

Zhang, Ziyun

Article

Attitudes towards ambiguous information and stock returns

Cogent Economics & Finance

Provided in Cooperation with:

Taylor & Francis Group

Suggested Citation: Zhang, Ziyun (2019) : Attitudes towards ambiguous information and stock returns, Cogent Economics & Finance, ISSN 2332-2039, Taylor & Francis, Abingdon, Vol. 7, Iss. 1, pp. 1-33,
<https://doi.org/10.1080/23322039.2019.1693678>

This Version is available at:

<https://hdl.handle.net/10419/270699>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



<https://creativecommons.org/licenses/by/4.0/>

Attitudes towards ambiguous information and stock returns

Ziyun Zhang |

To cite this article: Ziyun Zhang | (2019) Attitudes towards ambiguous information and stock returns, Cogent Economics & Finance, 7:1, 1693678, DOI: [10.1080/23322039.2019.1693678](https://doi.org/10.1080/23322039.2019.1693678)

To link to this article: <https://doi.org/10.1080/23322039.2019.1693678>



© 2019 The Author(s). This open access article is distributed under a Creative Commons Attribution (CC-BY) 4.0 license.



Published online: 13 Dec 2019.



Submit your article to this journal [↗](#)



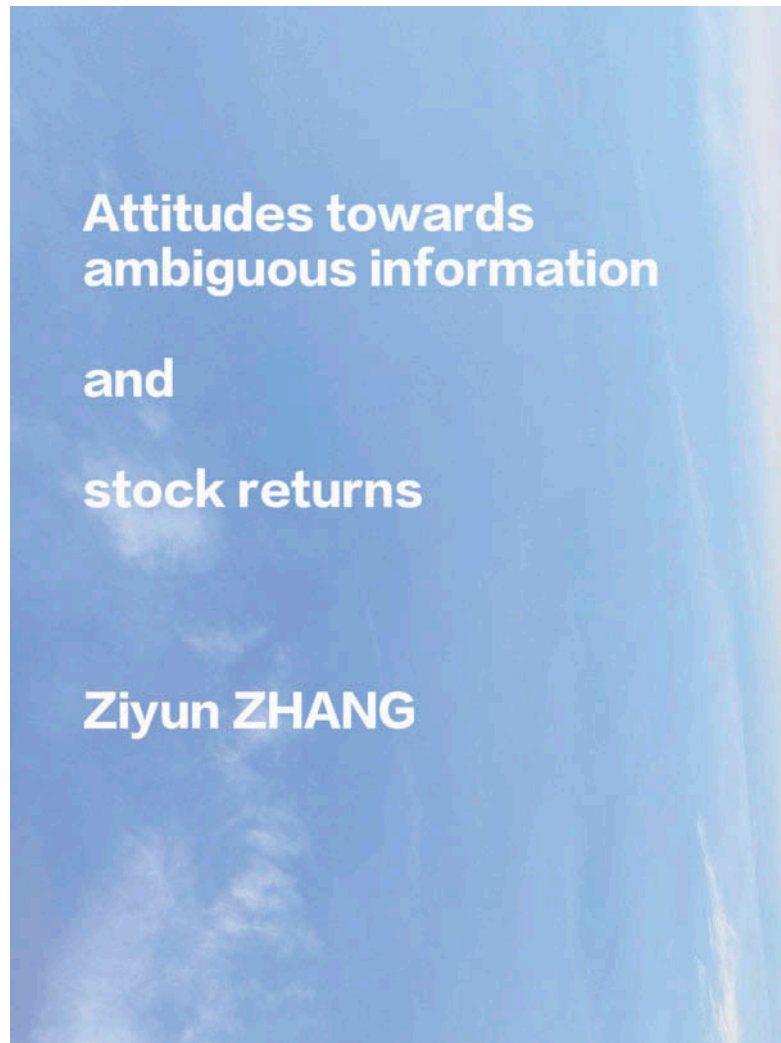
Article views: 680



View related articles [↗](#)



View Crossmark data [↗](#)



GENERAL & APPLIED ECONOMICS | RESEARCH ARTICLE

Attitudes towards ambiguous information and stock returns

Ziyun Zhang

Cogent Economics & Finance (2019), 7: 1693678



Received: 19 June 2019
Accepted: 15 September 2019
First Published: XXXXXXXX

*Corresponding author: Ziyun Zhang
Beijing Normal University, No. 19,
Xinjiekou Outer Street, Haidian District,
Beijing, P. R, China
E-mail: zhang11zy@163.com

Reviewing editor:
Numan Ulku, University of South
Australia, Adelaide Australia

Additional information is available at
the end of the article

GENERAL & APPLIED ECONOMICS | RESEARCH ARTICLE

Attitudes towards ambiguous information and stock returns

Ziyun Zhang*

Abstract: This study examines whether investors' attitudes toward ambiguity can explain cross-sectional stock returns by investigating the relationship between future stock returns and option-implied volatilities as well as implied third moments. We find that investors' attitudes toward different levels of ambiguous stocks help explain cross-sectional variations of stock returns during the 1996–2010 period in the U.S. stock market. In this study, investors' attitudes toward ambiguity are measured by stocks' option-implied third moments. Negative-skewed quintiles represent ambiguity aversion and vice versa. Different levels of ambiguity for stocks are distinguished by stocks' option-implied volatility. High volatility quintiles represent stocks with high information ambiguity. Independent two-dimension sorting results show that ambiguity averters are compensated for holding stocks with higher ambiguity. Meanwhile, ambiguity-loving investors are willing to give up some returns to hold stocks with lower levels of ambiguity. The results show that both types of ambiguity attitudes increase the factor model's explanatory power. The estimated monthly premiums for ambiguity-aversion and ambiguity-loving factors are 0.38% and 1.28%, respectively.

Subjects: Finance; Public Finance; Investment & Securities

Keywords: uncertainty; ambiguity-aversion premium; ambiguity-loving premium; stock returns; factor model

Subjects: G12

1. Introduction

In stock markets, investors make judgements about price innovations based on information that incorporates signals of stocks' fundamentals. However, the content of some information is difficult for investors to qualify. The information that includes unsure content is called "ambiguous information." For example, investors can infer certain return innovations from a company's earnings reports, but not from news reports on the company. Some studies propose capturing the effect of

ABOUT THE AUTHOR

The author is a PHD student in Beijing Normal University Business School. She got a Distinction Master degree of Finance from Warwick Business School. Her research interest is international finance and asset pricing. Main research work is to investigate the factors that affect the assets pricing, especially focus on the effect on stock prices and FX returns. The main subject of her PHD dissertation is about carry trade in foreign exchange market.

PUBLIC INTEREST STATEMENT

This study examines whether investors' attitudes toward ambiguity can explain cross-sectional stock returns. As a behavior bias, ambiguity aversion is a cognition bias that could affect investors decision during the process of investing. This study empirically finds the effect of investors' ambiguous aversion attitude could explain some portion of cross-section variations of stock returns. Even, further study could continue to study whether ambiguity aversion affect other pricing bias or deviation.

ambiguous information on decision makers' utility. Ellsberg (1961) defines decision makers' dislike for ambiguous signals as ambiguity aversion. Gilboa and Schmeidler (1989) propose the concept of maxmin expected utility (MEU) to incorporate investors' ambiguity-aversion attitude. According to the MEU model, decision makers tend to maximize utility under the worst-case situation, that is, ambiguity-averse investors consider ambiguous bad news as precise, and then react strongly to this bad news. Therefore, stock returns with more ambiguous information are negatively skewed. After the seminal work of Gilboa and Schmeidler (1989), some research has generalized the MEU and derived other models to capture ambiguity aversion, including Dow and da Costa Werlang (1992), Epstein and Wang (1994), and Chen and Epstein (2002). In addition, some studies capture other attitudes toward ambiguity, like ambiguity-loving preferences. Ghirardato, Maccheroni, and Marinacci (2004) extend the MEU of Gilboa and Schmeidler (1989) to consider "maxmax" expected utility, which models investors' love of ambiguity. According to maxmax expected utility, ambiguity lovers tend to maximize the utility of the best-case situation. Investors consider ambiguous good news as precise and bad news as imprecise. As ambiguity-loving investors react more strongly to ambiguous good information, stock returns with more ambiguous information are likely to be more positively skewed.

Epstein and Schneider (2008) document that skewness of stock returns is driven by the extent of ambiguity. With more negative-skewed returns, stocks tend to have more ambiguous information. Hence, ambiguity-averse investors would be compensated for holding negative-skewed stocks, and ambiguity-loving investors would be compensated for holding positively skewed stocks. However, Stilger, Kostakis, and Poon (2016) find a positive relationship between future stock returns and implied skewness. This contradicts the argument of Epstein and Schneider (2008). Based on theories of preference for ambiguity, the love of ambiguity might also be incorporated in stock returns.

Motivated by the dissent between the theory of Epstein and Schneider (2008) and the empirical results of Stilger et al. (2016), this study examines whether investors' attitudes toward ambiguity can explain cross-sectional stock returns by investigating the relationship between future stock returns and option-implied volatilities as well as implied third moments, which are both forward looking. As Epstein and Schneider (2008) states, ambiguity adds excess volatility to stock return distribution, and thus, stocks with high volatility usually are also characterized by more ambiguous information. The ambiguity-aversion premium is measured by returns of portfolios with long negative-skewed high volatility stocks and short negative-skewed low volatility stocks. The ambiguity-loving premium is measured by returns of portfolios with long positive-skewed low volatility stocks and short positive-skewed high volatility stocks.

The first step in this study is to sort stocks independently into volatility and third moment quintiles in order to investigate whether clear patterns exist across these subportfolios or not. The empirical findings suggest that a positive relationship exists between option-implied third moments and stock returns. This result is consistent with Stilger et al. (2016), who find a positive relationship between risk-neutral skewness and future stock returns. In the negative third moment quintiles, portfolio returns increase with volatility, and thus, ambiguity averters are compensated for holding stocks with higher ambiguity levels. In the positive quintiles, returns are negatively related to volatility, and thus, ambiguity lovers pay some premium to hold stocks with higher ambiguity levels.

In the second step, a Fama-Macbeth regression is conducted on subportfolios to examine the patterns of Carhart's α s across volatility-third moment quintiles. Abnormal returns exhibit a similar pattern to returns on volatility-third moment quintiles. These results contradict the findings of Cumming, Johanning, Ordu, and Schweizer (2015) of a negative relationship between Carhart's α s and skewness. However, Stilger et al. (2016) find a similar positive relationship between skewness and Carhart's α s, and provide the following rationale: hedging demand for overpriced stock options leads to a negatively skewed implied distribution. In other words, compensations for investors' attitudes toward ambiguity are not explained by Carhart's four factors.

Finally, ambiguity-aversion and ambiguity-loving factors are incorporated into Carhart's four-factor model and generate three different pricing models through different combinations of these two ambiguity attitude factors. The Fama–MacBeth regression approach is conducted on these models to examine premiums for ambiguity factors. The most suitable model estimates a significant monthly ambiguity-aversion premium of 0.38% and a significant monthly ambiguity-loving premium of 1.28%. As the prices of these factors cannot be explained by Carhart's four factors, investors' different attitudes toward ambiguity should be priced.

This study contributes to the existing literature in two ways. First, this study finds an effect of investors' different attitudes toward ambiguity on stock returns. Second, the study estimates monthly premiums for ambiguity-aversion and ambiguity-loving factors.

This rest of this paper is constructed as follows. In section 2, we discuss the existing literature focused on ambiguity. Section 3 presents details of the theory of ambiguity, data, and the methodology used in the study. In section 4, the empirical results are analyzed and compared with those of other studies. Finally, section 5 concludes.

2. Literature review

2.1. The ambiguity of information quality and ambiguity attitude

Knight (1921) proposes a distinction between risk and uncertainty. He defines risk as when investors know the distribution of possible outcomes with certainty, whereas uncertainty refers to the unassigned distribution of possible outcomes. This uncertainty is called Knightian uncertainty. Knight (1921) states that uncertainty plays an even more important role in decision-making than risk. Ellsberg (1961) develops a thought experiment to provide a formal explanation for ambiguity. The experiment indicates that people are more willing to deal with an event whose probability is known rather than an ambiguous event. Ellsberg (1961) defines investors' dislike of ambiguity as ambiguity aversion.

