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## Article

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# Mathematical Skills of Students With Special Educational Needs in the Area of Learning (SEN-L) in Pre-Vocational Programs in Germany

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## Abstract

**Context:** Students with special educational needs in the area of learning (SEN-L) attend vocational trainings to be provided with qualifications for the labor market. Competences in arithmetic operations and comparing quantities such as weights and lengths are indispensable for obtaining a vocational qualification. Therefore, the study investigates whether students with SEN-L in Germany convert job-relevant quantities and master arithmetic operations accurately and with a certain speed before beginning vocational training.

**Methods:** 152 students with SEN-L in pre-vocational training programs in southern Germany were examined using curriculum-related computer tests for formative assessment. Three skill-based tests were constructed as progress monitoring tests comparing weights and lengths and arithmetic operations including a speed component.

**Results:** All newly developed tests meet the requirements of the Rasch model. Students' performance on the lengths test and the weights test correlates with a significant linear relationship ( $r = 0.64$ ). Only weak correlations ( $r > 0.29 < 0.45$ ) are found between the performance in the arithmetic operations and the weights and lengths tests. For items in the weights and lengths tests, participants show on average significantly higher probability of solving than in the arithmetic operations test. Furthermore, students with SEN-L show slow processing speed (median processing speed weights test 6.37 seconds, lengths test 6.26 seconds, arithmetic operations test 33.97 seconds on average per item). The students solve more items

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of the weights and lengths tests correctly than of the operation test (median for weights 25 items, for lengths 24 items, for arithmetic operations 7 items).

**Conclusion:** The majority of the students with SEN-L were not able to solve the expected number of items correctly and show that the participants' accuracy in dealing with arithmetic operations and quantity comparison is low. It can be assumed that this might lead to difficulties in mastering mathematical items in the context of vocational training. This is the case, in particular, where a certain speed is required (fluency). Math instruction in pre-vocational programs needs a stronger focus on building up and expanding arithmetic operations and quantities and has to be complemented by monitoring the learning process of students with SEN-L.

**Keywords:** Mathematical Skills, Special Educational Needs in the Area of Learning, SEN-L, Germany, Pre-Vocational Training Programs, Vocational Education and Training, VET

## 1 Introduction

Students with special educational needs (SEN) require support in developing basic mathematical skills to qualify for the training and labor market. Several studies indicate associations between early mathematical competencies and later mathematics achievement (e.g., Duncan et al., 2007; Krajewski & Schneider, 2009; Nguyen et al., 2016; Seitz & Weinert, 2022). The lack of basic skills in the areas of reading, writing and arithmetic as well as an overall low level of learning and performance are considered to be key obstacles to the successful integration into the training and labor market especially for students with special education needs (van Essen, 2013; Wocken, 2007). Difficulties in reading and mathematical skills have long-term effects on academic achievement, the successful completion of secondary school, and thus on the educational careers of young adults. This mainly affects male students whose achievement scores in mathematical skills were lower the more special education support they received (Holopainen & Hakkarainen, 2019). So far, however, too little is known about the mathematical performance level of students with SEN before they enter vocational training (Gebhardt et al., 2015).

### 1.1 Transition Problems of Students With Special Educational Needs (SEN) Into Vocational Training in Germany

After attending inclusive schools or special schools, there are several reasons why students with SEN receive further support for their vocational training in Germany (Secretariat of the Standing Conference of the Ministers of Education and Cultural Affairs of the Länder

in the Federal Republic of Germany [KMK], 2021). On the one hand, these students are overburdened in the academic competences and rarely obtain qualifying vocational qualifications (van Essen, 2013; Wocken, 2007). On the other hand, these students fail in the open application process for such training. For this reason, many students with SEN undergo special measures and attend programs of vocational education (Gebhardt et al., 2011; Hofmann et al., 2021; Tretter et al., 2011).

The legal right to additional resources and support is linked to the identification of special educational needs in many school systems. But the approaches to special education vary significantly from state to state (O'Hanlon, 2018). In Germany, funding and resources are based on the determination of the need for support, with concepts differing between the German federal states. The same applies in the area of vocational education and training, which is regulated by law (Spichtinger & Valaikiene, 2013). For in-company vocational training, the legal regulations are primarily found in the Vocational Training Act (Berufsbildungsgesetz, BBiG) and the Handicrafts Act (Gesetz zur Ordnung des Handwerks – Handwerksordnung, HwO) (KMK, 2021).

Vocational training for young people receives financial support in Germany. According to financing statistics, 66.1 billion euros were spent on general and vocational schools in Germany in 2017. In the area of vocational education and training, about 9.6 billion euros were allocated to vocational schools and about 2.6 billion euros to funding in the transition system area (KMK, 2021). The goal is to keep youth unemployment as low as possible, as career prospects of young people with SEN are unfavorable and their risk of remaining unemployed or working in precarious employment is high (Jochmaring, 2019). As KMK (2021) declares, it is the "function of all the courses of education at lower secondary level [...] to prepare pupils for courses of education at upper secondary level, at the end of which a vocational qualification or the right to access higher education is acquired" (KMK, 2021, p. 123). For this reason, many measures in the area of vocational training are state-subsidized to successfully lead young people to vocational qualifications. But this aim is not reached for all.

