

Correia, Isabel; Melo, Teresa

**Research Report**

## The impact of flexible warehouse lease contracts on the configuration of a two-echelon distribution network

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Campus Rotenbühl  
Waldhausweg 14  
D-66123 Saarbrücken

Telefon: +49 (0) 681/ 5867 - 519

E-Mail: [fakultaet-wiwi@htwsaar.de](mailto:fakultaet-wiwi@htwsaar.de)

Internet: [www.htwsaar.de/wiwi](http://www.htwsaar.de/wiwi)

# The impact of flexible warehouse lease contracts on the configuration of a two-echelon distribution network

Isabel Correia<sup>a</sup>, Teresa Melo<sup>b\*</sup>

<sup>a</sup> *Centro de Matemática e Aplicações (CMA), Departamento de Matemática, Faculdade de Ciências e Tecnologia, Universidade NOVA de Lisboa, P 2829-516 Caparica, Portugal*

<sup>b</sup> *Business School, Saarland University of Applied Sciences, D 66123 Saarbrücken, Germany*

## Abstract

E-commerce has experienced a remarkable growth in recent years. This trend is gradually driving retailers to shift from typical warehouse rental contracts with rigid conditions and long-term commitments to flexible alternatives that provide storage space on demand. This strategy gives retailers the ability to respond dynamically to variations in demand, thereby improving customer service. In this paper, we address this trend by considering a two-echelon, multi-commodity distribution network operated by a retailer and whose configuration can be changed over a multi-period planning horizon. Location decisions concern the intermediate facilities and are framed by flexible and scalable conditions. Accordingly, the retailer needs to decide which company-owned facilities should be retained, and where, when and for how long new facilities should be leased from a set of potential sites with given capacities. In this setting, a facility can be leased at the same location for several non-consecutive periods. In addition to location decisions, also procurement, inventory, and distribution decisions need to be made to satisfy customer demand at minimum total cost. We develop a mixed-integer linear programming formulation for this problem and propose various families of additional inequalities that prove to be very useful to find high-quality solutions. To measure the impact of the new business model, we also consider alternative approaches with limited flexibility and scalability that are frequently encountered in practice. Our computational study gives useful insights on the trade-offs achieved by each of the different approaches with respect to total cost, network redesign decisions, and various logistics functions.

**Keywords:** distribution network redesign, short-term storage space rental, mixed-integer linear programming, valid inequalities

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\*Corresponding author. *E-mail address:* [teresa.melo@htwsaar.de](mailto:teresa.melo@htwsaar.de)

# 1 Introduction

Traditionally, retail companies set up and manage a distribution network with a number of warehouses or distribution centers operating at geographically dispersed locations. This framework is the outcome of strategic planning, thereby involving a sizeable capital investment for purchasing or building storage facilities and acquiring material handling equipment. In the last two decades, outsourcing warehouse operations to third-party logistics (3PL) providers has become a significant trend (Marasco, 2008; Leuschner et al., 2014; Zhu et al., 2017). External warehouse contracts negotiated between retailers and 3PL firms require a commitment of at least one year, and routinely range from three to five years (Tomese et al., 2020). In both cases, the operation of company-owned and third-party warehouses involves a long-term commitment, independently of demand growth, seasonal peaks, and demand variations faced by the retail company. However, in recent years, retailers have experienced increasing pressure to scale their storage capacities in the short-term, chiefly due to the remarkable growth in online retailing. E-retail sales worldwide have reached 4.28 trillion U.S. dollars in 2020, accounting for 18 percent of total retail sales (Statista, 2021a). Global e-commerce sales are expected to increase by almost fifty percent until 2024, up to 6.388 trillion U.S. dollars (Statista, 2021b). This trend is affecting in particular companies that adopt omni-channel strategies, that is, combine physical sales with the management of online orders, leading to shortages of warehouse space (Kembro et al., 2018).

On-demand warehousing has emerged as a flexible alternative to long-term warehouse agreements having minimum volume commitments and rigid pricing models. This new business model has become possible through digital business-to-business marketplaces like Flexe and Stockpots, that enable companies to access additional storage capacity whenever and wherever required (Tomese et al., 2020; Xenou et al., 2021). Lease commitments can be made on an annual, monthly or even weekly basis.

In this paper, we study the impact of on-demand warehousing on redesigning a two-echelon distribution network. To this end, we consider the general setting in which a retail company operates several warehouses at fixed locations to serve the demands of a set of customer zones for multiple product families. Seasonal demand variations as well as demand peaks triggered by marketing campaigns, such as Black Friday and Cyber Monday, require the retailer to lease additional warehouse space on a temporary basis. It is assumed that short-term space rental can be provided by a set of potential warehouses with given capacities. Storage capacity

planning needs to be aligned not only with demand fluctuations but also with the limited availability of different products purchased by the company from various suppliers. Hence, over a planning horizon divided into a finite number of periods, the retailer must decide which company-owned warehouses are to be retained and where, when, and for how long new warehouse lease agreements are to be signed. The latter can be in place at the same location for non-consecutive periods of time. In addition, the amount of each product family to be held in stock at the storage facilities along with the material flow from the upstream sources (i.e., suppliers) to customer zones via company-owned and leased warehouses are to be determined in each time period. The objective is to find the least-costly network configuration.

The contributions of the present work are threefold. First, we study a new multi-period distribution network redesign problem that integrates temporary warehouse lease contracts as a flexible strategy for sizing storage space to accommodate future market needs. We develop a mixed-integer linear programming (MILP) model for this problem, and show that typical lease agreements having restrictive clauses on rental commitments are particular cases of the proposed formulation. Second, we develop several sets of valid inequalities that significantly enhance the computational performance of a general-purpose optimization solver. Third, we report on the results of an extensive computational study and assess the extent to which a company can benefit from on-demand warehousing as compared to the adoption of warehouse lease agreements with limited flexibility and scalability. Our comparative analysis also provides managerial insights into the trade-offs that are achieved under different options for network redesign.

The remainder of this paper is organized as follows. In Section 2, we review and examine how our study relates to the relevant literature. This is followed, in Section 3, by a formal description of our problem and the development of a mathematical formulation. We also discuss particular cases involving warehouse lease contracts with restrictive features frequently encountered in practice, and propose various sets of valid inequalities to enhance the original formulation. Computational results are presented in Section 4. Finally, a summary of the key findings is given in Section 5 and future research directions are identified.

## **2 Literature review**

Given that the problem that we study shares some common features with the multi-period capacitated facility location problem, we review in this section selected research articles that

have this problem at their core. Our aim is not to provide a systematic review of every study undertaken in this field but rather to examine the main features of the problems investigated and their relation to our work. In addition, we also highlight the extent to which our work goes beyond the classical approach taken in multi-period facility location.

Following the seminal paper by Van Roy and Erlenkotter (1982), the multi-period, or dynamic, facility location problem (MFLP) has gained increasing attention over the past few decades (Arabani and Farahani, 2012; Nickel and Saldanha da Gama, 2019). Its relevance has been prompted, especially in recent years, by computational advancements and the need to improve the realism in modeling practical problems. This is the case, for example, in the field of supply chain network design (SCND), where the MFLP is at the core of many complex problems (Melo et al., 2009; Cordeau et al., 2021).

Given a set of candidate locations for establishing facilities and a set of customers with time-dependent and deterministic demand, the aim of the MFLP is to determine the spatial distribution of the facilities at each time period so as to minimize the total cost for meeting the customer demand over time. Typically, location decisions have a strategic nature due to the sizeable capital investment required for building or purchasing new facilities. As a result, a common property of many studies is that once a facility is opened during a time period, it will remain open until the end of the planning horizon (Jacobsen, 1990; Gourdin and Klopfenstein, 2008). An extension of this setting is to also consider initially existing facilities that can be subject to closure decisions. Similar to the pure phase-in case, the status of these facilities can change at most once over the planning horizon due to the associated large removal cost. Therefore, once closed, it is not allowed to re-open an existing facility (Correia and Melo, 2016; Marufuzzaman et al., 2016; Sauvey et al., 2020).

In order to respond to demand and cost changes, opening new facilities and keeping the existing facilities open or closed is a natural way of making adjustments in the network configuration over time. Various other strategies have been devised in an attempt to provide a better response to demand and cost variations in real-world applications, leading to different extensions of the MFLP. These include facility expansion for strictly increasing demand (Shulman, 1991), facility expansion and contraction for both growing and declining demand (Antunes and Peeters, 2001; Wilhelm et al., 2013; Correia and Melo, 2017), and facility relocation (Melo et al., 2006; Torres-Soto and Ülster, 2011). Some authors, like Becker et al. (2019) and Allman and Zhang (2020), addressed location problems that allow for opening, closing, expanding, contracting, and relocating facilities over time. The possibility of closing facilities and re-opening

them more than once over the planning horizon was studied by Dias et al. (2007), who have taken into account that, in general, the cost of reactivating a facility is smaller than the cost of opening the facility for the first time. Jena et al. (2015, 2016) extended this problem by considering capacity expansion and capacity reduction, along with partial closing and re-opening of facilities. In our extension of the MFLP, we also model capacity adjustments in the network configuration over time, but from the perspective of temporary capacity acquisition. The latter is provided by flexible conditions for leasing storage space at different locations. Accordingly, location decisions are reversible, being made on a medium or even short-term basis (e.g., quarterly or monthly), as opposed to the classic approach shared by the above studies and many others on facility location, where these decisions are strategic and therefore have a long-lasting impact.

As aforementioned, the MFLP is at the core of many real-world problems arising in SCND. Extensions of this problem integrate various logistics functions, either independently or combined, such as the procurement of raw materials, the production of intermediate and finished items, the management of inventories, and the choice of transportation modes (Melo et al., 2009; Cordeau et al., 2021). In this context, a supply chain network is typically modeled with two or more facility echelons where multiple commodities are processed. The problem studied by Hinojosa et al. (2008) falls into this category by including manufacturing and storage facilities. An initial network configuration is considered whose total capacity can be adjusted over the planning horizon through opening new facilities and closing initially existing facilities (i.e., plants and warehouses). Inventory-related decisions at the warehouse echelon are modeled and customer demand may be (partially) satisfied from external sources. The latter feature was also captured by the network design model developed by Cortinhal et al. (2015), which includes procurement and production decisions, and the selection of transportation modes for the distribution of raw materials, intermediate products, and end items in a four-stage network. In their extension of the MFLP, Emirhüseyinoğlu and Ekici (2019) focus on supplier selection and the choice of a quantity discount level from a set of available options for purchasing a single commodity. Among the strategies that have been adopted in multi-period SCND problems to respond to fluctuations in the demand for multiple products, capacity expansion has been widely used (Thanh et al., 2008; Bashiri et al., 2012; Badri et al., 2013; Correia et al., 2013; Cortinhal et al., 2015). The multi-period distribution network redesign problem that we study in this paper includes several of the features listed above, namely two facility echelons, representing a set of suppliers and a set of warehouses with limited capacities, and a set of



customers with demand for multiple items. Decisions on procurement, inventory management, warehouse location, and distribution are modeled in a context that is particularly suitable for retail companies needing access to scalable warehousing space and flexible rental agreements.

As pointed out by the recent review by Kumar et al. (2021), warehouse location and capacity planning remains one of the critical problems faced by many companies. Flexible warehouse contracts for temporary use of storage space enable companies to dynamically respond to variable or seasonal demand, and thus improve customer service (Tomese et al., 2020). There is limited research regarding the integration of these features into the MFLP. Hybrid models, combining privately owned and leased warehouses, partially capture the requirements of omnichannel retailers. Thanh et al. (2008) redesign a logistics network comprising suppliers, plants, warehouses, and customers. At the beginning of the planning horizon, a set of plants and a set of private warehouses are in place. The status of each one of these facilities can be changed at most once over time, either by closing the facility or expanding its initially available capacity. New plants can also be established and additional storage space can be rented at public warehouses for several non-consecutive periods. However, a minimum two-period gap is enforced between two rental contracts at the same public warehouse. This condition is not imposed in our problem as we allow total flexibility in deciding the frequency and the duration of a lease contract at a particular location. A similar problem was studied by Bashiri et al. (2012) and Badri et al. (2013) for determining the network configuration that maximizes the total net income subject to side constraints related to the location and expansion of plants and warehouses, and various logistics functions for processing multiple products at different facilities. No limit is imposed on the amount of capacity that can be rented at a public warehouse in each period, which, in our opinion, is not a realistic assumption. Due to their strategic nature, location and capacity expansion decisions can only be made at selected time periods in the planning horizon, while all other decisions related to the procurement, production, inventory accounting, and distribution of materials are considered to be tactical, and thus can be made at any period. This approach was also used in other extensions of the MFLP as a means of integrating strategic and tactical decisions in a single model (e.g., Correia and Melo (2016, 2017)). Recently, Darvish and Coelho (2018) modeled a multi-period, multi-product, two-echelon production-distribution network design problem, where the intermediate facilities (i.e., distribution centers, DCs) can be leased for a limited number of consecutive periods. The duration of a lease contract is assumed to be the same at each potential DC location and a fixed rental fee is charged per period that depends on the selected location. By contrast, we

adopt a more realistic setting in our problem since the length of a lease contract at a selected warehouse location is an endogenous decision whose fixed cost depends on the tenancy term and the amount of capacity rented. Furthermore, capacity is not only provided by leased warehouses but also by company-owned facilities, whose operation can be discontinued over time. At the operational planning level, the location-routing problem studied by Darvish et al. (2019) selects on a daily basis the DC locations to be operated and determines a vehicle route for each open DC to deliver one single commodity to a subset of customers. A fixed daily fee for renting a DC is incurred in addition to variable costs for shipping the commodity from a central depot to the open DCs, variable vehicle traveling costs, and variable penalty costs for late deliveries to customers. In this setting, a DC can be rented at the same location for several non-consecutive days.

Despite the significant literature on the MFLP and its many extensions addressing different features encountered in real-world applications, to the best of our knowledge the problem that we study has never been examined, thus making our contribution original. In particular, our work will assist omni-channel retailers in understanding how their distribution networks can benefit from the flexibility provided by new business models such as on-demand warehousing.

### 3 Problem statement and formulation

In this section, we present the general setting in which our two-echelon distribution network redesign problem arises and motivate the assumptions made. Subsequently, we propose a MILP formulation for this problem in Section 3.2. Particular warehouse lease commitments that are encountered in practice are discussed in Section 3.3. Furthermore, several sets of valid inequalities are developed in Section 3.4 that strengthen the proposed formulation.

We consider a retail company that operates a set of warehouses at fixed locations to serve the demands of customer zones for multiple product families over a planning horizon divided into a finite number of time periods. Without loss of generality, these facilities will be termed *company-owned warehouses* hereafter, even if they are rented from a 3PL firm. In addition, short-term space rental can be provided by a set of potential warehouses with given capacities. Flexible contractual conditions are assumed that permit leasing storage space at these facilities over multiple periods that need not be consecutive. While seasonal demand peaks provide the most obvious reason for a retail company to access space and warehousing services for limited periods of time, there are also other sources. These include new product launches, entry into new

markets, high-demand campaigns, and omni-channel retail operations. An increasing number of companies is making use of on-demand warehousing to access new markets or introduce new products. This strategy avoids making major investments in new storage facilities without knowing if a position in the new market can be secured or how consumer demand for a new product will evolve. Short, high-intensity periods triggered by promotional campaigns (e.g., Valentine's Day, Black Friday, Cyber Monday, post-Christmas promotions) also lead to the need for scaling storage capacity in the short-term. Moreover, retailers try to position their products closer to the final customers without having to commit to long-term lease contracts, and so enabling them to respond to rapid changes in buying patterns. In addition, they take advantage of lower last-mile delivery costs and reduced time to delivery.