To deal with the well-known Ellsberg paradox¹, some researchers promote utility models to incorporate the effect of ambiguity aversion. Gilboa and Schmeidler (1989) propose the MEU model for investors' ambiguity aversion. The implication of the utility model is that ambiguity-averse investors maximize utility under the most unfavorable situation. The utility model is shown in the following representation:

$$f > g \text{ if and only if } \min_{p \in \wp} E_p[U(f)] \geq \min_{p \in \wp} E_p[U(g)]$$

where probability p belongs to \wp , which is a convex set that indicates the precision of information quality. Following the seminal work of the atemporal model of Gilboa and Schmeidler (1989), many theoretical models, which generalize the MEU, display variant different versions of ambiguity aversion in various areas. For instance, Dow and da Costa Werlang (1992) examine the effect of ambiguity on portfolio choice. They study the two-period problem and find that ambiguity causes a wide interval for prices.

Although some researchers emphasize modelling ambiguity aversion, others formulate investors' other attitudes toward ambiguity. Ghirardato et al. (2004) extend the MEU with ambiguity-loving attitude, which makes ambiguity aversion in MEU zero. Ambiguity-averse investors maximize utility under the best case, and then, the utility model is the maxmax expected utility model:

$$f > g \text{ if and only if } \max_{p \in \wp} E_p[U(f)] \geq \max_{p \in \wp} E_p[U(g)]$$

Klibanoff, Marinacci, and Mukerji (2005) characterize a decision-making model to capture investors' preference for ambiguity. In their model,

$f > g$ if and only if $E_{\mu} \phi [E_{\pi} U(f)] \geq E_{\mu} \phi [E_{\pi} U(g)]$

where μ is subjective probability, U is the utility function, and ϕ is an increasing transformation that captures investors' ambiguity attitude. A concave ϕ implies an ambiguity-aversion attitude and a convex ϕ implies an ambiguity-loving attitude.

There are many theoretical studies on the subject of asset pricing under ambiguity. As the Ellsberg paradox contradicts the Savage model, which does not distinguish between risk and uncertainty, Epstein and Wang (1994) extend the Lucas equilibrium and propose an intertemporal asset-pricing model that incorporates the effect of Knightian uncertainty. The study of Epstein and Wang (1994) is developed by Chen and Epstein (2002), who derive a continuous-time intertemporal recursive multiple-priors utility, which is a form of utility that allows a distinction between risk aversion and ambiguity aversion. The model places constraints on stock returns that permit a risk premium and additional premium for ambiguity. Epstein and Schneider (2003) axiomatize the intertemporal recursive multiple-priors utility on a dynamic setting without restricting investors' response to data. Epstein and Schneider (2003) model investors' behavior when they face ambiguity.

Epstein and Schneider (2008) propose a three-period model based on their earlier intertemporal model Epstein and Schneider (2003) in order to obtain insights into the effect of ambiguity of signals. Their thought experiment shows that ambiguity-averse investors react asymmetrically to ambiguous information, as they adopt worst-case assessments of uncertainty for information. The experiment also indicates that people are more willing to choose an event with less prior ambiguity, which is the uncertainty of fundamentals before the arrival of ambiguous information. Therefore, ambiguity-averse investors have more dislike for stocks with ambiguous information if they have high volatility fundamentals.

2.2. Information quality, ambiguity premium, and returns

In addition to theoretical utility frameworks of the effects of ambiguity, many researchers show the relationship between information quality and returns. Caskey (2009) shows that ambiguity-averse investors prefer aggregated information, which has less exposure to ambiguity, even at the cost of loss of information. Furthermore, Caskey (2009) finds that investors prefer summary signals in order to reduce ambiguity, as even disaggregated information is freely available to investors. The cross-sectional analysis of Easley and O'Hara (2004) posits that information structure could affect stock premiums. If information structure shifts from public information to private information, returns tend to rise, as uninformed investors cannot infer the return innovation from this information. Consistent with the findings of Caskey (2009), they also conclude that investors like aggregated information, even that with less content, for low ambiguity.

Although many studies analyze the effect of ambiguity on investors' behavior and asset-pricing theory, many researchers have attempted to develop an exact method to quantify ambiguity. However, there is no comprehensive acceptable method to measure the extent of ambiguity yet. Anderson, Ghysels, and Juergens (2009) propose an approach to measure ambiguity by employing the degree of disagreement of forecasters. Their empirical results show that high correlation exists between market excess returns and uncertainty (ambiguity). After investigating the role played by uncertainty in asset pricing, they conclude that the uncertainty premium is significantly positive and improves the explanatory power of the Fama-French three-factor model. Moreover, they document that uncertainty is a more important determinant of returns than risk. This result proves Knight (1921) conclusion that investors are compensated for bearing uncertainty instead of risk. Differently, Jiang, Lee, and Zhang (2005) define information uncertainty (IU) as value ambiguity. For firms with high IU, it is more difficult for investors to interpret their intrinsic values from available information. Jiang et al. (2005) conclude that fundamental analysis of high IU firms is costly and less reliable, and thus, returns tend to exhibit higher volatility. In addition, they find that momentum strategy is more profitable among high IU stocks.

As numerous studies endeavor to investigate sources of the equity premium puzzle, some researchers have attempted to examine whether the risk premium anomaly can be explained by

uncertainty. Veronesi (2000) adopts a dynamic asset-pricing model to examine the relationship between public information quality and stock market returns. They find that with less ambiguous information about the state of the economy, the risk premium tends to increase. Therefore, the situation is even more puzzling after considering ambiguity.

However, other studies find the opposite. Olsen and Troughton (2000) state that risk has limited power to explain return variations. The authors gather data on ambiguity and risk from a questionnaire and survey, and their results show that most investors are ambiguity averse, and that risk aversion and ambiguity aversion are not correlated. Their analysis of risk premium anomaly and ambiguity indicates that investors require higher premiums for outcomes whose values are difficult to estimate. In addition, Rieger and Wang (2012) document a similar result—that ambiguity aversion provides an explanation for the equity premium puzzle.

Epstein and Schneider (2008) adopt the information classification introduced by Daniel and Titman (2006), which defines intangible information² as ambiguous information. Epstein and Schneider (2008) show that ambiguous information (signals with low information quality) induces ambiguity premium. Furthermore, they find that the skewness of stock returns is driven by ambiguity of information, and thus, the return distributions would have more negative skewness if signals for these stocks were more ambiguous.

2.3. The relationship between high moments and returns

Epstein and Schneider (2008) state that stocks with more ambiguity have more negative skewness, and thus, skewness can be an indicator of ambiguity. If skewness is priced in stock returns, the ambiguity premium is then supported by the empirical evidence. Recently, many studies incorporate skewness as a pricing factor. Kraus and Litzenberger (1976) first derive a three-moment capital asset pricing model (CAPM), which includes unconditional systematic skewness. Empirical evidence shows that the estimated coefficients are consistent with their expectations: there is an insignificant intercept, a significantly positive coefficient for market beta, and a significantly negative coefficient for systematic skewness. Incorporating systematic skewness also increases the R^2 of the CAPM. In addition, Lim (1989) employs Kraus and Litzenberger (1976) three-moment CAPM and concludes that systematic skewness is priced in stock returns.

There is some empirical evidence that echoes the model of Epstein and Schneider (2008). Harvey and Siddique (2000) examine the role of conditional skewness³ in asset pricing mainly by employing monthly U.S. equity returns. They conclude that conditional skewness indeed significantly explains variations of cross-sectional returns, which size and the book-to-market ratio cannot explain. The authors estimate an annual 3.6% risk premium for systematic skewness and state the economic importance of skewness. Further tests show that skewness for momentum losers is higher than for momentum winners. Recently, some studies have focused on innovation in high moments of returns. Chang, Christoffersen, and Jacobs (2013) investigate the pricing effect of innovations in market volatility, market skewness, and market kurtosis by using option-implied moments, similar to Ang, Hodrick, Xing, and Zhang (2006). Chang et al. (2013) document that stocks with higher exposures to innovations in market volatility and market skewness have lower returns, while stocks with higher exposures to innovations in market kurtosis have higher returns. Furthermore, their robustness test shows that market skewness is a significant factor explaining cross-sectional stock returns and cannot be explained by other factors of Carhart's four-factor model.

3. Method

3.1. Theory of ambiguity

Epstein and Schneider (2008) introduce a model that incorporates ambiguity to help explain stock prices and returns by measuring information quality. This model is built on Epstein and Schneider (2003) recursive multiple-priors utility, and assumes that investors are risk neutral and ambiguity averse.

3.1.1. Price and return under ambiguity

Epstein and Schneider (2008) derive a model that incorporates the premium induced by ambiguous information, rather than the ambiguity of fundamentals, on which the previous literature focused. Epstein and Schneider (2008) decompose stock returns into three terms. The first term of returns captures ambiguity-averse investors' asymmetric response to current intangible information. The second term captures forecasting error, which is the difference between current dividend innovation and response to the past signal. Finally, the third term of stock returns captures the present value of premiums on future ambiguous information. The premium of the third part in the return increases with both the information quality range and the volatility of fundamentals. Furthermore, in Epstein and Schneider (2008) model, with the true content of information (γ^*), the expected excess return of stocks is defined as

$$E^*(R_{t+1}) = (\bar{\gamma} - \underline{\gamma}) \frac{1}{\sqrt{2\pi\gamma}} \left(1 + \frac{r}{r + \kappa} \sqrt{\gamma/\gamma^*} \right) \sigma_u$$

where $\bar{\gamma}$ and $\underline{\gamma}$ are upper and lower bounds of γ_t , respectively. With good news, the volatility of the signal is high, and ambiguity-averse investors consider the news as having lower quality; thus, γ_t takes the lowest value of $\underline{\gamma}$. With bad news, the volatility of the signal is low, and ambiguity-averse investors regard the news as having high quality; thus, the γ_t takes the highest value of $\bar{\gamma}$. r is the interest rate and κ is the persistence level, which reflects the speed at which dividends converge to the mean. σ_u is the volatility of the dividend shock or the volatility of fundamentals. Obviously, the ambiguous information causes a premium to excess return; with unambiguous information, information quality is certain, $\bar{\gamma} = \underline{\gamma}$, $E^*(R_{t+1}) = 0$. This zero return is generated because of the assumption that investors are risk neutral. Again, stock returns increase with both the range of information quality $\bar{\gamma} - \underline{\gamma}$ and the volatility of fundamental σ_u .

3.1.2. Volatility of return under ambiguity

Epstein and Schneider (2008) state that ambiguity of information quality creates additional volatility to returns. In their model, the variance of return is defined as

$$\text{var}(R_t) = \sigma_u^2 \left(\frac{1+r}{r+\kappa} \right)^2 \left\{ 1 - \bar{\gamma} - \underline{\gamma} + \frac{1+\beta^2}{2\gamma^*} \left(\bar{\gamma}^2 + \underline{\gamma}^2 - \frac{1}{\pi} (\bar{\gamma} - \underline{\gamma})^2 \right) \right\}$$

where β is the discount factor. In the case of no information, the variance is related only to the volatility of foundational, σ_u^2 . From the model, the increase in either $\bar{\gamma}$ or $\underline{\gamma}$ leads to an increase of volatility of returns. At the Bayesian benchmark, without ambiguous news, $\gamma^* = \underline{\gamma} = \bar{\gamma}$, so that a change in volatility of returns is decided not only by fundamental volatility. Unlike the Bayesian benchmark, the existence of ambiguous information increases $\bar{\gamma}$ and decreases $\underline{\gamma}$, so that ambiguous information increases the range of $\bar{\gamma} - \underline{\gamma}$, and contributes large volatility to stock returns.

3.1.3. The third moment of return under ambiguity

One stylized fact in stock markets is that most stock returns have negative skewness. In the model, this is explained by ambiguity-averse investors' dislike of ambiguous information; they respond more strongly to bad ambiguous news than to good ambiguous news. Those investors' asymmetrical response leads to negatively skewed returns. The model of Epstein and Schneider (2008) defines the third moment of stock returns as

$$\mu_3(R_{t+1}) = \left(\frac{1+r}{r+\kappa} \right)^3 \left\{ -(1-\beta^3)\mu_3(\gamma_t s_t) - \frac{\sigma_u^3}{\sqrt{2\pi\gamma^*}} (\bar{\gamma} - \underline{\gamma}) (\bar{\gamma} + \underline{\gamma} - \gamma^*) \right\}$$

Obviously, the second term in the big brackets is negative. With ambiguous information in the market, $\gamma_t s_t$ is also negative, and thus, the term $-(1-\beta^3)\mu_3(\gamma_t s_t)$ is negative with a high discount factor β , or a lower discount rate⁴. For stocks without information, skewness is zero, because $\underline{\gamma} = \bar{\gamma} = 0$. By contrast, an ambiguous signal usually causes stock returns to be negatively skewed.