To receive financial state support, a distinction is drawn between people who need support during training in order to complete it successfully to lead an independent professional life and people who also need support in life after completion of the training (Bundesministerium für Bildung und Forschung [BMBF], 2022). Therefore, the vocational education system distinguishes between students with special educational needs in the area of learning (SEN-L) or with learning disabilities and students with intellectual disabilities (SEN-ID). After completing vocational training, state support for young adults with learning difficulties ends, whereas adults with intellectual disabilities continue to receive life-long support.

Different vocational education programs are offered for students with SEN-L depending on the support they need. There are three main types:

1. Students without SEN usually take up a so-called *dual system vocational training*. This is also possible for students with low support needs. Combining theoretical and practical learning, the training takes place on a dual basis in a company and at a vocational school (Anselmann et al., 2022). The young people primarily acquire the practical content and skills in a company subsidized, if applicable, by the German Federal Employment agency. A vocational school complements this training on-the-job with theoretical content (Wieland, 2015). In some federal states in Germany (e.g., Baden-Württemberg, Nordrhein-Westfalen), students with SEN-L can receive individual support at the vocational school. In other German federal states (e.g., Bavaria, Sachsen), students with SEN-L have the option of choosing a special education vocational school as a dual learning place instead of a vocational school or can attend special classes at the vocational school. The main differences between special education vocational schools and vocational schools are reduced class sizes, special pedagogical staff, individualized instruction, intensive training support and targeted support measures for vocational preparation. The students primarily end up in craft occupations (e.g., bricklaying, painting), simple sales occupations, or cleaning and hospitality occupations (Autorengruppe Bildungsberichterstattung, 2020). Many of these occupations as for instance painter, hairdresser or carpenter are regulated by law in the Handicrafts Act (HwO) and supplemented in the Vocational Training Act (BBiG), which applies throughout Germany (KMK, 2021). They require plenty of manual skills and are less technical in nature. Vocational education and training lead to a vocational qualification for skilled work as qualified staff, e.g. in accredited occupations requiring formal training. Towards the end of the vocational training, an exam is held under the supervision of a competent body (§ 71 BBiG) as e.g. the Chamber of Trades and Industry or the Chambers of Handicrafts (Wieland, 2015). About 50 percent of the students in any given year opt for a two-, three- or three-and-a-half-year apprenticeship in one of the 324 occupations requiring vocational training (Bundesinstitut für Berufsbildung [BiBB], 2021; KMK, 2021).
2. Students with SEN-L who would be able to start vocational training but do not have sufficient theoretical knowledge attend a state-financed vocational preparation year (Spichtinger & Valaikienė, 2013). *Pre-vocational programs* provide a basis for subsequent vocational training or employment. Students gain experience in vocational fields such as e.g. business and administration, horticulture, home economics, building, metal or wood technology. Various programs offer young people the opportunity to improve their individual chances of starting an apprenticeship, enable them to catch up on (missing) school-leaving qualifications and bridge the time until the prospective entry into training. This is important in that the majority of young people in the transition area have comparatively low or no school-leaving qualifications; 26.4 percent do not have a lower secondary school leaving certificate (BMBF, 2022). It is about 21 percent of the students

who participate in a vocational preparation measure after leaving school (Autorengruppe Bildungsberichterstattung, 2020).

3. If the need for support is so high that no dual training can be taken up, there is the option of attending special vocational training (§ 66 BBiG, § 42r HwO). Often, special vocational training courses are started after the completion of one (or more) vocational preparation programs. They are intended to be completed by persons for whom training in an accredited training occupation is not an option due to the nature and severity of their disability (BMBF, 2022). Special vocational training courses that are based on the corresponding standard occupations are mostly theory-reduced (Jochmaring, 2019), e. g. wood technician. In 2021, almost 7,000 new training contracts were concluded nationwide on the basis of § 42r HwO in so-called specialist trainee or worker training programs, corresponding to 1.5% of all training contracts concluded in that year (BMBF, 2022).

The training regulations, e.g. the Handicrafts Act, specify for each accredited training what must be met in order to pass the examinations after two or three years of training (Deissinger, 1996). Anyone who successfully completes the training acquires a qualification for skilled work as qualified staff accredited (KMK, 2021). In the vocational school, skills, knowledge and abilities that determine the occupational profile are taught in various so-called learning fields (Riedl & Schelten, 2010). For example, the learning field "preparing and coating surfaces" includes the competence to prepare surfaces according to customer orders, to carry out an initial coating and to comply with requirements for the set-up and safe use of workplaces (Bayerisches Staatsministerium für Unterricht und Kultus [StMUK], 2021a). The descriptions of the competences in the individual learning fields are binding, indicate the level of qualification at the end of the learning process and represent the minimum scope (Bayerisches Staatsministerium für Unterricht und Kultus [StMUK], 2021a).