We assume that warehouse lease costs reflect economies of scale regarding volume and commitment duration. In addition, both company-owned and leased warehouses can hold inventory that can be carried from one period to the next for the duration of the warehouse's operation. Demand fluctuations and suppliers' capacities can make it beneficial to build inventory in some periods to be used in later ones. This setting for inventory management was also adopted in other studies (e.g., Melo et al. (2006); Hinojosa et al. (2008); Bashiri et al. (2012); Darvish and Coelho (2018)). In addition to deciding which potential warehouses should be leased and for how long, the retailer also has the option to discontinue the operation of some or all company-owned warehouses at some point during the planning horizon. In this case, a fixed cost is incurred (e.g., for terminating the rental agreement with the 3PL provider) and the facility cannot be re-opened. The latter feature is motivated by the significant capital and time investment needed for acquiring new storage space or negotiating a new partnership with a 3PL firm.

The products handled by the warehouses are purchased from a set of suppliers whose manufacturing capacities vary throughout the planning horizon. In addition to determining a schedule for operating company-owned warehouses and leasing additional storage space, the retailer also needs to decide on the allocation of warehouses to suppliers and customer zones, along with the flow of products through the network over the planning horizon. The objective is to identify the network configuration that satisfies customer demands at minimal total cost. Fixed costs are incurred for operating and discontinuing company-owned warehouses as well as for temporarily leasing storage space at new locations. Furthermore, variable costs are considered for storing and handling the product families at the warehouses and distributing them through the network.

### 3.1 Notation

In this section, we introduce the notation that will be used hereafter. We start by defining the sets used in our model.

$T$	Set of discrete time periods in the planning horizon.
$S$	Set of suppliers.
$I^e$	Set of company-owned warehouses that are operating at the beginning of the planning horizon.
$I^n$	Set of potential warehouses where storage space can be leased on a temporary basis.
$I$	Set of all warehouse locations, $I = I^e \cup I^n$ , $I^e \subseteq I$ , $I^n \subseteq I$ and $I^e \cap I^n = \emptyset$ .
$J$	Set of customer zones.
$P$	Set of product families.

The following parameters are defined for suppliers, warehouses, and customer zones.

$SQ_{sp}^t$	Amount of product family $p \in P$ available at supplier $s \in S$ in time period $t \in T$ .
$Q_i^t$	Storage capacity of warehouse $i \in I$ in time period $t \in T$ .
$\beta_p$	Amount of capacity used by one unit of product family $p \in P$ stored at warehouse $i \in I$ .
$u_{ip}^0$	Amount of product family $p \in P$ held in stock in warehouse $i \in I$ at the beginning of the planning horizon. At the potential locations, it is assumed that $u_{ip}^0 = 0$ ( $i \in I^n$ ; $p \in P$ ).
$d_{jp}^t$	Demand of customer zone $j \in J$ for product family $p \in P$ in time period $t \in T$ .

The following cost parameters are introduced:

$OC_i^t$	Fixed cost of operating the company-owned warehouse $i \in I^e$ at time period $t \in T$ .
$FC_i^t$	Fixed cost of discontinuing the operation of the company-owned warehouse $i \in I^e$ at the <i>beginning</i> of time period $t \in T$ .
$L_i^{t,t'}$	Total fixed cost of leasing warehouse $i \in I^n$ from the beginning of time period $t$ until the end of time period $t'$ ( $t, t' \in T$ ; $t' \geq t$ ).
$SC_{sip}^t$	Cost of distributing one unit of product family $p \in P$ from supplier $s \in S$ to warehouse $i \in I$ at time period $t \in T$ .

- $DC_{ijp}^t$  Cost of distributing one unit of product family  $p \in P$  from warehouse  $i \in I$  to customer zone  $j \in J$  at time period  $t \in T$ .
- $IC_{ip}^t$  Cost of storing one unit of product family  $p \in P$  in warehouse  $i \in I$  at time period  $t \in T$ .

### 3.2 Mixed-integer linear programming formulation

The mathematical formulation to be presented next uses two sets of binary variables and three sets of non-negative continuous variables. The former represent warehouse decisions, while the latter model the product flow through the network as well as inventory holding decisions at the warehouses.

$$y_i^t = 1 \text{ if the company-owned warehouse } i \text{ ceases to be operated at the beginning of time period } t, \text{ and } 0 \text{ otherwise } (i \in I^e, t \in T). \quad (1)$$

$$z_i^{t,t'} = 1 \text{ if warehouse } i \text{ is leased from the beginning of time period } t \text{ until the end of time period } t', \text{ and } 0 \text{ otherwise } (i \in I^n, t \in T, t' = t, t + 1, \dots, |T|). \quad (2)$$

$$v_{sip}^t : \text{quantity of product family } p \text{ distributed from supplier } s \text{ to warehouse } i \text{ at time period } t (s \in S, i \in I, p \in P, t \in T). \quad (3)$$

$$x_{ijp}^t : \text{quantity of product family } p \text{ distributed from warehouse } i \text{ to customer zone } j \text{ at time period } t (i \in I, j \in J, p \in P, t \in T). \quad (4)$$

$$u_{ip}^t : \text{quantity of product family } p \text{ held in stock in warehouse } i \text{ at the end of time period } t (i \in I, p \in P, t \in T). \quad (5)$$

The MILP formulation is as follows:

$$\begin{aligned} \text{Min} \quad & \sum_{i \in I^e} \sum_{t \in T} OC_i^t \left( 1 - \sum_{t'=1}^t y_i^{t'} \right) + \sum_{i \in I^e} \sum_{t \in T} FC_i^t y_i^t + \sum_{i \in I^n} \sum_{t \in T} \sum_{t'=t}^{|T|} L_i^{t,t'} z_i^{t,t'} + \\ & \sum_{s \in S} \sum_{i \in I} \sum_{p \in P} \sum_{t \in T} SC_{sip}^t v_{sip}^t + \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} DC_{ijp}^t x_{ijp}^t + \\ & \sum_{i \in I} \sum_{p \in P} \sum_{t \in T} IC_{ip}^t u_{ip}^t \end{aligned} \quad (6)$$

subject to

$$\sum_{t \in T} y_i^t \leq 1 \quad i \in I^e \quad (7)$$

$$\sum_{t'=t}^{|T|} z_i^{t,t'} \leq 1 \quad i \in I^n, t \in T \quad (8)$$

$$\sum_{t'=1}^t z_i^{t',t} \leq 1 \quad i \in I^n, t \in T \setminus \{1\} \quad (9)$$

$$\begin{aligned} \sum_{m=1}^{t-1} \sum_{m'=t}^{t'} z_i^{m,m'} + \sum_{m=t+1}^{t'} \sum_{m'=m}^{|T|} z_i^{m,m'} \\ \leq (t' - t + 1) \left(1 - z_i^{t,t'}\right) \end{aligned} \quad \begin{aligned} i \in I^n, t \in T \setminus \{|T|\}, \\ t' = t + 1, \dots, |T| \end{aligned} \quad (10)$$

$$\sum_{i \in I} v_{sip}^t \leq SQ_{sp}^t \quad s \in S, p \in P, t \in T \quad (11)$$

$$u_{ip}^{t-1} + \sum_{s \in S} v_{sip}^t = u_{ip}^t + \sum_{j \in J} x_{ijp}^t \quad i \in I, p \in P, t \in T \quad (12)$$

$$\sum_{p \in P} \beta_p u_{ip}^{t-1} + \sum_{s \in S} \sum_{p \in P} \beta_p v_{sip}^t \leq Q_i^t \left(1 - \sum_{t'=1}^t y_i^{t'}\right) \quad i \in I^e, t \in T \quad (13)$$

$$\sum_{p \in P} \beta_p u_{ip}^{t-1} + \sum_{s \in S} \sum_{p \in P} \beta_p v_{sip}^t \leq Q_i^t \sum_{m=1}^t \sum_{m'=t}^{|T|} z_i^{m,m'} \quad i \in I^n, t \in T \quad (14)$$

$$\sum_{p \in P} \beta_p u_{ip}^{t-1} \leq Q_i^t \left(1 - \sum_{t'=t}^{|T|} z_i^{t',t'}\right) \quad i \in I^n, t \in T \quad (15)$$

$$\sum_{p \in P} \beta_p u_{ip}^t \leq Q_i^t \left(1 - \sum_{t'=1}^t z_i^{t',t}\right) \quad i \in I^n, t \in T \quad (16)$$

$$\sum_{i \in I} x_{ijp}^t = d_{jp}^t \quad j \in J, p \in P, t \in T \quad (17)$$

$$y_i^t \in \{0, 1\} \quad i \in I^e, t \in T \quad (18)$$

$$z_i^{t,t'} \in \{0, 1\} \quad \begin{aligned} i \in I^n, t \in T, \\ t' = t, t + 1, \dots, |T| \end{aligned} \quad (19)$$

$$v_{sip}^t \geq 0 \quad s \in S, i \in I, p \in P, t \in T \quad (20)$$

$$x_{ijp}^t \geq 0 \quad i \in I, j \in J, p \in P, t \in T \quad (21)$$

$$u_{ip}^t \geq 0 \quad i \in I, p \in P, t \in T \quad (22)$$

The objective function (6) minimizes the total cost over the planning horizon. The first two components represent the fixed costs to operate and to stop using company-owned warehouses, respectively. The third component gives the total fixed cost of leasing storage space in new locations. The total costs of distributing the product families from the suppliers to the warehouses and from the latter to the customer zones are represented by the fourth and fifth components, respectively. The last component determines the total inventory holding cost at the warehouses.

Constraints (7) ensure that the operation of a company-owned warehouse can be discontinued at most once throughout the planning horizon. Constraints (8) guarantee that lease contracts for additional storage space at a given warehouse can only be in place during the planning horizon. In addition, different lease contracts for a particular location cannot start in the same time period. Constraints (9) prevent multiple lease contracts from terminating in the same time period in a fixed location. Constraints (10) make sure that the time windows of different lease contracts for a given location do not overlap. Specifically, if a lease contract is in place in location  $i$  from period  $t$  until period  $t'$  (i.e.,  $z_i^{t,t'} = 1$  for  $t' > t$ ) then no other lease agreements can be signed for periods that partly fall within this time window, as illustrated in Figure 1. If the opposite occurs (i.e.,  $z_i^{t,t'} = 0$  for  $t' > t$ ) then not more than  $t' - t + 1$  lease

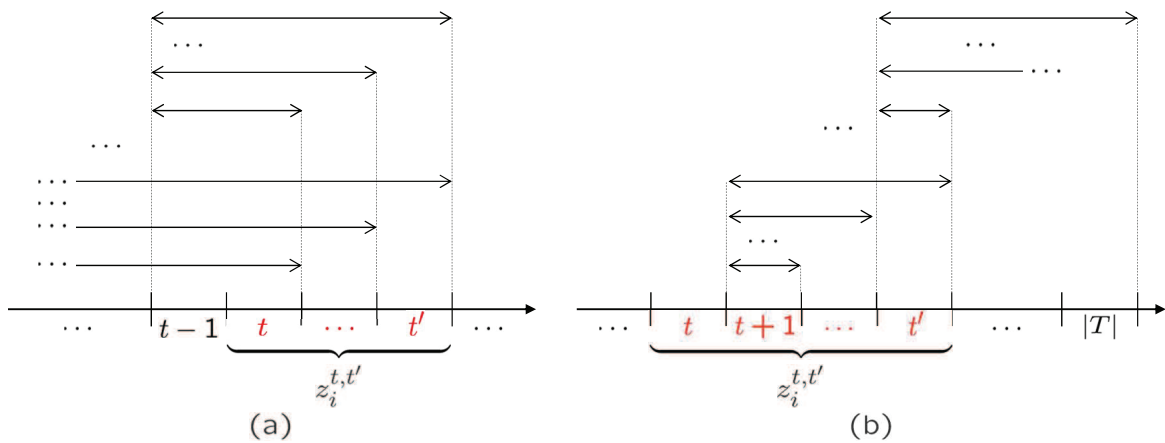


Figure 1: Multiple warehouse lease contracts covering at least one period from  $t$  to  $t'$  ( $t' > t$ ). (a) Contracts starting before period  $t$ . (b) Contracts starting in period  $t + 1$  or later.

agreements can be selected at location  $i$ . Observe that at most one contract can be in place in the time window  $[m, m']$  for  $m = 1, \dots, t - 1$  and  $m' = t, \dots, t'$ . Furthermore, at most  $t' - t$  lease agreements can be selected in the time window  $[m, m']$  for  $m = t + 1, \dots, t'$  and

$m' = m, \dots, |T|$ . Hence, in total, not more than  $t' - t + 1$  lease contracts can be signed for location  $i$ . We note that it is not necessary to define constraints (10) for  $t' = t$  because in this case they reduce to inequalities (9).

Supplier capacity constraints are enforced by inequalities (11). Constraints (12) are inventory balance conditions. Constraints (13) and (14) guarantee that the capacities of operating company-owned warehouses and leased warehouses are not exceeded, respectively. According to constraints (15), no inventory can be held in a new warehouse at the beginning of its lease agreement. Moreover, when the lease contract of a warehouse expires, its inventory level must drop to zero as stated by constraints (16). Customer demands are satisfied by constraints (17). Finally, binary and non-negativity conditions are defined by constraints (18)–(22).

The problem formulated by (6)–(22) includes the classical multi-period uncapacitated facility location problem (MUFLP) as a special case and thus, it is NP-hard. The reduction of our problem to the MUFLP is straightforward by taking  $I^e = \emptyset$ ,  $I^n = I$ ,  $S = \emptyset$ ,  $|P| = 1$ , and  $Q_i^t = +\infty$  for every  $i \in I$  and  $t \in T$ . Moreover, the sets of variables (1), (3), and (5) are removed. The binary variables (2) are redefined as  $z_i^t = 1$  if warehouse  $i$  is leased from the beginning of time period  $t$  until the end of the planning horizon, and  $z_i^t = 0$ , otherwise ( $i \in I$ ,  $t \in T$ ).

The particular ‘greenfield’ situation, in which the company does not own any warehouses at the beginning of the planning horizon (i.e.,  $I^e = \emptyset$ ), is easily modeled by the proposed formulation. This approach corresponds to solely using warehousing resources on-demand and applies, for example, to startup businesses in their early growth stage. By contrast, we have decided to focus on the more general ‘brownfield’ case by assuming that a distribution network is already in place, which needs to be redesigned, for instance, due to changes in the spatial distribution of the customers or as a consequence of significant demand variations over the planning horizon. A pure expansion scenario is also easily modeled by setting  $y_i^t = 0$  for every  $i \in I^e$  and  $t \in T$ , and thus retaining the company-owned warehouses. In this case, additional storage space becomes available through leasing warehouses. Our formulation can also accommodate the availability of various warehouse size alternatives at a particular location. This is accomplished by extending the set  $I^n$  with as many copies of the location as storage sizes that are offered for leasing.