However, ambiguity-loving investors have the opposite attitude toward ambiguity. According to Ghirardato, Maccheroni and Marinacci's (2004) maxmax expected utility, ambiguity lovers maximize their utility in the best case. They regard good news as precise and bad news as imprecise and respond more strongly to good news. They prefer assets with more ambiguous information and even pay a premium to take a gamble, because they tend to consider good ambiguous information as precise and as increasing stock prices. Thus, they have different asymmetrical responses to ambiguity averters when they obtain ambiguous information. Thus, the asymmetrical response leads to positively skewed returns.

3.2. Test of ambiguity premium

In this subsection, the data and β method used to explore the role of ambiguity in asset pricing are explained in detail. The ambiguity-aversion premium and ambiguity-loving premium are tested through the following steps. First, sort stocks' volatilities and third moments into portfolios independently to analyze post-ranking return variations across sorted volatility-third moment portfolios. Similar with Stilger et al. (2016), this study uses the option-implied volatility and third moment data, which aims to explore the forward looking results. Then, regress time-series returns of each portfolio on the market risk factor, size factor, and book-to-market factor proposed by Fama and French (1992) as well as an additional momentum factor proposed by Carhart (1997), to compare abnormal returns of these subportfolios. Next, test whether premiums for attitudes to ambiguity are significant or not, construct mimicking portfolios, and include them in Carhart's four-factor model. Finally, compare the regression results of different pricing models, and choose the best one.

3.2.1. Data description

This study uses monthly option-implied volatility and third moment data for the time period from January 1996 to December 2010. Individual stocks' high moments' data with 1-month maturity are collected from OptionMetrics. In addition, dividend, split and other treatments are automatically adjusted in OptionMetrics. There are 9303 individual companies in the chosen time period with their CUSIP identifiers. The volatility, measured by $\sigma(R_t)$, is obtained by taking the square root of implied variances.

Monthly individual holding period returns from February 1996 to January 2011⁵ are from the Center for Research in Security Prices. To perform the following analysis, we merge these 1-month post-ranking monthly returns with their individual volatility and third moments by both the corresponding dates and their CUSIP identifiers. However, after the merging process, only 7255 companies remain in total, owing to the unmatched date of two data sets. In addition, some missing returns from the downloaded data set cause the unmatched data problem.

To build the asset-pricing model, monthly factor premiums of market risk factor, size factor, book-to-market factor, and momentum factor, for February 1996 to January 2011, are collected from the Fama French & Liquidity Factors database of Wharton Research Data Services (WRDS). The 1-month Treasury bill rate is the proxy for the risk-free rate, as samples of high moments and returns are for the U.S. market.

3.2.2. Double-sorting based on stocks' individual volatilities and skewness

As stated by Epstein and Schneider (2008), stocks with more ambiguous information have higher volatility than do those with less ambiguous information. Ambiguity-averse investors respond more strongly to ambiguous bad news, and the stock returns in which they invest are mostly negatively skewed. However, ambiguity-loving investors have the opposite attitude toward ambiguous information and generate positively skewed returns.

The double-sorting method introduced by Fama and French (1993) is applied to test the theory of Epstein and Schneider (2008). Stocks are independently sorted into two dimensions of portfolios

by their individual volatilities and third moments. The compositions of portfolios are rebalanced monthly to ensure that each volatility-third moment portfolio is dynamic and accurate at any time.

First, stocks are sorted into five volatility quintiles based on their individual volatilities in ascending order from quintiles 1 to 5⁶. Hence, stocks in portfolio 1 have the lowest volatilities and those in portfolio 5 have the highest volatilities. Second, stocks are sorted into five third moment quintiles based on their individual third moments, also in ascending order from portfolios 1 to 5⁷. Stocks in portfolio 1 have the most negative skewness and those in portfolios 5 have the most positive skewness.

After assigning each stock into volatility and third moment quintiles, 25 subportfolios are constructed based on these two independent dimensions. Within each subportfolio, post-ranking monthly returns of its composite stocks are recorded. The cross-sectional average of these post-ranking returns is taken as the portfolio's return. Thus, these 25 volatility-third moment portfolios are equal-weighted average portfolios. With the monthly returns of 25 subportfolios from February 1996 to January 2011, the time-series average of each portfolio is calculated to present the return of each portfolio.

If the implications of Epstein and Schneider (2008) asset-pricing model hold, some clear trends of returns should be evident. On the one hand, the most negatively skewed returns imply that ambiguity-averse investors have the most asymmetrical response to ambiguous signals, and thus, negative third moments reflect the behavior of the ambiguity-aversion premium. According to Epstein and Schneider (2008) model, stocks with higher volatility tend to have higher levels of ambiguity. Hence, stocks with higher volatility are expected to have higher returns in the negative third moment quintile, as ambiguity averters dislike ambiguity. On the other hand, positively skewed returns reflect the behavior of the ambiguity-loving premium. As ambiguity-loving investors prefer stocks with more ambiguous signals, they need higher returns for holding assets with less ambiguity, and thus, a negative relationship between return and volatility is expected.

3.2.3. Sources of returns variations across subportfolios

This subsection aims to explore the sources of return trends and patterns shown in the previous section. A large body of empirical evidence in the literature supports the notion that Carhart's four factors can explain cross-sectional stock returns. In this subsection, we test whether Carhart's four factors can explain the patterns of returns across volatility-third moment portfolios in subsection 3.2.2. Fama and MacBeth (1973) two-pass regression method is conducted to test sources of variations across subportfolios' returns.

The first pass is a time-series regression for each portfolio for the time period rolling window $t = 1, \dots, T$. The form to be estimated is

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt} Z_{mkt,t} + \beta_{i,SMB} Z_{SMB,t} + \beta_{i,HML} Z_{HML,t} + \beta_{i,UMD} Z_{UMD,t} + \epsilon_{i,t} \quad (1)$$

where $R_{i,t}$ is the return of subportfolio i ⁸ for time period t . $R_{f,t}$ is the risk-free return of window t . In this study, the 1-month Treasury bill rate is the proxy for the risk-free rate. $Z_{mkt,t}$ is the market premium for time period t ; $Z_{SMB,t}$, $Z_{HML,t}$, and $Z_{UMD,t}$ are factor premiums for the size factor, book-to-market factor, and momentum factor, respectively, at rolling window t .

In order to keep enough variations to perform the first-pass time-series regression, the length of the rolling window is 5 years (T is 60). Thereafter, the window is rolled monthly. After running the first-pass regression, the collected abnormal returns (α_s) are used to compare their differences between subportfolios. Abnormal returns are the part of returns that cannot be explained by factors in the model.

If investors' attitude toward ambiguity is actually priced in the stock market, Carhart's four-factor model does not capture premiums for investors' attitudes. Therefore, we anticipate that α_s increase with volatility for the negative third moment quintile, similarly to subsection 3.2.2. In addition, α_s are expected to increase with the decrease of volatility for the positive third moment quintile.

In order to obtain factor premiums, loadings of time-series regressions, β s, are collected to regress the second-pass regression. The regression form is as follows:

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt}\hat{\beta}_{i,mkt,t-1} + \lambda_{SMB}\hat{\beta}_{i,SMB,t-1} + \lambda_{HML}\hat{\beta}_{i,HML,t-1} + \lambda_{UMD}\hat{\beta}_{i,UMD,t-1} + \epsilon_i \quad (2)$$

where $\bar{Z}_{i,t}$ is the time-series average excess return of portfolio i in rolling window t . $\hat{\beta}$ s are collected estimates from the first-pass regression. λ s are the estimated factor premiums from the cross-sectional regression.

Under the hypothesis of ambiguity-aversion and ambiguity-loving premiums, the constant term λ_0 is expected to be significantly different from zero. Risk premiums for the four risk factors are expected to be positive and significantly different from zero.

3.2.4. Extension of the four-factor model with ambiguity

To test whether ambiguity-aversion and ambiguity-loving premiums are priced in stock returns, factors that capture investors' attitudes toward ambiguity are incorporated into a four-factor model. Following the approach introduced by Fama and French (1993), this study constructs two mimicking portfolios to build two factors that measure aversion and loving attitudes to ambiguity, respectively. Meanwhile, according to Epstein and Schneider (2008), volatility is a good proxy for ambiguity, as ambiguity adds volatility to stock returns.

For the most negative-skewed quintile 1, stocks accurately capture the asymmetric response of ambiguity-averse investors. Therefore, the mimicking portfolio that longs high volatility quintiles and shorts low volatility quintiles captures the ambiguity-aversion premium in the most negative third moment quintile. The return of the ambiguity-aversion factor is considered as a premium or compensation for ambiguity-averse investors to hold stocks for which more ambiguous signals are relevant.

Third moment quintile 5 reflects investors' preference for assets with more ambiguous information, as ambiguity-loving investors react strongly to ambiguous good news. The second mimicking portfolio takes a long position on low volatility portfolios and a short position on high volatility portfolios in third moment quintile 5. Hence, the mimicking portfolio is taken as an ambiguity-loving factor that captures premiums for ambiguity lovers to hold stocks for which less ambiguous information is relevant.

Both of these ambiguity factors are incorporated into the four-factor model to test whether they can help explain stock returns. The regression method introduced by Fama and MacBeth (1973) is used. The first-pass regression is conducted on the following forms for rolling window $t = 1, \dots, T$:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \beta_{i,AVE}Z_{AVE,t} + \epsilon_{i,t} \quad (3)$$

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \beta_{i,LOV}Z_{LOV,t} + \epsilon_{i,t} \quad (4)$$

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \beta_{i,AVE}Z_{AVE,t} + \beta_{i,LOV}Z_{LOV,t} + \epsilon_{i,t} \quad (5)$$

where $Z_{AVE,t}$ and $Z_{LOV,t}$ are the returns of the ambiguity-aversion factor and the ambiguity-loving factor, respectively, at rolling window t . $\beta_{i,AVE}$ and $\beta_{i,LOV}$ are subportfolio i 's exposures to the ambiguity-aversion factor and the ambiguity-loving factor, respectively. Each rolling window contains 5-year data (T is 60) and is rolled over each month. After running the first-pass regression, time-varying exposures β s are collected in order to run the second-pass regression.

Different from Equation (1), the α s of Equations (3)–(5) are expected to be insignificantly different from zero. In addition, trends of α s across subportfolios are expected to be less clear than those of Equation (1), since the ambiguity-aversion factor and the ambiguity-loving factor are expected to explain the α s of Equation (1) to some extent. Loadings to ambiguity-aversion and ambiguity-loving factors are expected to be significant.

After collecting time-varying exposures from the first-pass regression, the cross-sectional regression is conducted in the following forms:

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt}\hat{\beta}_{i,mkt,t-1} + \lambda_{SMB}\hat{\beta}_{i,SMB,t-1} + \lambda_{HML}\hat{\beta}_{i,HML,t-1} + \lambda_{UMD}\hat{\beta}_{i,UMD,t-1} + \lambda_{AVE}\hat{\beta}_{i,AVE,t-1} + \varepsilon_i \quad (6)$$

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt}\hat{\beta}_{i,mkt,t-1} + \lambda_{SMB}\hat{\beta}_{i,SMB,t-1} + \lambda_{HML}\hat{\beta}_{i,HML,t-1} + \lambda_{UMD}\hat{\beta}_{i,UMD,t-1} + \lambda_{LOV}\hat{\beta}_{i,LOV,t-1} + \varepsilon_i \quad (7)$$

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt}\hat{\beta}_{i,mkt,t-1} + \lambda_{SMB}\hat{\beta}_{i,SMB,t-1} + \lambda_{HML}\hat{\beta}_{i,HML,t-1} + \lambda_{UMD}\hat{\beta}_{i,UMD,t-1} + \lambda_{AVE}\hat{\beta}_{i,AVE,t-1} + \lambda_{LOV}\hat{\beta}_{i,LOV,t-1} + \varepsilon_i \quad (8)$$

where λ_{AVE} and λ_{LOV} are estimated premiums for ambiguity-aversion and ambiguity-loving factors, respectively. Since the two factors are expected to help to explain stock returns, both λ_{NMP} and λ_{LOV} are anticipated to be positive and significant. In addition, premiums for the other risk factors, λ_{mkt} , λ_{SMB} , λ_{HML} , and λ_{UMD} , are expected to be positive and significant. Finally, the constant term λ_0 is expected to be insignificantly different from zero.