Meeting the standards of complex knowledge- and technology-based jobs, requiring general skills and increased specific occupational and professional qualifications (Carnevale & Desrochers, 2002) is very difficult for students with SEN-L. They often fail because of poorer performances and slow working speed paired with a lack of school qualifications and basic general skills (van Essen, 2013; Wocken, 2007). Therefore, it is necessary that basic skills must be acquired as a solid foundation.

## 1.2 Difficulties of Students With SEN-L Meeting Mathematical Requirements in Vocational Education in Germany

Catching up on lacking basic skills is therefore an important step in gaining a foothold in professional life: All apprenticeship occupations require the mastery of basic mathematical

skills. As soon as material requirements have to be determined, goods purchased, invoices issued or measurements taken, dealing with these units is indispensable (StMUK, 2021a). Learning basic mathematical skills and thus also logical thinking is therefore not linked to technical professions, but can be found in all areas of the working world. Practice and consolidation of basic mathematical knowledge and skills must be conducted to a sufficient extent throughout the vocational training (StMUK, 2021a).

For math, the curriculum for vocational preparation states four objectives (StMUK, 2021b): 1. Grasping basic mathematical structures and written calculation procedures in the area of basic arithmetic, the rule of three, fractions and percentages, 2. acting independently on the basis of a real (vocational) situation with quantities, measures and units of measurement, 3. developing a notion of geometric constructions and shapes with calculations, 4. acquiring basic skills for solving linear equations and formulas. These requirements concerning basic mathematical skills described in the curriculum for vocational preparation represent requirements that are needed in all occupations and must be mastered by young people, regardless of their disability or special educational needs, to enable them to successfully complete vocational training.

In addition to accuracy, the percentage of correctly solved tasks, fluency, the speed with which arithmetic tasks are solved, are considered essential for the development of computational strategies and mathematical competence (Carr & Alexeev, 2011). The reliable mastering of arithmetic operations and the converting and comparing of quantities, especially quantities relevant to everyday life and work such as weights and lengths are of enormous importance in all occupations (see e.g., vocational school curriculum for the occupation painter and varnisher, StMUK, 2021a). These skills must be mastered confidently and automatically, and also at a certain speed to succeed in professional life.

Weakly developed basic skills are a predictor of later mathematical achievement (Ennemoser et al., 2015). Failure to build a sustainable mathematical understanding and basic mathematical skills during the elementary school years often leads to difficulties in acquiring secondary school subject matter, as difficulties in learning mathematics manifest during the school years (Scherer et al., 2016). Basic skills are not limited to early developmental stages, but continue to evolve and differentiate in higher grades (Ennemoser et al., 2011). Building on a solid concept based on natural numbers, learners first develop an understanding of decimals and then fractions, but only a limited number of students have understood the dense structure of rational numbers by the end of the elementary school years (van Hoof et al., 2018). Children have difficulties in estimating quantities and working on factual tasks before entering school, and also still in their third and fourth school years and as adults (Häsel, 2001; Lobemeier, 2005; Stinken, 2015).

For students with SEN-L, one of the challenges is the development of basic mathematical skills (Gebhardt et al., 2013; Lehmann & Hoffmann, 2009; Moser Opitz, 2013). Difficulties

in learning mathematics become evident in the three central content areas of basic arithmetic that characterize mathematics learning in elementary school and have implications for mathematics learning in secondary school (Moser Opitz, 2013; Rensing et al., 2016): 1. Understanding natural numbers, 2. understanding the decimal place value in the number system, 3. understanding arithmetical operations (Gaidoschik et al., 2021). Arithmetic skills are a prerequisite for the conversion of units of measurement (Franke & Ruwisch, 2010) and a lack of skills is often the reason for difficulties of students with SEN-L in factual arithmetic (Häsel-Weide, 2020). One third of students with SEN-L who completed special education have difficulties in dealing appropriately with numbers and solving simple division tasks (Lehmann & Hoffmann, 2009).

Despite various interventions, effective school-based support has only been successful to a limited extent for students with SEN-L. Several studies, such as the PISA 2012 Oversample of special schools, display insufficient mathematical competencies for students with SEN-L in special schools after grade 9 (Gebhardt et al., 2015). In the 2018 PISA assessment, 21.1% of 15-year-old students (including students with and without SEN) remained below level 2 of mathematics proficiency, making it difficult for them to interpret without direct instruction and to recognize how a simple real-life-situation can be represented mathematically. They are considered to be particularly at risk and do not have adequate mathematical literacy to make decisions in personal or professional situations (Organisation for Economic Co-operation and Development [OECD], 2019). To qualify for the training and labor market, students with SEN-L require support in developing basic mathematical skills.

Students with SEN-L need adaptive instruction that addresses their difficulties. Formative assessment as monitoring of learning progress is particularly effective for students with SEN-L (Fuchs & Fuchs, 2006; Hosp et al., 2016; Vaughn et al., 2003). In 2012, the European Agency for Development in Special Needs Education elaborated European Patterns of Successful Practice in Vocational Education and Training. It drew particular attention, among other things, to learner-centered approaches that are influenced by the use of innovative teaching and assessment methods, the implementation of individualized plans and flexible curricula focusing on learners' capabilities (European Agency for Development in Special Needs Education, 2013). In order to address each student's individual abilities and, as teachers, to determine if they need to modify their curriculum, materials, or instructional procedures, it is useful to monitor student progress (Deno, 2003; Fuchs & Fuchs, 2006; Jungjohann et al., 2018). With progress monitoring, small learning successes can be made visible and comprehensible. Experience with assessment formats that serve to monitor the learning process within the framework of a course (formative assessment) or to check results after completion of a course (summative assessment) plays a role for only around three tenths of vocational school teachers (Autorengruppe Bildungsberichterstattung, 2020).