Formulation (6)–(22) can also be extended in several ways to include various additional features of practical situations. By reversing the inequality sign, constraints (13) and (14) can be used to enforce lower limits on the total quantity processed by each warehouse in every period.



Accordingly, a minimum amount must be handled for a warehouse to be economically viable. A similar condition can also be imposed on product acquisition, when a minimum quantity of a product family must be purchased from a supplier in order to obtain a quantity discount. In this case, constraints (11) with reversed inequality sign are added to the formulation either for all or a given subset of suppliers. Single-assignment conditions for customer zones can also be incorporated by replacing the flow variables  $x_{ijp}^t$  by binary variables  $\tilde{x}_{ijp}^t$ , and taking  $d_{jp}^t \tilde{x}_{ijp}^t$  in (6), (12), and (17) instead of  $x_{ijp}^t$ . Finally, the variable distribution costs  $DC_{ijp}^t$  between warehouses and customer zones may include other expenses in addition to shipment costs (e.g., handling and packing costs). Moreover, manufacturing costs can also be added to the distribution costs  $SC_{ijp}^t$  from suppliers to warehouses.

### 3.3 Lease contracts with restrictive clauses

Formulation (6)–(22) models a problem with flexible lease warehouse contracts, without minimum commitments or special conditions being enforced on lease agreements. By contrast, in traditional warehousing, lease contracts are framed by rigid terms. For example, often a minimum lease term is imposed. This particular case is easily integrated into the proposed formulation by adding inequality (23), where  $\lambda_i$  denotes the minimum number of consecutive time periods of a lease contract at the potential warehouse  $i \in I^n$ .

$$\sum_{i \in I^n} \sum_{t \in T} \sum_{t'=t}^{\min\{t+\lambda_i-2, |T|\}} z_i^{t,t'} = 0 \quad (23)$$

Accordingly, any term length shorter than  $\lambda_i$  periods is excluded by constraint (23) when a lease agreement is selected to begin in period  $t$  at location  $i$ .

Most multi-period facility location problems specify that a new facility may be opened in any time period, but once opened, it must be operated until the end of the planning horizon (Arabani and Farahani, 2012; Nickel and Saldanha da Gama, 2019). This feature together with the minimum lease term condition described above can easily be modeled by adding the following constraint to the formulation in Section 3.2:

$$\sum_{i \in I^n} \sum_{t=1}^{|T|-\lambda_i+1} \sum_{t'=t}^{|T|-1} z_i^{t,t'} + \sum_{i \in I^n} \sum_{t=|T|-\lambda_i+2}^{|T|} \sum_{t'=t}^{|T|} z_i^{t,t'} = 0 \quad (24)$$

The above equality excludes lease agreements that start in some period  $t$  and do not run until period  $|T|$  (first component on the left-hand side) and lease contracts ending at period  $|T|$  that are shorter than the minimum length requirement (second component on the left-hand side).

In some practical situations, the duration of a lease contract must be a multiple of a base fixed-term tenancy, e.g., 3 months. Let  $\alpha_i$  denote the base length of a contract at location  $i \in I^n$ . In this case, all lease agreements whose total duration is not a multiple of  $\alpha_i$  must be excluded. As a result, the following condition is added to the original formulation:

$$\sum_{i \in I^n} \sum_{t \in T} \sum_{\substack{t' = t \\ t' \neq (t-1) + \alpha_i \\ m = 1, \dots, \lfloor \frac{|T|}{\alpha_i} \rfloor}} z_i^{t,t'} = 0 \quad (25)$$

For example, for  $|T| = 10$  and  $\alpha_i = 3$  for a given warehouse location  $i \in I^n$ , lease agreements starting in period 2 ( $t = 2$ ) can last until the end of period 4 (3-period duration), or period 7 (6-period duration), or period 10 (9-period duration). Other lease term lengths cannot be selected and this is ensured by the condition  $\sum_{t'=2, t' \neq 1+3m: m=1,2,3}^{10} z_i^{2,t'} = z_i^{2,2} + z_i^{2,3} + z_i^{2,5} + z_i^{2,6} + z_i^{2,8} + z_i^{2,9} = 0$  according to (25).

### 3.4 Model enhancements

In an attempt to improve the polyhedral description of the set of feasible solutions to formulation (6)–(22), several families of additional inequalities are proposed in this section. These may contribute to more problem instances and with larger sizes being solved to (near-)optimality with general-purpose optimization software within reasonable computing time. Often, the performance improvement results from the decrease in the number of nodes to be explored in the branch-and-cut tree.

#### 3.4.1 Feasibility conditions

Even though the following inequalities are redundant for the linear relaxation of formulation (6)–(22), they prove to be computationally very useful (see Section 4). These inequalities ensure that sufficient global capacity is provided by open warehouses to satisfy the demand for

all product families in every period of the planning horizon.

$$\sum_{i \in I^e} Q_i^t \left( 1 - \sum_{t'=1}^t y_i^{t'} \right) + \sum_{i \in I^n} Q_i^t \sum_{m=1}^t \sum_{m'=t}^{|T|} z_i^{m,m'} \geq \sum_{j \in J} \sum_{p \in P} \beta_p d_{jp}^t \quad t \in T. \quad (26)$$

Inequalities (26) together with the following requirements define necessary conditions for feasible solutions to our problem to exist.

$$\sum_{t'=1}^t \sum_{s \in S} S Q_{sp}^{t'} \geq \sum_{t'=1}^t \sum_{j \in J} \beta_p d_{jp}^t \quad p \in P, t \in T.$$

The above inequalities guarantee that enough supplier capacity is available to satisfy the demand requirements for each product family over the planning horizon.

### 3.4.2 Limiting the selection of warehouse lease contracts

We note that constraints (10) can be disaggregated into two groups of inequalities, (27) and (28). Together, they limit the number of lease agreements that can be signed at each potential warehouse location over the planning horizon.

$$\sum_{m=1}^{t-1} \sum_{m'=t}^{t'} z_i^{m,m'} \leq 1 - z_i^{t,t'} \quad i \in I^n, t \in T \setminus \{1, |T|\},$$

$$t' = t + 1, \dots, |T|. \quad (27)$$

$$\sum_{m=t+1}^{t'} \sum_{m'=m}^{|T|} z_i^{m,m'} \leq (t' - t) \left( 1 - z_i^{t,t'} \right) \quad i \in I^n, t \in T \setminus \{|T|\},$$

$$t' = t + 1, \dots, |T|. \quad (28)$$

For illustration purposes, suppose that the planning horizon is divided into  $|T| = 7$  periods, and no lease contract is signed for periods 3 ( $t = 3$ ) through 5 ( $t' = 5$ ) at a particular location  $i$ , i.e.,  $z_i^{3,5} = 0$ . Inequalities (27) rule all contracts that start either in period 1 or in period 2, and terminate in periods 3, 4 or 5 as displayed in Figure 2 (a). Specifically, at most one contract can be selected within these options, i.e.,  $(z_i^{1,3} + z_i^{1,4} + z_i^{1,5}) + (z_i^{2,3} + z_i^{2,4} + z_i^{2,5}) \leq 1$ . By contrast, if  $z_i^{3,5} = 1$  then none of these lease agreements can be selected. A closer examination of this condition reveals that we can further strengthen it by including the variables  $z_i^{1,6}$ ,  $z_i^{1,7}$ ,

$z_i^{2,6}$ , and  $z_i^{2,7}$  on the left-hand side. In this way, we restrict the selection of additional lease agreements covering periods 3, 4, and 5. Hence, we can use the following inequalities (27'), which strengthen conditions (27), through replacing the upper limit  $t'$  in the second sum symbol by the total length of the planning horizon,  $|T|$ .

$$\sum_{m=1}^{t-1} \sum_{m'=t}^{|T|} z_i^{m,m'} \leq 1 - z_i^{t,t'} \quad i \in I^n, t \in T \setminus \{1, |T|\}, t' = t + 1, \dots, |T|. \quad (27')$$

Using the previous example, we show that constraints (28) guarantee that not more than two lease contracts can be selected between periods 3 and 5 ( $t' - t = 5 - 3$ ) when  $z_i^{3,5} = 0$ . In this case, several options are available, namely contracts can start in period 4 or later, but not later than period 5, and span one or several periods up to the end of the planning horizon. This case is displayed in Figure 2 (b). Accordingly,  $(z_i^{4,4} + z_i^{4,5} + z_i^{4,6} + z_i^{4,7}) + (z_i^{5,5} + z_i^{5,6} + z_i^{5,7}) \leq 2$ . Recall that constraints (8) enforce  $z_i^{4,4} + z_i^{4,5} + z_i^{4,6} + z_i^{4,7} \leq 1$  and  $z_i^{5,5} + z_i^{5,6} + z_i^{5,7} \leq 1$ . If the opposite decision is made, i.e. if  $z_i^{3,5} = 1$ , then inequalities (28) guarantee that no other lease contracts can be in place that (partially) cover periods 3, 4, and/or 5.

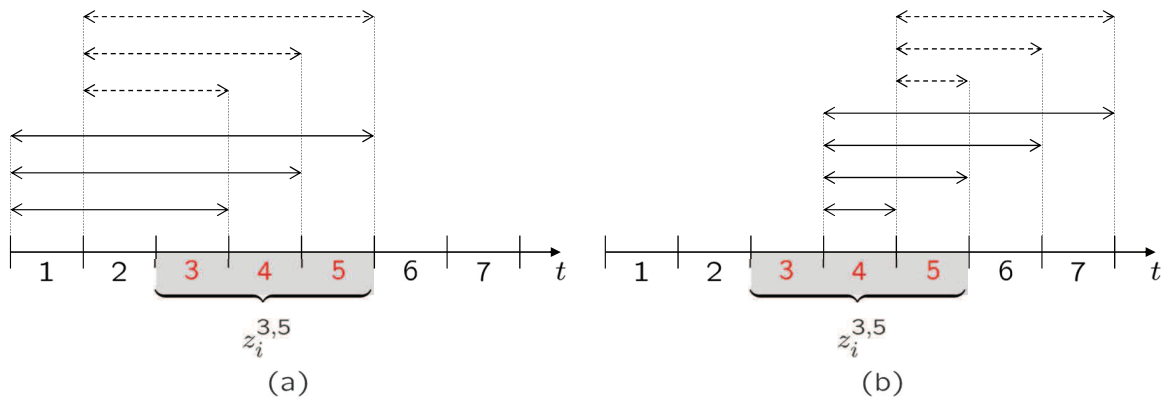


Figure 2: (a) Lease contracts starting in period 1 (double-sided arrows) and period 2 (dashed double-sided arrows), and ending in periods 3, 4, and 5. (b) Lease contracts starting in period 4 (double-sided arrows) and period 5 (dashed double-sided arrows).

In the presence of economies of scale in the warehouse lease costs, as in our case (see Appendix 1), it will not be optimal to sign multiple lease contracts, which together cover consecutive periods because the selection of a single contract spanning the same set of periods will always be less expensive. For example, if a contract starts in period 2 and ends in period 3 at a certain location, then it will not be optimal to sign an additional contract from period 4

onwards in that site. This observation gives rise to the following set of inequalities:

$$\sum_{t'=t+1}^{|T|} z_i^{t+1,t'} \leq 1 - \sum_{t'=1}^t z_i^{t',t} \quad i \in I^n, t \in T \setminus \{|T|\}.$$

For instance, if  $|T| = 5$  and  $t = 3$  then  $z_i^{4,4} + z_i^{4,5} \leq 1 - (z_i^{1,3} + z_i^{2,3} + z_i^{3,3})$ . Hence, for  $z_i^{2,3} = 1$  (and  $z_i^{1,3} = z_i^{3,3} = 0$ ), this condition imposes  $z_i^{4,4} = z_i^{4,5} = 0$ , which excludes the possibility of signing an additional lease contract starting in period 4 and terminating in period 4 or 5. Observe that we do not prevent from renting storage space again in location  $i$  at a later point in time (i.e., in period 5).

The above inequalities can be further strengthened by considering all contracts starting in period 1 or later, but not later than period  $t$ , in the left-hand side. This observation yields the set of additional inequalities (29).

$$\sum_{m=1}^t \sum_{m'=t+1}^{|T|} z_i^{m,m'} + \sum_{t'=t+1}^{|T|} z_i^{t+1,t'} \leq 1 - \sum_{t'=1}^t z_i^{t',t} \quad i \in I^n, t \in T \setminus \{|T|\}. \quad (29)$$

Using again the same example, the enhanced condition is:  $(z_i^{1,4} + z_i^{1,5} + z_i^{2,4} + z_i^{2,5} + z_i^{3,4} + z_i^{3,5}) + (z_i^{4,4} + z_i^{4,5}) \leq 1 - (z_i^{1,3} + z_i^{2,3} + z_i^{3,3})$ . The variables added to the left-hand side are associated with contracts that have overlapping periods with contracts on the right-hand side.

A further enhancement is obtained by observing that the duration of a lease contract cannot exceed the total length of the planning horizon. This requirement is expressed by the following inequalities:

$$\sum_{t=1}^{|T|} \sum_{t'=t}^{|T|} (t' - t + 1) z_i^{t,t'} \leq |T| \quad i \in I^n.$$

Recall that  $(t' - t + 1)$  gives the total number of periods covered by a lease agreement that starts in period  $t$  and terminates at the end of period  $t'$ . By applying the Chvátal-Gomory rounding procedure a finite number of times  $p$  ( $1 \leq p \leq |T|$ ) to the above inequalities (Wolsey, 1998), we obtain

$$\sum_{t=1}^{|T|} \sum_{t'=t}^{|T|} \left\lfloor \frac{t' - t + 1}{p} \right\rfloor z_i^{t,t'} \leq \left\lfloor \frac{|T|}{p} \right\rfloor \quad i \in I^n, p = 1, \dots, |T|. \quad (30)$$

A close analysis shows that some inequalities (30) are dominated by others because we obtain many such inequalities with the same right-hand side. Therefore, for a given right-hand side, we only need to consider the inequality with the 'stronger' left-hand side, that is, the inequality with the largest number of variables on the left-hand side. For example, if  $|T| = 5$  then  $\lfloor 5/p \rfloor = 1$  for  $p = 3, 4, 5$ . In this case, the condition for  $p = 3$  includes six variables on the left-hand side, the condition for  $p = 4$  has three variables on the left-hand side, and the condition for  $p = 5$  has one variable on the left-hand side. Therefore, only the inequality associated with  $p = 3$  needs to be considered since it dominates the other two conditions.