4. Results

4.1. Double-sorting based on stocks' individual volatilities and third moments

As Table A1 shows, the 25 volatility-third moment portfolios have a wide range of average returns from -0.15% to 1.75%. Each volatility quintile shows a positive relationship between portfolio returns and third moment quintiles, which is consistent with the result of Stilger et al. (2016). The positive relationship provides evidence contrary to that of Epstein and Schneider (2008) in which negatively skewed returns reflect more ambiguity and ambiguity-averse investors are compensated more for holding stocks with more ambiguous information.

At the negative third moment quintiles, portfolios' returns have a positive relationship with volatility. Ambiguity-averse investors usually react strongly to ambiguous bad news, as they consider it to be more precise than good news. The most negative third moment quintile captures the attitudes of ambiguity averters in the market. Because ambiguity averters need to be compensated for holding assets with more ambiguous information (high volatility quintiles), they would obtain higher returns for holding high volatility quintiles. Therefore, for the most negative third moment quintile, there should be a positive relationship between return and volatility.

At the most positive third moment quintiles, however, the returns of subportfolios have a negative relationship with volatility quintiles. On the contrary to ambiguity-averse investors, ambiguity-loving investors prefer assets for which more ambiguous signals are relevant. Ambiguity lovers are willing to give up some returns to hold stocks with more ambiguous information, which have high volatility. In addition, they are compensated for holding stocks with lower volatility and for not taking an ambiguity gamble. Hence, there is a negative relationship between returns and volatility for the most positive third moment quintiles.

To test whether these average returns are significant or not, each portfolio's monthly returns are regressed on a constant term to obtain the t-statistics. In Table A1, the t-statistics for this time-series regression indicate that most returns are significant. However, all returns of high volatility quintiles and negative third moment quintiles are insignificant. The return differences between volatility quintiles are also not significant.

4.2. Results of Fama-Macbeth regression in the four-factor model

Table A2 displays the results of the first-pass regression of the Fama-Macbeth approach discussed in subsection 3.2.3. The regression form is

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \varepsilon_{i,t} \quad (1)$$

The results in panel A of Table A2 show that abnormal returns have a similar pattern to the volatility-third moment portfolio returns described in subsection 4.1. The abnormal returns have a wide range from -1.11% to 1.14% , with a length of 2.25% . Similar to the patterns of returns in subsection 4.1, α s have a positive relationship with third moment quintiles for the majority of volatility quintiles. Furthermore, for negative third moment quintiles, all abnormal returns are negative, and thus, stocks in negative third moment quintiles are overpriced by the four-factor model. Stilger et al. (2016) find a positive relationship between skewness and stock returns, and provides the following explanation: hedging demand for overpriced stocks raises the prices of these stocks' put options and lowers the prices of their call options, leading to negative skewness. However, Cumming et al. (2015) document a negative relationship between Carhart's alpha and skewness. They take Carhart's alphas as ambiguity-aversion premiums.

Again, the relationship between volatility and abnormal returns differs among third moment quintiles. For the negative third moment quintile, abnormal returns increase with volatility quintiles; for the positive third moment quintile, α s are negatively related to volatility quintiles. This result indicates that the return pattern in subsection 4.1 cannot be explained by Carhart's four factors.

Since the method of Fama and MacBeth (1973) does not correct the statistics calculation for the time-series regression, t-statistics are calculated by the following formula:

$$t_i = \frac{\bar{\alpha}_i}{\overline{SE}(\alpha_i)}$$

where $\bar{\alpha}_i$ is the time-series average of estimated α_i in the first-pass regression. $\overline{SE}(\alpha_i)$ s is the time-series average of the Newey–West standard error of $\bar{\alpha}_i$ to correct the autocorrelation problem. These t-statistics show that most abnormal returns in third moment quintiles 1 and 5 (negative and positive quintiles) are significant. In particular, the abnormal returns of negative third moment quintiles are more significant than those of double sorting in subsection 4.1.

The time-series averages of exposures to four factors are shown in panel B. Exposures to the market risk factor increase with volatility quintiles. It is reasonable that market risk is one component of the volatility. All exposures to the market risk factor are significantly different from zero. These significant slopes suggest that the market risk factor captures cross-sectional variations of stock excess returns.

In terms of exposures to size factor, portfolios of the lowest volatility quintiles have the smallest and most insignificant exposures to size factor. Both size factors' exposures and their t-statistics increase with volatility quintiles. As the previous empirical evidence suggests that small companies usually have higher size factor exposures, the high volatility quintiles may include more companies with small market capitalization. As Epstein and Schneider (2008) state, small stocks with more prior ambiguity have higher volatility and as a result have a higher premium for coming ambiguous information. The results of the size factor exposures echo the theory of Epstein and Schneider (2008).

Exposures to the book-to-market factor mostly decrease significantly with volatility quintiles. However, there is no clear trend across subportfolios for momentum factor loadings. Therefore, the momentum factor may have lower power to explain variations of stock excess returns in this study. Furthermore, exposures to the momentum factor are less significant.

To estimate the premiums for the four factors, the following model is employed to run the second-pass regression

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt}\hat{\beta}_{i,mkt,t-1} + \lambda_{SMB}\hat{\beta}_{i,SMB,t-1} + \lambda_{HML}\hat{\beta}_{i,HML,t-1} + \lambda_{UMD}\hat{\beta}_{i,UMD,t-1} + \varepsilon_i \quad (2)$$

The results are shown in Table A3. The t-statistics of the regression coefficients are calculated by the following formula, which is introduced by Fama and MacBeth (1973):

$$t(\bar{\lambda}_j) = \frac{\bar{\lambda}_j}{s(\hat{\lambda}_j)/\sqrt{n}}$$

where $s(\hat{\lambda}_j)$ is the standard deviation of the estimates of λ_j , and n is the number of cross-sectional regressions, which is 120 here.

The risk premiums for the market risk factor and book-to-market factor are positive and significant, which is in line with the expectation. Hence, the market risk factor and book-to-market factor have the ability to predict stock returns. In addition, the market risk factor has a risk premium of 1.01% per month and that of the book-to-market factor is 0.87% per month.

However, there are several empirical results that throw doubt on the validity of the four-factor model. On the one hand, premiums for the size factor and momentum factor are significant but with an unexpected negative sign. On the other hand, the constant term λ_0 is significantly different from zero. This suggests that there are risks other than these four factors to explain the cross-sectional variations of stock excess returns. Hence, there are other factors that need to be included in the pricing model.

\bar{R}^2 shown in the table is the time-series average of R^2 for 120 cross-sectional regressions and $s(\bar{R}^2)$ is the standard deviation for \bar{R}^2 . R_a^2 is the adjusted R^2 . The 48% \bar{R}_a^2 suggests that the factor model explains 48% of cross-sectional variations of stocks' excess returns.

Therefore, abnormal return patterns of the first-pass regression show that the four-factor model does not explain the clear patterns of returns across subportfolios discussed in subsection 4.1. Significant constant terms of the second-pass regression prove this point again.

4.3. Results of Fama–Macbeth regression on models incorporating ambiguity

Figure A1 shows cumulative returns for Carhart's four factors, the ambiguity-aversion factor, and the ambiguity-loving factor. All these six factors have positive cumulative returns from February 1996 to January 2011. The ambiguity-aversion factor has a cumulative return of 60.91% during the 15 years; the ambiguity-loving factor has the highest cumulative return of 136.62% during the 15 years; those of the other five factors range from 56.89% to 78.17%. From Figure A1, the cumulative return of the ambiguity-aversion factor is obviously negatively related to that of the ambiguity-loving factor. The formal correlation test indicates that the returns of these two factors are negatively related to the -0.7022 correlation coefficient.

The conclusion of subsection 4.2 suggests that factors that other than Carhart's four factors are expected to explain part of the cross-sectional variations of stock excess returns. To investigate whether these two new factors related to ambiguity could help explain the return pattern across subportfolios in subsection 4.1 and the abnormal return pattern in subsection 4.2, we run the first-pass regression on the following models and compare their α s:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \beta_{i,AVE}Z_{AVE,t} + \epsilon_{i,t} \quad (3)$$

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \beta_{i,LOV}Z_{LOV,t} + \epsilon_{i,t} \quad (4)$$

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \beta_{i,AVE}Z_{AVE,t} + \beta_{i,LOV}Z_{LOV,t} + \epsilon_{i,t} \quad (5)$$

Table A4 shows constant term α_i of the time-series regressions in models (3) to (5). Panels A, B, and C display average α_i s for models (3), (4), and (5), respectively. Three facts support the view that

the ambiguity-aversion factor and the ambiguity-loving factor help capture abnormal returns of model (1). First, compared with α 's 2.25% range of Carhart's four-factor model, these three new models have a narrower range of abnormal returns of 1.85%, 2.24%, and 1.68%, respectively.

Second, abnormal returns among subportfolios are much more random for models (3), (4), and (5) than for model (1). After adding new factors, clear patterns of α s of Equation (1) vanish. In terms of α s' differences between negative and positive third moment quintiles, models (3), (4), and (5) all have a less clear pattern than does model (1) across volatility quintiles. This also holds for the α s' differences between high and low volatility quintiles. In particular, none of α s' differences between high and low volatility quintiles are significant in model (5).

Finally, t-statistics calculated through the average Newey–West standard errors show that α s are less significant for the new models. Models (1), (3), (4), and (5) have 7, 6, 8, and 5 α s, respectively, which are significant at the 10% level. Hence, the model with both factors shows that attitudes toward ambiguity perform best at explaining the abnormal returns produced in model (1). These constant terms provide evidence that part of α s produced in Carhart's four-factor model can be explained by investors' aversion and loving attitudes to ambiguous signals.

Table A5 presents the average exposures to risk factors in the three new models. Panels A, B, and C show the average factor exposures for models (3), (4), and (5), respectively. In panel A, loadings to the ambiguity-aversion factor move from negative to positive with low to high volatility quintiles. Hence, ambiguity averters have high exposures to ambiguity-aversion risk if they hold high volatility risk, regardless of the third moment quintiles. They expect to earn high returns for exposure to this risk factor. However, only seven slopes are significant at least at the 10% level of confidence. Thus, the power of the ambiguity-aversion factor to explain cross-sectional variations is not significant, as expected.

In panel B of Table A5, loadings to ambiguity-loving factors have the opposite pattern to that of ambiguity-aversion factors. For high volatility quintiles, exposures to the ambiguity-loving factor are negative; they become positive at low volatility quintiles. Thus, when they hold low volatility quintiles, ambiguity lovers have high levels of ambiguity-loving risk and expect to obtain high return for this part of risk. Again, the factor does not have the expected significant t-statistics and only five slopes are significant at the 1% level of confidence.

In panel C of Table A5, both ambiguity-aversion and ambiguity-loving factors have a similar pattern and significant performance. In all three panels, exposure to Carhart's four factors and their t-statistics do not change much in the three new models. Exposures to market risk factor and size factor increase with volatility quintiles. All loadings to the market risk factor are significant at the 1% level of confidence. Loadings to the size factor for high volatility quintiles are more significant than those for low volatility quintiles. The slopes for the book-to-market factor decrease with the increase of volatility quintiles. However, most loadings to the momentum factor are insignificant and do not display any clear pattern across subportfolios.

In panel B of Table A2, subportfolios' average adjusted R^2 s for the four-factor model have a range from 47% to 92%, which is high enough. In Table A4, the three new models have even higher level of goodness of fit. Thus, the three new models can explain more excess return variations. Furthermore, average adjusted R^2 for model (3) and model (5) do not exhibit any distinction across the 25 subportfolios. Hence, these two models have similar explanatory power for different portfolios.