## **2 Research Questions**

As basic mathematical skills are indispensable for obtaining a vocational qualification, we developed new web-based tests to measure three occupationally relevant skills: Arithmetic operations and comparing quantities such as weights and lengths. To date, only a few studies are available that demonstrate these skills in students especially with SEN in the area of learning (SEN-L) prior to entering vocational training. For this reason, the study focuses on arithmetic operations and comparing weights and lengths within a curriculum-based measurement framework for students with SEN-L in pre-vocational programs. It describes test development and analyzes task difficulty. In addition, the study explores what arithmetic skills young people with SEN-L have before entering vocational training and how this performance in the areas of arithmetic operations compares with their weight and length comparison skills.

The following research questions can be specified:

1. Do the newly developed instruments fit the heterogeneous group of students with SEN-L in pre-vocational programs?
2. What skills do students with SEN-L possess and are they measurable by item range as a power test on the first 30 items?
3. Which skills can be processed sufficiently quickly and confidently within five minutes to be sufficiently competent for the job?

## **3 Materials and Methods**

To answer the research questions, three skill-based speeded tests comparing weights and lengths and arithmetic operations were developed. They were used to examine 152 students with SEN-L in vocational preparation programs in southern Germany.

### **3.1 Instruments**

To assess and compare the performance in the areas of arithmetic operations, length and weight comparison, three skill-based speeded tests were constructed as progress monitoring tests with a drawing algorithm for multiple measurement time points. They can be conducted digitally on a tablet and were implemented in the online platform [www.levumi.de](http://www.levumi.de) ("Levumi"). Levumi is a self-operated, free online platform for learning progress assessment operating since 2015, which complies with the data protection regulations of the German federal states (Gebhardt et al., 2016).

The performance of students in Levumi speeded tests is determined not only by the number of correctly and incorrectly solved items, but also by their processing speed and thus the total number of items solved. The completion time of the speeded tests we constructed was set to five minutes, as this duration had been found to be optimal for students with SEN-L in previous analyses (Ebenbeck et al., 2021; Schurig et al., 2021).

#### *Weights and Lengths Test*

A test for comparing weights and a test for comparing lengths were newly constructed as speeded tests based on the curriculum because there are no tests available in German-speaking countries that exclusively offer items for comparing quantities with a time component. In common test procedures for school performance, for example, only individual items on variables are found. The weights and lengths tests are designed as speeded tests with a time limit of five minutes, which makes them suitable for learning progress monitoring and formative assessment (Jungjohann et al., 2018; Schurig et al., 2021). They are conceived as multiple-choice tests. Participants enter the relation sign.

To design the tests as fairly as possible for heterogeneous groups, item categories with different difficulties are formed. If the quantity comparison items include a decimal number, they fall into the hard item category. The comma notation poses an additional challenge (Franke & Ruwisch, 2010) and can lead to misinterpretations if the student has no idea of it (Peter-Koop & Nührenbörger, 2008). In addition, the relation sign to be used in each case is included as a further rule (equal (=), smaller (<), larger (>)). Thus, a total of six groups of items were formed and a pool of 240 items was created for each of the two comparison tests.

#### *Arithmetic Operations Test*

There are no comparative tests of arithmetic operations for older students that take into account both accuracy and fluency, as required in everyday working life. Therefore, a test on arithmetic operations, similar to tests in elementary school, such as the Heidelberg Arithmetic Test (Heidelberger Rechentest - HRT) (Haffner et al., 2005), was newly developed. It can also be used with older students and includes a speed component.

The goal of the test is to determine the value of the placeholder in the four arithmetic operations addition, subtraction, multiplication and division in the number range up to 100. Research results on the development of mathematical competencies show that the position of the placeholder in the item has an influence on the difficulty. Determining the placeholder of the first position or the second position requires a higher level of mathematical competence than determining the result (Gebhardt et al., 2013, 2014; Mittelberg, 2004). The difficulty of an addition or subtraction task is further influenced by crossing the tens boundary (Anderson et al., 2022; Beishuizen et al., 1997; Benz, 2007). Correspondingly, the types of multiplication tables up to ten also have an influence on the difficulty of the item (Gaidoschik, 2014;

Gasteiger & Paluka-Graham, 2013; Padberg & Benz, 2021). In total, 16 item categories with different difficulties were formed.

A pre-test was conducted with individual students in advance. The item formats and individual test items were developed with experts from vocational schools and special education.