### 3.4.3 Minimum number of selected warehouses

Let  $n^t$  denote the minimum number of warehouses that must be operating at period  $t$ . These facilities must have in total sufficient capacity to satisfy all demand requirements for that period. Hence, the following inequalities must hold:

$$\sum_{i \in I^c} \left( 1 - \sum_{t'=1}^t y_i^{t'} \right) + \sum_{i \in I^n} \sum_{m=1}^t \sum_{m'=t}^{|T|} z_i^{m,m'} \geq n^t \quad t \in T. \quad (31)$$

In order to determine an appropriate value for the lower bound  $n^t$ , we consider the total demand that must be satisfied in time period  $t$ , i.e.  $D^t = \sum_{j \in J} \sum_{p \in P} \beta_p d_{jp}^t$ . In addition, all company-owned and potential warehouses are sorted by non-increasing order of their capacities. Let us denote this sequence by  $Q_{[1]}^t \geq Q_{[2]}^t \geq \dots \geq Q_{[|I|]}^t$ , where  $Q_{[1]}^t$  is the largest capacity and  $Q_{[|I|]}^t$  is the smallest capacity available in time period  $t \in T$ . Therefore, parameter  $n^t$  is determined by the following conditions:

$$\sum_{k=1}^{n^t-1} Q_{[k]}^t < D^t \leq \sum_{k=1}^{n^t} Q_{[k]}^t$$

Finally, we remark that it is computationally inexpensive to add inequalities (26) and (31) to the original formulation. Inequalities (29) and (30) contribute with  $|I^n| \cdot (|T| - 1)$  and  $|I^n| \cdot |T|$  additional constraints, respectively. However, since some of the inequalities (30) are dominated, the size increase of the model is frequently smaller. The largest effect in terms of the total number of constraints is obtained when constraints (10) are replaced by inequalities (27') and (28). In this case, the formulation is extended with  $|I^n| \cdot (|T| - 1) \cdot (|T| - 2) / 2$  additional constraints.

## 4 Computational study

Nowadays, many organizations resort to state-of-the-art commercial solvers for decision support. The reasons are multifold. Modern optimization solvers have become powerful and reliable tools for obtaining near-optimal or even optimal solutions to many difficult MILP problems within an acceptable computing time. Additionally, not every company employs optimization specialists or can afford to outsource the design and implementation of a solution procedure specially-tailored to its optimization problems. As pointed out by Alumur et al. (2016), often optimality is not of paramount importance to practitioners due to errors resulting from data collection and/or forecasting. Hence, in this section, we assess the suitability of a general-purpose solver such as CPLEX to return high-quality solutions to a set of randomly generated instances within a pre-specified time limit. In Section 4.1, we describe the settings considered to create these instances. Numerical results are reported in Section 4.2 and relevant insights into the characteristics of the best solutions identified by the solver are discussed in Section 4.3. In particular, we compare the flexible setting for leasing warehouse space on a temporary basis against the restrictive approaches described in Section 3.3.

### 4.1 Test instances

Since benchmark instances are not available with all the required data for the problem that we study, we generated two reference data sets that differ in the data profile considered. Table 1 gives the cardinality of the various index sets. For each combination of these parameters and each demand profile, five instances were generated, yielding a total of 90 instances. In all instances, the planning horizon represents a time frame of one year divided into 12 months. This choice is in accordance with lease commitments for short-term warehouse use (Tomese et al., 2020), and contrasts with the classical setting taken in (many extensions of) the MFLP, where a time period often represents one year and the total number of periods in the planning horizon is relatively small (e.g., Melo et al. (2006); Thanh et al. (2008); Correia et al. (2013); Cortinhal et al. (2015)).

The instances in the first reference data set have a demand profile, hereafter called A, that accounts for seasonality, with the individual product families exhibiting demand peaks at different time periods in the planning horizon. Specifically, product family 2 is impacted by advertising campaigns such as Black Friday, Cyber Monday, and Christmas promotions that boost sales in the last three periods ( $t = 10, 11, 12$ ). By contrast, the demand peak of product

Description	Parameter	Value(s)
No. of periods	$ T $	12
No. of suppliers	$ S $	3
No. of warehouses	$ I $	10, 15, 20
No. of company-owned and potential warehouses	$( I^e ,  I^n )$	(5,5), (8,7), (10,10)
No. of customer zones	$ J $	$2 I $ , $5 I $ , $10 I $
No. of product families	$ P $	3

Table 1: Cardinality of index sets.

family 3 occurs in the middle of the planning horizon (i.e.,  $t = 6, 7, 8$ ), as it is often the case with products that have higher sales in the summer season (e.g., air conditioners and air coolers). Product family 1 undergoes irregular demand variations that range from -5% to +5% between two consecutive periods. The instances in the second reference data set follow demand profile B, which is characterized by all product families having demand peaks in the last three periods. Detailed information on the generation of the two demand profiles is provided in Appendix 1. All other parameters (i.e., supplier and warehouse capacities, fixed and variable costs) are also described in Appendix 1. We note that the capacities of the suppliers for each product family are aligned with the demand profiles. Moreover, economies of scale are explicitly taken into account in the generation of the various fixed facility costs. In all instances, the unit capacity consumption factor,  $\beta_p$ , is equal to 1 for every product family  $p \in P$  handled by a warehouse.

Table 5 in Appendix 2 presents the sizes of the instances for formulation (6)–(22). The total number of binary (continuous) variables ranges from 450 to 900 (8,640 to 146,880). The total number of constraints varies between 1,938 and 9,528, with an average of 4,812 constraints. When the enhancements (26), (27'), and (28)–(31) are added to the formulation, the total number of constraints grows by about 25%, on average.

## 4.2 Summary of numerical results

Formulation (6)–(22) was coded in C++ and embedded in IBM ILOG Concert Technology with IBM ILOG CPLEX 12.6. This framework was also used for the special cases discussed in Section 3.3 and the enhancements proposed in Section 3.4. Table 2 presents the models that were tested and classifies them according to their degree of flexibility with respect to



leasing warehouse capacity. All experiments were performed on a workstation with a multi-core Intel Xeon E5-2650V3 processor (2.3 GHz, 10 cores), 32 GB RAM, and running the Ubuntu operating system (64-bit). A limit of 12 hours of CPU time was set for each solver run, which corresponds to about 3.5 hours of wall clock time. Moreover, CPLEX was used with default settings under a deterministic parallel mode.

Notation	Description	Formulation
$(P)$	Total flexibility in warehouse lease contracts	(6)–(22)
$(P_{23})$	Restrictive flexibility due to lease contracts having a minimum lease duration	(6)–(22), (23)
$(P_{24})$	Restrictive flexibility due to lease contracts having a minimum lease duration and running until the end of the planning horizon	(6)–(22), (24)
$(P_{25})$	Restrictive flexibility due to lease contracts being multiples of a based-fixed term tenancy	(6)–(22), (25)

Table 2: Problems with varying degree of flexibility with respect to accessing warehouse capacity on a temporary basis.

Figures 3 and 4 summarize the performance of formulation  $(P)$  with different families of additional inequalities for the reference data sets having demand profiles A and B, respectively. The results reported in each figure are based on 225 experiments that were carried out on 45 instances. Tables 6 and 7 in Appendix 2 give detailed information on these runs. We also tested formulation  $(P)$  with more combinations of valid inequalities but opted to provide a summary of the most relevant enhancements. The bars in Figures 3–4 indicate the total number of instances that were solved to optimality within the pre-specified time limit and for a particular formulation. The average optimality gaps are also displayed, with  $\text{MIP gap (\%)} = (z^{UB} - z^{LB})/z^{UB} \times 100\%$ ,  $z^{UB}$  denoting the objective value of the best feasible solution available for an individual instance, and  $z^{LB}$  representing the best lower bound identified by CPLEX for that instance. We note that the performance measure ‘MIP gap (%)’ is calculated *only* for those instances that could not be solved to proven optimality, and not for all 45 instances in a data set. Therefore, the results reported in Figures 3–4 (as well as in Tables 6–7) for  $(P)$  and the various sets of additional inequalities are obtained for different numbers of instances. The average integrality gaps are shown in the figures by the dashed lines. The latter give the relative percent deviation between the objective value of the best feasible solution available ( $z^{UB}$ ) and the lower bound provided by the linear relaxation ( $z^{LP}$ ), i.e.,  $\text{LP gap (\%)} = (z^{UB} - z^{LP})/z^{UB} \times 100\%$ .

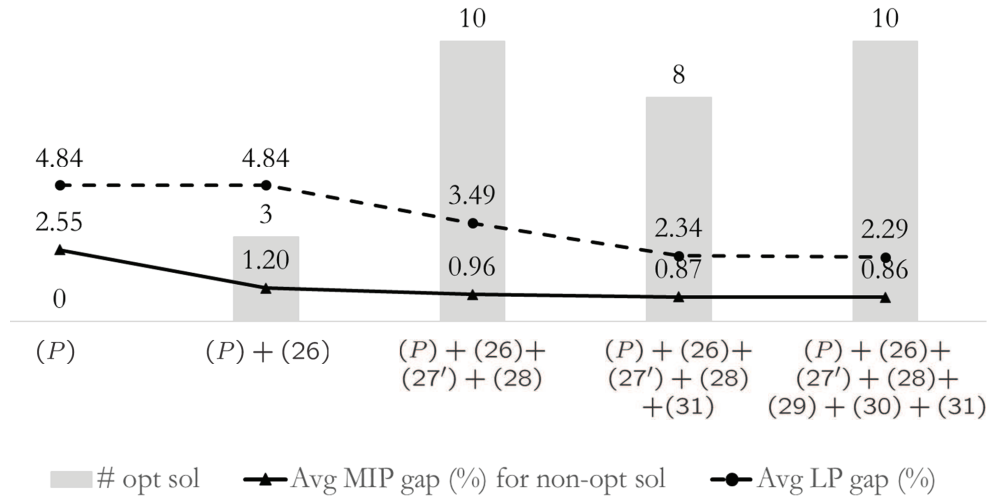


Figure 3: Total number of optimal solutions identified within the time limit, and average optimality and integrality gaps for flexible warehouse lease contracts under demand profile A.

Finding the optimal solution within at most 12 h with formulation  $(P)$  proves to be challenging. In fact, only three instances (out of 90), all with demand profile B, could be solved to proven optimality. Nevertheless, the optimality gaps returned by CPLEX are, on average, reasonable: 2.55% for demand profile A and 2.06% for demand profile B. A similar observation also applies to the quality of the lower bound provided by the linear relaxation. Remarkable improvements are achieved in two performance measures when the redundant constraints (26) are added to  $(P)$ . Not only more optimal solutions are identified, especially for instances with the demand profile B, but also the optimality gaps decrease, on average, by more than 50%, compared to  $(P)$ . As expected, the LP gaps are not affected by these constraints. Inequalities (27') and (28), that disaggregate the original constraints (10), further improve the three performance measures as shown in Figures 3–4. In particular, this case yields the overall best average optimality gap for instances with demand profile B (0.81%). Adding the lower bound (31) on the total number of open warehouses in each time period also proves to be useful to strengthen the integrality gap, particularly for instances with demand profile A. For the latter instances, the best average results are achieved when all families of inequalities are used. Compared to the original formulation  $(P)$ , they yield a reduction of 66% and 53% of the optimality gap and integrality gap, respectively. Moreover, the total number of nodes that are explored in the branch-and-cut tree declines by 76% (123,655 nodes for  $(P)$  against 29,772 nodes for  $(P)$  with all families of inequalities, on average). As reported in Table 6 (Appendix 2), optimal solutions

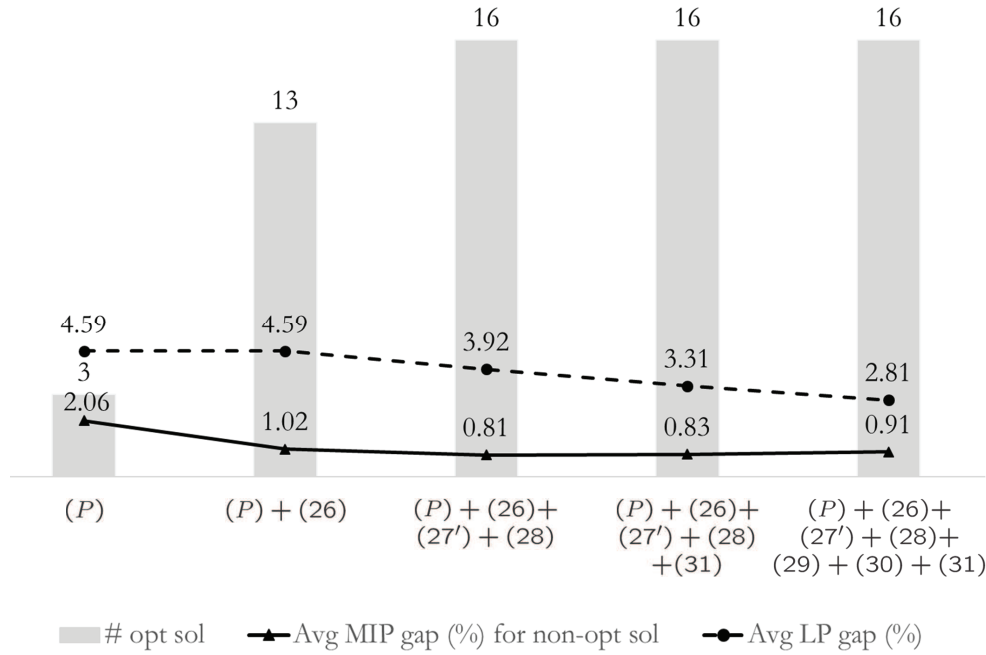


Figure 4: Total number of optimal solutions identified within the time limit, and average optimality and integrality gaps for flexible warehouse lease contracts under demand profile B.

are available to 8 of the smaller instances with  $|I| = 10$  and 2 instances with  $|I| = 15$ .

Interestingly, for instances with demand profile B, the trade-off between computational effort and solution quality is noticeable when all the proposed enhancements are used. In this case, an average MIP gap of 0.91% is obtained, which is slightly greater than the optimality gaps achieved when different subsets of the additional inequalities are considered. Even though the average size of the branch-and-cut tree decreases by 71% compared to  $(P)$ , the exploration of each node requires more effort as the size of the associated sub-problem is larger. Nonetheless, high-quality feasible solutions are still obtained and at the same time, the LP-bound is the most strengthened when all inequalities are introduced.

Figure 5 displays the average CPU time for the instances that were solved to proven optimality, with and without different combinations of valid inequalities. For the original formulation  $(P)$ , no bar appears in the figure for instances with demand profile A because an optimal solution was not identified to any of these instances. The type of demand profile affects the time needed to find an optimal solution, with instances with demand profile B requiring less computing time. This feature may be explained by the fact that all product families follow the same demand pattern (although with distinct demand requirements), while the data set with

demand profile A exhibits higher variations over the planning horizon as the demand peaks of two of the families occur at different periods. The additional inequalities enable not only more optimal solutions to be identified, but also the CPU time is considerably less compared to the original formulation ( $P$ ). Regarding the linear relaxation, its optimal solution is always found in less than 3.6 CPU minutes for both demand profiles and all combinations of additional inequalities. Hence, solving the linear relaxation is computationally inexpensive (see also Tables 6–7 in Appendix 2).