In order to obtain the factor premiums for five, cross-sectional regressions of the Fama–Macbeth approach are run on the following models:

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt} \hat{\beta}_{i,mkt,t-1} + \lambda_{SMB} \hat{\beta}_{i,SMB,t-1} + \lambda_{HML} \hat{\beta}_{i,HML,t-1} + \lambda_{UMD} \hat{\beta}_{i,UMD,t-1} + \lambda_{AVE} \hat{\beta}_{i,AVE,t-1} + \varepsilon_i \quad (6)$$

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt}\hat{\beta}_{i,mkt,t-1} + \lambda_{SMB}\hat{\beta}_{i,SMB,t-1} + \lambda_{HML}\hat{\beta}_{i,HML,t-1} + \lambda_{UMD}\hat{\beta}_{i,UMD,t-1} + \lambda_{LOV}\hat{\beta}_{i,LOV,t-1} + \varepsilon_i \quad (7)$$

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt}\hat{\beta}_{i,mkt,t-1} + \lambda_{SMB}\hat{\beta}_{i,SMB,t-1} + \lambda_{HML}\hat{\beta}_{i,HML,t-1} + \lambda_{UMD}\hat{\beta}_{i,UMD,t-1} + \lambda_{AVE}\hat{\beta}_{i,AVE,t-1} + \lambda_{LOV}\hat{\beta}_{i,LOV,t-1} + \varepsilon_i \quad (8)$$

Table A6 shows the result of the second-pass regression of models (2), (6), (7), and (8). In model (8), consistent with the expectation, the market risk factor, size factor, and book-to-market factor have a positive and significant monthly factor premium of 0.55%, 0.24%, and 0.70%, respectively. For each unit increase in exposure to the market risk factor, size factor, and book-to-market factor, the expected compensation for excess monthly returns is 0.55%, 0.24%, and 0.70%, respectively. The premiums for these three factors are also positive and significant in models (6) and (7). In all three models, the prices of the momentum factor are negative and significant, which is contrary to with expectation.

In all three models, ambiguity-aversion and ambiguity-loving factors have significant and positive premiums as expected. Monthly prices for the ambiguity-aversion factor are 0.31% and 0.38% in models (6) and (8), respectively. Monthly prices for the ambiguity-loving factor are 1.21% and 1.28% in models (7) and (8), respectively. In model (8), for each unit increase in exposure to ambiguity-aversion and ambiguity-loving factors, the monthly excess return would increase about 0.38% and 1.28%, respectively. Hence, the evidence support the fact that these two factors can significantly predict stock returns.

In terms of the models' adjusted R^2 , model (8) has the highest R^2 of 58.35%. Thus, 58.35% of the cross-sectional variations of stock returns can be explained by the six risk factors in model (8). Since these exposures used in the second-pass regression are estimated betas from the first-pass regression, but not the true betas, the results of R^2 are considered to be overstated. However, it still can be a criterion when comparing asset-pricing models.

However, in Table A6, all models have significant λ_0 , which weaken the validity of six-factor model (8). The result suggests that there are other factors that could explain stock returns that not included in the six-factor model.

Therefore, the results imply that the ambiguity-aversion factor and the ambiguity-loving factor should be included in the asset-pricing model. On the one hand, they significantly explain excess return variations that Carhart's four factors cannot. On the other hand, the pricing model exhibits higher explanatory power after including these two factors.

5. Conclusion

This study investigates the importance of ambiguity-aversion and ambiguity-loving factors in pricing stock returns by measuring these two factors through stocks individual option-implied volatilities and third moments. The empirical findings of sorting stocks into volatility-third moment quintiles show that a positive relationship exists between stocks' individual third moments and future stock returns. Stilger et al. (2016) document similar results—that a positive relationship exists between individual skewness and future stock returns. In negative third moment quintiles, the positive relationship between stock returns and volatility implies that ambiguity-averse investors are compensated for holding assets for which more ambiguous information is relevant. In positive third moment quintiles, stock returns are negatively related to volatility, and thus, ambiguity-loving investors give up some returns in order to hold assets with greater ambiguity. Carhart's alphas of these volatility-third moment portfolios have the same patterns as returns of these subportfolios. Thus, previous return patterns across these 25 subportfolios cannot be explained by Carhart's four factors. Therefore, other factors that can explain premiums for investors' ambiguity attitudes should be included in the factor model.

To investigate whether the ambiguity-aversion premium and ambiguity-loving premium are priced in stock returns, the prices of these two factors are the returns of mimicking portfolios. A negative relationship exists between ambiguity-aversion and ambiguity-loving premiums with a correlation coefficient

-0.7022. After adding these new factors into Carhart's model, patterns of alphas across portfolios vanish. In addition, the model that incorporates both ambiguity-aversion and ambiguity-loving premiums has the highest goodness of fit. The estimated premium for the ambiguity-aversion factor is 0.38% per month; that for the ambiguity-loving factor is 1.28% per month.

Acknowledgements

I'd like to thank Dr. Roman Kozhan and Dr. Liping He for their support throughout the process of writing this paper.

Funding

The author received no direct funding for this research.

Author details

Ziyun Zhang
E-mail: zhang11zy@163.com
Beijing Normal University Business School, Beijing,
Chaoyang District, China.

Cover image

Source: Bank Indonesia.

Citation information

Cite this article as: Attitudes towards ambiguous information and stock returns, Ziyun Zhang, *Cogent Economics & Finance* (2019), 7: 1693678.

Notes

1. The Ellsberg paradox, popularized by Daniel Ellsberg, indicates that people prefer a risk situation in which the distribution of outcomes' possibilities is known rather than an uncertainty situation in which the distribution of probabilities is ambiguous.
2. Daniel and Titman (2006) define concrete information, such as earnings reports and financial statements, as tangible information and all other information, like news, as intangible. Investors cannot infer the exact innovations for dividends or prices from intangible information, but they can do so from tangible information.
3. Harvey and Siddique (1999) propose a method to estimate time-varying skewness and find the importance of conditional skewness.
4. The role of discount factor is as follows. Epstein and Schneider (2008) show the response to future intangible signal induces negative skewness to return distribution, but the forecasting error induces positive skewness. With high discount factor, ambiguous information about future dividends is discounted at a lower discount rate, and generated negative skewness is high enough to cancel the positive skewness caused by forecasting error.
5. The time period of return data has a 1-month lag, as the option-implied high moment is forward looking. Therefore, the 1-month post-return is collected and merged with its volatility and third moment.
6. Volatility quintile 1 is called low quintile and volatility quintile 5 is called high quintile throughout the paper.
7. Third moment quintile 1 is called negative quintile and third moment quintile 5 is called positive quintile throughout the paper.
8. Portfolio i is the equal weighted subportfolio i from the cross-sectional average of returns of subportfolio i 's compositions, as discussed in subsection 3.2.2.

References

- Anderson, E. W., Ghysels, E., & Juergens, J. L. (2009). The impact of risk and uncertainty on expected returns. *Journal of Financial Economics*, 94(2), 233–263. doi:10.1016/j.jfineco.2008.11.001
- Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2006). The cross-section of volatility and expected returns. *The*

- Journal of Finance*, 61(1), 259–299. doi:10.1111/j.1540-6261.2006.00836.x
- Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of Finance*, 52(1), 57–82. doi:10.1111/j.1540-6261.1997.tb03808.x
- Caskey, J. A. (2009). Information in equity markets with ambiguity-averse investors. *Review of Financial Studies*, 22(9), 3595–3627. doi:10.1093/rfs/hhn062
- Chang, B. Y., Christoffersen, P., & Jacobs, K. (2013). Market skewness risk and the cross section of stock returns. *Journal of Financial Economics*, 107(1), 46–68. doi:10.1016/j.jfineco.2012.07.002
- Chen, Z., & Epstein, L. (2002). Ambiguity, risk, and asset returns in continuous time. *Econometrica*, 70(4), 1403–1443.
- Cumming, D. J., Johannang, L., Ordu, U., & Schweizer, D. (2015). Ambiguity in option markets—Evidence from SEOs. *Journal of Financial Management, Markets and Institutions*. doi:10.12831/87060
- Daniel, K., & Titman, S. (2006). Market reactions to tangible and intangible information. *The Journal of Finance*, 61(4), 1605–1643. doi:10.1111/j.1540-6261.2006.00884.x
- Dow, J., & da Costa Werlang, S. R. (1992). Uncertainty aversion, risk aversion, and the optimal choice of portfolio. *Econometrica*, 60(1), 197–204.
- Easley, D., & O'Hara, M. (2004). Information and the cost of capital. *The Journal of Finance*, 59(4), 1553–1583. doi:10.1111/j.1540-6261.2004.00672.x
- Ellsberg, D. (1961). Risk, ambiguity, and the Savage axioms. *The Quarterly Journal of Economics*, 75(4), 643–669. doi:10.2307/1884324
- Epstein, L. G., & Wang, T. (1994). Intertemporal asset pricing under knightian uncertainty. *Econometrica*, 62(2), 283–322.
- Epstein, L. G., & Schneider, M. (2003). Recursive multiple-priors. *Journal of Economic Theory*, 113(1), 1–31. doi:10.1016/S0022-0531(03)00097-8
- Epstein, L. G., & Schneider, M. (2008). Ambiguity, information quality, and asset pricing. *The Journal of Finance*, 63(1), 197–228. doi:10.1111/j.1540-6261.2008.01314.x
- Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. *The Journal of Finance*, 47(2), 427–465. doi:10.1111/j.1540-6261.1992.tb04398.x
- Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1), 3–56. doi:10.1016/0304-405X(93)90023-5
- Fama, E. F., & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *The Journal of Political Economy*, 81(3), 607–636. doi:10.1086/260061
- Ghirardato, P., Maccheroni, F., & Marinacci, M. (2004). Differentiating ambiguity and ambiguity attitude. *Journal Of Economic Theory*, 118(2), 133–173.
- Gilboa, I., & Schmeidler, D. (1989). Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics*, 18(2), 141–153. doi:10.1016/0304-4068(89)90018-9
- Harvey, C. R., & Siddique, A. (1999). Autoregressive conditional skewness. *Journal of Financial and Quantitative Analysis*, 34(04), 465–487. doi:10.2307/2676230

- Harvey, C. R., & Siddique, A. (2000). Conditional skewness in asset pricing tests. *The Journal of Finance*, 55(3), 1263–1295. doi:10.1111/0022-1082.00247
- Jiang, G., Lee, C. M., & Zhang, Y. (2005). Information uncertainty and expected returns. *Review of Accounting Studies*, 10(2–3), 185–221. doi:10.1007/s11142-005-1528-2
- Klibanoff, P., Marinacci, M., & Mukerji, S. (2005). A smooth model of decision making under ambiguity. *Econometrica*, 73(6), 1849–1892. doi:10.1111/ecta.2005.73.issue-6
- Knight, F. (1921). *Risk, ambiguity, and profit*. Boston, MA: Houghton Mifflin.
- Kraus, A., & Litzenberger, R. H. (1976). Skewness preference and the valuation of risk assets. *The Journal of Finance*, 31(4), 1085–1100.
- Lim, K.-G. (1989). A new test of the three-moment capital asset pricing model. *Journal of Financial and Quantitative Analysis*, 24(2), 205–216.
- Olsen, R. A., & Troughton, G. H. (2000). Are risk premium anomalies caused by ambiguity? *Financial Analysts Journal*, 56(2), 24–31. doi:10.2469/faj.v56.n2.2341
- Rieger, M. O., & Wang, M. (2012). Can ambiguity aversion solve the equity premium puzzle? Survey evidence from international data. *Finance Research Letters*, 9(2), 63–72. doi:10.1016/j.frl.2012.02.001
- Stilger, P. S., Kostakis, A., & Poon, S.-H. (2016). What does risk-neutral skewness tell us about future stock returns? *Management Science*, 63(6), 1814–1834. doi:10.1287/mnsc.2015.2379
- Veronesi, P. (2000). How does information quality affect stock returns? *The Journal of Finance*, 55(2), 807–837.