### **3.2 Sample**

Participants were students with SEN-L in pre-vocational programs in southern Germany ( $n = 152$ ). 29% reported themselves as female, 69% as male and 2% as non-binary. Students were on average 17.16 years old ( $SD = 1.28$ ). The test was conducted at the end of the school year during which arithmetic operations and exercises on various quantities had been practiced in class. All participants worked on their own on a tablet. Trained instructors provided guidance on which strategies students should use to perform optimally in the speeded tests. After sample comprehension clarification items, the items of the three tests were to be completed in any order as fast as the participants were able to. Participants were advised to enter the result zero if no result could be calculated in an arithmetic operation test. This opened up the possibility of completing further items in the given time. All tests had a time limit of five minutes and were administered consecutively in class. A variation in the order of completion was made possible for the participants as they selected the test order freely.

### **3.3 Statistical Analysis**

To ensure that the constructed test procedures are appropriate for use in vocational schools with students with SEN-L, the test goodness is first checked using the Rasch model. The Rasch model is an item response theory (IRT) model. In tests that conform to the Rasch model, a person's likelihood of solving an item depends on the difficulty of the particular item (item difficulty) and the person's overarching ability (person ability). If speeded tests are constructed on the basis of the Rasch model, on the one hand a good fit between item difficulty and person ability and on the other hand an item difficulty that is as homogeneous as possible within a test are aimed for (Bock & Gibbons, 2021).

Next, student performance is contrasted for the three different tests. For this purpose, the mean sum scores are compared by means of a one-factor analysis of variance (ANOVA) and the relation between the test results at student level is analyzed by means of Pearson correlations. In a Latent Profile Analysis, different groups of students are additionally singled out depending on their performance patterns in the three tests.

## 4 Results

In the first part of the results section, we describe, if the newly constructed tests meet the requirements of the Rasch model. Moreover, we examine the solution probabilities of students. In the second part, we figure out different students' performances in a latent profile analysis<sup>1</sup>.

### 4.1 Power Test Analysis

The two newly constructed tests for comparing weights and lengths meet the requirements of the Rasch model. For a power analysis of the tests, the first 30 items of the item pool are used, since this item range has all difficulty-generating characteristics and can be answered by most students in five minutes. In a power analysis, both tests show good test reliability (Lengths: WLE = 0.86, Weights: WLE = 0.84). The infit and outfit statistics of the items are within the required range between 0.5 and 1.5 (Lengths: Outfit Min = 0.6, Outfit Max = 1.4, Infit Min = 0.7, Infit Max = 1.3; Weights: Outfit Min = 0.6, Outfit Max = 1.5, Infit Min = 0.7, Infit Max = 1.4). Item difficulty coincides with students' estimated person abilities, suggesting a fit between the tests and student performance (Figure 1). The items of both tests show a homogenous item difficulty, since all items are in the logit range between -2 and 2. For a median split of persons, the graphical model test does show only little irregularities on the item level, which indicates a fit of the item pools to the Rasch model (Figure 2). Two items seem to be easier for students with high abilities (Weights:  $3 t > 3 kg$ , Lengths:  $2 cm > 2 mm$ ). These items contain the same single-digit number and differ only in the unit of measurement. Thus, they control the knowledge of units of measurements in a small number space, no conversion is necessary. Because these items are particularly simple, they are not expected to test unfairly with respect to low-achieving students. They may just be less meaningful than other items in the item pool. To make for having simple items in the item pool that can be solved by many students and lead to a sense of accomplishment, these items are kept in the item pool.



Figure 1: Item-Person-Map of the First 30 Items of Weight and Lengths Tests

<sup>1</sup> Data and Code for R are available on OSF.io: [https://osf.io/nkyxt/?view\\_only=75ed8232c84c478b86dc7f9b2fcc7667](https://osf.io/nkyxt/?view_only=75ed8232c84c478b86dc7f9b2fcc7667)

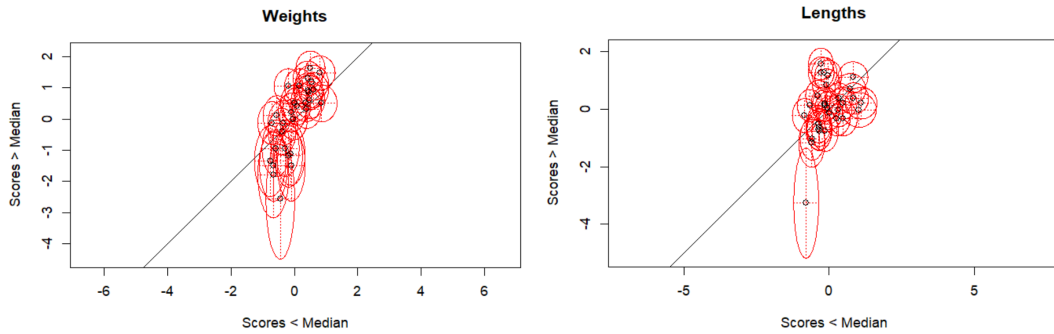


Figure 2: Graphical Model Test of the First 30 Items of Weight and Lengths Tests

Note: The graphical model test compares item fit for two median split groups of subjects tested. The red circles show the standard deviation of every item. If the red circle intersects with the bisecting angle, the respective item shows no difference for the two groups.