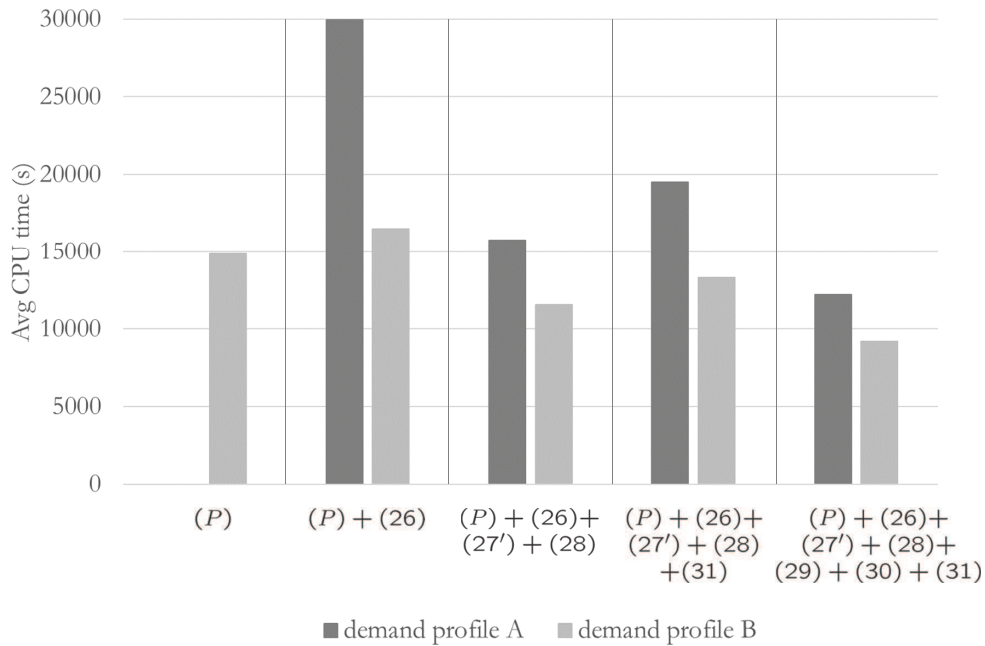


Figure 5: Average CPU time for instances solved to optimality.

In summary, the proposed enhancements are very useful, since they improve the quality of the lower bound provided by the linear relaxation and help find optimal solutions to a larger number of instances. Moreover, they also make an important contribution towards identifying high-quality feasible solutions when the time limit is reached, despite the large size of the test instances. This in turn results in smaller optimality gaps being obtained. Even though no specific combination of additional inequalities consistently yields the best values for all performance measures and demand profiles, tighter LP-bounds are obtained when all inequalities are considered. This feature can be very useful to evaluate the quality of a feasible solution, e.g., provided by a heuristic method specially-tailored to the problem. For this reason, the subsequent analysis on the restrictive models ( $P_{23}$ ), ( $P_{24}$ ), and ( $P_{25}$ ) focuses on the formulations without any additional inequalities and the formulations with all proposed enhancements. We note

that inequalities (29) were not added to formulation ( $P_{24}$ ) because in this case, once a lease contract is initiated it must run until the end of the planning horizon. Therefore, multiple lease agreements cannot be in place at the same location for non-consecutive periods.

A summary of the results obtained is presented in Figures 6, 7, and 8. The information displayed stems from a total of 540 experiments, with 180 runs being associated with each individual figure. Detailed results are given in Tables 8–10 in Appendix 2. A minimum lease duration of 6 periods is considered in ( $P_{23}$ ) and ( $P_{24}$ ), i.e.,  $\lambda_i = 6$  for every  $i \in I^n$ . For ( $P_{25}$ ), the base fixed-term tenancy is also 6 periods ( $\alpha_i = 6, i \in I^n$ ). These settings greatly affect the total number of binary variables ruling the lease contracts and that are fixed at zero according to (23)–(25). As a result, the total number of variables  $z_i^{t,t'}$  whose values need to be determined is also impacted. Since the planning horizon spans 12 periods, 78 variables are defined for each single potential warehouse location  $i$  ( $i \in I^n$ ). When the length of a lease agreement must be 6 periods or longer, as in ( $P_{23}$ ), only 28 variables  $z_i^{t,t'}$  become ‘active’ for each warehouse  $i$  because the remaining 50 (64%) variables are associated with contracts that violate this condition, and therefore, are fixed at zero by (23). The conditions ruling lease agreements in ( $P_{24}$ ) and ( $P_{25}$ ) result in setting 71 (91%) and 70 (90%) variables  $z_i^{t,t'}$  equal to zero, respectively, for each  $i \in I^n$ . Hence, the values of only 7 and 8 variables need to be determined for every new warehouse, respectively. Due to this significant decrease in the number

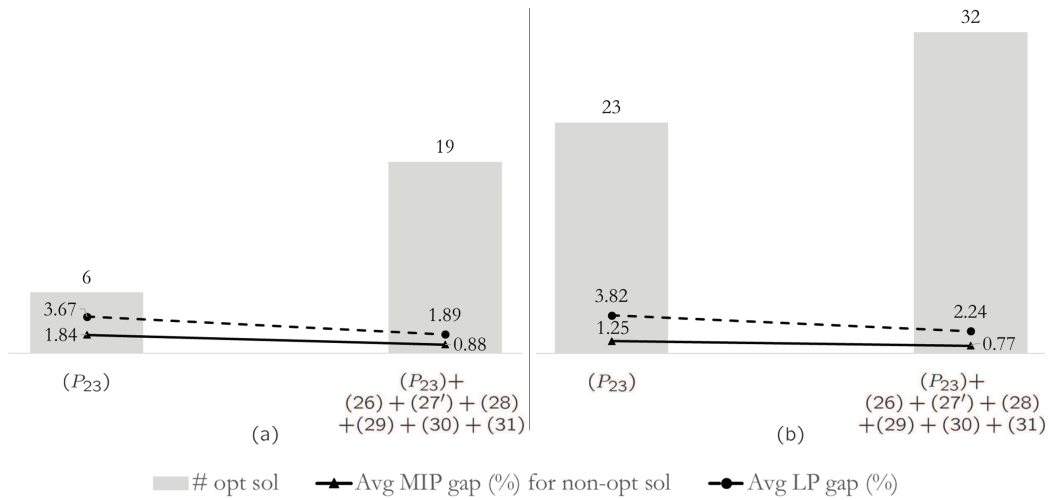


Figure 6: Performance of the problem variant ( $P_{23}$ ) with warehouse contracts having a minimum commitment of 6 periods. (a) Demand profile A. (b) Demand profile B.

of active binary variables  $z_i^{t,t'}$ , the problems become easier to solve. Accordingly, a remarkably large number of optimal solutions can be identified within the time limit, especially when the

proposed enhancements are used, as demonstrated by Figures 6–8. This feature is stronger for the data set with demand profile B, for which optimality is achieved in 71%, 82%, and 93% of the instances, depending on the type of restrictive clauses in place. As reported in Tables 8–10, this also indicates that optimal solutions are available for some of the large instances having 20 warehouse locations and 200 customer zones. The additional inequalities also help finding high-quality solutions to those instances that reach the given time limit. This is evidenced by the very small average optimality gap ( $< 1\%$ ) that is obtained for any demand profile and lease conditions. Furthermore, the improvement of the integrality gap is also noteworthy when the proposed enhancements are used, with the average LP gap being lower than 2.3% for all data sets and warehouse lease conditions. As shown in Tables 8–10, the linear relaxation of the models investigated is solved in short CPU times.

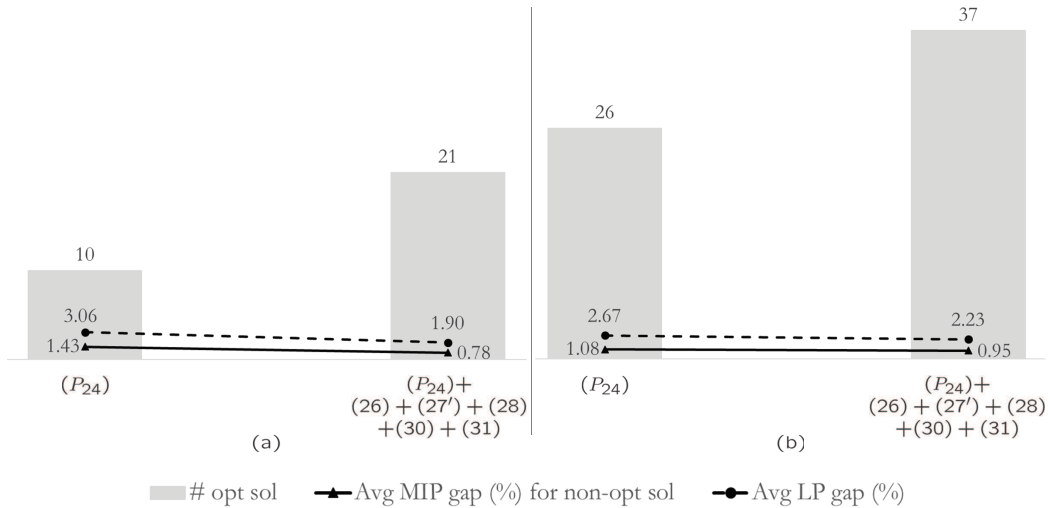


Figure 7: Performance of the problem variant  $(P_{24})$  with warehouse lease contracts having a minimum commitment of 6 periods and running until the end of the planning horizon. (a) Demand profile A. (b) Demand profile B.

Figure 9 presents the average CPU time for the instances that were solved to proven optimality using the restrictive conditions on warehouse lease contracts. As expected, the time required to identify an optimal solution decreases, on average, as stronger conditions are imposed on the length of the lease agreements. In other words, the models based on  $(P_{23})$  are more difficult to solve than the formulations based on  $(P_{25})$ . In general, using the additional inequalities results in lower computing times to achieve optimality. One exception occurs for  $(P_{24})$  with demand profile B, where the computational effort is slightly lower when no inequalities are considered. This is mainly due to the relatively large CPU time required by CPLEX to solve six instances

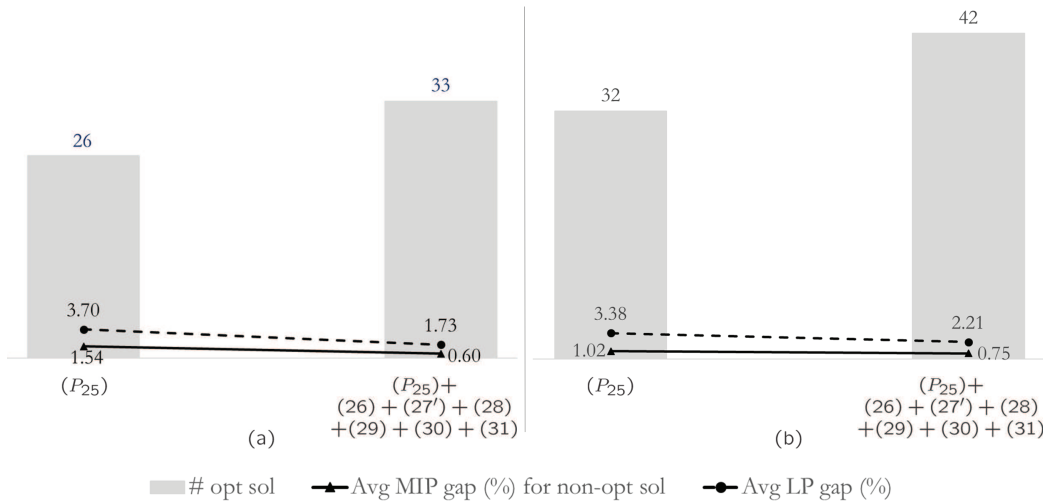


Figure 8: Performance of the problem variant ( $P_{25}$ ) with warehouse contracts being multiples of a based-fixed term tenancy of 6 periods. (a) Demand profile A. (b) Demand profile B.

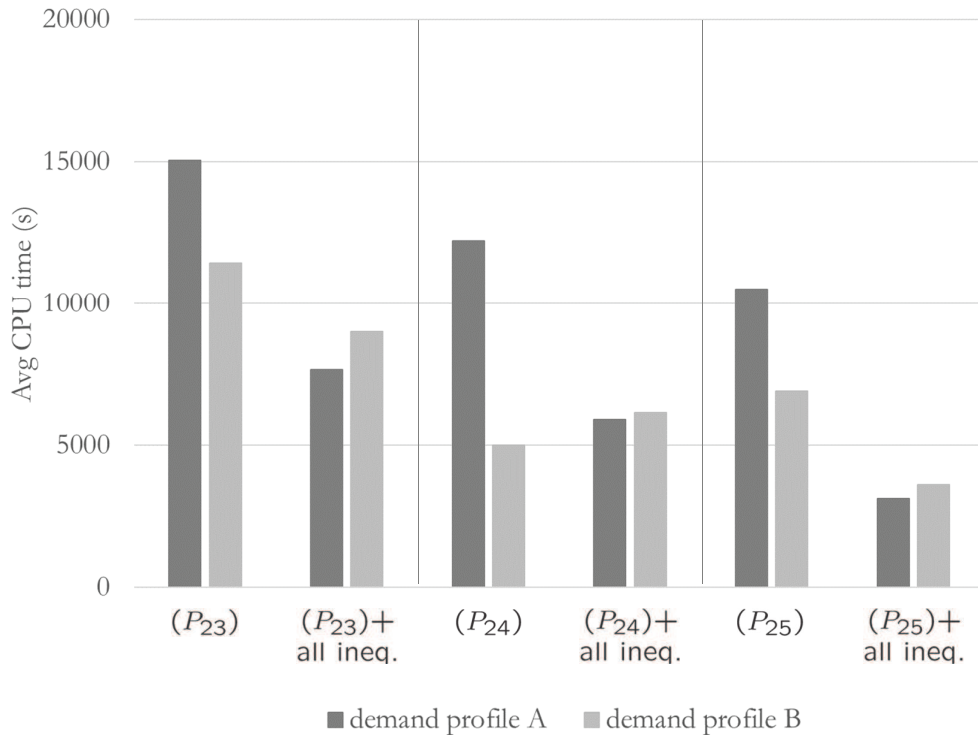


Figure 9: Average CPU time for instances solved to optimality when restrictive lease agreements are enforced.

in this data set to optimality with the proposed enhancements. By contrast, the solver was unable to return an optimal solution to any of these six instances with formulation ( $P_{24}$ ).

### 4.3 Measuring the value of flexible warehouse lease commitments

This section offers a broad insight into the features of the best solutions identified by CPLEX to the four problems given in Table 2. Accordingly, we examine how the configuration of the distribution network and related costs are affected by the different conditions in place to lease warehouse space. This analysis will help a decision maker become aware of the trade-offs achieved by each of the four strategies and in particular, the value of the flexibility provided by the on-demand warehousing business model.

Let  $n$  denote an instance belonging to a given data set. Moreover, let  $z_n^{UB,k}$  denote the objective value of the best solution available for problem  $k$  and the  $n$ -th instance, with  $k = (P), (P_{23}), (P_{24}), (P_{25})$ . The ratio  $(z_n^{UB,k} - z_n^{UB,(P)}) / z_n^{UB,(P)} \times 100\%$  gives the relative deviation of the total cost of the best solution to the restrictive problem  $k = (P_{23}), (P_{24}), (P_{25})$  to that of the flexible problem  $(P)$  with respect to the  $n$ -th instance. Table 3 reports the average value of this ratio over the instances belonging to demand profile A and B (column 3). In addition, a similar ratio is also calculated for each individual cost component in (6), and the associated averages are given in columns 4–9.

