

Ceesay, Muhammed

Preprint

## Suspecting Collusion

*Suggested Citation:* Ceesay, Muhammed (2023) : Suspecting Collusion, ZBW - Leibniz Information Centre for Economics, Kiel, Hamburg

This Version is available at:

<https://hdl.handle.net/10419/270874>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

---

# SUSPECTING COLLUSION

---

 **Ceesay Muhammed** \*

May 2, 2023

## ABSTRACT

When collusion is analyzed for Independent private value auctions, it is implicitly assumed that ring presence is commonly known to colluding and non-colluding bidders. We drop this assumption and analyze a simple model of a first price Independent Private Value auction with uniformly distributed values where a single bidder knows privately of the existence of collusion by others. We show that this knowledge leads him to bid shading (weakly) in the first price auction compared to what he would have bid otherwise. This in turn yields the result that the second price auction dominates the first price auction in terms of seller revenue. This contrasts results from the literature showing that under our framework, when bidding is done while the presence of colluding bidders is common knowledge, the first price auction dominates the second price auction.

**Keywords** Almost-All-Inclusive Ring · Informational Structures

**JEL Classification** D44

---

\*Center for Economic and International Studies - CEIS, Tor Vergata - University of Rome Tor Vergata.  
Email Address: [muhammed.ceesay@uniroma2.it](mailto:muhammed.ceesay@uniroma2.it)

This work is based on parts of Chapter 2 of my PhD Thesis [Ceesay \(2019\)](#) at the University of Naples Federico II, and my poster (available at [https://siecon3-607788.c.cdn77.org/sites/siecon.org/files/media\\_wysiwyg/ceesay.pdf](https://siecon3-607788.c.cdn77.org/sites/siecon.org/files/media_wysiwyg/ceesay.pdf) Accessed: 2021-08-03 ) at the 60th meeting of the Italian Economic Society in 2019.

Declarations of interest: None.

## 1 Introduction

How much does it hurt seller revenue if some bidders know that others are colluding?

Evidence for the prevalence of colluding bidders in auctions abound.<sup>2</sup> Typically, when collusion is analyzed for Independent Private Value Auctions, it is implicitly assumed that ring presence is common knowledge among colluding and non-colluding bidders.<sup>3</sup> However, since bid rigging is mostly considered illegal, bidding rings make considerable effort to conceal their presence. Therefore, it is reasonable to think that they go into auctions believing they are concealed. Nonetheless, the ubiquity of collusion means that bidders who participate tend to harbor suspicion that they are bidding against cartels.<sup>4</sup> In this vein, if non colluding members discover that there is a bidding ring, this would be while the ring is oblivious to it.

We drop the assumption that ring presence is commonly known, and assume instead that the ring believes it's presence is concealed. We analyze a simple model of collusion with a single non-ring bidder in an Independent Private Value First Price auction with uniformly distributed bidder values (with the same support), where the non-ring bidder knows privately of the ring presence and show that the common conclusion that the first price auction revenue-dominates the second price auction under collusion no longer holds.

The assumption of uniformly distributed bidder values is common to many analyses of collusion in Independent private value first price auctions.<sup>5</sup> The conditions established by Kirkegaard (2012)<sup>6</sup> implies that under this assumption, when bidding is done while the presence of colluding bidders is common knowledge, the first price auction dominates the second price auction.

This implies that when deciding between first price and second price auctions, practitioners should take into consideration that the superiority of the first price auction only holds when ring presence is assumed to be common knowledge.

The rest of the article is organized as follows; Section 2 describes the framework and the ring mechanism. Sections 3 and 4 state bidder strategies in the First and Second Price auctions respectively. In section 5, we compare revenues from both auction formats. Section 6 concludes.

## 2 Framework

There is a single item for sale to  $N \geq 3$  risk-neutral bidders  $\mathcal{N} = \{1, 2, \dots, N\}$ , each  $i \in \mathcal{N}$  assigning independent private values  $X_i$  uniformly distributed on the interval  $[0, 1]$ . The bidding ring is assumed to be an *almost all-inclusive ring* - all but one of the bidders are members. The objective of the seller is to maximize revenue.

### 2.1 Informational Structure

The ring believes it is concealed. Therefore, our non-ring bidder has private knowledge that the ring is present.

<sup>2</sup>Kawai and Nakabayashi (2022), OFT (2009), (Athey, 2001), Baldwin et al. (1997), Asker (2010), Porter (2005), Pesendorfer (2000).