The test for arithmetic operations measures four different operations: Addition, subtraction, multiplication and division. However, since the items are processed as placeholder items, the mathematical operations of the addition and subtraction items as well as the multiplication and division items can hardly be separated. Therefore, the items are divided into two dimensions and estimated in a two-dimensional Rasch model. For a power analysis of the tests in a two-dimensional Rasch Model, the first 32 items of the item pool are used, as this item range covers all difficulty-generating characteristics to the same extent (Figure 3). In the two dimensions, the test shows good reliability (addition & subtraction: EAP = 0.85, multiplication & division: EAP = 0.84).

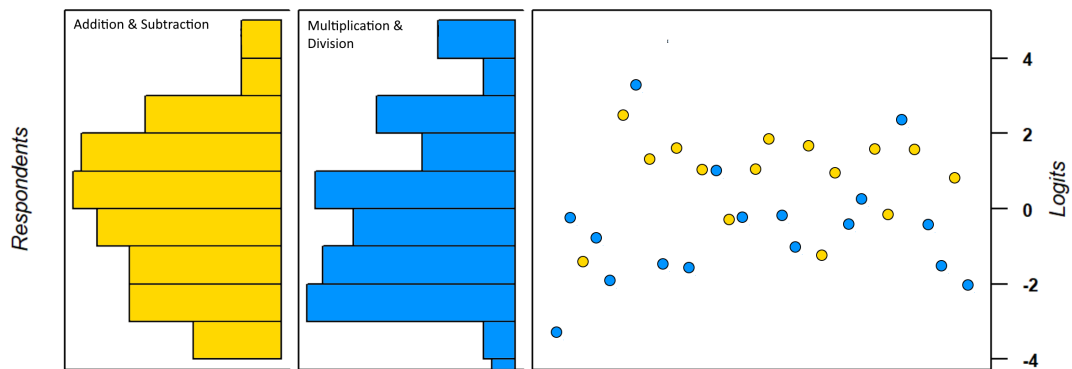


Figure 3: Item-Person-Map of the First 32 Items of the Operations Tests With Two Dimensions

For items in the weights and lengths tests, students show on average significantly higher probability of solving than in the arithmetic operations test (Figure 4).

The solution probabilities of students differ significantly between the tests ( $F(3) = 71.5$ ,  $p < 0.001$ ) with the highest differences in solution probabilities lying between the tests of arithmetic operations and weights or lengths ( $\text{diff} > 15.39 < 34.33$ ,  $p < 0.001$ ). Only between tests or dimensions with the same instructions (lengths and weights, addition & subtraction and multiplication & division) no significant difference could be detected. In the context of arithmetic operations, students work on the multiplication and division problems with the highest probability of solution (Figure 4). Students' performance on the lengths test and the weights test correlate ( $r = 0.64$ ). Only weak correlations ( $r > 0.29 < 0.45$ ) are found between the performance in the arithmetic operations and the weights and lengths tests. Performances within the arithmetic operations test between the different dimensions (addition & subtraction, multiplication & division) are moderately correlated ( $r > 0.6 < 0.82$ ) because these are fundamentally independent scales that provide different values and information for educators.

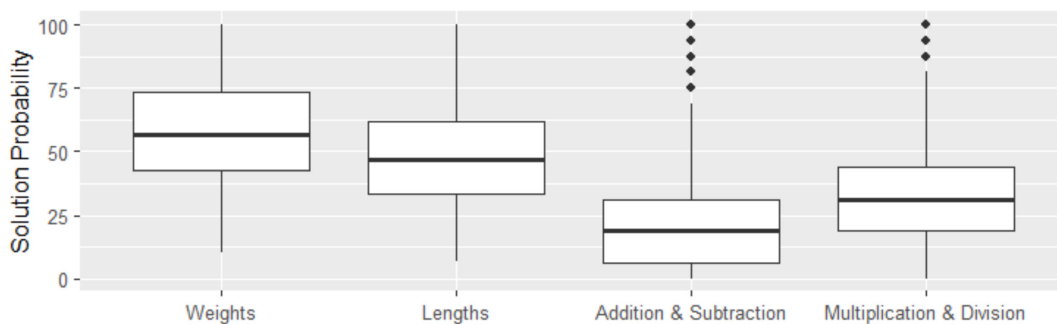


Figure 4: Range of Solution Probabilities

The average solution probabilities are between 55% and 20%, with only a few students showing more than 75% solution probability in the tests. Especially in the arithmetic operations, the solution probabilities are low and only few students solve all items correctly. Since the construction of the tests on comparing lengths and weights allows a guess probability of 30%, this 30% must be subtracted from the calculated solution probability. Thus, the solution probabilities of all tests are between 20% and 35%.

## 4.2 Speed Test Analysis

When the item pools are evaluated as speed tests, the increase in item difficulty is evident as the test progresses (Figure 5). The necessary speed component of students' performance can be seen, as items in the back get harder. A fast student can solve more items and only a fast student can solve those items in the back.