Demand	Problem	Total	Company-owned wareh.		New wareh.	Distribution cost		Inventory
profile		cost	closing cost	operating cost	lease cost	suppliers to wareh.	wareh. to cust. zones	cost
A	$(P_{23})$	1.3	0.2	11.1	3.5	-0.1	-0.1	0.6
	$(P_{24})$	1.3	0.7	27.8	0.9	-0.1	0.0	-0.1
	$(P_{25})$	1.6	-2.2	18.9	1.9	-0.2	-0.1	0.3
B	$(P_{23})$	8.2	17.1	-0.5	43.5	-0.2	-0.2	-2.6
	$(P_{24})$	8.2	26.8	-0.9	44.0	-0.2	-0.2	-2.4
	$(P_{25})$	8.2	27.5	-1.5	44.8	-0.2	-0.2	-2.6

Table 3: Average percent difference of the total cost and individual cost categories between each of the restrictive models and the flexible model  $(P)$ .

As expected, more expensive distribution network configurations are obtained when warehouse lease agreements have restrictive clauses. This feature is more evident for the instances with demand profile B, which have 8.2% higher total cost on average. Recall that in this case, the demands of all product families grow considerably in the last three periods of the planning horizon. The results in columns 4 and 6 (Table 3) reveal that this significant cost increase is due to discontinuing the operation of a larger number of company-owned warehouses and at the same time making a higher investment in warehouse leases, compared to the flexible model  $(P)$ . By retaining fewer company-owned warehouses than in  $(P)$ , the total warehouse



operating cost decreases albeit the additional cost for removing some warehouses. Naturally, this strategy requires renting storage space to compensate for the capacity that is lost due to dropping company-owned warehouses. As a result, the total warehouse lease cost greatly increases (43.5%-44.8%). Since any lease term is a minimum of six periods in  $(P_{23})$ ,  $(P_{24})$ , and  $(P_{25})$ , new lease contracts have to start in period 7 at the latest. In this case, not only are lease costs incurred until the end of the planning horizon, but also part of the rented storage capacity remains unused, especially in periods 7, 8, and 9. This characteristic is less present in the instances with demand profile A because their demand peaks occur in five of the last six periods of the planning horizon, more specifically, in periods 7 and 8 for product family 3 and in periods 10 to 12 for family 2. Accordingly, there is a tendency to retain more company-owned warehouses, which leads to higher costs for operating them. At the same time, there is less need to lease additional capacity. In both demand profiles, these decisions yield a slightly lower total distribution cost as well as a small reduction of the total inventory holding cost. This analysis is complemented by Table 11 in Appendix 2, which reports the average relative contribution of the individual cost components to the overall cost for each of the four strategies. It can be seen that regardless of the demand profile and the problem type, the total warehouse lease cost and the total distribution cost from open warehouses to customer zones have the greatest share in the total cost. By contrast, the total inventory holding cost and the total cost of removing company-owned warehouses have the lowest weight.

To further analyze the differences between the flexible model ( $P$ ) and the three restrictive models for leasing storage capacity, Table 4 reports the values of several statistics related to warehouse decisions. Again, the values presented in this table refer to the best solutions identified by CPLEX and which were obtained using the proposed enhancements. For each demand profile and model, column 3 gives the average number of company-owned warehouses that are kept over the entire planning horizon. Column 4 shows the average number of company-owned warehouses whose operation is terminated, while column 5 presents the average number of periods during which such a warehouse was operated before being discontinued. For the new warehouses, the average number of selected locations is indicated in column 6 and the average number of lease agreements signed is reported in column 7. Moreover, the average duration of a lease contract is presented in column 8.

The last three columns of Table 4 indicate how the various settings for accessing warehouse space affect network redesign decisions. The impact is more marked for instances with demand profile B due to their special characteristics. Specifically, slightly more new warehouse locations

Demand profile	Problem	Company-owned warehouses			New warehouses		
		retained	not retained		avg no. selected locations	avg no. lease contracts	avg duration lease contract (no. periods)
		avg no.	avg no.	avg no. working periods			
A	( <i>P</i> )	1.5	6.2	0.1	6.4	6.7	8.6
	( <i>P</i> <sub>23</sub> )	1.5	6.2	0.2	6.2	6.2	9.4
	( <i>P</i> <sub>24</sub> )	1.5	6.2	0.4	6.0	6.0	9.6
	( <i>P</i> <sub>25</sub> )	1.7	6.0	0.1	6.1	6.1	9.5
B	( <i>P</i> )	4.1	3.6	0.0	7.3	7.4	4.8
	( <i>P</i> <sub>23</sub> )	3.5	4.1	1.3	7.1	7.1	7.0
	( <i>P</i> <sub>24</sub> )	3.2	4.4	2.1	7.1	7.1	7.1
	( <i>P</i> <sub>25</sub> )	3.2	4.4	2.1	7.2	7.2	7.1

Table 4: Characteristics of best feasible solutions with respect to warehouse decisions.

are selected in (*P*) compared to (*P*<sub>23</sub>), (*P*<sub>24</sub>), and (*P*<sub>25</sub>). While in the latter three strategies a single warehouse lease contract is signed at each new site, due to the conditions in place, in model (*P*) there may be more than one lease agreement at the same site, each spanning different periods over the planning horizon. More importantly, the average commitment duration of a warehouse contract is significantly shorter in model (*P*) than the average contract length in the other three cases (4.8 periods vs 7.0/7.1 periods). This feature clearly shows the advantages of the flexible model as it enables a company to align warehouse capacity to match customer demand. During periods of peak demand, additional warehouse space is leased which is then shrunk during periods of lower demand requirements. In this way, storage resources are not wasted and costs are reduced. Lease agreements with restrictive clauses do not have these characteristics due to their limited degree of flexibility. These observations also apply to the data set with demand profile A, although being less prominent.

Columns 3–5 in Table 4 provide information about decisions related to company-owned warehouses. For demand profile B and the restrictive models (*P*<sub>23</sub>), (*P*<sub>24</sub>), and (*P*<sub>25</sub>), these warehouses remain in operation for a longer period of time before being removed, which contrasts with the strategy followed by the flexible model (*P*) where the operation of some of these warehouses is discontinued right at the beginning of the planning horizon. Hence, it is cost effective to ‘exchange’ some of the company-owned warehouses for new ones because the latter can be leased temporarily, while the former cannot be re-opened after having being removed. These characteristics are less evident in instances with demand profile A because their demand peaks are more distributed, which is not the case in demand profile B.

We selected two representative instances to show in detail how warehouse decisions are

affected by the different settings for redesigning the network. Figure 10 displays the optimal schedules for choosing new warehouses for an instance with 50 customer zones, 5 company-owned warehouses, and 5 potential new warehouses. Demands for the three product families follow profile A. Figure 11 shows the optimal choices for an instance with demand profile B, 100 customer zones, 5 company-owned warehouses, and 5 potential new warehouses.

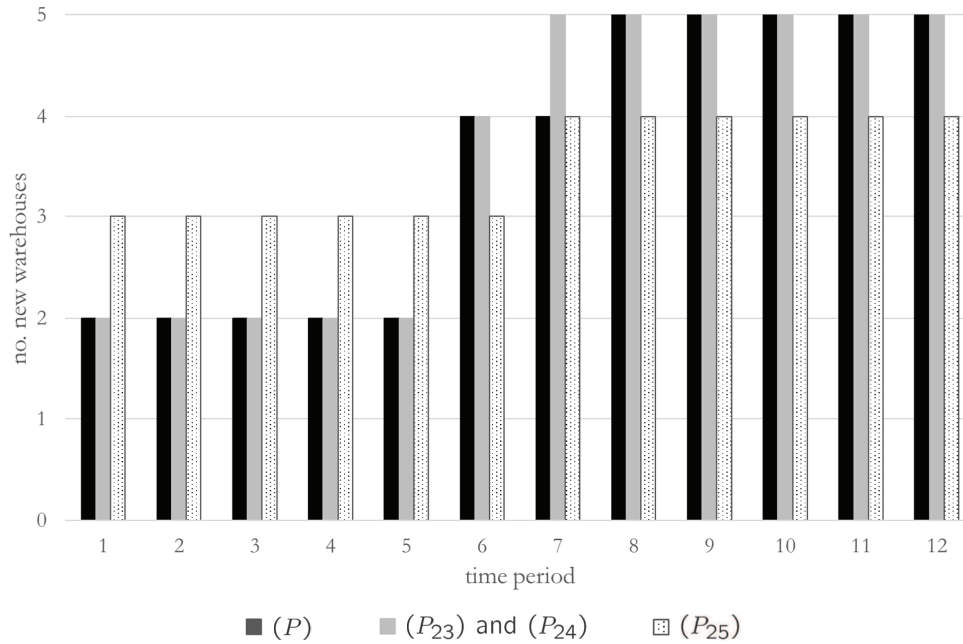


Figure 10: Optimal number of leased warehouses for an instance with demand profile A,  $|I^e| = 5$ ,  $|I^n| = 5$ , and  $|J| = 50$ .

According to Figure 10, for the flexible model ( $P$ ), two warehouses are leased in the first period which are operated until the end of the planning horizon. In addition, but not shown in the figure, four company-owned warehouses are removed directly in  $t = 1$ , and only one warehouse of this type is retained. In period 6, additional storage capacity is acquired by leasing two more warehouses in order to respond to the demand peak for product family 3 from that period on, and to account for having discontinued the operation of the company's sole warehouse that had been retained. In period 8, one more warehouse is leased, possibly to handle the increased capacity provided by the supplier of product family 2 and which later declines (see Appendix 1). In total, all five new available locations are operating from period 8 onward. Recall that additional storage capacity is needed for product family 2 in periods 10-12, so that the lease agreements in all new warehouses must run until the last period. Models ( $P_{23}$ ) and ( $P_{24}$ ) adopt the same strategy as ( $P$ ) in periods 1-6, but since tenancies must have a duration

of at least 6 periods, they are required to select one more warehouse in period 7, i.e., one period earlier than in model ( $P$ ). Naturally, this results in an increase of the total lease cost, namely 2.4%. For model ( $P_{25}$ ), a different strategy is taken by operating a single company-owned warehouse across all periods and starting three new lease contracts in period 1. A fourth contract is initiated in period 7, the latter being the last period in which this kind of decision can be made. Compared to the flexible model ( $P$ ), the total costs for discontinuing company-owned warehouses and leasing new warehouses decrease by 18.4% and 3.6%, respectively. However, 112.2% higher warehouse operating costs are incurred. Hence, the resulting schedule is 4% more expensive.

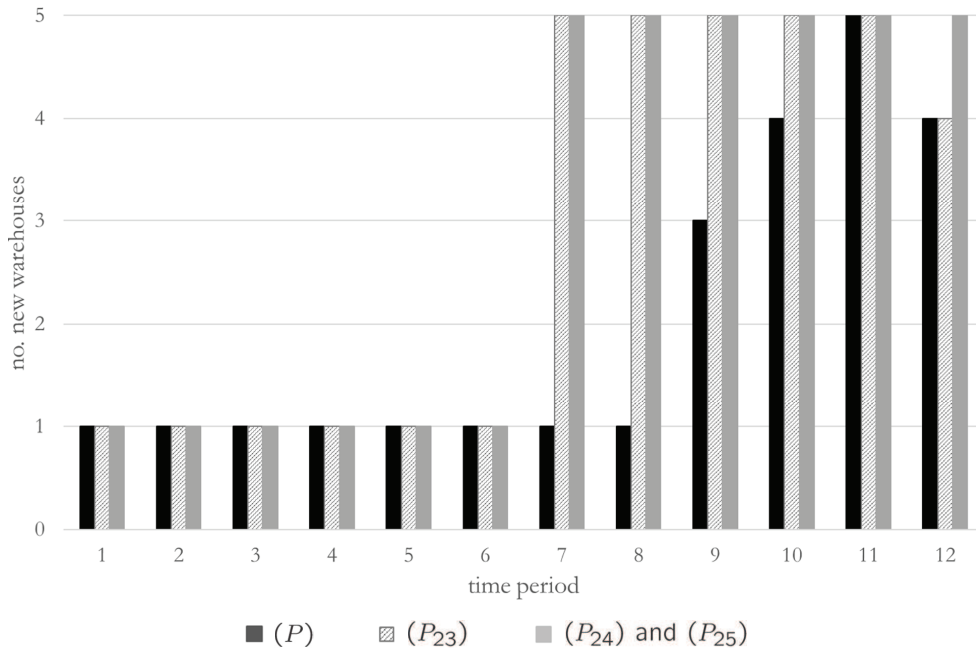


Figure 11: Optimal number of leased warehouses for an instance with demand profile B,  $|I^e| = 5$ ,  $|I^n| = 5$ , and  $|J| = 100$ .

Regarding the example in Figure 11, all models adopt the same decision for the selection of new warehouses in periods 1 through 6, namely a single lease contract is in place. In addition to the information provided by the figure, three of the five warehouses owned by the company remain in operation across all periods, and for all problems. Distinct choices are made according to the type of setting available for leasing capacity. Temporary capacity adjustments are clearly present in model ( $P$ ), through the gradual increase of the number of lease contracts in periods 9, 10, and 11, and the removal of capacity in the last period. This means that one of the contracts ends in period 11. This strategy is in line with the expected flexibility of this model.

By contrast, when a lease lasts 6 or 12 periods, as in  $(P_{25})$ , there is only one option to deal with the demand peaks in periods 10–12, namely to start four more leases in period 7. Accordingly, the total cost is 6.64% higher than the optimal network configuration determined by  $(P)$ . Model  $(P_{24})$  uses the same strategy, even though it offers slightly more flexibility by allowing any lease agreement to last six periods or longer. However, once a new warehouse is selected it must be operated until the end of the planning horizon, which prevents capacity from being removed in the last period. The latter aspect is captured by model  $(P_{23})$ , which terminates one of the lease contracts in period 11. Nevertheless, a minimum six-period tenancy leads to a network configuration that is 6.60% more costly than the one determined by the flexible model  $(P)$ .

## 5 Conclusions

In this paper, we introduced a new and relevant extension of the MFLP in the context of redesigning a two-echelon distribution network comprising suppliers, warehouses, and customer zones. In contrast to classical facility location problems, flexible conditions are considered that allow leasing intermediate facilities on a temporary basis. This setting, which stems from on-demand warehousing, is particularly suitable for companies that need to rescale capacity for a limited time in order to respond to peaks in demand, and without having to make a substantial investment on a long-term commitment, e.g., by building new facilities or commissioning a 3PL firm. We provided a MILP formulation for this problem and developed several families of additional inequalities to enhance the original model. Computational testing on randomly generated instances with moderate sizes indicated that a general-purpose optimization solver such as CPLEX is a suitable option for obtaining high-quality solutions within a given time limit. However, its success relies heavily on using the proposed enhancements as they enable the solver to identify a significantly larger number of optimal solutions. Moreover, even when optimality cannot be proven, considerably better feasible solutions are returned within the same time limit.

