselves near to the centre of the town, whereas the latter refers to the case where the firms cannot position themselves close to the extremities of the town. It is shown that when the precision of information is quite low, it is better for a welfare-maximizing regulator to impose peripheral zoning, but when the precision of information is high, the regulator prefers imposing central zoning. In other words, the diffusion of Big Data is likely to alter the optimal shape of city zoning.

This paper offers a number of empirically testable predictions. In particular, the degree of spatial separation as a function of the information accuracy could be tested. Indeed, our model predicts that those industries where firms have a huge amount of private information about the consumers' location should be characterized by spatial closeness (in contrast with those industries where only uniform pricing is possible and maximal separation emerges in equilibrium). Second, one could test the relation between the bias of the regulator and the characteristics of zoning. As our model predicts that peripheral zoning is more likely to occur when the regulator is biased toward consumer surplus, it could be possible to test empirically the relation between the political leanings of the local government (as a proxy of the bias toward the consumer surplus or the firms' profits) and the type of zoning regulation which is currently implemented.

The present model has some limitations and could be extended in several directions. First, the firms might be asymmetric with respect to the amount of information they have (Colombo et al., 2021). Such an asymmetry might alter the strategic behaviour of firms and, then, the optimal zoning regulation which is adopted by the city authorities. Second, it is assumed that firms have information about the location of customers and they make use of them. On the other side, the local authorities, while having the same information the firms have, do not use them directly (in other words, in the present set-up, information is valuable for firms, but not for the local authorities, which adjust optimal zoning by anticipating the behaviour of firms). However, there might be other types of information which are valuable for the local authorities but not for firms (as for example GPS individual data which are used to support urban planning and design but do not affect the purchasing behaviour of the residents). Such heterogeneous information is likely to affect optimal zoning. Third, the consumers are not strategic players, that is, they do not change their behaviour as a consequence of the decision of firms and local authorities. However, zoning policies by local authorities as well as pricing policies by firms might induce consumers to change their purchasing habits and residents to change their location in the city (or to modify their votes in the next local election). In this case, the zones should be set by taking account the possible reaction of consumers/residents. These questions are left for future research.

## Declarations of competing interest

None.

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<sup>12</sup> For example, when  $\alpha = 6/10$ , the most-to-the-left optimal location of Firm A is 0.39 (when  $n = 10$ ); when  $\alpha = 9/10$ , the optimal location is always  $1/2$ .

<sup>13</sup> From (11), it can be observed that when  $\alpha \leq \frac{80-36n+18n^2}{168-72n+45n^2}$ , the optimal locations are the endpoints of the city.

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