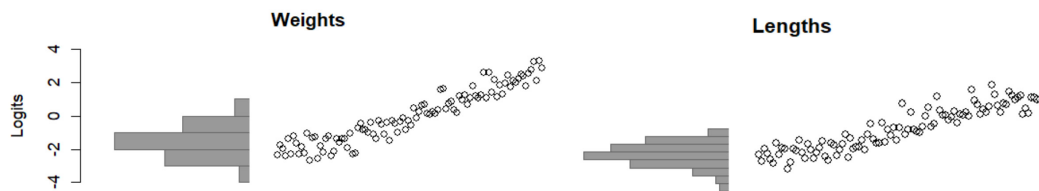


Figure 5: Item-Person-Map for Weights and Lengths Tests as Speed Test

In comparison, the students solve more items of the weights and lengths tests correctly than of the operation test (Figure 6). The median for weights is 25 items, for lengths 24 items and for operations 7 items. Four different performance classes emerge. The largest class is class 4 (purple) with a total of 82 students, who show the lowest abilities in all three tests. The second largest class is class 2 (blue) with a total of 50 students, who also show the lowest abilities in operations, but medium abilities in lengths and weights calculation. Class 1 (red) comprises 9 students, who have the highest sum scores in regards to the other students and low to medium abilities in operations. Class 3 (green) with a total of 8 students is the smallest class. These students show the best scores in arithmetic operations, but medium abilities in lengths and weights.

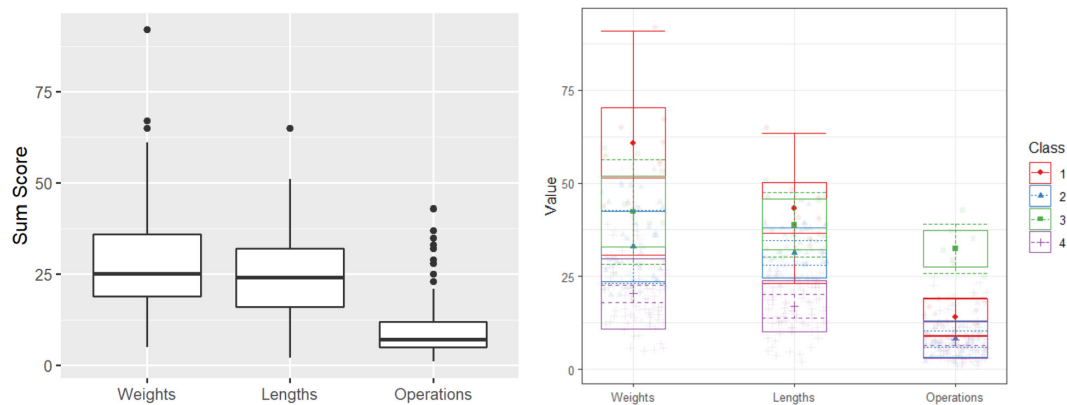


Figure 6: Range of Sum Scores of the Speed Tests as Box Plots (Left) and in a Latent Profile Analysis With Four Profiles (Right)

Also, students show slow processing speed. In the weights test, the median processing speed is 6.37 seconds, with the fastest 10% needing 3.8 seconds and the slowest 10% 11.9 seconds on average per item. The lengths test is processed somewhat faster on average. The median processing speed is 6.26 seconds, the fastest 10% need 3.1 seconds and the slowest 10% 13.3 seconds on average per item. The test on arithmetic operations is processed the slowest. On average, 17.15 seconds are needed per item, the fastest 10% need 6.9 seconds and the slowest 10% 33.97 seconds on average per item.

## 5 Discussion

The three newly constructed tests of weights and lengths comparison as well as the arithmetic operation test for students with SEN-L in pre-vocational training programs are appropriate for this population and correspond to the test-theoretical assumptions of the Rasch model. The tests are designed to be used both cross-sectionally and longitudinally as curriculum-based measurement (Schurig et al., 2021). The test items are relatively easy and correspond to the learning content of the third and fourth grade of the German elementary school in mathematics. Students should therefore be familiar both with comparing lengths and weights and with applying the basic arithmetic operations and a solution probability of 100 percent should be possible in the vocational school, since even fourth graders at the lower proficiency levels are quite capable of ordering representatives of different quantities (Lobemeier, 2005). With a certain amount of errors or technical problems, such as pressing an answer incorrectly, we expect that 90 percent of the items should be solved correctly (accuracy). The majority of the sample, however, was not able to solve the expected number of items. Although the students tested are an at-risk group due to their special needs, these results are only partly in line with our expectations, as their performance in all areas was weaker than anticipated. From this we conclude that they are likely to have difficulties in the later vocational theoretical tests.

One item in each of the two quantity tests was particularly easy, especially for students with higher abilities, and can be seen as differential item functioning (DIF). However, since these two items were very easy, this may also be due to a sampling effect. These items control the knowledge of units of measurements in a small number space. Items with a solution probability of  $\approx .80$  are referred to as "icebreaker items" (Moosbrugger & Kevala, 2020) and provide a sense of achievement for a large proportion of participants. Therefore, they will be kept in the item pool for motivational reasons.