As shown by our computational study, on-demand warehousing can be an important driver in increasing supply chain responsiveness to changes in demand. Having access to scalable storage space and flexible warehouse lease agreements results in lower costs for redesigning a distribution network. Our study also revealed that this cost advantage is not achievable with restrictive warehouse lease clauses frequently encountered in practice. Furthermore, our model

identifies over the planning horizon the best mix between capacity provided by company-owned warehouses and capacity leased on a temporary basis at additional facilities. From a managerial viewpoint these insights help decision-makers understand the opportunities that arise from a business model such as on-demand warehousing. The flexibility offered by this setting comes at the cost of an increase in the difficulty of solving the problem, even though it portrays a trend in modern warehousing and therefore, better depicts the business environment. To overcome the computational burden of solving large-sized instances with an off-the-shelf optimization solver, a solution technique specially tailored to the problem at hand would need to be developed. Since the proposed enhancements result in a tight LP-bound in a short computing time, as demonstrated by our numerical study, this line of future research could exploit the information provided by the optimal solution to the linear relaxation. Another future research venue would be to incorporate the uncertainty associated to future demand in the redesign of the distribution network. In such a context, the study carried out in this paper would also be relevant for the development of a stochastic model.

### Acknowledgements

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## Appendix 1: Data generation

In what follows, we denote by  $\mathcal{U}[a, b]$  the generation of random numbers over the range  $[a, b]$  according to a continuous uniform distribution. Two demand profiles were generated that allow for different types of seasonality. For this purpose, random numbers are drawn from three different uniform distributions, namely  $\delta_j^t \in \mathcal{U}[0.95, 1.05]$ ,  $\gamma_j^t \in \mathcal{U}[1.2, 1.3]$ , and  $\rho_j^t \in \mathcal{U}[0.8, 0.9]$ , for  $t \in T$  and  $j \in J$ . In demand profile A, each product family follows an individual demand pattern which is in accordance with changes experienced by many consumer products in practice. The demand of each customer zone  $j \in J$  for product family 1 is obtained as follows:

$$\begin{aligned} d_{j1}^1 &= \mathcal{U}[20, 100] \\ d_{j1}^t &= \delta_j^t d_{j1}^{t-1} \quad t = 2, \dots, 12. \end{aligned}$$

Hence, demand can change up to  $\pm 5\%$  from one period to the next for a given customer zone. While demand product family 1 follows an irregular pattern, the other two families face demand patterns with seasonal peaks at different time periods in the planning horizon. Specifically, in the first nine periods, the scheme defined for family 1 is also used for family 2, followed by a seasonal peak in the last three periods of the planning horizon. Under the assumption that a time period represents one month, this demand pattern is related to a product family that faces a demand peak in the last quarter of the year. Marketing events such as Black Friday, Cyber Monday, and Christmas promotions fall into this period of the year, during which the sales of various products (e.g., consumer electronics) experience a sharp increase. Accordingly, for every  $j \in J$  we set:

$$\begin{aligned} d_{j2}^1 &= \mathcal{U}[20, 100] \\ d_{j2}^t &= \delta_j^t d_{j2}^{t-1} & t = 2, \dots, 9 \\ d_{j2}^t &= \gamma_j^t d_{j2}^{t-1} & t = 10, 11, 12. \end{aligned}$$

Product family 3 faces a demand peak in the middle of the planning horizon, namely in periods 6, 7, and 8. The following generation scheme is used for every  $j \in J$ :

$$\begin{aligned} d_{j3}^1 &= \mathcal{U}[20, 100] \\ d_{j3}^t &= \delta_j^t d_{j3}^{t-1} & t = 2, \dots, 5 \\ d_{j3}^t &= \gamma_j^t d_{j3}^{t-1} & t = 6, 7, 8 \\ d_{j3}^t &= \rho_j^t d_{j3}^{t-1} & t = 9, 10, 11 \\ d_{j3}^t &= \delta_j^t d_{j3}^{t-1} & t = 12. \end{aligned}$$

Thus, assuming again that a time period represents one month, this case is related to a product family with increasing sales in the summer months (e.g., air conditioners and air coolers).

In demand profile B, the three product families have similar demand patterns, corresponding to the generation scheme of family 2 in profile A.

The suppliers' capacities for the different product families are linked up with the demand profiles, and use random numbers  $\tilde{\gamma}^t$  drawn from  $\mathcal{U}[1.25, 1.50]$ . In profile A, the capacity of supplier  $s \in S$  in period  $t \in T$  for product family 1 is 25% to 50% larger than the average

demand per period and supplier, namely

$$SQ_{s1}^t = \tilde{\gamma}^t \frac{\sum_{t \in T} \sum_{j \in J} d_{j1}^t}{|T| \cdot |S|}.$$

For product families facing seasonal demand variations, suppliers increase their production capacities close to the peak season. Therefore, for family 2, which has a demand peak in the last three periods, the capacity of supplier  $s \in S$  grows in periods 7, 8, and 9 as follows:

$$SQ_{s2}^t = \tilde{\gamma}^t \frac{\min_{t \in T} \left\{ \sum_{j \in J} d_{j2}^t \right\}}{|S|} \quad t = 1, \dots, 6 \text{ and } t = 10, 11, 12$$

$$SQ_{s2}^t = \tilde{\gamma}^t \frac{\max_{t \in T} \left\{ \sum_{j \in J} d_{j2}^t \right\}}{|S|} \quad t = 7, 8, 9.$$

For product family 3, the capacity of supplier  $s \in S$  is adjusted in such a way that it increases in the three periods just before the demand peak occurs, as described next.

$$SQ_{s3}^t = \tilde{\gamma}^t \frac{\sum_{t=1}^2 \sum_{j \in J} d_{j3}^t}{2|S|} \quad t = 1, 2$$

$$SQ_{s3}^t = \tilde{\gamma}^t \frac{\max_{6 \leq t \leq 8} \left\{ \sum_{j \in J} d_{j3}^t \right\}}{|S|} \quad t = 3, 4, 5$$

$$SQ_{s3}^t = \tilde{\gamma}^t \frac{\sum_{t=6}^{12} \sum_{j \in J} d_{j3}^t}{7|S|} \quad t = 6, \dots, 12.$$

The above scheme is also used in demand profile B for generating the suppliers' capacities for each of the three product families.

The capacity of each warehouse  $i \in I$  takes into account the average demand per time



period and facility. This value is perturbed by a random number  $\tilde{\delta}_i \in \mathcal{U}[2, 3]$ .

$$Q_i^1 = \tilde{\delta}_i \frac{\sum_{j \in J} \sum_{p \in P} \sum_{t \in T} d_{jp}^t}{|T| \cdot |I|}$$

$$Q_i^t = Q_i^1 \quad t = 2, \dots, |T|.$$

The fixed costs of operating company-owned warehouses are generated in order to reflect economies of scale. Hence, the larger the capacity size of a warehouse  $i \in I^e$ , the lower the associated operating cost.

$$OC_i^1 = \mathcal{U}[0, 100] + \mathcal{U}[2000, 2100] \sqrt{Q_i^1}$$

$$OC_i^t = \mu^t OC_i^{t-1} \quad t = 2, \dots, |T|,$$

with  $\mu^t \in \mathcal{U}[1, 1.01]$ . The fixed operating cost of a company-owned warehouse can grow up to 1% between two consecutive periods. Moreover, the fixed cost for discontinuing the operation of the warehouse corresponds to 20 percent of its operating cost,  $FC_i^t = 0.2 OC_i^t$  ( $i \in I^e$ ;  $t \in T$ ).

At a new warehouse location  $i \in I^n$ , economies of scale are included in the fixed lease costs by taking into account the capacity of the warehouse and the duration of the lease agreement as follows:

$$L_i^{1,1} = \mathcal{U}[0, 100] + \mathcal{U}[2200, 2300] \sqrt{Q_i^1}$$

$$L_i^{t,t} = \mu^t L_i^{t-1,t-1} \quad t = 2, \dots, |T|$$

$$L_i^{t,t'} = L_i^{t,t} + 0.8 \sum_{m=t+1}^{t'} L_i^{m,m} \quad t = 2, \dots, |T|,$$

with  $\mu^t \in \mathcal{U}[1, 1.01]$ . The cost incurred by a lease contract for  $t' - t + 1$  periods is lower than the sum of the costs for one-period leases, i.e.,  $L_i^{t,t'} < \sum_{m=t}^{t'} L_i^{m,m}$ .

The unit distribution costs from supplier  $s \in S$  to warehouse  $i \in I$  and from the latter to customer zone  $j \in J$  for product family  $p \in P$  in time period  $t \in T$  are generated according to the following scheme:

$$SC_{sip}^1 = \mathcal{U}[15, 20] \quad DC_{ijp}^1 = \mathcal{U}[25, 30]$$

$$SC_{sip}^t = \xi^t SC_{sip}^{t-1} \quad DC_{ijp}^t = \xi^t DC_{ijp}^{t-1} \quad t = 2, \dots, |T|,$$

with  $\xi^t \in \mathcal{U}[1.01, 1.03]$ . This means that the distribution costs increase at a rate ranging from 1 to 3 percent per period. We also assume that shipments from the suppliers to the warehouses are less expensive than shipments to the customer zones per unit delivered, as the former are associated with large quantities compared to the latter.

Finally, the unit inventory holding cost for product family  $p \in P$  in warehouse  $i \in I$  at time period  $t \in T$  is generated as follows:

$$\begin{aligned} IC_{ip}^1 &= \mathcal{U}[1, 2] \\ IC_{ip}^t &= \mu^t IC_i^{t-1} \quad t = 2, \dots, |T|, \end{aligned}$$

with  $\mu^t = \mathcal{U}[1, 1.01]$ . Therefore, inventory holding costs are monotonic increasing, with a growth rate not higher than 1 percent per period.

## Appendix 2: Complementary results

Table 5 presents the total number of variables and constraints in the instances generated.

$ I^e $	$ I^n $	$ J $	no. of variables		no. of constraints
			binary	continuous	
5	5	20	450	8640	1938
5	5	50	450	19440	3018
5	5	100	450	37440	4818
8	7	30	642	18360	2791
8	7	75	642	42660	4411
8	7	150	642	83160	7111
10	10	40	900	31680	3768
10	10	100	900	74880	5928
10	10	200	900	146880	9528

Table 5: Size of test instances for formulation ( $P$ ).

Tables 6–10 report the values of various performance measures for the different formulations that were tested. Each row gives the average results over five instances. The measure ‘Avg MIP gap (%)’ was calculated for those instances that could not be solved to optimality within the time limit. Accordingly, an empty cell means that optimal solutions are available to all the associated instances. In this case, the average optimality gap is 0% even though it is not explicitly stated.

Finally, Table 11 reports the contribution of the individual cost components (cf. (6)) to the overall cost for each problem type and the two demand profiles. The results shown in this table refer to averages determined with respect to the best feasible solution available for each instance. Typically, such a solution is identified when all the proposed enhancements are added to the associated original formulation.

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$ I^e $	$ I^n $	$ J $	Formulation	# opt sol/ # non-opt sol	Avg MIP		Avg LP	
					Gap (%)*	CPU (s)**	Gap (%)	CPU (s)
5	5	20	(P)	0/5	3.70	–	7.81	13.1
			(P)+(26)	0/5	1.98	–	7.81	6.3
			(P)+(26)+(27′)+(28)	4/1	1.12	13530.0	6.89	8.0
			(P)+(26)+(27′)+(28)+(31)	3/2	0.48	12295.2	3.22	15.7
			(P)+(26)+(27′)+(28)+(29)+(30)+(31)	4/1	0.71	10096.2	3.20	5.0
5	5	50	(P)	0/5	2.86	–	5.91	7.0
			(P)+(26)	0/5	0.68	–	5.91	3.2
			(P)+(26)+(27′)+(28)	2/3	0.62	29134.2	4.94	4.4
			(P)+(26)+(27′)+(28)+(31)	1/4	0.65	32783.4	3.02	19.5
			(P)+(26)+(27′)+(28)+(29)+(30)+(31)	2/3	0.61	22622.1	3.01	11.9
5	5	100	(P)	0/5	3.32	–	5.18	12.4
			(P)+(26)	2/3	1.16	32875.6	5.18	12.0
			(P)+(26)+(27′)+(28)	2/3	0.89	12892.3	4.37	17.7
			(P)+(26)+(27′)+(28)+(31)	2/3	0.85	22301.2	3.14	34.6
			(P)+(26)+(27′)+(28)+(29)+(30)+(31)	2/3	0.92	13400.1	3.11	17.0
8	7	30	(P)	0/5	2.04	–	5.68	6.5
			(P)+(26)	1/4	0.81	24069.6	5.68	1.4
			(P)+(26)+(27′)+(28)	2/3	1.05	9658.9	3.16	8.8
			(P)+(26)+(27′)+(28)+(31)	2/3	0.87	20923.1	2.11	9.9
			(P)+(26)+(27′)+(28)+(29)+(30)+(31)	2/3	0.61	4845.4	2.00	25.9
8	7	75	(P)	0/5	2.11	–	3.88	29.9
			(P)+(26)	0/5	1.20	–	3.88	22.9
			(P)+(26)+(27′)+(28)	0/5	0.93	–	2.49	20.5
			(P)+(26)+(27′)+(28)+(31)	0/5	1.04	–	2.01	45.6
			(P)+(26)+(27′)+(28)+(29)+(30)+(31)	0/5	1.00	–	1.97	53.7
8	7	150	(P)	0/5	2.12	–	3.20	35.0
			(P)+(26)	0/5	1.51	–	3.20	50.9
			(P)+(26)+(27′)+(28)	0/5	1.20	–	2.31	46.7
			(P)+(26)+(27′)+(28)+(31)	0/5	1.11	–	1.75	57.1
			(P)+(26)+(27′)+(28)+(29)+(30)+(31)	0/5	1.11	–	1.72	64.1
10	10	40	(P)	0/5	2.46	–	5.12	18.7
			(P)+(26)	0/5	0.80	–	5.12	9.3
			(P)+(26)+(27′)+(28)	0/5	0.75	–	2.96	18.9
			(P)+(26)+(27′)+(28)+(31)	0/5	0.73	–	2.14	42.8
			(P)+(26)+(27′)+(28)+(29)+(30)+(31)	0/5	0.63	–	2.02	28.3
10	10	100	(P)	0/5	2.22	–	3.51	51.1
			(P)+(26)	0/5	1.06	–	3.51	47.2
			(P)+(26)+(27′)+(28)	0/5	1.08	–	2.22	31.4
			(P)+(26)+(27′)+(28)+(31)	0/5	0.96	–	1.92	83.1
			(P)+(26)+(27′)+(28)+(29)+(30)+(31)	0/5	0.98	–	1.89	18.4
10	10	200	(P)	0/5	2.12	–	3.25	100.7
			(P)+(26)	0/5	1.47	–	3.25	55.0
			(P)+(26)+(27′)+(28)	0/5	1.00	–	2.09	73.1
			(P)+(26)+(27′)+(28)+(31)	0/5	0.90	–	1.71	132.5
			(P)+(26)+(27′)+(28)+(29)+(30)+(31)	0/5	0.89	–	1.69	39.2

Table 6: Summary of the results obtained for flexible warehouse lease contracts under demand profile A; \*average MIP gap for instances *not* solved to optimality within 12 h; \*\*average CPU time for instances solved to optimality; – time limit (12 h) reached by all instances.