Appendices

Table A1. Double sorting on individual volatilities and third moments. Stocks are sorted into five quintiles based on their individual volatility in ascending order. Quintile 1 contains stocks with the lowest volatility and quintile 5 contains those with the highest volatility. Independently, these stocks are sorted into five quintiles based on their individual third moments in ascending order. Therefore, quintile 1 contains stocks with the most negative third moments and quintile 5 contains those with the most positive third moments. The time period of option-implied individual volatilities and third moments is from January 1996 to December 2010. Compositions of each portfolio are rebalanced monthly. After assigning the stocks, we record stock returns in the next month. The post-ranking cross-sectional simple average of stock returns is calculated to represent the portfolio's return in this month. After gathering the time-series returns from February 1996 to January 2011, the time-series average of each portfolio's returns is calculated to represent the portfolio's return. These returns and standard deviations for the 25 portfolios are shown. Newey–West standard errors are in brackets and t-statistics are calculated through these robust standard errors. ***, **, and * indicate statistical significance at a confidence level of 1%, 5%, and 10%, respectively

Third moment quintiles	Volatility quintiles					High-Low
	Low	2	3	4	High	
	Mean return					
Negative	-0.15% (0.0043)	0.30% (0.0047)	0.33% (0.0049)	0.53% (0.0060)	0.19% (0.0082)	0.34% (0.0068)
2	0.54% (0.0034)	0.84%** (0.0037)	0.93%** (0.0045)	0.75% (0.0055)	0.20% (0.0076)	-0.34% (0.0064)
3	0.67%** (0.0032)	0.83%** (0.0040)	0.94%** (0.0043)	0.97%* (0.0051)	0.89% (0.0067)	0.23% (0.0055)
4	1.04%*** (0.0031)	1.05%** (0.0041)	1.26%*** (0.0045)	1.07%* (0.0055)	0.36% (0.0077)	-0.68% (0.0067)
Positive	1.53%*** (0.0047)	1.66%*** (0.0048)	1.75%*** (0.0050)	1.39%** (0.0058)	0.77% (0.0074)	-0.76% (0.0065)
Negative-Positive	1.67%*** (0.0040)	1.36%*** (0.0037)	1.41%*** (0.0036)	0.87%*** (0.0028)	0.57%* (0.0031)	

Table A2. First-pass regression results on four-factor model.
 At each rolling window of 60 months, the time-series regression is run on the following model:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt} Z_{mkt,t} + \beta_{i,SMB} Z_{SMB,t} + \beta_{i,HML} Z_{HML,t} + \beta_{i,UMD} Z_{UMD,t} + \epsilon_{i,t}$$

 The regression is run on each volatility-third moment portfolio and the rolling window is rolled monthly. The length of the rolling window is 60 months. The results of constant term α_i are displayed in Panel A. There are only 121 observations for the time-series regression. Then, the time-series average of these 121 periods of α_i is displayed. Their average Newey–West standard errors are shown in bracket. t-statistics are calculated by dividing the average return by average standard errors. ***, **, and * indicate statistical significance at a confidence level of 1%, 5%, and 10%, respectively. The exposures to risk factors are shown in Panel B. The time-series average of these 121 periods of exposures are shown in Panel B. t-statistics are calculated by dividing the average return by average Newey–West standard errors. ***, **, and * indicate statistical significance at a confidence level of 1%, 5%, and 10%, respectively. The results of adjusted R^2 for the 25 regressions are shown in Panel C.

Third moment quintiles	Volatility quintiles					High-Low
	Low	2	3	4	High	
	Mean abnormal return					
Panel A						
Negative	-1.11%** (0.0051)	-0.60% (0.0039)	-0.59%* (0.0033)	-0.40% (0.0030)	-0.56% (0.0039)	0.55% (0.0045)
2	-0.24% (0.0021)	0.02% (0.0019)	0.07% (0.0024)	-0.11% (0.0033)	-0.61% (0.0048)	-0.37% (0.0061)
3	-0.09% (0.0016)	-0.01% (0.0018)	0.16% (0.0022)	-0.05% (0.0034)	0.19% (0.0054)	0.28% (0.0053)
4	0.43%* (0.0022)	0.25% (0.0026)	0.47%* (0.0027)	0.15% (0.0035)	-0.17% (0.0052)	-0.60% (0.0058)
Positive	0.98%* (0.0055)	0.76%** (0.0039)	1.14%*** (0.0040)	0.66% (0.0041)	0.24% (0.0048)	-0.74%** (0.0030)

(Continued)

Table A2. (Continued)

Third moment quintiles	Volatility quintiles								High-Low	
	Low		2		3		4			High
	Mean abnormal return									
Negative-Positive	2.08%*** (0.0078)	1.37%* (0.0073)	1.73%* (0.0100)	1.06%** (0.0047)	0.79% (0.0062)					
Third moment quintiles	Volatility quintiles									
	Low		2		3		4		High	
	Average exposures to market risk factor									
	0.83*** (0.1500)	0.92*** (0.0988)	1.08*** (0.0857)	1.23*** (0.0881)	1.43*** (0.1153)	0.35* (0.1957)	0.39*** (0.1264)	0.45*** (0.1075)	0.74*** (0.1306)	
2	0.79*** (0.0571)	0.97*** (0.0564)	1.12*** (0.0699)	1.22*** (0.0930)	1.42*** (0.1403)	0.11 (0.0704)	0.15** (0.0580)	0.39*** (0.0923)	0.54*** (0.1101)	
3	0.82*** (0.0458)	0.94*** (0.0628)	1.08*** (0.0638)	1.27*** (0.1004)	1.29*** (0.1421)	-0.02 (0.0514)	0.14** (0.0586)	0.31*** (0.0783)	0.49*** (0.1203)	
4	0.81*** (0.0620)	1.03*** (0.0835)	1.08*** (0.0825)	1.29*** (0.1063)	1.28*** (0.1462)	0.01 (0.0598)	0.20** (0.0858)	0.28*** (0.0957)	0.61*** (0.1373)	
Positive	0.97*** (0.1465)	1.09*** (0.1141)	1.12*** (0.1073)	1.29*** (0.1188)	1.46*** (0.1306)	0.21 (0.1534)	0.43*** (0.1280)	0.59*** (0.1305)	0.69*** (0.1326)	

(Continued)

Table A2. (Continued)

Third moment quintiles	Volatility quintiles									
	Low		2		3		4		High	
	Average exposures to market risk factor					Average exposures to size factor				
	Average exposures to book-to-market factor					Average exposures to momentum factor				
Negative	0.24 (0.1950)	0.26 (0.1653)	0.08 (0.1460)	-0.05 (0.1290)	-0.44*** (0.1387)	0.15 (0.1192)	0.17** (0.0837)	0.17** (0.0684)	0.12** (0.0582)	-0.08 (0.0844)
2	0.43*** (0.0682)	0.38*** (0.0693)	0.25*** (0.0933)	0.00 (0.1304)	-0.36** (0.1647)	0.08* (0.0420)	0.10** (0.0411)	0.09* (0.0479)	0.07 (0.0541)	-0.03 (0.1071)
3	0.35*** (0.0650)	0.38*** (0.0912)	0.30*** (0.0860)	0.14 (0.1279)	-0.38* (0.1979)	0.08** (0.0408)	0.01 (0.0339)	-0.02 (0.0511)	0.12* (0.0665)	-0.09 (0.1006)
4	0.33*** (0.0740)	0.39*** (0.1122)	0.33*** (0.1009)	0.09 (0.1321)	-0.44** (0.2072)	-0.06 (0.0423)	-0.02 (0.0596)	-0.06 (0.0507)	-0.06 (0.0726)	-0.17* (0.1035)
Positive	0.32* (0.1743)	0.47*** (0.1420)	0.15*** (0.1326)	0.01 (0.1345)	-0.54*** (0.1677)	-0.24** (0.1023)	-0.12 (0.0990)	-0.17* (0.0962)	-0.15 (0.0905)	-0.20* (0.1026)
	Average adjusted R ²									
Negative	0.47	0.70	0.82	0.92	0.92					
2	0.79	0.88	0.88	0.89	0.87					
3	0.88	0.89	0.89	0.85	0.83					
4	0.84	0.84	0.87	0.88	0.85					
Positive	0.65	0.77	0.83	0.88	0.89					

Table A3. Second-pass regression results on four-factor model

After the first-pass regression, the following model is regressed:

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt} \hat{\theta}_{i,mkt,t-1} + \lambda_{SMB} \hat{\theta}_{i,SMB,t-1} + \lambda_{HML} \hat{\theta}_{i,HML,t-1} + \lambda_{UMD} \hat{\theta}_{i,UMD,t-1} + \epsilon_i$$

The betas are collected estimates from the first-pass regression. As the first-pass regression is run on 121 rolling windows, there are 120 collected betas employed to run the second-pass regression. The average of estimates of λ s and the standard deviation of λ $s(\lambda)$, are shown in the table. t-statistics for the estimates are calculated through the Fama-MacBeth method by the following formula:

$$t(\bar{\lambda}_j) = \frac{\bar{\lambda}_j}{s(\lambda_j)/\sqrt{n}}$$

where n is number of observations. As we use current exposures to predict the excess return for the next period, there are only 120 observations.

\bar{R}^2 is the time-series average of R^2 of 175 cross-sectional regressions. $s(\bar{R}^2)$ represents their standard deviations.

	$\bar{\lambda}$	$s(\lambda)$	t-statistics
Description of premium for four factors			
Market risk factor, λ_{mkt}	0.0101	0.0068	16.23
Size factor, λ_{SMB}	-0.0016	0.0068	-2.53
Book-to-market factor, λ_{HML}	0.0087	0.0071	13.42
Momentum factor, λ_{UMD}	-0.0214	0.0099	-23.61
Abnormal return, λ_0	-0.0085	0.0092	-10.10
Goodness of fit for the four-factor model			
\bar{R}^2	0.57	$s(\bar{R}^2)$	0.2376
\bar{R}_a^2	0.48	$s(\bar{R}_a^2)$	0.2851

Figure A1. Cumulative returns on factor portfolios.

The data for MKT, SMB, HML, and UMD are collected from the WRDS website. The data for the ambiguity-aversion factor and the ambiguity-loving factor are calculated from the returns of volatility-third moment portfolios. Panel A displays premiums for ambiguity-aversion and ambiguity-loving factors. Panel B shows premiums for Carhart's four factors.