<sup>3</sup>Marshall et al. (1994)

<sup>4</sup>Marshall and Marx (2012)

<sup>5</sup>See for instance, Marshall et al. (1994)

<sup>6</sup>

**Lemma 1.1.** Kirkegaard (2012) Let  $F_S$  and  $F_W$  denote the distributions of the Strong and Weak bidder respectively such that  $\forall x < y$  in their common support,  $\frac{F_S(x)}{F_S(y)} < \frac{F_W(x)}{F_W(y)}$ . If

$$\int_x^{F_S^{-1}(F_W(x))} (f_W(x) - f_S(\psi))d\psi \geq 0$$

$\forall x$ , the seller's revenue from the first price auction will be higher than the revenue from the second price auction.

In our case,  $F_S(x) = x^{N-1}$ , and  $F_W(x) = x$ . So that

$$\int_x^{F_S^{-1}(F_W(x))} (f_W(x) - f_S(\psi))d\psi = \int_x^{x^{\frac{1}{N-1}}} (1 - (N-1)\psi^{N-2})d\psi = 0$$

implying that seller's revenue is higher under the first price auction.

The ring selects the member with the highest value as its representative, who bids competitively against non-ring bidders, other ring members bidding zero, and proceeds being shared equally afterwards. We abstract away from ring members incentive to be untruthful in reporting their values.

### 3 The First-Price Auction

#### Benchmark: Non Cooperation

When every bidder is bidding competitively, the symmetric equilibrium increasing bidding strategy  $\beta(\cdot)$  is characterized by<sup>7</sup>

$$\beta(x) = \frac{N-1}{N}x, \text{ with inverse } \phi(b) = \frac{N}{N-1}b.$$

#### The Ring Representative

**Lemma 3.1.** *When the ring representative goes into the auction believing the ring is concealed, she bids*

$$b_R(x_R) = \frac{1}{2}x_R$$

*Proof.* Since the ring believes that it is concealed, the ring representative believes it is competing against a non-ring bidder who bids  $\beta(x_n) = \frac{N-1}{N}x_n$ . Firstly, the ring representative bids no more than  $\frac{N-1}{N}$ , as any bid  $b = \frac{N-1}{N} + \epsilon$  is strictly dominated by  $b' = \frac{N-1}{N} + \frac{\epsilon}{k}$ ,  $k > 1$ . Secondly, the ring representative wins with a bid  $b < \frac{N-1}{N}$  provided  $\frac{N-1}{N}x_n < b$ , which implies  $x_n < \frac{N}{N-1}b$ . Therefore, this  $b$  is chosen such that

$$b = \arg \max_b (x_R - b) \cdot \frac{N}{N-1}b, \text{ which yields } b = \frac{1}{2}x_R$$

The ring representative's strategy is therefore

$$b_R(x_R) = \min \left\{ \frac{1}{2}x_R, \frac{N-1}{N} \right\} = \frac{1}{2}x_R$$

□

This means when the ring representative is bidding on behalf of the ring believing the ring is concealed, her bid  $b_R(x_R)$  is lower than her bid  $\beta(x_R)$  when she is bidding non-cooperatively.

#### The Non-ring Bidder

Since his knowledge of collusion is private, the non-ring bidder chooses  $b_n$  to best respond to  $b_R(x_R) = \frac{1}{2}x_R$  by the ring representative.

**Lemma 3.2.** *This best response  $b_n$  is*

$$b_n(x_n) = \min \left\{ \frac{N-1}{N}x_n, \frac{1}{2} \right\}$$

*Proof.* Firstly, given that  $b_R(x_R) = \frac{1}{2}x_R$ , the non-ring bidder bids no higher than  $\frac{1}{2}$ , as any bid  $b = \frac{1}{2} + \epsilon$ ,  $\epsilon > 0$  is strictly dominated by  $b' = \frac{1}{2} + \frac{\epsilon}{k}$ ,  $k > 1$ . Therefore,  $b_n(x_n) \leq \frac{1}{2}$ . Secondly, upon submitting a bid  $b$ , the non-ring bidder wins if  $b_R(x_R) < b \iff \frac{x_R}{2} < b \iff x_R < 2b$ , and this happens with probability  $(2b)^{N-1}$ . Therefore  $b$  is chosen such that

$$b_n \equiv \arg \max_b (x_n - b)(2b)^{N-1} \iff b_n = \frac{N-1}{N}x_n$$

---

<sup>7</sup>Riley and Samuelson (1981)