$ I^e $	$ I^n $	$ J $	Formulation	# opt sol/ # non-opt sol	Avg MIP		Avg LP	
					Gap (%)*	CPU (s)**	Gap (%)	CPU (s)
5	5	20	(P)	1/4	3.00	3327.4	7.83	19.1
			(P)+(26)	3/2	1.75	10743.0	7.83	23.1
			(P)+(26)+(27')+(28)	5/0		6561.4	7.14	18.0
			(P)+(26)+(27')+(28)+(31)	5/0		5679.2	5.77	29.9
			(P)+(26)+(27')+(28)+(29)+(30)+(31)	5/0		1669.7	5.17	31.9
5	5	50	(P)	1/4	2.78	6045.5	5.44	24.3
			(P)+(26)	3/2	0.48	5848.2	5.44	12.9
			(P)+(26)+(27')+(28)	3/2	0.31	2033.7	4.94	9.6
			(P)+(26)+(27')+(28)+(31)	3/2	0.32	2330.2	4.94	18.1
			(P)+(26)+(27')+(28)+(29)+(30)+(31)	5/0		7728.0	4.64	19.9
5	5	100	(P)	0/5	2.58	–	5.73	39.9
			(P)+(26)	2/3	0.56	39325.7	5.73	26.0
			(P)+(26)+(27')+(28)	2/3	0.46	32679.7	5.29	17.9
			(P)+(26)+(27')+(28)+(31)	3/2	0.72	37125.5	3.52	34.2
			(P)+(26)+(27')+(28)+(29)+(30)+(31)	2/3	0.77	27361.9	3.18	33.8
8	7	30	(P)	0/5	1.03	–	4.01	28.1
			(P)+(26)	3/2	0.46	11700.0	4.01	6.9
			(P)+(26)+(27')+(28)	3/2	0.46	4863.3	3.01	5.5
			(P)+(26)+(27')+(28)+(31)	3/2	0.58	7530.3	1.91	29.9
			(P)+(26)+(27')+(28)+(29)+(30)+(31)	3/2	0.40	4937.2	1.37	37.2
8	7	75	(P)	0/5	1.87	–	3.99	46.1
			(P)+(26)	1/4	1.26	40347.5	3.99	28.0
			(P)+(26)+(27')+(28)	2/3	1.30	29148.1	3.34	34.9
			(P)+(26)+(27')+(28)+(31)	1/4	1.08	34437.5	2.60	56.4
			(P)+(26)+(27')+(28)+(29)+(30)+(31)	0/5	1.13	–	2.15	55.5
8	7	150	(P)	1/4	1.17	35177.8	2.61	83.2
			(P)+(26)	0/5	0.72	–	2.61	50.0
			(P)+(26)+(27')+(28)	0/5	0.57	–	2.31	49.3
			(P)+(26)+(27')+(28)+(31)	0/5	0.58	–	1.97	92.0
			(P)+(26)+(27')+(28)+(29)+(30)+(31)	0/5	0.65	–	1.44	81.6
10	10	40	(P)	0/5	2.05	–	4.34	44.7
			(P)+(26)	1/4	1.19	10544.5	4.34	22.6
			(P)+(26)+(27')+(28)	1/4	1.14	7757.0	3.38	22.4
			(P)+(26)+(27')+(28)+(31)	1/4	1.12	9027.9	3.33	31.7
			(P)+(26)+(27')+(28)+(29)+(30)+(31)	1/4	1.22	30640.4	2.71	23.1
10	10	100	(P)	0/5	2.21	–	4.04	88.3
			(P)+(26)	0/5	1.14	–	4.04	55.6
			(P)+(26)+(27')+(28)	0/5	0.84	–	3.14	62.7
			(P)+(26)+(27')+(28)+(31)	0/5	0.84	–	3.14	68.3
			(P)+(26)+(27')+(28)+(29)+(30)+(31)	0/5	0.93	–	2.60	70.0
10	10	200	(P)	0/5	1.98	–	3.28	97.6
			(P)+(26)	0/5	1.30	–	3.28	66.7
			(P)+(26)+(27')+(28)	0/5	0.98	–	2.73	115.2
			(P)+(26)+(27')+(28)+(31)	0/5	0.97	–	2.57	147.2
			(P)+(26)+(27')+(28)+(29)+(30)+(31)	0/5	0.98	–	1.99	122.8

Table 7: Summary of the results obtained for flexible warehouse lease contracts under demand profile B; \*average MIP gap for instances *not* solved to optimality within 12 h; \*\*average CPU time for instances solved to optimality; – time limit (12 h) reached by all instances.

Demand profile	$ I^e $	$ I^n $	$ J $	Formulation	# opt sol/ # non-opt sol	Avg MIP		Avg LP	
						Gap (%)*	CPU (s)**	Gap (%)	CPU (s)
A	5	5	20	$(P_{23})$	3/2	2.14	10066.6	6.50	1.6
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		846.8	2.17	3.0
	5	5	50	$(P_{23})$	2/3	1.78	26557.7	4.58	3.2
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	4/1	0.30	3999.7	2.13	5.6
	5	5	100	$(P_{23})$	0/5	2.26	–	4.35	7.2
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	4/1	0.66	10631.5	2.30	9.8
	8	7	30	$(P_{23})$	1/4	1.25	6949.7	3.54	4.0
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	4/1	0.85	9677.8	1.61	13.5
	8	7	75	$(P_{23})$	0/5	1.62	–	2.96	17.2
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	1/4	1.17	31626.7	1.91	12.1
	8	7	150	$(P_{23})$	0/5	2.12	–	2.74	32.3
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	0/5	1.18	–	1.79	56.7
10	10	40	$(P_{23})$	0/5	1.87	–	3.35	17.1	
			$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	1/4	0.48	12260.1	1.80	17.6	
10	10	100	$(P_{23})$	0/5	1.87	–	2.57	10.1	
			$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	0/5	0.79	–	1.74	17.2	
10	10	200	$(P_{23})$	0/5	1.71	–	2.41	30.6	
			$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	0/5	0.92	–	1.56	86.9	
B	5	5	20	$(P_{23})$	5/0		4433.3	5.90	2.6
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		261.4	3.51	2.9
	5	5	50	$(P_{23})$	5/0		10847.1	4.82	2.5
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		2541.1	4.00	2.1
	5	5	100	$(P_{23})$	3/2	1.34	25259.3	4.95	5.6
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		21936.3	2.82	5.1
	8	7	30	$(P_{23})$	5/0		4627.4	3.60	5.5
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		2130.2	1.48	7.7
	8	7	75	$(P_{23})$	4/1	2.12	21107.8	3.34	16.2
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	4/1	2.19	7907.0	1.74	20.1
	8	7	150	$(P_{23})$	1/4	0.89	3253.5	2.28	31.9
				$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	2/3	0.59	17238.9	1.16	43.5
10	10	40	$(P_{23})$	0/5	0.97	–	3.64	23.8	
			$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	3/2	0.50	9192.4	2.11	13.3	
10	10	100	$(P_{23})$	0/5	1.49	–	3.26	26.0	
			$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	1/4	0.70	22209.7	1.96	31.1	
10	10	200	$(P_{23})$	0/5	1.35	–	2.58	109.1	
			$(P_{23})+(26)+(27')+(28)+(29)+(30)+(31)$	2/3	0.74	18983.1	1.38	86.8	

Table 8: Summary of the results obtained for warehouse contracts with a *minimum lease term* of 6 periods, under demand profiles A and B; \*average MIP gap for instances *not* solved to optimality within 12 h; \*\*average CPU time for instances solved to optimality; – time limit (12 h) reached by all instances.

Demand profile	$ I^e $	$ I^n $	$ J $	Formulation	# opt sol/ # non-opt sol	Avg MIP		Avg LP	
						Gap (%)*	CPU (s)**	Gap (%)	CPU (s)
A	5	5	20	$(P_{24})$	4/1	1.21	9268.4	6.08	1.1
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	5/0		696.2	2.24	0.8
	5	5	50	$(P_{24})$	3/2	1.24	10977.9	4.16	1.4
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	4/1	0.28	3133.8	2.20	1.4
	5	5	100	$(P_{24})$	1/4	1.86	38725.6	3.79	5.6
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	3/2	0.48	409.8	2.29	4.8
	8	7	30	$(P_{24})$	2/3	0.72	6782.5	2.67	7.1
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	5/0		13459.3	1.75	2.5
	8	7	75	$(P_{24})$	0/5	1.27	–	2.31	18.1
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	1/4	0.91	14892.1	1.86	6.7
	8	7	150	$(P_{24})$	0/5	1.71	–	2.13	20.5
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	0/5	1.18	–	1.65	18.5
10	10	40	$(P_{24})$	0/5	1.20	–	2.57	2.7	
			$(P_{24})+(26)+(27')+(28)+(30)+(31)$	3/2	0.30	8175.3	1.82	2.7	
10	10	100	$(P_{24})$	0/5	1.59	–	2.03	9.7	
			$(P_{24})+(26)+(27')+(28)+(30)+(31)$	0/5	0.74	–	1.76	7.7	
10	10	200	$(P_{24})$	0/5	1.62	–	1.80	33.3	
			$(P_{24})+(26)+(27')+(28)+(30)+(31)$	0/5	0.75	–	1.49	19.1	
B	5	5	20	$(P_{24})$	5/0		848.3	4.61	1.3
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	5/0		207.8	3.46	1.4
	5	5	50	$(P_{24})$	5/0		5297.4	3.99	1.3
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	5/0		1886.5	3.99	3.3
	5	5	100	$(P_{24})$	4/1	1.29	11620.8	3.99	4.2
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	5/0		9988.7	2.81	4.8
	8	7	30	$(P_{24})$	5/0		1940.6	2.18	5.1
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	5/0		708.9	1.45	1.3
	8	7	75	$(P_{24})$	4/1	1.75	8440.6	2.18	18.7
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	4/1	1.69	3606.5	1.69	4.8
	8	7	150	$(P_{24})$	2/3	0.78	989.8	1.41	27.6
				$(P_{24})+(26)+(27')+(28)+(30)+(31)$	2/3	0.67	4533.4	1.16	18.3
10	10	40	$(P_{24})$	1/4	0.72	7607.4	2.20	16.7	
			$(P_{24})+(26)+(27')+(28)+(30)+(31)$	5/0		13026.3	2.19	20.1	
10	10	100	$(P_{24})$	0/5	1.16	–	1.94	22.1	
			$(P_{24})+(26)+(27')+(28)+(30)+(31)$	2/3	0.66	18550.3	1.94	42.2	
10	10	200	$(P_{24})$	0/5	1.28	–	1.52	60.1	
			$(P_{24})+(26)+(27')+(28)+(30)+(31)$	4/1	1.91	9465.1	1.37	37.0	

Table 9: Summary of the results obtained for warehouse contracts with a *minimum lease term* of 6 periods and running until the end of the planning horizon, under demand profiles A and B; \*average MIP gap for instances *not* solved to optimality within 12 h; \*\*average CPU time for instances solved to optimality; – time limit (12 h) reached by all instances.

Demand profile	$ I^e $	$ I^n $	$ J $	Formulation	# opt sol/ # non-opt sol	Avg MIP		Avg LP	
						Gap (%) <sup>*</sup>	CPU (s) <sup>**</sup>	Gap (%)	CPU (s)
A	5	5	20	$(P_{25})$	5/0		1176.0	6.33	1.3
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		217.0	1.74	0.9
	5	5	50	$(P_{25})$	5/0		3931.6	4.94	1.2
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		450.9	1.58	1.4
	5	5	100	$(P_{25})$	4/1	1.65	18356.1	4.62	4.7
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		610.6	1.23	3.0
	8	7	30	$(P_{25})$	5/0		7318.0	3.66	4.3
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		459.3	2.02	2.3
	8	7	75	$(P_{25})$	5/0		17751.9	3.06	12.8
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		3418.1	2.23	15.6
	8	7	150	$(P_{25})$	0/5	1.98	–	2.80	41.1
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	2/3	0.72	12866.7	1.83	39.4
10	10	40	$(P_{25})$	2/3	0.92	24440.6	3.28	2.7	
			$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		7991.3	1.77	25.2	
10	10	100	$(P_{25})$	0/5	1.35	–	2.44	8.3	
			$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	1/4	0.61	11228.9	1.82	33.6	
10	10	200	$(P_{25})$	0/5	1.65	–	2.13	28.0	
			$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	0/5	0.51	–	1.40	29.3	
B	5	5	20	$(P_{25})$	5/0		279.5	5.34	1.9
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		24.4	3.45	1.4
	5	5	50	$(P_{25})$	5/0		2206.4	4.47	2.0
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		246.3	3.98	1.8
	5	5	100	$(P_{25})$	5/0		2636.8	4.61	6.3
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		573.0	2.81	2.9
	8	7	30	$(P_{25})$	5/0		1045.7	3.08	1.9
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		146.5	1.45	1.5
	8	7	75	$(P_{25})$	4/1	0.82	3960.2	2.92	8.6
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		9991.8	1.69	18.7
	8	7	150	$(P_{25})$	3/2	0.87	21497.7	1.93	29.2
				$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		7224.9	1.15	24.9
10	10	40	$(P_{25})$	4/1	0.72	21954.2	3.12	8.6	
			$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	5/0		3935.4	2.16	6.2	
10	10	100	$(P_{25})$	1/4	0.90	22163.7	2.74	38.3	
			$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	3/2	0.52	9186.8	1.92	17.4	
10	10	200	$(P_{25})$	0/5	1.27	–	2.17	64.4	
			$(P_{25})+(26)+(27')+(28)+(29)+(30)+(31)$	4/1	1.22	3379.0	1.30	47.6	

Table 10: Summary of the results obtained for warehouse lease contracts whose lengths are *multiples* of a base-fixed term tenancy of 6 periods, under demand profiles A and B; <sup>\*</sup>average MIP gap for instances *not* solved to optimality within 12 h; <sup>\*\*</sup>average CPU time for instances solved to optimality; – time limit (12 h) reached by all instances.



Demand profile	Problem	Percent of total cost					
		Company-owned wareh.		New wareh.	Distribution cost		Inventory cost
		closing cost	operating cost	lease cost	suppliers to wareh.	wareh. to cust. zones	
A	(P)	0.8	12.9	34.0	19.8	32.4	0.1
	(P <sub>23</sub> )	0.8	13.3	34.3	19.5	32.0	0.1
	(P <sub>24</sub> )	0.8	14.4	34.1	19.1	31.5	0.1
	(P <sub>25</sub> )	0.8	14.0	33.8	19.4	31.9	0.1
B	(P)	0.4	30.6	20.1	18.4	30.3	0.2
	(P <sub>23</sub> )	0.5	28.0	26.3	17.0	28.0	0.2
	(P <sub>24</sub> )	0.5	27.9	26.4	17.0	28.0	0.2
	(P <sub>25</sub> )	0.5	27.7	26.6	17.0	28.0	0.2

Table 11: Average percent contribution of different cost categories to total cost for the best solutions returned by CPLEX.

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