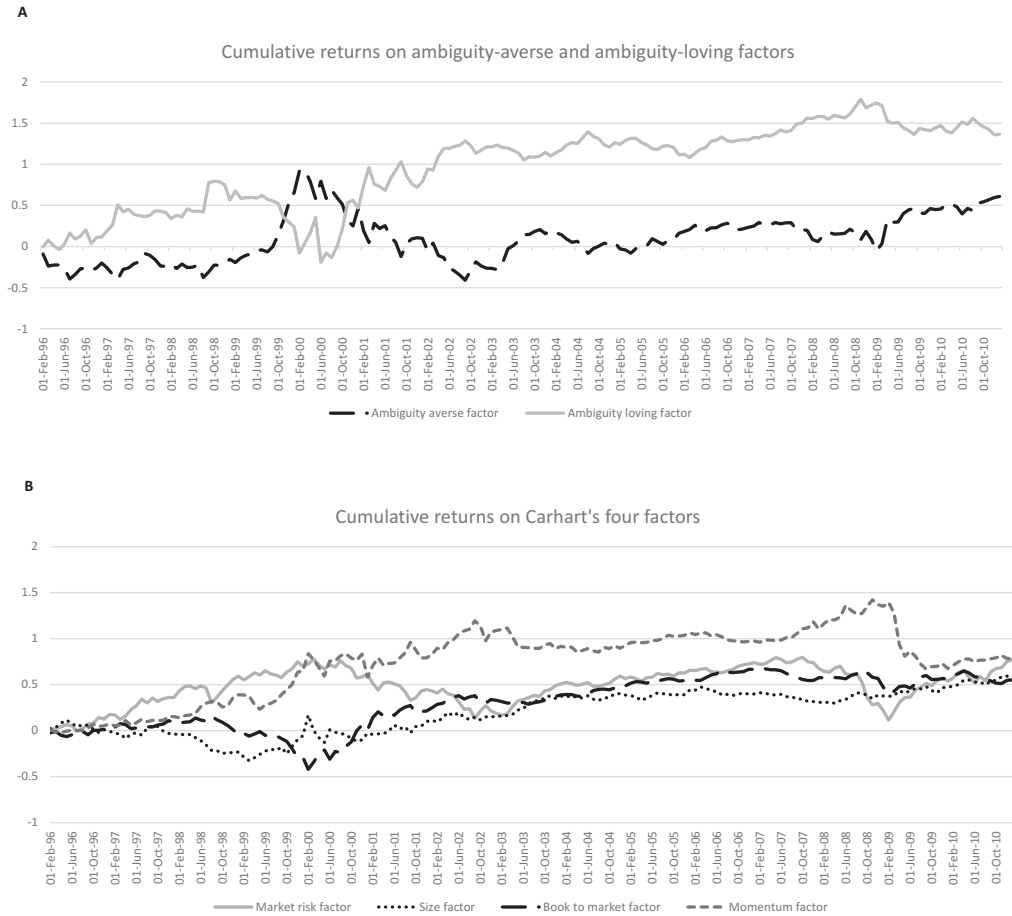


Table A4. First-pass regression's abnormal returns on model (3), (4), and (5).
 At each rolling window of 60 months, the time-series regression is run on the following forms:
 $R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \beta_{i,AVE}Z_{AVE,t} + \epsilon_{i,t}$ (3)
 $R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \beta_{i,LOV}Z_{LOV,t} + \epsilon_{i,t}$ (4)
 $R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,mkt}Z_{mkt,t} + \beta_{i,SMB}Z_{SMB,t} + \beta_{i,HML}Z_{HML,t} + \beta_{i,UMD}Z_{UMD,t} + \beta_{i,AVE}Z_{AVE,t} + \beta_{i,LOV}Z_{LOV,t} + \epsilon_{i,t}$ (5)
 The regression is run on each volatility-third moment portfolio and the rolling window is rolled monthly. The constant term results of models (3), (4), and (5) are displayed in Panels A, B, and C, respectively. In each model, there are only 121 observations for the time-series regression. Then, the time-series average of these 121 periods of α_i is displayed. Their average Newey–West standard errors are shown in brackets. t-statistics are calculated by dividing average returns by average standard errors. ***, **, and * indicate statistical significance at a confidence level of 1%, 5%, and 10%, respectively.

Third moment quintiles	Volatility quintiles					High-Low
	Low	2	3	4	High	
Mean abnormal return						
Panel A						
Negative	-0.74%** (0.0031)	-0.58% (0.0040)	-0.54% (0.0033)	-0.49% (0.0030)	-0.74%** (0.0031)	0.00% (0.0000)
2	-0.19% (0.0020)	0.06% (0.0019)	0.09% (0.0024)	-0.19% (0.0033)	-0.72% (0.0046)	-0.53% (0.0074)
3	-0.05% (0.0016)	0.01% (0.0018)	0.16% (0.0023)	-0.10% (0.0035)	0.18% (0.0053)	0.23% (0.0054)
4	0.43%* (0.0022)	0.27% (0.0026)	0.43% (0.0027)	0.07% (0.0036)	-0.30% (0.0050)	-0.73% (0.0072)
Positive	1.04%* (0.0058)	0.75%** (0.0038)	1.11%*** (0.0038)	0.54% (0.0036)	0.08% (0.0042)	-0.96%** (0.0038)
Negative-Positive	1.78%*** (0.0064)	1.33%* (0.0071)	1.65%* (0.0088)	1.03%** (0.0042)	0.82% (0.0060)	
Panel B						
Negative	-1.11%** (0.0055)	-0.61% (0.0040)	-0.59%* (0.0034)	-0.37% (0.0030)	-0.35% (0.0036)	0.76%* (0.0040)
2	-0.31% (0.0020)	0.00% (0.0018)	0.08% (0.0024)	-0.06% (0.0034)	-0.38% (0.0044)	-0.08% (0.0063)
3	-0.12% (0.0016)	-0.03% (0.0018)	0.20% (0.0022)	0.01% (0.0035)	0.30% (0.0052)	0.41% (0.0049)
4	0.38%* (0.0022)	0.24% (0.0026)	0.46% (0.0028)	0.22% (0.0036)	0.04% (0.0049)	-0.34% (0.0064)
Positive	0.63%* (0.0037)	0.71%* (0.0040)	1.13%** (0.0042)	0.70%* (0.0042)	0.63%* (0.0037)	0.00% (0.0000)
Negative-Positive	1.74%** (0.0077)	1.32% (0.0081)	1.72% (0.0107)	1.07%** (0.0050)	0.98% (0.0061)	
Panel C						
Negative	-0.58%* (0.0031)	-0.59% (0.0042)	-0.54% (0.0036)	-0.48% (0.0030)	-0.58%* (0.0031)	0.00% (0.0000)
2	-0.27% (0.0019)	0.02% (0.0019)	0.10% (0.0024)	-0.14% (0.0033)	-0.47% (0.0044)	-0.20% (0.0071)
3	-0.08% (0.0016)	0.00% (0.0019)	0.19% (0.0023)	-0.04% (0.0036)	0.31% (0.0052)	0.38% (0.0053)

(Continued)

Third moment quintiles	Volatility quintiles					High-Low
	Low	2	3	4	High	
	Mean abnormal return					
4	0.37%* (0.0022)	0.28% (0.0026)	0.41% (0.0028)	0.13% (0.0036)	-0.05% (0.0048)	-0.42% (0.0075)
Positive	0.54% (0.0035)	0.69%* (0.0040)	1.09%** (0.0040)	0.58% (0.0038)	0.54% (0.0035)	0.00% (0.0000)
Negative-Positive	1.12%* (0.0059)	1.28% (0.0081)	1.63%* (0.0095)	1.06%** (0.0043)	1.12%* (0.0059)	

Table A5. First-pass regression's factor exposure returns on model (3), (4), and (5). At each rolling window of 60 months, time-series regressions are run on the following forms:

$$R_{i,t} - R_{f,t} = \alpha_j + \theta_{i,mkt} Z_{mkt,t} + \theta_{i,SMB} Z_{SMB,t} + \theta_{i,HML} Z_{HML,t} + \theta_{i,UMD} Z_{UMD,t} + \theta_{i,AVE} Z_{AVE,t} + \epsilon_{i,t} \quad (3)$$

$$R_{i,t} - R_{f,t} = \alpha_j + \theta_{i,mkt} Z_{mkt,t} + \theta_{i,SMB} Z_{SMB,t} + \theta_{i,HML} Z_{HML,t} + \theta_{i,UMD} Z_{UMD,t} + \theta_{i,LOV} Z_{LOV,t} + \epsilon_{i,t} \quad (4)$$

$$R_{i,t} - R_{f,t} = \alpha_j + \theta_{i,mkt} Z_{mkt,t} + \theta_{i,SMB} Z_{SMB,t} + \theta_{i,HML} Z_{HML,t} + \theta_{i,UMD} Z_{UMD,t} + \theta_{i,AVE} Z_{AVE,t} + \theta_{i,LOV} Z_{LOV,t} + \epsilon_{i,t} \quad (5)$$

The regression is run on each volatility-third moment portfolio and the rolling window is rolled monthly. The factor exposure results of models (3), (4), and (5) are shown in Panels A, B, and C, respectively. In each model, there are only 121 observations for the time-series regression. Then, the time-series average of these 121 periods of exposures is displayed. Their average Newey–West standard errors are shown in brackets. t-statistics are calculated by dividing the average return by average standard errors. Average adjusted R²s are also displayed. *, **, and * indicate statistical significance at a confidence level of 1%, 5%, and 10%, respectively.**

Third moment quintiles	Volatility quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Average exposures to market risk factor					Average exposures to size factor				
Panel A										
Negative	1.22*** (0.1036)	0.95*** (0.0591)	1.13*** (0.0512)	1.15*** (0.0676)	1.22*** (0.1625)	0.73*** (0.13)	0.42*** (0.13)	0.48*** (0.12)	0.67*** (0.11)	0.73*** (0.13)
2	0.84*** (0.1141)	1.00*** (0.0581)	1.14*** (0.0633)	1.17*** (0.0814)	1.29*** (0.1176)	0.14* (0.07)	0.16*** (0.06)	0.40*** (0.10)	0.50*** (0.12)	0.64*** (0.15)
3	0.86*** (0.0952)	0.96*** (0.0741)	1.07*** (0.0703)	1.22*** (0.0805)	1.24*** (0.1056)	0.02 (0.05)	0.16*** (0.06)	0.30*** (0.08)	0.45*** (0.13)	0.88*** (0.19)
4	0.81*** (0.0898)	1.06*** (0.0982)	1.06*** (0.1004)	1.23*** (0.1022)	1.15*** (0.1112)	0.03 (0.07)	0.24*** (0.09)	0.28*** (0.10)	0.57*** (0.14)	0.92v (0.18)
Positive	1.01*** (0.1036)	1.09*** (0.1535)	1.10*** (0.1605)	1.20*** (0.1498)	1.28*** (0.1289)	0.24 (0.16)	0.43*** (0.14)	0.60*** (0.14)	0.64*** (0.14)	0.85*** (0.16)

(Continued)

Table A5. (Continued)

Third moment quintiles	Volatility quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Average exposures to book-to-market factor					Average exposures to momentum factor				
Negative	-0.20 (0.14)	0.23 (0.18)	-0.01 (0.16)	0.05 (0.15)	-0.20 (0.14)	0.00 (0.08)	0.16* (0.09)	0.15** (0.07)	0.15** (0.06)	0.00 (0.08)
2	0.35*** (0.08)	0.32*** (0.08)	0.21* (0.11)	0.09 (0.15)	-0.18 (0.18)	0.06 (0.04)	0.09** (0.04)	0.08 (0.05)	0.09 (0.06)	0.01 (0.11)
3	0.30*** (0.08)	0.36*** (0.10)	0.30*** (0.10)	0.22 (0.16)	-0.29 (0.23)	0.07 (0.04)	0.00 (0.03)	-0.02 (0.05)	0.14* (0.07)	-0.08 (0.11)
4	0.34*** (0.08)	0.39*** (0.12)	0.40*** (0.13)	0.19 (0.18)	-0.21 (0.22)	-0.07 (0.04)	-0.03 (0.06)	-0.05 (0.05)	-0.03 (0.08)	-0.13 (0.11)
Positive	0.25 (0.21)	0.49*** (0.17)	0.23 (0.18)	0.21 (0.16)	-0.29 (0.20)	-0.26** (0.10)	-0.12 (0.10)	-0.17* (0.09)	-0.12 (0.09)	-0.14 (0.10)
	Average exposures to ambiguity-aversion factor					Average adjusted R²				
Negative	-0.65*** (0.07)	-0.05 (0.09)	-0.06 (0.08)	0.13** (0.06)	0.35*** (0.07)	0.81	0.69	0.83	0.92	0.95
2	-0.07* (0.04)	-0.04 (0.04)	-0.02 (0.05)	0.09 (0.07)	0.20** (0.10)	0.81	0.89	0.88	0.89	0.88
3	-0.07* (0.04)	-0.04 (0.03)	0.02 (0.05)	0.07 (0.09)	0.06 (0.11)	0.89	0.89	0.89	0.85	0.83
4	-0.02 (0.04)	-0.06 (0.03)	0.02 (0.05)	0.09 (0.09)	0.16 (0.11)	0.84	0.84	0.87	0.88	0.86

(Continued)

Table A5. (Continued)

Third moment quintiles	Volatility quintiles									
	Low	2	3	4	High	Low	2	3	4	High
Positive	(0.04)	(0.05)	(0.06)	(0.09)	(0.10)	0.64	0.77	0.83	0.89	0.90
	-0.07	0.00	0.02	0.12	0.25**	0.64	0.77	0.83	0.89	0.90
	(0.09)	(0.08)	(0.10)	(0.09)	(0.11)					
Panel B										
Negative	0.84***	0.94***	1.09***	1.21***	1.29***	0.35	0.40***	0.44***	0.69***	0.70***
	(0.16)	(0.10)	(0.09)	(0.09)	(0.12)	(0.24)	(0.14)	(0.12)	(0.12)	(0.13)
2	0.84***	0.99***	1.12***	1.19***	1.26***	0.17**	0.16**	0.37***	0.47***	0.50***
	(0.06)	(0.06)	(0.07)	(0.10)	(0.14)	(0.07)	(0.06)	(0.11)	(0.12)	(0.15)
3	0.85***	0.95***	1.05***	1.23***	1.20***	0.00	0.15**	0.26***	0.41***	0.79***
	(0.05)	(0.06)	(0.07)	(0.11)	(0.16)	(0.06)	(0.06)	(0.08)	(0.14)	(0.19)
4	0.82***	1.04***	1.08***	1.25***	1.11***	0.05	0.19*	0.28**	0.52***	0.74***
	(0.06)	(0.09)	(0.09)	(0.11)	(0.15)	(0.07)	(0.10)	(0.11)	(0.15)	(0.18)
Positive	1.17***	1.12***	1.11***	1.25***	1.17***	0.55***	0.49***	0.60***	0.64***	0.55***
	(0.11)	(0.12)	(0.11)	(0.12)	(0.11)	(0.13)	(0.15)	(0.16)	(0.16)	(0.13)
							Average exposures to momentum factor			
Negative	0.18	0.21	0.01	-0.04	-0.30**	0.15	0.17**	0.18***	0.12**	-0.08
	(0.22)	(0.18)	(0.16)	(0.14)	(0.14)	(0.12)	(0.08)	(0.06)	(0.06)	(0.07)
2	0.34***	0.31***	0.21**	0.02	-0.11	0.09**	0.11***	0.09*	0.08	-0.05
	(0.08)	(0.08)	(0.11)	(0.14)	(0.16)	(0.04)	(0.04)	(0.05)	(0.05)	(0.10)