Therefore, the strategy of the non-ring bidder is

$$b_n(x_n) = \min \left\{ \frac{N-1}{N} x_n, \frac{1}{2} \right\}$$

□

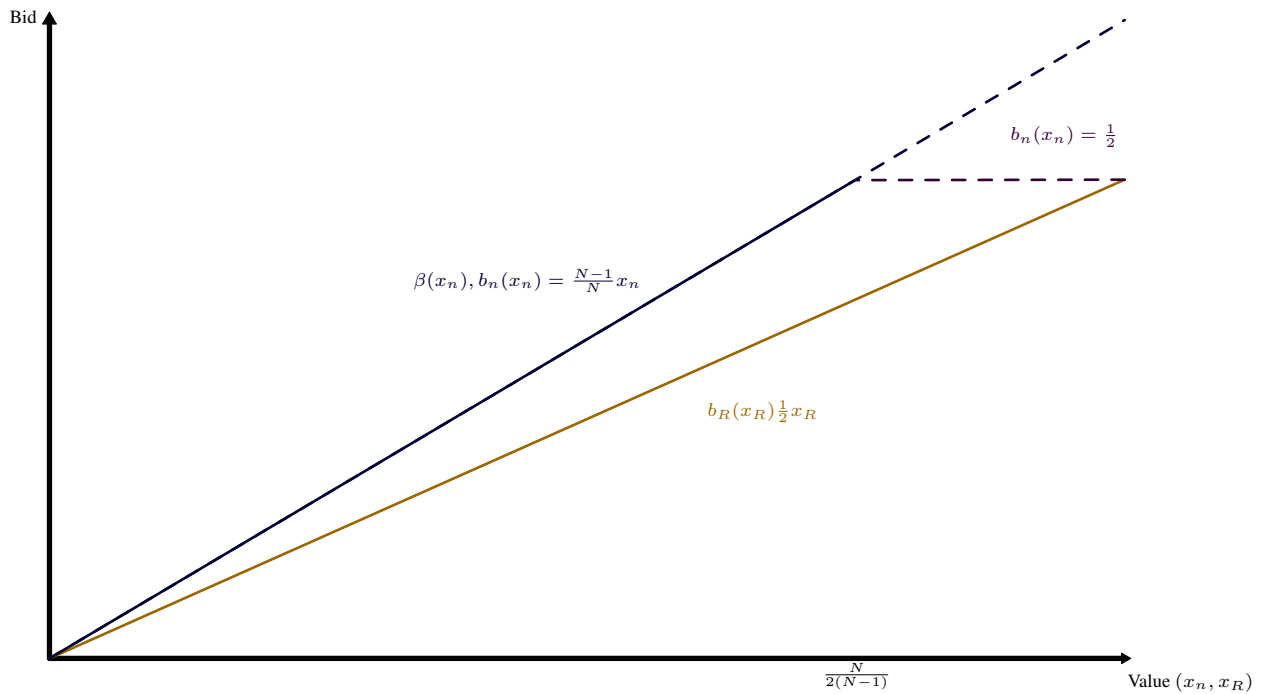


Figure 1: Bids Under Non-Cooperation and Under Ring Presence.

Therefore, knowing that a ring is present at the auction causes the non-ring bidder to shade (weakly) his bid compared to under Non-Cooperation.

## 4 The Second-Price Auction

The symmetric increasing equilibrium strategy involves active bidders bidding truthfully, regardless of the number of bidders or the asymmetry of their distributions.<sup>8</sup> That is,  $\forall i$

$$\beta(x_i) = x_i$$

## 5 Expected Revenue Comparison

We compare expected revenues for each auction format.

### First Price

The bids submitted are  $b(x_n) = \min \left\{ \frac{N-1}{N}x_n, \frac{1}{2} \right\}$ , and  $b_R(x_R) = \frac{1}{2}x_R$ . The expected revenue of the seller can be expressed as the sum of the ex-ante expected payments of both bidders.

The Non-ring bidder wins provided  $\frac{1}{2}x_R < \left\{ \frac{N-1}{N}x_n, \frac{1}{2} \right\}$ . This happens with probability

$$\mathbb{P} \left( x_R < 2 \cdot \min \left\{ \frac{N-1}{N}x_n, \frac{1}{2} \right\} \right) = \begin{cases} \left( \frac{2(N-1)}{N}x_n \right)^{N-1} & \text{for } x_n \in \left[ 0, \frac{N}{2(N-1)} \right) \\ 1 & \text{for } x_n \in \left[ \frac{N}{2(N-1)}, 1 \right] \end{cases}$$

So that his expected payment is

$$\int_0^{\frac{N}{2(N-1)}} \frac{N-1}{N}x_n \cdot \left( \frac{2(N-1)}{N}x_n \right)^{N-1} dx_n + \int_{\frac{N}{2(N-1)}}^1 \frac{1}{2}dx_n = \frac{N^2 - 2}{4(N^2 - 1)} \quad (1)$$