All students complete fewer items correctly in the operations test than in the weights and lengths tests. The lower number of items processed and the resulting slower processing speed can be attributed to the different instructions of the tests. Whereas the tests for weights and lengths are multiple-choice, in the operation test a number must be actively entered via a number field. A longer processing time may therefore be decisive for the lower total score.



In addition, there is a lower guess probability for the operational tests. However, since a significantly lower number of solution probabilities is also found for the operation test in all dimensions, difficulties in arithmetic skills and calculating with operations can be proven for all students, which fits the findings for ninth graders of Gebhardt et al. (2013, 2014).

In the context of the operational test, multiplication items in particular are solved with a higher solution probability. Multiplication items are learned by heart and the result is therefore not calculated but retrieved from long-term memory. The use of placeholder tasks impedes recall from memory and was also found to be difficult for some seventh- and eighth-graders in secondary school (Mittelberg, 2004). However, students in these high grades are presumably already so familiar with placeholder items that they should not represent a significantly higher hurdle than regular arithmetic items. A confident use of arithmetic operations, as demanded in occupational life (StMUK, 2021a), requires a flexible application of arithmetic logic, e.g. to solve reversal tasks, as demanded in placeholder tasks. Therefore, all arithmetic skills must be covered intensively in vocational school instruction as well.

Since the students showed very heterogeneous performances, further profile analyses were conducted. We used latent profile analysis to identify groups of students with similar patterns in their mathematical skills. Comparing the three tests, four different performance profiles can be carved out:

One group of students (6%) consistently performs best, but also completes fewer arithmetic operations correctly. Thus, it is questionable here whether this top group of students can live up to the theoretical expectations or whether this group's performance, with approximately 40 to 50 correctly solved items per test, is already too weak in the tested areas. With a high solution probability, there is potential for improvement here, especially in calculation speed. We believe this group to have the best chance of progressing to an apprenticeship.

The second group of students (33%) shows medium to good performance in the weights and lengths test, but weak performance in the arithmetic operations. This profile includes students who show an understanding of quantities and, potentially, have also memorized the conversion of quantities. However, there is a calculation weakness or a problem in arithmetic, which is why the items cannot be solved correctly. This may lead to problems when it comes to successfully completing vocational training.

The third group of students (5%) shows the best scores in arithmetic operations, but medium skills in lengths and weights. This profile includes students who have a secure grasp of arithmetic operations but do not yet have a solid understanding of quantities. Increased treatment of length measurement and items and activities in curricula that specifically address student challenges are also called for by Smith et al. (2013). Through targeted exercises in comparing lengths as well as weights, existing gaps can perhaps be closed so that vocational training can be started.

The fourth and largest group of students (55%) shows weak performance in all three tests. There is neither a suitable understanding of weights and lengths nor are arithmetic

operations correctly mastered and applied. Fundamental difficulties are evident in all areas. Here, support must be provided with regard to all basic mathematical skills. For example, a flexible application of multiple strategies such as the strategy "subtraction by addition" in addition to the direct subtraction strategy could be useful here, for all children, not only those with mathematical learning disabilities, as suggested by Peters et al. (2014). For the fourth group, it is to be expected that successful completion of vocational training is more difficult.

All four groups clearly show that the participants' accuracy in dealing with arithmetic operations and quantity comparison is low. This is consistent with OECD (2019) findings on at-risk students. It can be assumed that this might lead to difficulties in mastering mathematical items in the context of vocational training. This is the case, in particular, where a certain speed is required to solve the item, as all participants show slow processing speed in all of the three tests. The core question of the study, namely whether, at the end of a pre-vocational training program, students with SEN-L will have all the skills for the conversion of occupationally relevant quantities and the reliable mastering of arithmetic operations to enable them to successfully start and complete a vocational training, has to be answered in the negative for the largest group of the participants in this study.

Here the limitation of the study becomes evident. The sample is not representative of all students with SEN-L in pre-vocational training programs in Germany. In addition, the present study did not examine students without SEN as a comparison group in order to derive, for example, solution probability. In further studies, the reported results should therefore be tested and compared with data on students without SEN. Likewise, testing with a broad item pool of the same dimensions could also be conducted in other grades or at other times, in addition to students in pre-vocational training programs. This would be easily possible because the items are relatively simple and correspond to the learning content of the third and fourth grades of German elementary school in mathematics.

It has been demonstrated by this study that math instruction in pre-vocational programs needs more focus on building up and expanding arithmetic operations and quantities. Learning field-oriented teaching in vocational schools for developing occupational competencies is important (Riedl & Schelten, 2010), but needs adaptive support for students with SEN-L, especially with regard to basic mathematical skills. Evidence-based programs and progress monitoring can be used to support at-risk children (Mononen et al., 2014). Targeted support with direct instruction (Scherer et al., 2016) combined with strategy instruction (Swanson & Hoskyn, 1998) and a monitoring of learning through learning progress assessment appear to be effective methods. Therefore, we intend to provide structured support for occupationally relevant mathematical skills for a similar group of students with SEN-L in addition to their vocational instruction. To this effect, the newly developed curriculum-based progress monitoring tests will be used in formative assessments.

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