(Continued)

Table A5. (Continued)

Third moment quintiles	Volatility quintiles										Average adjusted R ²
	Low	2	3	4	High	Low	2	3	4	High	
3	0.29*** (0.07)	0.36*** (0.10)	0.31*** (0.10)	0.20 (0.15)	-0.25 (0.23)	0.09** (0.04)	0.01 (0.03)	-0.02 (0.05)	0.13** (0.06)	-0.09 (0.10)	
4	0.29*** (0.09)	0.36*** (0.12)	0.32*** (0.11)	0.12 (0.15)	-0.20 (0.22)	-0.06 (0.04)	-0.02 (0.06)	-0.06 (0.05)	-0.06 (0.08)	-0.19** (0.09)	
Positive	-0.14 (0.16)	0.45 (0.16)	0.17 (0.15)	0.07 (0.15)	-0.14 (0.16)	-0.22*** (0.08)	-0.12 (0.10)	-0.18* (0.10)	-0.15* (0.09)	-0.22*** (0.08)	
	Average exposures to ambiguity-loving factor										
Negative	0.01 (0.13)	0.02 (0.08)	0.01 (0.08)	-0.05 (0.07)	-0.27*** (0.08)	0.46	0.70	0.83	0.92	0.93	
2	0.08** (0.04)	0.04 (0.04)	-0.01 (0.05)	-0.08 (0.07)	-0.31*** (0.08)	0.82	0.90	0.88	0.89	0.89	
3	0.04 (0.04)	0.01 (0.03)	-0.06 (0.04)	-0.09 (0.08)	-0.16 (0.11)	0.89	0.89	0.89	0.85	0.83	
4	0.05 (0.05)	0.01 (0.05)	0.01 (0.05)	-0.09 (0.06)	-0.31*** (0.10)	0.85	0.84	0.87	0.88	0.87	
Positive	0.46*** (0.09)	0.07 (0.08)	0.00 (0.08)	-0.07 (0.08)	-0.54*** (0.09)	0.84	0.77	0.83	0.89	0.95	

(Continued)

Table A5. (Continued)

Third moment quintiles	Volatility quintiles									
	Low	2	3	4	High	Low	2	3	4	High
Panel C										
Negative	1.15*** (0.10)	0.95*** (0.12)	1.13*** (0.10)	1.15*** (0.09)	1.15*** (0.10)	0.59*** (0.14)	0.42*** (0.14)	0.46*** (0.12)	0.65*** (0.12)	0.59*** (0.14)
2	0.86*** (0.06)	1.00*** (0.06)	1.13*** (0.08)	1.15*** (0.10)	1.20*** (0.15)	0.18** (0.07)	0.17*** (0.06)	0.38*** (0.11)	0.44*** (0.12)	0.46*** (0.15)
3	0.87*** (0.05)	0.96*** (0.06)	1.05*** (0.07)	1.20*** (0.11)	1.18*** (0.17)	0.03 (0.05)	0.16** (0.07)	0.25*** (0.08)	0.39*** (0.14)	0.78*** (0.20)
4	0.82*** (0.07)	1.05*** (0.08)	1.06*** (0.08)	1.20*** (0.10)	1.05*** (0.15)	0.06 (0.07)	0.22** (0.09)	0.28** (0.11)	0.50*** (0.15)	0.72*** (0.18)
Positive	1.11*** (0.10)	1.11*** (0.12)	1.09*** (0.11)	1.17*** (0.12)	1.11*** (0.10)	0.52*** (0.13)	0.49*** (0.15)	0.60*** (0.16)	0.61*** (0.15)	0.52*** (0.13)
Average exposures to book-to-market factor										
Negative	-0.14 (0.13)	0.21 (0.18)	-0.04 (0.17)	0.04 (0.16)	-0.14 (0.13)	-0.01 (0.07)	0.16* (0.09)	0.16** (0.06)	0.16*** (0.06)	-0.01 (0.07)
2	0.30*** (0.08)	0.28*** (0.09)	0.19 (0.12)	0.10 (0.15)	-0.04 (0.18)	0.07* (0.04)	0.10** (0.04)	0.09* (0.05)	0.10 (0.06)	-0.02 (0.10)
3	0.28*** (0.08)	0.35*** (0.10)	0.31*** (0.11)	0.25 (0.17)	-0.22 (0.24)	0.07* (0.04)	0.01 (0.04)	-0.01 (0.05)	0.14* (0.07)	-0.09 (0.10)
4	0.32*** (0.09)	0.37*** (0.13)	0.39*** (0.13)	0.20 (0.18)	-0.07 (0.22)	-0.06 (0.04)	-0.03 (0.06)	-0.05 (0.05)	-0.04 (0.08)	-0.16* (0.09)
Positive	-0.05 (0.16)	0.48*** (0.18)	0.23 (0.19)	0.22 (0.17)	-0.05 (0.16)	-0.20** (0.08)	-0.12 (0.10)	-0.17* (0.10)	-0.12 (0.09)	-0.20** (0.08)
Average exposures to ambiguity-aversion factor										
Average exposures to ambiguity-loving factor										

(Continued)

Table A5. (Continued)

Third moment quintiles	Volatility quintiles									
	Low	2	3	4	High	Low	2	3	4	High
Negative	-0.70*** (0.07)	-0.04 (0.10)	-0.05 (0.08)	0.13** (0.06)	0.30*** (0.07)	-0.19*** (0.07)	0.01 (0.09)	-0.01 (0.09)	-0.02 (0.07)	-0.19*** (0.07)
2	-0.05 (0.04)	-0.02 (0.05)	-0.02 (0.05)	0.08 (0.07)	0.11 (0.09)	0.07* (0.04)	0.03 (0.04)	-0.01 (0.05)	-0.06 (0.07)	-0.28*** (0.08)
3	-0.06 (0.04)	-0.04 (0.03)	0.01 (0.05)	0.05 (0.09)	0.01 (0.12)	0.02 (0.03)	0.00 (0.03)	-0.05 (0.05)	-0.08 (0.08)	-0.15 (0.11)
4	0.00 (0.04)	-0.06 (0.06)	0.03 (0.07)	0.08 (0.09)	0.07 (0.10)	0.05 (0.05)	-0.01 (0.05)	0.01 (0.05)	-0.08 (0.06)	-0.30*** (0.11)
Positive	0.10 (0.08)	0.01 (0.08)	0.02 (0.10)	0.11 (0.09)	0.10 (0.08)	0.48*** (0.09)	0.07 (0.09)	0.00 (0.08)	-0.05 (0.08)	-0.52*** (0.09)
Third moment quintiles	Volatility quintiles					Volatility quintiles				
	Low	2	3	4	Highs	Low	2	3	4	Highs
	Average adjusted R ²									
Negative	0.83	0.69	0.83	0.83	0.95	0.83	0.92	0.89	0.89	0.95
2	0.83	0.90	0.88	0.88	0.89	0.88	0.89	0.85	0.88	0.89
3	0.89	0.89	0.89	0.89	0.83	0.89	0.85	0.85	0.88	0.83
4	0.85	0.84	0.87	0.84	0.87	0.87	0.88	0.88	0.87	0.87
Positive	0.84	0.77	0.83	0.83	0.95	0.83	0.89	0.89	0.89	0.95

Table A6. Second-pass regressions' results on model (2), (6), (7), and (8).

After the first-pass regression, the following models are regressed:

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt} \hat{\theta}_{i,mkt,t-1} + \lambda_{SMB} \hat{\theta}_{i,SMB,t-1} + \lambda_{HML} \hat{\theta}_{i,HML,t-1} + \lambda_{UMD} \hat{\theta}_{i,UMD,t-1} + \lambda_{AVE} \hat{\theta}_{i,AVE,t-1} + \epsilon_t \quad (6)$$

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt} \hat{\theta}_{i,mkt,t-1} + \lambda_{SMB} \hat{\theta}_{i,SMB,t-1} + \lambda_{HML} \hat{\theta}_{i,HML,t-1} + \lambda_{UMD} \hat{\theta}_{i,UMD,t-1} + \lambda_{LOV} \hat{\theta}_{i,LOV,t-1} + \epsilon_t \quad (7)$$

$$\bar{Z}_{i,t} = \lambda_0 + \lambda_{mkt} \hat{\theta}_{i,mkt,t-1} + \lambda_{SMB} \hat{\theta}_{i,SMB,t-1} + \lambda_{HML} \hat{\theta}_{i,HML,t-1} + \lambda_{UMD} \hat{\theta}_{i,UMD,t-1} + \lambda_{AVE} \hat{\theta}_{i,AVE,t-1} + \lambda_{LOV} \hat{\theta}_{i,LOV,t-1} + \epsilon_t \quad (8)$$

The betas are collected estimates from the first-pass regression. As first-pass regression is run on 121 rolling windows, there are 120 collected betas employed to run the second-pass regression. The average of estimates of λ s and the standard deviation of λ , $s(\lambda)$ are shown in the table. The t-statistics for the estimates are calculated by the Fama-MacBeth method using the following formula:

$$t(\bar{\lambda}) = \frac{\bar{\lambda}}{s(\lambda)/\sqrt{n}}$$

where n is the number of observations. As we use current exposures to predict the excess return for the next period, there are only 120 observations.

	(2)	(6)	(7)	(8)
Market factor	0.0101 (16.23)	0.0050 (7.67)	0.0111 (17.80)	0.0055 (8.10)
Size factor	-0.0016 (-2.53)	0.0002 (0.30)	0.0002 (0.39)	0.0024 (3.89)
Book-to-market factor	0.0087 (13.42)	0.0090 (10.79)	0.0077 (9.64)	0.0070 (6.75)
Momentum factor	-0.0214 (-23.61)	-0.0190 (-22.66)	-0.0208 (-22.71)	-0.0173 (-21.64)
Ambiguity-aversion factor		0.0031 (7.26)		0.0038 (8.30)
Ambiguity-loving factor			0.0121 (21.42)	0.0128 (23.08)
Intercept	-0.0085 (-10.10)	-0.0042 (-5.85)	-0.0099 (-12.33)	-0.0052 (-7.52)
Adjusted R^2	47.93%	49.58%	54.67%	58.35%



© 2019 The Author(s). This open access article is distributed under a Creative Commons Attribution (CC-BY) 4.0 license.



You are free to:

Share — copy and redistribute the material in any medium or format.

Adapt — remix, transform, and build upon the material for any purpose, even commercially.

The licensor cannot revoke these freedoms as long as you follow the license terms.

Under the following terms:

Attribution — You must give appropriate credit, provide a link to the license, and indicate if changes were made.

You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use.

No additional restrictions

You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits.

***Cogent Economics & Finance* (ISSN: 2332-2039) is published by Cogent OA, part of Taylor & Francis Group.**

Publishing with Cogent OA ensures:

- Immediate, universal access to your article on publication
- High visibility and discoverability via the Cogent OA website as well as Taylor & Francis Online
- Download and citation statistics for your article
- Rapid online publication
- Input from, and dialog with, expert editors and editorial boards
- Retention of full copyright of your article
- Guaranteed legacy preservation of your article
- Discounts and waivers for authors in developing regions

Submit your manuscript to a Cogent OA journal at www.CogentOA.com