The ring representative wins when  $\frac{N-1}{N}x_n < \frac{1}{2}x_R \implies x_n < \frac{N}{2(N-1)}x_R$ , and this occurs with probability  $\frac{N}{2(N-1)}x_R$ . Therefore ex-ante, the expected payment for the ring representative is

$$\int_0^1 \frac{x_R}{2} \cdot \frac{N}{2(N-1)}x_R \cdot (N-1)x_R^{N-2} dx_R = \frac{N}{4(N+1)} \quad (2)$$

The seller's expected revenue is thus

$$\mathbb{E}[\mathbf{R}]_{[\text{First}]} = (1) + (2) = \frac{2N^2 - N - 2}{4(N^2 - 1)}. \quad (3)$$

### Second Price

**Vickrey (1961)** implies that whenever there is a ring, both the ring representative and the non-ring bidder bid their respective valuations. The expected revenue is thus the expected first order statistic of  $\{x_n, x_R\}$ . This has distribution  $Q(\theta) = \theta + \theta^{N-1} - \theta^N$ , with density  $q(\theta) = 1 + (N-1)\theta^{N-2} - N\theta^{N-1}$ .

Therefore,

$$\mathbb{E}[\mathbf{R}]_{[\text{Second}]} = \int_0^1 \theta q(\theta) d\theta = \frac{(N+2)(N-1)}{2N(N+1)}. \quad (4)$$

However, since

$$\frac{2N^2 - N - 2}{4(N^2 - 1)} < \frac{(N+2)(N-1)}{2N(N+1)},^9$$

we have that

<sup>8</sup>Vickrey (1961)

<sup>9</sup>This is because  $\frac{2N^2 - N - 2}{4(N^2 - 1)} - \frac{(N+2)(N-1)}{2N(N+1)} = \underbrace{-\frac{(N-2)^2}{4N(N^2 - 1)}}_{<0}$

$$\mathbb{E}[\mathbf{R}]_{\text{First}} < \mathbb{E}[\mathbf{R}]_{\text{Second}}$$

Therefore,

**Proposition 1.** *When the ring presence will be known by the non-ring bidder, the seller is better off under the second price auction.*

This means that the conclusion that the first price auction dominates the second price auction in the presence of colluding bidders is not robust to the case where the non-ring bidder finds out privately about the ring's presence.

## 6 Conclusion

In this short paper, we have analyzed a simple model of collusion with one non-colluding bidder in a single object independent private value first price auction. Different from the norm, we assume that when collusion is prevalent, the ring takes precaution to hide its presence. We analyze subsequently the strategies and revenues when the non-ring bidder is aware of the ring's presence while the ring is oblivious to this fact. Then we compare the seller's revenue to that under a second price auction. We find that expected revenues are larger in the second price auction compared to the first price auction. This is in contrast to earlier results comparing revenues under collusion where the first price auction dominates.

The consequence is that the statement that first price auctions revenue dominate second price auctions under collusion is not very robust.

**References**

- Asker, J. (2010). A Study of the Internal Organization of a Bidding Cartel. *American Economic Review*, 100(3):724–62.
- Athey, S. (2001). Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information. *Econometrica*, 69(4):861 – 899.
- Baldwin, L. H., Marshall, R. C., and Richard, J. (1997). Bidder Collusion at Forest Service Timber Sales. *Journal of Political Economy*, 105(4):657–699.
- Ceesay, M. (2019). *Collusion in Auctions: Secret vs Public Rings*. PhD thesis, University of Naples Federico II.
- Kawai, K. and Nakabayashi, J. (2022). Detecting large-scale collusion in procurement auctions. *Journal of Political Economy*, 130(5):1364–1411.
- Kirkegaard, R. (2012). A Mechanism Design Approach to Ranking Asymmetric Auctions. *Econometrica*, 80(5):2349 – 2364.
- Marshall, R. and Marx, L. (2012). *The Economics of Collusion*. The MIT Press.
- Marshall, R. C., Meurer, M. J., Richard, J.-F., and Strmoquist, W. (1994). Numerical Analysis of Asymmetric First Price Auctions. *Games and Economic Behavior*, 7:193–220.
- OFT (2009). Decision of the Office of Fair Trading No. CA98/02/2009 Bid rigging in the construction industry in England. Technical report.
- Pesendorfer, M. (2000). A Study of Collusion in First-Price Auctions. *Review of Economic Studies*, 67:381–411.
- Porter, R. H. (2005). Detecting Collusion. *Review of Industrial Organization*, 26(2):147–167.
- Riley, J. and Samuelson, W. (1981). Optimal Auctions. *American Economic Review*, 71(3):381–392.
- Vickrey, W. (1961). Counterspeculation, Auctions, and Competitive Sealed Tenders. *Journal of Finance*, 16(1):8 – 37.